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DESIGNING AN AUTOMATIC CONTROL SYSTEM
FOR A SUBMARINE

by

Orhan K. Babaoglu

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Thesis Advisor George J. Thaler

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Designing an Automatic Control System for a Submarine

by

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Lieutenant Junior Grade, Turkish Navy
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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

from the

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ABSTRACT

The purpose of this thesis is to linearize given non-linear differential equations and design a complete automatic control system for the three dimensional motions of a submarine. Automatic control systems are designed using a steady state decoupling scheme for vertical and horizontal motion. Both designs are simulated using the Dynamic Simulation Language (DSL) for both linear and non-linear models and compared. Cross-coupling effect between horizontal and vertical motions due to the rudder deflections is also investigated.

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I. INTRODUCTION

Since they are operated in three dimensions and because of their different body structure and operational conditions, submarines always present a great challange for automatic control engineers. Especially for submarines with extremely high underwater speeds, it is very important to have automatic controls which can be used effectively.

In this study, using the equations of motions in six degrees of freedom which were developed by Naval Ship Research and Development Center (NSRDC), a linearized submarine model was derived for both horizontal and vertical motions. It was obvious that working with a linear model is much simpler then with a complete nonlinear model. Also the automatic control system design procedures which are used in this study require a linear model for decoupling. Even though the linearized model does not introduce a cross-coupling effect between horizontal and vertical motion, as would a real submarine, it works in almost the same way the nonlinear model does.

In designing an automatic controller for both vertical and horizontal motions, a MIMO (Multi-input Multi-output) system representing the submarine, has to be investigated. Inputs are propeller which creates the forward speed, rudder for horizontal motion, and the bow and stern planes for vertical motion. The outputs are the three speed components u, w, v and roll, yaw, pitch angles around three axes of the submarine. Also a ballast system can be used to maneuver the submarine but it is not included in this study assuming the submarine is always in trim.

The pitch and yaw angles and the depth have the main importance for maneuvering a submerged submarine. Therefore the automatic control system is designed to control these three states.

After obtaining valid linear models for both horizontal and vertical motions, the method of the automatic control design has to be chosen. One of the most popular design method is optimal control theory but it requires feedback of both position and rate information. This information is available for submarines which are equipped with an inertial guidance system. For the small coastal submarines which do not have an inertial guidance system, a different design approach must be carried out. A possible way would be the design of cascaded compensators using only position (such as depth) feedback.

There is always a cross-coupling effect between vertical and horizontal motion in a submerged submarine which is also called a squatting effect. The cross-coupling effect is simply the rudder effect on vertical plane which makes the submarine pitch up and change depth when a rudder angle is applied. The cross-coupling effect is also investigated in this study.

II. EQUATIONS OF MOTIONS IN SIX DEGREES OF FREEDOM

A. BACKGROUND

With diving capability, submarines differ from surface ships. They also have completely different hull structures, hydrodynamic specifications and relatively complex control and stability problems. A submarine can be operated in all six degrees of freedom. To maneuver usually three sets of plane surfaces, the propulsion system consisting of one or two propellers, and a ballast system consisting of two or three ballast tanks for different type of submarines are used.

To control horizontal motion the submarine has a usual rudder such as surface ships do. But in vertical motion, a submerged submarine needs at least one more control surface to maintain the desired depth and pitch angle.A classic submarine has bow planes, which can be used to keep ordered depth, and stern planes, which can be used to tilt the submarine to an ordered pitch angle. Depending on the submarines's speed and condition these planes can have an appreciable interaction.

Modern submarines usually have bow planes on their sails, which are called fairwater planes. However, high underwater speeds reduce the necessity of bowplanes. It is possible to keep ordered depth without using bow planes while operating with higher underwater speeds. Since the numbers presented by NSRDC [Ref. 1: p. 88] are for an American submarine, bow and fairwater planes were both considered in this study.

An illustrative picture of a submarine with axes, velocity and plane definitions is given in Fig.1. The arrows are pointed in the positive motion direction.This coordinate system is the right hand orthogonal system which is fixed in the submarine and moves with it. The origin of the coordinates is located at the center of gravity with x-axis along the center plane. The positive x direction is forward, the positive y direction is horizontally to the right, and the positive z direction is down. [Ref. 2: p. 438]

The heading of the submarine is the direction of its x-axis, and this is measured as an angle with respect to the geographic coordinate system. The heading angle, also called the yaw angle, is defined to be the angle between the direction of the ships x-axis and the direction of the x-axis of the geographic coordinate system. The symbol used for the yaw angle is ψ.

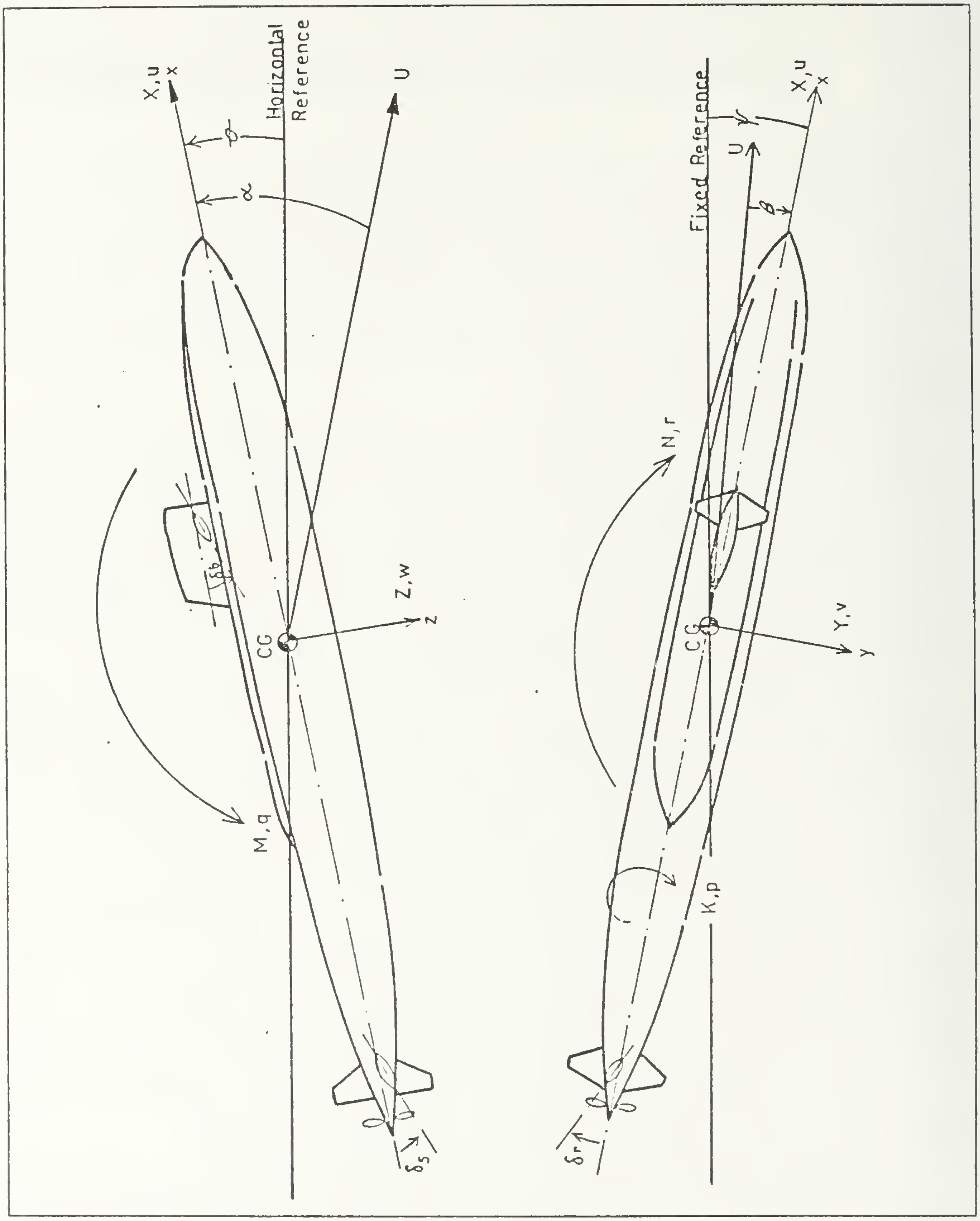

Figure 1. A Submarine with Axes of Motion

The pitch angle of the ship is the rotation around its y-axis. It is defined to be the angle between the direction of the ships x-axis and the horizontal reference line. The symbol used for the yaw angle is θ .

The roll angle of the submarine is the rotation around its x-axis. It is measured from the vertical reference to the direction of the submarine z-axis. The symbol used for the roll angle is ϕ.

Velocities for the x, y and z directions are u, v and w respectively, which can be called velocity components of linear velocity of body axes relative to an earth-fixed axis system.

Definitions for all symbols used in this study are given in Appendix A.

B. DERIVATION OF THE LINEARIZED MODEL

The equations of motion are derived by summing the applicable forces and moments in each degree of freedom: surge(x), sway(y), heave(z), roll(ϕ), pitch(θ) and yaw(ψ). Reference 1 presents the standard sets of equations of motion developed for submarine motion studies by NSRDC. These equations are general enough to simulate the trajectories and responses of submarines in the six degrees of freedom resulting from various types of maneuvers. They simulate motion of a given ship design upon insertion of the nondimensionalized hydrodynamic coefficients developed for that particular design. In addition values must be supplied for propulsion force and rudder and diving plane angles. A complete set of hydrodynamic coefficients and other required data used in this thesis is given on Appendix B.

The derivation of equations of motions in six degrees of freedom which are to be linearized, was discussed in several earlier studies. [Ref. 3 , Ref. 4 .] The authors were satisfied that these equations are valid and can simulate a submarine's motion effectively.

1. Assumptions

Forward speed can be taken as constant. Linearizing about the axial speed,u, which affects nearly every term in the standard equations, could be very complex, so the forward speed was assumed to be constant. This also reduces the degrees of freedom to five.

Roll angle is assumed to be small. Under normal circumtances in submarine maneuvering, the roll angle usually stays within $\pm 5°$. Large roll angles are only caused by high speed plus hard over rudder. Therefore, the roll angle can be neglected.

Cross-products of inertia can be neglected. This assumption is common to all submarine simulations because the hull and interior layout of submarines is approximately symmetric.

All terms including W_i can be discarded. Since it is assumed that the submarine is in trim, weight of water blown from a particular ballast tank, W_o, must equal zero.

All terms involving nonlinearity are neglected.

Vertical motion is decoupled from horizontal motion. As a result of the first five assumptions it also has to be assumed that there is no coupling between vertical and horizontal motion.

2. Derivation of the linear equations of Motion

a. Linearization on the vertical plane

The linearized form of the equations on vertical plane are:

1) Equation of Motion Along z-axis (Normal Force):

$$m\dot{w} - umq = \frac{\rho}{2} l^4 Z_{\dot{q}}\dot{q} + \frac{\rho}{2} l^3 (Z_{\dot{w}}\dot{w} + Z_q uq) + \frac{\rho}{2} l^2 (Z_w uw + u^2(Z_{\delta s}\delta s + Z_{\delta b}\delta b)) \quad (1)$$

where

$\rho = 2.0 \ \frac{slug}{ft^3}$, mass density of sea water,

$l = 415 \ ft.$, submarine length, and

$m = 6.25x10^6 \ slugs$, submarine weight.

All values for the hydrodynamic coefficients are given in Appendix B. Substituting these numbers into the equation, and after performing the required algebra

$$\dot{w} = -5.11\dot{q} - 1.632x10^{-3}uw + 0.261uq - 7.416x10^{-4}u^2\delta s - 3.71x10^{-4}u^2\delta b \quad (2)$$

2) Equation of Motion About y-axis:

$$I_y\dot{q} = \frac{\rho}{2} l^5 M_{\dot{q}}\dot{q} + \frac{\rho}{2} l^4 (M_q uq + M_{\dot{w}}\dot{w}) + \frac{\rho}{2} l^3 (M_w uw + u^2(M_{\delta s}\delta s + M_{\delta b}\delta b)) + Bz_B\theta \quad (3)$$

After substituting appropriate numbers and required algebra

$$\dot{q} = -4.975x10^{-4}\dot{w} - 6.219x10^{-3}uq + 1.798x10^{-5}uw - 1.5x10^{-5}u^2\delta s + 3.0x10^{-6}u^2\delta b + 2.516x10^{-3}\theta \quad (4)$$

If these two equations are substituted into each other

$$\dot{w} = 0.294uq - 1.728x10^{-3}uw - 6.667x10^{-4}u^2\delta s - 3.873x10^{-4}u^2\delta b - 0.01280 \theta \qquad (5)$$

$$\dot{q} = 1.884x10^{-5}uw - 6.365x10^{-3}uq - 1.465x10^{-5}u^2\delta s + 3.193x10^{-6}u^2\delta b + 2.522x10^{-3}\theta \qquad (6)$$

These two equations describe the state variable representation of the linearized, vertical plane equations of motion. However they do not have the depth as a state variable. In order to make the depth a state variable, these equations are to be modified by using linearized auxilary equations which are given in Appendix C. Therefore the auxilary equation used for the modification is

$$\dot{z}_0 = -uSin\theta + vCos\theta \sin\phi + w\cos\theta\cos\phi$$

Using our assumptions the linearized equation will be

$$\dot{z}_0 = -u\theta + w$$

Then the modified linear equations of motion have the following form

$$\begin{aligned}\ddot{z} = \; & -1.728x10^{-3}u\dot{z} - 0.706uq + (0.01283 - 1.728x10^{-3}u^2)\theta - 6.667x10^{-4}u^2\delta s \\ & -3.873x10^{-4}u^2\delta b\end{aligned} \qquad (7)$$

$$\begin{aligned}\dot{q} = \; & 1.884x10^{-5}u\dot{z} - 6.365x10^{-3}uq - 1.465x10^{-5}u^2\delta s + 3.193x10^{-6}u^2\delta b \\ & + (1.884x10^{-5}u^2 - 2.522x10^{-3})\theta\end{aligned} \qquad (8)$$

As it was mentioned before the forward speed u is not a state variable but a constant which can be changed as desired. A complete block diagram for vertical motion is given in Figure 2.

b. Linearization on the Horizontal Plane

The linearized form of the equations on horizontal plane are:

1) Equation of motion along y-axis (Lateral Force):

$$\begin{aligned} m\dot{v} - mur = \; & \frac{\rho}{2} l^4 (Y_{\dot{r}}\dot{r} + Y_{\dot{p}}\dot{p}) + \frac{\rho}{2} l^3 (Y_{\dot{v}}\dot{v} + Y_r ur + Y_p up) \\ & + \frac{\rho}{2} l^2 (Y_v uv + u^2 Y_{\delta r}\delta r)\end{aligned} \qquad (9)$$

Using same set of numbers and hydrodynamic coefficients, the final form of the equation is

$$\dot{v} = 1.89\dot{r} - 6.3\dot{p} - 0.291ur - 0.035up - 2.563x10^{-3}uv + 7.568x10^{-4}u^2\delta r \qquad (10)$$

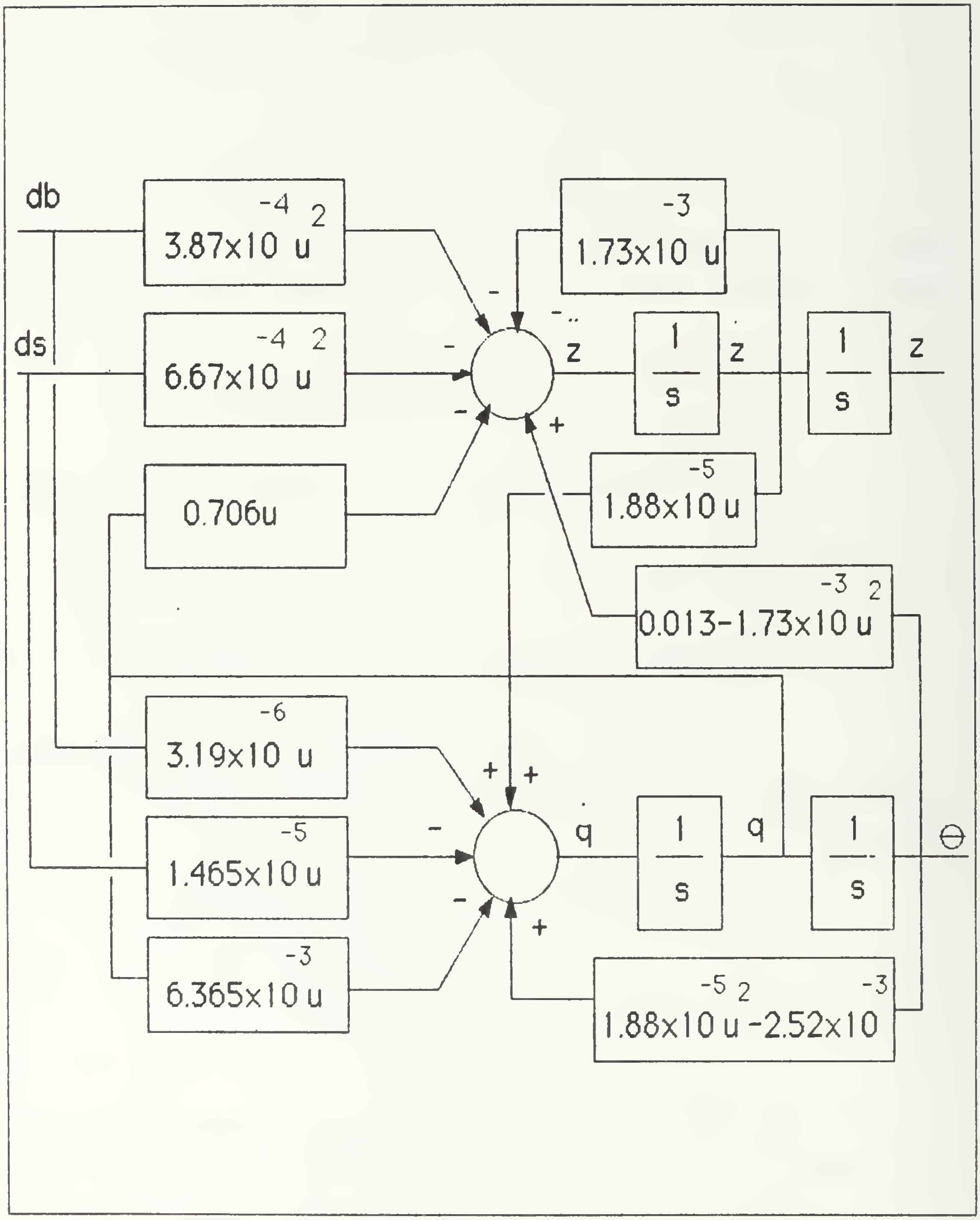

Figure 2. Block Diagram for the Linearized Model on the Vertical Plane

$$I_x\dot{p} = \frac{\rho}{2} l^5(K_{\dot{p}}\dot{p} + K_{\dot{r}}\dot{r}) + \frac{\rho}{2} l^4(K_p up + K_r ur + K_{\dot{v}}\dot{v}) + \frac{\rho}{2} l^3(K_v uv + u^2 K_{\delta r}\delta r) + Bz_B\phi \quad (11)$$

The final form of the equation is

$$\begin{aligned}\dot{p} = & -0.679\dot{r} - 0.0584\dot{v} - 8.179x10^{-3}up - 9.347x10^{-3}ur - 3.942x10^{-4}uv \\ & + 3.942x10^{-5}u^2\delta r - 0.236\phi\end{aligned} \quad (12)$$

3) Equation of motion about z-axis (Yawing Moment)

$$I_z\dot{r} = \frac{\rho}{2} l^5(N_{\dot{r}}\dot{r} + N_{\dot{p}}\dot{p}) + \frac{\rho}{2} l^4(N_r ur + N_p up + N_{\dot{v}}\dot{v}) + \frac{\rho}{2} l^3(N_v uv + u^2 N_{\delta r}\delta r) \quad (13)$$

The final form of the equation is

$$\begin{aligned}\dot{r} = & -6.553x10^{-3}\dot{p} + 6.767x10^{-4}\dot{v} - 6.767x10^{-3}ur - 4.511x10^{-6}up - 4.076x10^{-5}uv \\ & -1.631x10^{-5}u^2\delta r\end{aligned} \quad (14)$$

These three equations are supposed to describe a submerged submarine motion in the horizontal plane. The only difference from the equations for vertical motion is the equations for the horizontal motion have the order of the highest derivative of all the variables such as v, p and r in each particular equation. [Ref. 4: p. 48]

Having all of the highest derivatives in each particular equation creates an algebraic loop problem for the simulation. To solve this problem it is possible to manipulate the equations to eliminate the highest order derivative from one of the equations which includes the other derivative as it was done before for the vertical plane equations of motion. This was done very nicely for the case of two equations but does not seem to be very attractive when there are three or more equations involved.

There are some other possible ways to solve algebraic loop problems. But since the new version of DSL [Ref. 5] can take care of this problem automatically, it is preferred to use those equations in simulation.

A complete block diagram for horizontal motion is given in Figure 3.

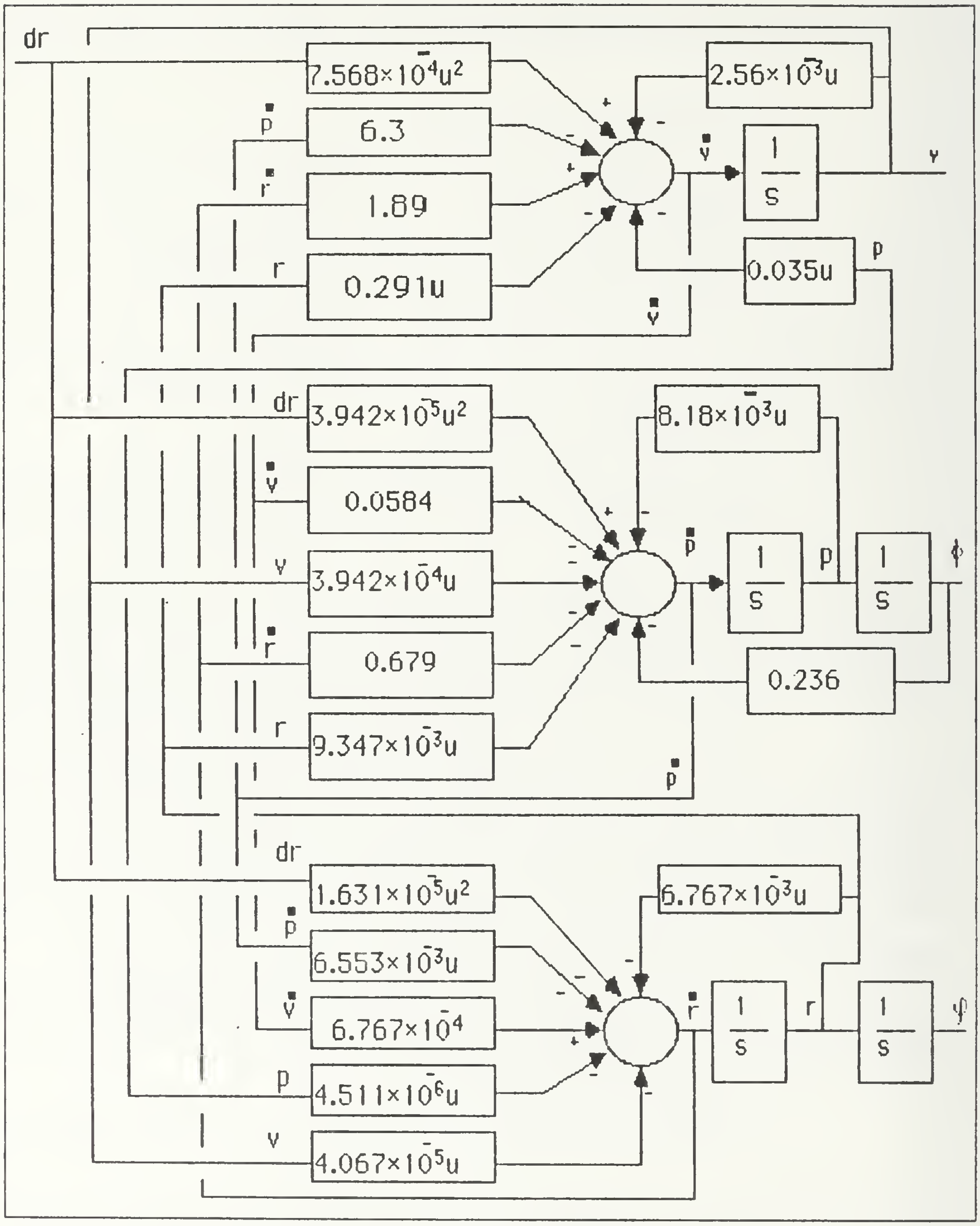

Figure 3. Block Diagram for the Linearized Model on the Horizontal Plane

C. VALIDATION OF LINEAR MODEL

The objective of this section is to compare the dynamics of the standard model with the derived linear model in both vertical and horizontal planes.

In order to compare both models they should be in the same initial state and both models have to be in trim. In trim has the meaning that the submarine maintains depth at a given speed with the desired pitch angle without using bow or stern planes. When making linearizing assumptions the terms which are related to trim are already ignored. Therefore the linearized model will be in trim at all times. Because of the submarine hull and sail structure it is required to adjust ballast tanks for given speed. The corrections for trim which are used in the simulation for this study are obtained from an earlier thesis study. [Ref. 6: p. 184]

To validate the linear model it is preferred to obtain both the initial condition and forced response in order to make sure that the linear model is working properly.

1. Validation of Linear Model on Vertical Plane

a. Initial Condition Response

It was expected that for small perturbations the deviations between models should be small. Therefore initial conditions of 5° in pitch were tested first. For the linear model it is also required to give an initial value for depth change which was defined as:

$$z = -u\sin\theta$$

Test runs up to 360 sec. in the speed range 5 to 25 Knots were performed simultaneously for both models. Maximum differences for each run were obtained from data files and given in Table 1. The pitch and depth behaviors for both models were given in Fig. 4-8.

Table 1. INITIAL CONDITION RESPONSE TO 5 DEGREE PITCH ANGLE

Run No.	Speed (Kts.)	Maximum Deviation In						Fig.
		Pitch		$\dot{Z}$		Depth		
		Deg.	%	Ft. sec.	%	Feet	%	
1	5	0.0901	1.8	0.0017	0.2	0.1050	0.1	4
2	8	0.0608	1.2	0.069	0.6	0.6740	0.7	5
3	12	0.0302	0.6	0.0113	0.6	1.0960	1.1	6
4	18	0.0486	1.0	0.0275	1.0	3.4200	1.4	7
5	25	0.1908	3.8	0.1599	4.3	11.960	4.8	8

As can be easily seen from the figures and Table 1 all deviations are very small for this initial condition. That means dynamics for both model are nearly identical for small perturbations.

In normal operational conditions a submarine never exceeds 20° pitch angle. But theoretically maximum allowed pitch angle is limited to about 45°. Therefore three more runs were performed with 45° initial pitch to see large perturbation effects on system dynamics. Simulation results for 45° initial pitch angle are given on Figures 9-11. Maximum deviations for pitch angle, speed in the z direction and depth are given in Table 2. Maximum deviation does not exceed 7% for this case as can be seen in Table 2.

Table 2. INITIAL CONDITION RESPONSE TO 45 DEGREE PITCH ANGLE

Run No.	Speed (Kts.)	Maximum Deviation In						Fig.
		Pitch		$\dot{Z}$		Depth		
		Deg.	%	Ft./sec.	%	Feet	%	
6	5	0.8449	1.9	0.2408	4.0	18.470	3.7	9
7	8	1.1988	2.7	0.1997	2.1	9.690	1.9	10
8	12	2.486	5.5	0.9760	6.8	65.260	6.5	11

The deviations between both models for a second set of initial conditions are much bigger but still leads to very similar dynamic behavior. This was expected as the angle approximation

$$\sin\theta = \theta \tag{15}$$

is not as valid as for 5° initial pitch angle. In general for both sets of initial conditions it is observed that increasing speed tends to increase the deviations between trajectories.

b. Forced Response

Both stern and bow planes can be used in different combinations to keep the ordered depth and pitch angle. In order to validate the linear model it is required to include some control plane commands in the simulation. Since the mechanical limit for the planes is about 35°, test runs were performed up to this angle. It is also desired to keep the submarine in maximum allowed pitch and depth limits. For the simulation runs which are performed only with bow plane, 5, 15 and 35 degree plane angles were applied after the first ten seconds.

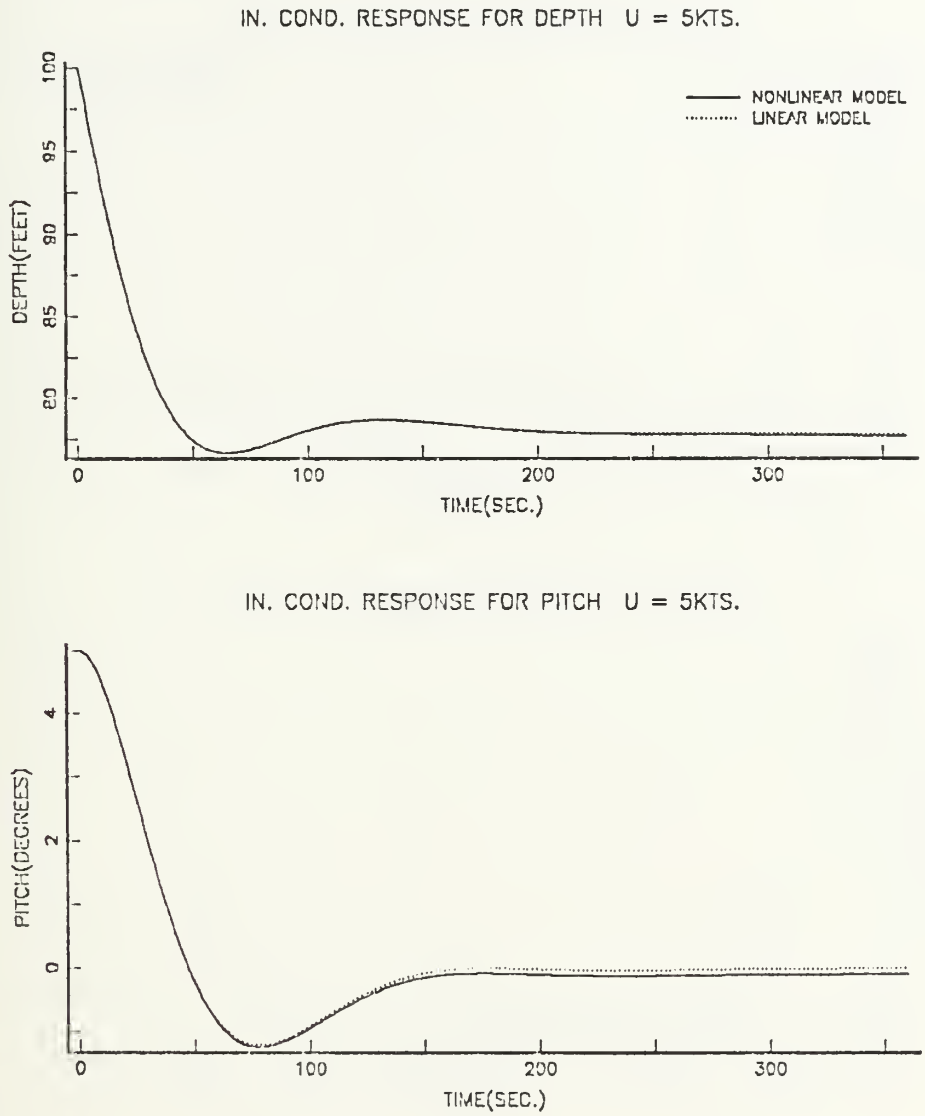

Figure 4. Initial Condition Response Init. Pitch = 5 Deg. U = 5 Kts.

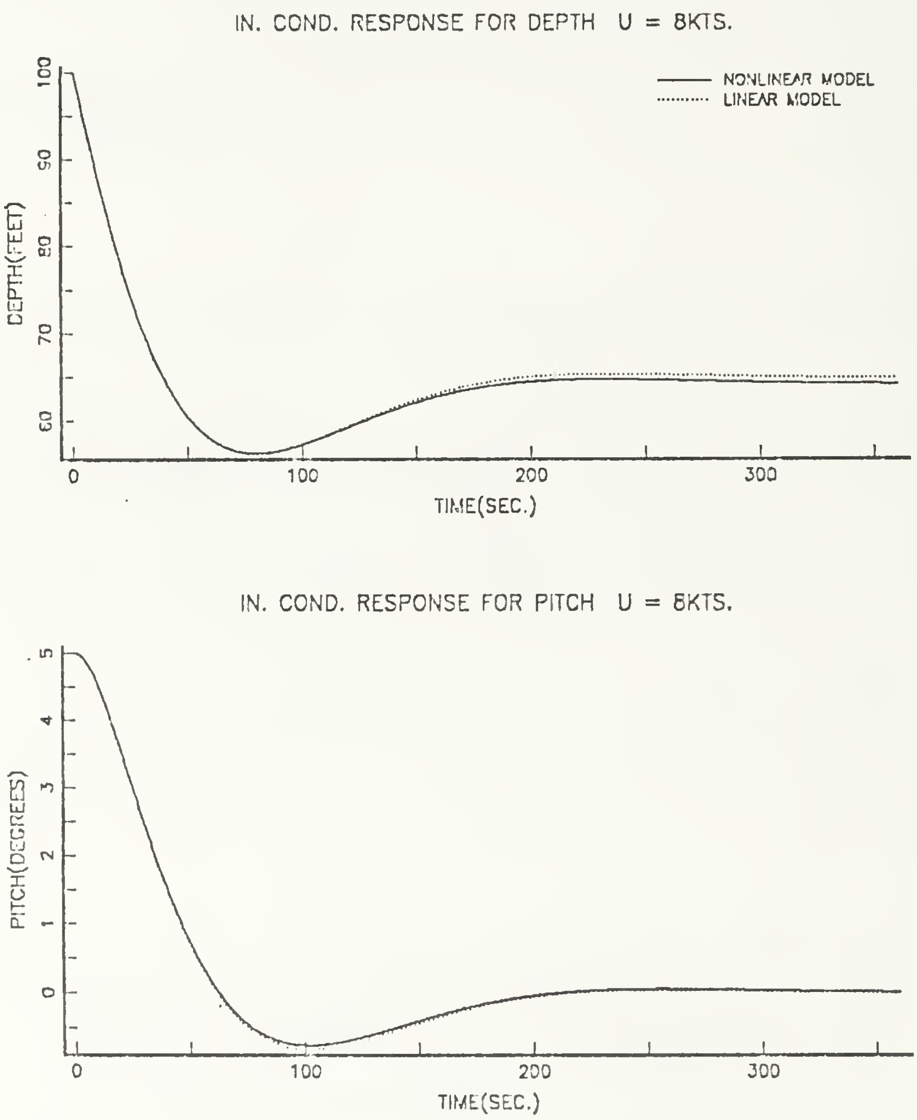

Figure 5. Initial Condition Response Init. Pitch = 5 Deg. U = 8 Kts.

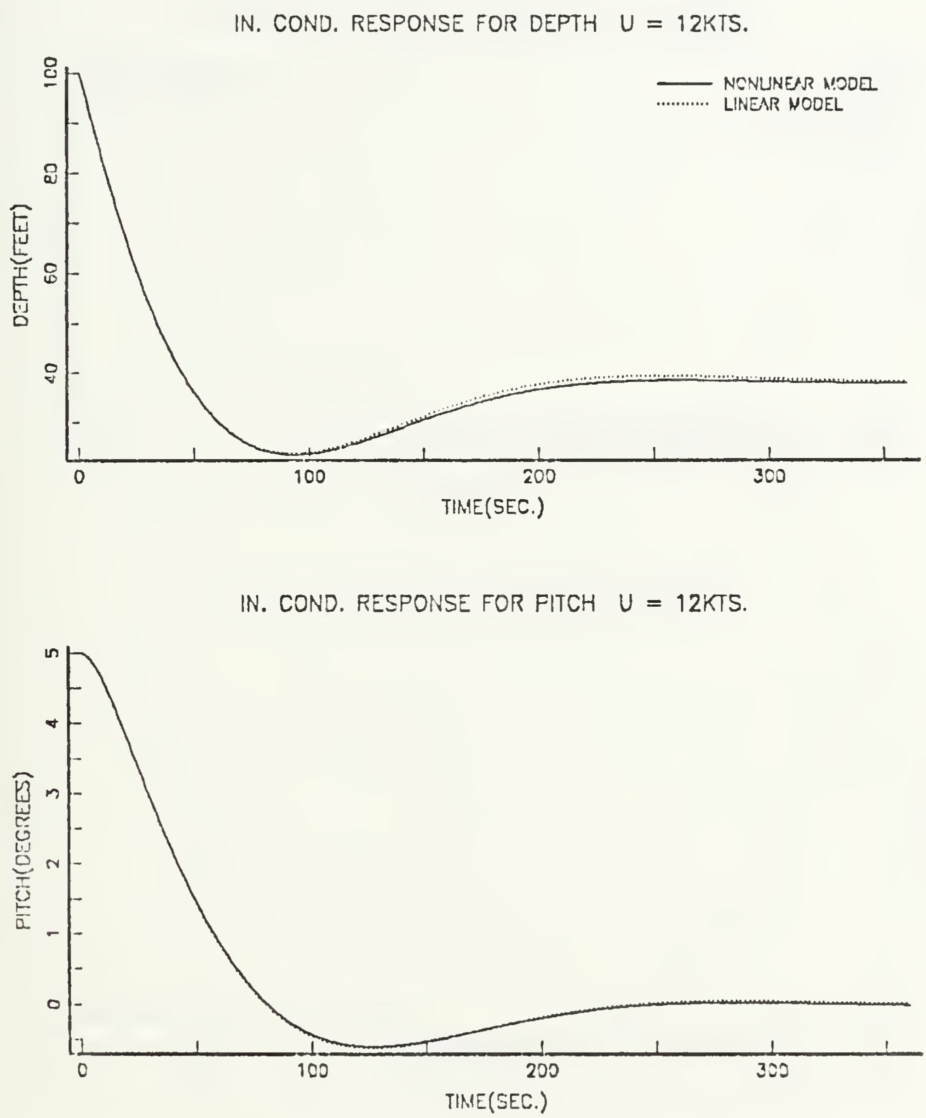

Figure 6. Initial Condition Response Init. Pitch = 5 Deg. U = 12 Kts.

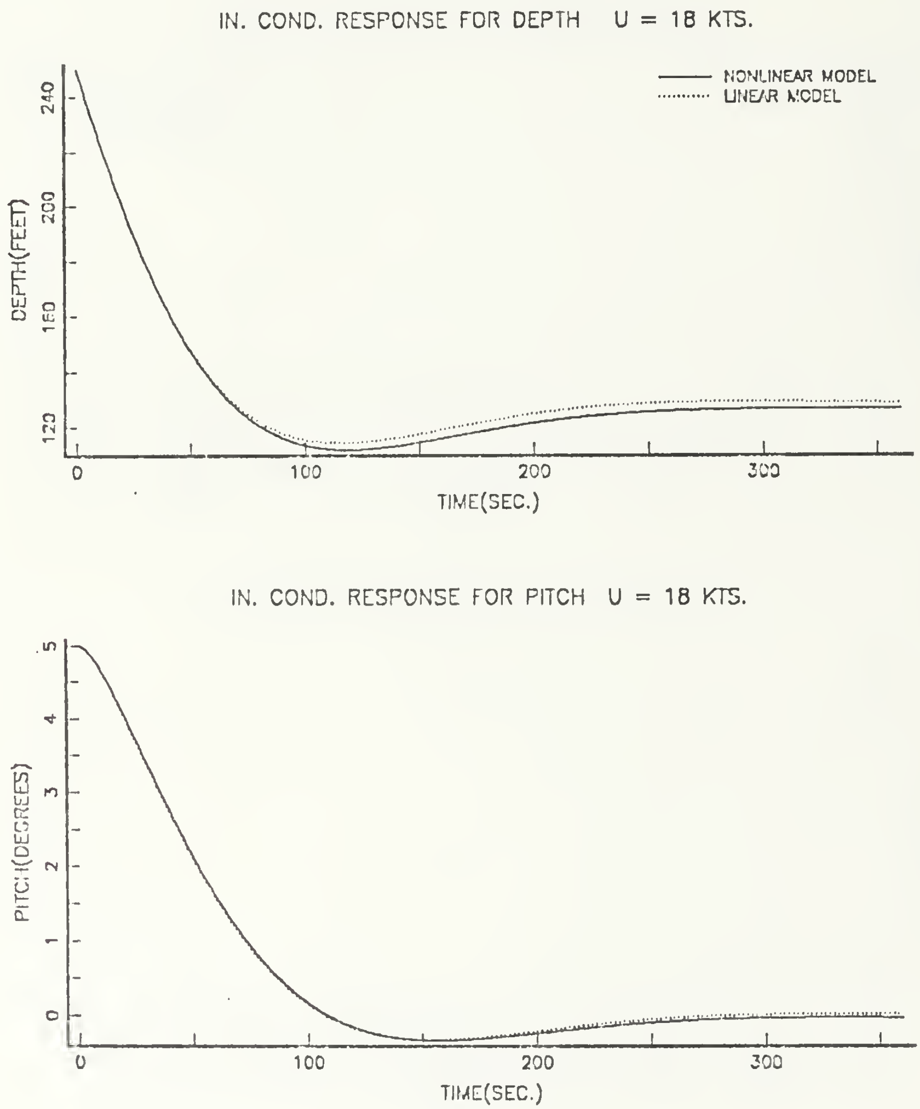

Figure 7. Initial Condition Response Init. Pitch = 5 Deg. U = 18 Kts.

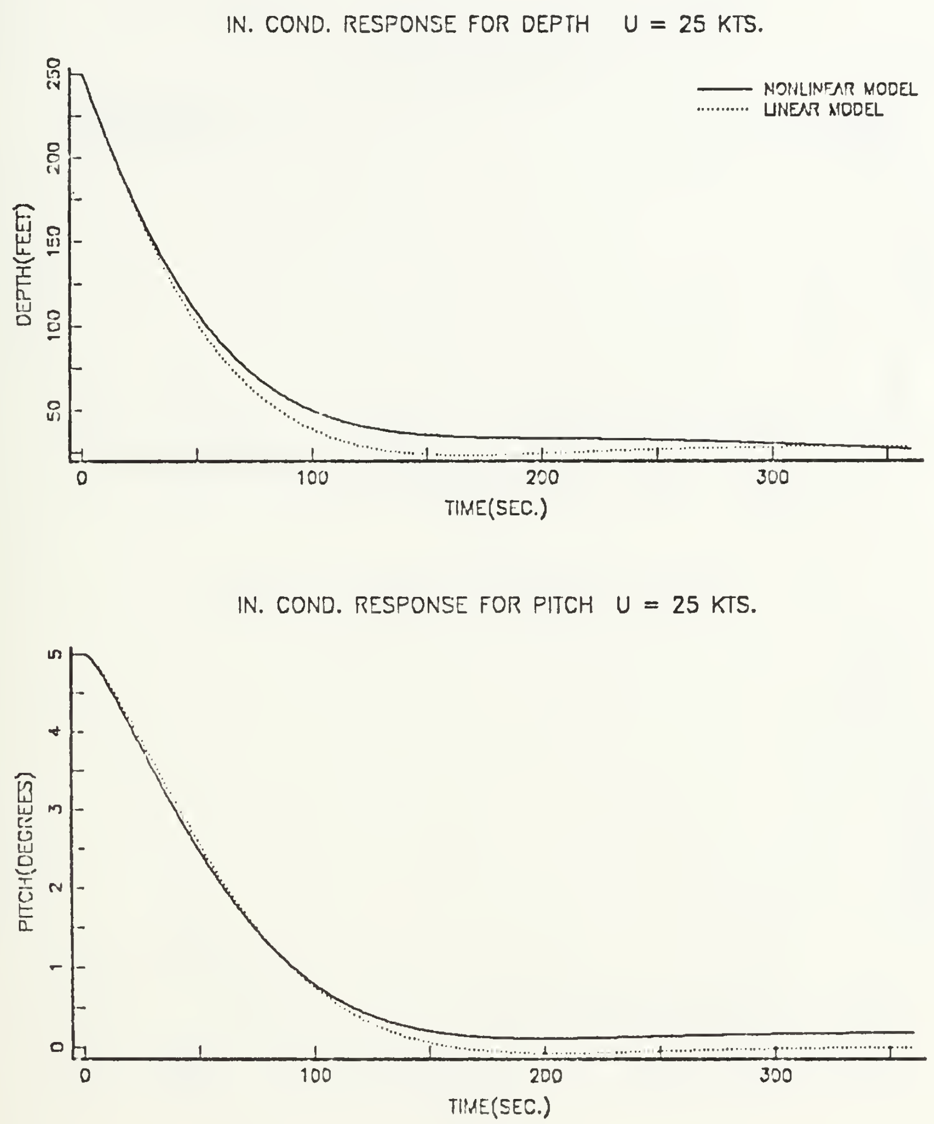

Figure 8. Initial Condition Response Init. Pitch = 5 Deg. U = 25 Kts.

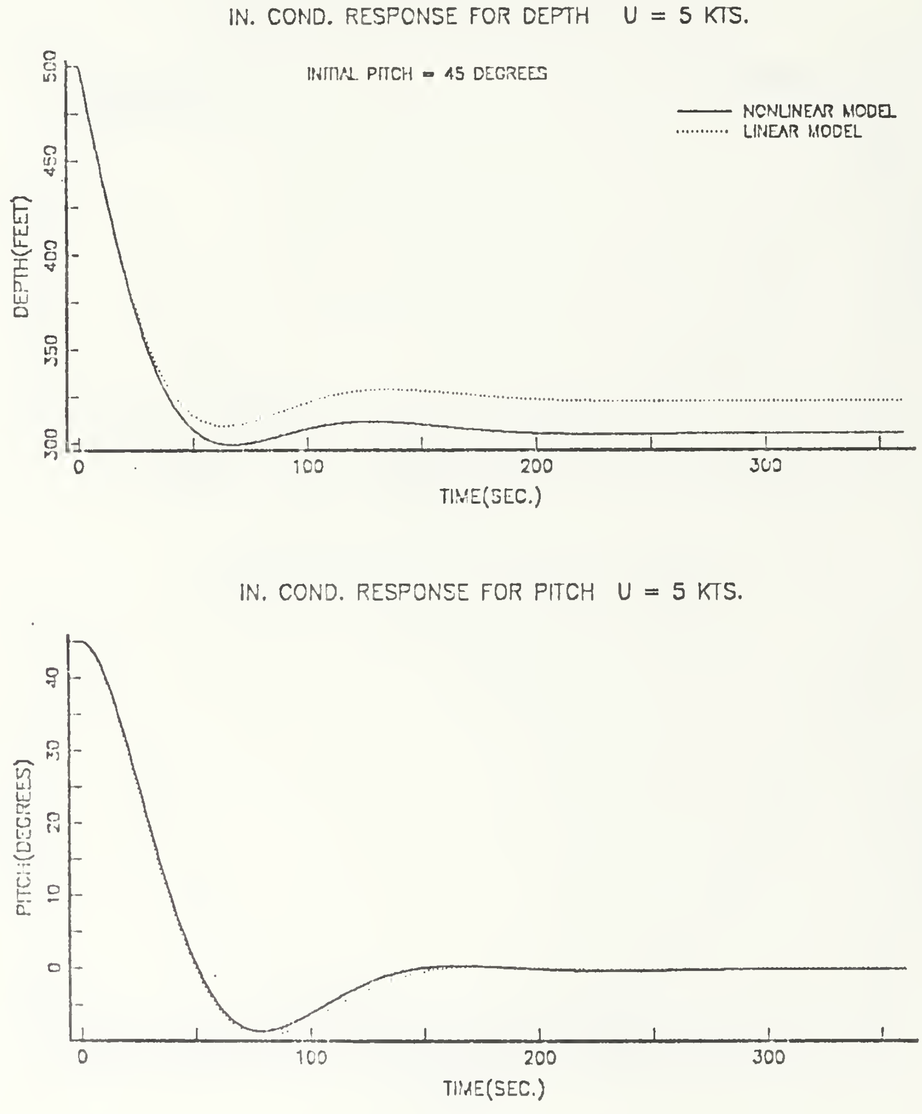

Figure 9. Initial Condition Response Init. Pitch = 45 Deg. U = 5 Kts.

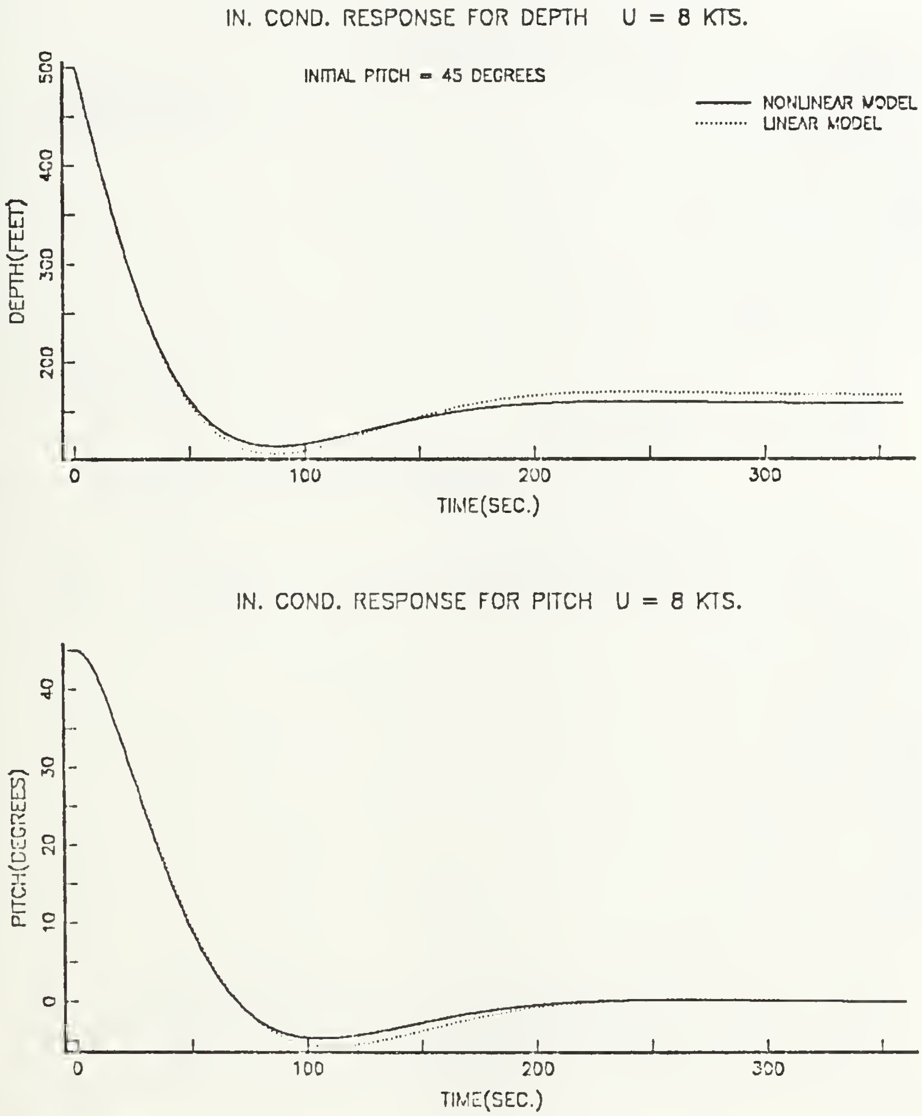

Figure 10. Initial Condition Response Init. Pitch = 45 Deg. U = 8 Kts.

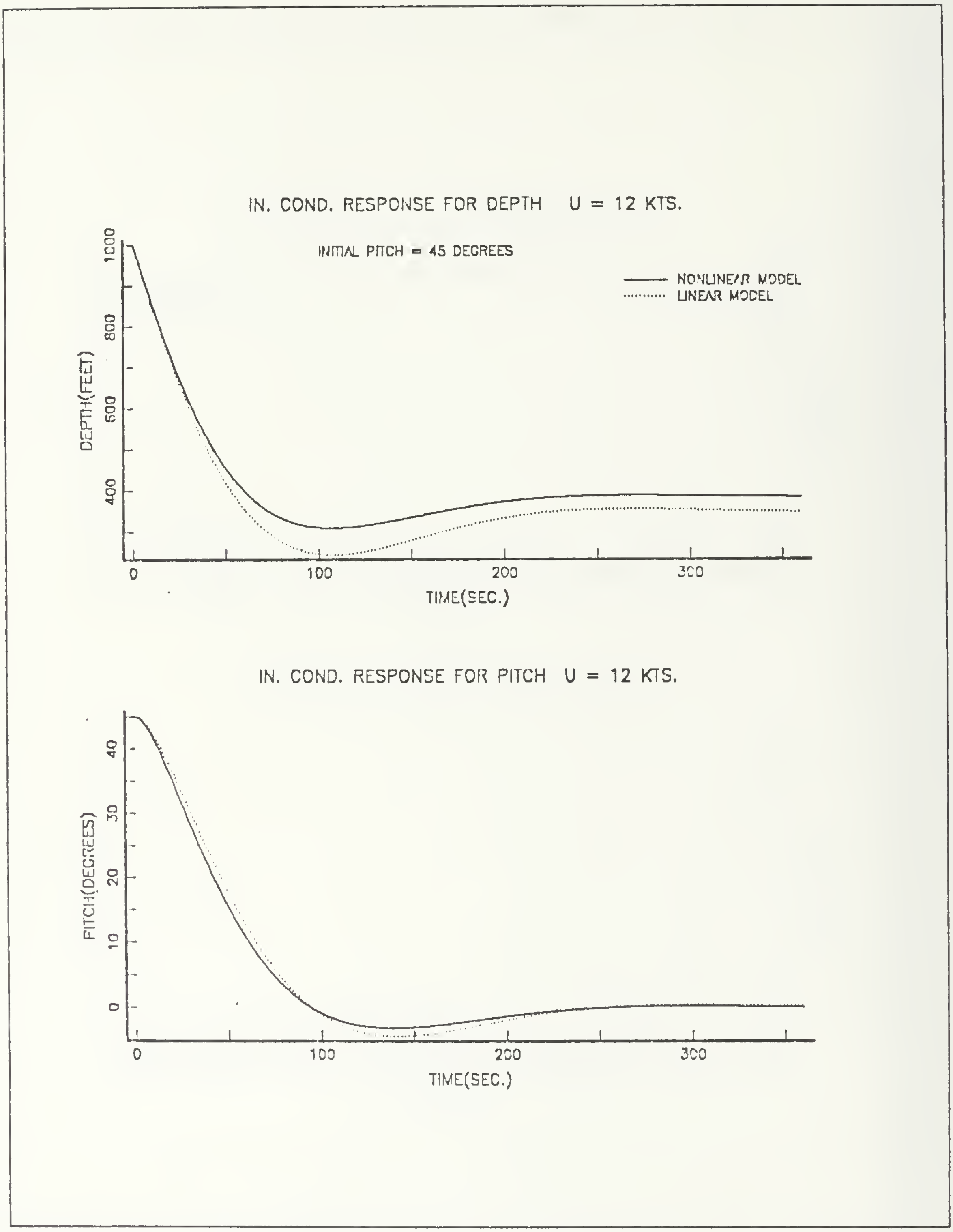

Figure 11. Initial Condition Response Init. Pitch = 45 Deg. U = 12 Kts.

Because of the enormous effect of the stern plane on submarine pitch angle, it was concluded to use reverse angles for 30 seconds each and then to bring the stern plane to the neutral condition.

Maximum deviations were obtained from data files by a FORTRAN program and given in Table 3, 4 and 5 for bow planes, stern planes and both planes respectively.

The test simulation results which were obtained with the bow planes, are given in Figures 12 through 20. Figures 12, 15 and 18 represent small perturbations for three different speeds and it can also be observed from Table 3 that maximum deviation is not more than 23 ft. for depth and not more than 0.4 degree for pitch angle. Figures 13, 16 and 19 were given for 15 degree bow plane and except for Figure 19 which represents the simulation with 18 Kts. forward speed, the linear model is acceptable. For 35 degree bow planes, the linear model is valid only for lower speeds as can be seen from Figures 14, 17 and 20.

