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The Combinatorial Retention Auction Mechanism (CRAM)

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THE COMBINATORIAL RETENTION AUCTION MECHANISM

(CRAM)

by

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ABSTRACT

We propose a reverse uniform price auction called Combinatorial Retention Auction Mechanism (CRAM) that integrates both monetary and non-monetary incentives (NMIs). CRAM computes the cash bonus and NMIs to a single cost parameter, retains the lowest cost employees and provides them with compensation equal to the cost of the first excluded employee. CRAM is dominant strategy incentive compatible. We provide optimal bidding strategy, and show that there is cost saving compared to a benchmark auction (monetary retention auction). Because CRAM and the benchmark may retain different employees, we provide for whom and under what conditions the utility may increase or decrease by CRAM. Finally, we show that there is an increase in the total social welfare by utilizing CRAM to the benchmark.

Keywords: Combinatorial Auction; Labor Markets; Compensation; Defense Economics

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EXECUTIVE SUMMARY

As the Department of Defense (DoD) prepares for austere future budgets, all elements of the DoD's budget face scrutiny. Military pay and benefits represents approximately one third of the defense budget. Military compensation is a reasonable area to seek savings in defense expenditures. However, it is critical to maintain service member satisfaction given the all-volunteer force and recent increases in operational tempo and deployments.

This report examines an auction (a market mechanism) incorporating individualized packages of non-monetary incentives (NMIs) as part of military retention packages. While this discussion focuses on retention incentives, the concepts are easily extended to other areas of military compensation and to private sector applications. Optimizing individual incentive packages eliminates the waste associated with undervalued NMIs while providing service members a voice in determining their compensation. Furthermore, utilizing a market to determine the proper incentives to pay reduces error by estimating the optimal incentive to provide.

More specifically, the Combinatorial Retention Auction Mechanism (CRAM) is developed to improve control in 1) reducing retention cost, 2) accurately retaining the proper number of service members, and 3) improving the effectiveness of NMIs. Prior research indicated the difficulty of identifying any NMI that has significant value for even 50 percent of the service members surveyed, but also showed that approximately 80 percent of the surveyed service members expressed a significant value for at least one NMI. The key to exploiting the potential of incorporating NMIs as part of total compensation is personalizing the employees' NMI packages to reflect their individual preferences. If the value of NMI is greater than the cost of providing the NMI, the service members will self-select into the NMI. If the value of NMI is less than the cost, then they will simply not request the NMI and ask for additional cash compensation.

CRAM combines monetary compensation with the costs of an individualized set of NMIs to create a single total retention cost parameter. CRAM shows the employees the possible NMIs available and how much each one will cost. The only information CRAM requests from the employee is the set of NMIs he desires and the monetary requirement. CRAM retains the least expensive total cost employees, providing each a compensation package with a cost equal to the cost of the first excluded bid (cheapest not-retained employee).

This research explores CRAM's technical performance. CRAM is never more expensive to the employer than strictly monetary compensation, and is generally less expensive. CRAM also provides at least as much surplus value as monetary compensation, and generally more, where surplus value is the difference between the total value of compensation for the retained employees, outside compensation for those not retained, and the minimum compensation required to secure the labor services for each retained employee. In short, CRAM creates higher value for the society as a whole and reduces waste.

However, employee outcomes under CRAM are more complicated, in part because CRAM generally retains different employees than pure monetary compensation. In fact, some employees will be better off under CRAM (e.g., those retained under CRAM but not with monetary compensation); some employees are better off with monetary compensation (e.g., those retained with monetary compensation but not under CRAM); some employees are indifferent (e.g., those not retained in either case); some employees can be better or worse off depending on how much the first excluded bid changes (e.g., those who are retained in both CRAM and monetary compensation).

We also show that the optimal strategy is for individual employees to select the set of NMIs that maximizes their net value (total value less the total provision costs), and request a cash premium so that the bid's total value equals the employee's opportunity cost of employment (the minimum compensation required to accept that job). This has a nice characteristic, in that there is no gaming involved for the participants; being truthful is the best strategy for the employee.

We conclude by describing future research plans to verify CRAM's performance and extend this general approach to other areas of military force management.

A SHORT EXAMPLE: Table 1 provides a short example that shows that the Department of the Navy (DoN) will save on cost even as total value received by the Sailor increases. Additional examples are provided in the technical report.

Table 1. CRAM Example

Sailor #	Min. $ to Retain	Incentive 1 Value	Incentive 2 Value	Total Incentive Cost	Total Incentive Value	Revised Min. $ to Retain	Total Cost to Retain	Cash Bonus	Total Value Received
1	$80K	$40K	$10K	$20K	$40K	$40K	$60K	$60K	$100K
2	$90K	$10K	$30K	$20K	$30K	$60K	$80K	-	-
3	$100K	$30K	$40K	$40K	$70K	$30K	$60K	$40K	$110K

- Suppose Navy wants to retain 2 out of these 3 sailors
- Outcome with cash bonus only under uniform-price auction
  - Each retainee receives 1st-excluded cash bid = $100K
  - Total cost to retain = 2 X $100K = $200K
  - Total value received by the sailors = 2 X $100K = $200K
- Suppose cost to Navy of each of 2 non-monetary incentives = $20K
- Outcome with Combinatorial Retention Auction Mechanism (CRAM)
  - Cost per retainee = 1st-excluded total cost to retain = $80K
  - Total cost to retain = 2 X $80K = $160K
  - Total value received by the sailors = $100K + $110K = $210K

I. INTRODUCTION

We propose a mechanism called the Combinatorial Retention Auction Mechanism (CRAM), which an organization can utilize to provide both monetary and non-monetary incentives (NMIs) for retention. CRAM is a reverse uniform price auction where a single employer retains the desired number of employees. The mechanism is simple: CRAM elicits employees' reservation value by asking them how much cash bonus and what set of NMIs they require to be retained. The cost of providing the requested cash bonus and the set of NMIs is calculated and presented as a single cost parameter. Then, CRAM selects the preannounced number of the lowest costing employees to retain. Finally, the benefit each retained employee receives is determined by the cost of the first excluded employee. In other words, any retained employee receives the set of NMIs and the cash bonus he requested. Furthermore, every retained employee receives an additional cash bonus that equals the cost of the first excluded employee minus the cost of retaining the particular retained employee. Therefore, as with the standard uniform price auction, the cost of retaining each employee equals the cost of retaining the first excluded employee. CRAM is dominant strategy incentive compatible; it is weakly dominant for every employee to reveal their true reservation value by announcing the bonus required and selecting a set of NMI that maximizes the difference between the value of NMIs and the cost of NMIs. We will work under the retention framework, although CRAM may be generalized to designing compensation packages for newly hired employees.

A general discussion about the combinatorial auction and the application of CRAM to the U.S. Department of Defense (DoD)'s retention problem will be presented in the following sections. Readers with sufficient background in either field may choose to skip over the particular section.

Our contributions are as follows: we provide general framework, characterization and properties of CRAM. CRAM provides a simple and straightforward way of determining the retention cost and the benefits to the employees. This process lessens the burden on the auctioneer (the employer), as well as the participants (the employees). We provide an optimal bidding strategy for the employee that is dominant strategy incentive compatible and show that any optimal bidding strategy must take this form. The employee should reveal their true reservation value, select the set of NMIs that maximizes the total surplus, and specify how many dollars he needs to receive in addition to NMIs to be retained. The dollar amount plus the value of the NMIs should equal the reservation value.

Next, we show that the cost of retaining employees via CRAM is (weakly) cheaper than retaining employees purely by monetary retention auction. This result is driven by the fact that CRAM takes advantage of the surplus generated by providing NMIs instead of cash. Because CRAM may retain a different set of individuals compared to the monetary auction, with a different cut-off cost, determining whom the mechanism benefits is not straightforward. Therefore, we compare an employee's utility under CRAM to the monetary retention auction and show which sets of employees are better off and worse off.

The employees are broken into four sets. An employee not retained under either mechanism is indifferent. An employee retained under the monetary retention auction and not CRAM is better off under monetary retention auction because he receives higher than the reservation utility when retained. Similarly, an employee retained under CRAM but not under the monetary retention auction prefers CRAM. For anyone who is retained under both mechanisms, he may be better or worse off depending on how much the cutoff cost decreases. An employee will be better off under CRAM if the cutoff value does not drop more than the gain in surplus from the NMIs.

Finally, we compare the social welfare, the sum of both retained and not retained employees' utility minus employer's cost, and show that CRAM's social welfare is (weakly) greater than the monetary retention auction.

A. COMBINATORIAL AUCTIONS

Combinatorial auctions generally deal with bidding on multiple objects. What makes combinatorial auctions interesting and difficult is the computational complexity. With n goods introduced, there are $2^n - 1$ possible combinations of goods that the auctioneer and the participants may have to consider. Formally, these problems are considered to be NP-complete, meaning that typical computers may have a difficult time finding an "optimal solution."

While combinatorial auctions have always been of interest, the field has seen the greatest growth with the application of the Federal Communication Commission (FCC) spectrum auctions. Between 1994 and 2003, the FCC has utilized some form of combinatorial auction 41 times, which raised over $40 billion in revenue (Kwasnica, Ledyard, Porter, and DeMartini 2005). Even prior to the major utilization by the FCC, combinatorial auctions had been utilized to enhance market and non-market transactions by public and private entities. Grether, Isaac, and Plott (1981) were one of the earlier proposers of using an auction type of design to solve airport time slot allocation problems for the FAA. Rassenti, Smith, and Bulfin (1982) further improved the use of a computer-assisted smart market way of solving the landing rights problem. Banks, Olson, Porter, Rassenti, and Smith (2003) provided a list of references analyzing various combinatorial auctions that have been utilized to solve complicated government and non-government allocation problems. These references are: Arizona Energy Exchange for energy trading, Federal Energy Regulatory Commission study for gas delivery, payload manifest for Space Shuttle, resource allocation for Cassini mission to Saturn, train scheduling, transportation services, pollution markets, and markets to exchange financial portfolios.

While we have a pretty good understanding of single-object auctions, combinatorial auctions are faced with other problems in addition to computational complexity. These include: exposure problem, threshold problem, and winner determination problem.[1]

Consider a combinatorial auction with three objects, {a} {b} {c}, and four participants. An exposure problem occurs when a participant values a combination of a good more than the sum of the individual good. If participant 1's value for {a}, {b}, and {ab} are 2, 2, and 6, respectively, and the highest bid for {a} and {b} are currently 2 and 2, should participant 1 increase his bid for object {a} and {b}? If participant 1 can only obtain object {a} and not {b}, then he will end up paying more than 2 for an object he only values at 2. Threshold problem occurs when the sum of the smaller bidder's valuation exceeds the larger bidder's valuation for the package of goods but cannot single-handedly outbid the larger bidder.

[1] Jump bidding: although some may state that jump bidding is a problem, as discussed by Isaac, Salmon,

Suppose that participant 1 bids 32 for {abc} and that participants 2, 3, and 4 bid 10 each for {a}, {b}, and {c}, respectively. Participant 1 should be awarded package {abc} and pay 32 for it. On the other hand, if participants 2, 3, and 4 value {a}, {b}, and {c} at 11 each, respectively, the revenue maximizing allocation would be to allocate the goods to the individuals and charge 11 per good. However, a single individual raising his bid to 11 will not change the winner and would require some coordination with the other two bidders to change the allocation (though colluding is often not allowed in most auction formats).

Simply put, it is extremely difficult to determine the winner of a combinatorial auction. To start with, the highest bid on a single package is not guaranteed to win if some alternative combination of bids can generate higher revenue. Finding the alternative combination of bids is also not a simple problem. Further problems arise with tie breaking rules. How are ties to be broken? Which group of people are winners if different combinations of goods by different sets of participants yield the same revenue? What if the highest revenue generating combination does not utilize all possible resources? These problems can cause lower revenue for the auctioneer and inefficient allocation of resources among the participants. Pekeč and Rothkopf (2003) provided an excellent overview of combinatorial auctions and its challenges.