Table 3. FORCED RESPONSE TO BOW PLANES

Run No.	Speed (Kts.)	Bow Plane (Deg.)	Maximum Deviation In						Fig.
			Pitch		Z		Depth		
			Deg.	%	Ft./sec.	%	Feet	%	
9	5	5	0.0850	19.8	0.0046	3.2	1.420	1.0	12
10	5	15	0.3751	29.2	0.0128	3.1	0.500	0.2	13
11	5	35	1.4706	68.2	0.2285	22.6	58.980	14.1	14
12	12	5	0.3598	35.6	0.1006	19.0	21.750	11.5	15
13	12	15	2.5177	82.5	0.6860	43.2	148.25	33.1	16
14	12	35	9.0961	127.7	1.9668	53.1	382.52	40.4	17
15	18	5	0.4103	19.0	0.1743	19.2	23.650	14.4	18
16	18	15	5.1356	79.3	2.4369	89.7	557.87	103.7	19
17	18	35	19.340	128.0	8.1425	128.5	1852.8	110.8	20

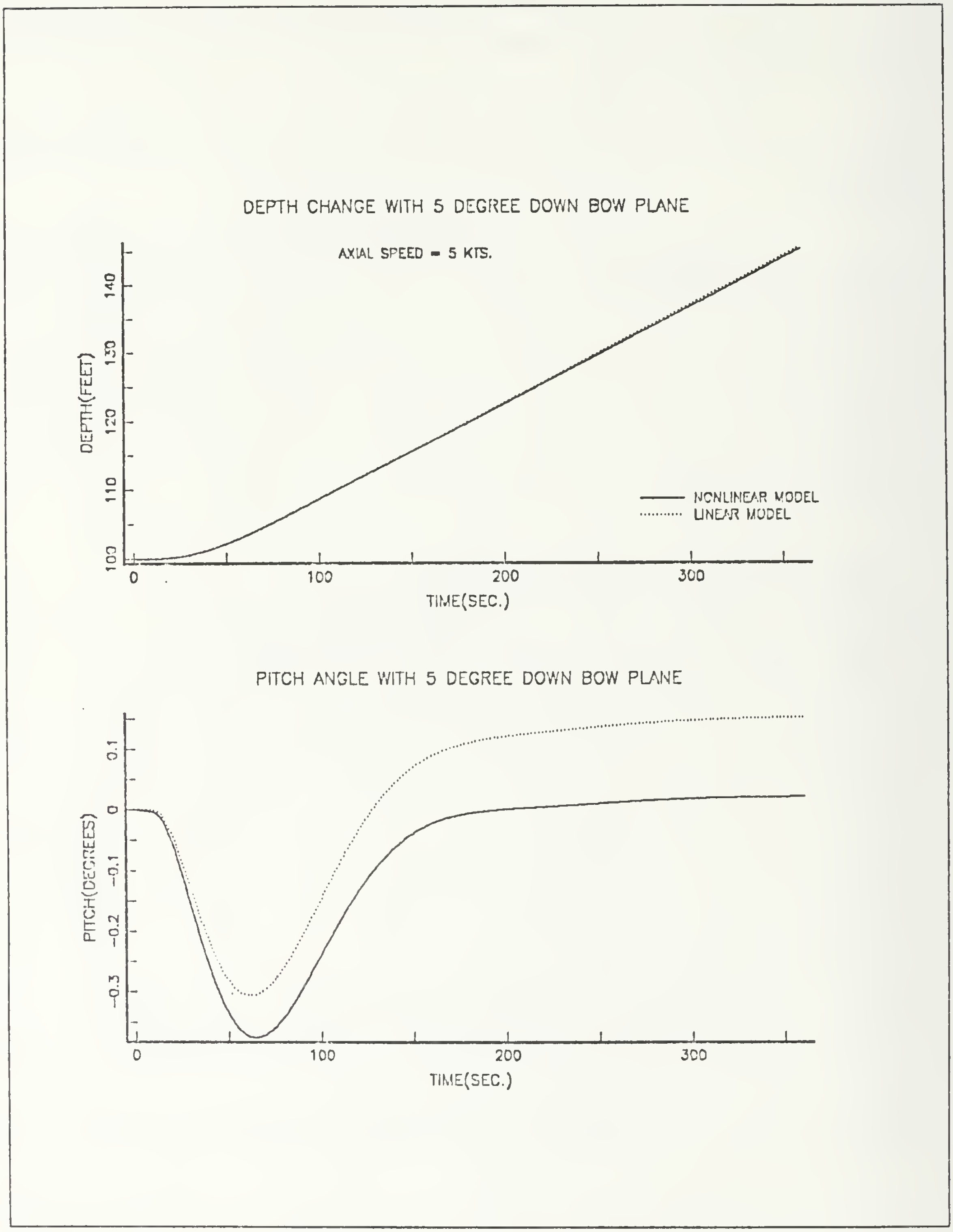

Figure 12. Forced Response. Bow Plane = 5 Deg. down. U = 5 Kts.

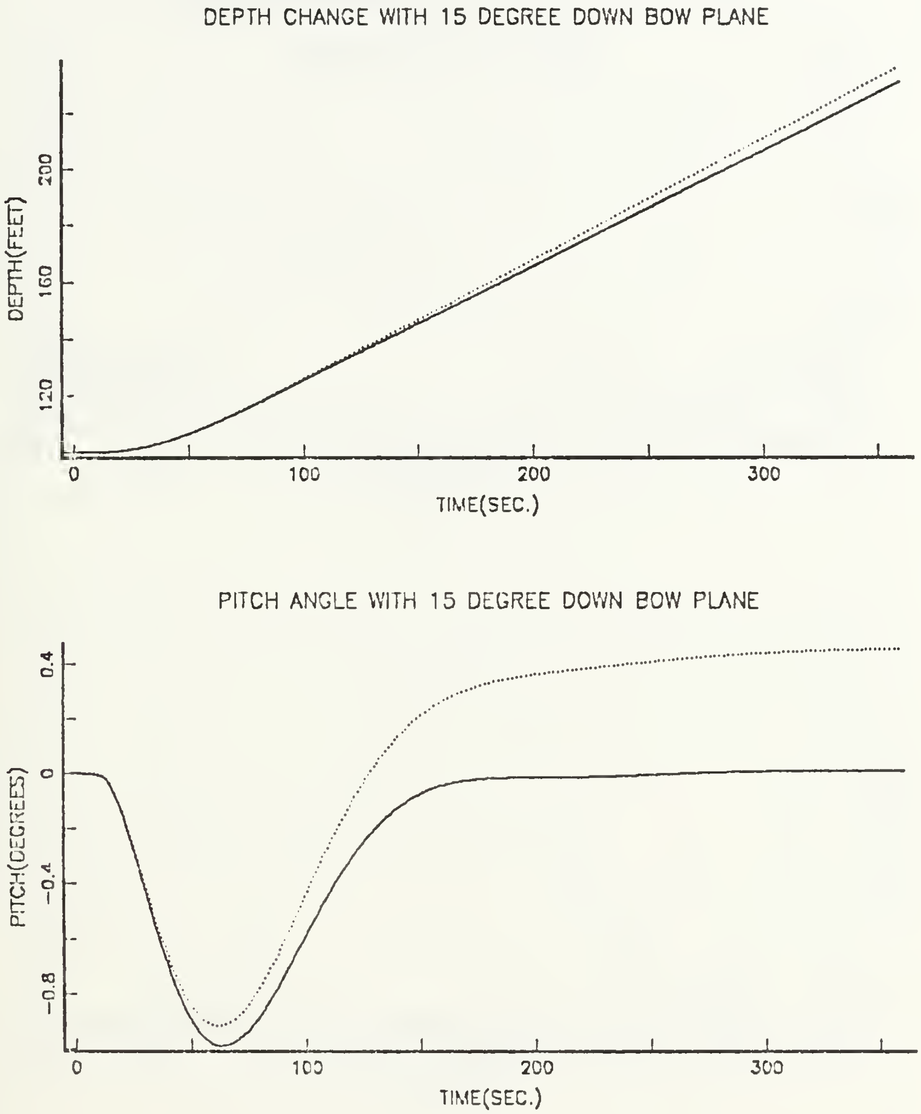

Figure 13. Forced Response. Bow Plane = 15 Deg. down. U = 5 Kts.

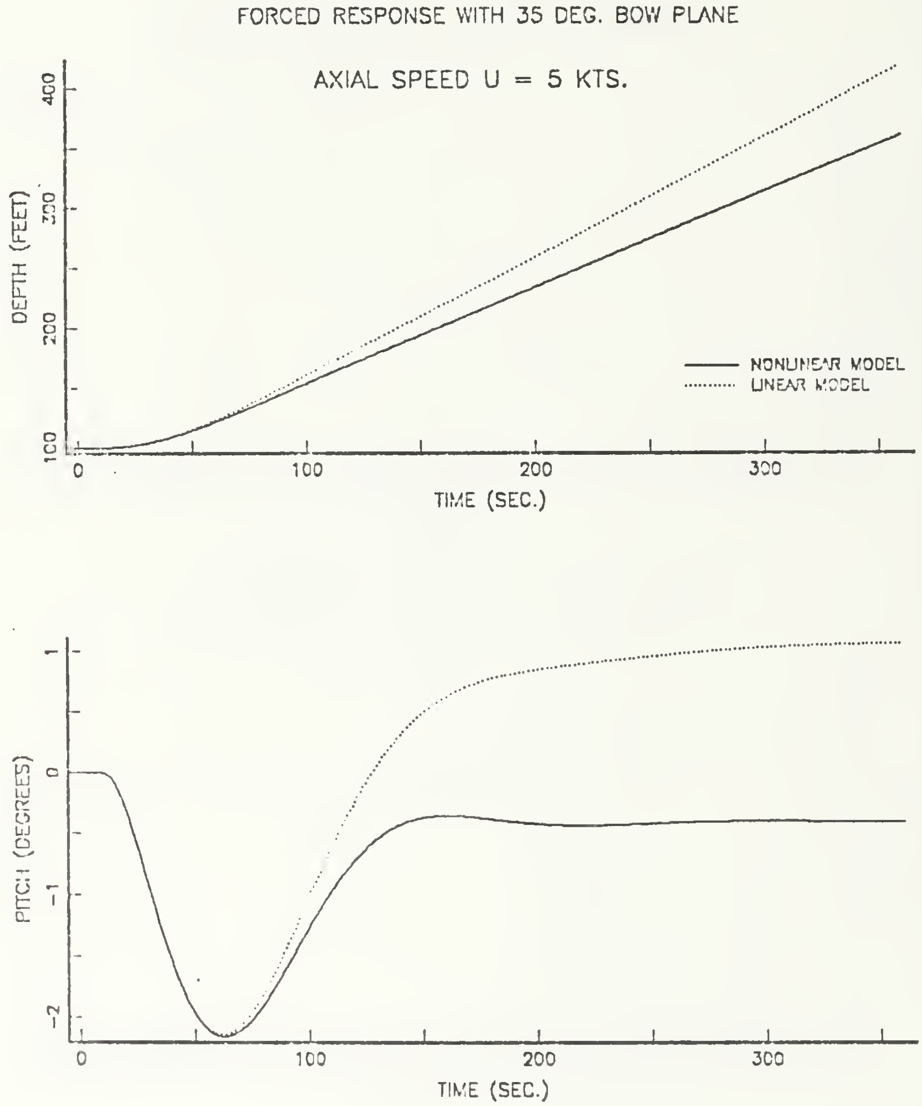

Figure 14. Forced Response. Bow Plane = 35 Deg. down. U = 5 Kts.

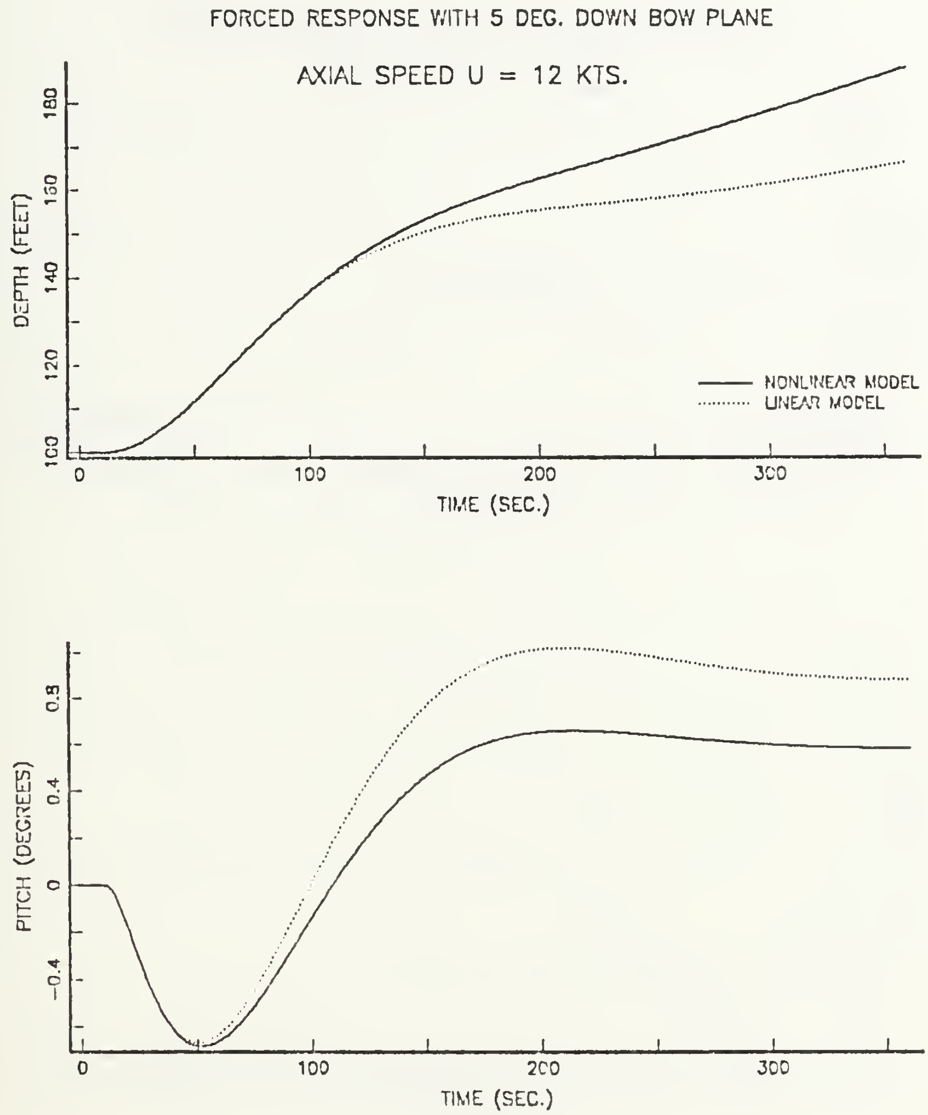

Figure 15. Forced Response. Bow Plane = 5 Deg. down. U = 12 Kts.

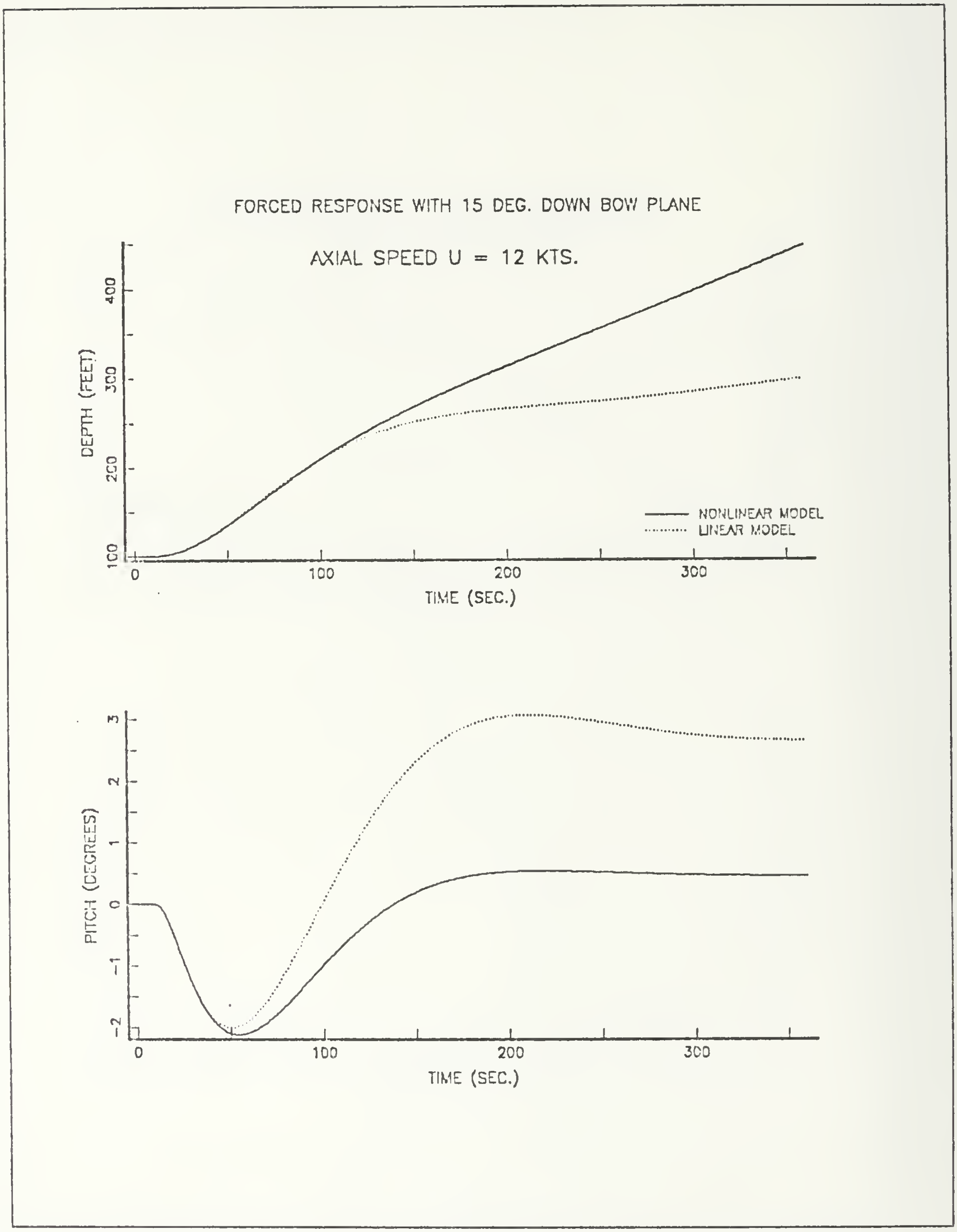

Figure 16. Forced Response. Bow Plane = 15 Deg. down. U = 12 Kts.

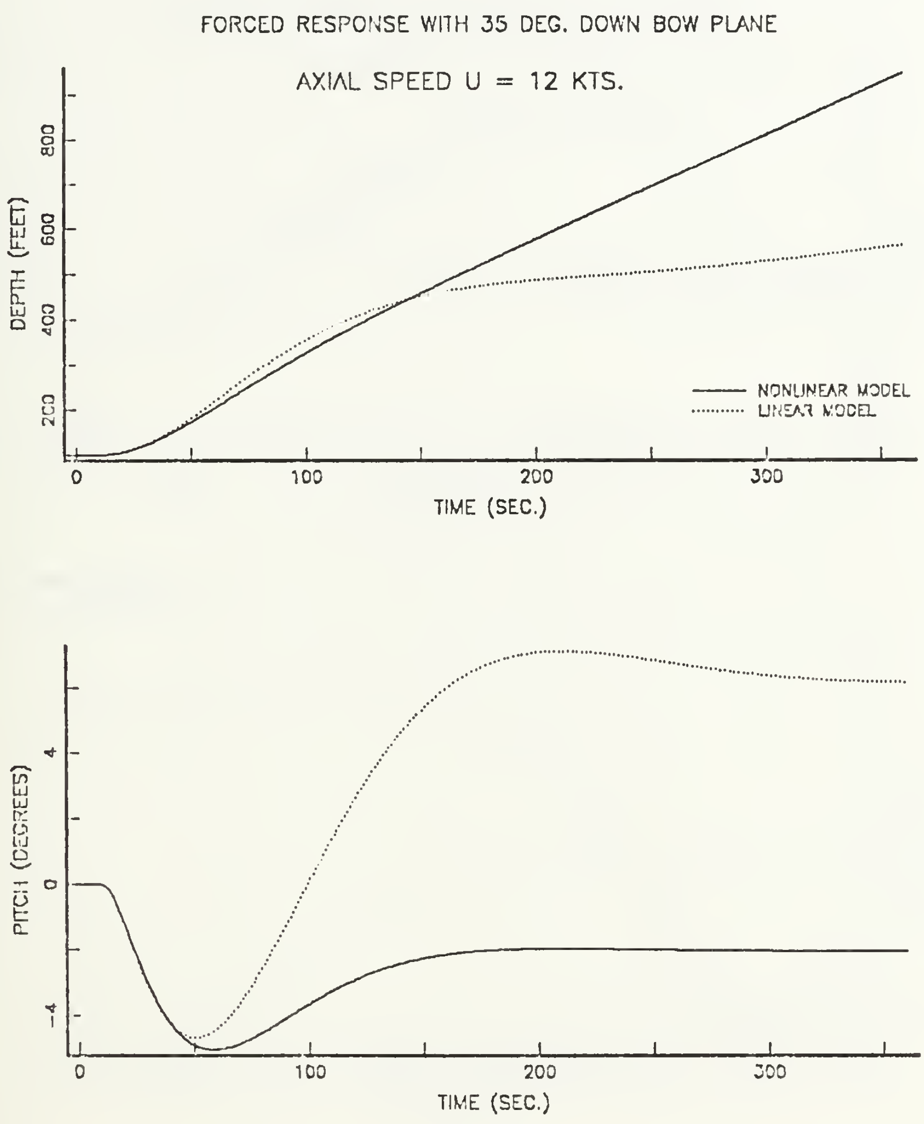

Figure 17. Forced Response. Bow Plane = 35 Deg. down. U = 12 Kts.

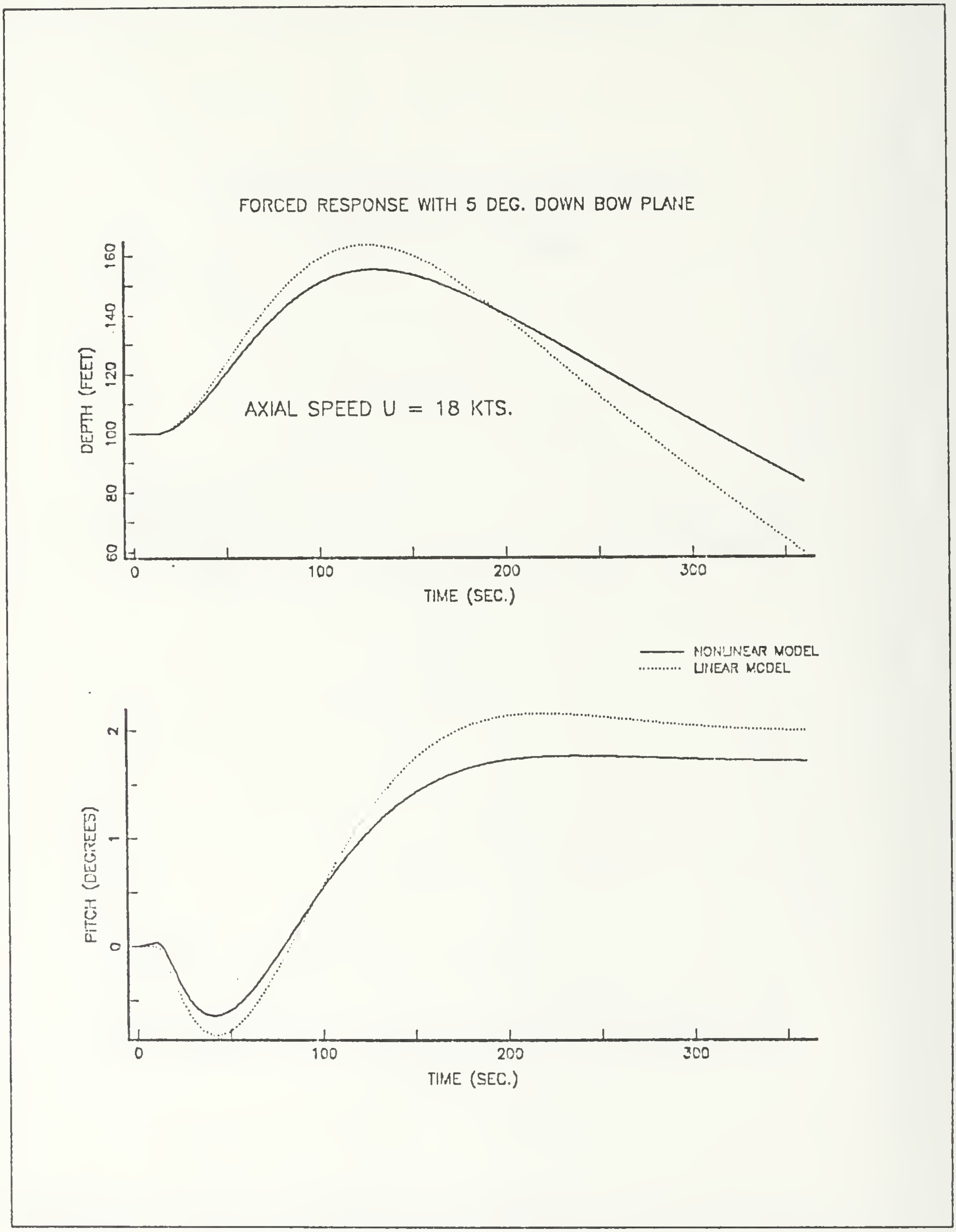

Figure 18. Forced Response. Bow Plane = 5 Deg. down. U = 18 Kts.

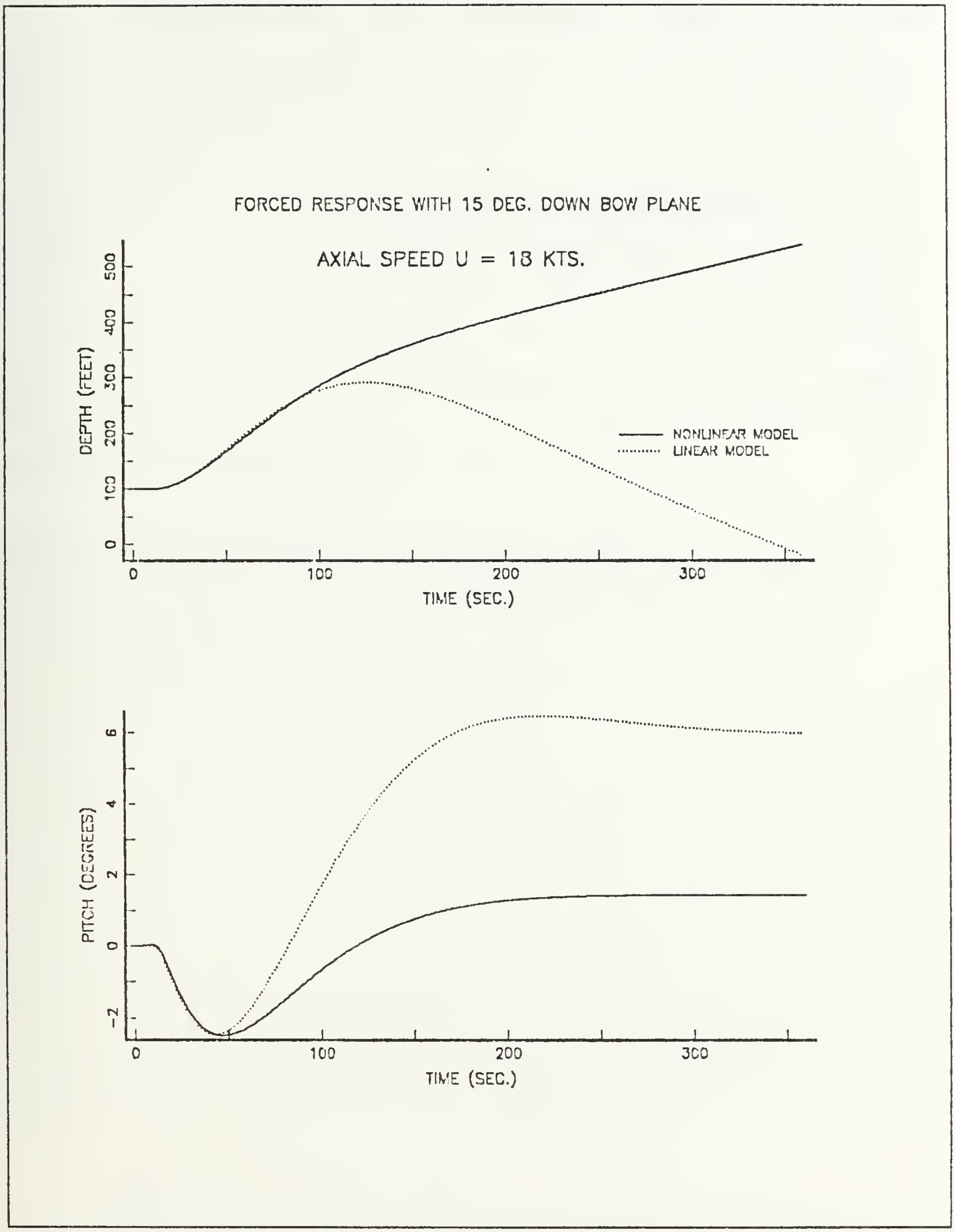

Figure 19. Forced Response. Bow Plane = 15 Deg. down. U = 18 Kts.

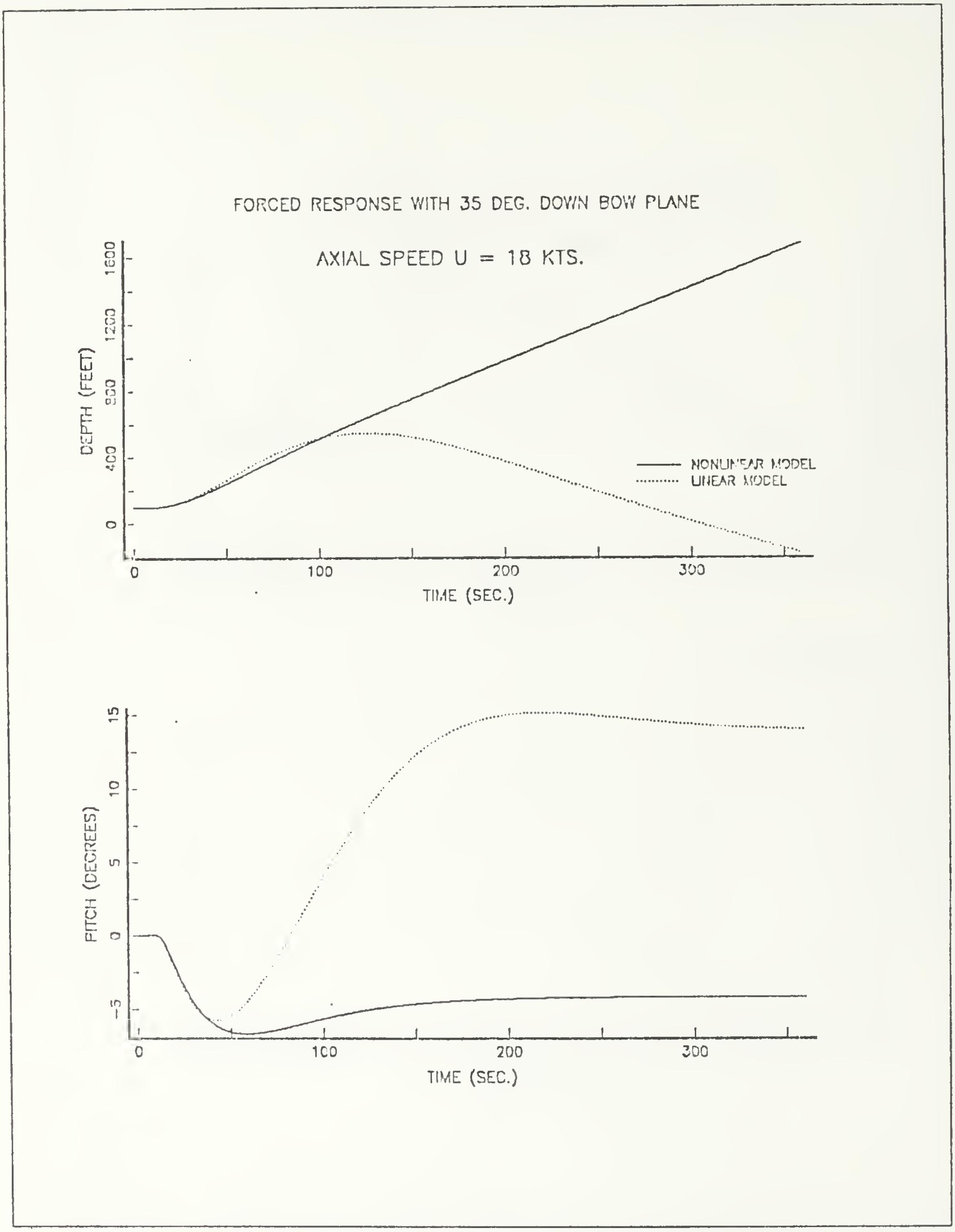

Figure 20. Forced Response. Bow Plane = 35 Deg. down. U = 18 Kts.

Therefore the linear model is valid for all speeds for small plane angles. And in general, it is possible to say that for the first 120 seconds the linear model does work well enough for large perturbations. In fact it is not very often that a watch officer wants to keep the bow planes full down for more than 120 seconds in a real submarine.

The test run results which are obtained with stern plane, are given in Figures 21 to 29 and Table 4. Similarly, deviations are acceptable for small and medium perturbations as can be seen from the figures. The only condition for which the linear model can not be accepted as valid, is displayed in Figure 29 which represents 35 degree bow planes with 18 Kts. forward speed. This is expected since two important linearizing assumption are invalid at this speed and resulting pitch angles are large. As mentioned before, the constant speed assumption for large plane angles and the sin(x) = x approximation for pitch angle are no longer valid for this run.

Table 4. FORCED RESPONSE TO STERN PLANES

Run No.	Speed (Kts.)	Stern Plane (Deg.)	Maximum Deviation In						Fig.
			Pitch		$\dot{Z}$		Depth		
			Deg.	%	Ft./sec.	%	Feet	%	
18	5	5	0.0910	5.6	0.0016	1.2	0.1030	0.1	21
19	5	15	0.1808	3.8	0.0126	3.3	0.7400	0.7	22
20	5	35	0.9021	8.2	0.1463	16.4	4.6610	3.8	23
21	12	5	0.3673	5.6	0.1070	6.0	5.000	3.0	24
22	12	15	2.5750	13.1	0.8569	15.9	35.680	12.1	25
23	12	35	12.010	26.1	5.2815	42.1	185.53	33.5	26
24	18	5	1.5585	12.6	0.7962	14.2	59.406	19.0	27
25	18	15	8.4160	22.6	4.8350	28.8	201.84	27.4	28
26	18	35	33.619	38.8	23.068	58.8	844.42	53.3	29

To be able to observe the effects of both planes on deviations between models, nine more runs were performed using stern and bow planes simultaneously. For each run the same bow and stern plane angles were applied in such a manner so they can suppress each other's effect in order not to exceed submarine depth and pitch

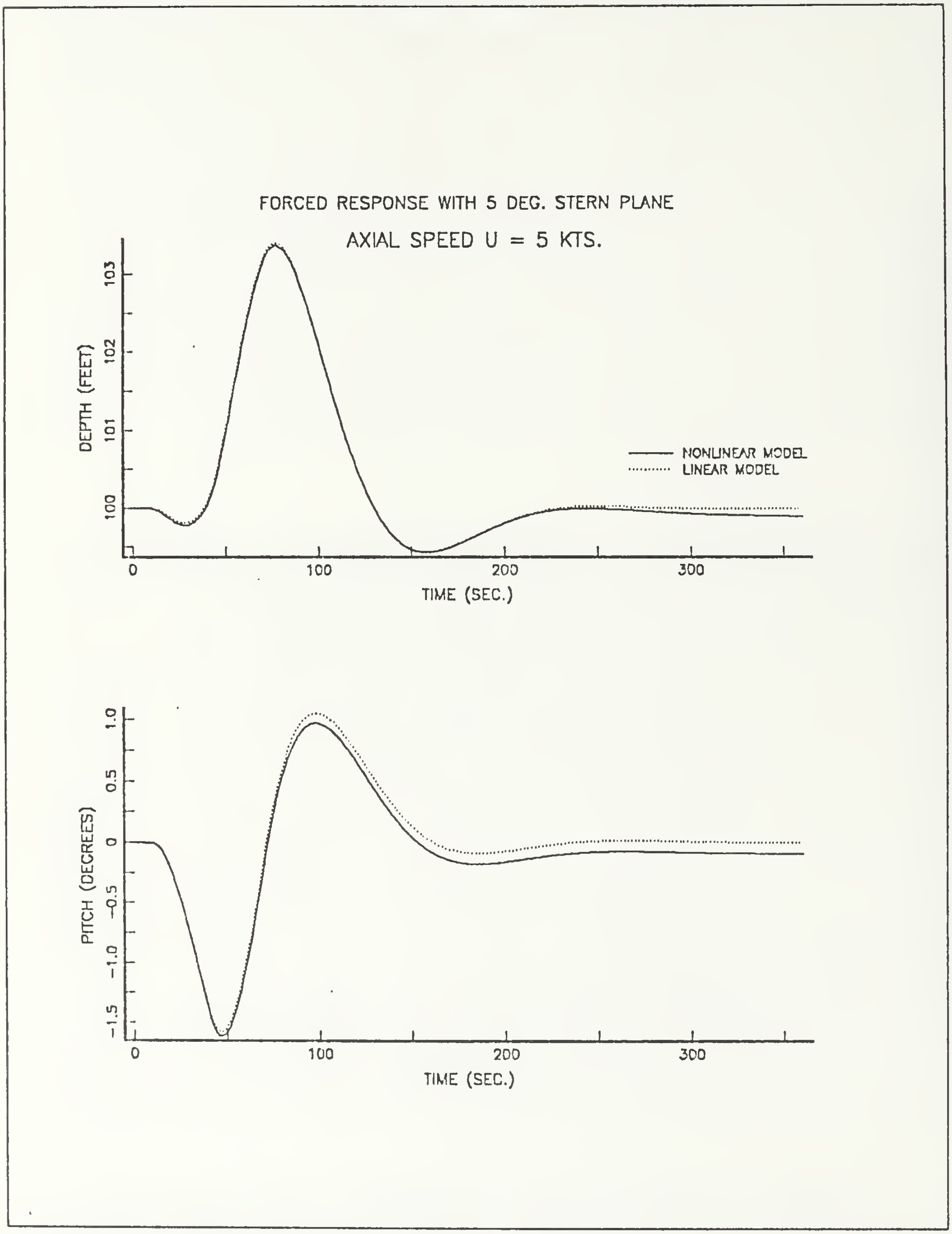

Figure 21. Forced Response. Stern Plane = 5 Deg. U = 5 Kts.

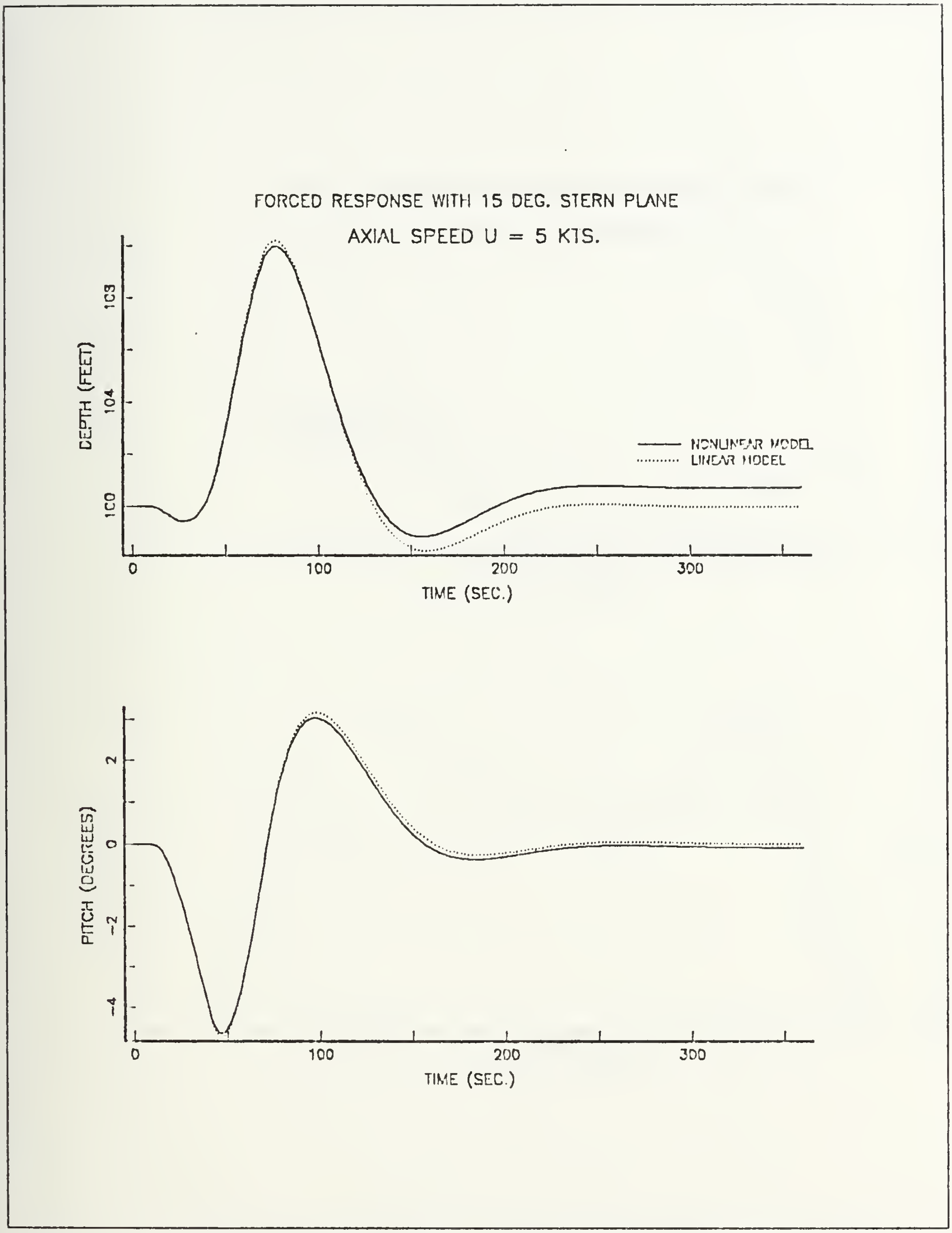

Figure 22. Forced Response. Stern Plane = 15 Deg. U = 5 Kts.

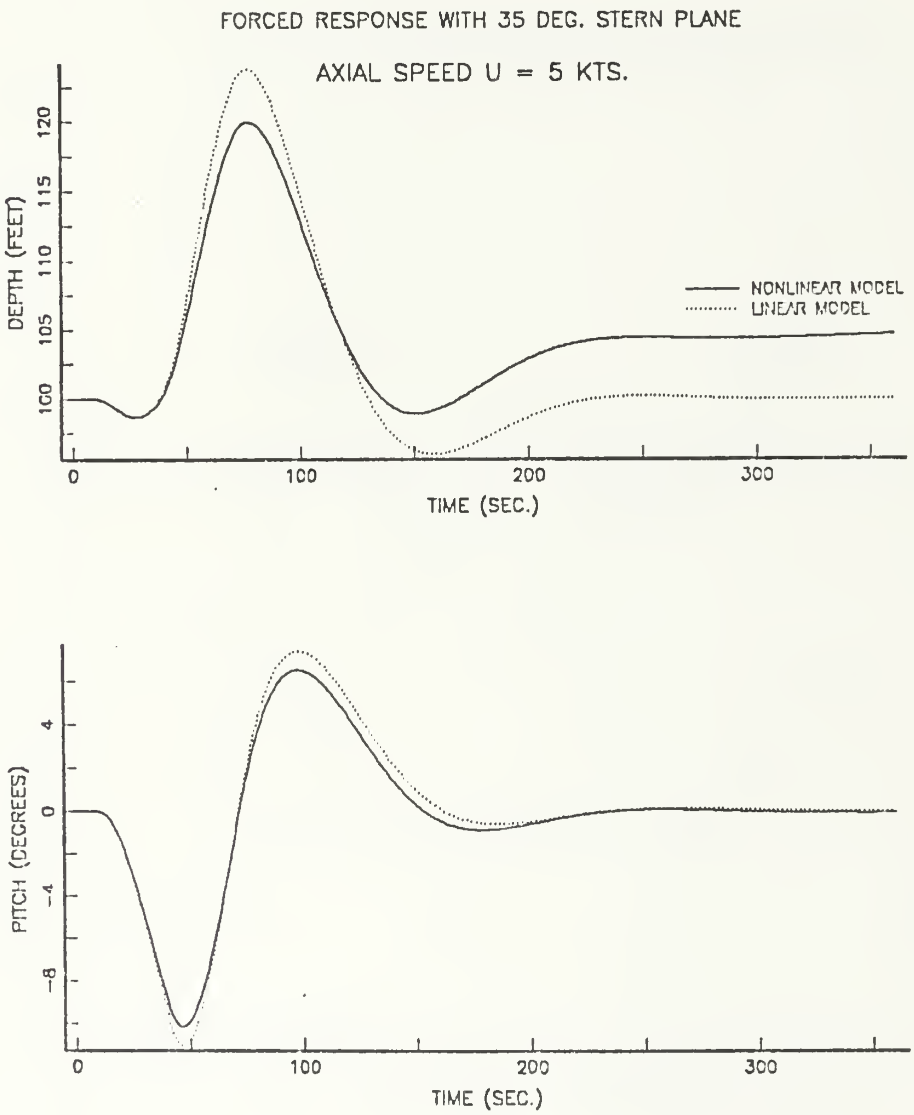

Figure 23. Forced Response. Stern Plane = 35 Deg. U = 5 Kts.

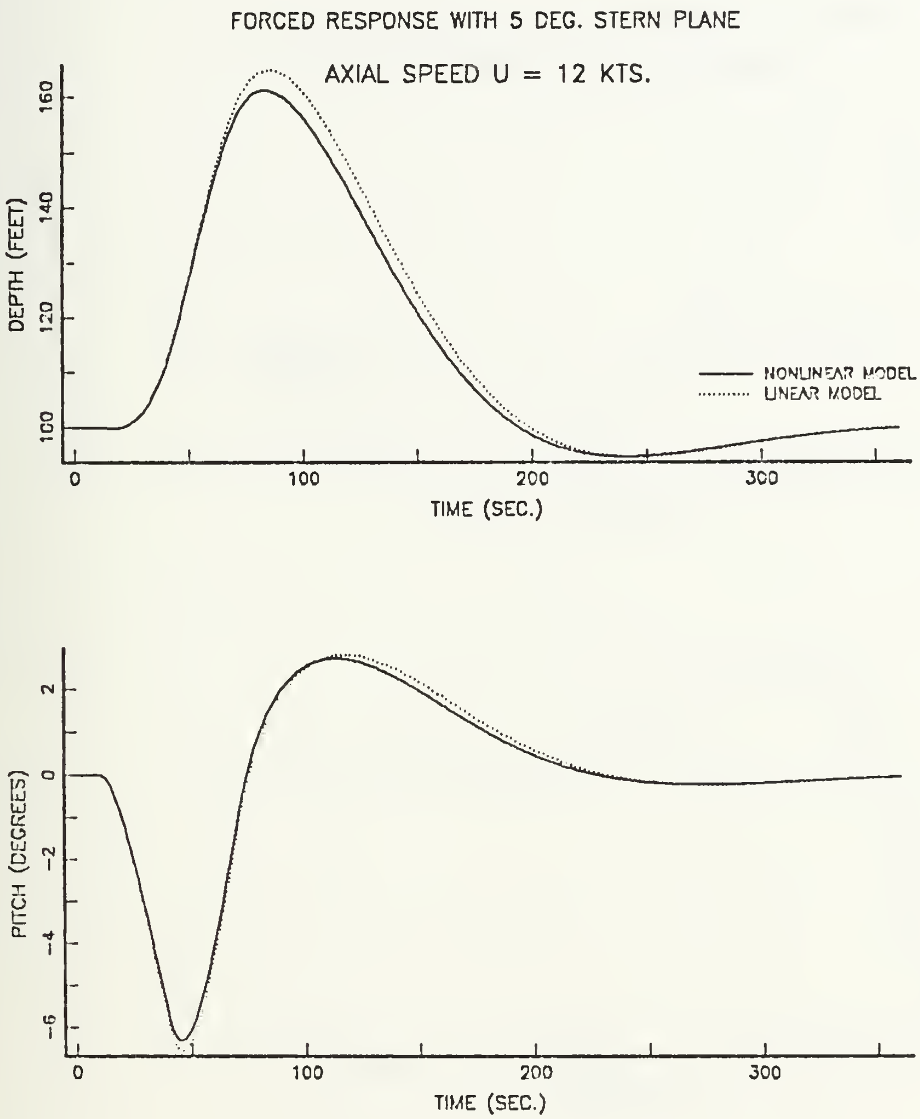

Figure 24. Forced Response. Stern Plane = 5 Deg. U = 12 Kts.

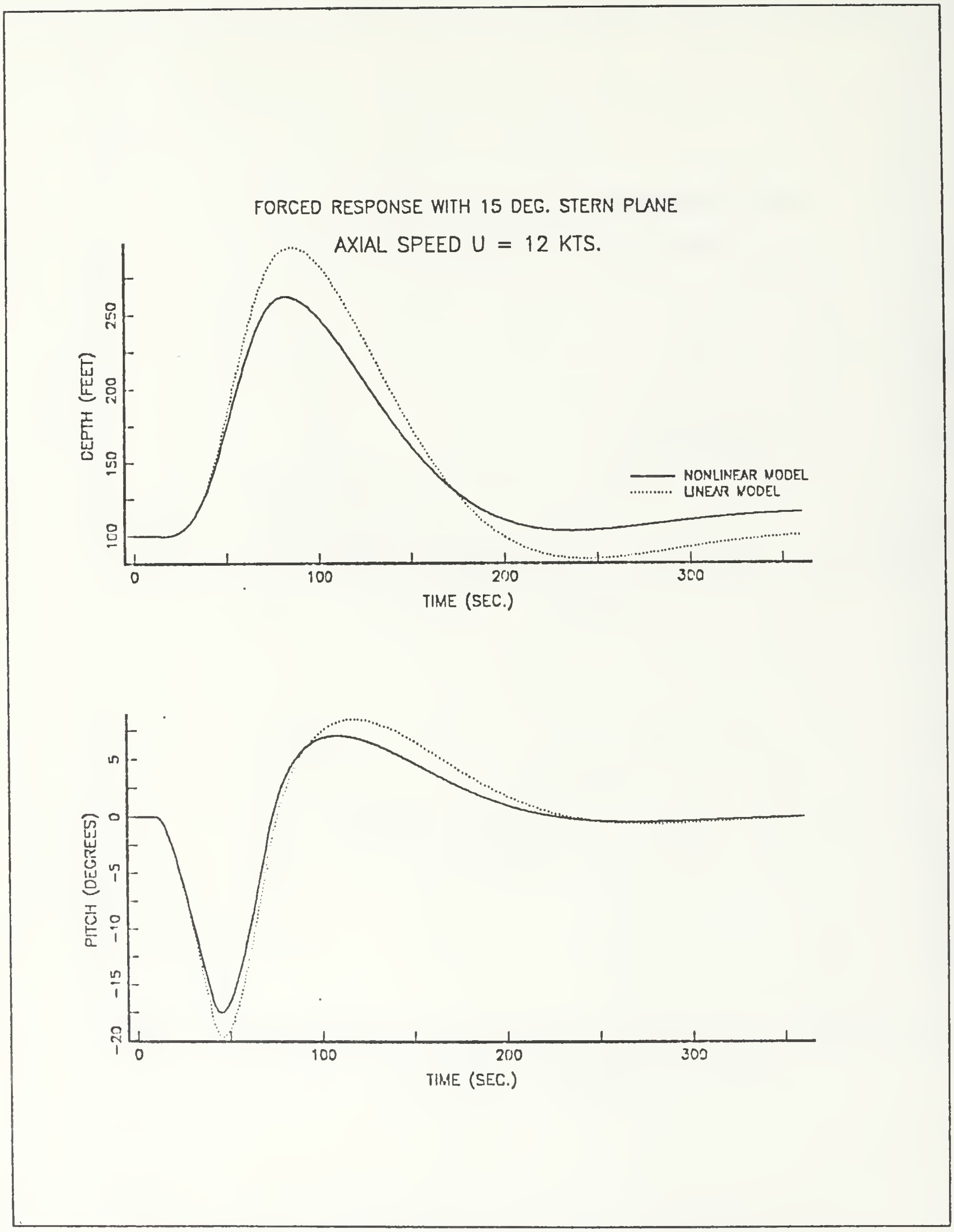

Figure 25. Forced Response. Stern Plane = 15 Deg. U = 12 Kts.

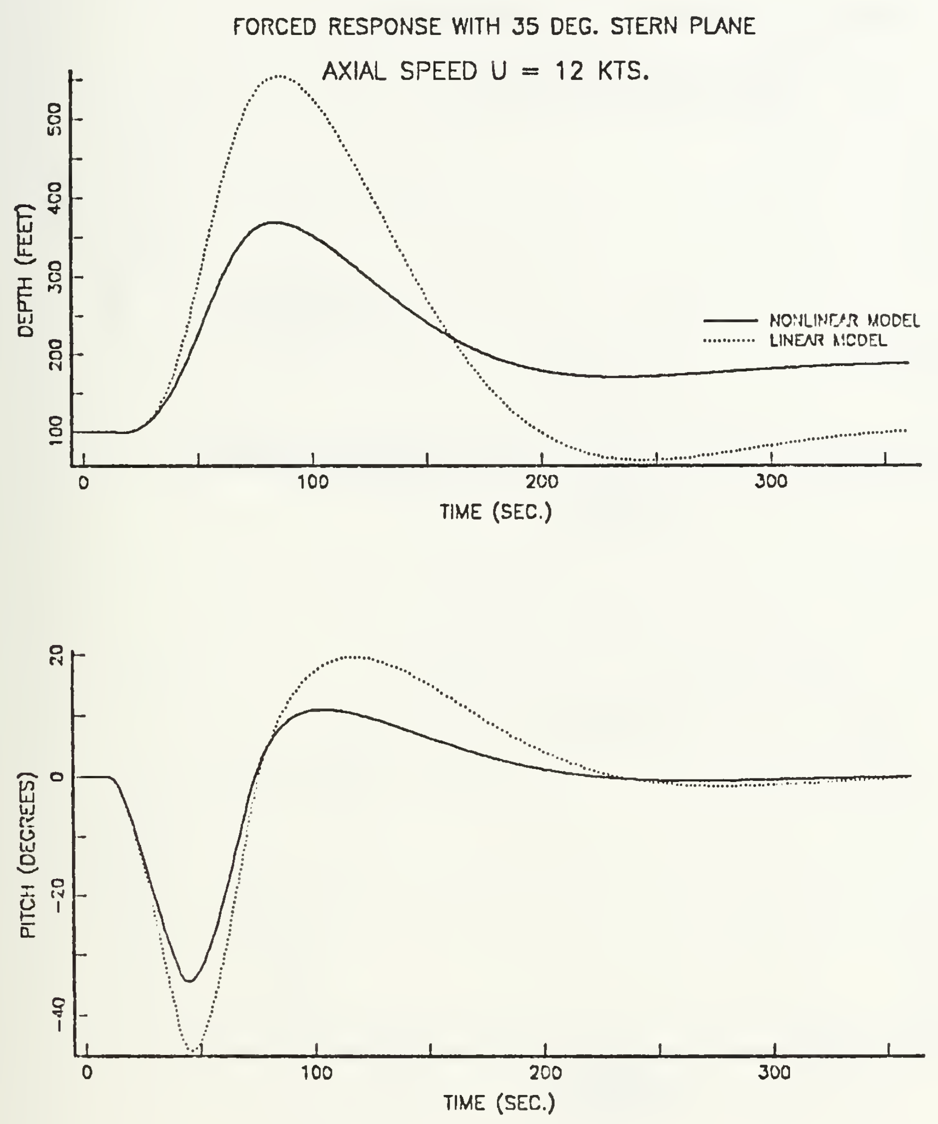

Figure 26. Forced Response. Stern Plane = 35 Deg. U = 12 Kts.

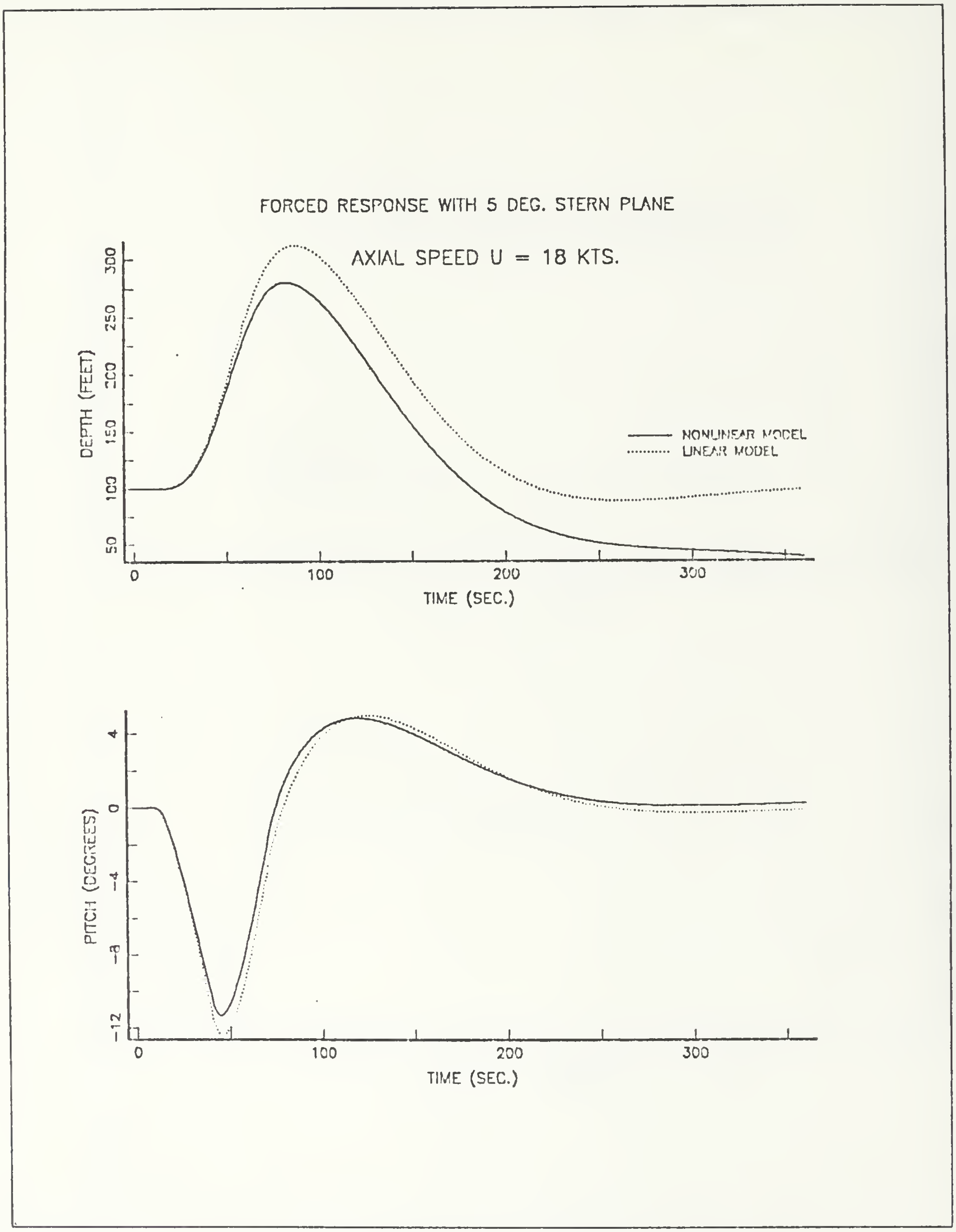

Figure 27. Forced Response. Stern Plane = 5 Deg. U = 18 Kts.

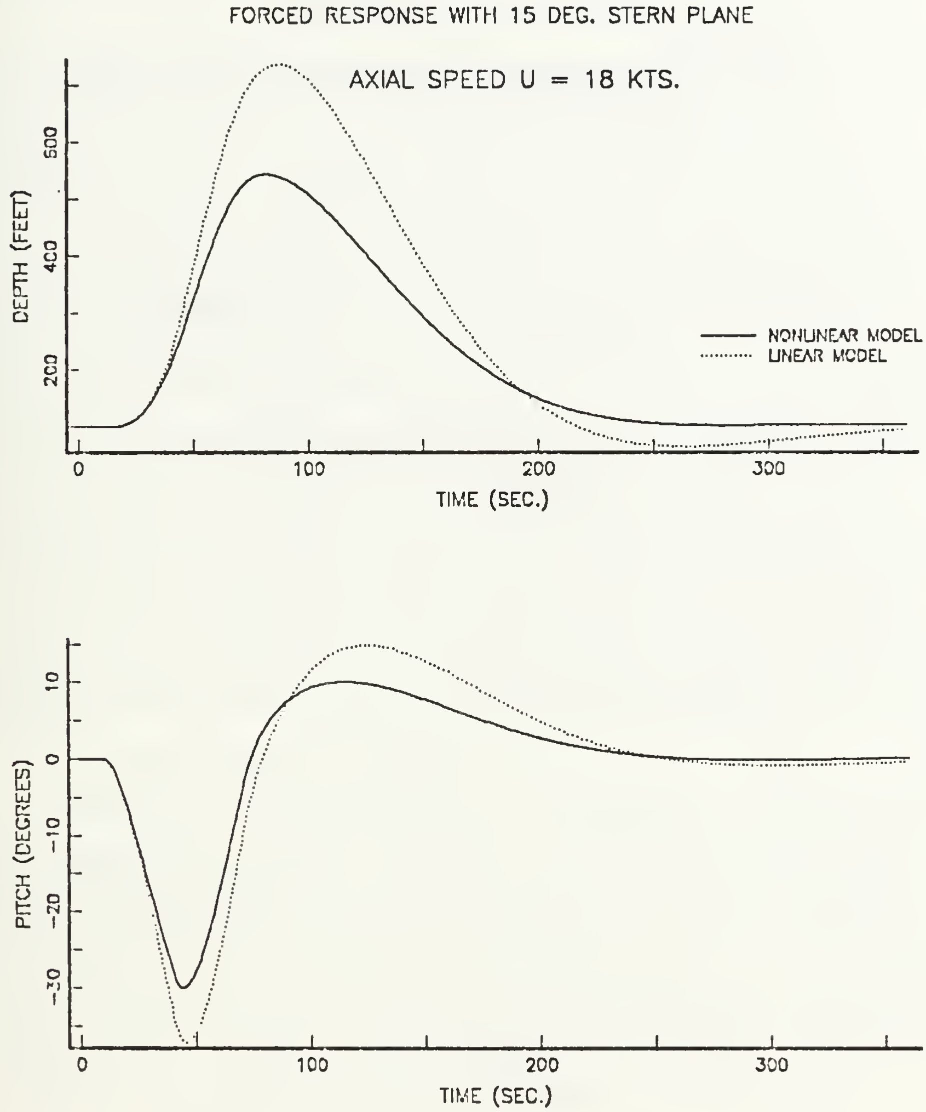

Figure 23. Forced Response. Stern Plane = 15 Deg. U = 18 Kts.

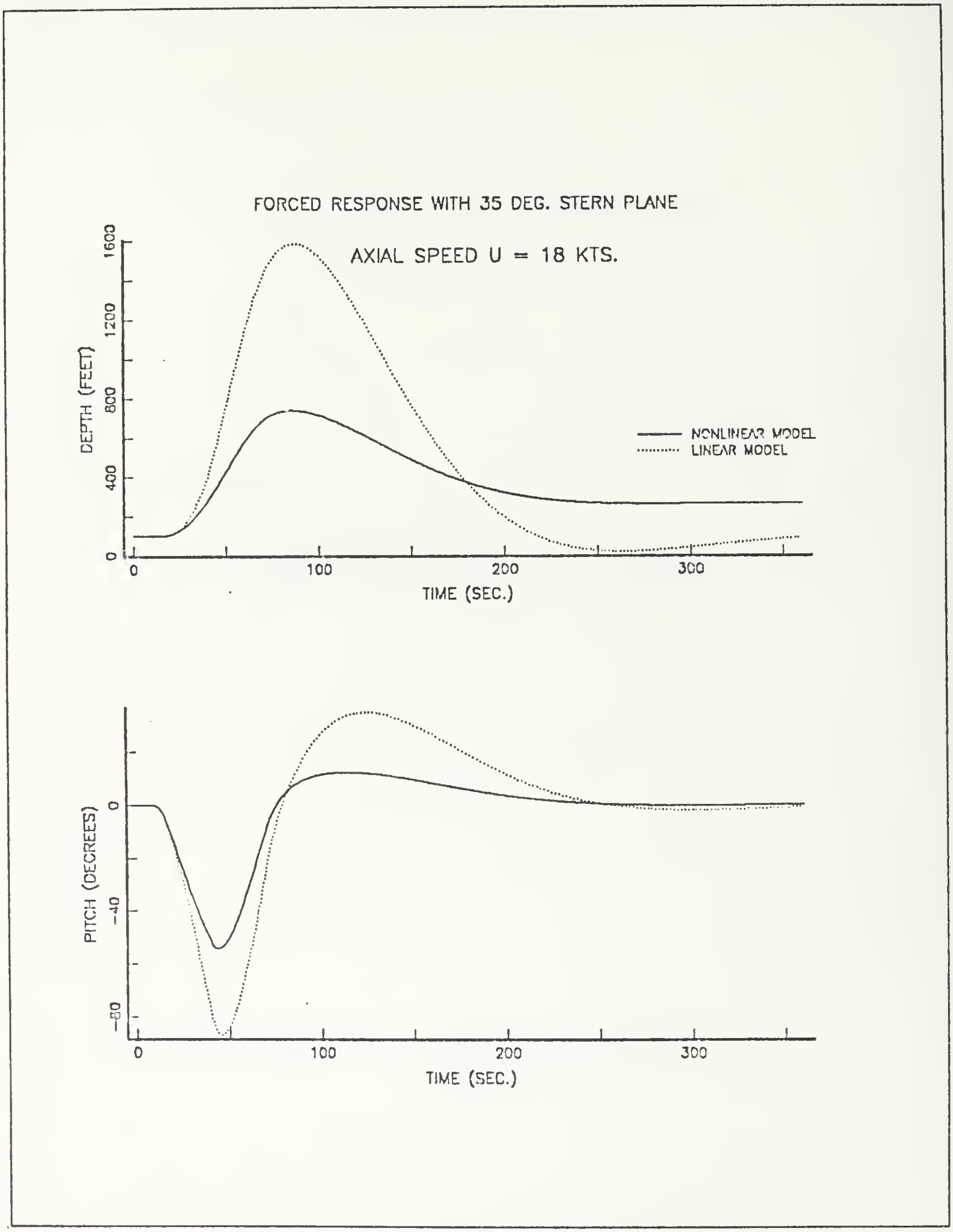

Figure 29. Forced Response. Stern Plane = 35 Deg. U = 18 Kts.

limitations. Figures were created but not supplied in this study since they are very similar to the preceeding results which were obtained using only stern planes. Deviations between models for this last set of runs are a little bit larger than the preceeding results. Using two sets of planes means more approximations for the linear model and greater deviations between linear and nonlinear models are expected.

Table 5. FORCED RESPONSE TO BOW AND STERN PLANES

Run No.	Speed (Kts.)	Bow & Stern Plane	Maximum Deviation In						Fig.
			Pitch		Z		Depth		
			Deg.	%	Ft./sec.	%	Feet	%	
27	5	5	0.0943	6.8	0.0012	1.1	0.0500	0.05	-
28	5	15	0.2448	6.0	0.0262	8.0	0.9030	0.8	-
29	5	35	1.3542	14.2	0.2891	37.7	8.0200	7.1	-
30	12	5	0.5151	8.4	0.1558	8.9	7.2100	4.4	-
31	12	15	3.6320	19.8	1.3423	25.6	53.000	18.3	-
32	12	35	16.329	38.2	7.1826	58.8	262.15	48.4	-
33	18	5	2.0117	16.8	1.0424	18.5	61.856	19.6	-
34	18	15	11.203	31.2	6.6167	39.1	267.52	35.8	-
35	18	35	43.214	51.6	27.816	70.4	1053.8	65.5	-

Obviously the linear model does not behave like the nonlinear model for large plane angles and high speeds. The most important reason for this is the constant speed assumption for the linear model. This assumption is no longer valid for large plane angles since planes reduce the forward speed of the actual submarine. Since the aim of this study is to validate the linear model for small perturbations, it is achieved for the vertical plane.

2. Validation of the Linear Model on the Horizontal Plane

A submarine behaves like a surface ship for most horizontal motions.There are some differences because of its submerged condition and sail structure. The main difference is in roll. A submarine rolls to inboard when a rudder angle is applied. Also the rudder has a squatting effect on the submarine which makes the submarine to pitch up and dive. Since the linear model assumes that there is no cross-coupling between vertical and horizontal motion it is not possible to compare the squatting effect with the linear model.

On the horizontal plane, roll and yaw angles and sway speed can be observed. Roll and yaw information are displayed on figures and tables for convenience. But the sway response is only supplied on tables as deviation between models.

a. Initial Condition Response

The simulations were carried out with a certain roll angle as initial condition. In order to see the small and large perturbations effects, 5 and 25 degrees initial roll angles were chosen and test runs were performed at 5, 8, 12, 18 and 25 Kts.

Since both models reach a steady state value after about 120 seconds, simulations up to 120 seconds were performed simultaneously for both the linear and nonlinear models. Maximum deviations for each run were obtained from data files and are given in Table 6.

Table 6. INITIAL CONDITION RESPONSE FOR HORIZONTAL PLANE

Run No.	Speed (Kts.)	Maximum Deviation In Roll				Figures
		Init. Roll=5 Deg.		Init. Roll=25 Deg.		
		Degree	%	Degree	%	
36-37	5	0.0972	1.9	3.2242	12.9	30-32
38-39	8	0.0677	1.3	2.0457	8.2	30-33
40-41	12	0.0365	0.7	1.3490	5.4	31-33
42-43	18	0.0342	0.7	1.1720	4.7	31-34
44-45	25	0.0252	0.5	1.0160	4.1	32-34

As can be seen from Figures 30, 31 and 32, it is obvious that there is almost no deviation on roll response for 5 degree initial roll angle. There are some slight deviations for 25 degree initial roll angle and unlike the vertical plane, deviations are decreasing with increasing axial speed. It is to be noted that the approximation by a linear model has not affected the period of rolling. Simulation results are given on Figures 32, 33 and 34 for 25 degree initial roll angle response.

Therefore it has been concluded that the linear model on horizontal plane is valid for small and large initial conditions.

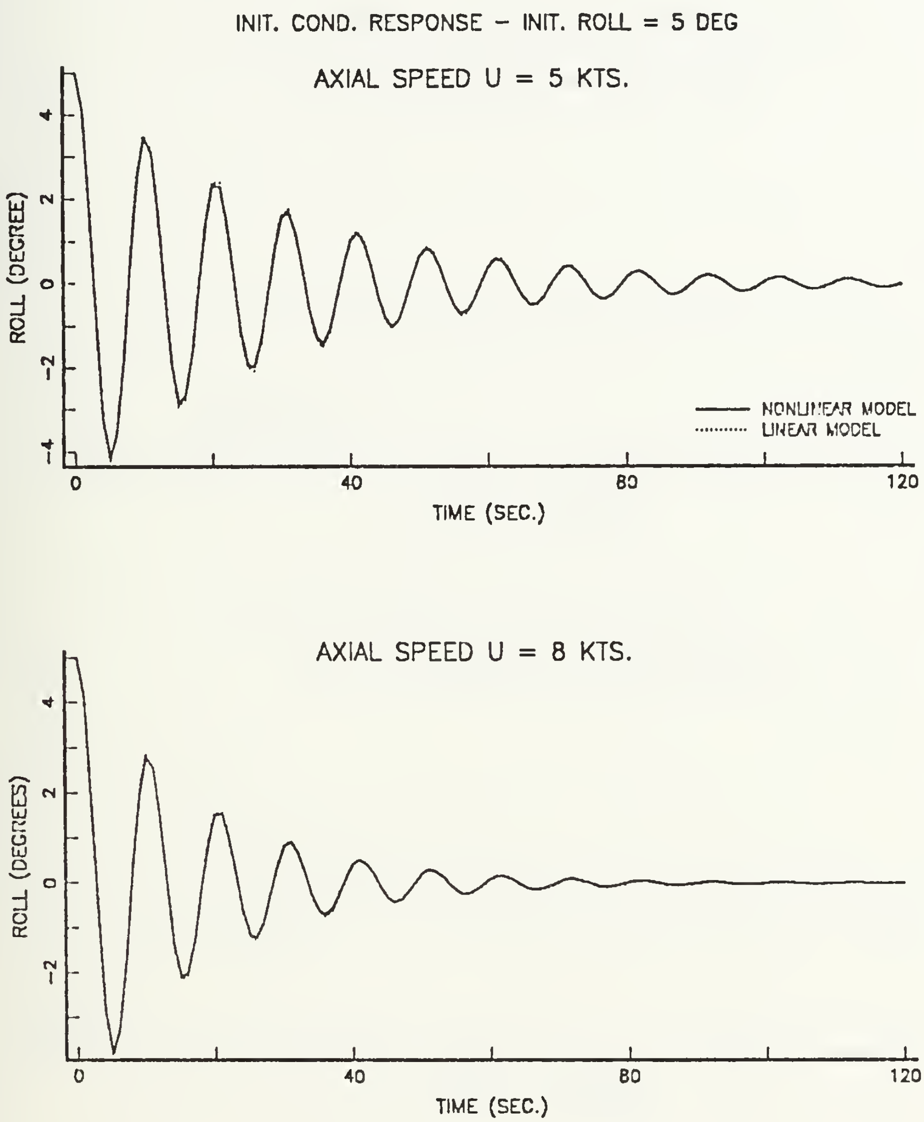

Figure 30. Initial Condition Response Init. Roll = 5 Deg. U = 5 and 8 Kts.

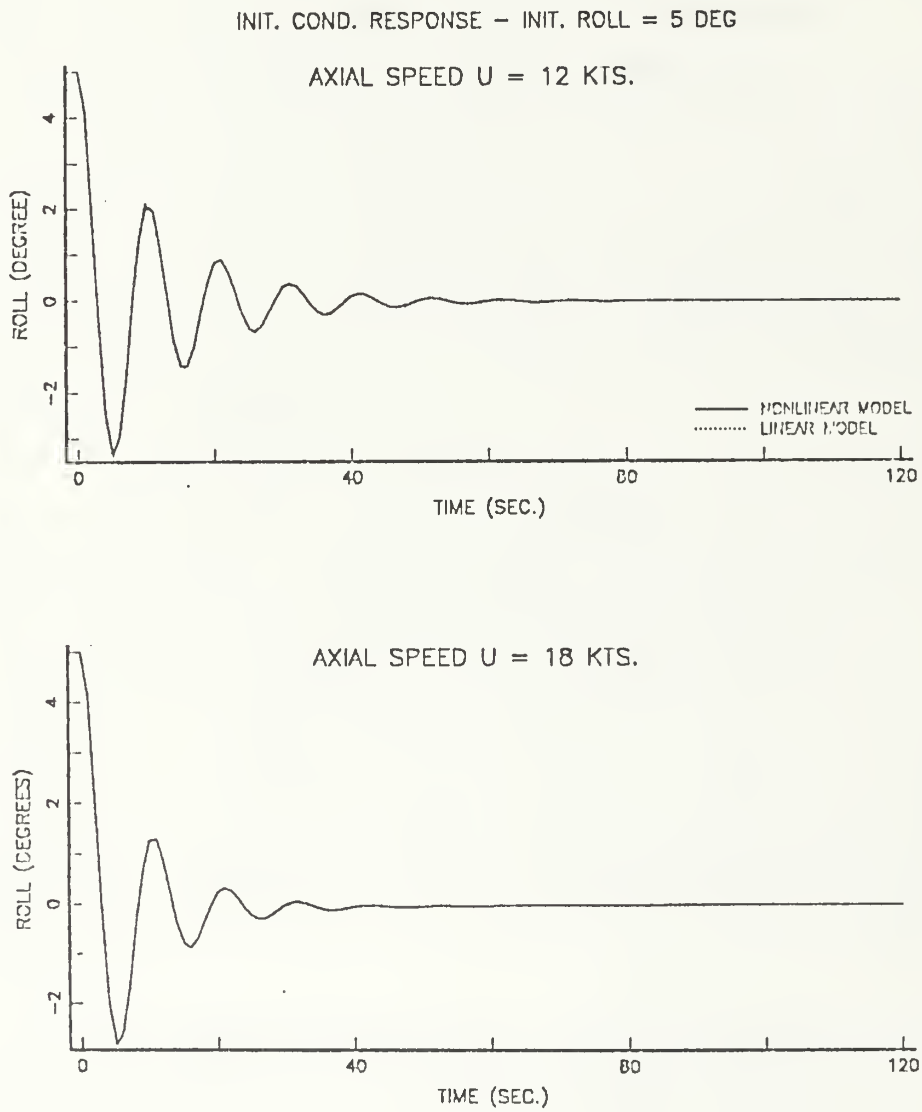

Figure 31. Initial Condition Response Init. Roll=5 Deg. U= 12 and 18 Kts.

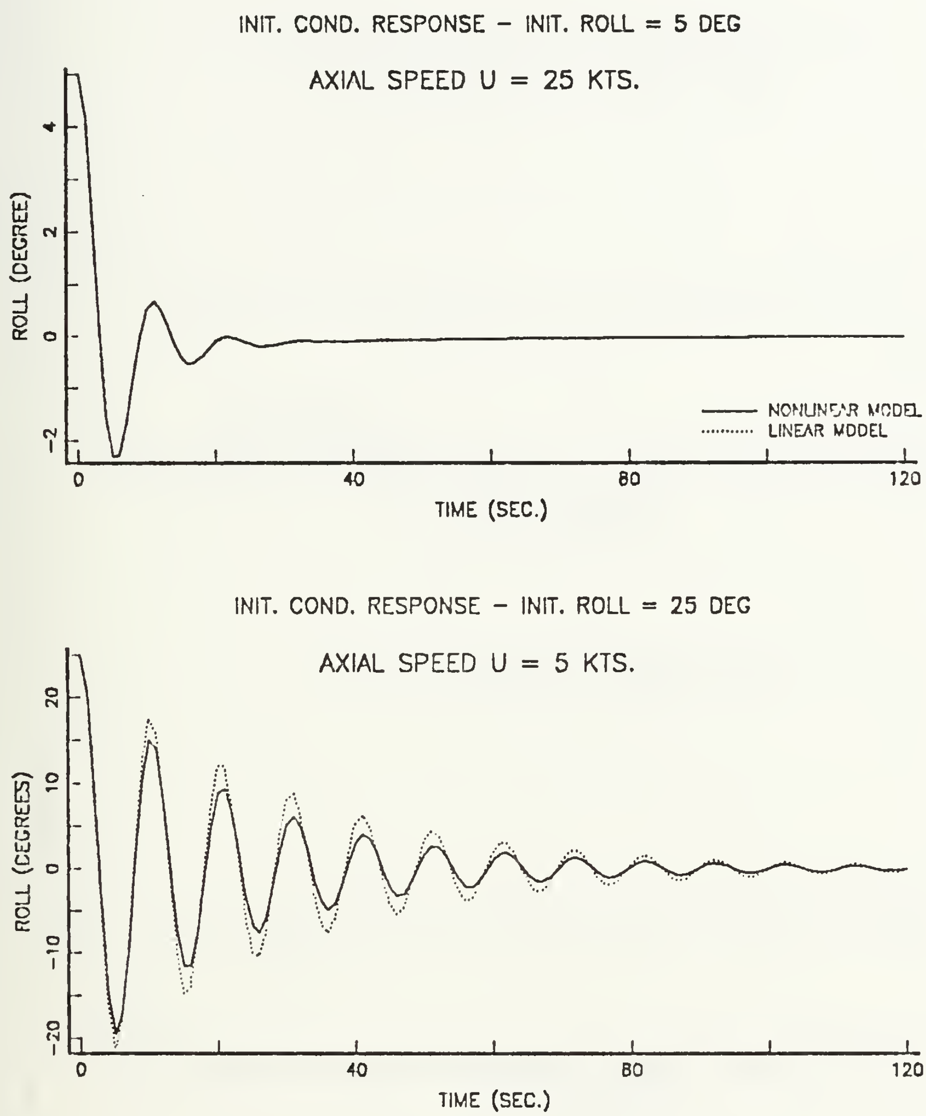

Figure 32. Init. Cond. Response Init. Roll = 5 and 25 Deg. U = 25 and 5 Kts.

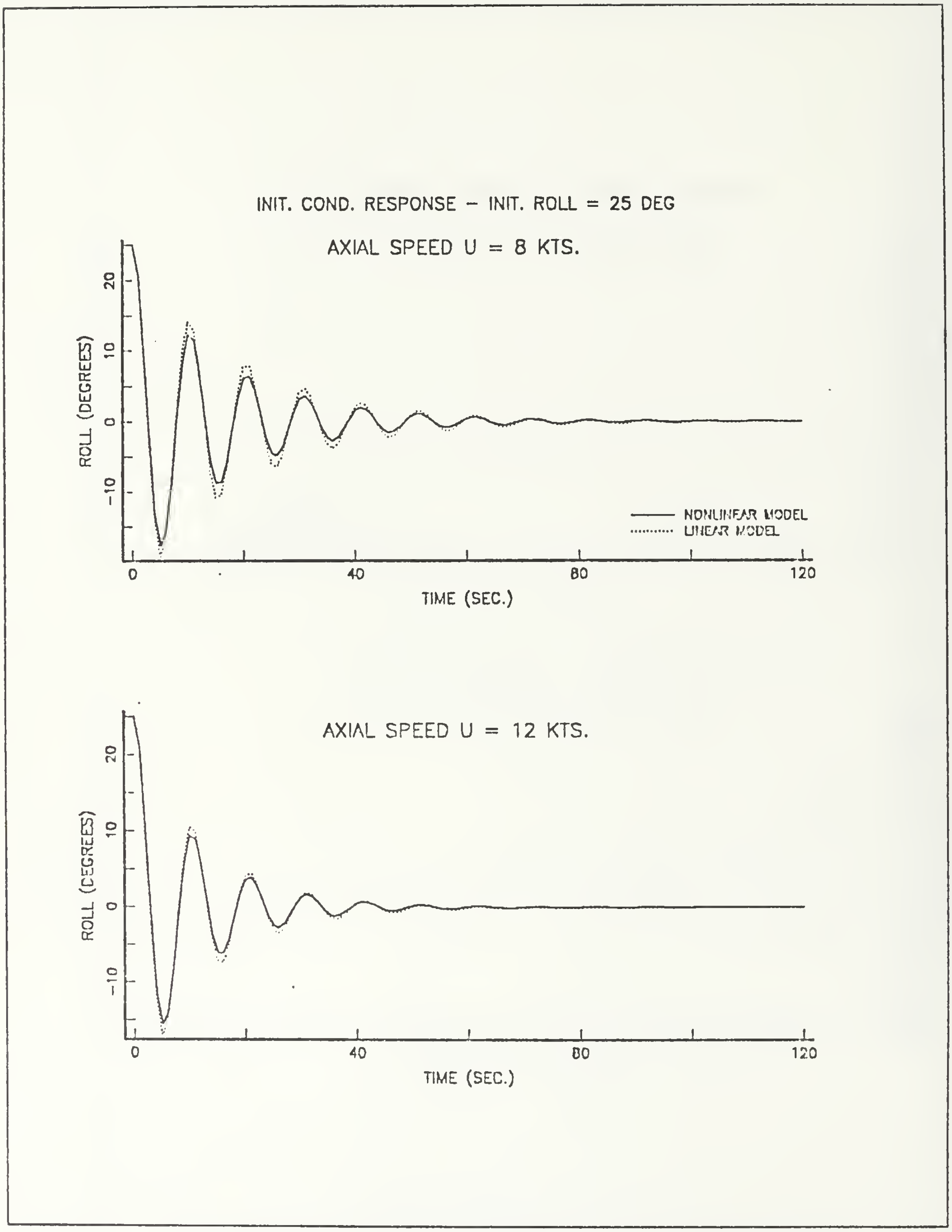

Figure 33. Initial Condition Response Init. Roll=25 Deg. U= 8 and 12 Kts.

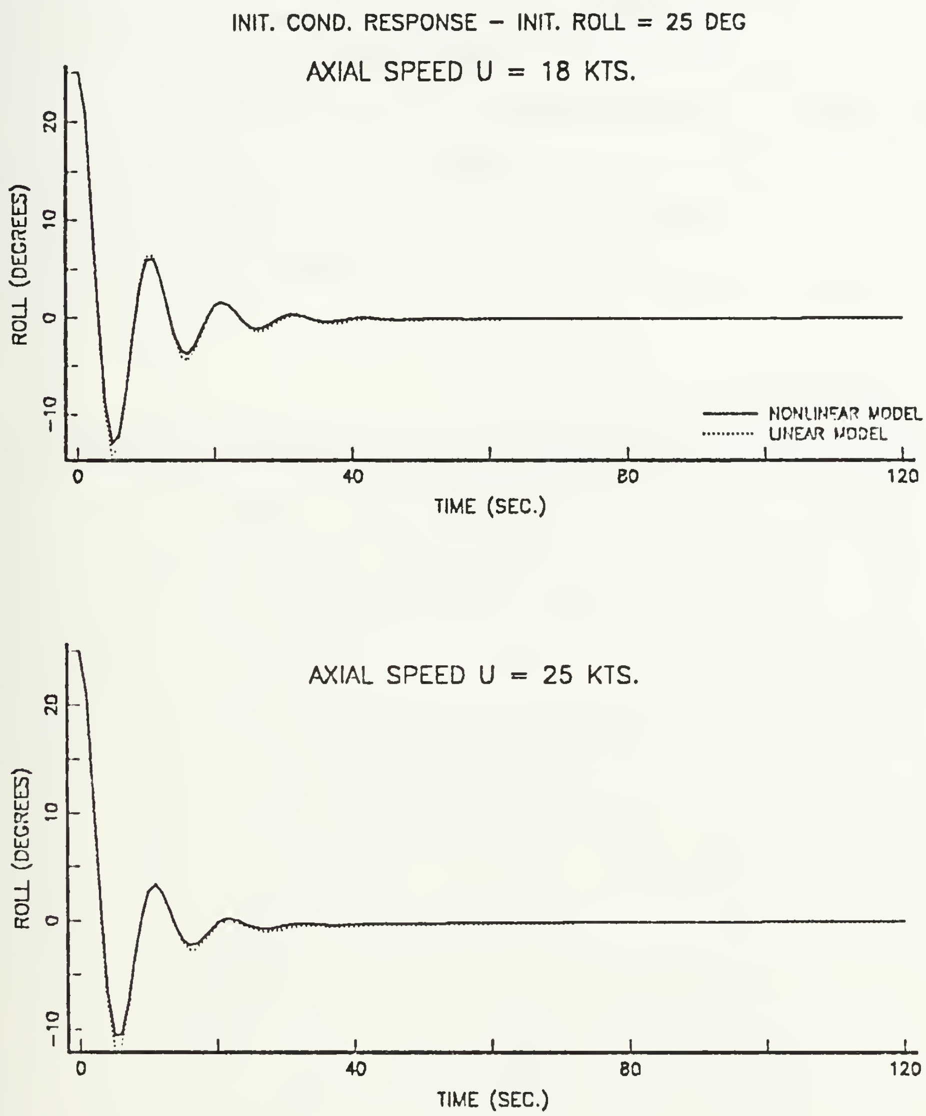

Figure 34. Initial Condition Response Init. Roll=25 Deg. U= 18 and 25 Kts.

b. *Forced Response*

The only relevant force beside propellers is created by the rudder on horizontal plane. The rudder also has an appreciable effect on vertical motion which is called the squatting effect. Even though the linear model assumes that there is no cross-coupling effect between vertical and horizontal motion, it was decided to display depth and pitch angle changes which were obtained by non-linear simulation for further study. Cross-coupling effects which are obtained by non-linear simulation at different speeds. are given in Figures 38, 40, 42, 44, 46, 48 and 50.

Simulation runs are abtained for three different speeds and rudder angles for this case. Plots for yaw and roll response are given in Figures 35-37, 39, 41, 43, 45, 47 and 49. Maximum deviations are given on Table 7. Again similar deviation behaviors can be observed as vertical motion.

Table 7. FORCED RESPONSE TO RUDDER

Run No.	Speed (Kts.)	Rudder (Deg.)	Maximum Deviation In						Fig.
			V		Yaw		Roll		
			Ft/sec.	%	Deg.	%	Deg.	%	
46	5	5	0.0060	3.6	0.3775	13.7	0.0041	3.0	35
47	5	15	0.0487	9.8	2.0360	24.7	0.0518	12.7	36
48	5	35	0.2413	20.8	3.3520	17.4	0.3633	38.3	37
49	12	5	0.1417	14.9	2.8058	23.8	0.2880	18.6	38
50	12	15	0.9515	33.6	13.27	37.5	2.6034	56.0	39
51	12	35	3.7804	57.2	42.223	51.1	10.345	95.4	40
52	18	5	.0.5104	25.3	6.97	30.3	1.6658	35.0	41
53	18	15	3.0963	51.2	35.065	50.8	11.262	78.9	42
54	18	35	10.424	73.8	106.83	66.4	34.525	103.6	43

As a result of this chapter it has been concluded that approximation by linear model is valid for small perturbations at all speeds for both motions. In addition. it has been observed that the linearized model is still valid for large perturbations applied over a short period of time.

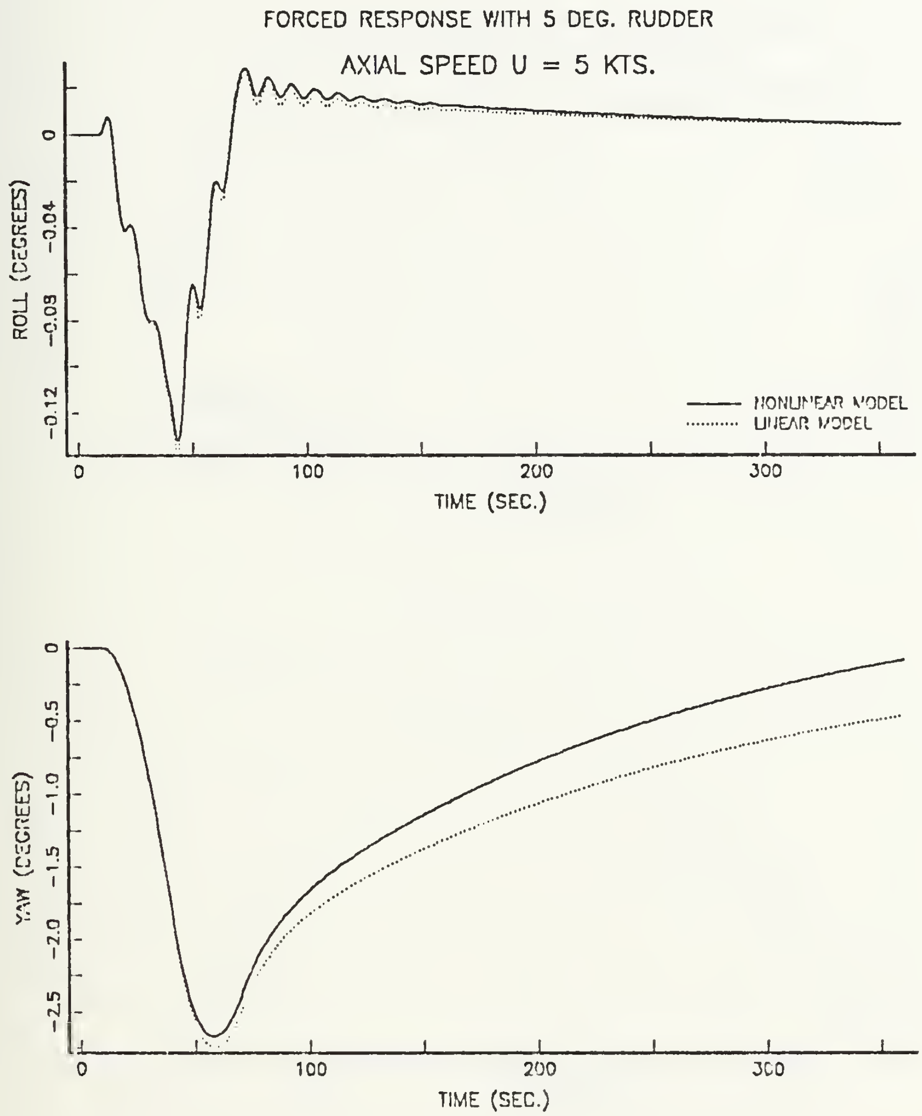

Figure 35. Forced Response. Rudder = 5 Deg. U = 5 Kts.

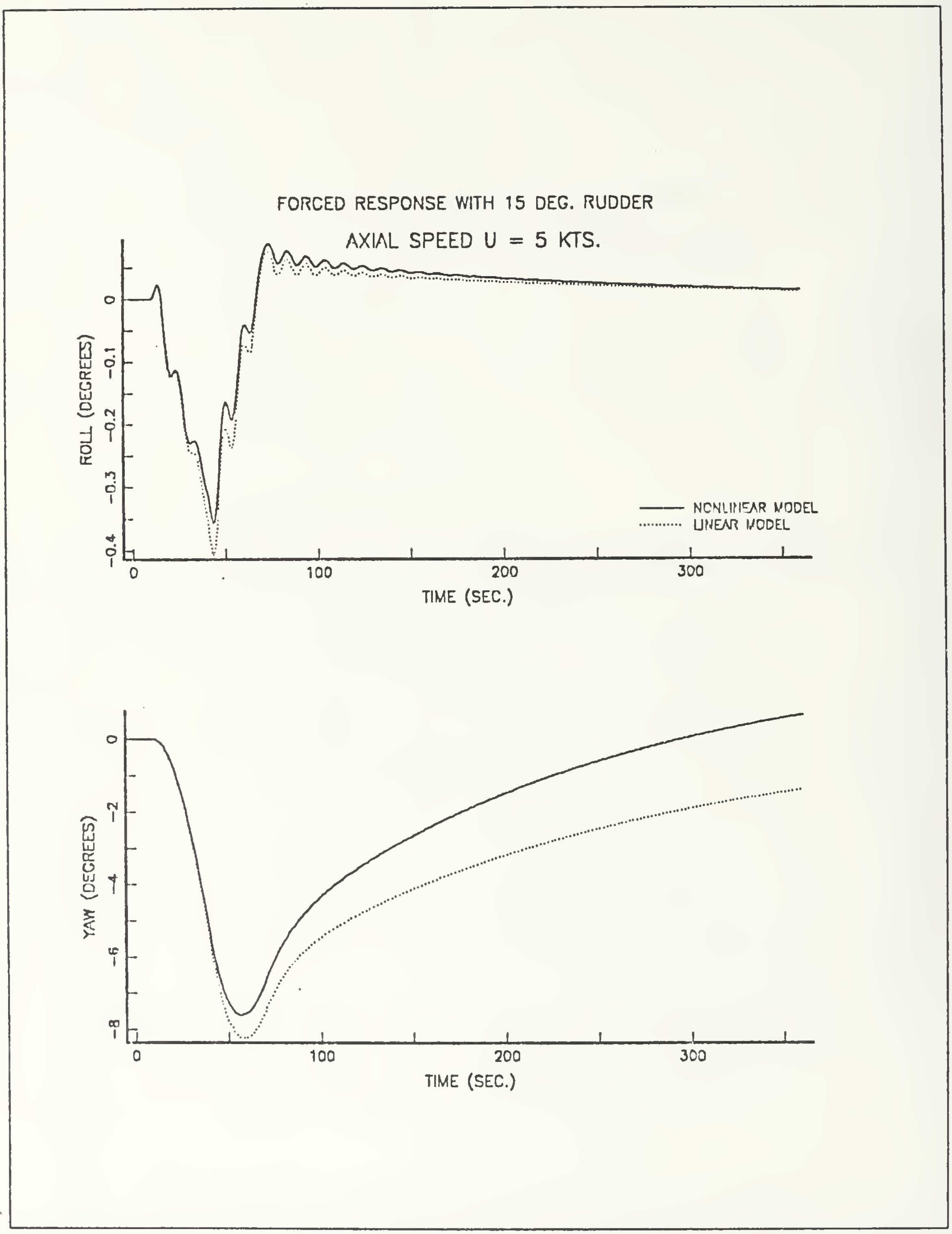

Figure 36. Forced Response. Rudder = 15 Deg. U = 5 Kts.

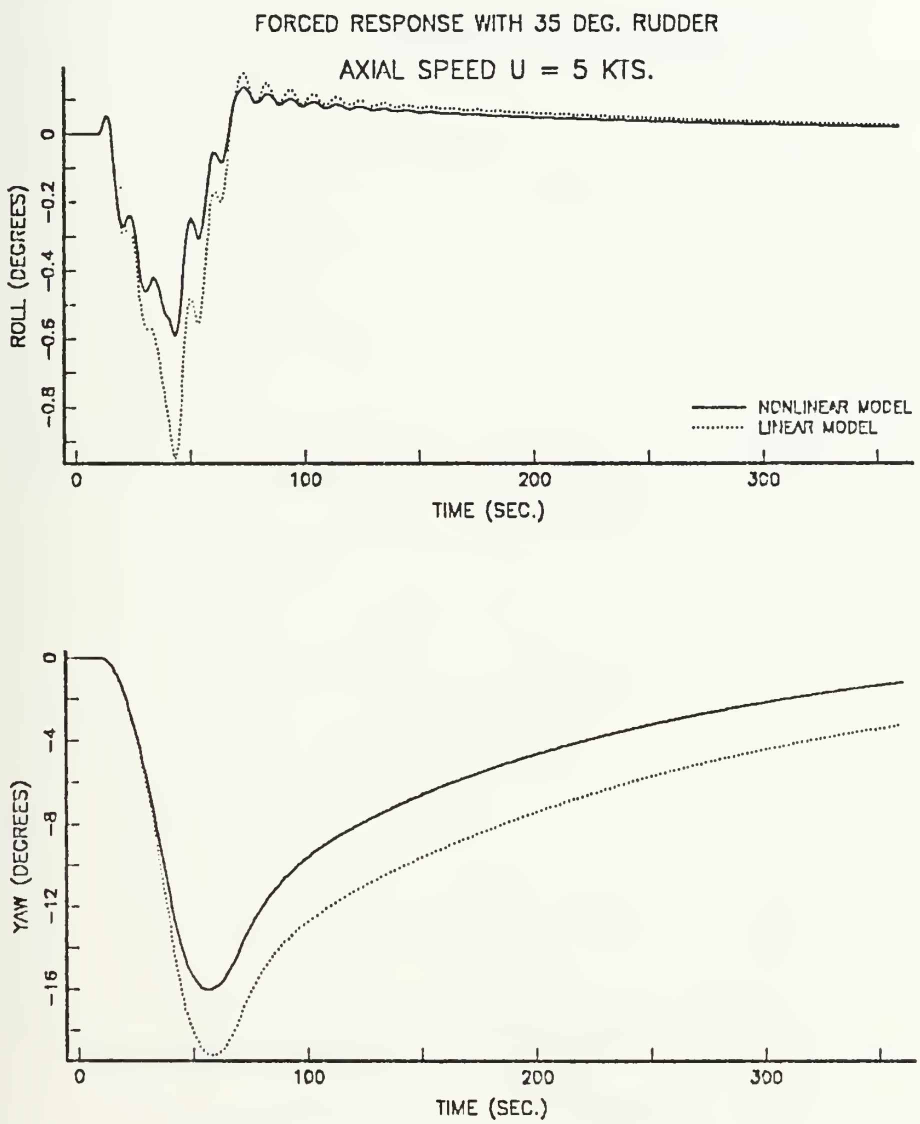

Figure 37. Forced Response. Rudder = 35 Deg. U = 5 Kts.

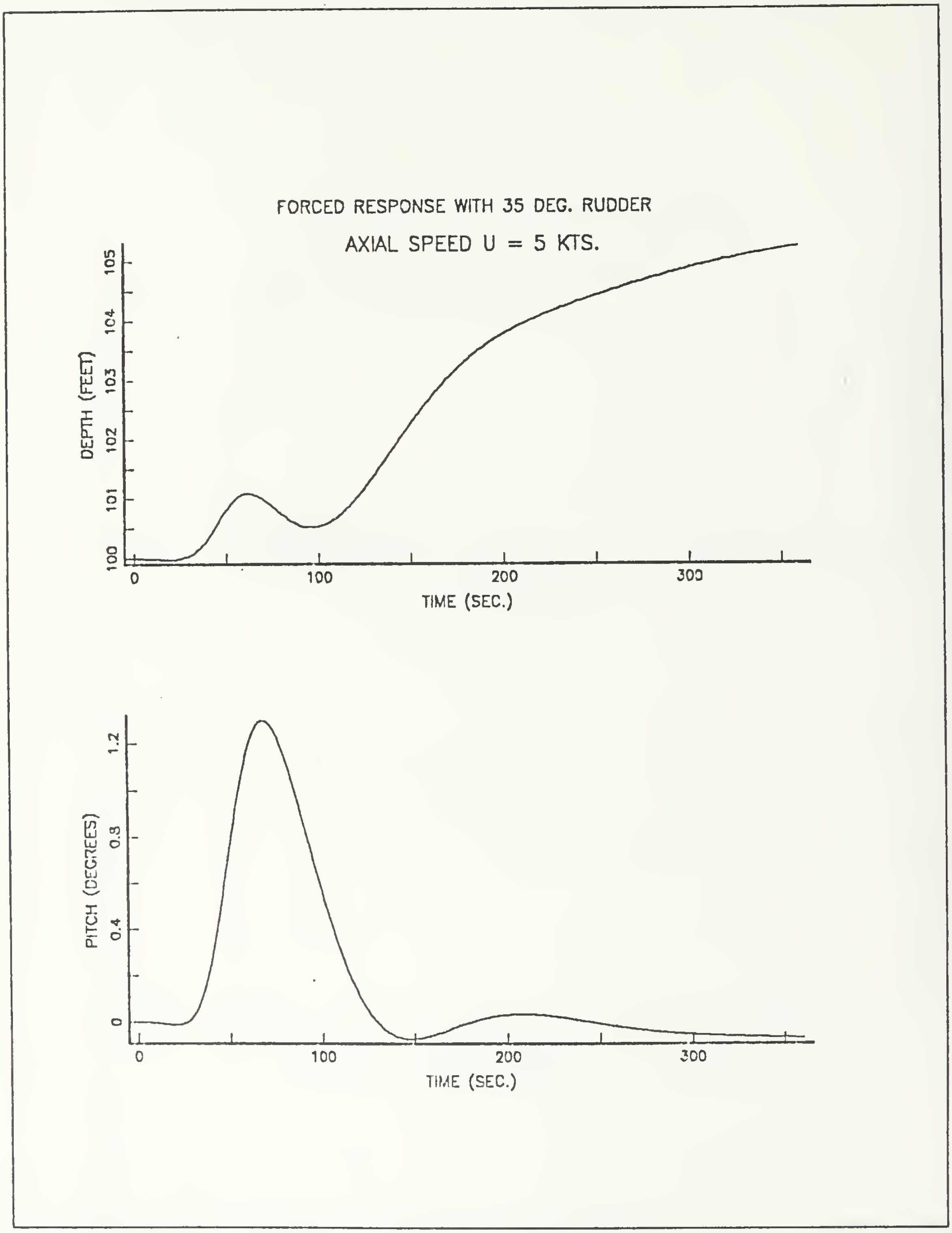

Figure 38. Cross-Coupling Effect on Vertical Plane. Rudder = 35 Deg. U = 5 Kts.

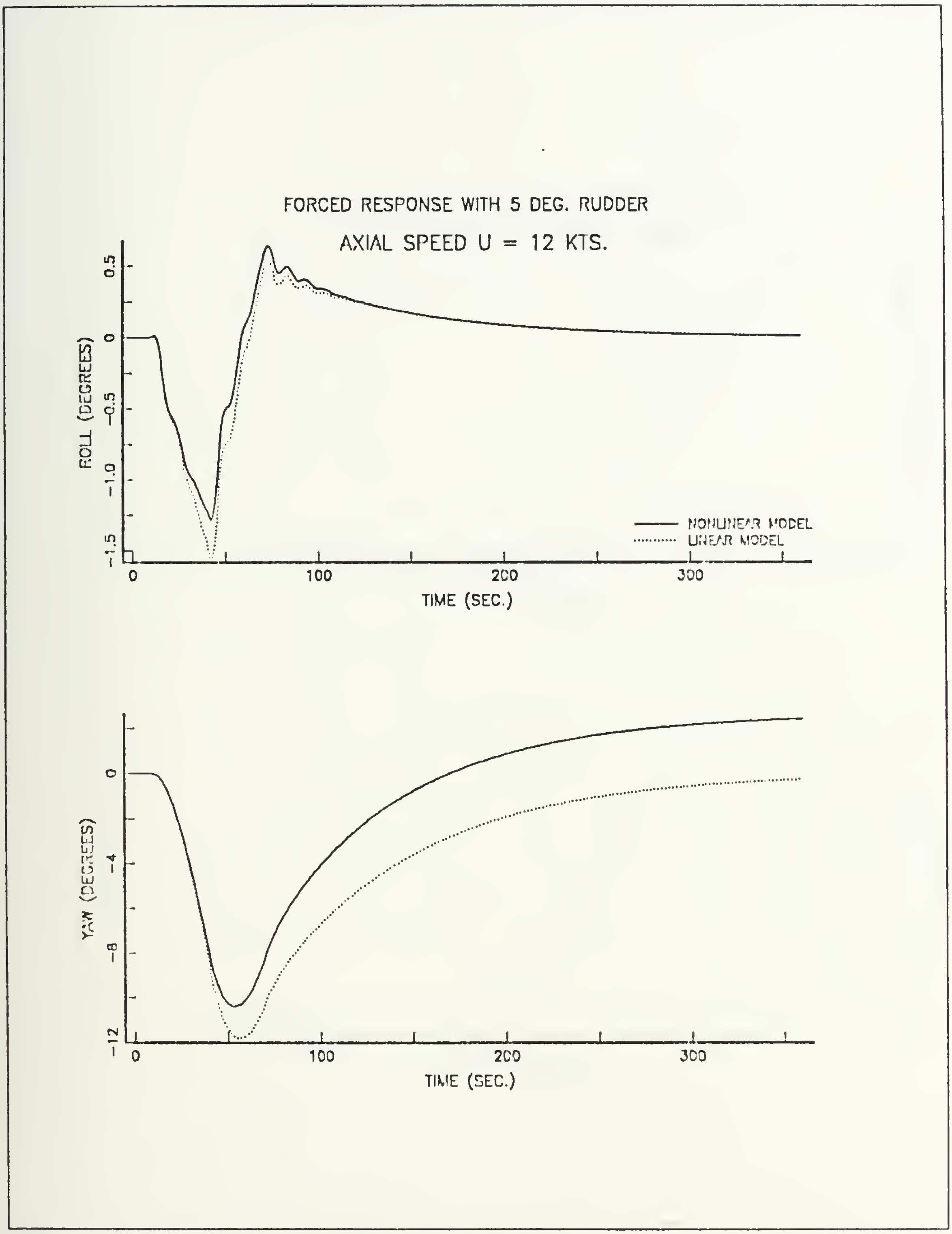

Figure 39. Forced Response. Rudder = 5 Deg. U = 12 Kts.

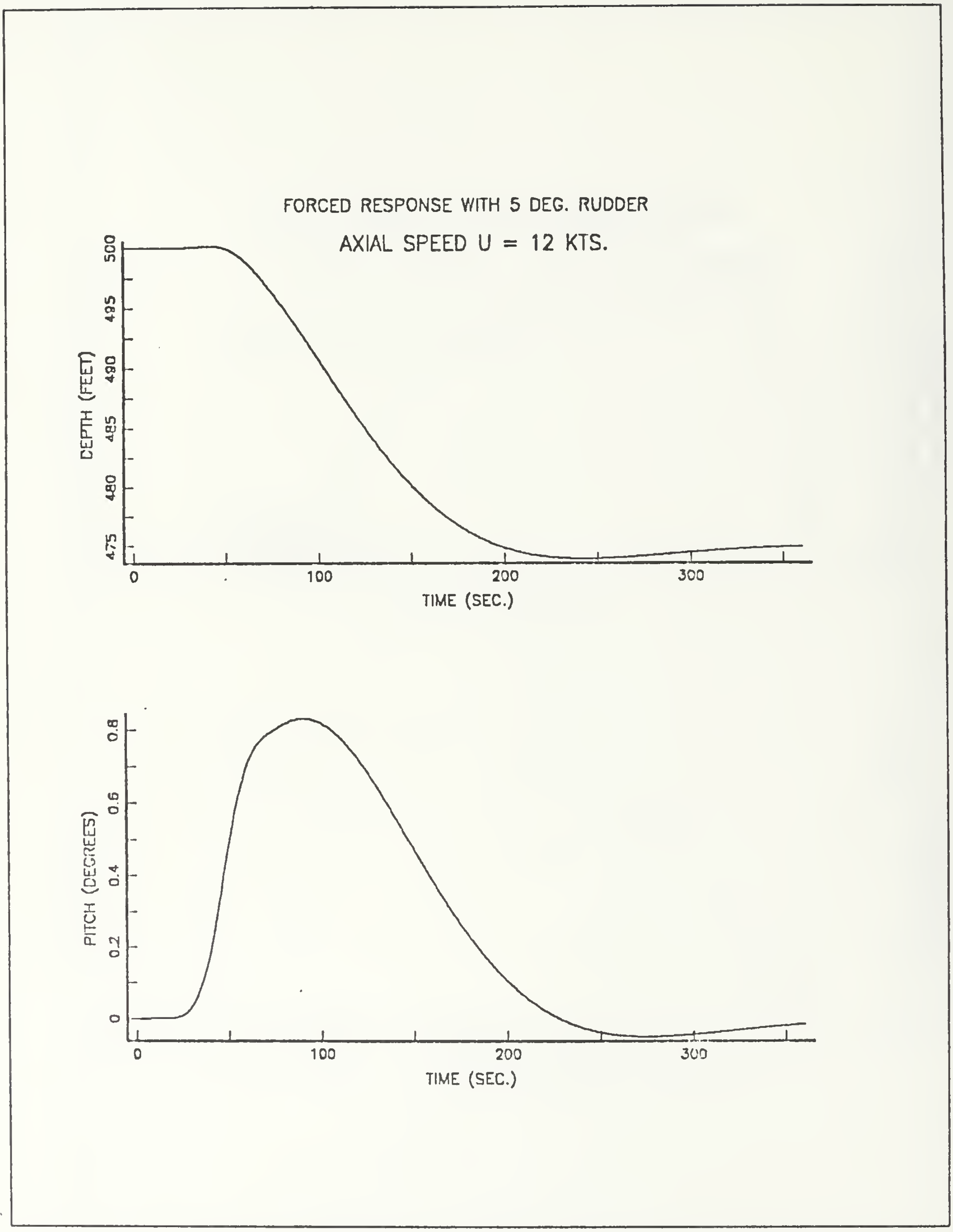

Figure 40. Cross-Coupling Effect on Vertical Plane. Rudder = 5 Deg. U = 12 Kts.

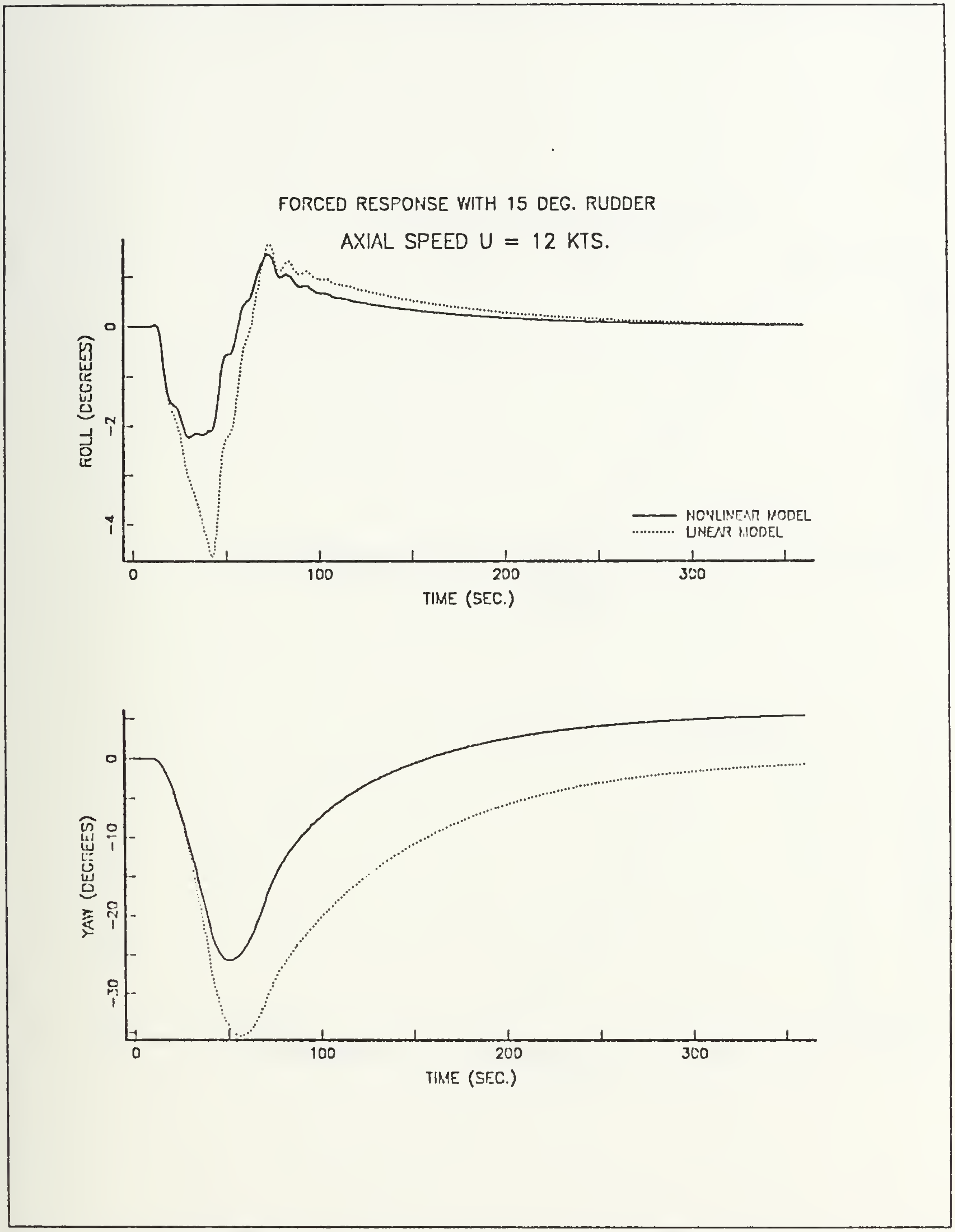

Figure 41. Forced Response. Rudder = 15 Deg. down. U = 12 Kts.