Within the combinatorial auction family, the following are some auction formats that have drawn considerable attention:

1. Simultaneous Multiple Round Auction (SMR): Format utilized by the FCC and often used as a benchmark comparison to other combinatorial auctions. This auction format does not allow for package bidding.
2. Adaptive User Selection Mechanism (AUSM): Developed by Banks, Ledyard, and Porter (1989), AUSM allows for package bidding in continuous time.
3. Resource Allocation Design (RAD): Developed by Kwasnica, at el. (2005), RAD is a hybrid of SMR and AUSM plus an additional pricing feature to guide bidders.
4. Combinatorial Clock Auction (CCA): Developed by Porter, Rassenti, Roopnarine, and Smith (2003) CCA uses a "clock" as a guide for bidding (similar to an English auction).

5. Simultaneous Multiple Round Package Bidding (SMRPB): Developed by the FCC as a variant of RAD, SMRPB includes the ability to utilize an "exclusive OR" function.

The details of each auction are excluded in this paper but references are provided for interested readers. Brunner, Goeree, Holt, and Ledyard (2010) summarized some of the commonly discussed combinatorial auctions mentioned above and compared their performance via experiments. Brunner et al. (2010) found that, when complementarities are present, package bidding is recommended and CCA generally yields the highest revenue.

Due to institutional restrictions, utilization of the auctions mentioned above is not a straightforward application to the retention framework. The above auction formats are called forward auctions and primarily deal with selling objects. Procurement auctions, or reverse auctions, are auctions where one is interested in buying goods and services instead of selling. Therefore, procurement auctions are closer to retention auctions. There are many differences between procurement auctions and retention auctions, however, again due to institutional features. One can procure half of the goods and services, or split the award among the multiple providers in order to keep the competitors competitive[2] (Chaturvedi, Beil, and Martinez-de-Albeniz 2013). However, in the active-duty military, it is not feasible to retain a portion of a person. Furthermore, NMIs are specific incentives that are salient for compensating employees but may not be salient in procurement or the standard forward auctions. The following section will further discuss some characteristics and institutional features that require changes to the known combinatorial auctions and the reason for developing CRAM.

Finally, it is worth noting that the combinatorial auction is also an extremely useful tool for aggregating information, as well as endogenously determining a market-clearing price. When the designer lacks information on which NMIs may or may not be sub or super modular, it may be best left for the decision maker to choose the best set of his own NMI. When it comes to price formation, instead of exogenously estimating the

[2] Awarding the procurement to only one vendor may make that vendor a monopoly in the future due to technological advancement.

price with a large room for error, these auctions will endogenously determine the market-clearing price.

B. APPLICATION TO THE U.S. DEPARTMENT OF DEFENSE

With over 1.4 million active duty and 1.1 million Reserve and National Guard service members in 2013 (DoD 2013), the U.S. military's labor force is unique because it is internally grown. As an example, if an Admiral retires and the Navy is in need of a replacement, it cannot simply go to the general labor market and hire a new Admiral. The Navy must promote from within. Therefore, the DoD and each of the services must carefully plan its force structure over the long term.

In terms of budget and compensation, approximately 51.4 percent of military compensation is cash compensation, while 20.5 percent of military compensation involves non-cash items (education, health care, etc.), and 28.1 percent of the compensation is deferred compensation (retirement pay accrual, etc.) (DoD 2012). Out of the $525 billion budget for the DoD in 2012, $181 billion was related to pay and benefits for military personnel (Harrison and Montgomery 2011). With cuts in the defense budget, the DoD also needs to find savings in pay and benefits.

Special and Incentive (S&I) pays are authorized by law to provide the military services the flexibility needed for force shaping (OSD Military Compensation 2013). There are currently over 60 authorized S&I pays. These pays can be significant. Examples include: 1) Selective Reenlistment Bonus (SRB), which authorizes the services to pay up to $90,000 for a minimum three-year reenlistment; 2) Surface Warfare Officer Continuation Pay that authorizes the Navy to pay up to $50,000 to eligible officers for committing to a Department Head tour;[3] and 3) Critical Skills Retention Bonus (CSRB), which authorizes up to $200,000 over a service member's career[4] for a skill-specific retention. Some S&I pays are much smaller, such as Demolition Duty Pay--a hazardous duty, which adds $150 per month for the assignment's duration. Of course, these S&I pays are reserved for very select groups of service members during a shortage of manpower. To provide perspective on a service member's base cash compensation during the 2013 calendar year, excluding S&I pays, an average Staff Sergeant in the U.S. Army (pay grade E-6) with 10 years of total service and three dependents would get annual cash

[3] Department Head tour is a type of leadership tour for the Navy's ship drivers.
[4] U.S. military service members are typically eligible for full retirement at 20 years of service.

compensation[5] of $60,520.08. An average Captain in the U.S. Marine Corps (pay grade O-3) with five years of total service and no dependents would get annual cash compensation of $80,107.68 (DFAS 2013). Therefore, these S&I pays can be a significant portion of the service member's cash income.

The CRAM is designed to support DoD's retention process. CRAM is developed to improve control in 1) reducing retention cost, 2) accurately retaining the proper number of service members, and 3) improving the effectiveness of NMIs. The DoD has been limited to utilizing a posted-price format for providing the S&I bonuses mentioned above, including selective retention bonuses.[6] Furthermore, these bonuses are provided as purely monetary compensation, thus forgoing any surplus that may be gained by including NMIs. Coughlan, Gates, and Myung (2013), CGM henceforth, described the additional surplus that the DoD can potentially gain by providing personalized NMIs. Furthermore, CGM stressed the importance of utilizing NMIs, the difficulty of providing a universal incentive package[7] of NMIs, and the variability in preference for NMIs by service members between and within communities. CGM found that, although none of the NMIs examined provided significant value to at least 50 percent of the service members surveyed, approximately 80 percent of the surveyed service members expressed a significant value for at least some NMIs.

As with designing any market, the market designer must consider important normative and positive characteristics that the customer values. For example, Pekeč and Rothkopf (2003) discussed that some of the key considerations of designing a combinatorial auction are allocative efficiency, cost minimization, low transaction cost, fairness, failure freeness, and transparency. In addition to the aforementioned considerations, our market design for the DoD emphasized the following normative characteristics as critical features of a combinatorial auction:

[5] Cash compensation is the Basic Pay (salary) plus additional cash payments for housing and allowance for subsistence. In addition, there are deferred and universal compensation elements, such as health insurance and tax advantages, but we do not include these in computing cash compensation.

[6] Posted-price format implies that the Service announces the bonus amount and the market determines how many service members accept the announced bonus. This method lacks control over the quantity of service members accepting the bonus and can be expensive if too many service members accept.

[7] Universal Incentive Package means that everyone receives the same set of NMIs.

1. Egalitarian: perception of equality. The military is of a strong mindset that everyone should get the same pay. Meaning, when S&I pay is being used, everyone under the specific S&I bonus should get the same bonus.
2. Dominant strategy incentive compatible mechanism: transparency and ease of using the mechanism. The military prefers a mechanism that is easy to understand and minimizes strategic gaming by the participants.
3. Low transaction cost: minimum time requirements for auction participants. This consideration is different from many other combinatorial auction designs. Unlike the FCC auction, which can take a form of ascending bid auction requiring participants to observe and interact for hours or days at a time, this is not feasible for the DoD. Different service members may be involved in operational activities throughout the world. A submariner may be undersea for an extended period of time and only have one chance to submit a single bid. An airman may be deployed in a hostile environment and unable to frequently check the current auction market status. Therefore, conducting a simultaneous ascending bid or clock type auction is not practical.

Taking these features into consideration, we developed CRAM for the DoD.

C. OUTLINE

We describe the general environment for the CRAM auction in Chapter II and formally define CRAM in Chapter III. Chapter IV discusses the employee's optimal bidding strategy. Chapter V introduces the monetary retention auction as a benchmark against which CRAM's characteristics are compared. Chapter VI compares the employer's cost under CRAM to the monetary retention auction, while Chapter VII compares the employees' utility, and Chapter VIII compares social welfare. We end with conclusions in Chapter IX.

II. THE ENVIRONMENT

A. THE RETENTION PROBLEM

Let I be a set of employees currently seeking retention with a given employer. The employer will retain $q \le |I|$ of these employees. The employer offers its employees both a monetary incentive, $m \in \mathbb{R}$, as well as a set of non-monetary incentives, N. Each employee ultimately retained by the employer receives a monetary incentive as well as some combination of NMIs as his or her retention package.

Denote by $S \subseteq N$ a subset of NMIs potentially received by any retained employee. Each employee can consume at most one of each of the $|N|$ NMIs. Therefore, there are $2^{|N|}$ different potential combinations of NMIs an employee could receive. We assume that each NMI is a non-rivalrous but excludable good (thus, each is a club good).

B. EMPLOYEE PREFERENCES

Each employee i is endowed with a utility for any combination of a monetary incentive (m_i) and a non-monetary incentive package (S_i), given by $U_i(m_i, S_i) = v_i(S_i) + m_i$. We normalize $v_i(\varnothing) = 0$ for all $i \in I$. Note that we explicitly allow for an employee's valuation, $v_i(S_i)$, of any package of non-monetary incentives to be additive, sub-additive, super-additive, or some combination thereof.

Each employee $i \in I$ is further endowed with a reservation value $r_i \in \mathbb{R}$, which reflects the employee's opportunity cost of being retained by the employer (or, alternatively, the employee's "willingness-to-retain" or the expected value of the employee's outside offer or opportunity). If not retained by the employer, each employee i will enjoy utility r_i. Each employee i's reservation value, r_i, and valuation of non-monetary incentives, v_i, are private information.

We denote the final retention package consisting of cash and a set of NMIs given to any retained employee i as $P_i = (m_i^*, S_i)$. Employee i's utility for this final retention package is then given by $U_i(P_i) = U_i(m_i^*, S_i) = v_i(S_i) + m_i^*$.

C. EMPLOYER COSTS

For each individual NMI, $s^n \in S \subseteq N$, the employer's cost to provide that NMI to any individual employee, $cost\left(s^n\right)$, is public knowledge (or at least communicated to all employees prior to the retention decision). Because each NMI is a non-rivalrous club good, provision of each NMI is characterized by constant marginal cost. Hence, the cost to provide any given NMI, s^n, to any given number of employees, *x*, is simply given by $x \bullet cost\left(s^n\right)$. Hence, there are neither economies nor diseconomies of scale in providing any particular NMI. We normalize $cost(\varnothing) = 0$. We further assume that there are neither economies nor diseconomies of *scope* in providing any *combination* of NMIs. That is, the total cost to provide any set of NMIs, *S*, is given by $cost\left(S\right) = \sum_{s^n \in S} cost(s^n)$.[8]

Therefore, the employer's total cost to provide a final retention package $P_i = \left(m_i^*, S_i\right)$ to any retained employee *i* is given by $cost\left(P_i\right) = m_i^* + cost\left(S_i\right)$ or $cost\left(P_i\right) = m_i^* + \sum_{s^n \in S_i} cost(s^n)$.

D. NMI SURPLUS

With this understanding of employee preference and employer cost, it is helpful to define the employee NMI surplus. Thus, for any bidder *i* and any set of NMIs *S*, let $surplus\left(i,S\right) = v_i\left(S\right) - cost\left(S\right)$.

Note that, for a given set of NMIs *S*, $surplus\left(i,S\right) \in \mathbb{R}$, is not necessarily positive. The following lemma guarantees, however, that employee NMI surplus will not be negative for all sets of NMIs, nor will NMI surplus be positive for all sets of NMIs.

LEMMA 1: For any set of NMIs *N* and any employee *i*, $\max_{S \subseteq N}\left(surplus\left(i,S\right)\right) \geq 0$ and $\min_{S \subseteq N}\left(surplus\left(i,S\right)\right) \leq 0$.

[8] We may relax the assumptions of constant marginal cost as well as the economies of scope in providing NMI combinations. However, this adds computational complexity without adding value to the introduction of our mechanism.