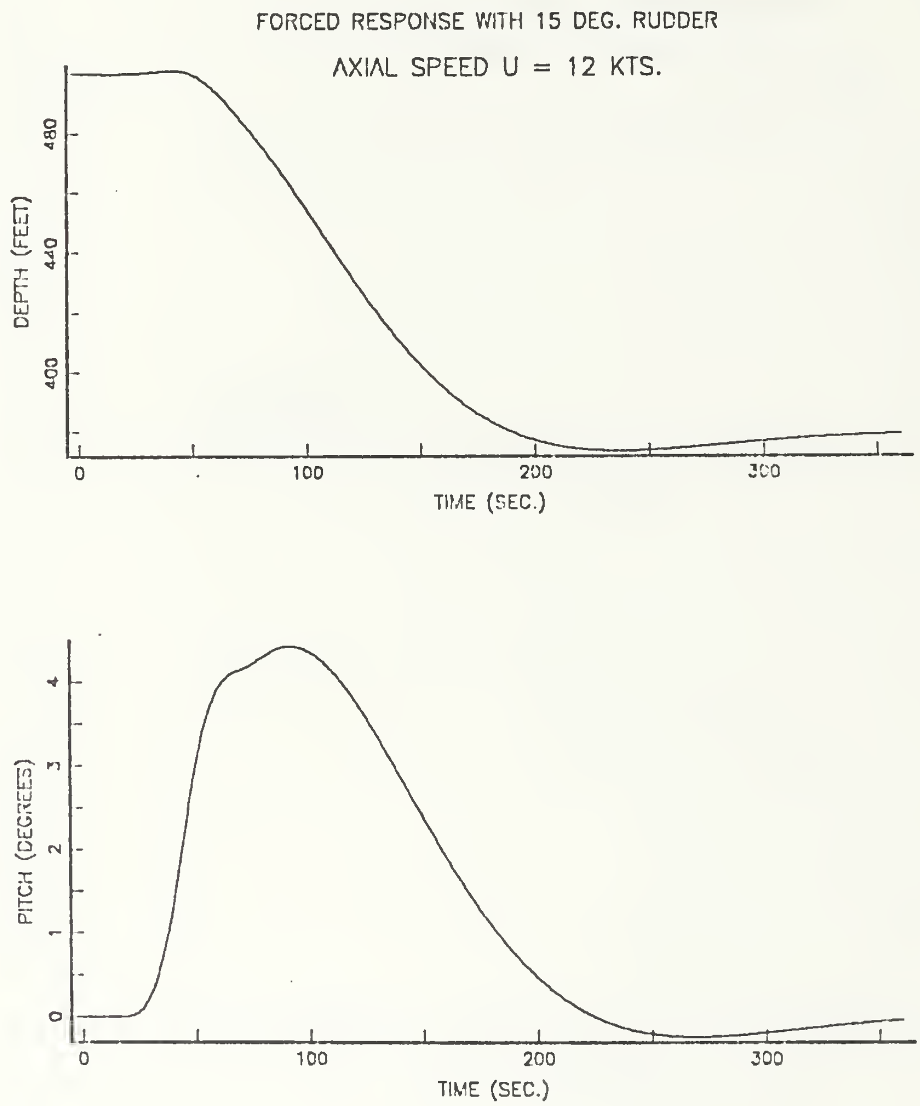

Figure 42. Cross-Coupling Effect on Vertical Plane. Rudder = 15 Deg. U = 12 Kts.

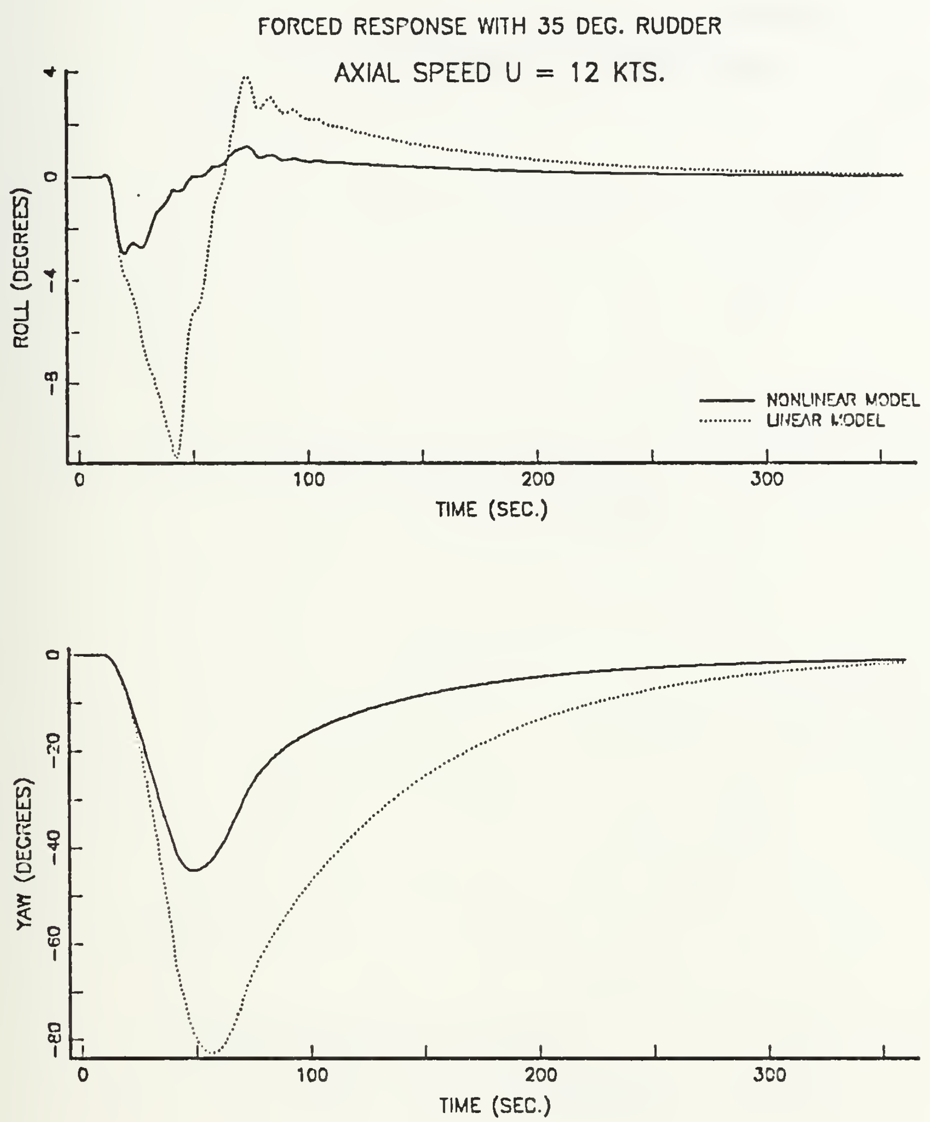

Figure 43. Forced Response. Rudder = 35 Deg. U = 12 Kts.

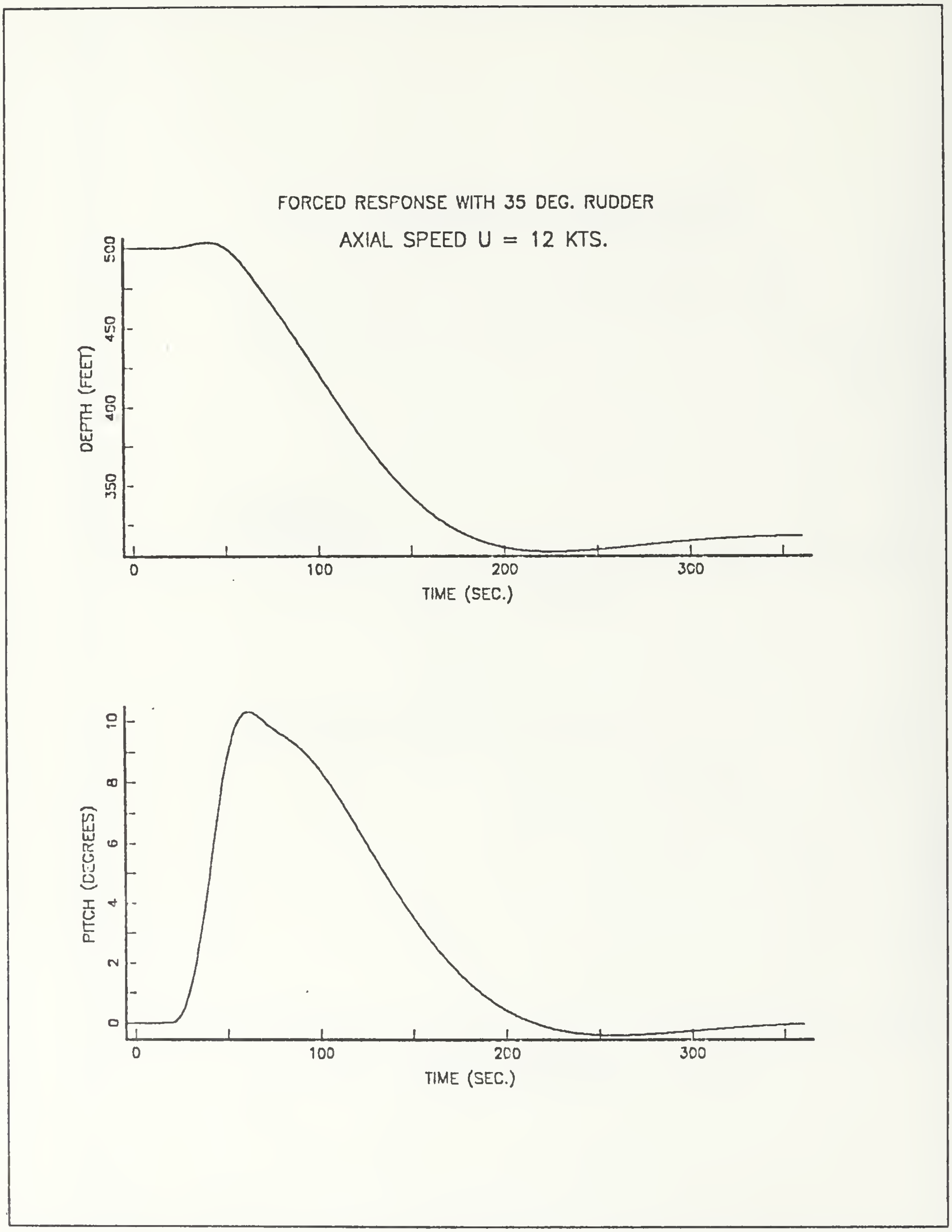

Figure 44. Cross-Coupling Effect on Vertical Plane. Rudder = 35 Deg. U = 12 Kts.

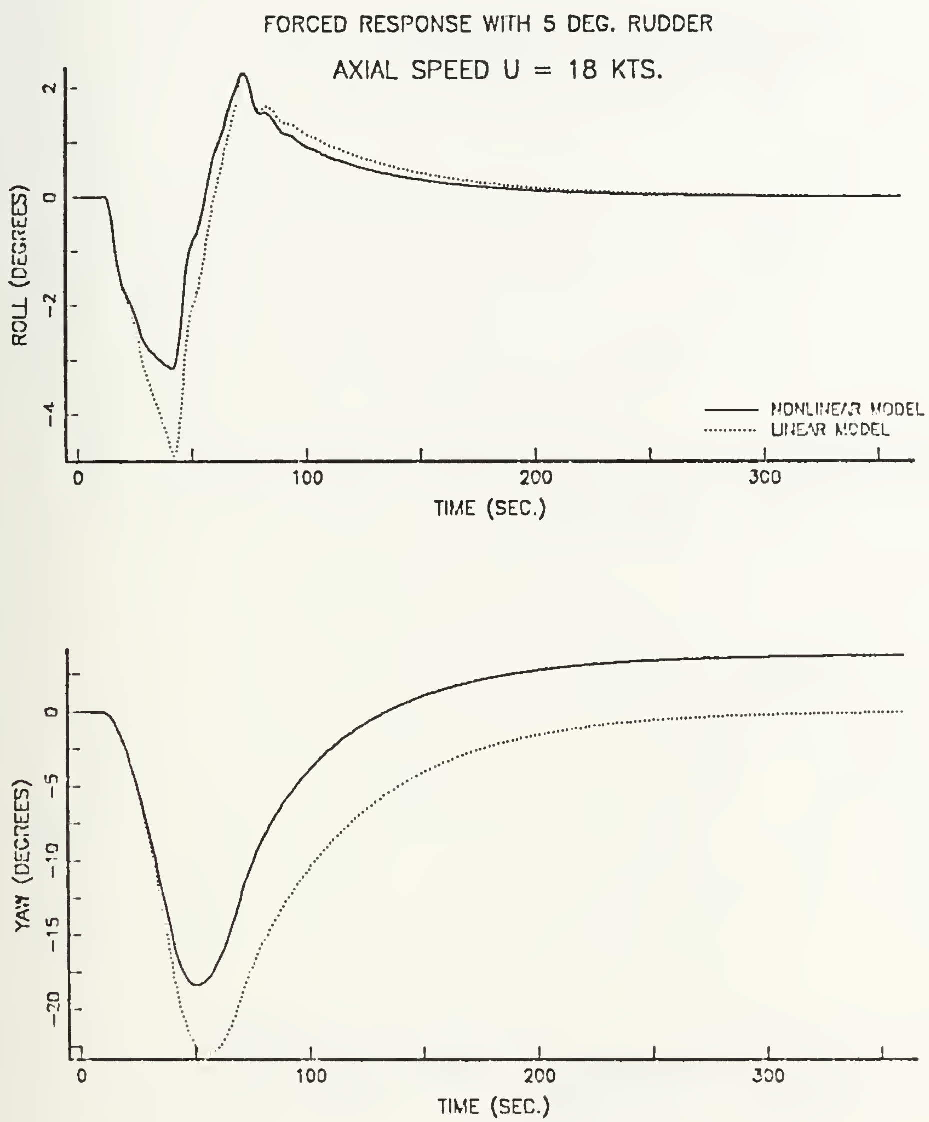

Figure 45. Forced Response. Rudder = 5 Deg. U = 18 Kts.

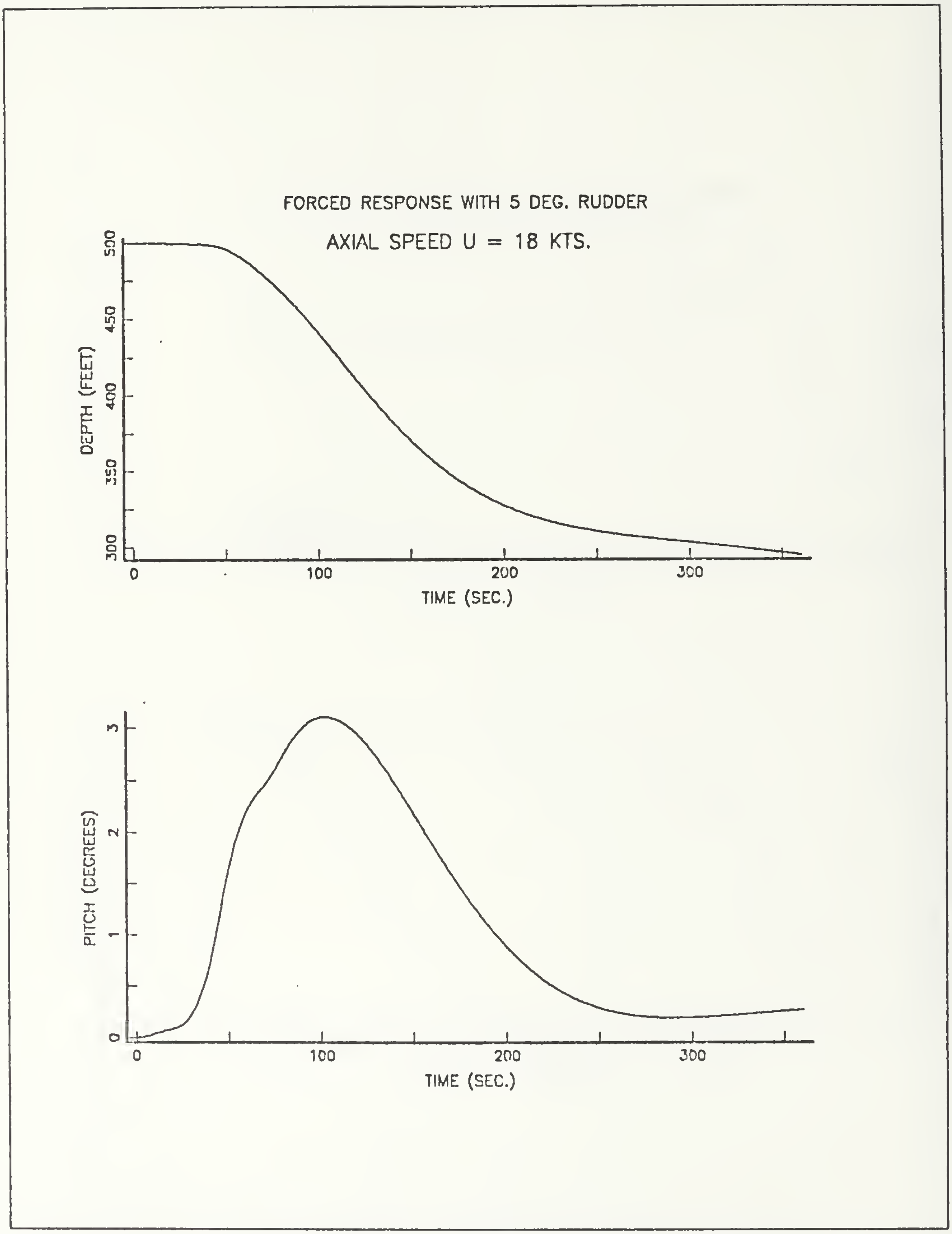

Figure 46. Cross-Coupling Effect on Vertical Plane. Rudder = 5 Deg. U = 18 Kts.

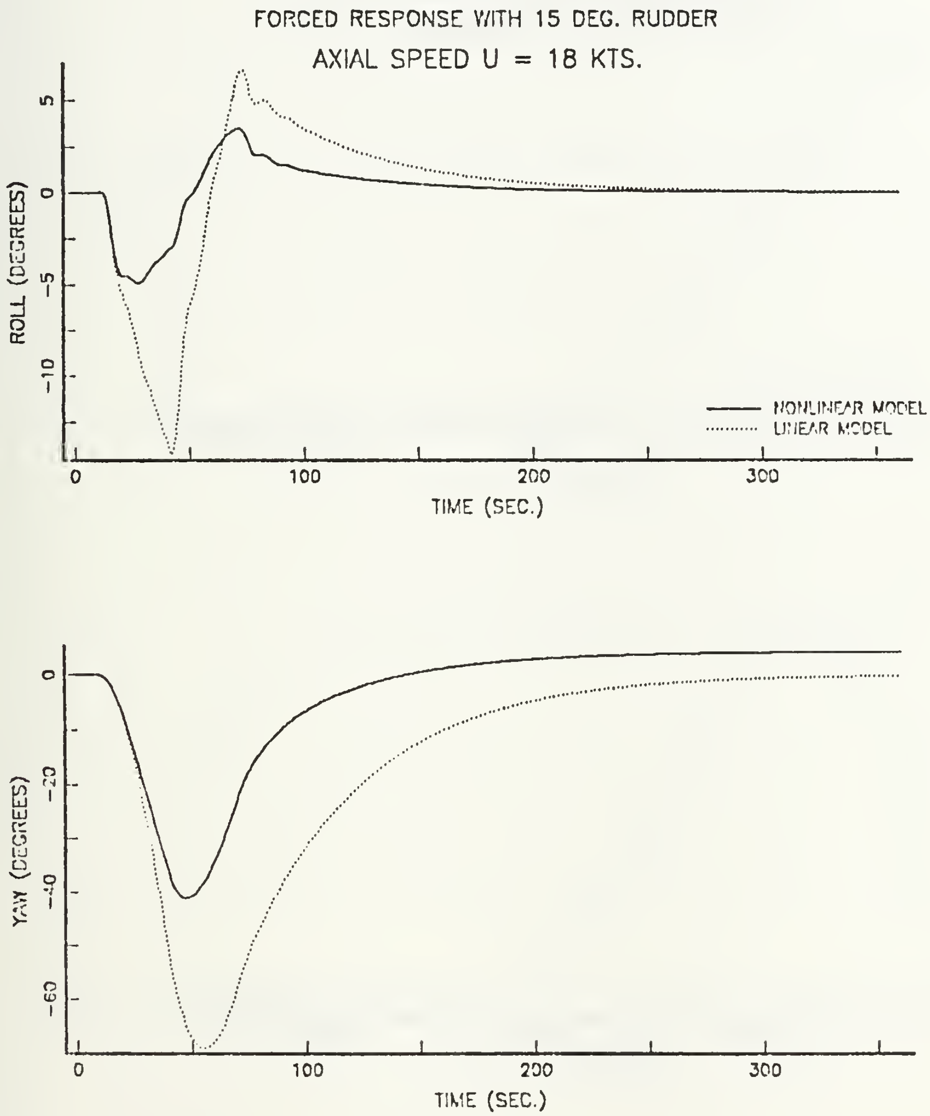

Figure 47. Forced Response. Rudder = 15 Deg. U = 18 Kts.

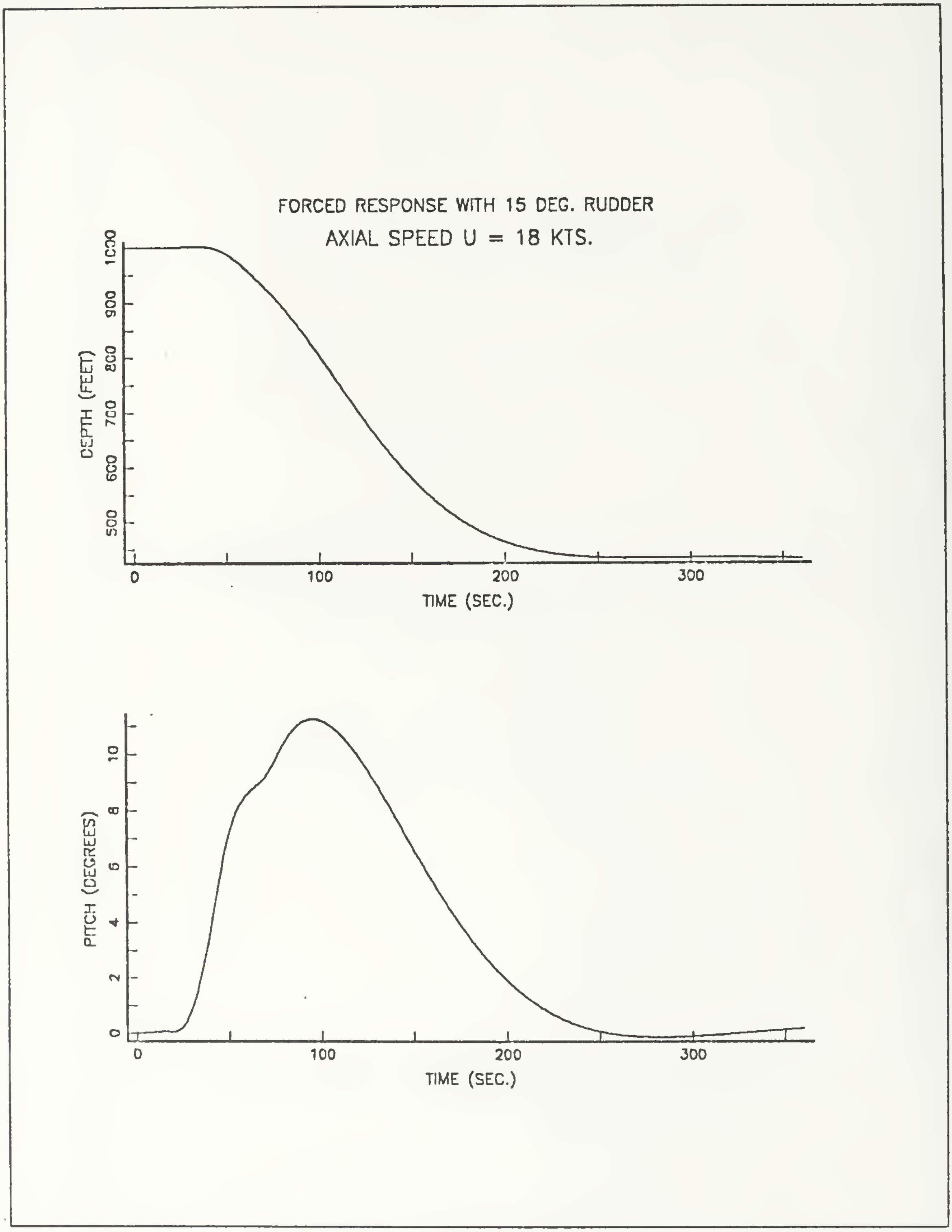

Figure 48. Cross-Coupling Effect on Vertical Plane. Rudder = 15 Deg. U = 18 Kts.

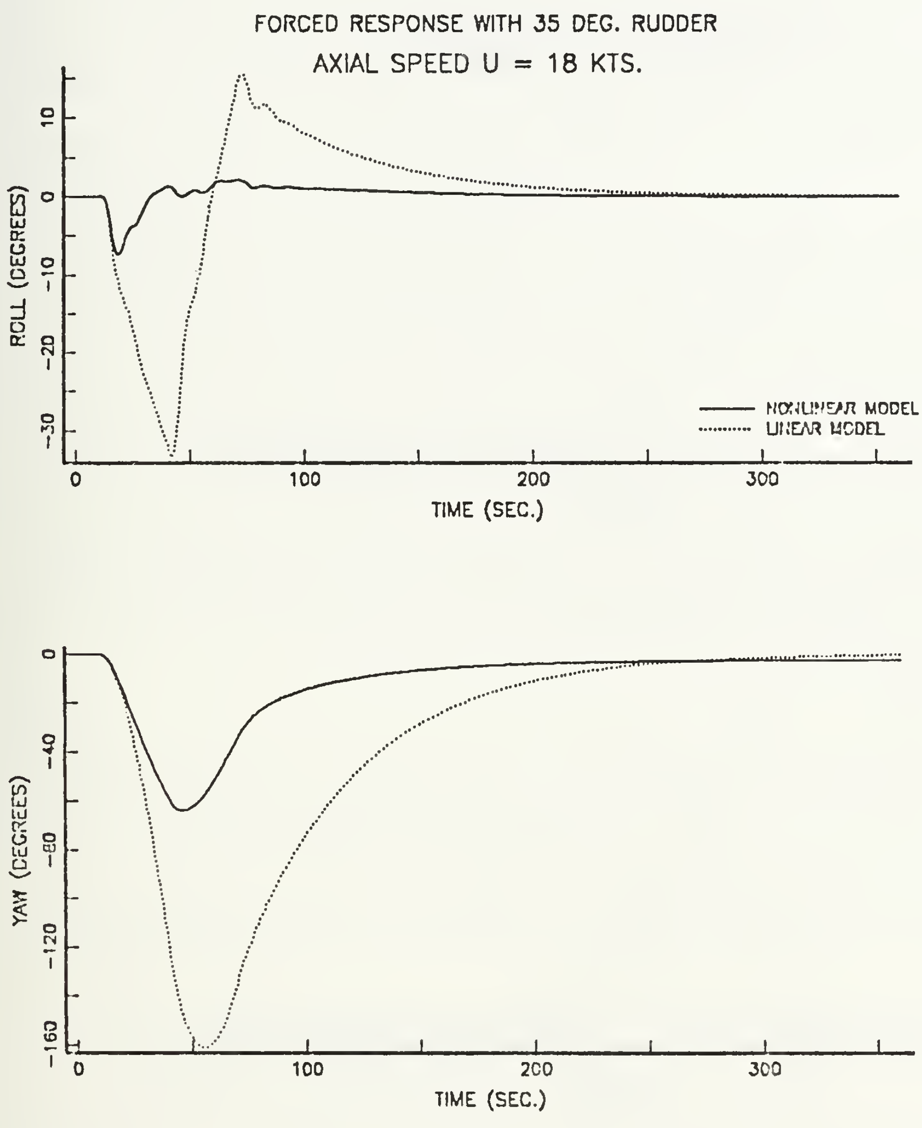

Figure 49. Forced Response. Rudder = 35 Deg. U = 18 Kts.

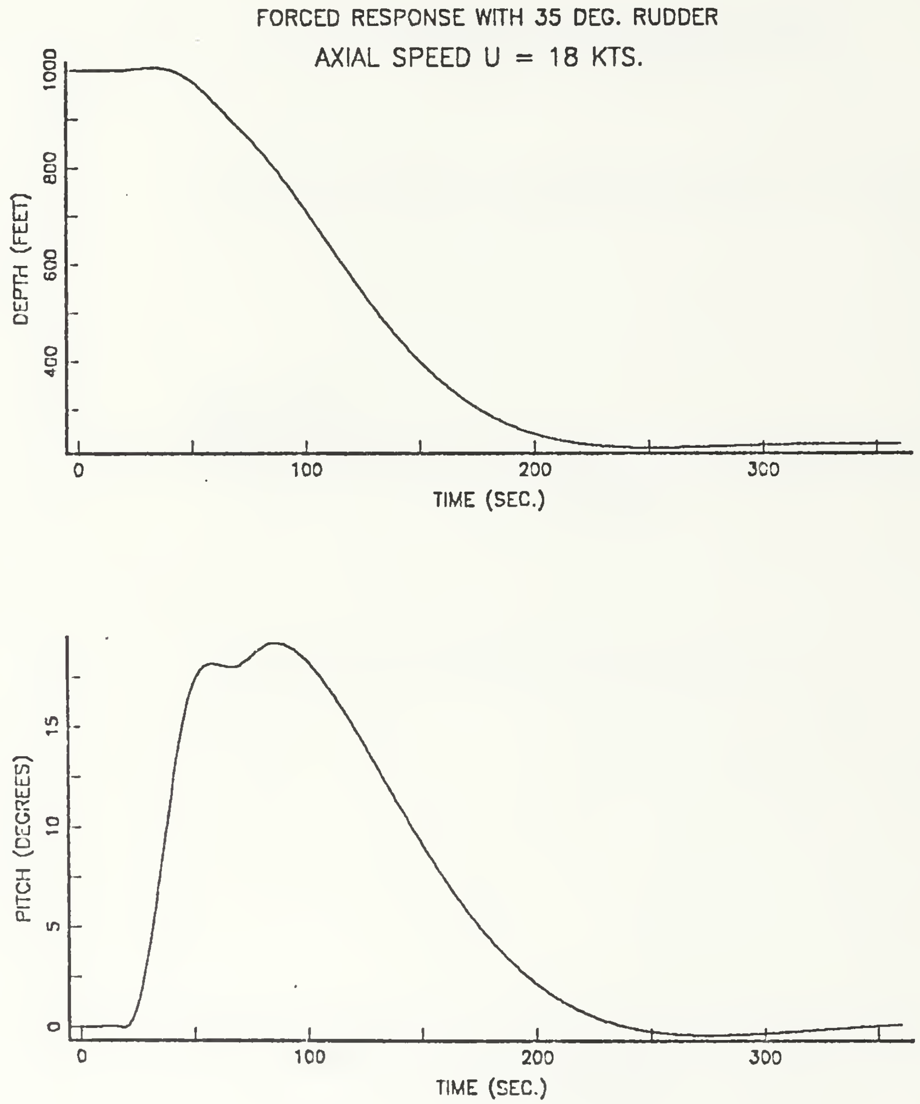

Figure 50. Cross-Coupling Effect on Vertical Plane. Rudder = 35 Deg. U = 18 Kts.

III. AUTOMATIC DEPTH AND PITCH CONTROL

A. DESIGN SPECIFICATIONS

After a valid linear model is defined, any type of design method for linear systems can be used. It is assumed that the submarine which is considered in this study has no inertial guidance system. That means rate information is not available and the only states to be used as feedback are depth, pitch and speed. Due to this limited instrumentation the controller will have to use cascaded filters.

Since a submarine maneuvering capability depends highly on the axial speed, it is very hard to satisfy some certain specifications for this kind of control systems. But basically it is acceptable if the control system can achieve 10 ft. depth change in 120 seconds and 100 ft. depth change in 240 seconds. Also more than 2 feet overshoot is not desirable for small depth changes and overshoot must stay within 5% for large depth changes. Deviation from ordered pitch must stay within 2 degrees.

For the control system to be designed, plane angle limits which are about 35 degrees, have to be taken into the consideration. A depth and pitch control system which requires more than 35 degrees plane angles to get the ordered depth or pitch is clearly not realizable.

B. DESIGN

The linearized equations of motion for the vertical plane are obtained and given in Chapter 2. They are repeated below for convenience

$$\begin{aligned}\ddot{z} = & -1.728x10^{-3}u\dot{z} - 0.706uq + (0.01283 - 1.728x10^{-3}u^2)\theta - 6.667x10^{-4}u^2\delta s \\ & -3.873x10^{-4}u^2\delta b\end{aligned} \tag{16}$$

$$\begin{aligned}\dot{q} = & \, 1.884x10^{-5}u\dot{z} - 6.365x10^{-3}uq - 1.465x10^{-5}u^2\delta s + 3.193x10^{-6}u^2\delta b \\ & + (1.884x10^{-5}u^2 - 2.522x10^{-3})\theta\end{aligned} \tag{17}$$

A signal flow graph can describe the corresponding input/output relations for these equations. Inputs will be the bow and stern plane angles and outputs of interest are depth and pitch for vertical motion and such a flow graph is given in Fig. 51 where

$a = -3.87x10^{-4}u^2$

$b = -1.73x10^{-3}u$

$c = -1.465x10^{-5}u^2$

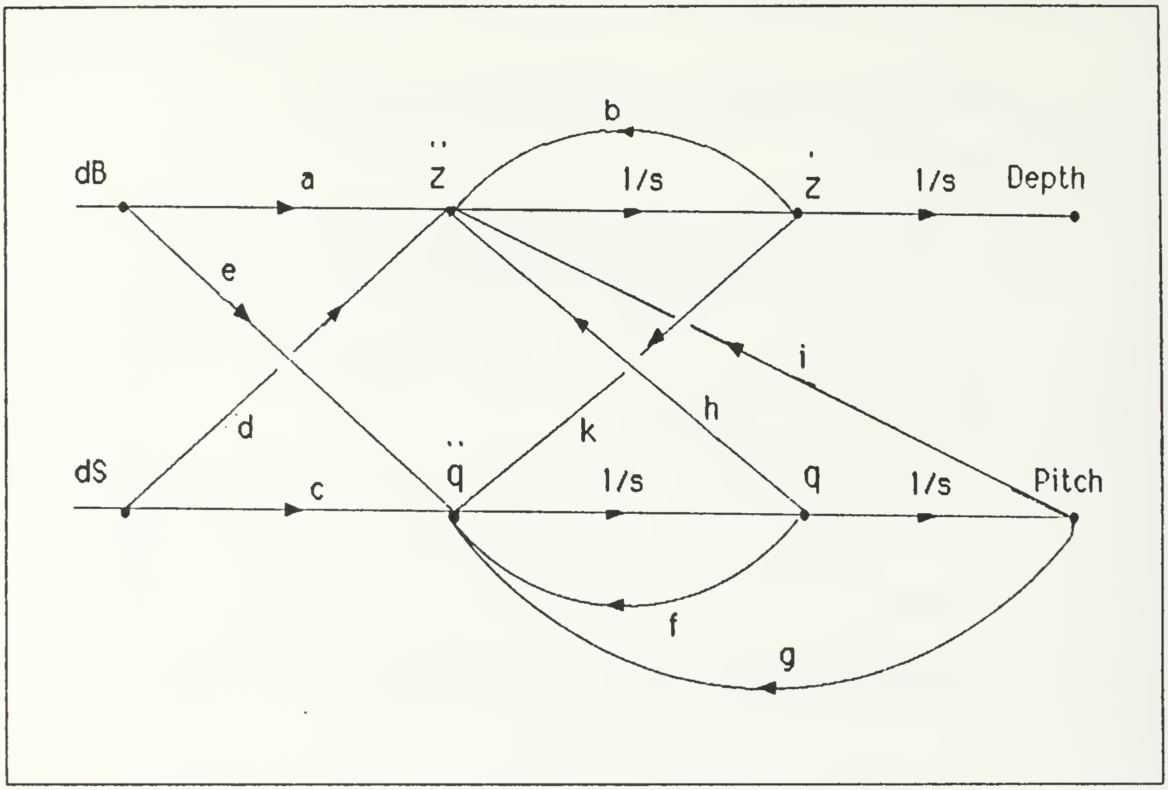

Figure 51. Signal Flow Graph for Vertical Equations of Motion

$d = -6.67x10^{-4}u^2$

$e = 3.19x10^{-6}u^2$

$f = -6.36x10^{-3}u$

$g = 1.88x10^{-5}u^2 - 2.52x10^{-3}$

$h = 0.706u$

$i = 0.013 - 1.73x10^{-3}u^2$

$k = 1.88x10^{-5}u$

1. Decoupling

In order to design a cascade compensator with a single loop technique, one must have the independent input-output relations for each input and output [Ref. 7]. In other words it is necessary to obtain two transfer functions for depth and two transfer functions for pitch which have the stern and bow planes as inputs.

Applying Mason's gain rule to the signal flow graph given in Fig.44 the input-output relations will be as follows [Ref. 8: p. 83].

$$\frac{depth}{\delta b}=\frac{as^2+(eh-af)s+ei-ag}{s^4-(f+b)s^3+(bf-g-kh)s^2+(gb-ki)s} \tag{18}$$

$$\frac{depth}{\delta s}=\frac{ds^2+(eh-df)s+ei-dg}{s^4-(f+b)s^3+(bf-g-kh)s^2+(gb-ki)s} \tag{19}$$

$$\frac{pitch}{\delta b}=\frac{es-be+ak}{s^3-(f+b)s^2+(bf-g-kh)s+(gb-ki)} \tag{20}$$

$$\frac{pitch}{\delta s}=\frac{es-be+dk}{s^3-(f+b)s^2+(bf-g-kh)s+(gb-ki)} \tag{21}$$

Substituting the corresponding numbers into these equations

$$\frac{depth}{\delta b}=\frac{-3.87x10^{-4}u^2s^2-2.1x10^{-7}u^3s-9.33x10^{-7}u^2+1.75x10^{-9}u^4}{s^4+8.09x10^{-3}us^3+(2.52x10^{-3}-2.11x10^{-5}u^2)s^2+4.12x10^{-6}us} \tag{22}$$

$$\frac{depth}{\delta s}=\frac{-6.67x10^{-4}u^2s^2-6.1x10^{-6}u^3s-1.87x10^{-6}u^2+3.78x10^{-8}u^4}{s^4+8.09x10^{-3}us^3+(2.52x10^{-3}-2.11x10^{-5}u^2)s^2+4.12x10^{-6}us} \tag{23}$$

$$\frac{pitch}{\delta b}=\frac{3.19x10^{-6}u^2s-1.75x10^{-9}u^3}{s^3+8.09x10^{-3}us^2+(2.52x10^{-3}-2.11x10^{-5}u^2)s+4.12x10^{-6}u} \tag{24}$$

$$\frac{pitch}{\delta s}=\frac{-1.465x10^{-5}u^2s-1.28x10^{-8}u^3}{s^3+8.09x10^{-3}us^2+(2.52x10^{-3}-2.11x10^{-5}u^2)s+4.12x10^{-6}u} \tag{25}$$

Transfer functions which are dealing with depth, have fourth order characteristic equations and they are type one systems with the same denominator. On the other hand, transfer functions for pitch are type zero and have third order characteristic equations with the same denominator. Also all transfer functions have the same poles except one at the origin. So it is expected that they might show similar frequency response and it may be possible to use only one cascade compensator to compansate the whole system.

In order to make further analysis on these transfer functions, the axial speed, u has to be defined as a number. It is always possible to design the control system for a specific speed and check the validation of design for a certain speed range. Since slower speeds make it harder to get desired depth and pitch angle, it is not very efficient to use an automatic control for less than 5 Knots. A possible approach would be to use 10 ft/sec. (5.9 Kts.) as axial speed. If the designed control system works for this speed, it will probably work for the higher speeds.

2. **Design**

With 10 ft./sec. axial speed, transfer functions become

$$\frac{depth}{\delta b} = \frac{-3.87x10^{-2}s^2 - 2.1x10^{-4}s - 9.33x10^{-5}}{s^4 + 8.09x10^{-2}s^3 + 0.41x10^{-3}s^2 + 4.12x10^{-5}s} \tag{26}$$

$$\frac{depth}{\delta s} = \frac{-6.67x10^{-2}s^2 - 6.06x10^{-3}s + 1.91x10^{-4}}{s^4 + 8.09x10^{-2}s^3 + 0.41x10^{-3}s^2 + 4.12x10^{-5}s} \tag{27}$$

$$\frac{pitch}{\delta b} = \frac{3.19x10^{-4}s - 1.75x10^{-6}}{s^3 + 8.09x10^{-2}s^2 + 0.41x10^{-3}s + 4.12x10^{-5}} \tag{28}$$

$$\frac{pitch}{\delta s} = \frac{-1.465x10^{-3} - 1.28x10^{-5}}{s^3 + 8.09x10^{-2}s^2 + 0.41x10^{-3}s + 4.12x10^{-5}} \tag{29}$$

It is more convenient to rename these transfer functions such as

$$g_{11}(s) = \frac{depth}{\delta b} \tag{30}$$

$$g_{12}(s) = \frac{depth}{\delta s} \tag{31}$$

$$g_{21}(s) = \frac{pitch}{\delta b} \tag{32}$$

$$g_{22}(s) = \frac{pitch}{\delta s} \tag{33}$$

Then the transfer function matrix becomes

$$G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix} \tag{34}$$

Using cascade compensation and a diagonal compensator matrix

$$G_c(s) = \begin{bmatrix} g_{c11}(s) & 0 \\ 0 & g_{c22}(s) \end{bmatrix} \tag{35}$$

The corresponding control model is sketched in Fig.52. The equivalent transfer function matrix will be

$$G_{eq} = G(s)G_c(s) = \begin{bmatrix} g_{11}(s)g_{c11}(s) & g_{12}(s)g_{c22}(s) \\ g_{21}(s)g_{c11}(s) & g_{22}(s)g_{c22}(s) \end{bmatrix} \tag{36}$$

Characteristic equation roots for these transfer functions are given in Table 8. Only one transfer function has roots in right half plane which is $g_{12}(s)$.

Table 8. CHARACTERISTIC EQUATION ROOTS FOR VERTICAL MOTION

Transfer Function	Roots
$g_{11}(s)$	-0.392 ± j0.188, -0.001 ± j0.045
$g_{12}(s)$	-0.030 ± j0.260, -0.0646, 0.043
$g_{21}(s)$	-0.014 ± j0.022, -0.078
$g_{22}(s)$	-0.008 ± j0.028, -0.065

The open loop Bode diagrams and root locus plots are given on Figures 53 to 60. As expected transfer functions for depth have similar frequency response with a small positive phase margin. Also both of them have root locations on the right half plane. On the other hand transfer functions for pitch show similar behavior. They are also stable with 50 and 60 degrees phase margin. The only transfer function which is stable for all gains is $g_{22}(s)$ as can be seen from Fig. 60.

From the root locus diagrams one can easily see that, except for $g_{22}(s)$, the other three transfer functions have many root locations on the right half plane which might make the cascade compensation design required. Since $g_{12}(s)$ has characteristic equation roots in the right half plane and also the root locus diagram shows one root location branch that extends along the positive real axis, it is clearly unstable. In particular $g_{22}(s)$ might not need any compensation other than a gain adjustment.

The effect of bow planes on pitch angle and the effect of stern planes on depth are rather small compared to the effect of bow planes on depth and stern planes on pitch. So it is considered best to focus on the transfer functions $g_{11}(s)$ and g_{22} while carrying out a design procedure. Even if the designed compensators for these two transfer functions are not satisfactory for the other two equations, the total system response might be sufficient.

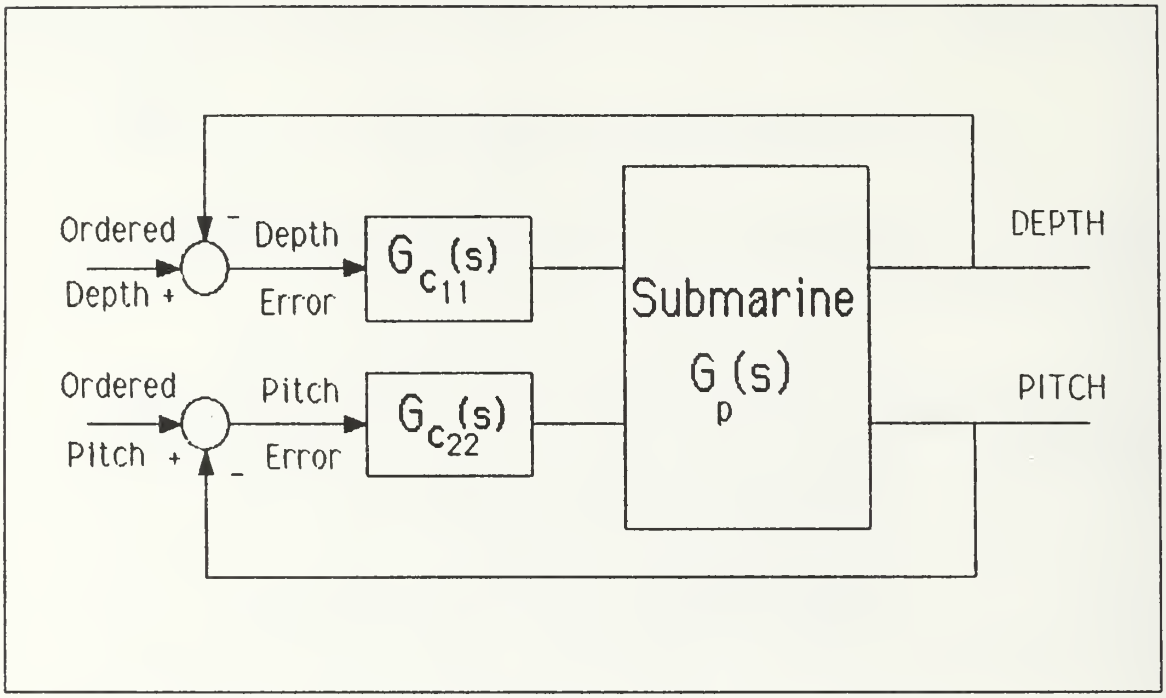

Figure 52. Cascade Compensated Control Model for Vertical Motion

Since the transfer functions for the vertical plane have poles in the right half plane, they do not represent minimum phase systems. For a non-minimum phase system it is more complicated to achieve a design which meets the required specifications. However it is possible to start with a very basic design and improve it after observing compensator effects on the system behavior.

From the requirements mentioned before, the settling time will be more than 100 seconds and the damping coefficient ζ is about 0.5 for sufficient damping. Using the formula for second order approximation

$$T_s = \frac{4}{\zeta\omega_n} \tag{37}$$

Solving for ω_n and substituting numbers

$$\omega_n = \frac{4}{0.5x100} = 0.08 \tag{38}$$

The gain crossover frequency for the uncompensated system is about 0.2 rad/sec. as can be seen from Fig. 54. Then the gain which is to be used for the first

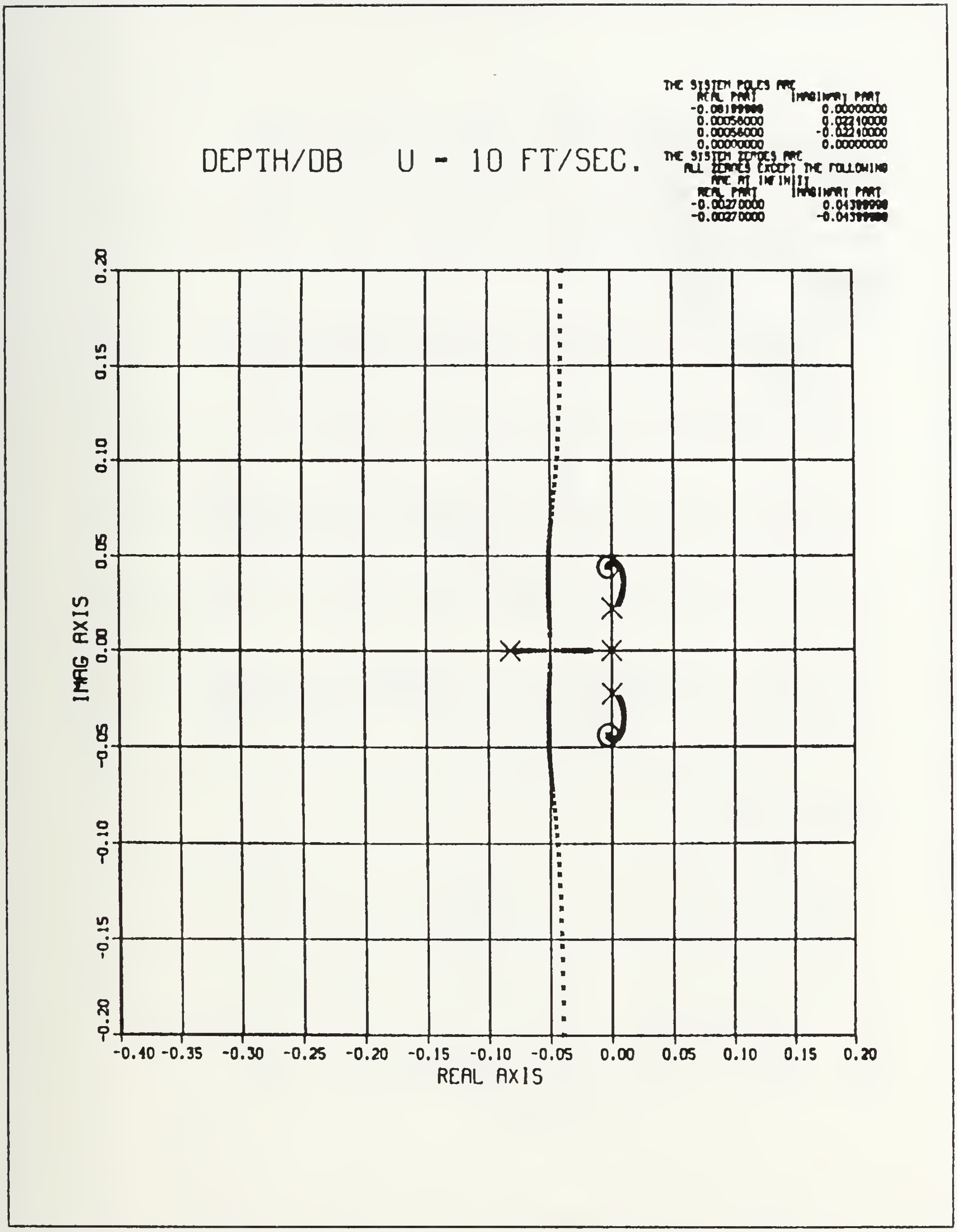

Figure 53. Root Locus Plot for $g_{11}(s)$.

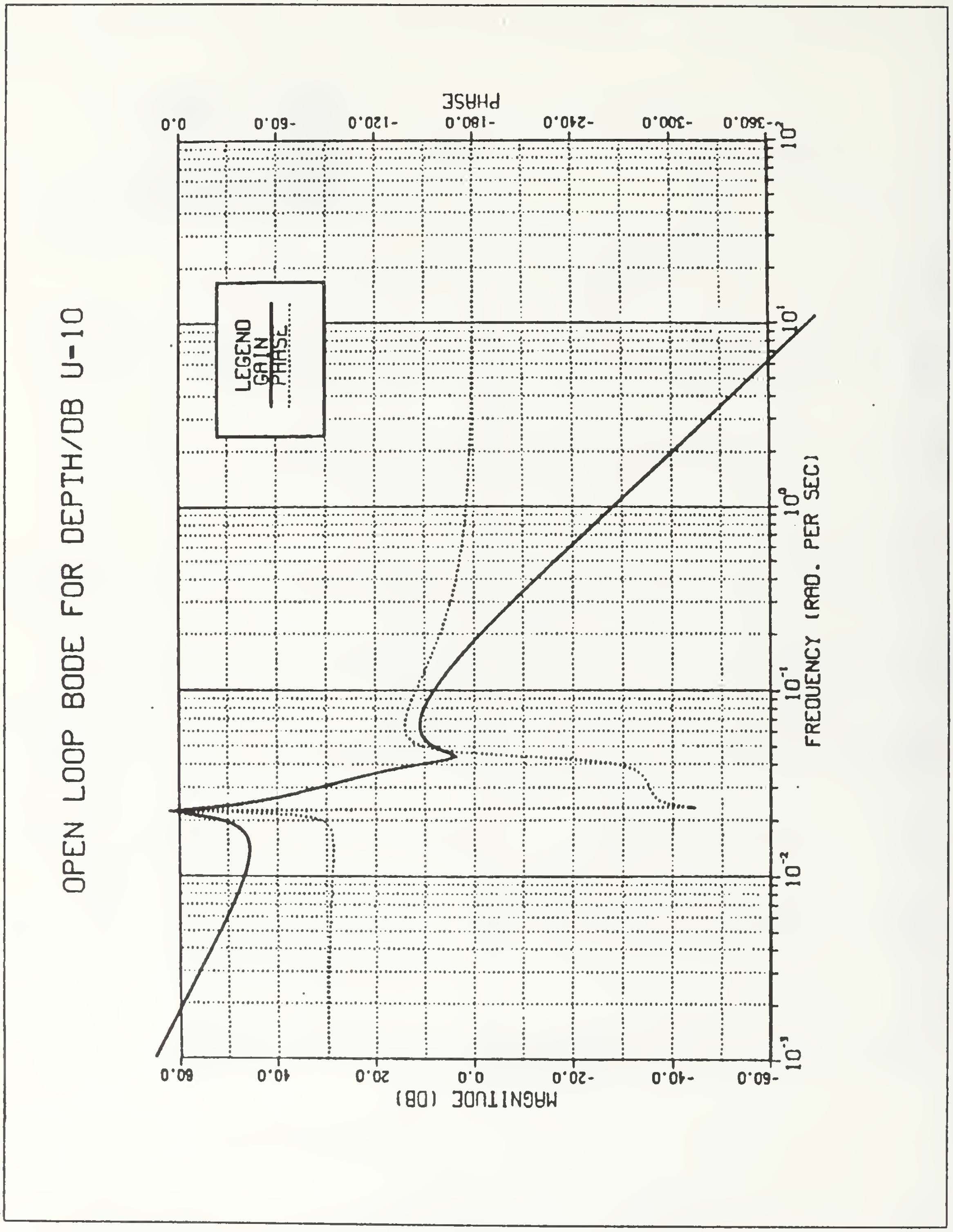

Figure 54. Open Loop Bode Plot for $g_{11}(s)$.