With this notion of NMI surplus, it is instructive to note that an employee *i*'s utility for the final retention package can now be written as $U_i(P_i) = U_i(m_i^*, S_i) = v_i(S_i) + m_i^* = m_i^* + cost(S_i) + surplus(i, S_i) = cost(P_i) + surplus(i, S_i)$. Hence, employee *i*'s utility for the final retention package is simply the employer's cost to provide that package plus the employee's NMI surplus.

Before proceeding, however, it is important to distinguish the notion of NMI surplus from total employee surplus (or supplier surplus). Recall that each employee $i \in I$ has a reservation value $r_i \in \mathbb{R}$ that reflects the employee's opportunity cost of being retained by the employer. While an employee's NMI surplus reflects how much that employee values a set of NMIs above and beyond the cost of providing those NMIs, employee total surplus reflects the utility for a total compensation package above and beyond that employee's reservation value. Hence, total employee surplus is equal to $U_i(P_i) - r_i = v_i(S_i) + m_i^* - r_i$.

III. COMBINATORIAL RETENTION AUCTION MECHANISM (CRAM)

A. MECHANISM

We first formally outline the mechanism in this section. Detailed explanation of the mechanism will be provided in the subsection to follow. First, define $B_i = (m_i, S_i)$ as the message or strategy being submitted by the employee to the employer. m_i is the monetary incentive and S_i is the employee's requested set of NMIs. The employer's cost of providing B_i is $b_i = cost(B_i) = m_i + cost(S_i)$. Without loss of generality, let b^* represent the $q+1$ lowest costing bid, or $b^* = b_{q+1}$. The CRAM mechanism $\Gamma = (B_1, ..., B_I, g(\bullet))$ is a collection of $|I|$ bids, $B_1, ..., B_I$ and an outcome function $g: B_1 \times ... \times B_I \rightarrow X$, where the outcome determines the retention and the compensation package in the following manner:

$$P_i = \begin{cases} (m^*_i, S_i) \text{ if } b_i < b^* \\ (0, \varnothing) \text{ if } b_i \geq b^* \end{cases} \quad \text{and retained if } b_i < b^*$$

where $m_i^* = b^* - cost(S_i)$. Therefore, persons with $b_i \geq b^*$ are not retained and receive their reservation value.

B. EMPLOYEE BIDS

Informally, the CRAM bidding process can be understood as involving two decision elements for each employee: (1) selecting NMIs and (2) submitting a minimum monetary incentive (or cash compensation) required to retain.

For the first decision element, employees must choose which NMIs they desire from a "menu" in which each NMI has an associated cost. As we will detail below, the employer will add the cost of each NMI selected to the employee's monetary incentive request to determine the cost of retaining that employee. Thus, the NMI cost, and not just its value to the employee, factors into the employee's decision regarding which combination of NMIs to select from the menu.

The second decision element of the bidding process involves requesting a monetary incentive or cash compensation incentive. Because retained employees receive each and every NMI they chose from the menu as part of the first bidding decision element, the monetary incentive bid reflects the minimum cash amount an employee must receive in order to retain, *conditional* on the fact that the retained employee will *also* receive all NMIs selected.

Thus, the CRAM bidding process can be formally described as follows. Each employee *i* submits a bid of the form $B_i = (m_i, S_i)$, where m_i is the monetary incentive (or cash compensation) and S_i is the combination of NMIs that employee *i* requests (or demands) to be retained. Let $B = (B_1, B_2, \ldots, B_{|I|})$ be the set of all submitted employee bids. Further, let $B_{-i} = (B_1, B_2, \ldots, B_{i-1}, B_{i+1}, \ldots, B_{|I|})$ denote the set of bids submitted by all employees other than employee *i*, or employee *i*'s competing bid set.

C. EMPLOYEE COST AND RETENTION

To retain employee *i* who has submitted bid $B_i = (m_i, S_i)$, the employer must provide that employee the set of NMIs, S_i, <u>and</u> cash compensation of m_i (or greater) to retain the employee. Thus, the cost to retain that employee is $b_i = cost(B_i) = m_i + cost(S_i)$.

The employer will retain the least expensive set of *q* employees. In other words, the employer will retain those *q* employees who submit the *q* lowest-cost bids. WLOG, let $b_i \leq b_j$ if $i < j$ for all $i, j \in I$. The employer will then retain employee *i* if and only if $i \leq q$ or, alternatively, if and only if $b_i \leq b_q$.

Note that a "tie" is possible, in which there exists more than one set of *q* lowest-cost bids. Whenever ties occur, multiple employees will have submitted bids that all have the q^{th} lowest cost. For simplicity, we assume that ties are not possible. An alternative is to break ties randomly.[9] However, this implies that only q employees will be retained, and therefore some employees who submitted a bid with the q^{th} lowest cost will be retained while others who submitted bids of the same cost will not be retained. Note that, whenever a tie occurs, we will have (at a minimum) that $b_q = b_{q+1}$. For our purpose, it is sufficient to assume that there are no ties between b_q and b_{q+1}.

[9] Accepting every tied bid may exceed the employer's budget constraint and encourage bidder collusion.

D. COMPENSATION FOR RETAINED EMPLOYEES

Because CRAM is a uniform-price auction mechanism, all retained employees will receive a total retention (or compensation) package of uniform cost to the employer. In particular, each retained employee will receive a retention package whose total cost is equal to the cost of the first-excluded bid, which is the lowest-cost bid submitted among those employees not retained. But each employee may not receive the same cash compensation.

Given our construction that $b_i \le b_j$ if $i < j$, the first-excluded bid is the bid submitted by agent (q+1). We shall refer to the cost of this first-excluded bid as the "cutoff cost" and will denote this cost by $b^* = b_{q+1}$.

Recall that the employer will provide compensation package $P_i = \left(m_i^*, S_i\right)$ to any retained employee *i*, and that the cost of this compensation package is given by $cost\left(P_i\right) = m_i^* + cost\left(S_i\right)$. Because we have specified that the compensation package for any retained employee *i* must satisfy $cost\left(P_i\right) = b^*$, we have that $m_i^* + cost\left(S_i\right) = b^*$ or that the cash compensation provided any retained employee *i* is given by $m_i^* = b^* - cost\left(S_i\right)$.

Hence, for each $i \le q$, employee *i's* retention package is given by $P_i = \left(m_i^*, S_i\right) = \left(b^* - cost\left(S_i\right), S_i\right)$. As the following lemma formalizes, it is important to recognize that every retained employee receives a monetary incentive greater than or equal to the amount requested in their bid.

LEMMA 2: For any employee *i* retained under CRAM, $m_i^* \ge m_i$.

Moreover, because each employee receives the exact set of NMIs requested, it is also the case that every retained employee's utility for the final retention package received will be greater than or equal to their utility for their package bid.

LEMMA 3: For any employee *i* retained under CRAM, $U_i\left(P_i\right) \ge U_i\left(B_i\right)$.

E. **DIFFERENCES ACROSS RETAINED EMPLOYEES**

Although the cost to the employer is exactly the same for every retained employee, not every retained employee receives the same compensation package. Different employees may have submitted different bids, $B_i = (m_i, S_i)$, requesting (and ultimately receiving) different NMI combinations.

Hence, if employees *i* and *j* are both retained with $B_i = (m_i, S_i)$ and $B_j = (m_j, S_j)$, these employees will receive different NMI packages whenever $S_i \neq S_i$. Furthermore, if $cost(S_i) \neq cost(S_j)$, these two retained employees will also receive different cash compensation, with $m_i^* = b^* - cost(S_i)$ and $m_j^* = b^* - cost(S_j)$.

Furthermore, even if two retained employees *i* and *j* do receive the exact same retention package, the utility enjoyed by these two retained employees will not necessarily be the same. Suppose we have $P_i = P_j = (m^*, S)$ for these two employees; they each receive the same cash compensation and same set of NMIs. But they may value these NMIs differently, so they will not necessarily receive the same utility despite identical compensation packages. More formally, if $v_i(S) \neq v_j(S)$, then $U_i(P_i) = v_i(S) + m^* \neq U_j(P_j) = v_j(S) + m^*$.

Finally, even if two retained employees do receive the same utility from their respective compensation packages, they do not necessarily enjoy the same total employee surplus, because they likely have different reservation values. Formally speaking, even if $U_i(P_i) = U_j(P_j)$, so long as $r_i \neq r_j$ we will have $U_i(P_i) - r_i \neq U_j(P_j) - r_j$ and, therefore, the two employees will receive different employee surpluses.

In sum, even though the cost of all compensation packages provided to retained employees will be the same under CRAM, (1) the NMIs received by retained employees may differ, (2) the cash compensation received by retained employees may differ, (3) the utility enjoyed by retained employees may differ, and (4) the surplus received by retained employees may differ.

IV. OPTIMAL BIDDING STRATEGY

Having fully described the Combinatorial Retention Auction Mechanism (CRAM) and even begun characterizing outcomes under this mechanism, we now turn to deriving the optimal bidding strategy for employees under CRAM. We conduct this derivation in two stages, first identifying the optimal monetary bidding strategy and then identifying the optimal non-monetary bidding strategy.

A. OPTIMAL MONETARY BIDDING STRATEGY

To understand an employee's optimal strategy for the monetary portion (m_i) of a CRAM bid, it is helpful to recall that the reservation value r_i reflects employee i's opportunity cost of being retained by the employer without any of the NMIs the employer has offered. Having selected a set of NMIs (S_i) as part of his CRAM bid, however, employee *i* will receive precisely those NMIs if retained by the employer.

Therefore, when determining the optimal monetary portion (m_i) of a CRAM bid, employee *i* must consider the revised opportunity cost of being retained with the chosen set of NMIs (S_i). Since these NMIs provide employee *i* a benefit of $v_i(S_i)$ if retained, the revised opportunity cost of being retained is given by $r_i' = r_i - v_i(S_i)$. In the lemma that follows, we show that employee *i*'s optimal bidding strategy involves submitting a monetary bid that truthfully reveals this revised opportunity cost.

LEMMA 4: Given any reservation value $r_i \in \mathbb{R}$, any set of competing bids B_{-i}, and any set of NMIs $S_i \subseteq N$, employee *i*'s utility under CRAM from bid $B_i = (m_i, S_i)$ will be maximized if m_i satisfies $m_i = r_i' = r_i - v_i(S_i)$.

Lemma 4 essentially says that submitting a monetary bid of $m_i = r_i' = r_i - v_i(S_i)$ is an optimal bidding strategy. With the next lemma, we show that such a monetary bid is, in fact, the only optimal bidding strategy.

LEMMA 5: Given any reservation value $r_i \in \mathbb{R}$ and any set of NMIs $S_i \subseteq N$, bid $B_i = (m_i, S_i)$ maximizes employee *i*'s utility under CRAM for any set of competing bids, *B-i*, if and only if $m_i = r_i' = r_i - v_i(S_i)$.

B. OPTIMAL NON-MONETARY BIDDING STRATEGY

In the previous sub-section, we demonstrated that the unique optimal monetary bid under CRAM is $m_i = r_i' = r_i - v_i(S_i)$, for any given set of NMIs $S_i \subseteq N$. In this sub-section, we characterize the optimal non-monetary bidding strategy to accompany the now-established optimal monetary bidding strategy. In particular, we show that the optimal non-monetary bidding strategy is to select a set of NMIs S_i that maximizes employee *i*'s NMI surplus, which, recall, is given by $surplus(i, S_i) = v_i(S_i) - cost(S_i)$.

LEMMA 6: For any reservation value $r_i \in \mathbb{R}$, submitting bid $B_i = (m_i, S_i)$ where $m_i = r_i - v_i(S_i)$ and $S_i \in \text{argmax}_{S \subseteq N}(surplus(i, S))$ maximizes employee *i*'s utility under CRAM for any set of competing bids *B-i*.

Lemma 6 essentially says that selecting a set of NMIs S_i that maximizes employee *i*'s NMI surplus while submitting a monetary bid of $m_i = r_i - v_i(S_i)$ is an optimal bidding strategy. With our first theorem, we show that this bidding strategy is, in fact, the only optimal bidding strategy.

THEOREM 1: Given any reservation value $r_i \in \mathbb{R}$, bid $B_i = (m_i, S_i)$ maximizes employee *i*'s utility under CRAM for any set of competing bids, *B-i*, if and only if $m_i = r_i' = r_i - v_i(S_i)$ and $S_i \in \text{argmax}_{S \subseteq N}(surplus(i, S))$.

As for the proof of Theorem 1, the "if" portion of this theorem was already covered in Lemma 6 and the "only if" portion of this portion of this theorem was partially covered in Lemma 5, with respect to the monetary bid m_i. Thus, we must only prove the "only if" portion for the NMI bid S_i. In other words, we must show that a bid $B_i^{'} = \left(m_i^{'}, S_i^{'}\right)$ with $S_i^{'} \notin \text{argmax}_{S \subseteq N}\left(surplus(i,S)\right)$ and $m_i^{'} = r_i - v_i\left(S_i^{'}\right)$ does not maximize utility under CRAM for all sets of competing bids *B*-*i*.

We have now proven that submitting a bid $B_i = \left(m_i, S_i\right)$ with $S_i \in \text{argmax}_{S \subseteq N}\left(surplus(i,S)\right)$ and $m_i = r_i - v_i\left(S_i\right)$ is the unique weakly dominant bidding strategy under CRAM.

C. COSTS AND UTILITY UNDER THE OPTIMAL BIDDING STRATEGY

Immediately following from Theorem 1, we have two corollaries that characterize the equilibrium employee costs-to-retain and retention utility under CRAM.

COROLLARY 1: The cost-to-retain associated with the optimal bid of any employee *i* under CRAM is given by $b_i = r_i - \max_{S \subseteq N}\left(surplus(i,S)\right)$.

Corollary 1 indicates that, the greater the maximum potential NMI surplus for any employee, the lower the employee's cost-to-retain and the more likely that the employee will be retained.

COROLLARY 2: Any employee *i* who submits the optimal bid under CRAM will receive a retention package P_i generating utility $U_i\left(P_i\right) = b^* + \max_{S \subseteq N}\left(surplus(i,S)\right)$ if retained.

Corollary 2 indicates that, the greater the maximum potential NMI surplus for any employee, the greater the employee's utility if retained. In combination, these two corollaries tell us that, the greater an employee's maximum NMI surplus, the more likely it is that employee will be retained under CRAM and the better off the employee will be if, in fact, retained.

V. BENCHMARK MECHANISM: MONETARY RETENTION AUCTION

To evaluate the relative performance of the Combinatorial Retention Auction Mechanism, we must compare CRAM to its logical alternative as a benchmark. In this case, the traditional method of motivating retention is to offer a uniform monetary incentive to all potential retainees.

In practice, the amount of any such monetary retention incentive is determined using some imperfect estimation method. However, an auction is the most cost-effective and welfare maximizing approach to setting a monetary retention incentive (and determining which employees to retain).

Therefore, let us formally describe the logical best alternative to CRAM as a monetary retention auction. Furthermore, for consistency, we will consider the uniform-price auction format. One can consider the monetary retention auction as a subset of CRAM, where the set of NMIs $N = \{\varnothing\}$. To distinguish the two auction mechanisms, we use the following notations for the monetary retention auction. Consider a monetary retention auction in which each employee *i* submits a single monetary bid $\hat{m}_i$ and the *q* employees retained by the employer are those who submit the *q* lowest monetary bids. With a uniform-pricing rule, each retained employee receives the same monetary retention incentive, which is set equal to the $(q+1)^{th}$–lowest bid. Let us denote the amount of this uniform monetary retention incentive (or, alternatively, the amount of the $(q+1)^{th}$–lowest bid) by $\hat{m}^*$.

The dominant strategy in a uniform-price monetary retention auction is for each bidder to truthfully-reveal their reservation value r_i by bidding $\hat{m}_i = r_i$. Thus, each retained employee *i* receives a monetary retention incentive equal to $\hat{m}^* = r_{q+1}$ and enjoys a surplus of $\hat{m}^* - r_i = r_{q+1} - r_i$. Further, the total retention cost for the employer is equal to $q\hat{m}^*$.

In the sections that follow, we will compare CRAM's performance to the just-described alternative of a monetary retention auction, considering employer cost, employee surplus, and overall social welfare.

VI. EMPLOYER COST

First we compare CRAM's performance to a monetary retention auction in terms of overall employer cost.

LEMMA 7: For any $i \in I$ and any set of NMIs *N*, the employer's cost to satisfy employee *i*'s optimal bid under CRAM (i.e., the minimum cost to retain employee *i*) is less than or equal to the cost to satisfy employee *i*'s optimal bid under a uniform-price monetary retention auction: $b_i \leq \hat{m}_i$.

The above lemma indicates that all employees will submit weakly lower-cost bids under CRAM than under a uniform-price monetary retention auction. One can consider CRAM as a monetary retention auction with more flexibility, which can therefore outperform the monetary auction. More specifically, from the logic of the proof, we can say the following: as long as an employee values some NMI (or combination of NMIs) greater than the cost to provide that NMI (or those NMIs), the employee will submit a strictly lower-cost bid under CRAM than under a uniform-price monetary retention auction. An employee will never (optimally) submit a higher-cost bid under CRAM than under the monetary auction, and the only scenario in which an employee would submit bids of identical cost under each mechanism is when no NMI (or combination of NMIs) provides value greater than the cost to provide that NMI (or combination of NMIs).

Knowing that employees will optimally submit weakly lower-cost bids under CRAM than under the monetary retention auction, it is not surprising that the actual total retention cost under CRAM is less than the cost under a monetary auction. The following Theorem formalizes this result.

THEOREM 2: Given any set of employees *I*, any number of retainees $q \leq |I|$, and any set of NMIs *N*, the cost-per-retainee under CRAM is less than or equal to the cost-per-retainee under a monetary retention auction. In other words, $b^* \leq \hat{m}^*$.

Theorem 2 indicates that CRAM will weakly outperform a uniform-price monetary retention auction in terms of minimizing employer cost. While the theorem only demonstrates that CRAM will cost no more than the monetary auction in this dimension, it is important to note that there are many scenarios in which CRAM will strictly outperform the monetary auction in terms of minimizing cost. Example 1 in Appendix B illustrates one such scenario. Furthermore, Example 1 illustrates that the employees are not necessarily better off under CRAM.

VII. EMPLOYEE UTILITY

In this section, we illustrate that CRAM may increase or decrease employees' utility relative to the benchmark uniform-price monetary auction. Example 1 showed that CRAM can produce lower combined employee surplus (40 vs. 60) and utility (160 vs. 180) than a monetary retention auction. Example 2 of Appendix B demonstrates that the opposite may be true. In other words, CRAM may strictly increase total employee utility and surplus relative to the benchmark uniform-price monetary retention auction under some conditions.

We now compare the utility level for all the employees under CRAM to the monetary retention auction in a general framework. For a given set of NMIs *N*, employees *I*, and number of retainees $q \le |I|$, denote $C = \{i \mid i \in I, b_i < b^*\}$ as the set of employees retained under CRAM and $M = \{i \mid i \in I, \hat{m}_i < \hat{m}^*\}$ as the set of employees retained under the monetary retention auction. There are four possible retention conditions: employees not retained under either CRAM or a monetary auction; employees retained under either a monetary auction or CRAM, but not under both; and employees retained under both auctions. We consider all four cases to completely contrast employee utility as well as a Pareto-improvement condition.

First, for employees who are not retained under either mechanism, $i \notin M \bigcup C$, they receive their reservation utility, r_i, under both auctions. For anyone retained under a monetary auction but not retained under CRAM, $i \in M \bigcap C^c$, utility will be lower under CRAM than the monetary auction: $U_i(\hat{m}^*) > r_i$. Employees not retained under CRAM receive their reservation utility, but receive utility greater than their reservation utility if retained under a monetary auction. For anyone retained under CRAM but not under a monetary auction, $i \in M^c \bigcap C$, utility will be higher under CRAM than the monetary auction: $U_i(P_i) > r_i$. These employees only receive their reservation value under a monetary auction, but receive utility greater than their reservation utility under CRAM.

Finally, consider employees who are retained under both mechanisms, $i \in M \bigcap C$. CRAM's first excluded bid cost is weakly smaller than the monetary auction's first excluded bid: $b^* \le \hat{m}^*$. Therefore, utility may increase or decrease between CRAM and monetary auction for employees retained under both mechanisms. The result is stated in the following proposition:

PROPOSITION 1: In order to have a Pareto-improvement for the employees $i \in M \cap C$ when switching from the monetary retention auction to CRAM, the following condition must hold: $\min_{i \in M \cap C} \max_{S \subseteq N} surplus(i,S) \ge \hat{m}^* - b^*$ with at least one person having $surplus(i,S) > \hat{m}^* - b^*$.

Proposition 1 states that the employees retained under both auctions will do at least as well by switching from monetary retention auction to CRAM if the employee with the smallest maximum surplus from the NMIs is at least as large as the decrease in the first excluded bid. Furthermore, the change would be a Pareto-improvement if at least one employee has an NMI surplus greater than the decrease in the first excluded bid. The following corollary provides a condition in which there will be a Pareto-improvement for all employees and the employer.

COROLLARY 3: If $b^* = \hat{m}^*$, then $C = M$. Furthermore, there is a Pareto-improvement by all retained employees, unretained employees, and the employer if at least one retained employee has $surplus(i,S) > 0$.

Corollary 3 states that if the first excluded CRAM bid equals the first excluded monetary retention auction bid, then both mechanisms must retain the same set of employees. Furthermore, if at least one of the retained employees has a strictly positive value from the NMIs received, this will ensure a Pareto-improvement, not just among a subset of people but over all employees (retained and unretained) and the employer.

Table 2 summarizes the result from this section. By switching from the monetary retention auction to CRAM:

Table 2. Changes in Employee's Utility by Switching from Monetary Retention Auction to CRAM

$i \notin M \cup C$	U_i: no change
$i \in M \cap C^c$	U_i: decreases
$i \in M^c \cap C$	U_i: increases
$i \in M \cap C$	U_i: can increase or decrease. See Proposition 1.

VIII. SOCIAL WELFARE

In the previous two sections, we demonstrated that CRAM generates lower employer costs than a monetary retention auction, but also that CRAM may generate lower employee utility. The critical remaining question, therefore, is whether CRAM maximizes social welfare. In particular, are CRAM's cost savings greater than or equal to any reduction in employee utility? In this section, we prove that the answer to this question is "Yes."

Since we compare social welfare under CRAM to social welfare under the monetary retention auction, we will continue to use the notation for people retained under CRAM and the monetary retention auction as $C = \{i \mid i \in I, b_i < b^*\}$ and $M = \{i \mid i \in I, \hat{m}_i < \hat{m}^*\}$, respectively.

First, we define social welfare in this environment as total employee utility (both retained and unretained) minus total employer costs: $\sum_{i \in C} \left(U_i(P_i) - b^* \right) + \sum_{i \in C^c} \left(r_i \right)$ for CRAM; and $\sum_{i \in M} \left(U_i(\hat{m}^*) - \hat{m}^* \right) + \sum_{i \in M^c} \left(r_i \right)$ for the monetary retention auction. This definition recognizes that we have explicitly defined utility functions for the employees, but we have not done so for the employer. We have only said that the employer's objective is to retain q employees at the lowest possible cost. Therefore, it is natural to measure social welfare as utility minus cost in this context.

To prove that social welfare is higher under CRAM than under a monetary retention auction, we compare social welfare for each of the four different sets of employees defined above. Our first lemma investigates the set of employees who are retained under both CRAM and a monetary retention auction, $i \in M \cap C$.

LEMMA 8: For any set of employees I, quantity of retainees q, and set of NMIs N, if $i \in M \cap C$, then $U_i(P_i) - b^* \geq U_i(\hat{m}^*) - \hat{m}^*$ for $\forall i \in M \cap C$. Furthermore, $\sum_{i \in M \cap C} \left(U_i(P_i) - b^* \right) \geq \sum_{i \in M \cap C} \left(U_i(\hat{m}^*) - \hat{m}^* \right)$.