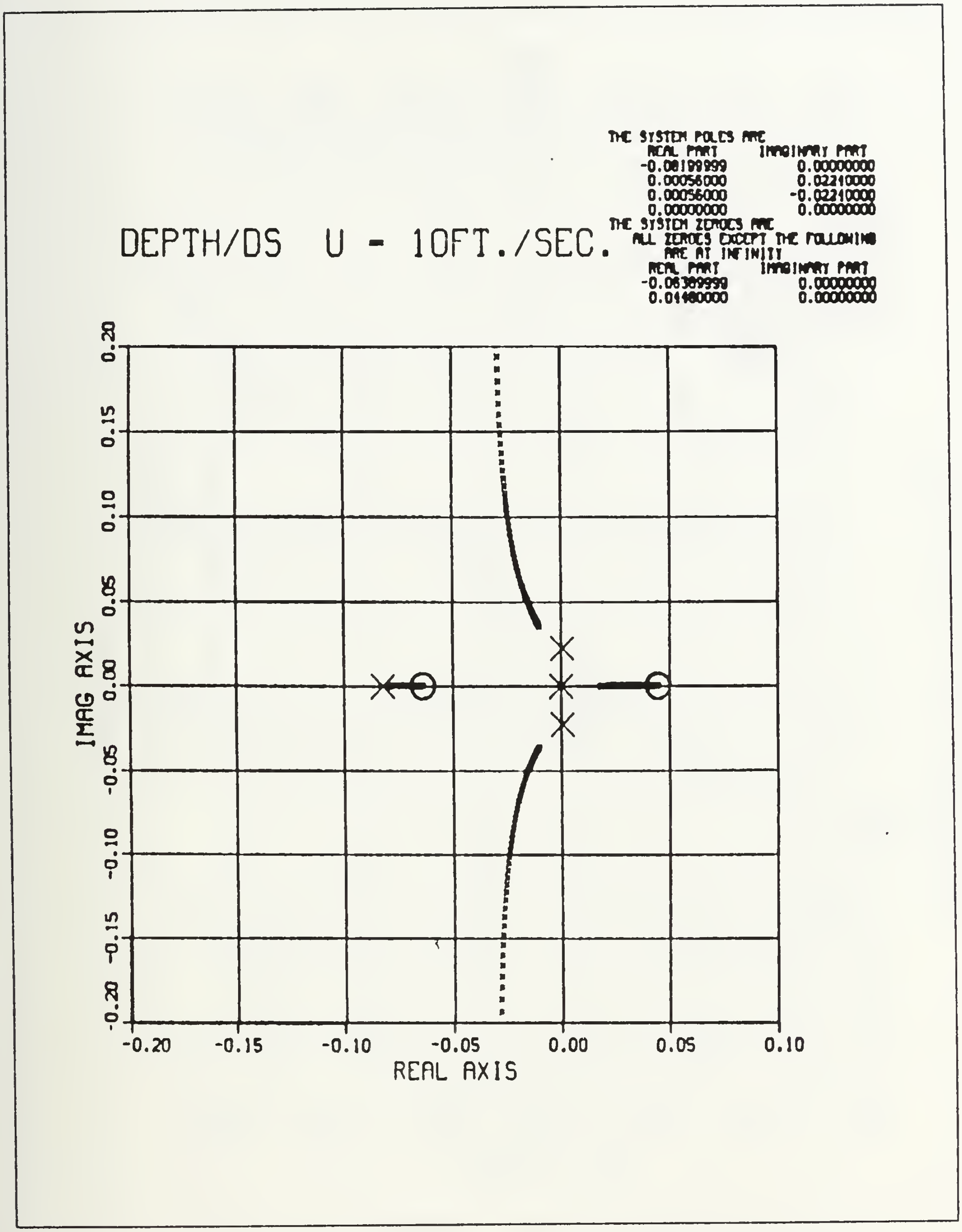

Figure 55. Root Locus Plot for $g_{12}(s)$.

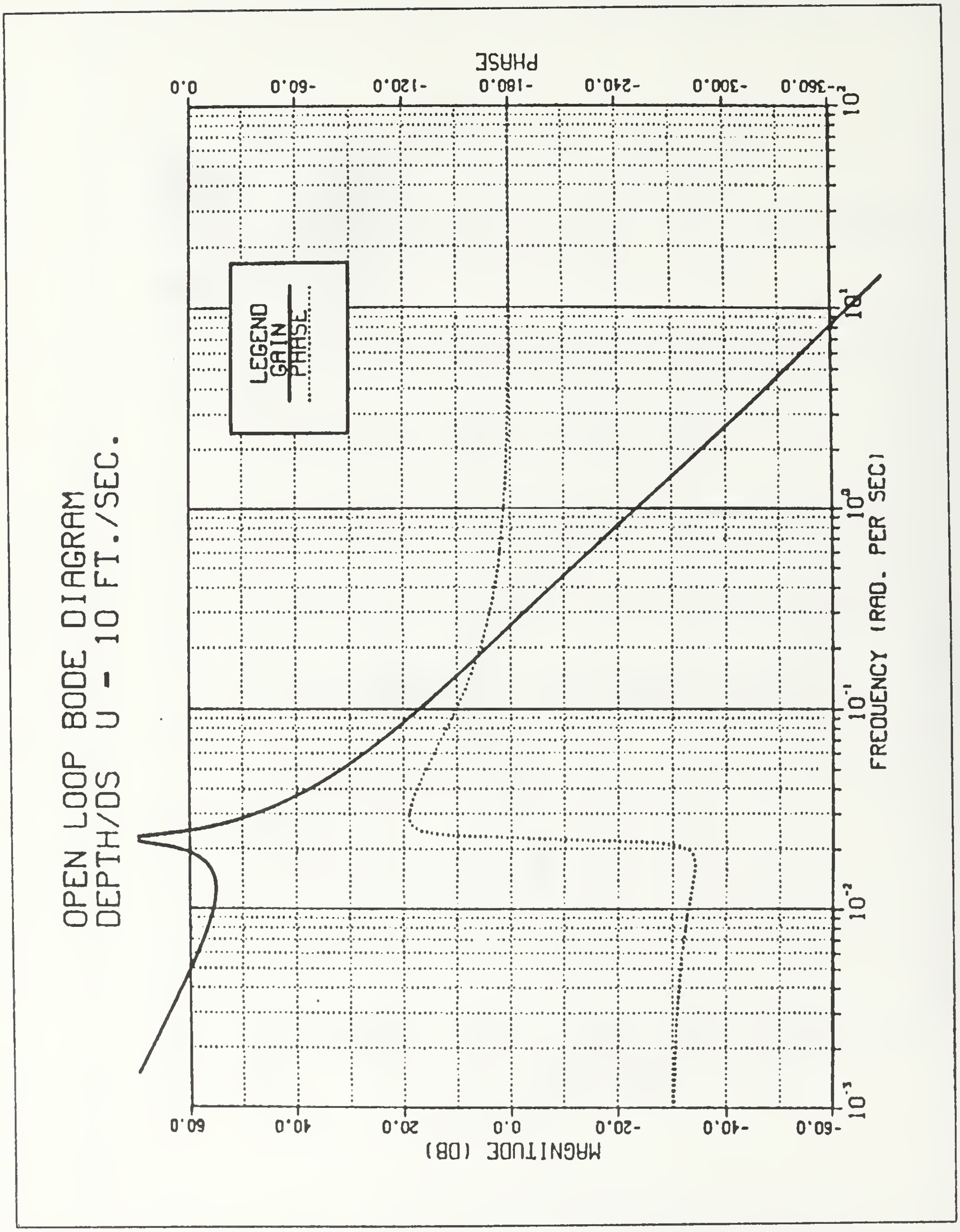

Figure 56. Open Loop Bode Plot for $g_{12}(s)$.

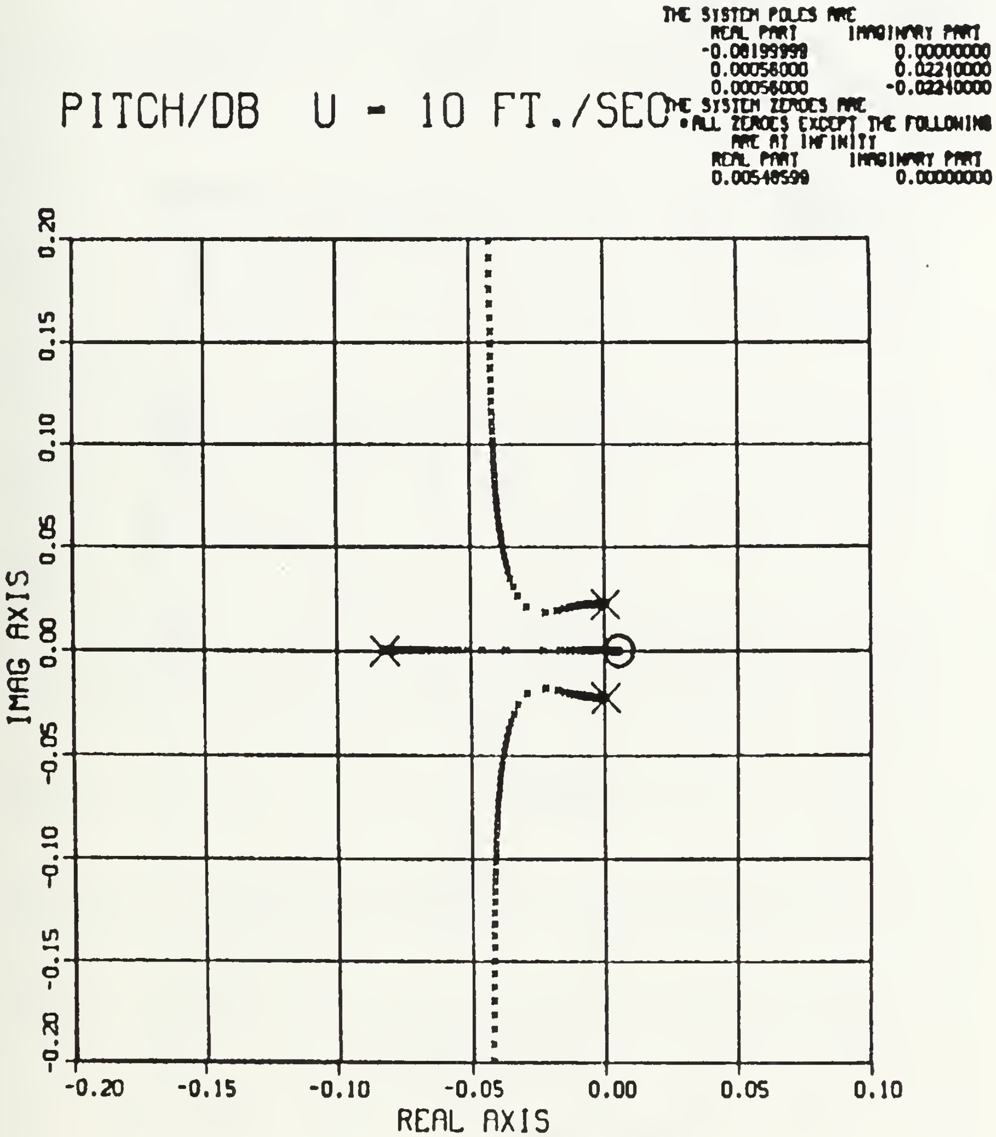

Figure 57. Root Locus Plot for $g_{21}(s)$.

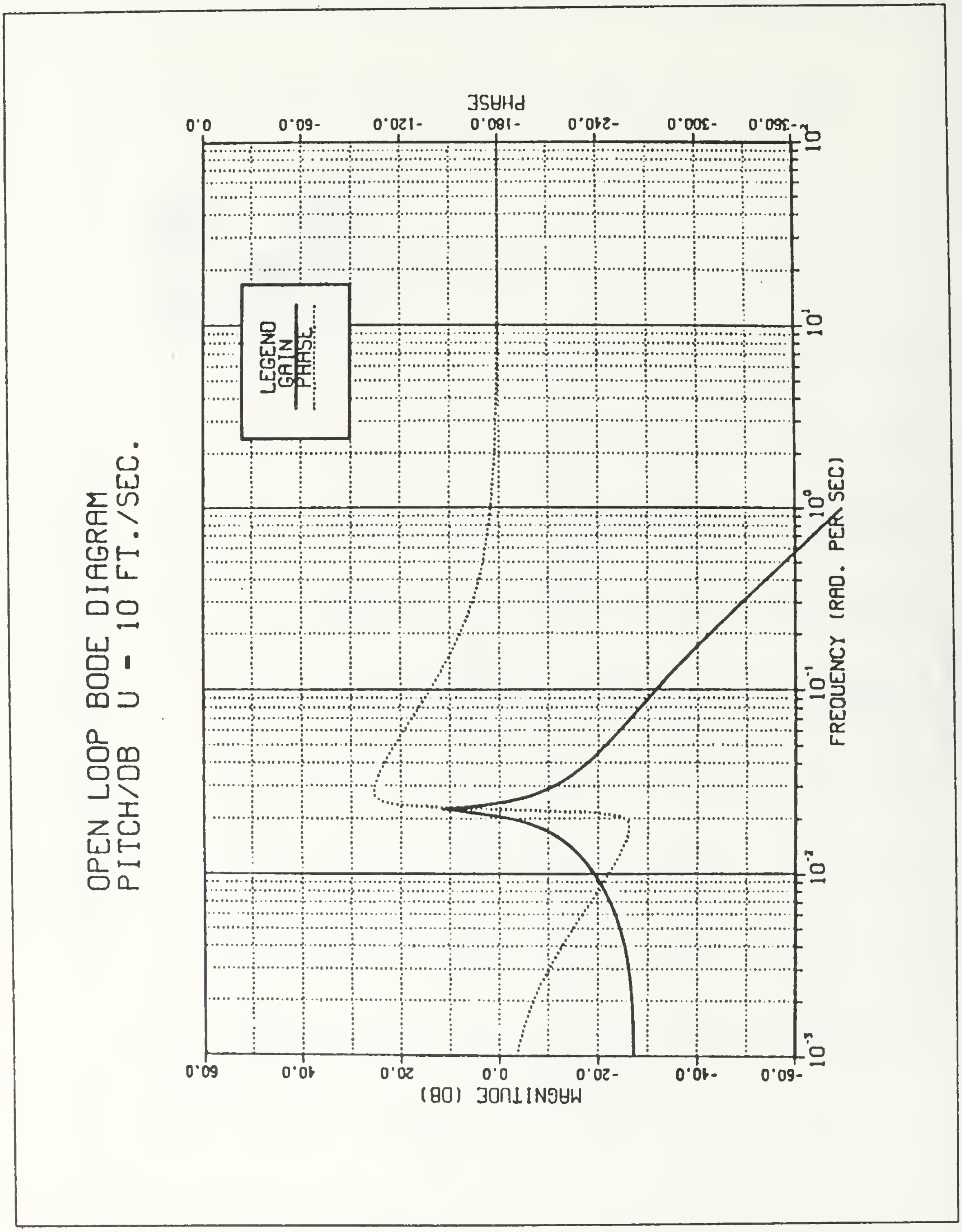

Figure 58. Open Loop Bode Plot for $g_{21}(s)$.

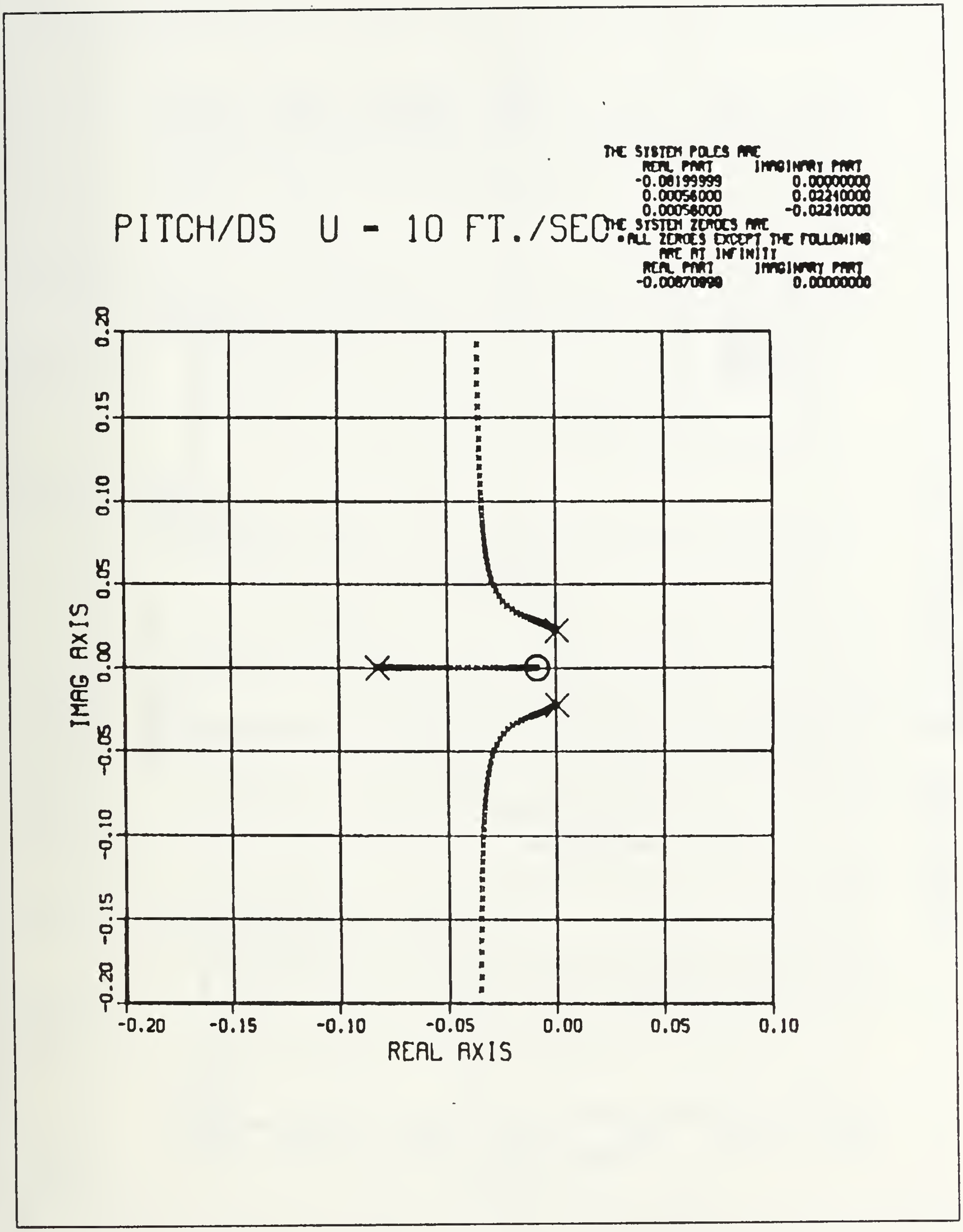

Figure 59. Root Locus Plot for $g_{22}(s)$.

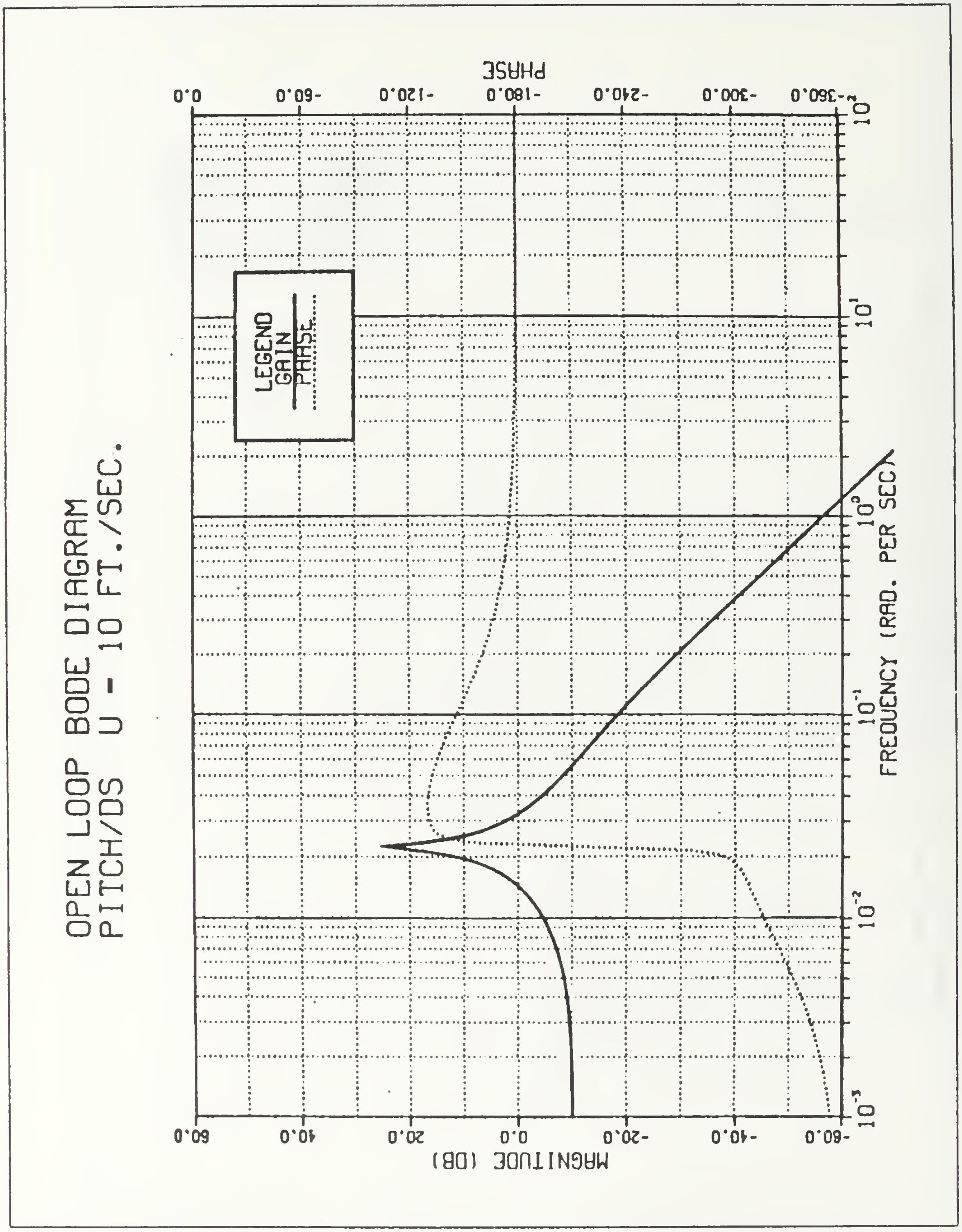

Figure 60. Open Loop Bode Plot for $g_{22}(s)$.

compensator, has to be less than one in order to get the desired response. This gain constant is called K1 and taken as 0.1 for the first trial.

In order to increase phase margin a first order lead compensator is to be added to the forward path. Such a compensator has the form

$$G_c = \frac{p}{z}\frac{(s+z)}{(s+p)} \tag{39}$$

The multiplier p/z is required to keep error coefficient constant. [Ref. 2]

Using cascade compensator design techniques the best choice for the first trial on g_{c11} will be

$$g_{c11} = \frac{1}{0.1}\frac{s+0.1}{s+1.0} = 10\frac{(s+0.1)}{s+1.0} \tag{40}$$

Multiplying with K1 the total compensator is

$$G_c = \frac{s+0.1}{s+1.0} \tag{41}$$

The root locus plot and open loop Bode diagram for the compensated system are given on Fig.61 and Fig. 62. The compensated system has about 75 degree phase margin which is obviously more than the specified requirements. This excess phase margin may cause a request for the large plane angles which it is not possible to supply. Since it is always possible to use limiters on plane angles it is concluded to leave the designed compensator as it is and use it for preliminary design procedures.

Since $g_{22}(s)$ is already very reasonable well damped, no compensator will be used and K2 will be taken as 1.0 for the first trial.

The next step is to put the compensator in the actual linear system and observe the response of the system. But before doing that the simulation program has to be updated in order to get more realistic results and accuracy.

a. Limiters

The mechanical limit for both plane deflections is 35 degree. But it is not desirable to use full plane angles for higher speeds. Also it is possible to limit planes and the error signals. The test runs which are achieved with limited planes led to unacceptable plane behavior such as very small deflections. Under these set of circumstances it was concluded to limit the error signal such as:

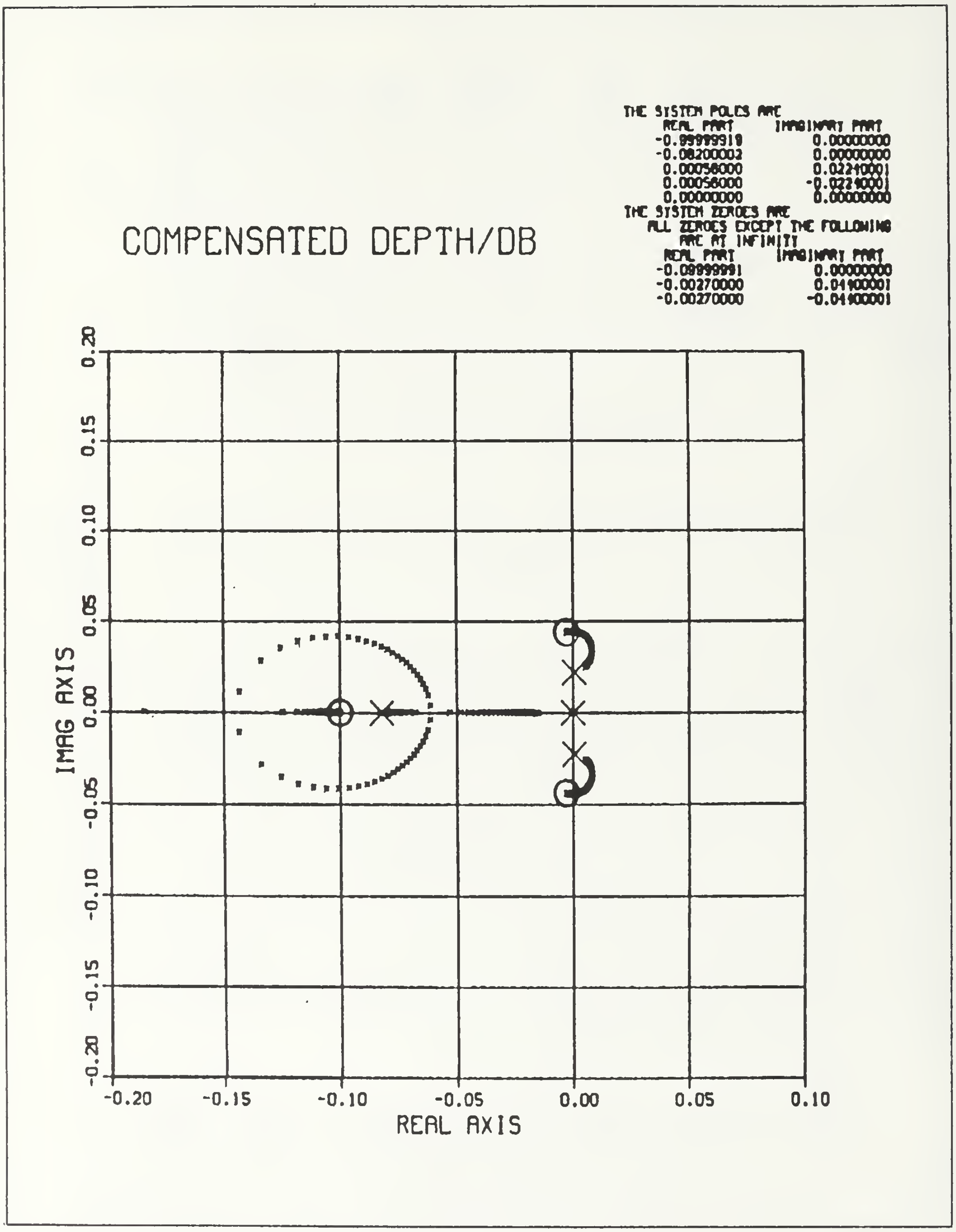

Figure 61. Root Locus Plot for $G_c g_{11}$.

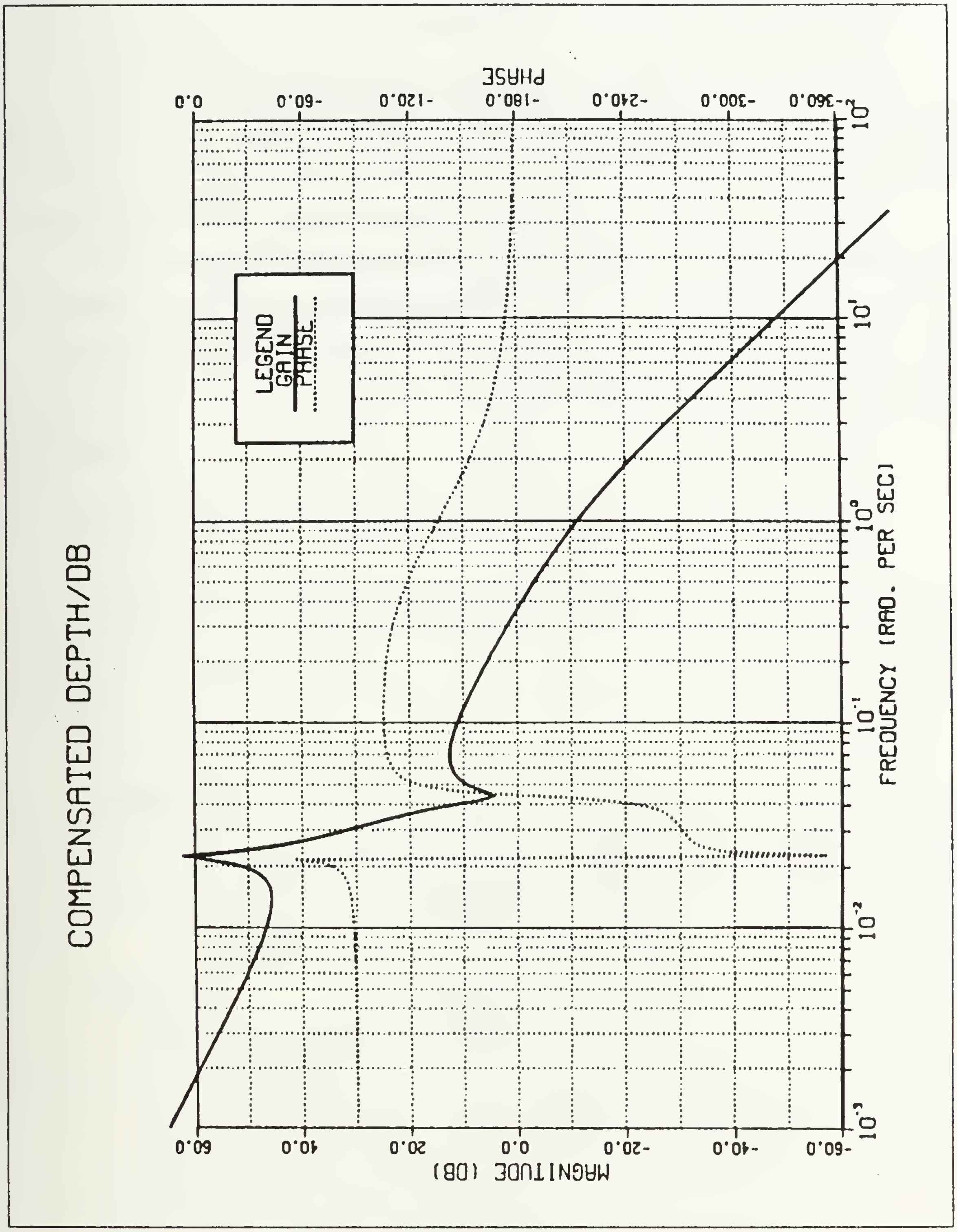

Figure 62. Open Loop Bode Plot for $G_c g_{11}$.

lim = 35 when u < 15 ft/sec.

lim = 25 when 15 < u < 30 ft/sec.

lim = 15 when u > 30 ft/sec.

Obviously this limiter does not have any effect for less than 15 ft. depth changes where there is no need for a limiter.

b. Actuators

The linearized model does not include the dynamics of the plane actuators, which are force and moment producers. The actuator dynamics were ignored in the model comparison part of this study. In order to have an accurate model for the design procedure, an actuator model has to be added to the system dynamics. Such an actuator model was developed by [Ref. 6] and represented as

$$G_{act} = \frac{1}{s + 0.667} \tag{42}$$

The complete model which is used in the simulation program is given in Fig. 63.

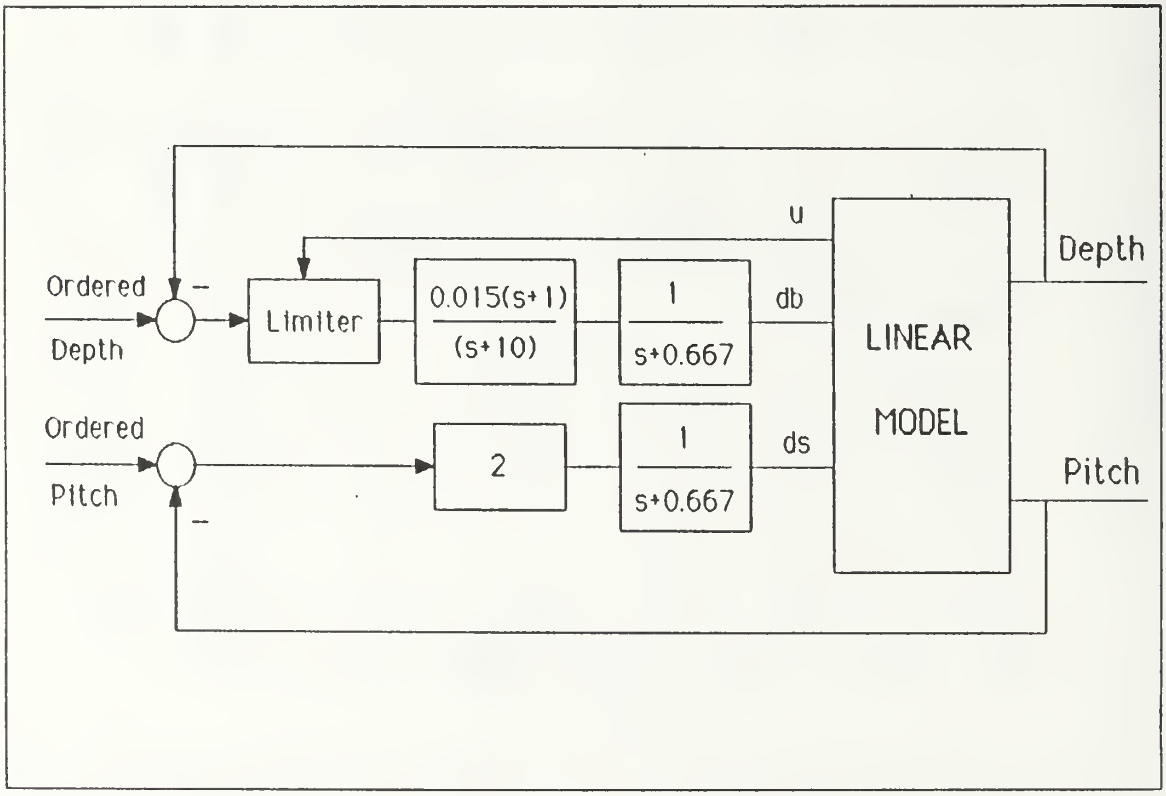

Figure 63. Block Diagram for Compensated Linear Model in Vertical Motion

3. Simulation

The simulation program is written based on the discussed subjects above. The first run was made with the preliminary design gains, poles and zeros. Then required corrections were made in order to meet the design specifications. The DSL simulation program with the final parameters is given in Appendix F.

Test run results which were achieved with different sets of parameters are given in Figures 64 to 74. Each run is explained briefly below:

Run No. 1:

The simulation program was run with the first set of parameters for 10 ft. depth change and 10 ft/sec. axial speed which is the lower limit for this compensation. With K1 = 0.1 the required bow plane angle was very large and overshoot was 25%. This run does not meet the specified requirements.

Run No. 2:

In order to get reasonable plane response it is decided to reduce K1 to the value of 0.01. This time the maximum bow plane deflection is 26 degree but the time required to reach 10 feet depth change is a little longer than the specification. This run is also discarded.

Run No. 3:

A third approach would be to change K1 to 0.015. The result was quite satisfactory except the 40 degree maximum bow plane angle. The main reason for this larger plane request is the excess phase margin on the system. The one possible way to reduce phase margin is to shift the cascade compensator one decade up in the frequency domain.

Run No. 4:

Using the new compensator with one zero at 1.0 and one pole at 10.0, the results are satisfactory. As can be seen from figure 66 the maximum required bow plane angle is 10 degrees, the time to complete 10 ft. depth change is 78 sec. and the overshoot is 10%. This excess overshoot is the payoff for reducing the phase margin but since it makes only one foot difference, it is acceptable.

Run No. 5:

It is desired to check the system response for large depth changes. The simulation program was run for a 100 ft. depth change. Maximum required bow plane angle is 34 degrees and overshoot is 5%. At this point it seems that the compensated linear model for depth control is acceptable.

It is also required to check the pitch response of the system. Test runs were performed with zero depth and some certain pitch angle change. Because of the bow plane effect (which tries to keep the submarine at the same depth) there was a steady state error on pitch angle. Since this pitch error relates very closely to K2, it is concluded to increase K2 to 2.0.

Runs No.6 and 7:

To make sure that there is no negative effect on depth behavior of the system created by the new K2 parameter, two more runs were achieved with K2 = 2.0 for 10 and 100 feet depth change. Since there was only a slight change on overshoot, the new K2 value is accepted and used for further study.

Run No. 8

In order to check the pitch response of the compensated system, a -5 degree pitch command was ordered while the depth change command was zero. System has reached the ordered pitch angle in 46 seconds and because of the bow plane effect, it settled on -4 degree. Increasing K2 might decrease this steady state error but at the same time it might create more overshoot and instability problems on depth behavior of the system. Since a 1 degree error is in the specifications limits, K2 = 2.0 will be used for further study.

Runs No. 9 and 10:

The next step is to check the designed system for a certain range of speed. For 15.2 ft./sec. (9 Kts.) two runs were performed with 10 and 100 ft. depth change. As it can be seen from Fig.s 69 and 70 there is 12% overshoot for 10 ft. depth change and 3% for 100 ft. depth change. Increasing the speed has a positive effect for large depth changes while having a negative effect for small ones.

Runs No. 11 and 12:

The axial speed was increased to 12 Kts. The system reaches the ordered depth in shorter time and has only 2% overshoot for 100 ft. depth change.

Runs No. 13 and 14:

Two more test runs were performed with 18 Kts. axial speed. As can be seen from Fig.s 21 and 22 the compensated control model is still valid and, in fact, works better with only 1% overshoot for large depth change.

Finally it is considered that the designed automatic control for the linearized vertical motion using cascade compensator design techniques is satisfactory and should be checked with actual non-linear model. After designing another cascade compensator for the horizontal motion, both models will be checked in order to see whether the design is completed or needs some alterations.

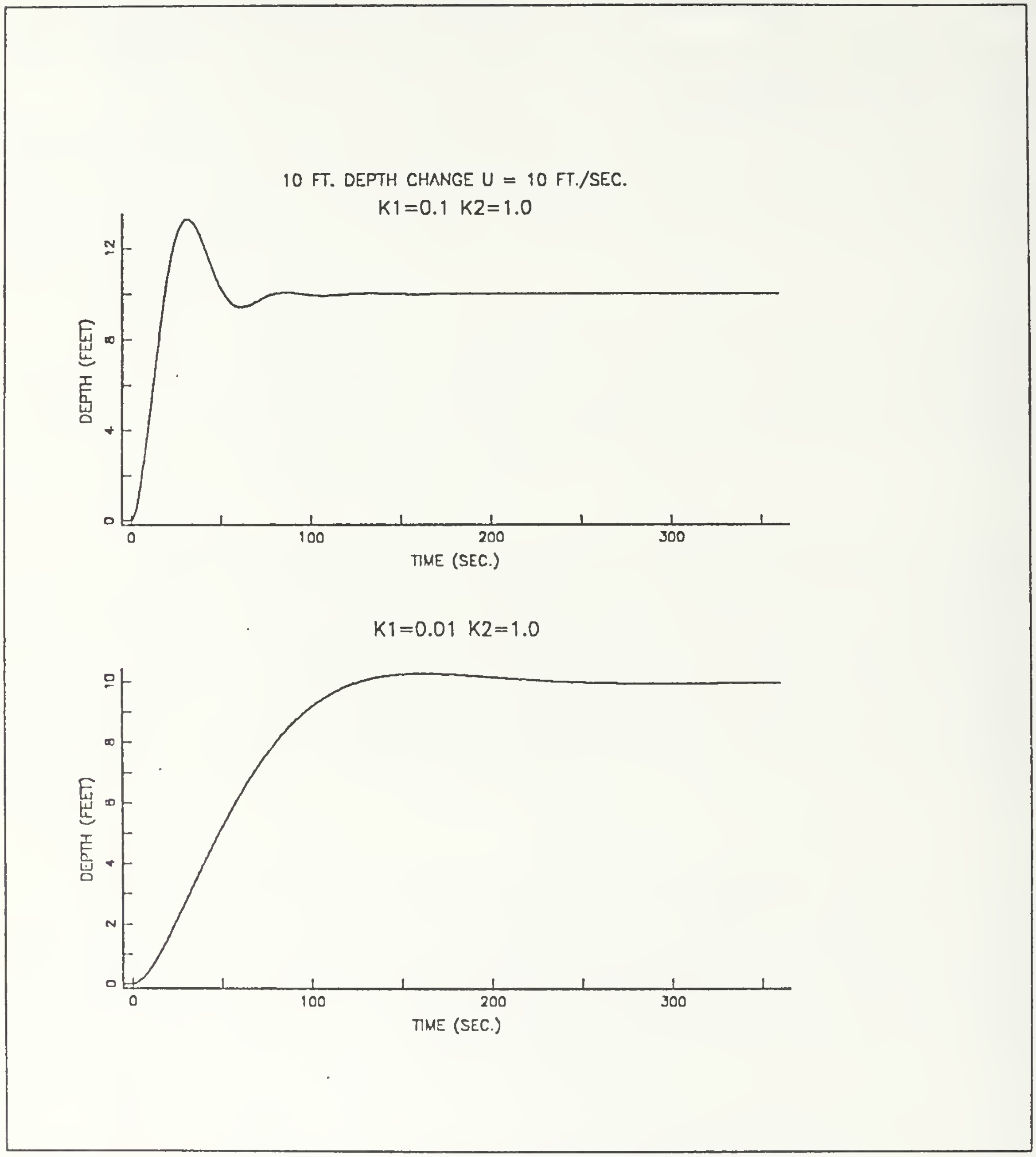

Figure 64. Compensated System Depth Response Z = 0.1, P = 1.0

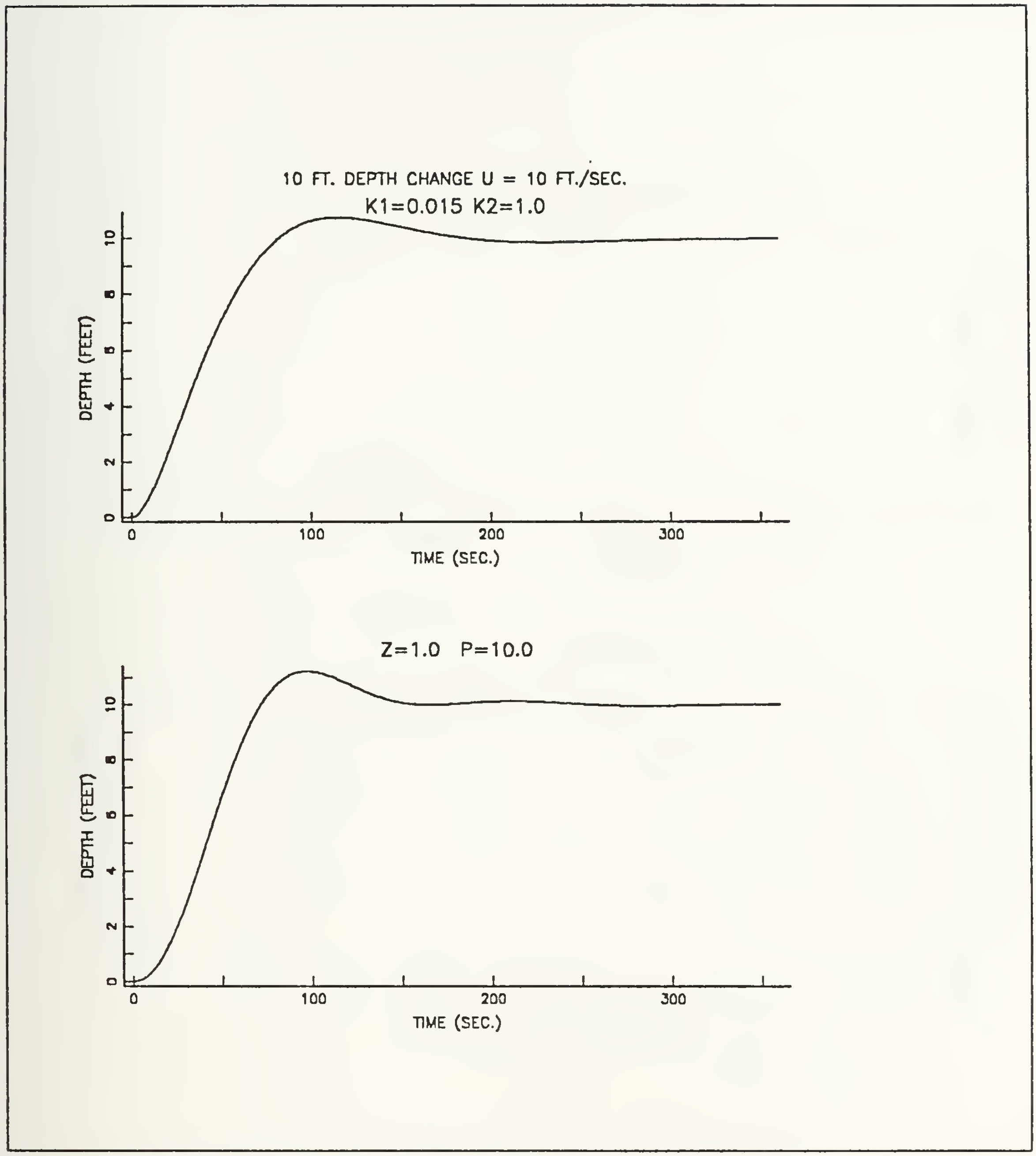

Figure 65. Compensated System Depth Response K1 = 0.015, K2 = 1.0

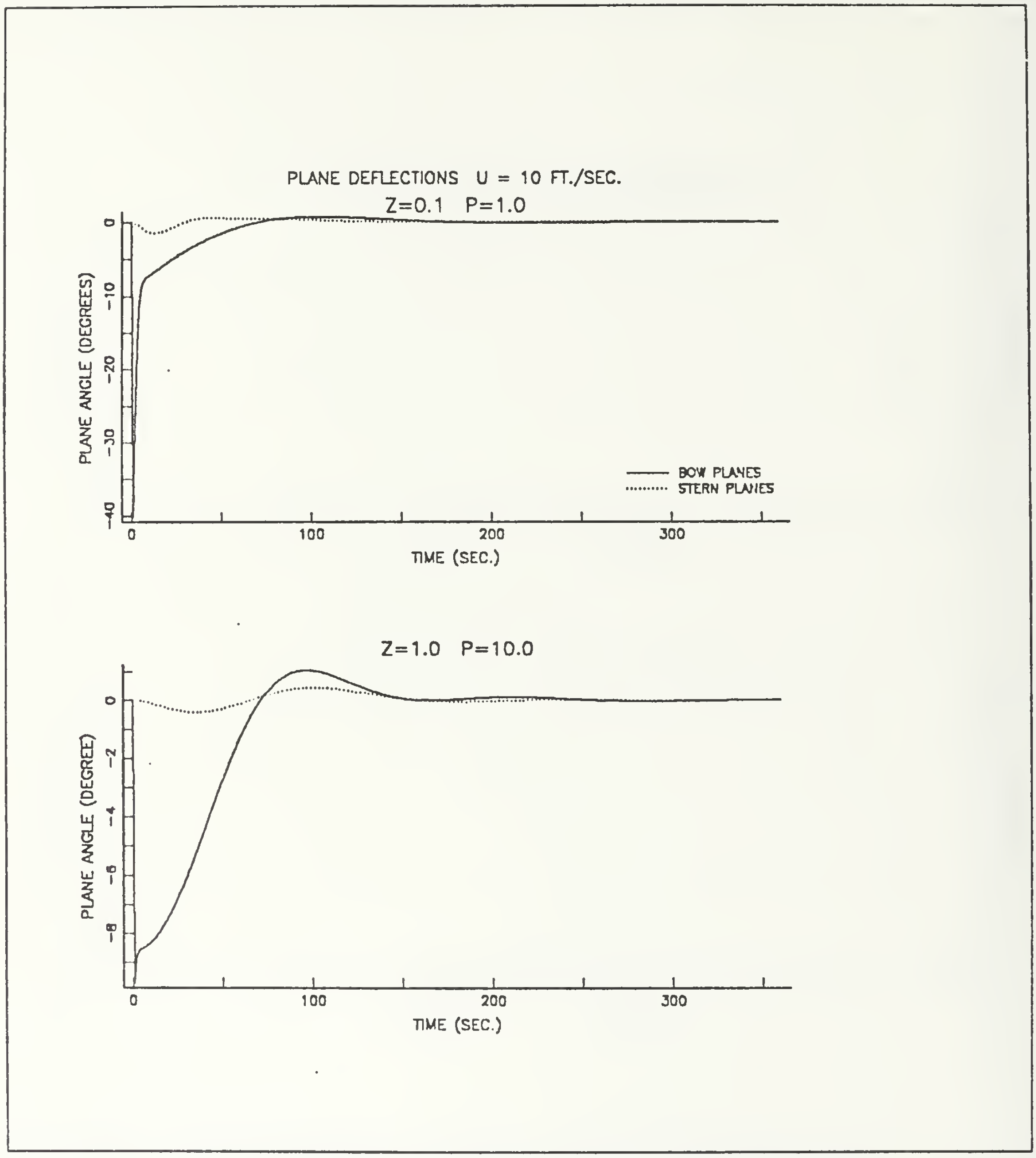

Figure 66. Plane Angle Deflections for first and second Compensator

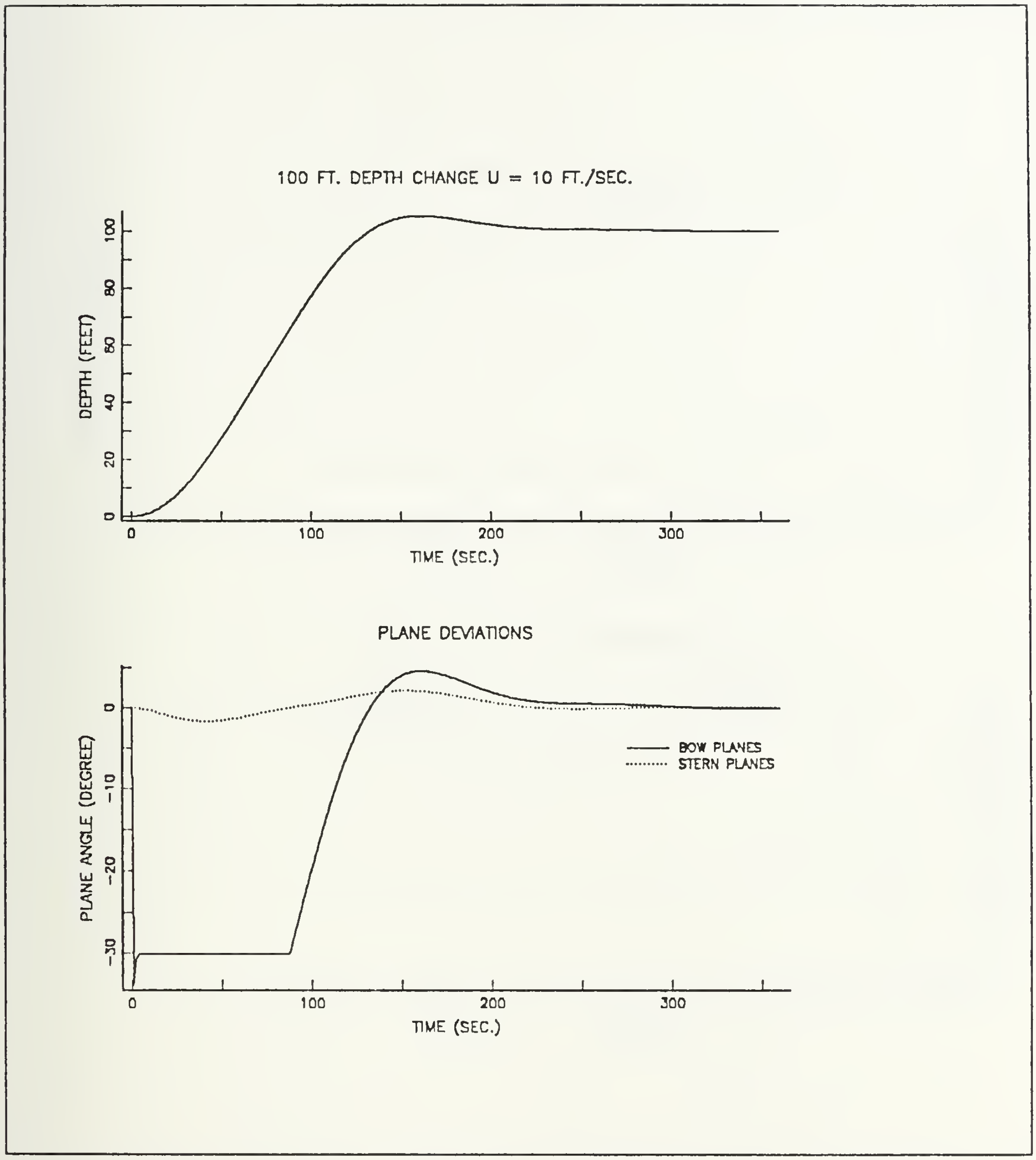

Figure 67. Compensated System Response to 100 ft. Depth Change

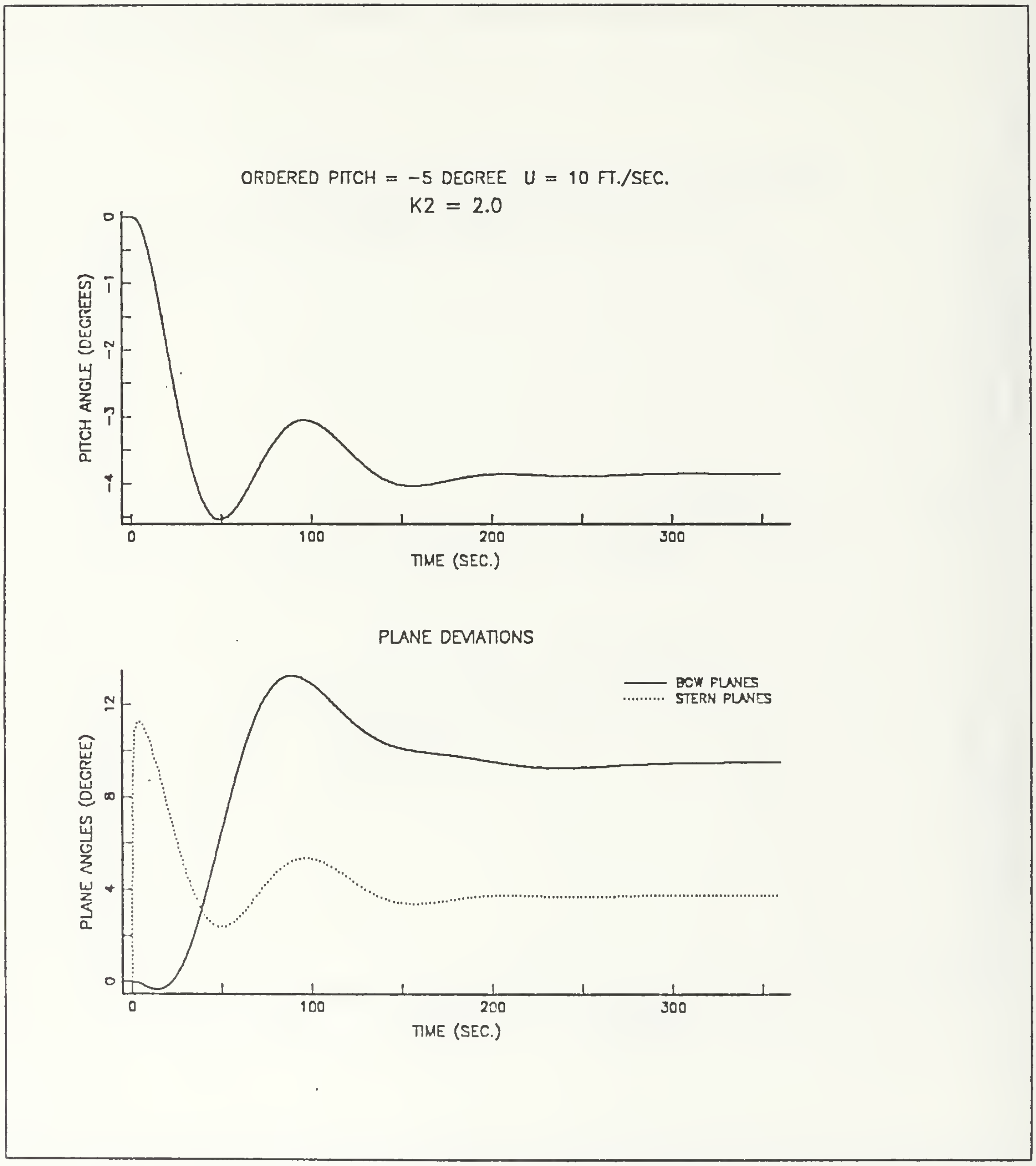

Figure 68. Compensated System Pitch Response for Commanded Pitch = -5 Deg.