Lemma 8 states that social welfare (employee utility minus employer cost) is higher under CRAM for any employee retained under both mechanisms. Because this is true for each individual employee belonging to the set $i \in M \bigcap C$, it is also true for the total social welfare associated with the entire set. This result is different from Proposition 1 because Lemma 8 states that the value of social welfare is at least as large under CRAM as under a monetary retention auction in all cases.

For the sets C and M, it is not necessarily true that the social welfare associated with either of these two sets individually is higher under CRAM. On the other hand, the following lemma tells that the total social welfare associated with the two sets combined is, in fact, higher under CRAM.

LEMMA 9: Suppose $i \in M^C \bigcap C$ and $j \in M \bigcap C^C$. Then, for any I, $q \leq |I|$, and N, the social welfare for i, j, and the employer are higher under CRAM than under the monetary retention auction: $U_i(P_i) - b^* + r_j \geq U_j(\hat{m}^*) - \hat{m}^* + r_i$. Furthermore,

$$\sum_{i \in M^C \cap C} \left(U_i(P_i) - b^* \right) + \sum_{j \in M \cap C^C} r_j \geq \sum_{j \in M \cap C^C} \left(U_j(\hat{m}^*) - \hat{m}^* \right) + \sum_{i \in M^C \cap C} r_i .$$

Lemma 9 says that, for any pair of employees, one who is only retained under CRAM and one who is only retained under the monetary auction, the total social welfare associated with this pair combined will always be higher under CRAM. Because there are an equal number of employees who fall into each category (i.e., $\left| M^C \bigcap C \right| = \left| M \bigcap C^C \right|$), the entirety of both sets can be broken into such pairs, each of which has higher social welfare under CRAM. Consequently, the social welfare associated with the combined set of employees belonging to either CRAM only or monetary retention auction only, $\left(M^C \bigcap C \right) \bigcup \left(M \bigcap C^C \right)$, is also higher under CRAM.

With these two lemmas in place, the following theorem becomes straightforward and shows the increase in social efficiency.

THEOREM 3: For any I, $q \leq |I|$, and N, the total social welfare for all employees I and the employer is weakly higher under CRAM than monetary retention auction: $\sum_{i \in C}\left(U_i(P_i) - b^*\right) + \sum_{i \in C^C}\left(r_i\right) \geq \sum_{i \in M}\left(U_i(\hat{m}^*) - \hat{m}^*\right) + \sum_{i \in M^C}\left(r_i\right)$.

Theorem 3 thus indicates that, not only does CRAM reduce employer cost, it also increases total social welfare. Hence, while there are some conditions in which CRAM might lower employee utility (relative to the monetary retention auction) as stated in Section IX, in net, the gain in social welfare outweighs the loss in the welfare. Moreover, there are many conditions (such as Example 2 in the previous section) in which CRAM will both reduce employer cost and increase employee utility.

IX. SUMMARY AND ISSUES FOR FURTHER RESEARCH

Employers often have an opportunity to offer employees non-monetary compensation that employees value well in excess of the employer's cost of provision. However, employee preferences across NMIs are diverse. What is valuable to some has little or no value to others. As stated earlier, surveys of military service members illustrate the difficulty of identifying any NMI that has significant value for even 50 percent of the service members surveyed, but also show that approximately 80 percent of the surveyed service members expressed a significant value for at least one NMI. These surveys show that employers could reduce compensation costs by relying more heavily on NMIs. However, the key to exploiting this potential is personalizing the employees' NMI packages to reflect their individual preferences.

CRAM provides a mechanism to accomplish this objective when setting employee retention bonuses, though it can easily be extended to voluntary separation incentives and other areas of employee compensation. CRAM is a reverse uniform price auction that combines monetary compensation with the costs of an individualized set of NMIs to create a single total retention cost parameter. CRAM retains the least expensive total cost employees, providing each a compensation package with a cost equal to the cost of the first excluded bid. Each employee receives their requested NMIs and a cash bonus equal to the total cost of the first excluded bid minus the total cost of that employee's package of NMIs.

This paper has demonstrated that CRAM is a dominant strategy incentive compatible mechanism. The weakly optimal strategy for any employee is to select the set of NMIs that maximize surplus value (the employee's value minus the total provision costs) and include a cash request so that the bid's total value to the employee equals the employee's reservation value of employment. Compared to a reverse uniform price monetary auction, CRAM is never more expensive than the purely monetary compensation, and often less expensive. Furthermore, CRAM provides at least as much, and often greater, total social welfare compared to a monetary auction.

However, the employee outcomes under CRAM are more complicated. This is most obvious considering that potentially different sets of employees are retained under

CRAM and a monetary auction. In fact, some employees will be better off under CRAM, including those retained under CRAM but not retained in a monetary auction; some employees are better off under a monetary auction, including those retained under a monetary auction but not under CRAM; some employees are indifferent, including those not retained under either auction; and some may be worse off or better off depending on how much the cost of first excluded bid has changed, including those who were obtained in both CRAM and monetary auction.

Considering the expected reduction in employer cost and increase in total social welfare, in conjunction with the truth-revealing attributes CRAM offers, CRAM appears to be an attractive approach to setting retention compensation in the military personnel system, and provides potential for a much broader range of applications. This is particularly important when there is an increase in pressure on the military budget.

Future research will include both simulating CRAM using survey data collected from active duty service members (CGM 2013), and conducting experiments to test bidding behavior under CRAM. The simulation and experimental data will be used to verify CRAM's attributes and project the potential employer savings, social welfare impacts, and impacts on employee utility.

One concern regarding both the current posted-price military retention process and CRAM or a simple monetary auction, observes that all three process retain the least expensive employees (most willing to serve or work). There may be cases where an employer would pay a premium to retain higher quality employees or to increase the flexibility of the type of employees retained. Quality Adjusted Uniform Price Auction (QUAD) (Myung 2013), is a mechanism developed precisely to control for quality of employees retained. QUAD improves the employer's ability to control cost and the number of employees retained, and also the quality of employees retained while still being a dominant strategy incentive compatible mechanism. Myung (2013) argued that, for the DoD's retention and separation problem, there are three important positive characteristics that the end user to should be able to control and adjust. These three are 1) cost of retention (cost), 2) number of employees being retained (quantity), and 3) quality of employees being retained (quality). CRAM can be modified to incorporate a QUAD-like mechanism process as well.

The ultimate goal for our research stream is to integrate market-based processes throughout the military personnel system, and apply these mechanisms more broadly as appropriate.

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APPENDIX A: PROOFS

LEMMA 1: For any set of NMIs *N* and any employee *i*, $\max_{S \subseteq N}\left(surplus(i,S)\right) \geq 0$ and $\min_{S \subseteq N}\left(surplus(i,S)\right) \leq 0$.

PROOF:

Note that the set of potential NMI packages, $S \subseteq N$, includes the empty set, $\varnothing$, for which $v_i(\varnothing)=0$ and $cost(\varnothing)=0$. Therefore, $surplus(i,\varnothing)) = v_i(\varnothing) - cost(\varnothing) = 0$. Thus, it must be the case that $\max_{S \subseteq N}\left(surplus(i,S)\right) \geq 0$ and that $\min_{S \subseteq N}\left(surplus(i,S)\right) \leq 0$.

LEMMA 2: For any employee *i* retained under CRAM, $m_i^* \geq m_i$.

PROOF:

Recall that $m_i^* = b^* - cost(S_i)$ and that $b_i = m_i + cost(S_i)$ or, in other words, $m_i = b_i - cost(S_i)$. Because employee *i* was retained, we must have $b^* \geq b_i$, which implies $b^* - cost(S_i) \geq b_i - cost(S_i)$ and, therefore, $m_i^* \geq m_i$.

LEMMA 3: For any employee *i* retained under CRAM, $U_i(P_i) \geq U_i(B_i)$.

PROOF:

Recall that $P_i = \left(m_i^*, S_i\right)$ and that $B_i = (m_i, S_i)$. Hence, $U_i(P_i) = U_i\left(m_i^*, S_i\right) = v_i(S_i) + m_i^*$ and $U_i(B_i) = U_i(m_i, S_i) = v_i(S_i) + m_i$. In other words, $U_i(P_i) = U_i(B_i) + m_i^* - m_i$. Because, as explained in Lemma 2, we have that $m_i^* \geq m_i$, it must also be the case that $U_i(P_i) \geq U_i(B_i)$.

LEMMA 4: Given any reservation value $r_i \in \mathbb{R}$, any set of competing bids *B-i*, and any set of NMIs $S_i \subseteq N$, employee *i*'s utility under CRAM from bid $B_i = (m_i, S_i)$ will be maximized if m_i satisfies $m_i = r_i^{'} = r_i - v_i(S_i)$.

PROOF:

The proof of this lemma follows the structure of the standard proof for the incentive-compatibility of a second-price auction. We demonstrate that, for all possible scenarios, employee *i* can never do better than to submit a bid $B_i = (r_i^{'}, S_i)$ with an associated retention cost of $b_i = cost(B_i) = r_i^{'} + cost(S_i)$.

In each scenario, we explore the implications of submitting an alternative bid $B_i^{'} = (m', S_i)$ where $m' \neq r_i^{''}$. Such an alternative bid has an associated retention cost of $b_i^{'} = cost(B_i^{'}) = m' + cost(S_i)$.

For each scenario, we explore separately the sub-scenarios where $m' > r_i^{'}$, where $m' < r_i^{'}$, and where the retention result is either changed or unchanged.

<u>Scenario 1:</u> Employee *i* is retained with a bid of $B_i = (r_i^{*}, S_i)$

Because employee *i* is retained with a bid of $B_i = (r_i^{'}, S_i)$ in this scenario, we must have that $b_i \leq b^*$. Furthermore, employee *i* will receive retention package $P_i = (b^* - cost(S_i), S_i)$ in this case. Recall that this retention package provides utility $U_i(P_i) = U_i(b^* - cost(S_i), S_i) = v_i(S_i) + b^* - cost(S_i) = b^* + surplus(i, S_i)$.

Note that $b_i = r_i^{'} + cost(S_i) = r_i - v_i(S_i) + cost(S_i) = r_i - surplus(i, S_i)$ or that $r_i = b_i + surplus(i, S_i)$. Because $b^* \geq b_i$, we thus have $U_i(P_i) = b^* + surplus(i, S_i) \geq b_i + surplus(i, S_i) = r_i$. In other words, whenever employee *i* is retained with a bid of $B_i = (r_i^{'}, S_i) = (r_i - v_i(S_i), S_i)$ for any set of NMIs S_i, it must be the case that he is better off (i.e., enjoys greater utility) than if he had not been retained.

<u>Sub-scenario 1A:</u> $m' < r_i^{'}$

With a bid of $B_i^{'} = (m', S_i)$, employee *i* will still be retained in this sub-scenario, since $b_i^{'} = m' + cost(S_i) < r_i^{'} + cost(S_i) = b_i \leq b^*$. Furthermore, with a bid of $B_i^{'} = (m', S_i)$, employee *i* also receives the same retention package $P_i = (b^* - cost(S_i), S_i)$, since the set of NMIs requested, S_i, is the same and the cutoff cost, b^*, remains unchanged. Thus, in this scenario, employee *i* cann<u>ot</u> do better by submitting a bid of $B_i^{'} = (m', S_i)$ where $m' < r_i^{'}$.

<u>Sub-scenario 1B:</u> $m' > r_i^{'}$ but employee *i* is still retained with a bid of $B_i^{'} = (m', S_i)$.

With $m' > r_i^{'}$, we have that $b_i^{'} = m' + cost(S_i) > r_i^{'} + cost(S_i) = b_i$. Therefore, because the cost of bid $B_i^{'}$ is higher, employee *i* may or may not be retained. This sub-scenario 1B specifies, however, that employee *i* is still retained with a bid of $B_i^{'} = (m', S_i)$. It must, therefore, be the case that $b_i^{'} \leq b^*$. Hence, with a bid of $B_i^{'}$, employee *i* also receives the same retention package $P_i = (b^* - cost(S_i), S_i)$, since the set of NMIs requested, S_i, is the same and the cutoff cost, b^*, remains unchanged. Thus, employee *i* cannot do better by submitting a bid of $B_i^{'} = (m', S_i)$ where $m' > r_i^{'}$ but employee *i* is still retained with this bid.

<u>Sub-scenario 1C:</u> $m' > r_i^{'}$ and employee *i* is not retained with a bid of $B_i^{'} = (m', S_i)$.

This sub-scenario 1C specifies that employee *i* is not retained with a bid of $B_i^{'} = (m', S_i)$. Hence, with a bid of $B_i^{'}$, employee *i* will receive only his reservation value of r_i. With the bid of $B_i = (r_i^{'}, S_i)$, however, employee *i* is retained and receives utility $U_i(P_i)$. Because $U_i(P_i) \geq r_i$, employee *i* cann<u>ot</u> do better (and could do worse) by submitting a bid of $B_i^{'} = (m', S_i)$, where $m' > r_i^{'}$ and employee *i* is not retained with this bid.

<u>Scenario 2:</u> Employee *i* is not retained with a bid of $B_i = (r_i^{'}, S_i)$

Because employee i is not retained with a bid of $B_i = (r_i^{'}, S_i)$ in this scenario, we must have that $b_i \geq b^*$. Furthermore, employee i will receive only his reservation value of r_i in this case.

Sub-scenario 2A: $m' > r_i^{'}$

If $m' > r_i^{'}$, employee i will not be retained with a bid of $B_i^{'} = (m', S_i)$, since $b_i^{'} = m' + cost(S_i) > r_i^{'} + cost(S_i) = b_i \geq b^*$, and he will, therefore, still receive only his reservation value of r_i. Thus, employee i cannot do better by submitting a bid of $B_i^{'} = (m', S_i)$ where $m' > r_i^{'}$.

Sub-scenario 2B: $m' < r_i^{'}$ but employee i is still not retained with a bid of $B_i^{'} = (m', S_i)$.

With $m' < r_i^{'}$, we have that $b_i^{'} = m' + cost(S_i) < r_i^{'} + cost(S_i) = b_i$. Thus, it is possible to have $b_i^{'} < b^*$ and have employee i retained with a bid $B_i^{'} = (m', S_i)$. Sub-scenario 2B specifies, however, that employee i is still not retained with a bid of $B_i^{'} = (m', S_i)$. This means that $b^* \leq b_i^{'} < b_i$. Hence, employee i still receives only his reservation value of r_i. Thus, in this sub-scenario, employee i can once again not do better by submitting a bid of $B_i^{'} = (m', S_i)$.

Sub-scenario 2C: $m' < r_i^{'}$ and employee i is retained with a bid of $B_i^{'} = (m', S_i)$.

This sub-scenario 2C specifies not only that $m' < r_i^{'}$, but also that employee i is retained with a bid of $B_i^{'} = (m', S_i)$. This means that $b_i^{'} \leq b^* \leq b_i$ (at least one of those inequalities must be strict, since $b_i^{'} < b_i$). With a bid of $B_i^{'} = (m', S_i)$, employee i would, therefore, receive a retention package $P_i = (b^* - cost(S_i), S_i)$, giving utility $U_i(P_i) = b^* + surplus(i, S_i)$. As $b^* \leq b_i$ and $r_i = b_i + surplus(i, S_i)$, we have $U_i(P_i) = b^* + surplus(i, S_i) \leq b_i + surplus(i, S_i) = r_i$. Thus, employee i will receive less utility (or, at best, the same utility) if he is retained with a bid of $B_i^{'} = (m', S_i)$ than if he was not retained with a bid of $B_i = (r_i^{'}, S_i)$.

LEMMA 5: Given any reservation value $r_i \in \mathbb{R}$ and any set of NMIs $S_i \subseteq N$, bid $B_i = (m_i, S_i)$ maximizes employee *i*'s utility under CRAM for any set of competing bids, *B-i*, if and only if $m_i = r_i^{'} = r_i - v_i(S_i)$.

PROOF:

The "if" portion of this lemma was already covered in Lemma 4, so we must only prove the "only if" part. In other words, we must show that a bid $B_i^{'} = (m_i^{'}, S_i)$ with $m_i^{'} \neq r_i^{'}$ does not maximize utility under CRAM for any set of competing bids *B-i*. The proof is by contradiction.

Suppose instead that bid $B_i^{'} = (m_i^{'}, S_i)$ maximizes employee *i*'s utility under CRAM for any reservation value r_i and any set of competing bids *B-i,* but that $m_i^{'} \neq r_i^{'} = r_i - v_i(S_i)$. Then we can prove this lemma via contradiction if we can find *some* set of competing bids *B-i* for which bid $B_i^{'} = (m_i^{'}, S_i)$ does not maximize employee *i*'s utility or, more specifically, for which the utility from bid $B_i = (r_i^{'}, S_i) = (r_i - v_i(S_i), S_i)$ exceeds the utility from bid $B_i^{'} = (m_i^{'}, S_i)$.

Before proceeding, note that the cost of bid $B_i^{'} = (m_i^{'}, S_i)$ is given by $b_i^{'} = m_i^{'} + cost(S_i)$, while the cost of bid $B_i = (r_i^{'}, S_i)$ is given by $b_i = r_i^{'} + cost(S_i) = r_i - v_i(S_i) + cost(S_i) = r_i - surplus(i, S_i)$. We will continue to denote the "cutoff cost" by b^*.

There are two scenarios to consider.

Scenario 1: $m_i^{'} > r_i^{'}$

In this scenario, we have

$b_i^{'} = m_i^{'} + cost(S_i) > r_i^{'} + cost(S_i) = r_i - v_i(S_i) + cost(S_i) = b_i$. Thus, consider some set of competing bids B_{-i} such that $b_i^{'} > b^* > b_i$. In that case, with bid $B_i^{'} = (m_i^{'}, S_i^{'})$ employee *i* would not be retained and would, therefore, receive only his reservation value r_i. In contrast, with bid $B_i = (r_i^{'}, S_i)$ employee *i* would be retained and receive retention package $P_i = (b^* - cost(S_i), S_i)$, giving utility

$U_i(P_i) = U_i(b^* - cost(S_i), S_i) = v_i(S_i) + b^* - cost(S_i)$. Because $b^* > b_i$, we have

$b^* > r_i - v_i(S_i) + cost(S_i)$ or $b^* - cost(S_i) > r_i - v_i(S_i)$.

Therefore, $U_i(P_i) = v_i(S_i) + b^* - cost(S_i) > v_i(S_i) + r_i - v_i(S_i) = r_i$ or $U_i(P_i) > r_i$. Hence, the utility from bid $B_i = (r_i^{'}, S_i) = (r_i - v_i(S_i), S_i)$ exceeds the utility from bid

$B_i^{'} = (m_i^{'}, S_i)$, yielding the contradiction.

Scenario 2: $m_i^{'} < r_i^{'}$

In this scenario, we have

$b_i^{'} = m_i^{'} + cost(S_i) < r_i^{'} + cost(S_i) = r_i - v_i(S_i) + cost(S_i) = b_i$. Thus, consider some set of competing bids B_{-i} such that $b_i^{'} < b^* < b_i$. In that case, with bid $B_i^{'} = (m_i^{'}, S_i)$ employee *i* would be retained and would, therefore, receive retention package

$P_i = (b^* - cost(S_i), S_i)$, thus providing utility

$U_i(P_i) = U_i(b^* - cost(S_i), S_i) = v_i(S_i) + b^* - cost(S_i)$. In contrast, with bid $B_i = (r_i^{'}, S_i)$ employee *i* would not be retained and would receive his reservation value r_i. Because $b^* < b_i$, we have $b^* < r_i - v_i(S_i) + cost(S_i)$ or $b^* - cost(S_i) < r_i - v_i(S_i)$.

Therefore, $U_i(P_i) = v_i(S_i) + b^* - cost(S_i) < v_i(S_i) + r_i - v_i(S_i) = r_i$ or $U_i(P_i) < r_i$. Hence, the utility from bid $B_i = (r_i^{'}, S_i) = (r_i - v_i(S_i), S_i)$ exceeds the utility from bid

$B_i^{'} = (m_i^{'}, S_i)$, once again yielding a contradiction.

Thus, for any $m_i^{'} \neq r_i^{'} = r_i - v_i(S_i)$, it cannot be the case that bid $B_i^{'} = (m_i^{'}, S_i)$ maximizes employee *i*'s utility under CRAM for any reservation value r_i and any set of competing bids B_{-i}.

LEMMA 6: For any reservation value $r_i \in \mathbb{R}$, submitting bid $B_i = (m_i, S_i)$ where $m_i = r_i - v_i(S_i)$ and $S_i \in \text{argmax}_{S \subseteq N}(surplus(i,S))$ maximizes employee *i*'s utility under CRAM for any set of competing bids B_{-i}.

PROOF:

From Lemmas 4 and 5, we already know that submitting a monetary bid of $m_i = r_i - v_i(S_i)$ maximizes employee *i*'s utility given any set of NMIs S_i. Thus, we must only prove that selecting a set of NMIs S_i, with $S_i \in \text{argmax}_{S \subseteq N}(surplus(i,S))$ in conjunction with such a monetary bid maximizes employee *i*'s utility under CRAM.

Once again, let $b^* = b_{q+1}$ be the cutoff cost and $b_i = m_i + cost(S_i) = r_i - v_i(S_i) + cost(S_i)$ be the cost of bid $B_i = (m_i, S_i)$. There are two scenarios to consider.

<u>Scenario 1:</u> Employee *i* is retained with bid $B_i = (m_i, S_i)$.

Since employee *i* is retained, his compensation package will be $P_i = (m_i^*, S_i) = (b^* - cost(S_i), S_i)$, which provides utility of $U_i(P_i) = v_i(S_i) + m_i^*$. First, note that whenever an employee submits a monetary bid of $m_i = r_i - v_i(S_i)$ for any set of NMIs S_i, his utility from being retained with such a bid will always match or exceed his utility from not being retained. This is because (using Lemma 2) $U_i(P_i) = v_i(S_i) + m_i^* \geq v_i(S_i) + m_i = v_i(S_i) + r_i - v_i(S_i) = r_i$. Thus, employee *i* must simply choose S_i to maximize $U_i(P_i)$ in this scenario. Because b^* is independent of the non-monetary bid S_i (and independent of monetary bid m_i as well), employee *i* maximizes

$U_i(P_i) = v_i(S_i) + m_i^* = v_i(S_i) - cost(S_i) + b^* = surplus(i, S_i) + b^*$ if, and only if, he chooses S_i where $S_i \in \text{argmax}_{S \subseteq N}(surplus(i, S))$.

Scenario 2: Employee *i* is not retained with bid $B_i = (m_i, S_i)$.

Since employee *i* is not retained with bid $B_i = (m_i, S_i)$, we know that $b_i \geq b^*$ and that employee *i* will receive his reservation value r_i. Proceeding to prove by contradiction, suppose there exists an alternative bid $B_i^{'} = (m_i^{'}, S_i^{'})$, with $S_i^{'} \notin \text{argmax}_{S \subseteq N}(surplus(i, S))$ and cost $b_i^{'} = m_i^{'} + cost(S_i^{'})$, which yields utility greater than r_i in this scenario. First of all, from Lemma 5, we know that utility from bid $B_i^{'} = (m_i^{'}, S_i^{'})$ is maximized if, and only if, $m_i^{'} = r_i - v_i(S_i^{'})$, so we can assume this to be true of $m_i^{'}$. Now, if $b_i^{'} > b^*$, employee *i* will still not be retained with bid $B_i^{'}$ and will still only receive his reservation value r_i. Therefore, for bid $B_i^{'}$ to yield utility greater than r_i, it must be the case that $b_i^{'} \leq b^*$. But, because $surplus(i, S_i^{'}) < surplus(i, S_i)$, we have

$$b_i^{'} = m_i^{'} + cost(S_i^{'}) = r_i - v_i(S_i^{'}) + cost(S_i^{'})$$
$$= r_i - surplus(i, S_i^{'}) > r_i - surplus(i, S_i^{'}) = r_i - v_i(S_i) + cost(S_i) \ = m_i + cost(S_i) = b_i \geq b^*.$$

Hence, $b_i^{'} > b^*$, which is a contradiction.