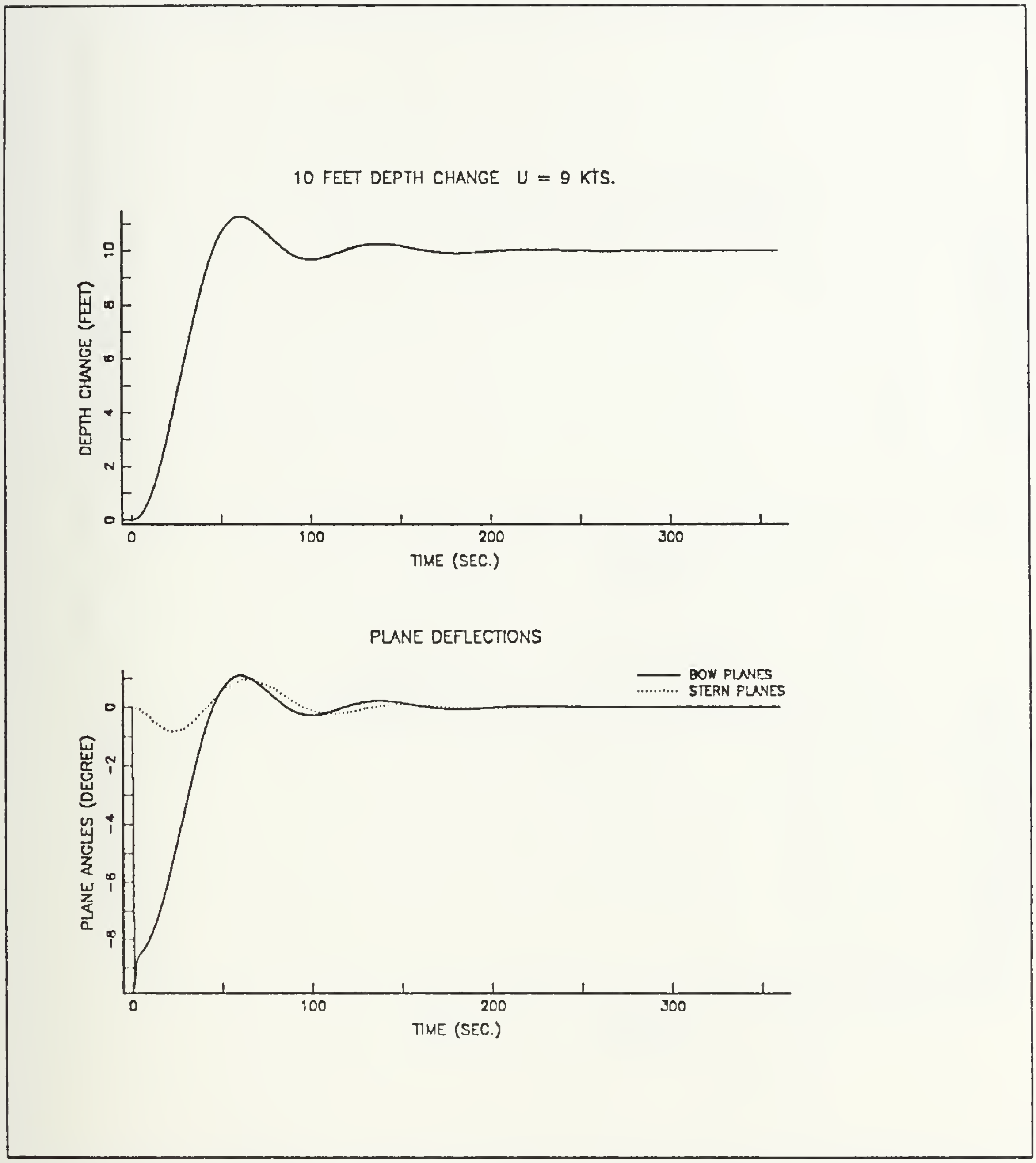

Figure 69. Compensated System Response to 10 ft. Depth Change U=9 Kts.

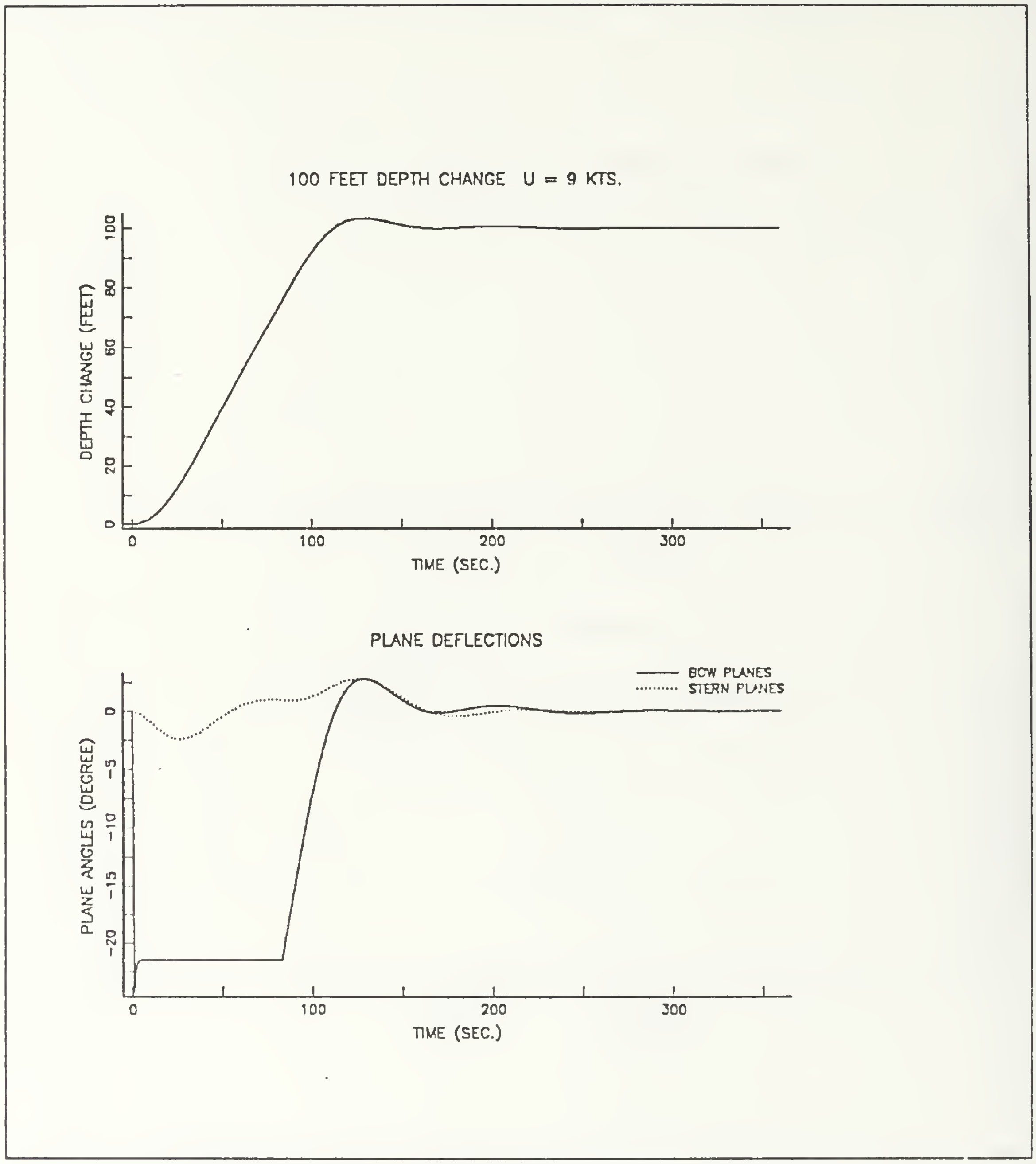

Figure 70. Compensated System Response to 100 ft. Depth Change U=9 Kts.

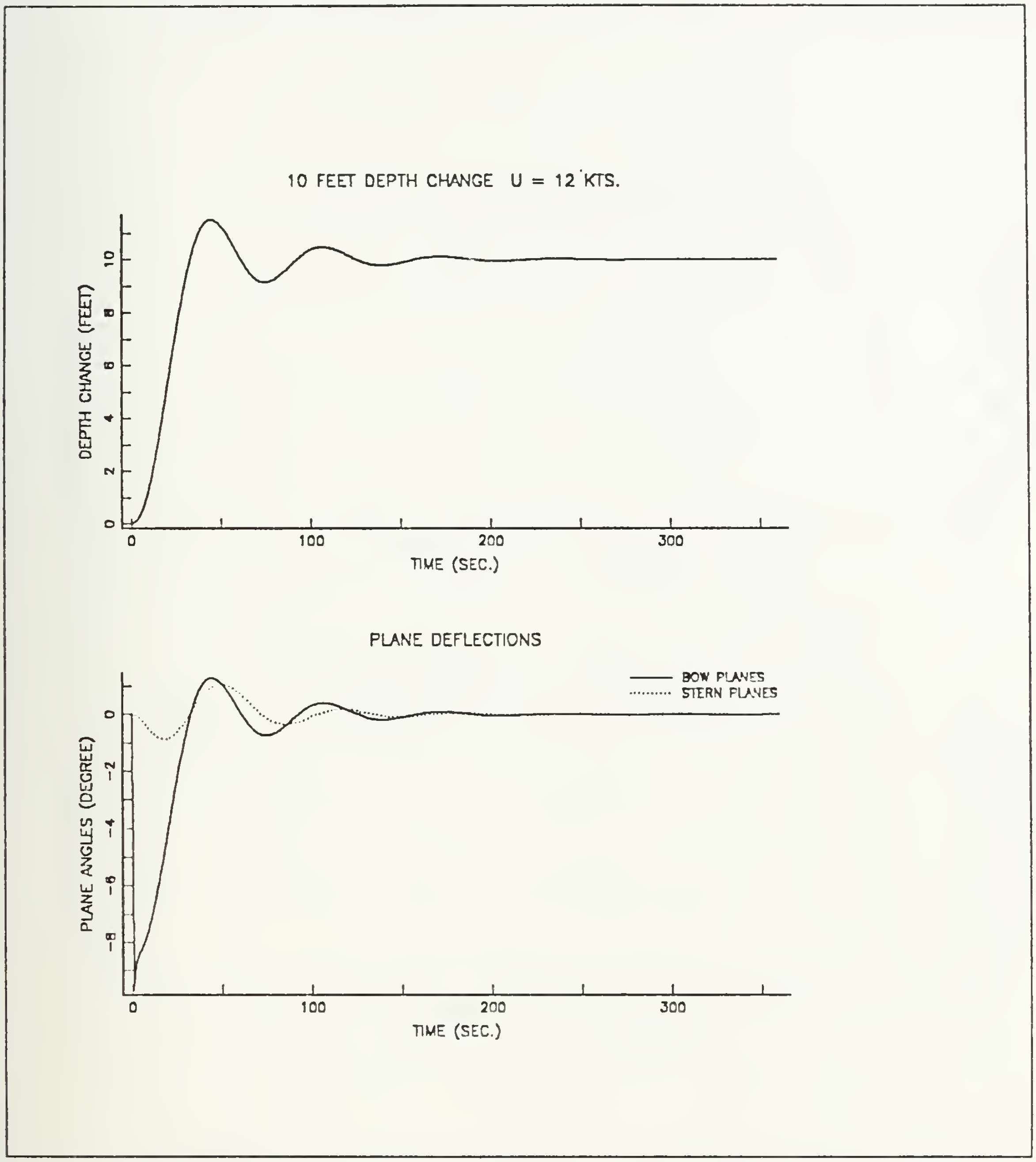

Figure 71. Compensated System Response to 10 ft. Depth Change U= 12 Kts.

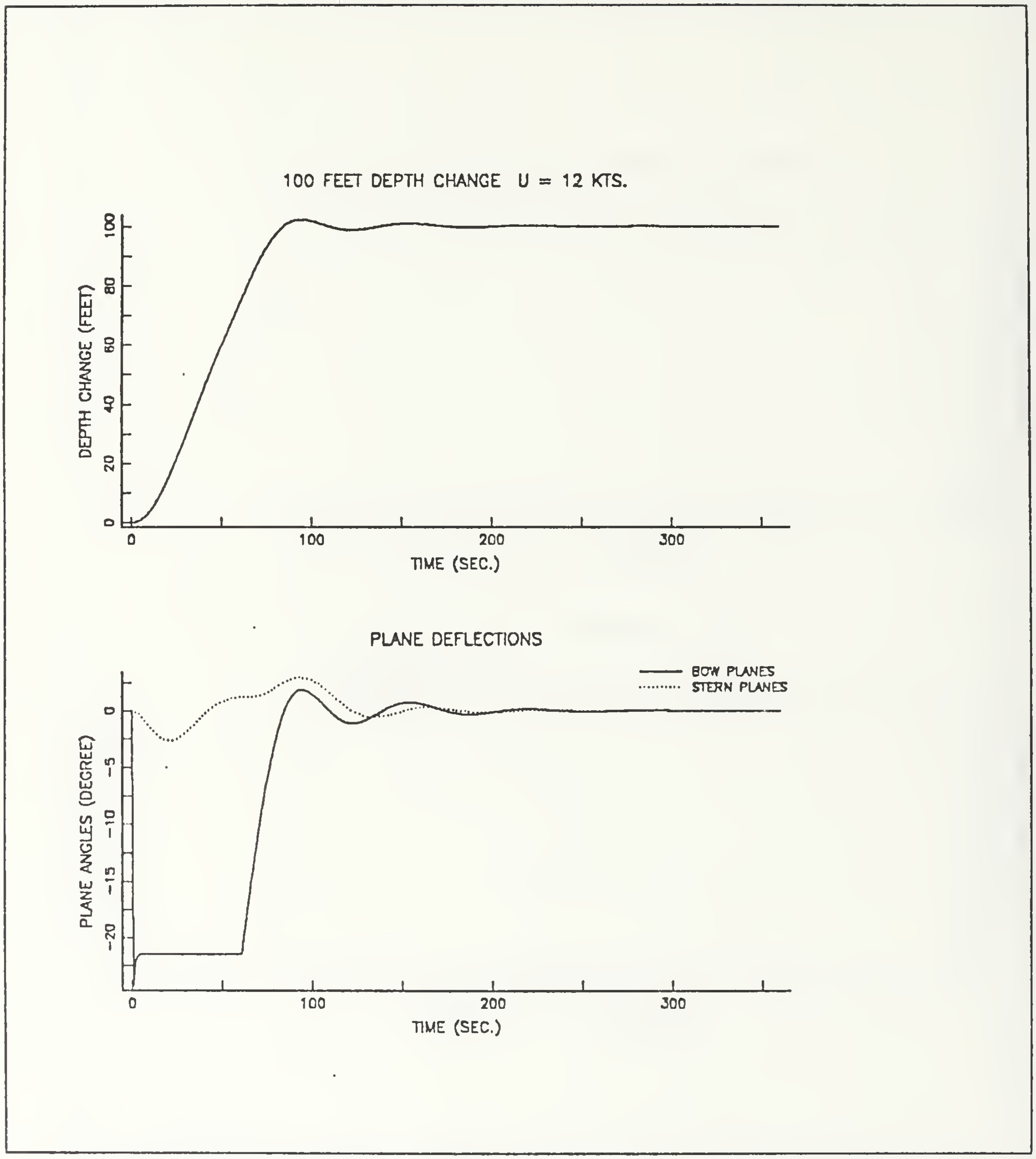

Figure 72. Compensated System Response to 100 ft. Depth Change U= 12 Kts.

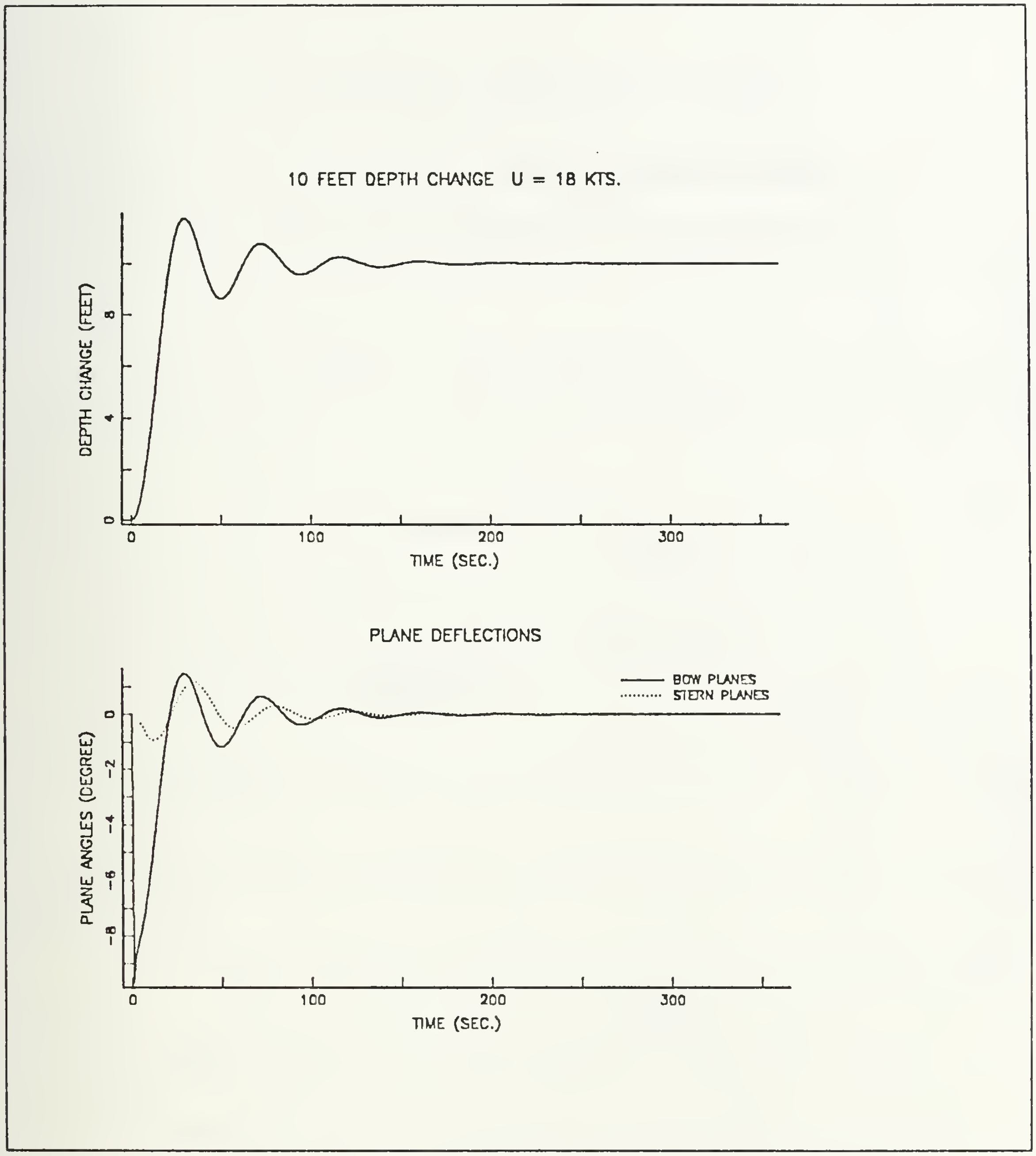

Figure 73. Compensated System Response to 10 ft. Depth Change U= 18 Kts.

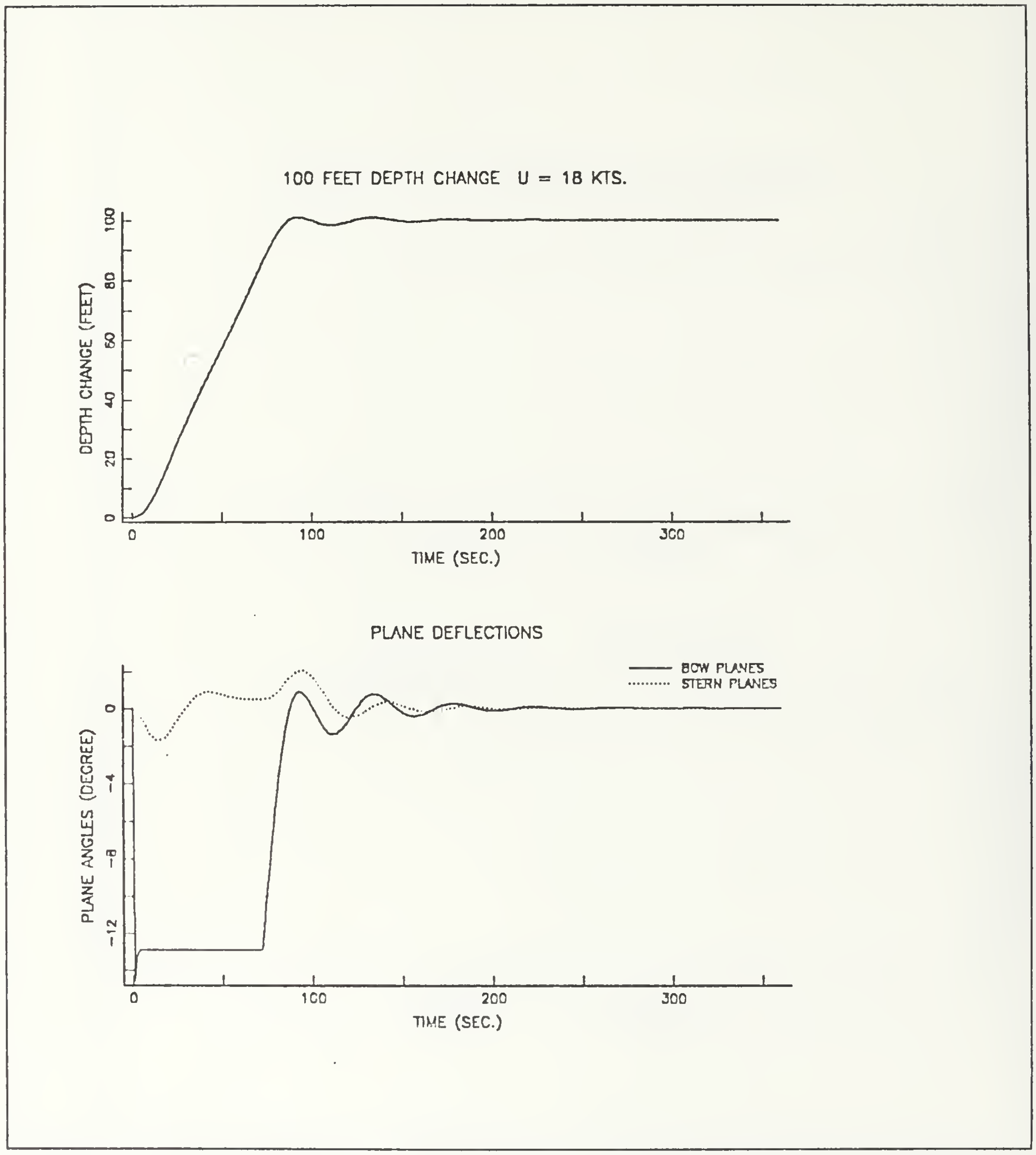

Figure 74. Compensated System Response to 100 ft. Depth Change U= 18 Kts.

IV. AUTOMATIC STEERING CONTROL

Turning characteristics of a surfaced submarine are very similar to a surface ship. But the situation in the submerged position shows big differences. Sail structure can be considered the main difference and the main source of rolling. But roll control is not considered in this study since the main purpose was to control depth change which is caused by the rudder.

In Chapter 2, three equations of motion were linearized and derived for the horizontal plane. Same equations will be used to design a steering control for a submerged submarine. But the algebraic loop problem has to be solved before using Mason's gain rule.

Three linear equations for horizontal motion are

$$\dot{v} = 1.89\dot{r} - 6.3\dot{p} - 0.291ur - 0.035up - 2.563x10^{-3}uv + 7.568x10^{-4}u^2\delta r \tag{43}$$

$$\begin{aligned}\dot{p} = & -0.679\dot{r} - 0.0584\dot{v} - 8.179x10^{-3}up - 9.347x10^{-3}ur - 3.942x10^{-4}uv \\ & + 3.942x10^{-5}u^2\delta r - 0.236\phi\end{aligned} \tag{44}$$

$$\begin{aligned}\dot{r} = & -6.553x10^{-3}\dot{p} + 6.767x10^{-4}\dot{v} - 6.767x10^{-3}ur - 4.511x10^{-6}up - 4.076x10^{-5}uv \\ & -1.631x10^{-5}u^2\delta r\end{aligned} \tag{45}$$

Substituting the highest derivative terms into each equation and after a great deal of algebraic work

$$\dot{v} = -0.437ur + 0.027up - 5.06x10^{-4}uv + 6.49x10^{-4}u^2\delta r - 2.389\phi \tag{46}$$

$$\dot{p} = 0.021ur - 9.8x10^{-3}up - 3.384x10^{-4}uv + 1.249x10^{-5}u^2\delta r - 0.378\phi \tag{47}$$

$$\dot{r} = -7.2x10^{-3}ur + 7.8x10^{-5}up - 3.888x10^{-5}uv - 1.595x10^{-5}u^2\delta r - 4.094x10^{-3}\phi \tag{48}$$

A. DESIGN SPECIFICATIONS

In general, the required time to achieve a course change in a ship depends on

1. The forward speed,
2. The difference between previous and commanded course,
3. Applied rudder angle,
4. Rudder area,

5. The length and hull structure of the ship.

The submerged condition is also a very important aspect since the required turning time is about three times greater for a submerged submarine than a surfaced one. Especially at lower speeds, it is very hard to achieve the desired course for a submerged submarine.

It is concluded that for the speeds which are less then 10 Kts., a control system must achieve every 10 degrees course change in 30 seconds. This allows 9 minutes to complete a 180 degrees turn and it is very reasonable for a low speed submerged submarine. For higher speeds this time limit would be 20 seconds. It is also considered that more than 2.5 degrees overshoot is not acceptable.

The mechanical limit angle for rudder is also 35 degree and has to be considered in the design process.

B. DESIGN

The cascade compensation method will be used for the horizontal motion. Since the aim of this chapter is to design a basic steering control, the roll response will not be investigated. The yaw response to the rudder is the only input-output relation of interest at this point. Figure 75 represents a control model for the horizontal motion.

A signal flow graph is given on Fig. 76 for the linearized equations of horizontal motion. The corresponding numbers for symbols in the flow graph are given below:

$a = 0.437u$

$b = 0.027u$

$c = -5.0x10^{-4}u$

$d = 6.5x10^{-4}u^2$

$e = -2.39$

$f = 0.021u$

$g = -9.8x10^{-3}u$

$h = -3.4x10^{-4}u$

$i = 1.25x10^{-5}u^2$

$j = -0.378$

$k = -7.2x10^{-3}u$

$l = -7.8x10^{-5}u$

$m = -3.9x10^{-5}u$

$n = -1.6x10^{-5}u$

$o = -4.1x10^{-3}$

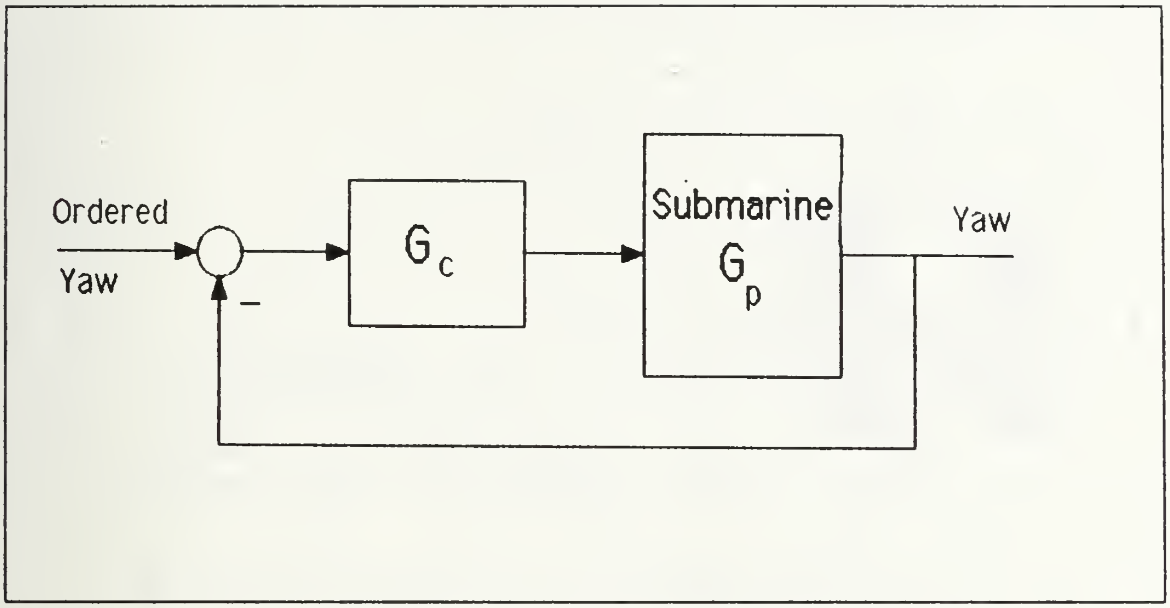

Figure 75. Cascade Compensated Control Model for Horizontal Motion

1. Decoupling

Since this signal flow graph creates 13 loops to be handled, it is considered to take u as 10 ft./sec. at the beginning of the calculation in order to reduce the amount of required algebraic work.

Applying Mason's gain rule to the signal flow graph given in Fig.76, the input-output relation for yaw will be as follows

$$\frac{yaw}{\delta r} = G_p = \frac{-1.6x10^{-4}s^3 - 5.4x10^{-5}s^2 - 6.8x10^{-5}s - 6.5x10^{-6}}{s^5 + 0.175s^4 + 0.3885s^3 + 0.022s^2 - 5.77x10^{-5}s} \tag{49}$$

In factorized form, the same equation will be

$$G_p = \frac{-1.6x10^{-4}(s + 0.1176 + j0.6222)(s + 0.1176 - j0.6222)(s + 0.102)}{s(s + 0.0587 + j0.6152)(s + 0.0587 - j0.6152)(s + 0.0602)(s - 0.0025)} \tag{50}$$

As can be seen from the transfer function, there is a real pole in the right half plane which is very near to the origin. The characteristic equation roots are

$-0.058 \pm j0.615$

$-0.018 \pm j0.020$

-0.1

-0.022

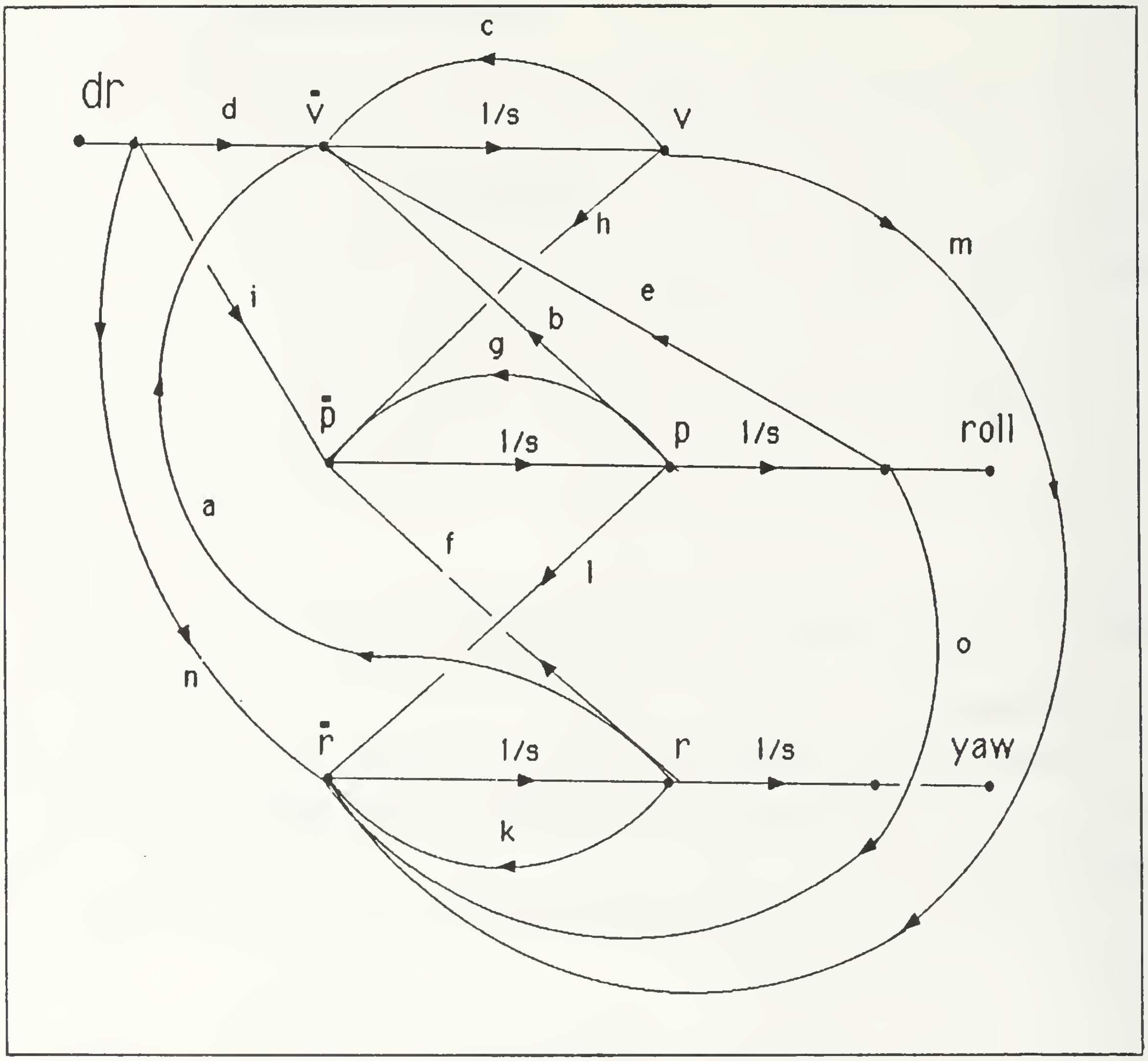

Figure 76. Signal Flow Graph for Horizontal Equations of Motion

The root locus plot and open loop Bode plot for G_p are given in Fig.s 77 and 78.

The root locus plot shows that there is a very small gain range where the system is stable. The Bode plot also agrees that the system is unstable with 15 degrees negative phase margin and there really is a small gain range over which the system will be stable with a small damping.

Since it is obvious that a gain adjustment will not be enough to get the desired response out of the system, the cascade compensation will be required. In order to

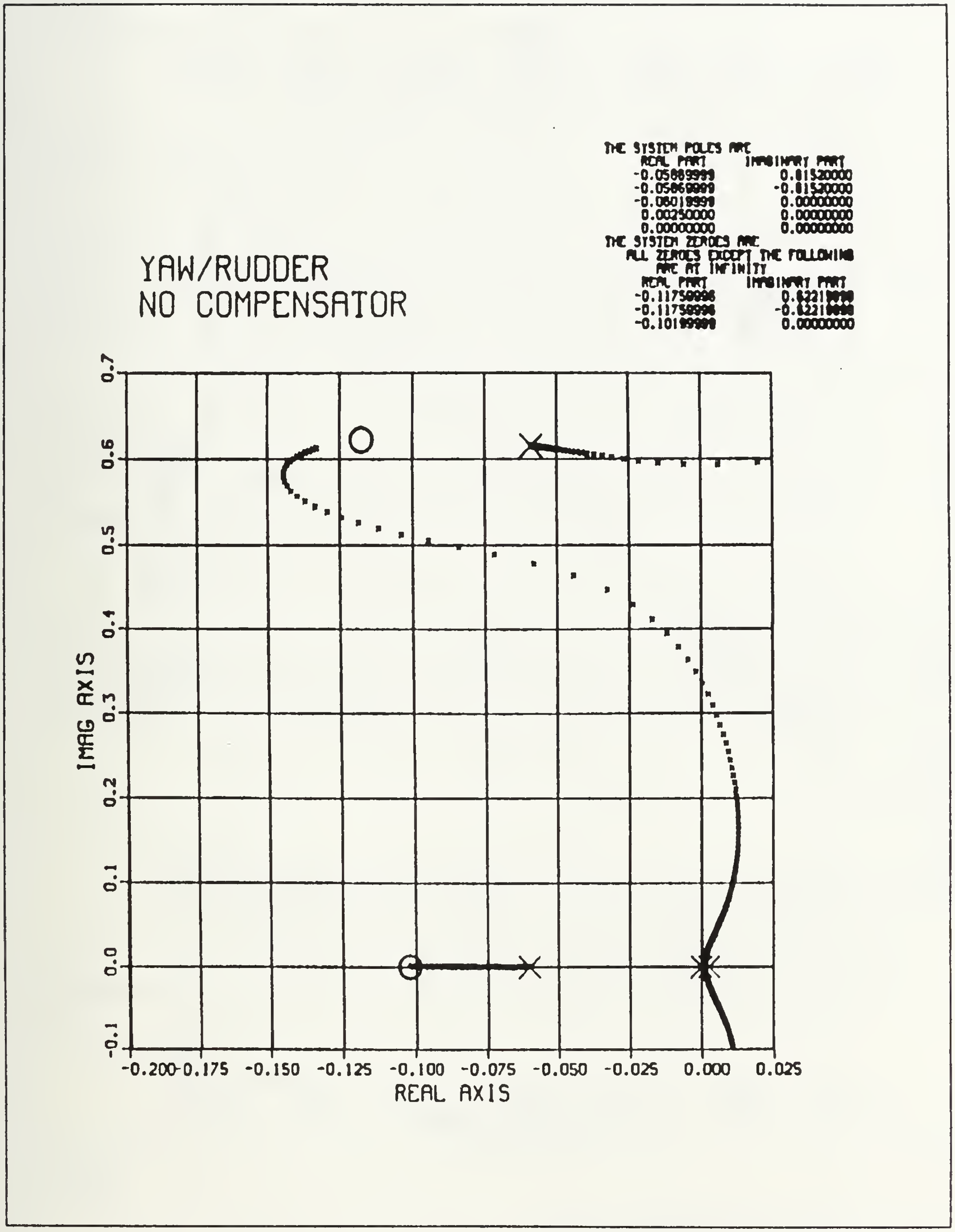

Figure 77. Root Locus Plot for $\frac{yaw}{\delta r}$.

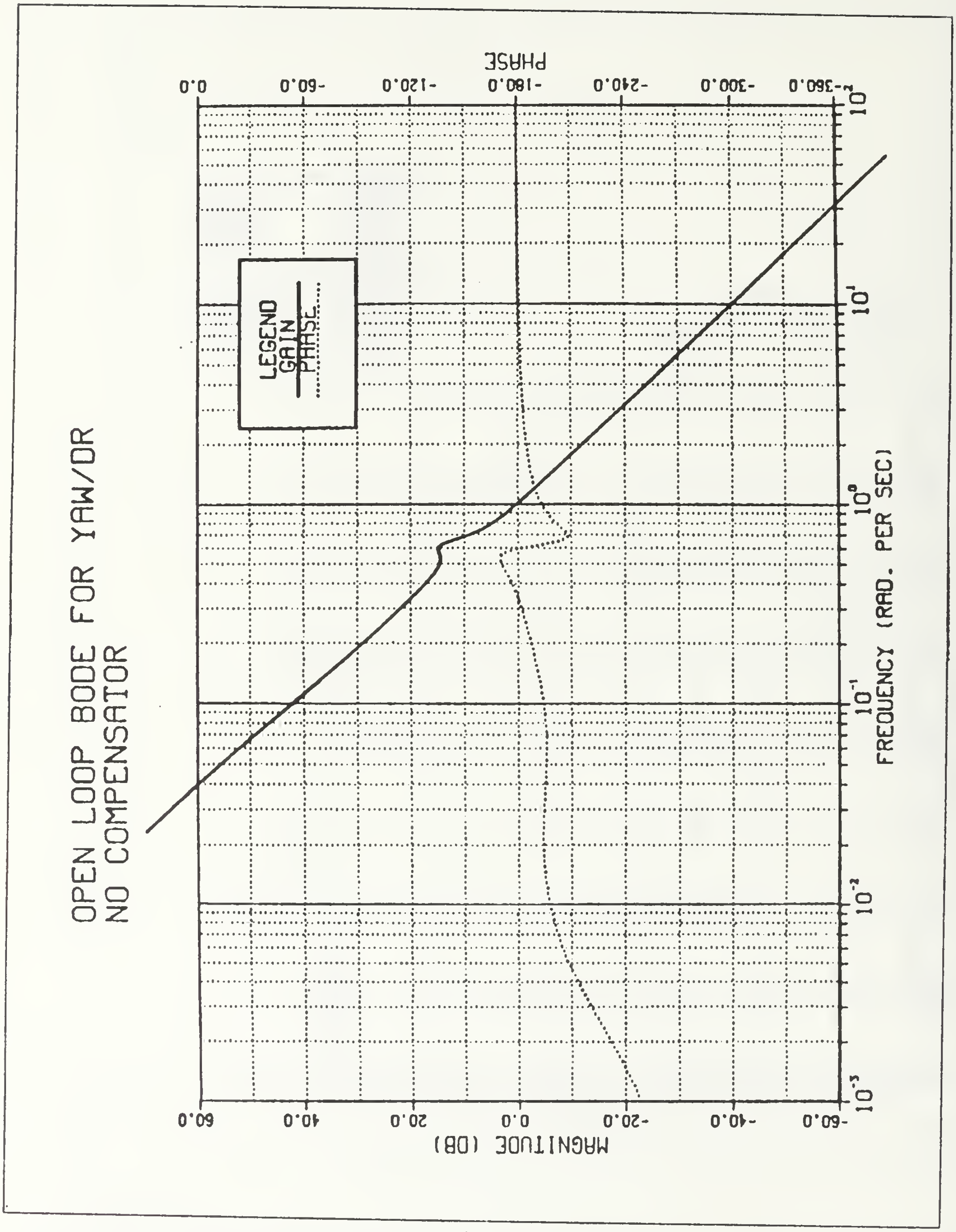

Figure 78. Open Loop Bode Plot for $\frac{yaw}{\delta r}$.

increase phase margin a first order lead compensator is to be added to the forward path. Using cascade compensator design techniques, the first trial would be

$$g_c = \frac{0.1}{0.01}\frac{s+0.01}{s+0.1} = 10\frac{(s+0.01)}{s+0.1} \tag{51}$$

There will be no gain adjusment at this point.

The Bode plot for the compensated system is given in Fig. 79. The phase margin is 40 degrees and that is about the maximum phase which can be acquired with only one cascade compensator. The root locus plot shows that the compensator has moved a lot of root locations to the left half plane and it is given in Fig. 80.

Since the root locus and Bode plots show very reasonable damping and stability, it is considered that the compensated system is ready for the simulation. The DSL simulation program which is used for the validation of the linear model (Appendix E), is updated with the designed cascade compensator. This program is given in Appendix G including required modifications.

2. Simulation

The same plane actuators which were used for the stern and bow planes, are used for the rudder in the simulation program. But since the input is totally different, it is necessary to design a different criteria for the limiter. The first run was made without any limiter, then using trial and error, the best limiter choice is appeared to be:

lim = 0.070	when $u < 12$ Kts.
lim = 0.050	when $12 < u < 18$ Kts.
lim = 0.035	when $u > 18$ Kts.

The complete model which is used in the simulation program is given in Fig. 81.

Test run results which were achieved with different sets of speed and course, are given in Figures 82 to 90. The roll response is also given in order to make sure that submarine does not exceed maximum allowable roll limits. Each run is explained briefly below:

Run No. 1:

Using 6 Kts. forward speed and 10 degrees course change, the maximum required rudder angle was 82 degrees. Therefore it is necessary to use a limiter on the error signal.

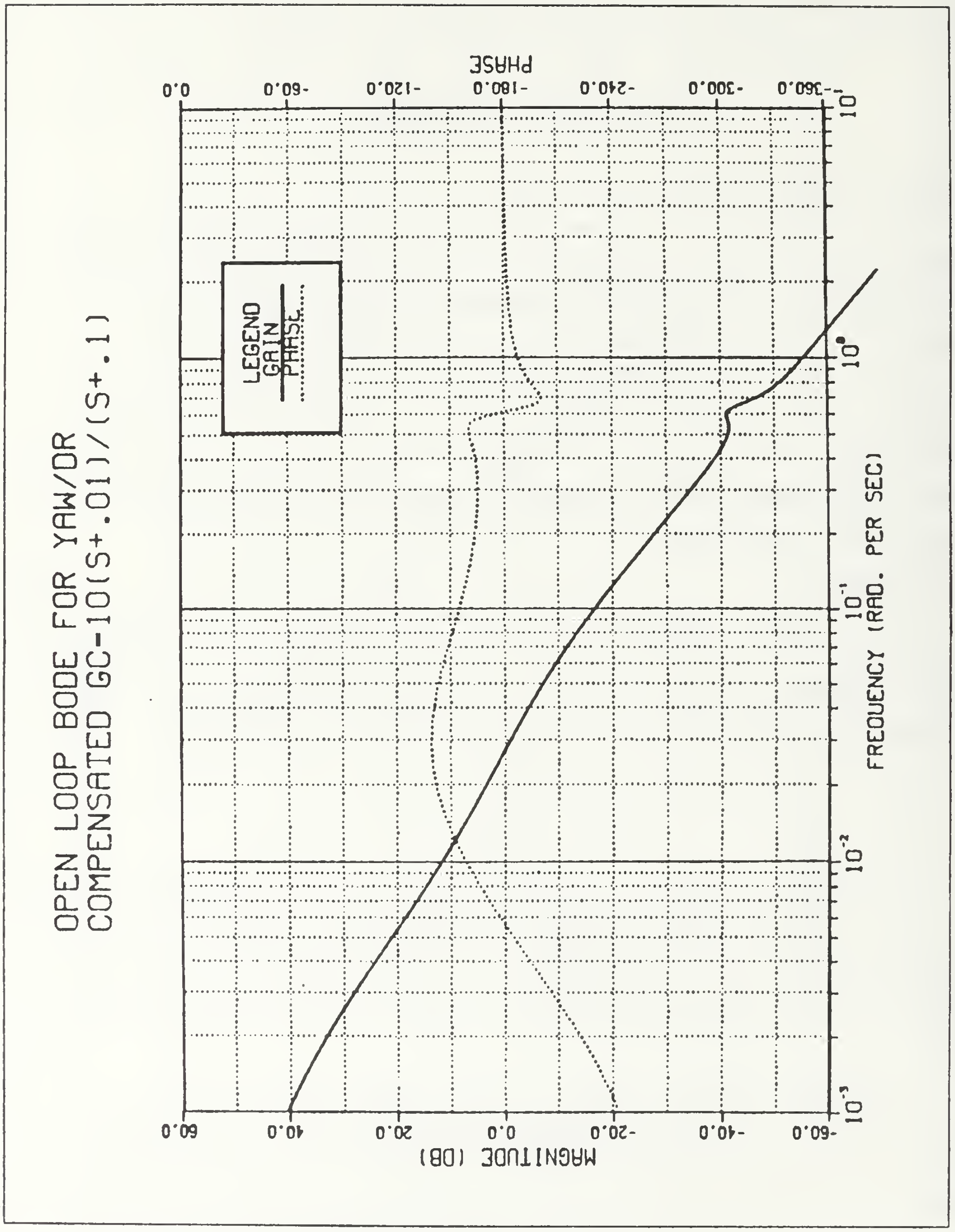

Figure 79. Open Loop Bode Plot for G_cG_p .

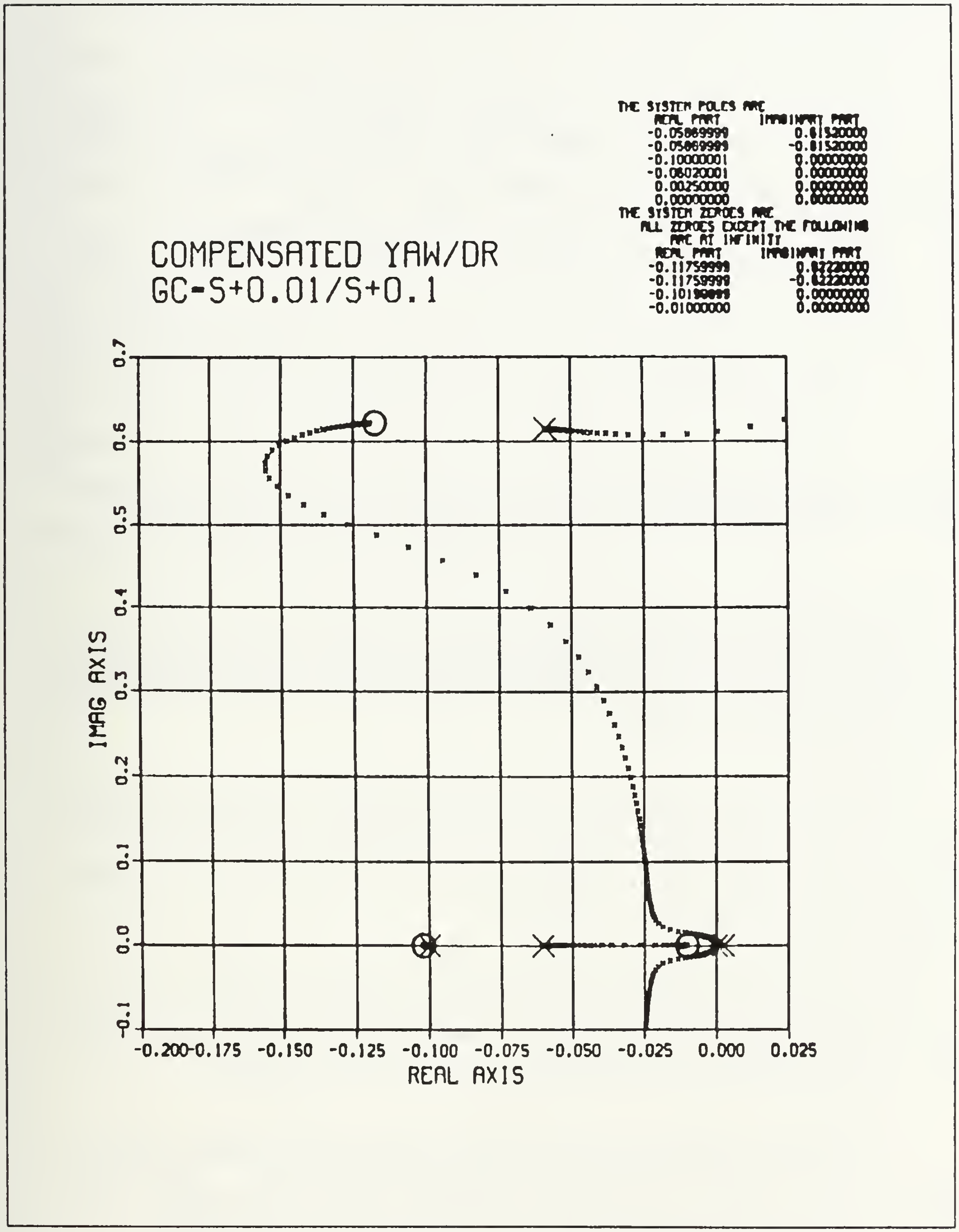

Figure 80. Root Locus Plot for $G_c G_p$.

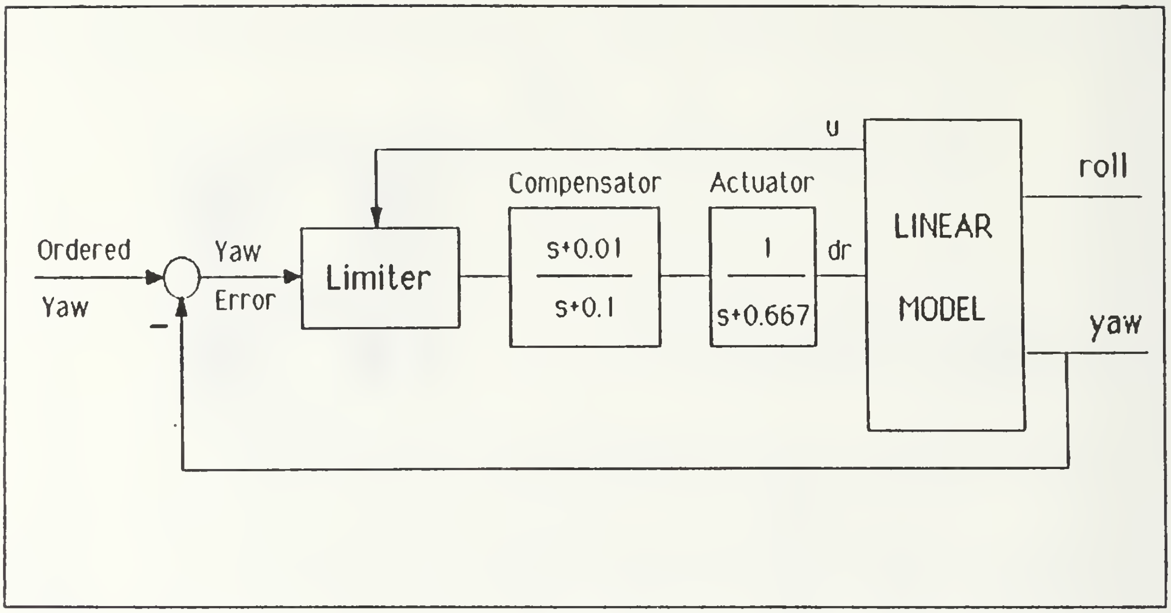

Figure 81. Block Diagram for Compensated Linear Model in Horizontal Motion

Run No. 2:

Using the limiter which was mentioned above and for 15 degrees course change, the maximum required rudder was 33 degrees. It takes 46 second to get 15 degrees course alteration with only 2.5% overshoot.

Run No. 3:

This time the system is tested with the same speed for a 90 degree course change which is one of the commonly used commands in a submarine. It takes 214 sec. to execute this command which is in the specified limits. The overshoot is 1.5% and maximum required rudder angle is also 33 degrees.

Run No. 4:

In order to get the speed range in which the compensated system stays in the required specifications, the forward speed is increased to 10 Kts. For a 15 degree course change the time to execute the command is 22 sec with 1.9 feet overshoot.

Run No. 5:

For 90 degrees course change with 10 Kts. forward speed the time to execute the command is 103 sec. with 1.5% overshoot. Maximum required rudder angle is still 33 degrees. As can also be seen from Fig. 86 the maximum roll is about 3 degrees.

Runs No.6 and 7:

These runs were made with 15 Kts. forward speed for 15 and 90 degrees course changes. While the required time decreases with increasing speed, the overshoot increases. But the results are still in the specification limit as can be seen from Fig.s 87 and 88. The maximum roll angle is 5 degrees for 15 Kts. forward speed which is also reasonable. The maximum required rudder angle is 23 degrees for this case.

Runs No. 8 and 9:

These runs were made with 20 Kts. forward speed also dictated the speed range for the compensated system because it becomes too oscillatory after 20 Kts. which is not desirable. It is necessary to add another cascade compensator to the forward path in order to get enough damping for speeds higher than 20 Kts. It is to be noted that using a limiter also helps to keep the roll angles small. In this case the maximum rudder angle is only 16 degrees because of the limiter effect. With this limited rudder angle the maximum roll is only 8 degrees. Even though it is not intended to control the roll, the limiter supplies an indirect control on the roll response.

Finally it is considered that the designed automatic control for the linearized horizontal motion using cascade compensator design techniques, is satisfactory for the speed range of 6 to 20 Kts. This design should be checked in the actual non-linear system.