Hence, bid $B_i = (m_i, S_i)$ where $m_i = r_i - v_i(S_i)$ and $S_i \in \text{argmax}_{S \subseteq N}(surplus(i, S))$ maximizes employee *i*'s utility under CRAM for any set of competing bids *B*-$_i$.

THEOREM 1: Given any reservation value $r_i \in \mathbb{R}$, bid $B_i = (m_i, S_i)$ maximizes employee *i*'s utility under CRAM for any set of competing bids, *B*-$_i$, if and only if $m_i = r_i^{'} = r_i - v_i(S_i)$ and $S_i \in \text{argmax}_{S \subseteq N}(surplus(i, S))$.

PROOF:

The "if" portion of this theorem was already covered in Lemma 6 and the "only if" portion of this theorem was partially covered in Lemma 5 with respect to the monetary bid m_i. Thus, we must only prove the "only if" part for the NMI bid S_i. In other words, we must show that a bid $B_i^{'} = \left(m_i^{'}, S_i^{'}\right)$ with $S_i^{'} \notin \text{argmax}_{S \subseteq N}\left(surplus\left(i,S\right)\right)$ and $m_i^{'} = r_i - v_i\left(S_i^{'}\right)$ does not maximize utility under CRAM for all sets of competing bids *B*-*i*.

In other words, we can prove this theorem if we can find *some* set of competing bids *B*-*i* for which any bid $B_i^{'} = \left(m_i^{'}, S_i^{'}\right)$, with $S_i^{'} \notin \text{argmax}_{S \subseteq N}\left(surplus\left(i,S\right)\right)$ and $m_i^{'} = r_i - v_i\left(S_i^{'}\right)$, does not maximize employee *i*'s utility. More specifically, we will show that, for any such bid $B_i^{'} = \left(m_i^{'}, S_i^{'}\right)$, there exists a cutoff cost b^* such that the utility from bid $B_i = \left(m_i, S_i\right)$ with $S_i \in \text{argmax}_{S \subseteq N}\left(surplus\left(i,S\right)\right)$ and $m_i = r_i - v_i\left(S_i\right)$ exceeds the utility from bid $B_i^{'} = \left(m_i^{'}, S_i^{'}\right)$.

Note that the cost of bid $B_i^{'} = \left(m_i^{'}, S_i^{'}\right)$ is given by $b_i^{'} = m_i^{'} + cost\left(S_i^{'}\right) = r_i - v_i\left(S_i^{'}\right) + cost\left(S_i^{'}\right) = r_i - surplus\left(i, S_i^{'}\right)$, while the cost of $B_i = \left(m_i, S_i\right)$ is given by $b_i = m_i + cost\left(S_i\right) = r_i - v_i\left(S_i\right) + cost\left(S_i\right) = r_i - surplus\left(i, S_i\right)$. Because $surplus\left(i, S_i^{'}\right) < surplus\left(i, S_i\right)$, we have $b_i^{'} = r_i - surplus\left(i, S_i^{'}\right) > r_i - surplus\left(i, S_i\right) = b_i$ or $b_i^{'} > b_i$.

Therefore, there exists some set of competing bids *B*-*i* such that $b_i^{'} > b^* > b_i$, meaning that employee *i* would not be retained with bid $B_i^{'} = \left(m_i^{'}, S_i^{'}\right)$, but would be retained with bid $B_i = \left(m_i, S_i\right)$. Thus, with bid $B_i^{'} = \left(m_i^{'}, S_i^{'}\right)$, employee *i* would simply receive his reservation value r_i. With bid $B_i = \left(m_i, S_i\right)$, on the other hand, he would receive compensation package $P_i = \left(m_i^*, S_i\right) = \left(b^* - cost\left(S_i\right), S_i\right)$, which provides utility of $U_i(P_i) = v_i(S_i) + m_i^* = v_i(S_i) + b^* - cost(S_i) > v_i(S_i) + b_i - cost(S_i) = v_i(S_i) + m_i$

$= v_i(S_i) + r_i - v_i(S_i) = r_i$ meaning $U_i(P_i) > r_i$. Hence, the utility from bid $B_i = (m_i, S_i)$ exceeds the utility from bid $B_i^{'} = (m_i^{'}, S_i^{'})$.

As further proof, consider a set of competing bids *B*-*i* such that $b^* > b_i^{'} > b_i$, under which employee *i* would be retained with either bid $B_i^{'} = (m_i^{'}, S_i^{'})$ or bid $B_i = (m_i, S_i)$. With bid $B_i = (m_i, S_i)$, employee *i* would, therefore, receive compensation package $P_i = (b^* - cost(S_i), S_i)$, which provides utility of $U_i(P_i) = v_i(S_i) + b^* - cost(S_i) = surplus(i, S_i) + b^*$. With bid $B_i^{'} = (m_i^{'}, S_i^{'})$, employee *i* would receive compensation package $P_i^{'} = (b^* - cost(S_i^{'}), S_i^{'})$, which provides utility of $U_i(P_i^{'}) = v_i(S_i^{'}) + b^* - cost(S_i^{'}) = surplus(i, S_i^{'}) + b^*$. Because $surplus(i, S_i^{'}) < surplus(i, S_i)$, we have $U_i(P_i) = surplus(i, S_i) + b^* > surplus(i, S_i^{'}) + b^* = U_i(P_i^{'})$. Hence, the utility from bid $B_i = (m_i, S_i)$ exceeds the utility from bid $B_i^{'} = (m_i^{'}, S_i^{'})$ under this set of competing bids.

In sum, bid $B_i^{'} = (m_i^{'}, S_i^{'})$ with $S_i^{'} \notin \operatorname{argmax}_{S \subseteq N}(surplus(i, S))$ and $m_i^{'} = r_i - v_i(S_i^{'})$ does not maximize utility under CRAM for all sets of competing bids *B*-*i*.

COROLLARY 1: The cost-to-retain associated with the optimal bid of any employee *i* under CRAM is given by $b_i = r_i - \max_{S \subseteq N}(surplus(i, S))$.

PROOF:

Recall that the cost-to-retain associated with a bid $B_i = (m_i, S_i)$ from employee *i* is given by $b_i = m_i + cost(S_i)$. If this bid includes the optimal monetary bid of $m_i = r_i - v_i(S_i)$, this cost becomes $b_i = r_i - v_i(S_i) + cost(S_i) = r_i - surplus(i, S_i)$. Finally, if employee *i* also selects the optimal set of NMIs, such that

$S_i \in \text{argmax}_{S \subseteq N}\left(surplus(i,S)\right)$, then the cost-to-retain associated with bid $B_i = (m_i, S_i)$ becomes $b_i = r_i - \max_{S \subseteq N}\left(surplus(i,S)\right)$.

COROLLARY 2: Any employee *i* who submits the optimal bid under CRAM will receive a retention package P_i, generating utility $U_i(P_i) = b^* + \max_{S \subseteq N}\left(surplus(i,S)\right)$ if retained.

PROOF:

If retained with a bid of $B_i = (m_i, S_i)$ under CRAM, recall that employee *i* will receive retention package $P_i = \left(b^* - cost(S_i), S_i\right)$, which provides utility of $U_i(P_i) = b^* - cost(S_i) + v_i(S_i) = b^* + surplus(i, S_i)$. If employee *i* has selected the optimal set of NMIs, such that $S_i \in \text{argmax}_{S \subseteq N}\left(surplus(i,S)\right)$, then this utility becomes $U_i(P_i) = b^* + \max_{S \subseteq N}\left(surplus(i,S)\right)$.

LEMMA 7: For any $i \in I$ and any set of NMIs *N*, the employer's cost to satisfy employee *i*'s optimal bid under CRAM (i.e., the minimum cost to retain employee *i*) is less than or equal to the cost to satisfy employee *i*'s optimal bid under a uniform-price monetary retention auction: $b_i \leq \hat{m}_i$.

PROOF:

As described in the previous section, under a uniform-price monetary retention auction, it is a dominant strategy for each bidder to truthfully reveal his or her reservation value r_i by bidding $\hat{m}_i = r_i$. Thus, the minimum cost to retain employee *i* under this monetary retention auction is equal to r_i.

Under CRAM, on the other hand, we know (from Theorem 1) that the unique weakly dominant bidding strategy is for employee *i* to submit a bid $B_i = (m_i, S_i)$ with

$S_i \in \text{argmax}_{S \subseteq N}\left(surplus\left(i,S\right)\right)$ and $m_i = r_i - v_i\left(S_i\right)$. Thus, the cost of employee *i*'s optimal bid under CRAM is equal to

$b_i = m_i + cost(S_i) = r_i - v_i(S_i) + cost(S_i) = r_i - surplus(i,S_i) = r_i - \max_{S \subseteq N}(surplus(i,S))$.

From Lemma 1, however, we know that $\max_{S \subseteq N}\left(surplus\left(i,S\right)\right) \geq 0$. Therefore, $b_i = r_i - \max_{S \subseteq N}\left(surplus\left(i,S\right)\right) \leq r_i$. Hence, for any employee *i*, the employer's cost to satisfy his optimal bid under CRAM is less than or equal to the cost to satisfy his optimal bid under a uniform-price monetary retention auction.

THEOREM 2: Given any set of employees *I*, any number of retainees $q \leq \left|I\right|$, and any set of NMIs *N*, the cost-per-retainee under CRAM is less than or equal to the cost-per-retainee under a monetary retention auction. In other words, $b^* \leq \hat{m}^*$.

PROOF:

Lemma 7 tells us that the cost of employee *i*'s optimal CRAM bid is less than or equal to the cost of his optimal uniform-price monetary auction bid. In other words, for all $i \in I$, $b_i \leq \hat{m}_i = r_i$. Therefore, the lowest-cost bid under CRAM must cost less than (or the same as) the lowest-cost bid under the monetary auction, the highest-cost bid under CRAM must cost less than (or the same as) the highest-cost bid under the monetary auction, and the n^{th} lowest-cost bid under CRAM must cost less than (or the same as) the n^{th} lowest-cost bid under the monetary auction for any $n \leq \left|I\right|$. Therefore, the cutoff cost b^*, which is equal to the cost of the $(q+1)^{\text{th}}$ lowest-cost bid under CRAM, must be less than $\hat{m}^*$, which is the cost of the $(q+1)^{\text{th}}$ lowest-cost bid under the monetary auction.

PROPOSITION 1: In order to have a Pareto-improvement for the employees $i \in M \cap C$ when switching from the monetary retention auction to CRAM, the following condition must hold: $\min_{i \in M \cap C} \max_{S \subseteq N} surplus(i,S) \geq \hat{m}^* - b^*$ with at least one person having $surplus(i,S) > \hat{m}^* - b^*$.

PROOF:

Recall that by Lemma 7, $b_i \leq \hat{m}_i$, and by theorem 2, $b^* \leq \hat{m}^*$. For $i \in M \bigcap C$, utility under CRAM is $U_i(P_i) = m_i^* + v_i(S_i) = b^* + \max_{S \subseteq N} surplus(i,S)$ and utility under monetary auction is $U_i(\hat{m}^*) = \hat{m}^*$. In order for $U_i(P_i) \geq U_i(\hat{m}^*)$, it must be that $\max_{S \subseteq N} surplus(i,S) \geq \hat{m}^* - b^*$ to have weak improvement for everyone. So, if $\min_{i \in M \bigcap C} \max_{S \subseteq N} surplus(i,S) \geq \hat{m}^* - b^*$, with at least one person having $surplus(i,S) > \hat{m}^* - b^*$ for some $S \subseteq N$, then $U_i(P_i) \geq U_i(\hat{m}^*)$ for all i, and $U_i(P_i) > U_i(\hat{m}^*)$ for some i.