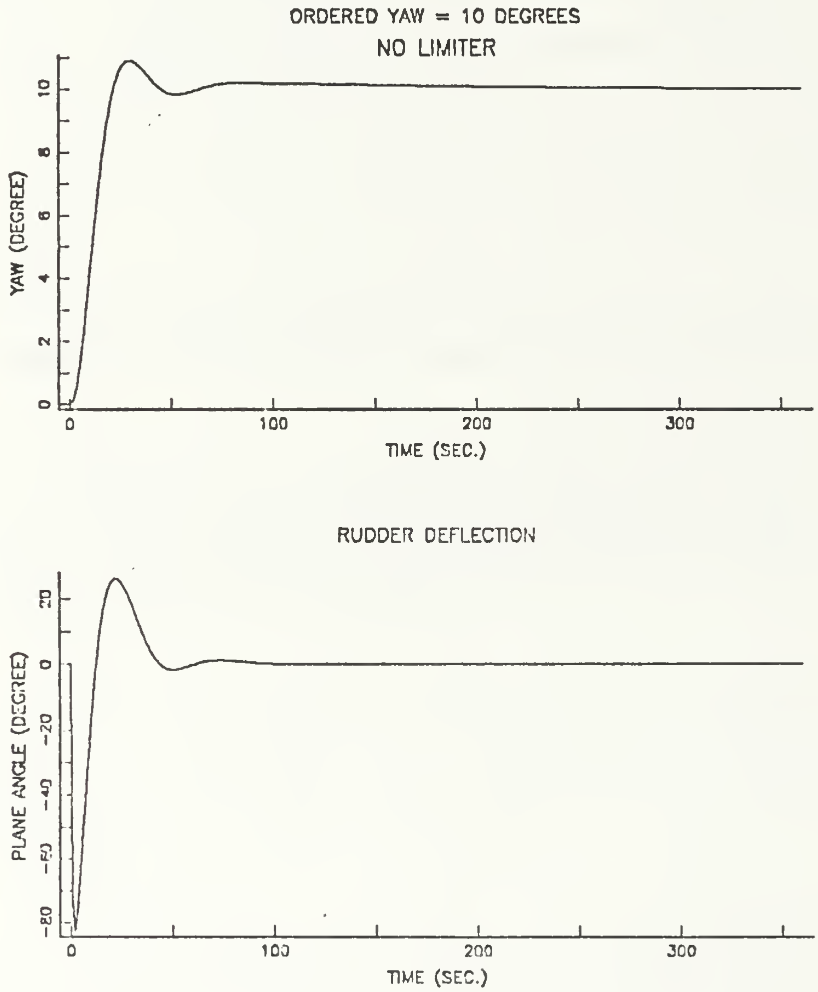

Figure 82. Yaw and Roll Response to 10 Degree Course Change, No Limiter

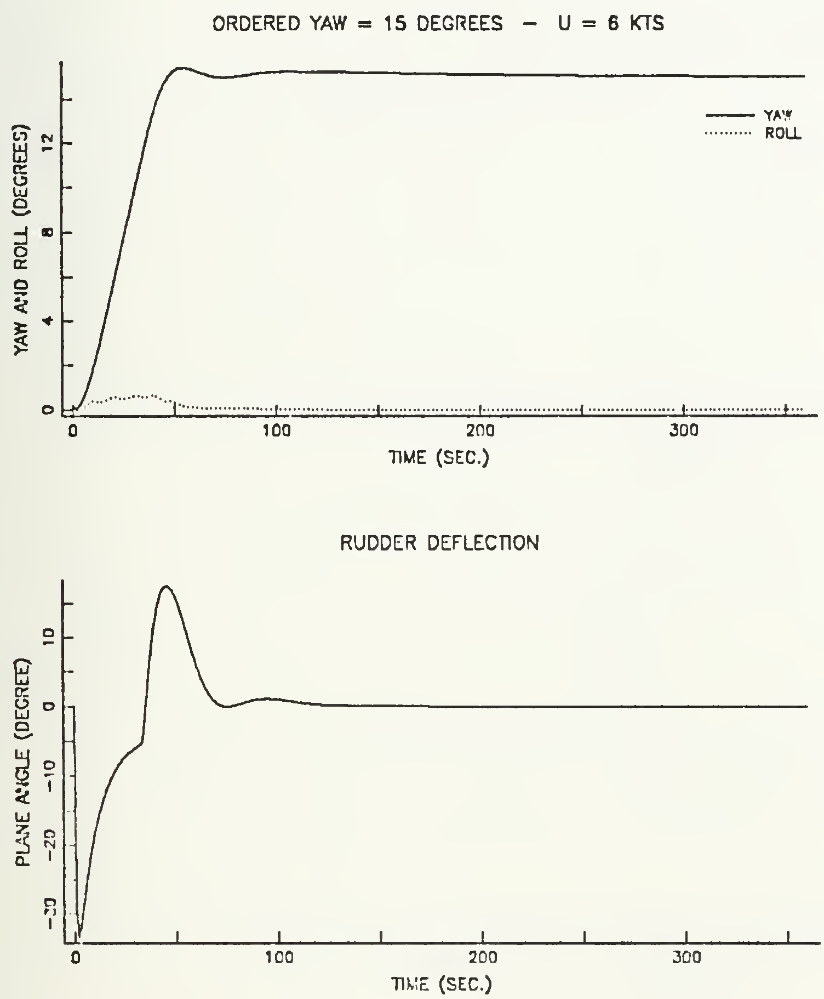

Figure 83. Yaw and Roll Response to 15 Degree Course Change, With Limiter

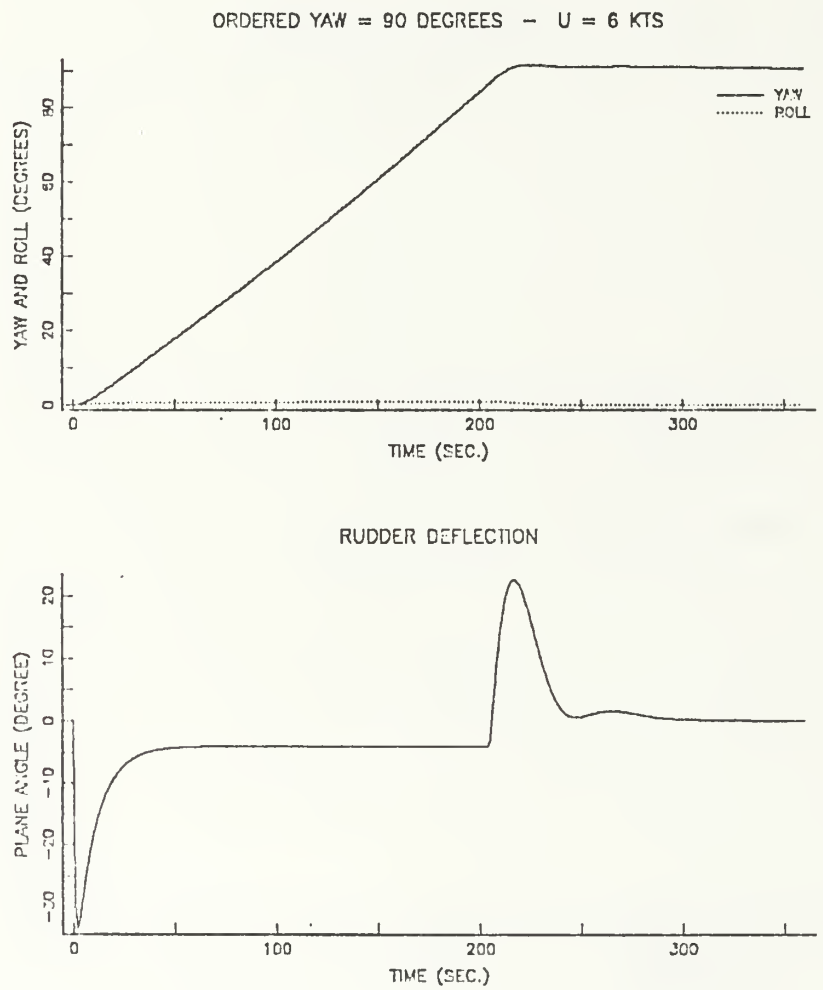

Figure 84. Yaw and Roll Response to 90 Degree Course Change. U = 6 Kts.

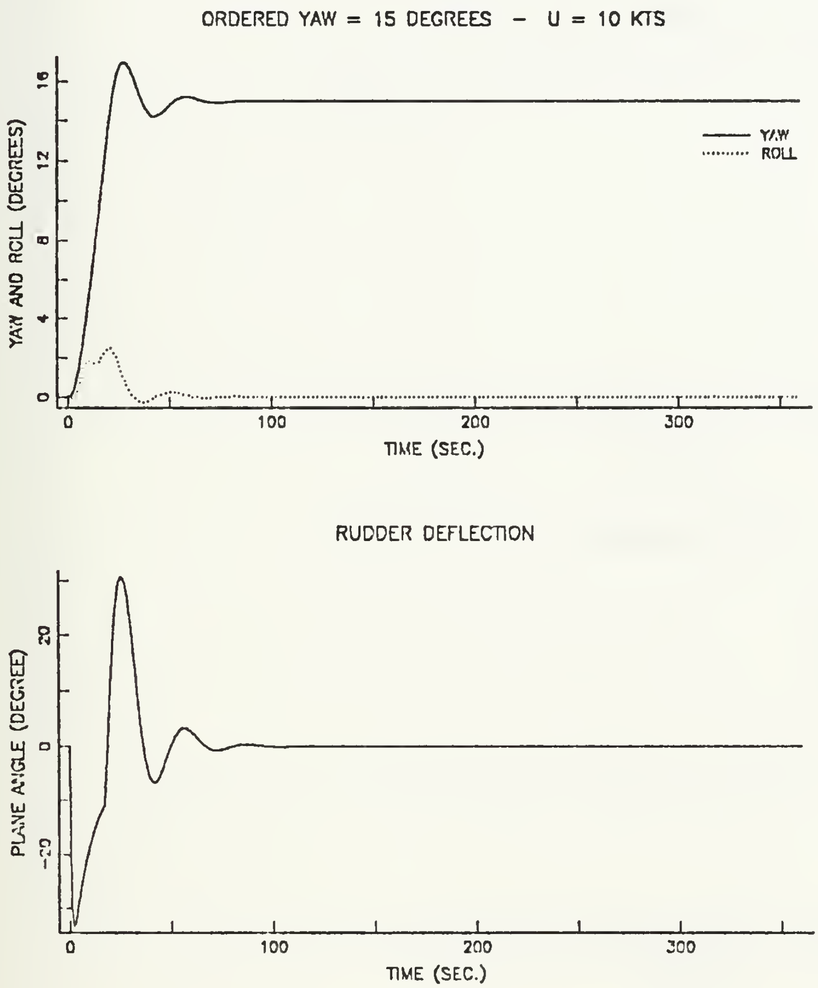

Figure 85. Yaw and Roll Response to 15 Degree Course Change. U = 10 Kts.

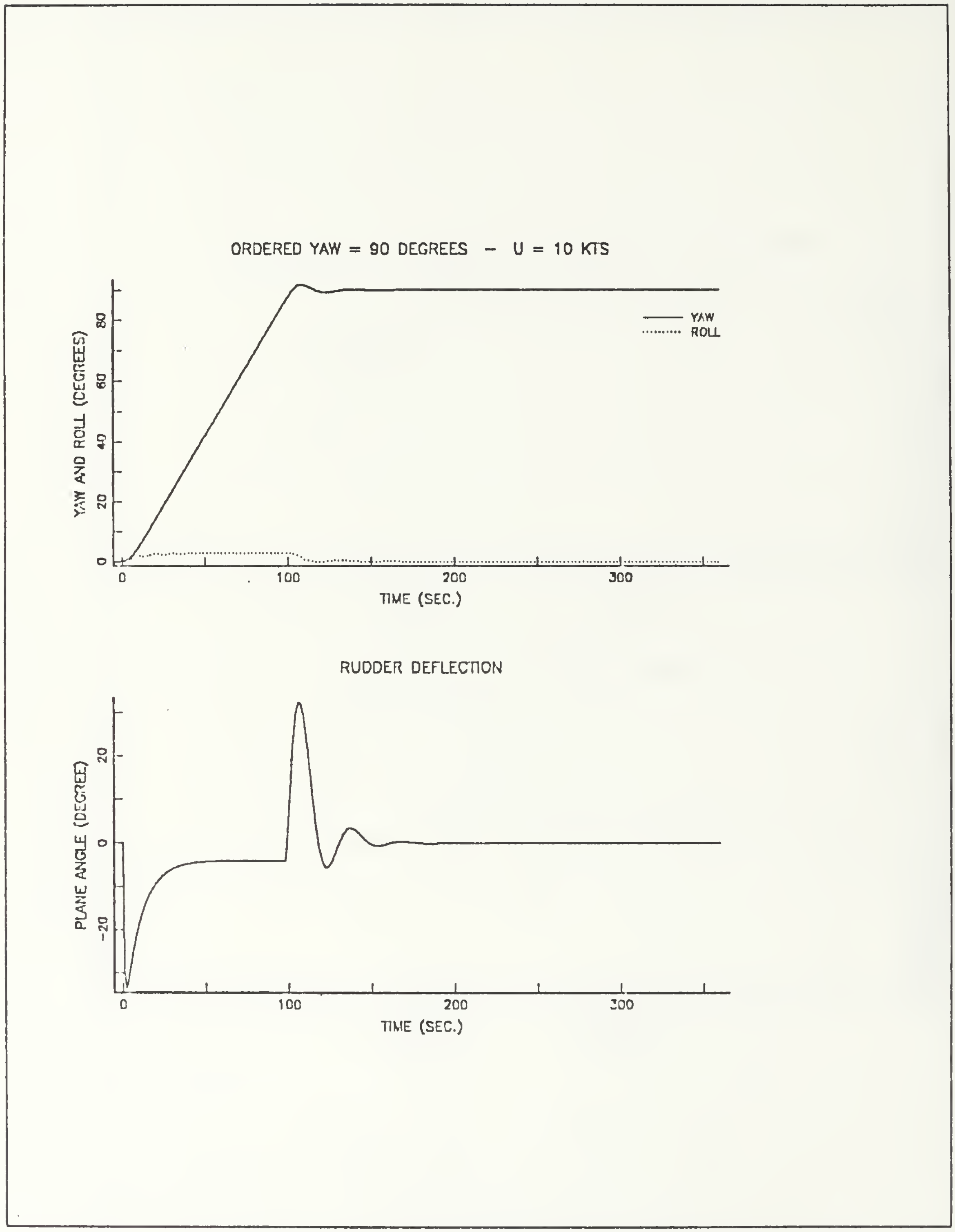

Figure 86. Yaw and Roll Response to 90 Degree Course Change. U = 10 Kts.

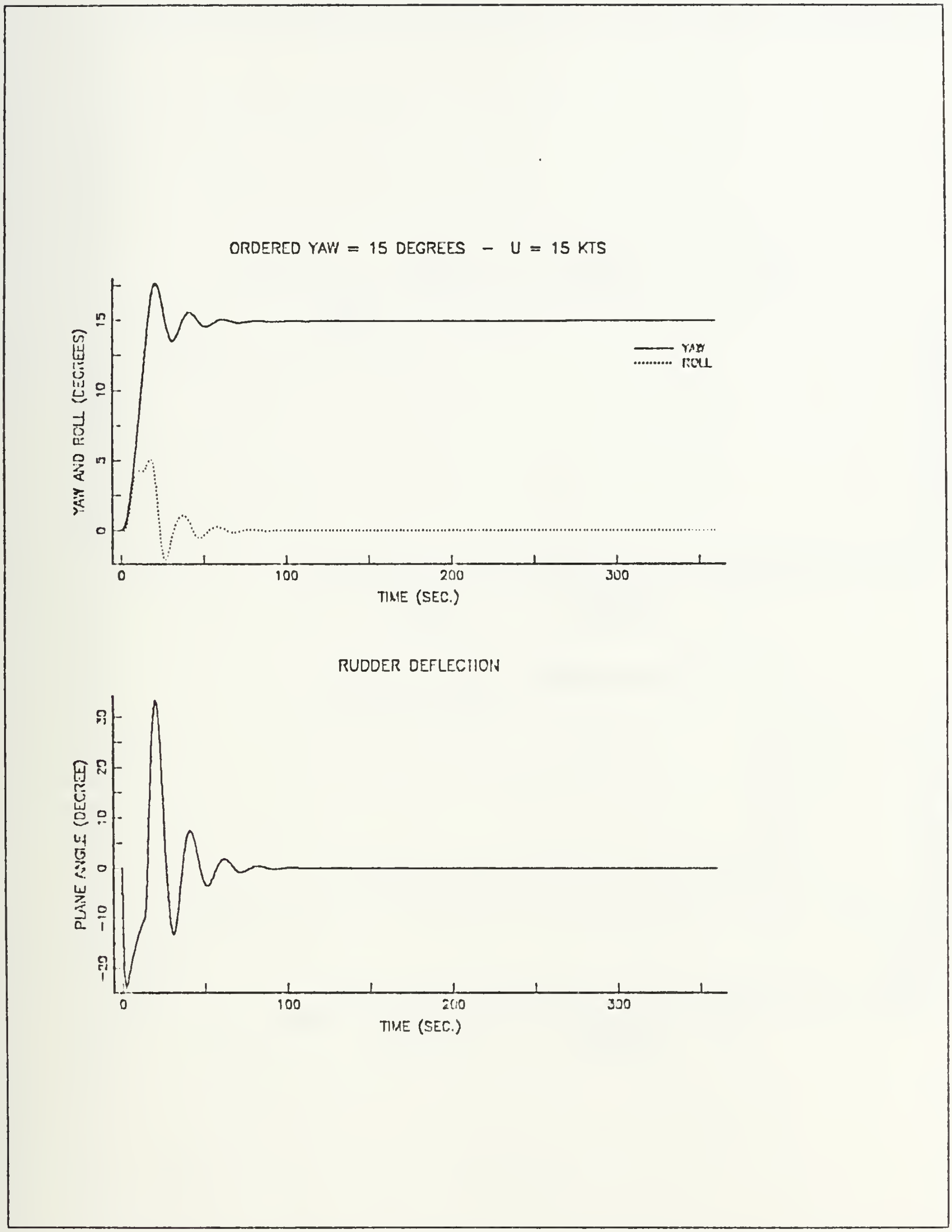

Figure 87. Yaw and Roll Response to 15 Degree Course Change. U = 15 Kts.

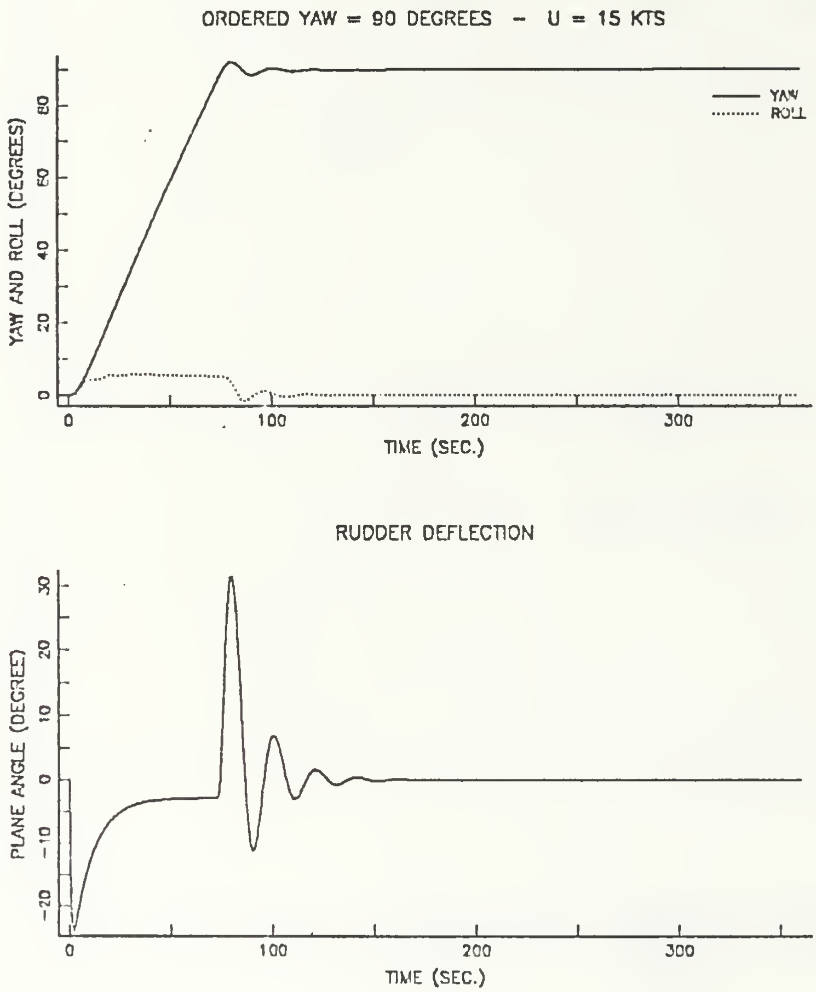

Figure 88. Yaw and Roll Response to 90 Degree Course Change. U = 15 Kts.

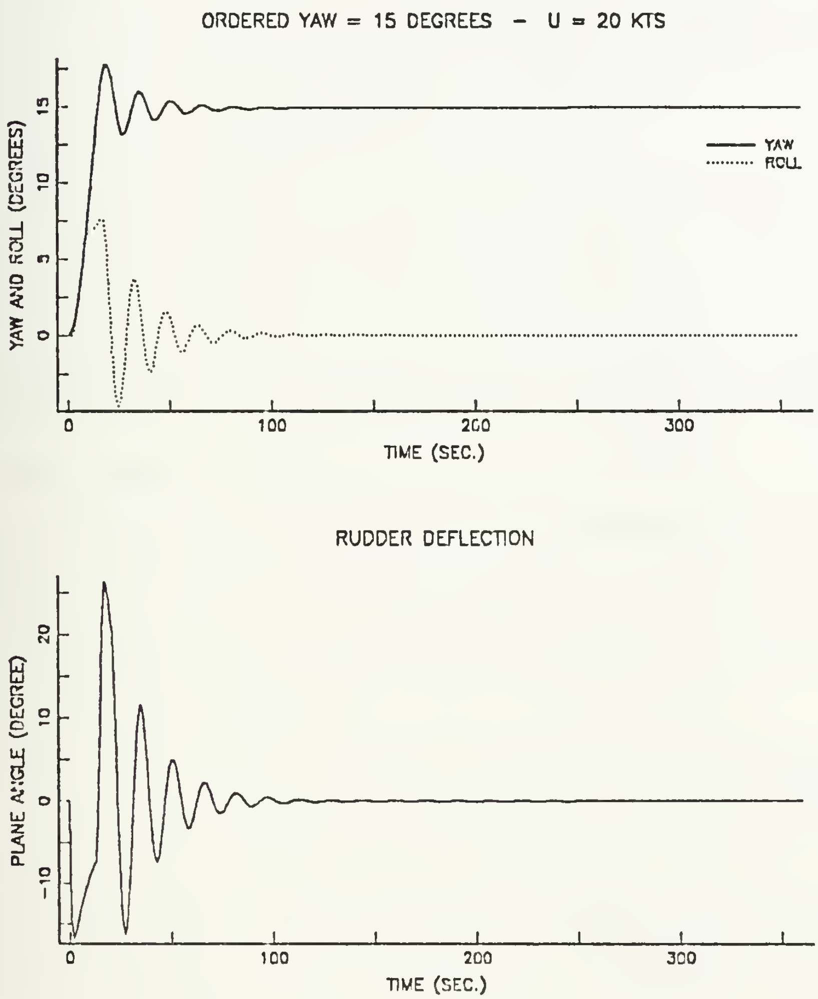

Figure 89. Yaw and Roll Response to 15 Degree Course Change. U = 20 Kts.

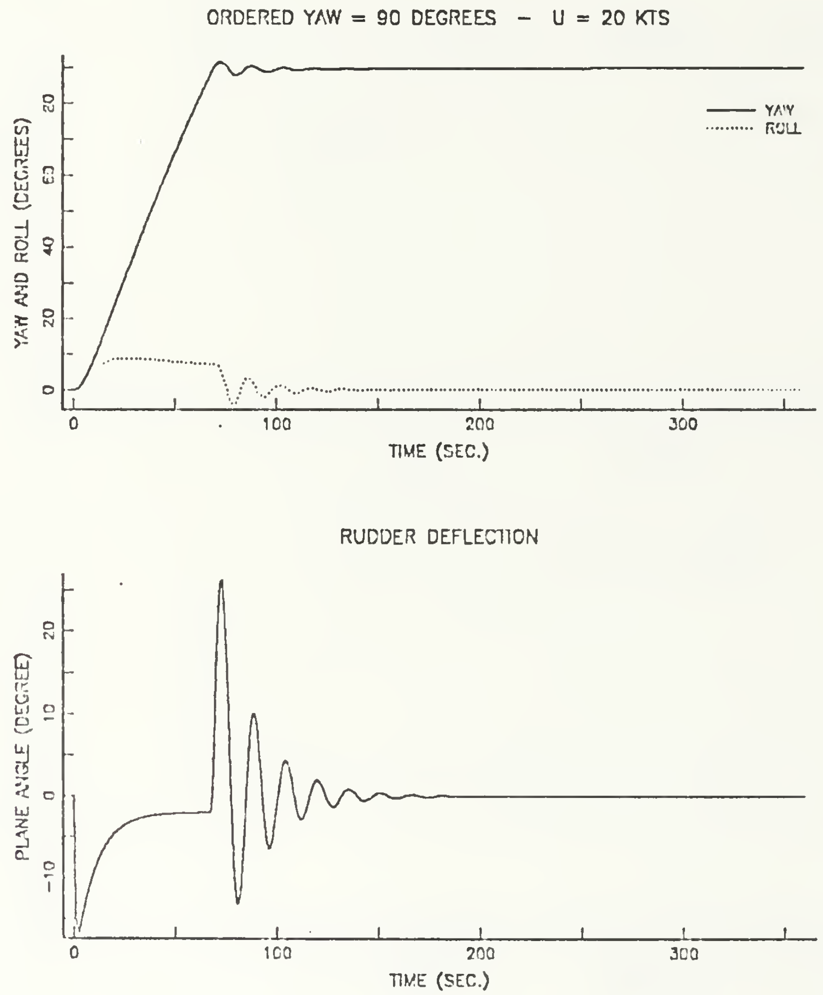

Figure 90. Yaw and Roll Response to 90 Degree Course Change. U = 20 Kts.

V. VALIDATION OF THE COMPENSATED NON-LINEAR MODEL

The main purpose of this study was to show that it is possible to design a compensator based on the linearized version of a non-linear model and then to compensate the actual non-linear model with this designed compensator. In the previous chapters the required compensators were designed for the linear models on vertical and horizontal motions. These compensators had to be checked with the actual non-linear model to see that the system will really work with them.

The complete DSL simulation program for the non-linear model was already written and used by Ref. 3 and Ref. 5. It is also used for this study to compare linearized models with the non-linear model. In order to check the validity of automatic control systems, the DSL simulation program is to be modified including the compensator and limiter algorithms in it. The modified version of the DSL program for the compensated non-linear model is given in Appendix I.

A. SIMULATION

For the test runs to check the designed compensators, the same limiter values are used. Since the actual and commanded velocities (U and UC) are two different parameters and U is always somewhat less then UC, it is concluded to take actual speed U as the parameter for the limiters. This will give more accurate plane deflections depending on actual forward speed.

The non-linear simulation program was run at 6, 10, 12, 15 and 20 Kts. for various depth, pitch and yaw commands. A diving submarine can give hundreds of maneuver variations in three dimensional motions. Since it is not possible to include all of them; only the most common commands and the commands which were used in Chapter 2,3 and 4 are included for comparison purposes.

The trim values for the ordered speed are carefully calculated from Ref. 5 and implemented in the non-linear model.

Test run results which were achieved for different sets of speeds and commands are given in Figures 91 to 106. Each run is explained briefly below:

Runs No. 1-4:

The simulation program was run for 10 and 100 ft. depth changes at 6 Kts. commanded forward speed. As can be seen from Fig. 91 the non-linear model completes a 10 ft. depth change 10 sec. after the linear model does. It completes a 100 ft. depth change 20 sec. after the linear model and overshoots for both case are on the specification limits. There is also a 1 ft. steady state error for both cases.

At the same speed the simulation program was run for 15 and 90 degree course changes. As can be seen from Fig. 92 the non-linear model takes about 100 seconds to achieve a 15 degree course change with no overshoot and a slight undershoot. Also the required time to make a 90 degree course change is more than 360 seconds for the non-linear model. These two case are also non-acceptable.

The main reason for this failure is the decreasing forward speed due to the plane deflections. The forward speed, depending on the amount of the rudder deflection, actually drops up to 4 Kts. while achieving a course change maneuver. The same thing also happens for a depth change maneuver due to the stern and bow planes. The compensators were designed for actual 6 Kts. and higher speeds and they do not meet the specifications for less than 6 Kts. forward speed.

Under these circumstances no more investigations were made at 6 Kts. At this point it is concluded to operate at 10 Kts.

Runs No. 5-8:

As can be seen in Fig. 93 the non-linear model completes a 10 ft. depth change in 42 seconds with 1 ft. overshoot at 10 Kts. It completes a 100 ft. depth change in 106 seconds with 3.5% overshoot. These numbers satisfied the required specifications and they are nearly the same as for the linear model with a little time lag.

As can be seen in Fig. 94 the non-linear model makes a 15 degree course change in 25 seconds with 0.8 degree overshoot which is less than the linear case. For a 90 degree course change the time is 205 seconds with no overshoot. Again due to the rudder drag force, the time to reach the commanded course is much larger but more realistic than the linear case. Since the constant speed assumption which is used for the linear model is no longer valid for large perturbations, this is really expected. On the other hand the designed cascade compensators can still control the actual non-linear system effectively enough and in fact, with less overshoot which is very important from the point of this study.

Runs No. 9-12:

In order to have an idea about speed deviation due to the plane deflections, four runs were performed for 10 and 100 feet depth change and 15 and 90 degree course change at 10 Kts. As can be seen in Fig. 95 there is an appreciable difference between the drag forces created by rudder and bow, stern planes. This is expected since the rudder has a lot more surface than the other planes. For a 15 degree course change the forward speed drops abruptly to 9.3 Kts. and goes back to it's original value in a relatively small time. For a 90 degree course change, the forward speed drops up to 9 Kts. and stays almost constant for about 150 seconds which is the greatest cause for the slower course change rate.

Runs No. 13 and 14:

Since the linear model assumes no cross-coupling between vertical and horizontal motion, these runs are performed only for the non-linear model. Cross-coupling effects on depth and pitch angle are shown in Fig. 96 at 10 Kts. for 15 and 90 degree course change commands. For both commands, the submarine stays in the 5 feet depth and 2 degrees pitch error limitations.

Run No. 15:

One of the most difficult maneuver for a submarine is to change depth while achieving a course command. Results of such a maneuver are given on Fig. 97 and Fig. 98. A simulation run for simultaneous 90 degree course and 100 ft. depth change commands, shows that the time to reach 90 degree course change is about 40 seconds longer than the usual condition but it does not affect the depth change. Because of the rudder effect only a small depth error appears until the submarine settles on the desired course.

As can be seen in Fig. 98 the non-linear submarine's roll and pitch responses are somewhat non-regular but still in reasonable limits for this case. The forward speed deviation due to the plane drag forces is also given in Fig. 98. Speed drops up to 8.3 Kts. and this gives an explanation for lower course change rate.

As a result for this run, even though the designed control systems interact, they can work well simultaneously.

Runs No. 16 and 17:

In order to be able to compare fixed rudder effects on depth and pitch angle, two simulations were performed for 15 degree and 35 degree fixed rudder commands in the same fashion as in Chapter 4 at 12 Kts. For 15 degree rudder the pitch and depth errors stay in specified limits but for 35 degree rudder these errors are not allowable. Simulation results for this case are given in Fig. 99.

Runs No. 18 and 19:

Figure 100 gives the simulation results for a 15 and a 90 degree course change at 15 Kts. for both linear and the non-linear models. The yaw response for 15 degree course change is almost the same as the response for 10 Kts. with a little more overshoot and oscillation. On the other hand the non-linear model shows a better response with less overshoot for both cases.

Runs No. 20 and 21:

Figure 101 gives the simulation results for a 10 and a 100 feet depth change at 15 Kts. Surprisingly there is almost no difference between linear and non-linear model for the 10 feet depth change. But for the 100 ft. depth change the non-linear model has a faster response than the linear model. This is unusual and created by different limiter behaviors on bow planes at this specific forward speed.

Runs No. 22 and 23:

In a real submarine a depth change command usually comes with a pitch command in order to reduce the time to get the desired depth. Figure 102 gives the results of such a command for 100 feet depth change with 5 degree down pitch angle at 6 and 15 Kts. For both cases the submarine reaches the desired depth 35 seconds before the case for which no pitch command is given. But as a trade-off the overshoots are over 10%. Also the pitch command has to be reduced to zero before the desired depth is reached in order to avoid too much overshoot and a steady state error on depth. This is done 10 feet before the desired depth is reached. for 6 Kts and 50 feet before for 15 Kts.

Runs No. 24 and 25:

Finally the compensated non-linear system was checked at 20 Kts. For a 15 and a 90 degree course change, the yaw responses of the compensated submarine are given in Fig. 103. Once again the non-linear model gives a better but slower response then the linear model. For the 15 degree course change the yaw responses of both models become too oscillatory due to the high speed. But the compensator still works well enough to control the submarine.

Runs No. 26 and 27:

Figure 104 gives the compensated submarine depth responses for a 10 and a 100 feet depth change at 20 Kts. There is a 0.8 feet steady state error for both cases which is created by the system dynamics due to the high speed. The control system design is based on 10 ft./sec. (6 Kts.) forward speed. At 20 Kts. the transfer functions which describes the submarine dynamics might have very different characteristics. Consequently it is concluded that the upper speed limit for this design is 20 Kts. In fact, the control system works up to 25 Kts. without exceeding design specification limits.

The compensated submarine pitch responses for the same runs are given in Fig. 105. The linear and non-linear models show very similar pitch behavior and pitch angles do not exceed the given 2 degree limit even for this high speed.

As a result of this chapter it has been shown that the designed automatic control system for the linearized model can also work effectively on the actual non-linear model.

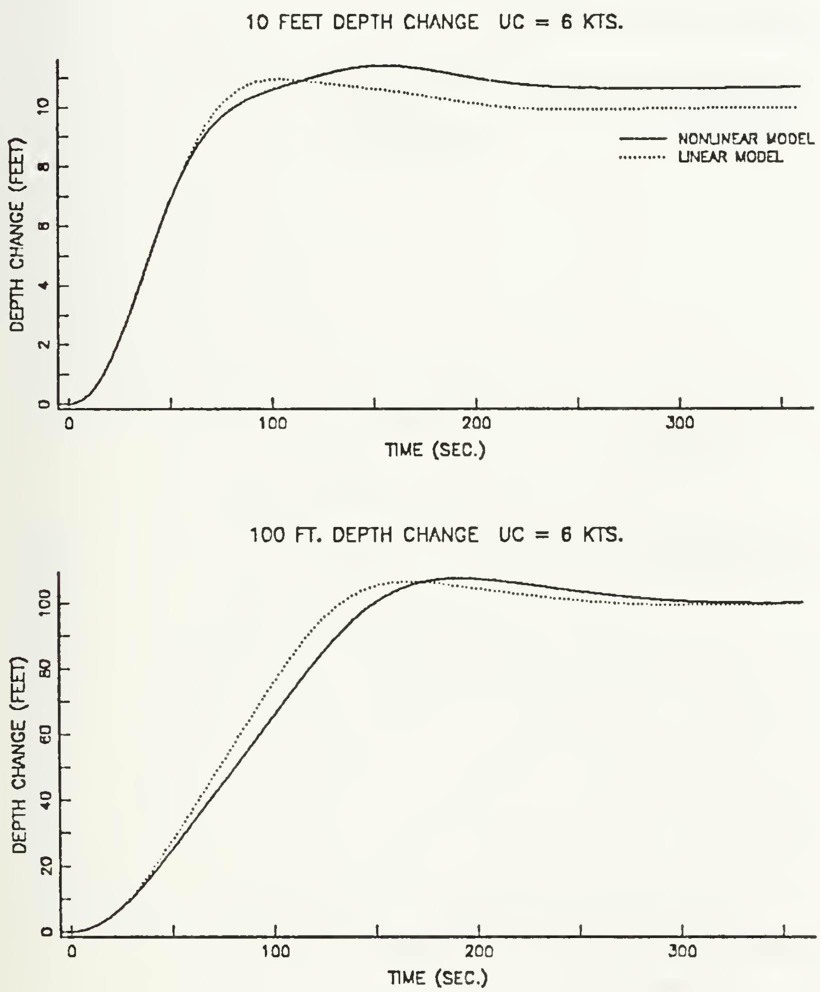

Figure 91. Compensated Submarine Depth Responses at 6 Kts.

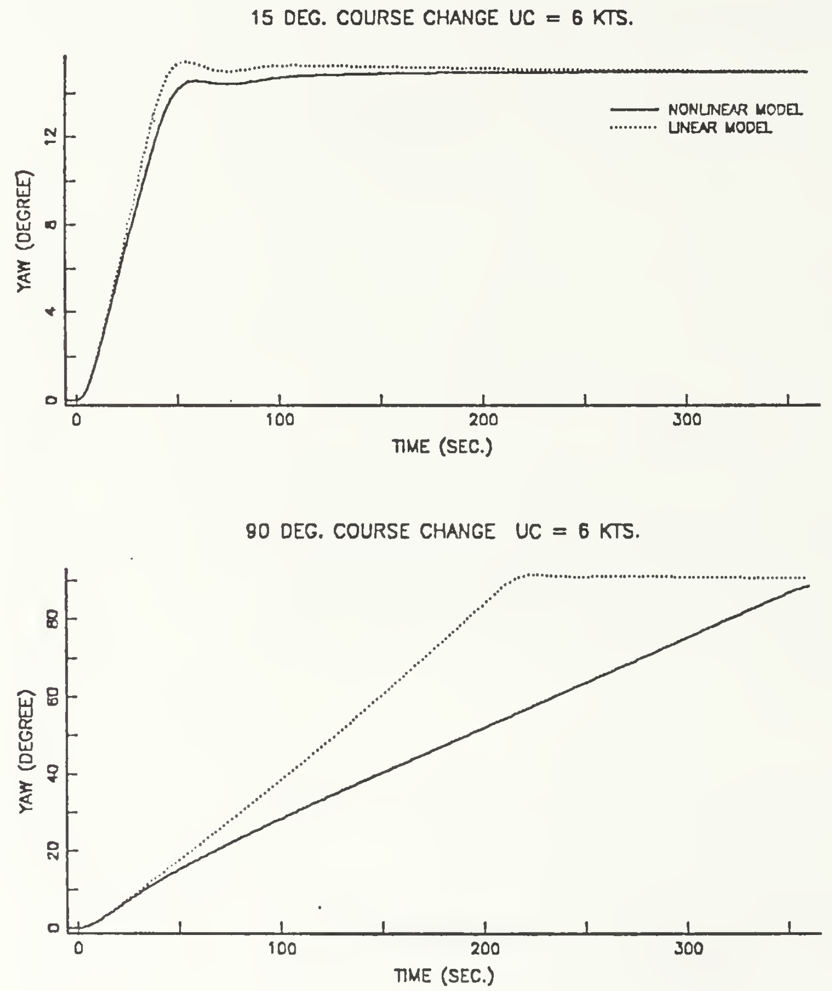

Figure 92. Compensated Submarine Yaw Responses at 6 Kts.

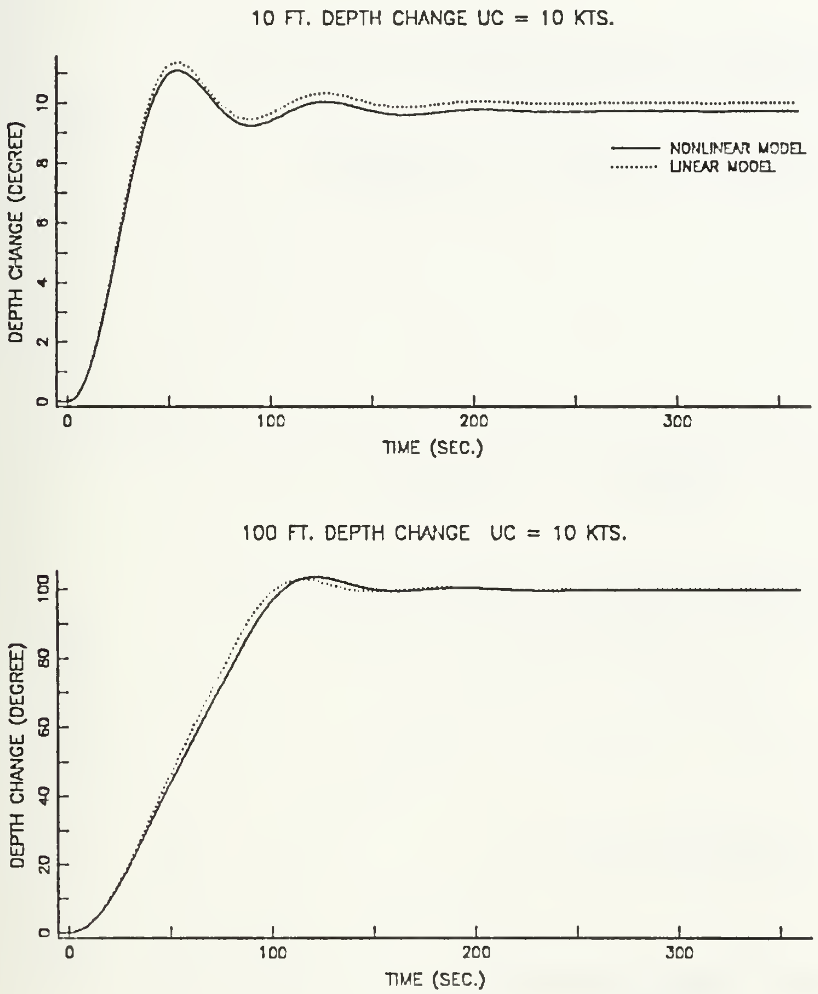

Figure 93. Compensated Submarine Depth Responses at 10 Kts.

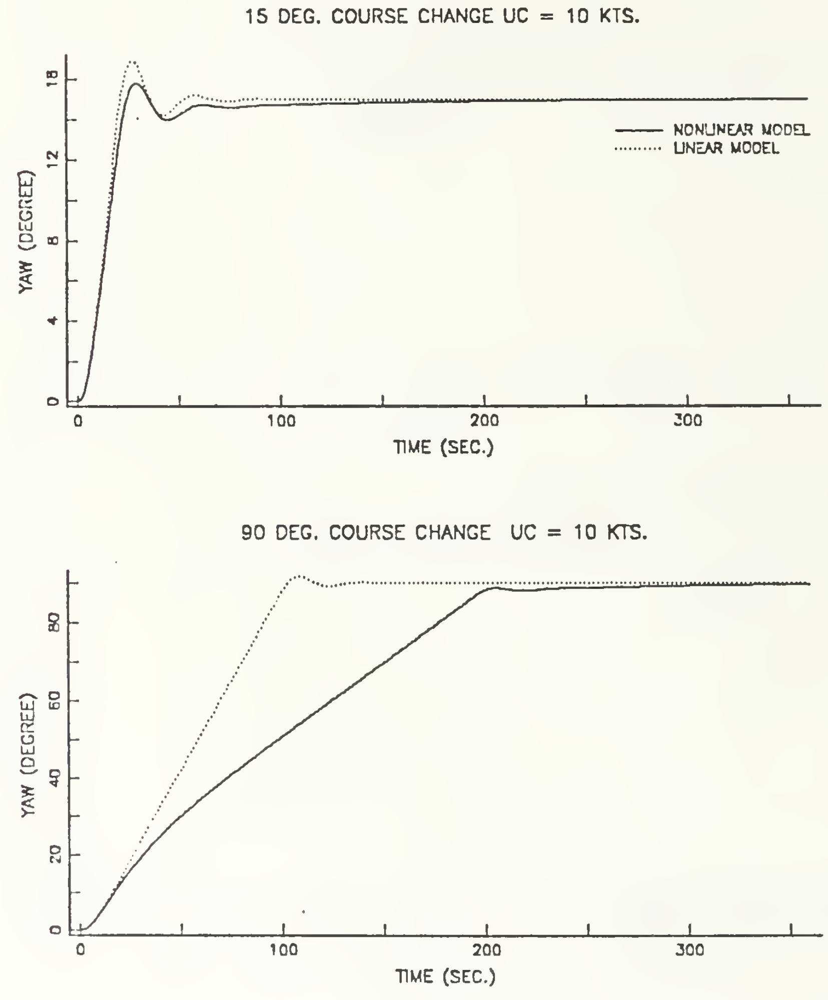

Figure 94. Compensated Submarine Yaw Responses at 10 Kts.

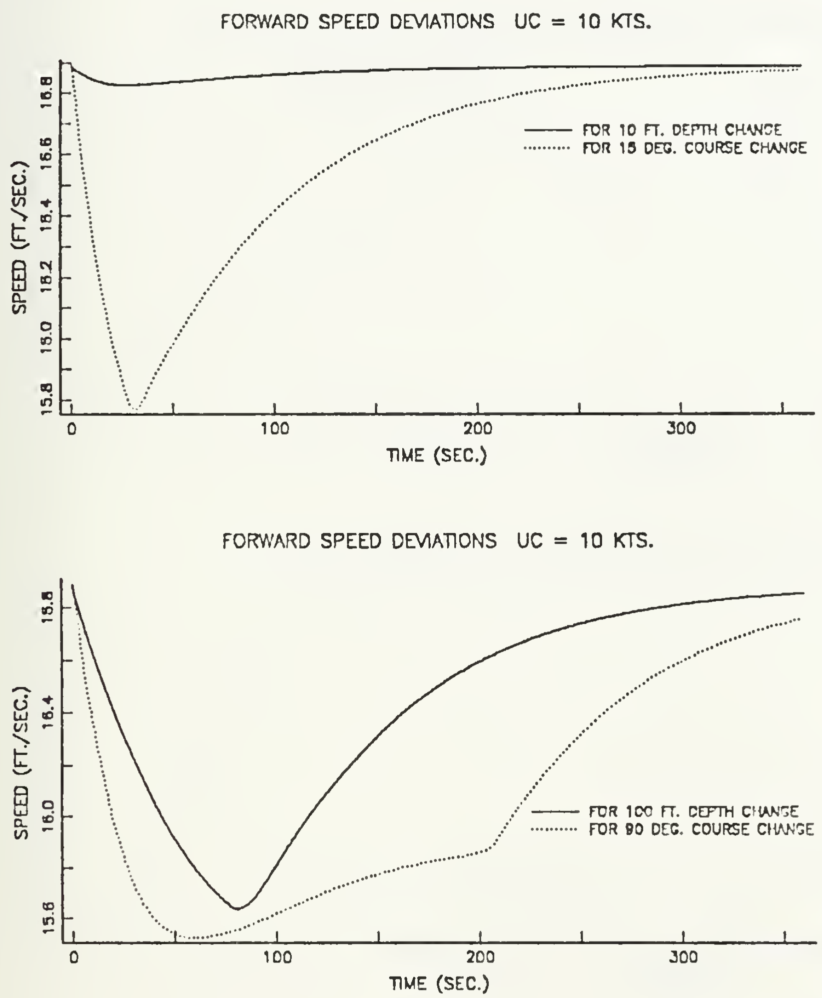

Figure 95. Deviations from the Commanded Speed for Non-Linear Submarine

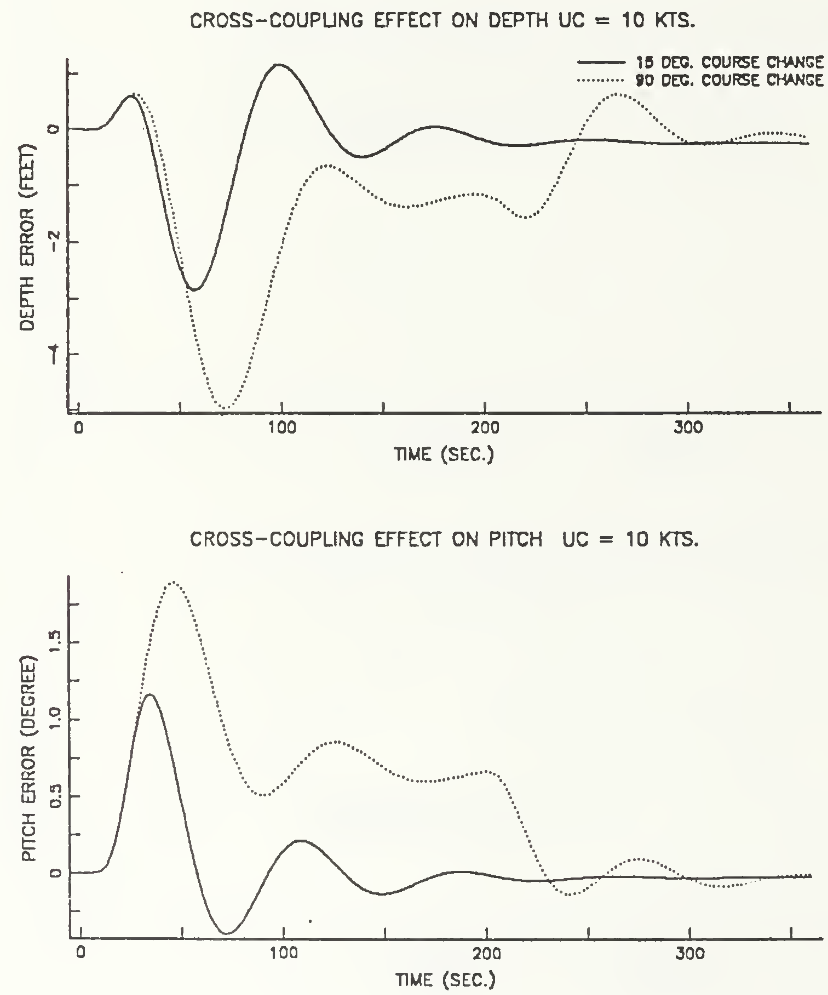

Figure 96. Cross-Coupling Effects for the Non-Linear Submarine at 10 Kts.

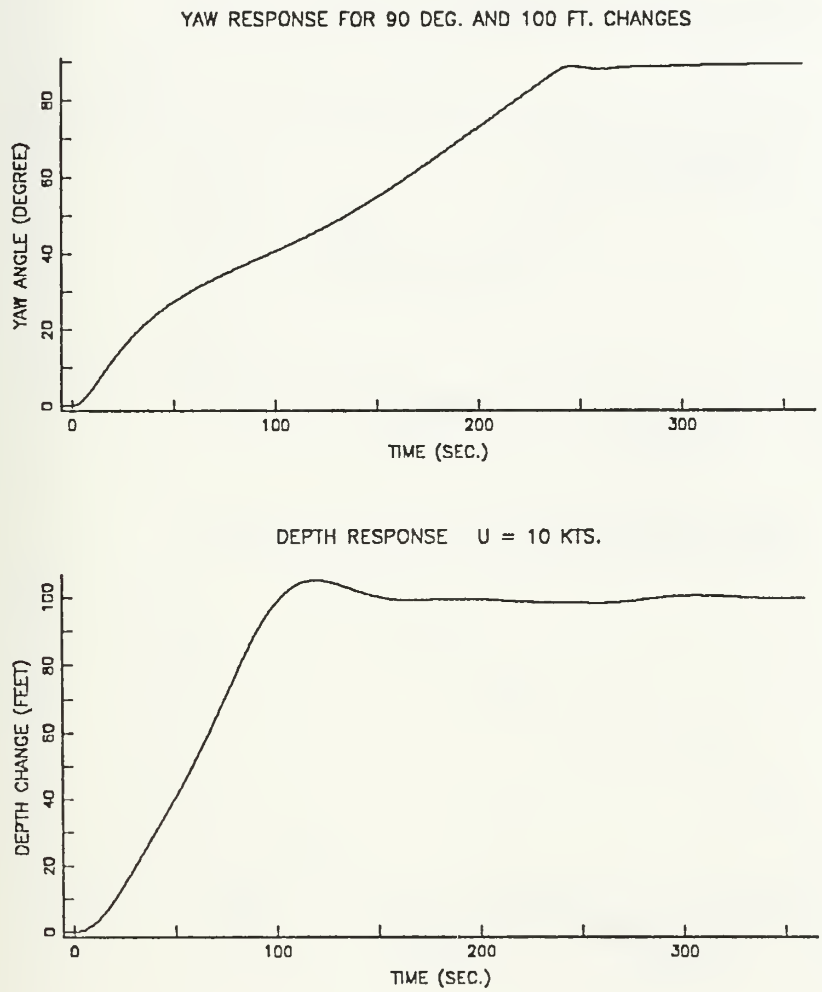

Figure 97. Course and Depth Change Commanded at the Same Time UC = 10 Kts.

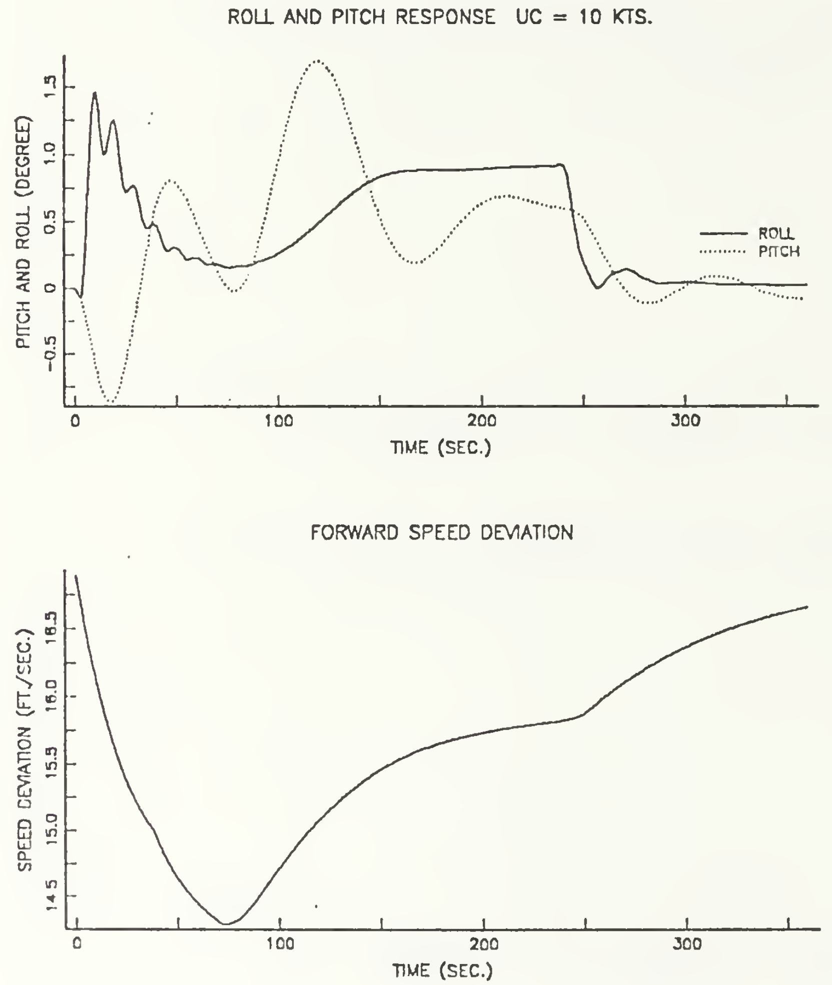

Figure 98. Roll, Pitch and Speed Response for Multi-Manevuer Submarine.

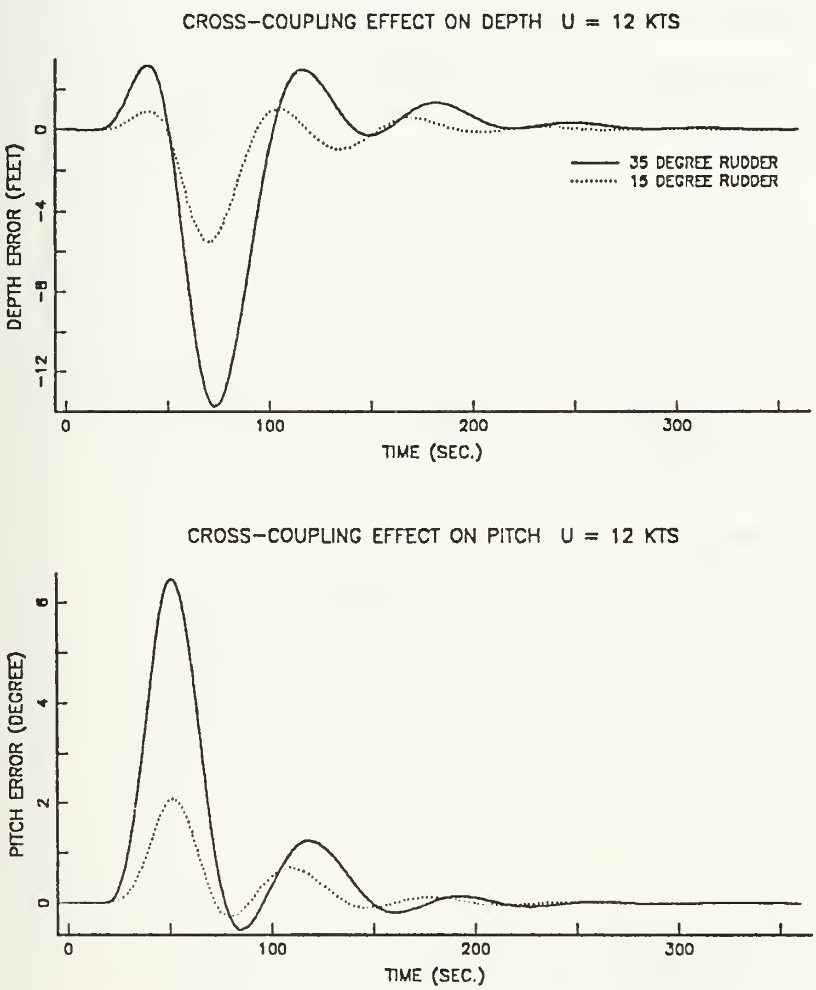

Figure 99. Depth and Pitch Response for Fixed Rudder Commands U= 12 Kts.