COROLLARY 3: If $b^* = \hat{m}^*$ then $C = M$. Furthermore, there is a Pareto-improvement by all retained employees, unretained employees, and the employer if at least one retained employee has $surplus(i,S) > 0$.

PROOF:

By Lemma 7, $b_i \leq \hat{m}_i$. If $b^* = \hat{m}^*$, and the employer only retains q people, in order for $C \neq M$, there must be at least one employee, j, who was retained under the monetary auction but not under CRAM. This means that $b^* \leq b_j$ and $\hat{m}_j < \hat{m}^*$, which implies $b^* \leq b_j \leq \hat{m}_j < \hat{m}^*$. But $b^* = \hat{m}^*$ so, therefore, it is not possible and $C = M$.

Now, to show that there is a Pareto-improvement: given that $C = M$, it must be that $C^c = M^c$. The individuals not retained always receive their reservation value r_i in either mechanism. As for the employer, in both mechanisms, it will cost $b^* = \hat{m}^*$ to retain each of the q employees in either mechanism. So for the unretained and the employer, they are equally as well off under either mechanism. Finally, as for the $i \in C = M$, these individuals were retained under both mechanisms. Since the change in cutoff value $\hat{m}^* - b^* = 0$, then $\max_{S \subseteq N} surplus(i,S) \geq 0 = \hat{m}^* - b^*$ for $\forall i$, by Lemma 1 and given that least one retained employee has $surplus(i,S)>0$, we satisfy the condition for the Proposition 1 and there is a Pareto-improvement among C. Therefore, in sum $C\bigcup C^C=I$ and the employer, there is a weak Pareto-improvement.

LEMMA 8: For any set of employees I, quantity of retainees q, and set of NMIs N, if $i\in M\bigcap C$, then $U_i(P_i)-b^*\geq U_i(\hat{m}^*)-\hat{m}^*$ for $\forall i\in M\bigcap C$. Furthermore, $\sum_{i\in M\bigcap C}\left(U_i(P_i)-b^*\right)\geq\sum_{i\in M\bigcap C}\left(U_i(\hat{m}^*)-\hat{m}^*\right)$.

PROOF:

Under CRAM, the surplus for employee *i* in equilibrium is $U_i(P_i)-b^*=b^*+\max_{S\subseteq N}(surplus(i,S))-b^*=\max_{S\subseteq N}(surplus(i,S))$. From Lemma 1, $\max_{S\subseteq N}(surplus(i,S))\geq 0$. Under monetary retention auction, the surplus for employee *i* in equilibrium is $U_i(\hat{m}^*)-\hat{m}^*=\hat{m}^*-\hat{m}^*=0$. Therefore, $U_i(P_i)-b^*\geq U_i(\hat{m}^*)-\hat{m}^*$. Finally, $\sum_{i\in M\bigcap C}(U_i(P_i)-b^*)\geq\sum_{i\in M\bigcap C}(U_i(\hat{m}^*)-\hat{m}^*)$ is true since $U_i(P_i)-b^*\geq U_i(\hat{m}^*)-\hat{m}^*$ for $\forall i\in M\bigcap C$.

LEMMA 9: Suppose $i\in M^C\bigcap C$ and $j\in M\bigcap C^C$. Then, for any I, $q\leq|I|$, and N, the social welfare for i, j, and the employer are higher under CRAM than under the monetary retention auction: $U_i(P_i)-b^*+r_j\geq U_j(\hat{m}^*)-\hat{m}^*+r_i$. Further, $\sum_{i\in M^C\bigcap C}\left(U_i(P_i)-b^*\right)+\sum_{j\in M\bigcap C^C}r_j\geq\sum_{j\in M\bigcap C^C}\left(U_j(\hat{m}^*)-\hat{m}^*\right)+\sum_{i\in M^C\bigcap C}r_i$.

PROOF:

Under CRAM, employee *i* is retained while employee *j* is not. Therefore, employee *i* receives a retention package P_i, generating utility $U_i\left(P_i\right)=b^*+\max_{S\subseteq N}\left(surplus\left(i,S\right)\right)$, while employee *j* receives his reservation value r_j. At the same time, the employer's cost to retain employee *i* is equal to b^*. Thus, under

CRAM, the combined utility minus employer cost is equal to

$b^* + \max_{S\subseteq N}\left(surplus(i,S)\right) + r_j - b^* = \max_{S\subseteq N}\left(surplus(i,S)\right) + r_j$.

Meanwhile, employee *j* is retained under the monetary retention auction while employee *i* is not. Therefore, employee *j* receives the uniform monetary retention incentive of $\hat{m}^*$ while employee *i* receives his reservation value r_i. At the same time, the employer's cost to retain employee *j* is equal to $\hat{m}^*$. Thus, under the monetary retention auction, the combined utility minus employer cost is equal to $\hat{m}^* + r_i - \hat{m}^* = r_i$.

Hence, to prove this proposition, we must show that

$\max_{S\subseteq N}\left(surplus(i,S)\right) + r_j \geq r_i$.

To show this to be true, note that employee *i* is retained under CRAM while employee *j* is not, so it must be the case that $b^* \geq b_i = r_i - \max_{S\subseteq N}\left(surplus(i,S)\right)$ and that $b^* \leq b_j = r_j - \max_{S\subseteq N}\left(surplus(j,S)\right)$. Putting these two inequalities together, we have that $r_i - \max_{S\subseteq N}\left(surplus(i,S)\right) \leq r_j - \max_{S\subseteq N}\left(surplus(j,S)\right)$ or that

$\max_{S\subseteq N}\left(surplus(i,S)\right) + r_j \geq r_i + \max_{S\subseteq N}\left(surplus(j,S)\right)$.

By Lemma 1, however, we know that $\max_{S\subseteq N}\left(surplus(i,S)\right) \geq 0$. Thus, our previous inequality implies $\max_{S\subseteq N}\left(surplus(i,S)\right) + r_j \geq r_i$, which demonstrates the first part of the proposition to be true.

For total social welfare, we know from the former part of the proof that for each unique match of i and j, $U_i(P_i) - b^* + r_j \geq U_j(\hat{m}^*) - \hat{m}^* + r_i$. Because $\left|M^c \cap C\right| = \left|M \cap C^c\right|$, we can match each unique i to a unique j, thus, there are an equal number of elements between $\sum_{i\in M^C\cap C}(U_i(P_i) - b^*) + \sum_{j\in M\cap C^C} r_j$ and $\sum_{j\in M\cap C^C}\left(U_j(\hat{m}^*) - \hat{m}^*\right) + \sum_{i\in M^C\cap C} r_i$. Therefore,

$\sum_{i\in M^C\cap C}\left(U_i(P_i) - b^*\right) + \sum_{j\in M\cap C^C} r_j \geq \sum_{j\in M\cap C^C}\left(U_j(\hat{m}^*) - \hat{m}^*\right) + \sum_{i\in M^C\cap C} r_i$.

THEOREM 3: For any I , $q \le |I|$, and N , the total social welfare for all employees I and the employer is weakly higher under CRAM than monetary retention auction: $\sum_{i\in C}\left(U_i(P_i)-b^*\right)+\sum_{i\in C^C}(r_i) \ge \sum_{i\in M}\left(U_i(\hat{m}^*)-\hat{m}^*\right)+\sum_{i\in M^C}(r_i)$.

PROOF:

Lemma 8 demonstrated that the total social welfare associated with the set $M \cap C$ is the same or greater under CRAM as under the monetary retention auction. Similarly, Lemma 9 demonstrated that total social welfare associated with the combined set $(M^C \cap C)\cup(M \cap C^C)$ is the same or greater under CRAM as under the monetary retention auction. Finally, because employees belonging to the set $M^C \cup C^C$ are not retained under either mechanism and receive only their reservation value, the social welfare associated with these employees is the same under each mechanism. Thus, total social welfare associated with the entire set of employees, $I=(M\cap C)\cup(M^C\cap C)\cup(M\cap C^C)\cup(M^C\cap C^C) = C\cup C^C = M\cup M^C$, is the same or greater under CRAM as under the uniform-price monetary retention auction:

$$\sum_{i\in C}\left(U_i(P_i)-b^*\right)+\sum_{i\in C^C}(r_i) \ge \sum_{i\in M}\left(U_i(\hat{m}^*)-\hat{m}^*\right)+\sum_{i\in M^C}(r_i).$$

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APPENDIX B: EXAMPLES

Example 1: Comparing CRAM to Monetary Retention Auction

Suppose there are three employees, such that $|I| = 3$ and $I = \{1,2,3\}$, and that two of these employees are to be retained (i.e., $q = 2$). Further, suppose that there is only a single NMI offered under CRAM, such that $|N| = 1$ and $N = \{s\}$, and that this NMI can be provided at a constant marginal cost of 10 $\left(\text{i.e., } cost(s) = 10\right)$.

Finally, suppose that each employee's reservation value (r_i) and value for the NMI offered are as indicated in columns two and three of Table 3. Under these conditions, the remaining columns of Table 3 indicate the optimal NMI choice, resulting NMI surplus, optimal CRAM monetary bid, and resulting CRAM bid cost for each employee.

Table 3. CRAM vs. Monetary Auction Example 1: Optimal Bid and Cost

Employee Number I	Reservation Value r_i	NMI Value $v_i(s)$	NMI Chosen S_i	NMI Surplus $Surplus(i,S_i)$	CRAM Money Bid m_i	CRAM Bid Cost b_i
1	20	0	$\varnothing$	0	20	20
2	40	0	$\varnothing$	0	40	40
3	60	20	s	10	40	50

Because $q = 2$, we have $b^* = b_{q+1} = b_3 = 50$ under CRAM. Similarly, we have $\hat{m}^* = r_{q+1} = r_3 = 60$ under the uniform monetary retention auction. Thus, $b^* < \hat{m}^*$ and the cost-per-retainee under CRAM is strictly less than the cost-per-retainee under a monetary retention auction in this example.

To further understand how the outcome in this example would differ under CRAM relative to a monetary retention auction, consider Table 4, which details the retention decision, utility, and surplus for each employee under the monetary auction and under CRAM.

Table 4. CRAM vs. Monetary Auction Example 1: Utility Comparison

Employee Number I	Retained in Monetary Auction?	Utility in Monetary Auction?	Surplus in Monetary Auction?	Retained under CRAM?	Utility under CRAM?	Surplus under CRAM?
1	Yes	60	40	Yes	50	30
2	Yes	60	20	Yes	50	10
3	No	60	0	No	60	-
TOTAL	-	180	60	-	160	40

Note that the employer is strictly better off under CRAM in this example, but every employee is not better off. In fact, employees 1 and 2 enjoy greater utility and surplus under the monetary retention auction in this example.

Example 2: Comparing CRAM to Monetary Retention Auction with an increase in employee's utility

Suppose the same situation as in Example 1, but the employees' NMI values have changed as now summarized in Table 5.

Table 5. CRAM vs. Monetary Auction Example 2: Optimal Bid and Cost

Employee Number I	Reservation Value r_i	NMI Value $v_i(s)$	NMI Chosen S_i	NMI Surplus $Surplus(i,S_i)$	CRAM Money Bid m_i	CRAM Bid Cost b_i
1	20	20	s	10	0	10
2	40	20	s	10	20	30
3	60	20	s	10	40	50

Because q = 2, we once again have $b^* = b_{q+1} = b_3 = 50$ under CRAM and $\hat{m}^* = r_{q+1} = r_3 = 60$ under the uniform monetary retention auction. Table 6 details the retention decision, utility, and surplus for each employee under the monetary auction and under CRAM for this example.

Table 6. CRAM vs. Monetary Auction Example 2: Utility Comparison

Employee Number *I*	Retained in Monetary Auction?	Utility in Monetary Auction?	Surplus in Monetary Auction?	Retained under CRAM?	Utility under CRAM?	Surplus under CRAM?
1	Yes	60	40	Yes	70	50
2	Yes	60	20	Yes	70	30
3	No	60	0	No	60	-
TOTAL	-	180	60	-	200	80

CRAM once again generates lower employer costs in this example, but it also produces a higher total employee surplus (80 vs. 60) and utility (200 vs. 180) than under a monetary retention auction.

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