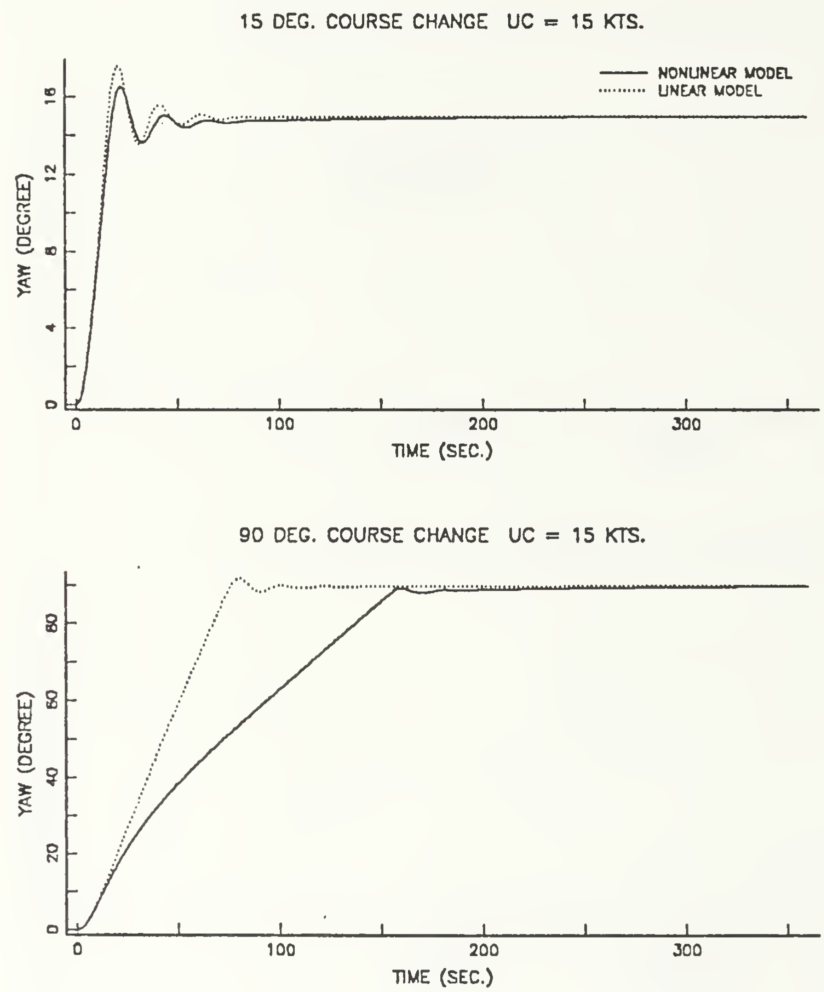

Figure 100. Compensated Submarine Yaw Responses at 15 Kts.

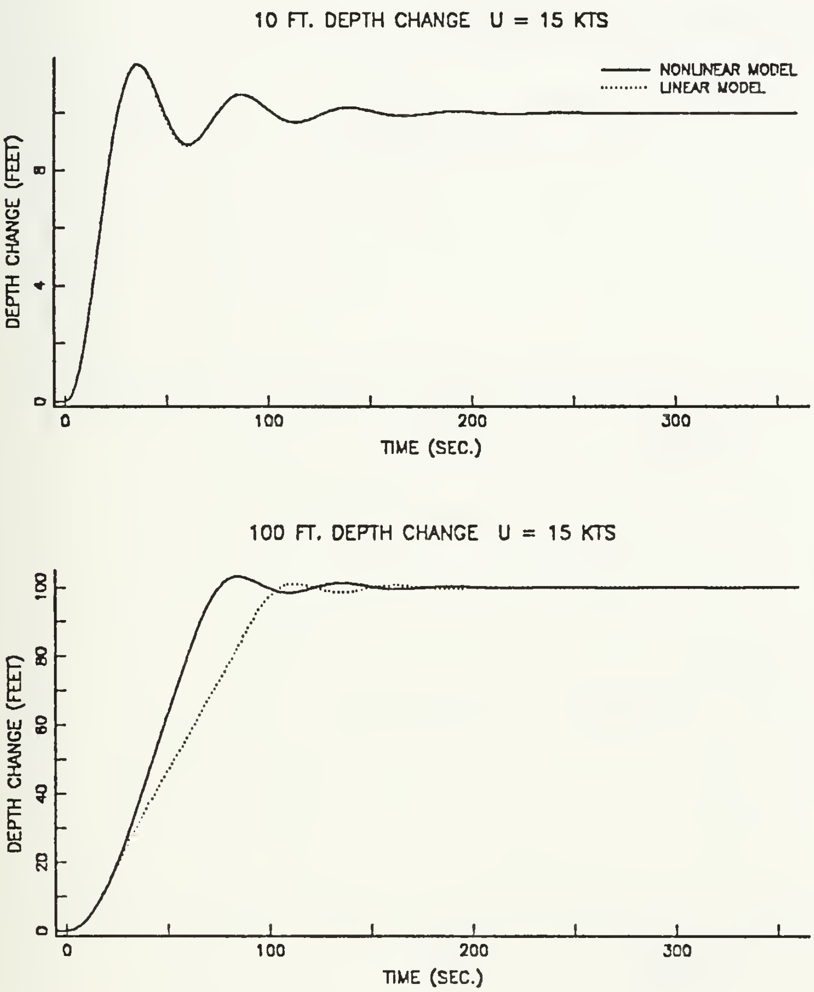

Figure 101. Compensated Submarine Depth Responses at 15 Kts.

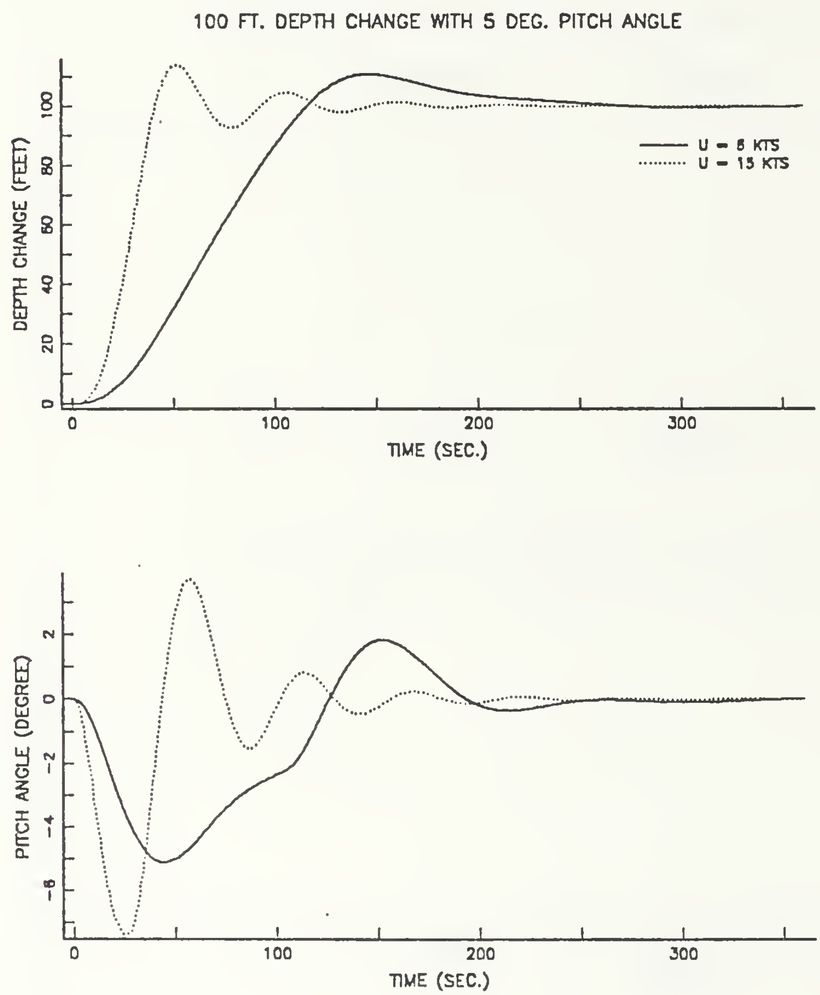

Figure 102. Depth Change with 5 Deg. Down Pitch Angle for Non-Linear Sub.

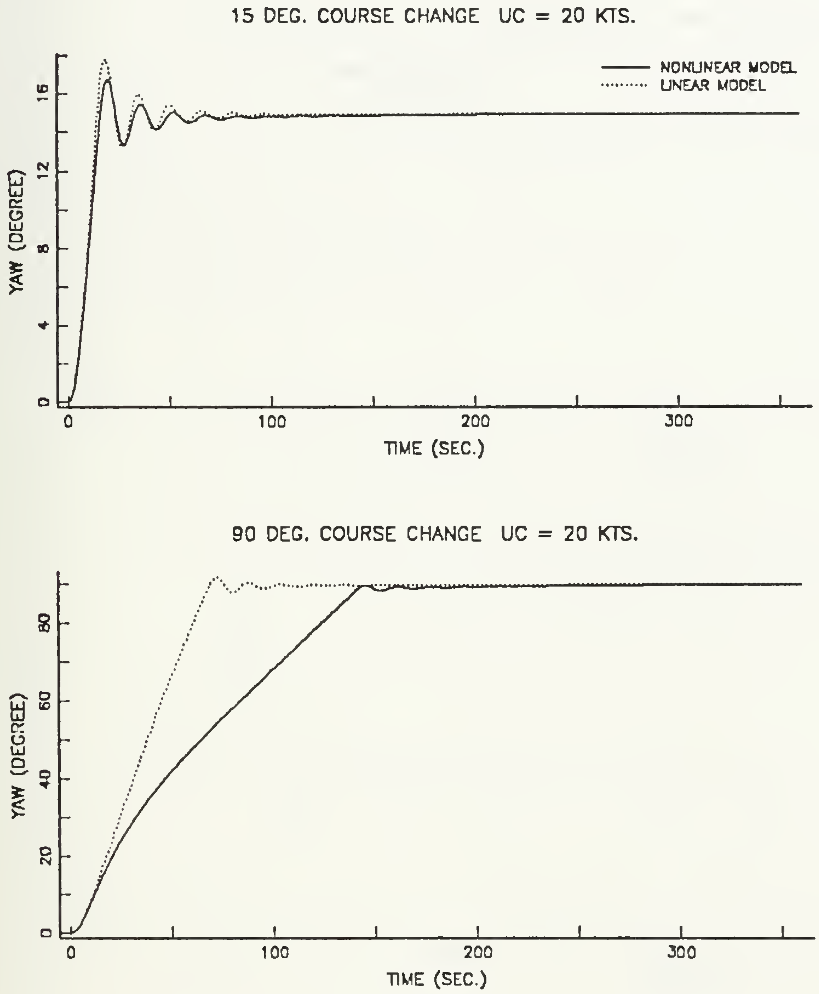

Figure 103. Compensated Submarine Yaw Responses at 20 Kts.

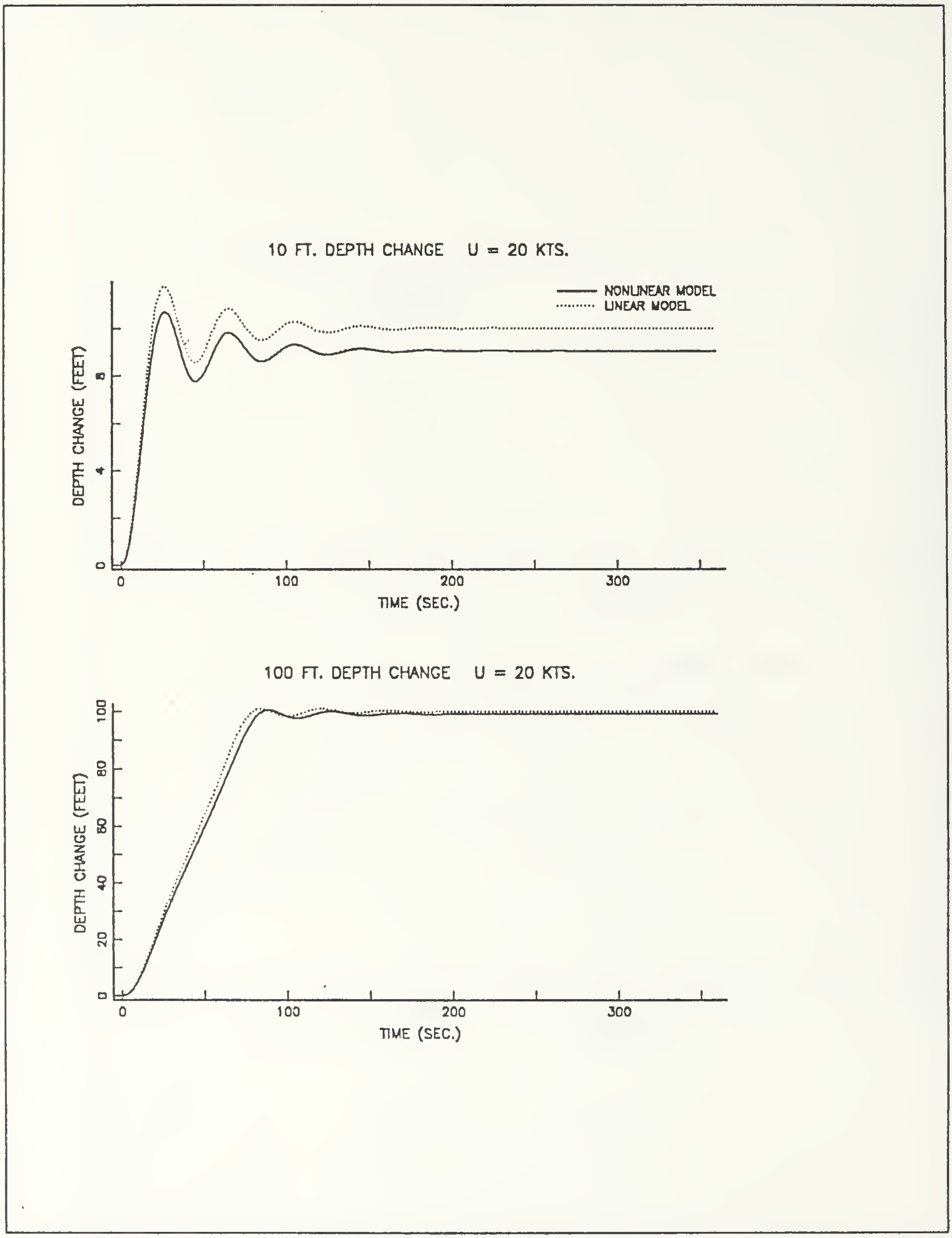

Figure 104. Compensated Submarine Depth Responses at 20 Kts.

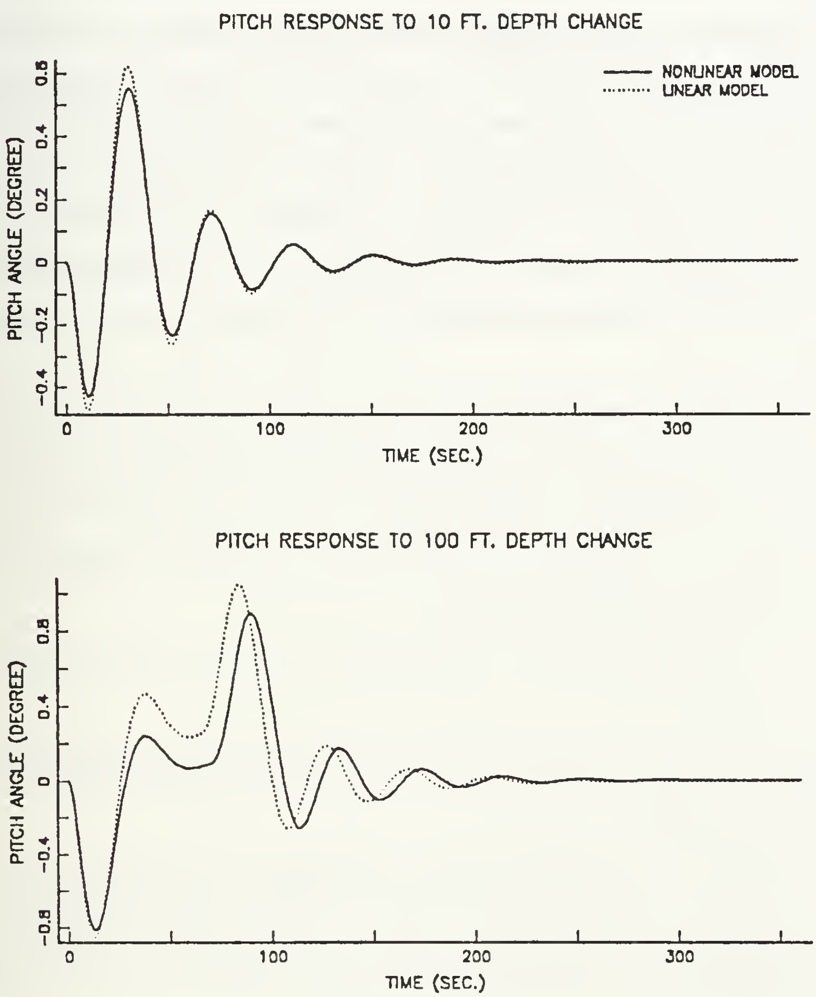

Figure 105. Compensated Submarine Pitch Response to Depth Change Commands

VI. CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER WORK

A. CONCLUSIONS

The linearization of given non-linear differential equations of motion in six degrees of freedom, designing two automatic control systems using cascade compensator design techniques for vertical and horizontal motion of a submarine and finally investigating cross-coupling effects due to the rudder deflections were the main concerns in this study.

It has been shown that using linearized equations to design an automatic control for the actual non-linear system is possible for the submarine problem. Also cascade compensation, using a single loop technique, which was mainly the Bode plot design in this study, is possible and practical for automatic pitch, depth and yaw control of small submarines.

The designed control systems for both planes satisfied the design specifications for a speed range from 8 to 20 Kts. That means the compensated system is rather insensitive to speed deviations. Therefore all problems related to gain switching, like cluttering and discontinuities in plane angles, are avoided. This is especially important because the forward speed changes significantly during maneuvers.

The implementation of the designed compensators into hardware has the following desirable features:

1. Minimal Instrumentation: Since rate information is not required, no inertial guidance system is necessary. Only a regular gyro for course and simple sensors for depth and pitch angle are needed.

2. Low Cost, Weight and Size: The simplicity of the compensator transfer functions makes them easily realizable in physical hardware at low manufacturing cost. Weight and size requirements are very small, another important factor especially for small coastal submarines. A wide speed range is covered by one fixed compensator and no changes in parameters are necessary.

3. Reliability: The automatic controller can be realized with a set of physical components with a well known high reliability. High component reliability and a small number of components will generally result in a high system reliability.

B. RECOMMENDATIONS FOR FURTHER WORK

1. The designed control system in this study can keep the pitch and depth errors in reasonable limits for small rudder deflections and course changes. But larger deflections still create an appreciable amount of depth and pitch error at high speeds which is not

desirable for near surface operations. It might be worthwhile to improve this design to get a sufficient control on cross-coupling effects for all kinds of heavy maneuvers. This can be done using different sets of parameters for the compensators and limiters and/or increasing the numbers of compensators for the vertical control of the submarine.

2. In some operational conditions it is very important to reach a desired depth as soon as possible in a submarine. Therefore an additional pitch angle command is given which has an enormous effect on depth change rate. For the present design it is possible to give both depth and pitch command at the same time but the watch officer has to decide where to change the pitch command to zero. Otherwise, depending on the forward speed and commanded pitch angle, the submarine might not stay on desired depth.

The present design can be modified using a new algorithm which can decide where and in what fashion to decrease the pitch angle automatically in order to get desired depth and stay there without any unacceptable overshoot and steady state error.

APPENDIX A. DEFINITIONS OF SYMBOLS

SYMBOL	DEFINITION
.	A dot over any symbol signifies differentiation with respect to time.
B	Buoyancy force which is positive upwards.
m	Mass of the submarine including the water in the free floating spaces.
l	Overall length of the submarine.
U	Linear velocity of origin of body axes relative to an earth-fixed axis system.
u	Component of U along the body x-axis.
v	Component of U along the body y-axis.
w	Component of U along the body z-axis.
u_c	Command speed.
x	Longitudinal axis of the body fixed coordinate axis system.
y	Transverse axis of the body fixed coordinate axis system.
z	Vertical axis of the body fixed coordinate axis system.

x_0	Distance along the x axis of an earth-fixed axis system.
y_0	Distance along the y axis of an earth-fixed axis system.
z_0	Distance along the z axis of an earth-fixed axis system.
p	Component of angular velocity about the body fixed x-axis.
q	Component of angular velocity about the body fixed y-axis.
r	Component of angular velocity about the body fixed z-axis.
z_B	The z coordinate of the center of buoyance (CB) of the submarine.
α	Angle of attack.
β	Angle of drift.
δb	Deflection of bow or fairwater planes.
δr	Deflection of rudder.
δb	Deflection of stern planes.
n	The ratio $\frac{u_c}{u}$.
θ	Pitch angle.

ψ	Yaw angle.
ψ	Roll angle.
ρ	Mass density of sea water.
W_i	Weight of water blown from a particular ballast tank identified by the integer assigned to the index i.
ω	Angular velocity.
t	Time.
x_{ti}	Location along the body x-axis of the center of mass of the i^{th} ballast tank when this tank is filled with sea water.
$(F_x)_p$	Propulsion force.
I_x	Moment of inertia of a submarine about the x-axis.
I_y	Moment of inertia of a submarine about the y-axis.
I_z	Moment of inertia of a submarine about the z-axis.
All K's	Non-dimensional constants each of which is assigned to a particular force term in the equation of motion about the body x-axis.
All M's	Non-dimensional constants each of which is assigned to a particular force term in the equation of motion about the body y-axis.

All N's	Non-dimensional constants each of which is assigned to a particular force term in the equation of motion about the body z-axis.
All X's	Non-dimensional constants each of which is assigned to a particular force term in the equation of motion along the body x-axis.
All Y's	Non-dimensional constants each of which is assigned to a particular force term in the equation of motion along the body y-axis.
All Z's	Non-dimensional constants each of which is assigned to a particular force term in the equation of motion along the body z-axis.

APPENDIX B. HYDRODYNAMIC COEFFICIENTS OF SIMULATION EQUATIONS

A. AXIAL FORCE

XQQ	=	-0.000200	XRR	=	-0.000090	XRP	=	0.000250
XUDOT	=	-0.000150	XVR	=	0.011000	XWQ	=	-0.007500
XUU	=	0.0	XVV	=	0.006500	XDRDR	=	-0.002800
XDSDS	=	-0.002500	XDBDB	=	-0.002600	XVVN	=	0.0
XWWN	=	0.0	XDR2N	=	0.0	XDS2N	=	0.0

B. LATERAL FORCE

YP/P/	=	0.0	YPQ	=	0.000200	YPDOT	=	-0.000300
YRDOT	=	0.000090	YVDOT	=	-0.011000	YV/R/	=	-0.007300
YWP	=	0.007500	YR	=	0.003000	YRDR	=	0.0
YP	=	-0.000700	YUU	=	0.0	YV/V/	=	-0.060000
YV	=	-0.021000	YDR	=	0.006200	YWV	=	-0.065000
YVS	=	0.0	YRN	=	0.0	YVN	=	0.0
YVAVN	=	0.0	YDRN	=	0.0			

C. NORMAL FORCE

ZRR	=	-0.001500	ZRP	=	-0.000900	ZQDOT	=	-0.000200
ZWDOT	=	-0.007500	ZVR	=	-0.008000	ZW/Q/	=	-0.006000
ZQ	=	-0.004500	ZQDS	=	0.0	ZVP	=	-0.007000
ZUU	=	-0.000100	ZVV	=	0.000650	ZW/W/	=	-0.030000
ZW	=	-0.011000	ZDS	=	-0.005000	ZDB	=	-0.002500
Z/W/	=	0.0	ZWW	=	0.0	ZVS	=	0.0
ZQN	=	0.0	ZWN	=	0.0	ZWAWN	=	0.0
ZDSN	=	0.0	Z+VP	=	0.0			

D. ROLLING MOMENT

KP/P/	= -0.0000008	KQR	= -0.000100	KPDOT	= -0.000003
KRDOT	= -0.000007	KVDOT	= -0.000250	KWP	= 0.000250
KR	= -0.000040	KP	= -0.000035	KUU	= 0.0
KV/V/	= -0.000900	KV	= -0.000700	KDR	= 0.000070
KWV	= 0.003500				

E. PITCHING MOMENT

MRR	= -0.0005500	MRP	= 0.000150	MQDOT	= -0.000400
M+RP	= 0.0	MWDOT	= -0.000200	MVR	= -0.002000
M/W/Q	= -0.002000	MQ	= -0.002500	MQDS	= 0.0
MVP	= 0.000900	MUU	= 0.000040	MVV	= 0.015000
MW/W/	= -0.005000	MW	= 0.003000	MDS	= -0.002500
MDB	= 0.000500	M/W/	= 0.0	MWW	= 0.0
MQN	= 0.0	MWN	= 0.0	MWAWN	= 0.0
MDSN	= 0.0				

F. YAWING MOMENT

NPQ	= -0.0004000	NPDOT	= -0.000007	NRDOT	= -0.000500
NVDOT	= 0.000300	N/V/R	= -0.004500	NWP	= -0.000200
NR	= -0.003000	NRDR	= 0.0	NP	= -0.000005
NUU	= 0.0	NV/V/	= 0.014000	NV	= -0.007500
NDR	= -0.003000	NWV	= 0.015000	NRN	= 0.0
NVN	= 0.0	NVAVN	= 0.0	NDRN	= 0.0

G. OTHERS

A1	= -0.001000	A2	= -0.000950	A3	= 0.001950
LC	= 415.0	ML	= 0.0087445	BZB	= 0.0010114
IX	= $7.311 x 10^{-6}$	IY	= $5.6867 x 10^{-4}$	IZ	= $5.6867 x 10^{-4}$

APPENDIX C. STANDARD EQUATIONS OF MOTION

A. AXIAL FORCE

$$m(\dot{u} - vr + wq) = \frac{\rho}{2} l^4 \left[X_{qq} q^2 + X_{rr} r^2 + X_{rp} rp \right]$$

$$+ \frac{\rho}{2} l^3 \left[X_{\dot{u}} \dot{u} + X_{vr} vr + X_{wq} wq \right]$$

$$+ \frac{\rho}{2} l^2 \left[X_{uu} u^2 + Xvvv^2 + X_{ww} w^2 \right]$$

$$+ \frac{\rho}{2} l^2 u^2 \left[X_{\delta r \delta r} \delta r^2 + X_{\delta s \delta s} \delta s^2 + X_{\delta b \delta b} \delta b^2 \right]$$

$$+ \frac{\rho}{2} l^2 X_{vvn} (n - 1) v^2$$

$$+ \frac{\rho}{2} l^2 X_{wwn} (n - 1) w^2$$

$$+ \frac{\rho}{2} l^2 u^2 X_{\delta s \delta s n} (n - 1) \delta_s^2$$

$$+ \frac{\rho}{2} l^2 u^2 X_{\delta r \delta r n} (n - 1) \delta_r^2$$

$$- \sum W_l \sin \theta$$

$$+ (F_x)_P$$

B. LATERAL FORCE

$$m(\dot{v} - wp + ur) = \frac{\rho}{2} l^4 [Y_{\dot{r}} \dot{r} + Y_{\dot{p}} \dot{p}]$$

$$+ \frac{\rho}{2} l^4 [Y_{pq} pq + Y_{p|p|} p|p|]$$

$$+ \frac{\rho}{2} l^3 \left[Y_{\dot{v}} \dot{v} + Y_{wp} wp + Y_{v|r|} \frac{v}{|v|} |(v^2 + w^2)^{\frac{1}{2}}||r| \right]$$

$$+ \frac{\rho}{2} l^3 [Y_r ur + Y_{|r|\delta r} u|r|\delta r + Y_p up]$$

$$+ \frac{\rho}{2} l^3 Y_{rn} (n - 1) ur$$

$$+ \frac{\rho}{2} l^2 \left[Y_{uu} u^2 + Y_v uv + Y_{v|v|} v|(v^2 + w^2)^{\frac{1}{2}}| \right]$$

$$+ \frac{\rho}{2} l^2 u^2 Ysub\delta r \delta r$$

$$+ \frac{\rho}{2} l^2 u^2 Y_{\delta rn} (n - 1) \delta r$$

$$+ \frac{\rho}{2} l^2 Y_{vn} (n - 1) uv$$

$$+ \frac{\rho}{2} l^2 Y_{v|v|n} (n - 1) v|(v^2 + w^2)^{\frac{1}{2}}|$$

$$+ \frac{\rho}{2} l^2 Y_{wv} wv$$

$$+ \frac{\rho}{2} l^2 (F_y)_{vs} \frac{v^2 + w^2}{U} (-w) \sin \omega t$$

$$+ \sum W_i \sin \phi \cos \theta$$

C. NORMAL FORCE

$$
\begin{aligned}
m(\dot{w} - uq + vp) = {} & \frac{\rho}{2} l^4 Z_{\dot{q}} \dot{q} \\
& + \frac{\rho}{2} l^4 \left[Z_{rr} r^2 + Z_{rp} rp \right] \\
& + \frac{\rho}{2} l^3 [Z_{\dot{w}} \dot{w} + Z_{vr} vr + Z_{vp} vp + \Delta Z_{vp} vp] \\
& + \frac{\rho}{2} l^3 \left[Z_q uq + Z_{|q|\delta s} u|q|\delta s + Z_{w|q|} \frac{w}{|w|} |(v^2 + w^2)^{\frac{1}{2}}||q| \right] \\
& + \frac{\rho}{2} l^3 Z_{qn} (n-1) uq \\
& + \frac{\rho}{2} l^2 \left[Z_{uu} u^2 + Z_w uw + Z_{w|w|} w|(v^2 + w^2)^{\frac{1}{2}}| \right] \\
& + \frac{\rho}{2} l^2 \left[Z_{|w|} u|w| + Z_{ww} |w(v^2 + w^2)^{\frac{1}{2}}| Z_{vv} v^2 \right] \\
& + \frac{\rho}{2} l^2 [Z_{\delta s} \delta s + Z_{\delta b} \delta b] \\
& + \frac{\rho}{2} l^2 \left[Z_{wn} (n-1) uw + Z_{w|w|n} (n-1) w|(v^2 + w^2)^{\frac{1}{2}}| \right] \\
& + \frac{\rho}{2} l^2 u^2 Z_{\delta sn} (n-1) \delta s \\
& + \frac{\rho}{2} l^2 (F_z)_{vs} \frac{v^2 + w^2}{U} v \sin \omega t \\
& + \sum W_i \cos \phi \cos \theta
\end{aligned}
$$

D. ROLLING MOMENT

$$I_x\dot{p} + (I_z - I_y)qr = \frac{\rho}{2} l^5 [K_p\dot{p} + K_{qr}qr + K_r\dot{r} K_{p|p|}p|p|]$$

$$+ \frac{\rho}{2} l^4 [K_p up + K_r ur + K_v\dot{v} + K_{wp}wp]$$

$$+ \frac{\rho}{2} l^3 \left[K_{uu}u^2 + K_v uv + K_{v|v|}v|(v^2 + w^2)^{\frac{1}{2}}| \right]$$

$$+ \frac{\rho}{2} l^3 K_{vw}vw$$

$$+ \frac{\rho}{2} l^3 u^2 K_{\delta r}\delta r$$

$$+ Bz_B \sin\phi \cos\theta$$

E. PITCHING MOMENT

$$
\begin{aligned}
I_y\dot{q} + (I_x - I_z)rp = {} & \frac{\rho}{2} l^5 \left[M_{\dot{q}}\dot{q} + M_{rr}r^2 + M_{rp}rp + \Delta M_{rp}rp \right] \\
& + \frac{\rho}{2} l^4 \left[M_q uq + M_{|q|\delta s} u|q|\delta s + M_{|w|q} |(v^2 + w^2)^{\frac{1}{2}}| q \right] \\
& + \frac{\rho}{2} l^4 \left[M_{\dot{w}}\dot{w} + M_{vr}vr + M_{vp}vp \right] \\
& + \frac{\rho}{2} l^4 M_{qn}(n-1)uq \\
& + \frac{\rho}{2} l^3 \left[M_{uu}u^2 + M_w uw + M_{w|w|} w |(v^2 + w^2)^{\frac{1}{2}}| \right] \\
& + \frac{\rho}{2} l^3 \left[M_{|w|} u|w| + M_{ww} |w (v^2 + w^2)^{\frac{1}{2}}| + M_{vv}v^2 \right] \\
& + \frac{\rho}{2} l^3 \left[M_{\delta s}\delta s + M_{\delta b}\delta b \right] \\
& + \frac{\rho}{2} l^3 M_{wn}(n-1)uw \\
& + \frac{\rho}{2} l^3 M_{w|w|n}(n-1) w |(v^2 + w^2)^{\frac{1}{2}}| \\
& + \frac{\rho}{2} l^3 u^2 M_{\delta sn}(n-1)\delta s \\
& + Bz_B \sin\theta \\
& + \sum W_i x_{ti} \cos\phi \cos\theta
\end{aligned}
$$

F. YAWING MOMENT

$$
\begin{aligned}
I_z\dot{r} + (I_y - I_x)pq = {} & \frac{\rho}{2} l^5 [N_{\dot{r}}\dot{r} + N_{pq}pq + N_{\dot{p}}\dot{p}] \\
& + \frac{\rho}{2} l^4 \left[N_r ur + N_{|r|\delta r} u|r|\delta r + N_{|v|r} |(v^2 + w^2)^{\frac{1}{2}}| r \right] \\
& + \frac{\rho}{2} l^4 [N_p up + N_{\dot{v}}\dot{v} + N_{wp} wp] \\
& + \frac{\rho}{2} l^4 N_{rn}(n - 1)ur \\
& + \frac{\rho}{2} l^3 \left[N_{uu}u^2 + N_v uv + N_{v|v|} v |(v^2 + w^2)^{\frac{1}{2}}| \right] \\
& + \frac{\rho}{2} l^3 u^2 N_{\delta r}\delta r \\
& + \frac{\rho}{2} l^3 u^2 N_{\delta rn}(n - 1)\delta r \\
& + \frac{\rho}{2} l^3 N_{vn}(n - 1)uv \\
& + \frac{\rho}{2} l^3 N_{v|v|n}(n - 1) v |(v^2 + w^2)^{\frac{1}{2}}| \\
& + \frac{\rho}{2} l^3 N_{wv} wv \\
& + \sum W_i x_{ti} \sin\phi \cos\theta
\end{aligned}
$$

G. AUXILARY EQUATIONS

$$\dot{\phi} = p + \dot{\psi} \sin \theta$$

$$\dot{\theta} = \frac{q - \dot{\psi} \cos \theta \sin \phi}{\cos \phi}$$

$$\dot{\psi} = \frac{r + \dot{\theta} \sin \phi}{\cos \phi \cos \theta}$$

$$\dot{x}_0 = u \cos \theta \cos \psi + v(\sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi) + w(\sin \phi \sin \psi + \cos \phi \sin \theta \cos \psi)$$

$$\dot{y}_0 = u \cos \theta \sin \psi + v(\cos \phi \cos \psi + \sin \phi \sin \theta \sin \psi) + w(\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi)$$

$$\dot{z}_0 = -u \sin \theta + v \cos \theta \sin \phi + w \cos \theta \cos \phi$$

$$(F_x)_P = \frac{\rho}{2} l^2 u^2 \left[a_1 + a_2 n + a_3 n^2 \right]$$

APPENDIX D. SIMULATION PROGRAM FOR LINEARIZED VERTICAL EQUATIONS OF MOTION

```
*   THIS PROGRAM SIMULATES THE LINEARIZED SUBMARINE EQUATIONS
*   IN THE VERTICAL PLANE.
*
*
TITLE SUBMARINE EQUATIONS FOR THE VERTICAL PLANE
*
*   AXIAL SPEED
CONST  U=8.445
INITIAL
       DS=0.0
       DB=0.0
DERIVATIVE
       DB=0.0*STEP(0)
       DS=0.0*STEP(0)
       THETA=INTGRL(0.08726,Q)
       DEPTH = INTGRL(.0,ZDOT)
       Q=INTGRL(.0,QDOT)
       ZDOT=INTGRL(-0.736,ZDDOT)
       Y1=-1.728E-3*U*ZDOT
       Y4=-0.706*U*Q
       Y5=(0.0128-(1.728E-3)*U*U)*THETA
       Y2=-6.667E-4*U*U*DS
       Y3=-3.873E-4*U*U*DB
       ZDDOT=Y1+Y2+Y3+Y4+Y5
       Y0=1.884E-5*U*ZDOT
       Y9=-6.365E-3*U*Q
       Y6=3.193E-6*U*U*DB
       Y7=-1.465E-5*U*U*DS
       Y8=((1.884E-5)*U*U-2.522E-3)*THETA
       QDOT=Y0+Y8+Y6+Y7+Y9
       DEP=INTGRL(.0,ZDOT)
CONTROL   FINTIM=360
PRINT  1.,THETA,DEPTH,ZDOT
SAVE   0.1,W,ZDOT,THETA,DEP
GRAPH(DE=TEK618)TIME,THETA,DEP
LABEL  INITIAL CONDITION RESPONSE IN.PITCH=5 DEG.   U=5 KTS.
GRAPH(DE=TEK618)TIME,ZDOT
LABEL  INITIAL CONDITION RESPONSE IN.PITCH=5 DEG.   U=5 KTS.
```

APPENDIX E. SIMULATION PROGRAM FOR LINEARIZED HORIZONTAL EQUATIONS OF MOTION

```
*   THIS PROGRAM SIMULATES THE LINEARIZED SUBMARINE EQUATIONS
*   IN THE HORIZONTAL PLANE.
*
*
*
TITLE SUBMARINE EQUATIONS FOR THE HORIZONTAL PLANE
*
*   AXIAL SPEED
CONST  U=30.4
INITIAL
       DR=0.0
DERIVATIVE
       Y1=1.89*RDOT
       Y2=-6.3*PDOT
       Y3=-0.291*U*R
       Y4=-0.035*U*P
       Y5=-2.563E-3*U*V
       Y6=7.568E-4*U*U*DR
       VDOT=Y1+Y2+Y3+Y4+Y5+Y6
       Y7=-0.679*RDOT
       Y8=-0.0584*VDOT
       Y9=-9.347E-3*U*R
       Y10=-8.179E-3*U*P
       Y11=-3.942E-4*U*V
       Y12=3.942E-5*U*U*DR
       Y13=0.236*PHI
       PDOT=Y7+Y8+Y9+Y10+Y11+Y12-Y13
       Y14=-6.553E-3*PDOT
       Y15=6.767E-4*VDOT
       Y16=-6.767E-3*U*R
       Y17=-4.511E-6*U*P
       Y18=-4.076E-5*U*V
       Y19=-1.631E-5*U*U*DR
       RDOT=Y14+Y15+Y16+Y17+Y18+Y19
       P=INTGRL(.0,PDOT)
       V=INTGRL(.0,VDOT)
       R=INTGRL(.0,RDOT)
*      PHI=INTGRL(0.43633,P)
       PHI=INTGRL(0.0,P)
       XI=INTGRL(.0,R)
DYNAMIC
       IF(TIME.GE.10) DR = 0.611
       IF(TIME.GE.40) DR = -0.611
       IF(TIME.GE.70) DR = 0.0
       DRDEG = DR*57.296
       ROLDEG= PHI*57.296
       YAWDEG= XI*57.296
CONTROL   FINTIM=360
```

```
PRINT   1. ,V,YAWDEG,ROLDEG
SAVE    0.1,V,YAWDEG,ROLDEG,DRDEG
GRAPH(DE=TEK618)TIME,YAWDEG,ROLDEG
LABEL   FORCED RESPONSE TO 35 DEG. RUDDER   U=18 KTS.
```

APPENDIX F. SIMULATION PROGRAM FOR THE COMPENSATED SYSTEM IN VERTICAL MOTION

```
*   THIS PROGRAM SIMULATES THE COMPENSATED SUBMARINE MOTIONS
*   IN THE VERTICAL PLANE.
*
*
*
TITLE SUBMARINE SIMULATION
PARAM K1 = 0.015
PARAM K2 = 2.0
PARAM UC = 10.4
PARAM ZOR= 100.
PARAM POR= 0.0
*
DERIVATIVE
       PITCH=INTGRL(0.0,Q)
       DEPTH = INTGRL(.0,ZDOT)
       Q=INTGRL(.0,QDOT)
       ZDOT=INTGRL(-0.736,ZDDOT)
       Y1=-1.728E-3*U*ZDOT
       Y4=-0.706*U*Q
       Y5=(0.0128-(1.728E-3)*U*U)*PITCH
       Y2=-6.667E-4*U*U*DS
       Y3=-3.873E-4*U*U*DB
       ZDDOT=Y1+Y2+Y3+Y4+Y5
       Y0=1.884E-5*U*ZDOT
       Y9=-6.365E-3*U*Q
       Y6=3.193E-6*U*U*DB
       Y7=-1.465E-5*U*U*DS
       Y8=((1.884E-5)*U*U-2.522E-3)*PITCH
       QDOT=Y0+Y8+Y6+Y7+Y9
       DEP=INTGRL(.0,ZDOT)
       ZER = ZOR - DEPTH
       PER = POR - PITCH
       ZERR= LIMIT(-LIM,LIM,ZER)
       PERR= LIMIT(-LIM,LIM,PER)
DYNAMIC
      IF(UC.LT.15.) LIM = 35.
      IF(UC.GE.15.) LIM = 25
      IF(UC.GE.25.) LIM = 15.
*COMPENSATOR GC11
      C11 = -K1*ZERR
      C12 = LEDLAG(0.,1.0,0.1,C11)
      DB  = REALPL(0.,.667,C12)
*COMPENSATOR GC22
      C21 = -K2*PERR
      DS  = REALPL(0.,.667,C21)
      DSDEG = 57.296*DS
      DBDEG = 57.296*DB
```

```
      PITDEG = 57.296*PITCH
CONTROL   FINTIM=360
SAVE   0.1,DEPTH,ZDOT,PITDEG,DSDEG,DBDEG
PRINT  1.,PITDEG,DEPTH,ZDOT,DSDEG,DBDEG
GRAPH(DE=TEK618)TIME,PITCH,DEPTH
LABEL  100 FT. DEPTH CHANGE   U=6 KTS.
GRAPH(DE=TEK618)DBDEG,DSDEG
LABEL  100 FT. DEPTH CHANGE   U=6 KTS.
```

APPENDIX G. SIMULATION PROGRAM FOR THE COMPENSATED SYSTEM IN HORIZONTAL MOTION

```
*   THIS PROGRAM SIMULATES THE COMPENSATED SUBMARINE MOTIONS
*   IN THE HORIZONTAL PLANE.
*
*
*
TITLE  SUBMARINE SIMULATION FOR THE HORIZONTAL PLANE
*
PARAM  K1  = 1.00
PARAM  LIM = 0.065
PARAM  U   = 30.4
PARAM  ORYAW=0.2618

DERIVATIVE
       Y1=1.89*RDOT
       Y2=-6.3*PDOT
       Y3=-0.291*U*R
       Y4=-0.035*U*P
       Y5=-2.563E-3*U*V
       Y6=7.568E-4*U*U*DR
       VDOT=Y1+Y2+Y3+Y4+Y5+Y6
       Y7=-0.679*RDOT
       Y8=-0.0584*VDOT
       Y9=-9.347E-3*U*R
       Y10=-8.179E-3*U*P
       Y11=-3.942E-4*U*V
       Y12=3.942E-5*U*U*DR
       Y13=0.236*PHI
       PDOT=Y7+Y8+Y9+Y10+Y11+Y12-Y13
       Y14=-6.553E-3*PDOT
       Y15=6.767E-4*VDOT
       Y16=-6.767E-3*U*R
       Y17=-4.511E-6*U*P
       Y18=-4.076E-5*U*V
       Y19=-1.631E-5*U*U*DR
       RDOT=Y14+Y15+Y16+Y17+Y18+Y19
       P=INTGRL(.0,PDOT)
       V=INTGRL(.0,VDOT)
       R=INTGRL(.0,RDOT)
       PHI=INTGRL(0.0,P)
       XI=INTGRL(.0,R)
DYNAMIC
       IF(U.LT.20.3) LIM=0.070
       IF(U.GE.20.3) LIM=0.050
       IF(U.GE.30.4) LIM=0.035
       ERR = ORYAW - XI
       LERR= LIMIT(-LIM,LIM,ERR)
*COMPENSATOR GC11
       LC1A = -K1*LERR
```

```
      LC1B = LEDLAG(0. ,100. ,10. ,LC1A)
      DR   = REALPL(0. ,.667,LC1B)
      DRDEG = DR*57.296
      ROLDEG= PHI*57.296
      YAWDEG= XI*57.296
CONTROL   FINTIM=360
PRINT  1. ,V,YAWDEG,DRDEG,ROLDEG
SAVE   0.1,V,YAWDEG,ROLDEG,DRDEG
GRAPH(DE=TEK618)TIME,YAWDEG,ROLDEG
LABEL  15 DEGREE COURSE CHANGE   U=18 KTS.
GRAPH(DE=TEK618)TIME,DRDEG
LABEL  RUDDER RESPONSE TO 15 DEG. COURSE CHANGE   U=18 KTS.
```

APPENDIX II. SIMULATION PROGRAM FOR NON-LINEAR EQUATIONS OF MOTION

```
*THIS PROGRAM SIMULATES NON-LINEAR EQUATIONS OF MOTION IN SIX DEGREES
*OF FREEDOM FOR A SUBMERGED SUBMARINE
*
TITLE NONLINEAR SIX DEGREE OF FREEDOM SUBMARINE SIMULATION
PARAM UC = 18.58
*
*BALLAST TANKS CONTAINS FOR DIFFERENT AXIAL SPEEDS
*
*FOR 5 KTS
*ARAM AT = -0.800E-5
*ARAM FT = 0.800E-5
*ARAM AU = 1.400E-5
*FOR 6 KTS
*ARAM AT = -1.03E-5
*ARAM FT = 1.03E-5
*ARAM AU = 2.500E-5
*FOR 8 KTS
*ARAM AT = -1.85E-5
*ARAM FT = 1.85E-5
*ARAM AU = 4.50E-5
*FOR 9 KTS
*ARAM AT = -2.35E-5
*ARAM FT = 2.35E-5
*ARAM AU = 5.70E-5
*FOR 10 KTS
PARAM AT = -2.85E-5
PARAM FT = 2.85E-5
PARAM AU = 7.00E-5
*FOR 12KTS
*ARAM AT = -4.138E-5
*ARAM FT = 4.138E-5
*ARAM AU = 9.77E-5
*FOR 18KTS
*ARAM AT = -8.400E-5
*ARAM FT = 8.400E-5
*ARAM AU = 1.80E-4
*FOR 25KTS
*ARAM AT = -9.080E-5
*ARAM FT = 9.080E-5
*ARAM AU = 2.100E-4
*
*PRECALCULATED COFACTORS
*
PARAM DEL=.18901E-16,    COFAA=.212502E-14, COFAB = 0.0,     COFAC = 0.0
PARAM COFAD = 0.0,       COFAE = 0.0,       COFAF = 0.0,     COFBA = 0.0
PARAM COFBB=.153152E-14, COFBC=0.0,    COFBD=-.186106E-10,   COFBE = 0.0
PARAM COFBF=.17543E-12,  COFCA=0.0,    COFCB = 0.0,   COFCC =.11665E-14
```

```
PARAM COFCD = 0.0,        COFCE=-.999506E-13,COFCF = 0.0,    COFDA = 0.0
PARAM COFDB=-.905797E-16,COFDC=0.0,    COFDD=.294191E-11,   COFDE = 0.0
PARAM COFDF=-.224359E-13,COFEA=0.0,    COFEB = 0.0,   COFEC=-.58035E-18
PARAM COFED = 0.0,        COFEE=.19562E-13,  COFEF = 0.0,    COFFA = 0.0
PARAM COFFB=.162929E-17, COFFC=0.0,    COFFD=-.318591E-13,  COFFE = 0.0
PARAM COFFF =.179521E-13
*
*HYDRODYNAMIC COEFFICIENTS AND SUBMARINE CHARACTERISTICS
*
PARAM LC = 415.0, ML = .0087445, A1 = -0.001,  A2 = -.00095, A3 = .00195
PARAM IX=7.3114E-6,IY=5.6867E-4, IZ = 5.6867E-4
PARAM XUDOT =-.00015, XVR = .011,  XWQ = -.0075, XVV = .0065, XWW =.002
PARAM XDRDR =-.0028, XDSDS=-.0025,XDBDB=-.0026, XQQ =-.0002, XRR=-.00009
PARAM XRP   = .00025
PARAM YVDOT =-.011,   YWP = .0075,  YV =-.021,  Y1V1V=-.06,    YR = .003
PARAM YV1R1 =-.0073,   YP =-.0007,YRDOT=.00009, YPDOT=-.0003, YDR =.0062
PARAM YPQ   =.0002,   YWV =-.065
PARAM ZWDOT =-.0075,  ZVP =-.007,   ZS =-.0001,   ZW =-.011, ZW1W1=-.03
PARAM ZVV   =.065,     ZQ =-.0045,ZW1Q1=-.006,   ZVR =-.008, ZRR =-.0015
PARAM ZDS   =-.005,   ZDB =-.0025,ZQDOT=-.0002, Z1W1 = 0.0,  ZWW = 0.0
PARAM ZRP   =-.0009
PARAM KPDOT =-3.E-6,  KQR =-.0001,KRDOT=-7.E-6, K1P1P=-8.E-7, KV =-.0007
PARAM K1V1V =-.0009,   KP =-3.5E-5, KR =-4.E-5, KVDOT=-.00025,KVW=.0035
PARAM KDR   =7.E-5,   KWP = 2.5E-4
PARAM MQDOT =-.0004,  MRP =.00015,  MS =4.E-5,    MW =.003, M1W1W=-.005
PARAM MVV   =.015,     MQ =-.0025,M1W1Q=-.002,   MVR =-.004, MRR=-.00055
PARAM MWDOT =-.0002,  MDS =-.0025, MDB =.0005,  M1W1 = 0.0,  MVP =.0009
PARAM NRDOT =-5.E-4,  NPQ =-4.E-4,NPDOT=-7.E-6,   NV =-.0075,N1V1V=.014
PARAM NR    =-.003, N1V1R =-.0045,  NP =-2.E-6,NVDOT =.0003, NDR =-.003
PARAM NWV   =.015,    NWP =-.0002
PARAM BZB   =1.011413E-3
INCON YADOT = 0.0, RODOT = 0.0, PIDOT = 0.0
INCON DS = 0.0 , DB = 0.0, DR =0.0
CONTRL FINTIM=360.,DELT=.01,DELS=.5
PRINT 1.,V,YAWGRA,ROLGRA,DEPTH,PITGRA
*
INITIAL
      LC2 = LC**2
      IZX = IZ-IX
      IYX = IY-IX
      IZY = IZ-IY
*
DYNAMIC
*
*     IF(TIME.GE.10) DR =0.611
*     IF(TIME.GE.10) DB =0.611
*     IF(TIME.GE.40) DR =-0.611
*     IF(TIME.GE.40) DB =-0.611
*     IF(TIME.GE.70) DR =0.0
*     IF(TIME.GE.70) DB =0.0
*
DERIVATIVE
*
*PRECALCULATION FOR EQUATIONS OF MOTION
*
      PA1 = XDRDR*U*U*DR*DR/LC
```

```
PA2 = XDSDS*U*U*DS*DS/LC
PA3 = XDBDB*U*U*DB*DB/LC
PB  = YDR*U*U*DR/LC
PC2 = ZDS*U*U*DS/LC
PC3 = ZDB*U*U*DB/LC
PD  = KDR*U*U*DR/LC2
PE2 = MDS*U*U*DS/LC2
PE3 = MDB*U*U*DB/LC2
PF  = NDR*U*U*DR/LC2
PA  = PA1 + PA2 + PA3
PC  = PC2 + PC3
PE  = PE2 + PE3
ABV = ABS(V)
ABW = ABS(W)
ABP = ABS(P)
ABQ = ABS(Q)
ABR = ABS(R)
VVWW= V*V + W*W
AVW = SQRT(VVWW)
ABWP=FCNSW(W,-1.,0.,1.)
ABVP=FCNSW(V,-1.,0.,1.)
SA1 =+LC*(XQQ*Q**2 + XRR*R**2 + XRP*R*P)
SA2 =+(ML*V*R + XVR*V*R + XWQ*W*Q -ML*W*Q)
SA3 =+(XVV*V**2 + XWW*W**2)/LC - SIN(PITCH)*(AT+FT+AU)
SA4 =+(A1*U**2 + A2*U*UC + A3*UC**2)/LC
SB1 =+LC*YPQ*P*Q
SB2 =+(YWP*W*P + YV1R1*ABR*AVW*ABVP +ML*W*P - ML*U*R)
SB3 =+(YWV*W*V + Y1V1V*AVW*V)/LC + SIN(ROLL)*COS(PITCH)*(AT+FT+AU)
SB4 =(YR*R +YP*P +YV*V/LC)*U
SC1 = LC*R*(ZRR*R + ZRP*P)
SC2 =+(ZVP*V*P + ZVR*V*R + ZW1Q1*ABQ*AVW*ABWP + ML*U*Q - ML*P*V)
SC3 =+(ZWW*W**2 + ZVV*V**2 + ZW1W1*W*AVW + U*Z1W1*ABW + U*U*ZS)/LC
SC4 = ZQ*U*Q + ZW*U*W/LC + COS(PITCH)*COS(ROLL)*(AT+FT+AU)
SD1 =+(KQR*Q*R + K1P1P*ABP*P) - IZY*Q*R
SD2 = (KWP*W*P-BZB*SIN(ROLL)*COS(PITCH))/LC
SD3 =+(K1V1V*V*AVW + KVW*V*W + KS*U**2)/LC2
SD4 = ((KP*P + KR*R)/LC + KV*V/LC2)*U
SE1 = (MRP*P + MRR*R + IZX*P)*R
SE2 = ((MVR*R + MVP*P)*V + M1W1Q*AVW*Q - BZB*SIN(PITCH))/LC
SE3 =(MVV*V**2 + MWW*W**2 + M1W1W*AVW*W +M1W1*U*AVW + U**2*MS)/LC2
SE4 = MQ*U*Q/LC + (MW*U*W -(175.5*FT-219.5*AT)*COS(PITCH)*...
COS(ROLL))/LC2
SF1 = (NPQ-IYX)*P*Q
SF2 =+(NWP*W*P + N1V1R*AVW*R)/LC
SF3 = (NWV*W + N1V1V*AVW)*V/LC2
SF4 = (NP*P+NR*R)*U/LC+(NV*U*V+(175.5*FT-219.5*AT)*COS(PITCH)*...
SIN(ROLL))/LC2
SA  = SA1 + SA2 + SA3 + SA4
SB  = SB1 + SB2 + SB3 + SB4
SC  = SC1 + SC2 + SC3 + SC4
SD  = SD1 + SD2 + SD3 + SD4
SE  = SE1 + SE2 + SE3 + SE4
SF  = SF1 + SF2 + SF3 + SF4
ZA  = SA + PA
ZB  = SB + PB
ZC  = SC + PC
```

```
      ZD  = SD + PD
      ZE  = SE + PE
      ZF  = SF + PF
*
*EQUATIONS OF MOTION
*
      UDOT =(COFAA*ZA+COFAB*ZB+COFAC*ZC+COFAD*ZD+COFAE*ZE+COFAF*ZF)/DEL
      VDOT =(COFBA*ZA+COFBB*ZB+COFBC*ZC+COFBD*ZD+COFBE*ZE+COFBF*ZF)/DEL
      WDOT =(COFCA*ZA+COFCB*ZB+COFCC*ZC+COFCD*ZD+COFCE*ZE+COFCF*ZF)/DEL
      PDOT =(COFDA*ZA+COFDB*ZB+COFDC*ZC+COFDD*ZD+COFDE*ZE+COFDF*ZF)/DEL
      QDOT =(COFEA*ZA+COFEB*ZB+COFEC*ZC+COFED*ZD+COFEE*ZE+COFEF*ZF)/DEL
      RDOT =(COFFA*ZA+COFFB*ZB+COFFC*ZC+COFFD*ZD+COFFE*ZE+COFFF*ZF)/DEL
*
*AUXILARY EQUATIONS
*
      ZODOT =-U*SIN(PITCH)+V*COS(PITCH)*SIN(ROLL)+W*COS(PITCH)*COS(ROLL)
      PIDOT = Q*COS(ROLL)-R*SIN(ROLL)
      YADOT = (R*COS(ROLL)+Q*SIN(ROLL))/COS(PITCH)
      RODOT = P+YADOT*SIN(PITCH)
      U = INTGRL(UC,UDOT)
      V = INTGRL(0.,VDOT)
      W = INTGRL(0.,WDOT)
      P = INTGRL(0.,PDOT)
      Q = INTGRL(0.,QDOT)
      R = INTGRL(0.,RDOT)
      DEPTH = INTGRL(0.,ZODOT)
*     ROLL  = INTGRL(0.43633,RODOT)
      ROLL  = INTGRL(0.0,RODOT)
*     PITCH = INTGRL(0.7854,PIDOT)
      PITCH = INTGRL(0.0,PIDOT)
      YAW   = INTGRL(0.,YADOT)
      DBGRA = DB*57.296
      DSGRA = DS*57.296
      DRGRA = DR*57.296
      PITGRA= PITCH*57.296
      ROLGRA= ROLL*57.296
      YAWGRA= YAW*57.296
SAVE  0.1,V,DEPTH,YAW,PITGRA,ROLL,ZODOT
GRAPH(DE=TEK618)TIME,DEPTH,ZODOT,PITGRA
LABEL  NI.PITCH0.04RAD. U=18.58 FT/SEC. NO PLANES
GRAPH(DE=TEK618)TIME,ROLL,YAW,V
LABEL  INI.ROLL=0.1 RAD. U=18.580 FT/SEC. NO PLANES
```

APPENDIX I. COMPENSATED NON-LINEAR MODEL

```
*
*THIS PROGRAM SIMULATES THE COMPENSATED NON-LINEAR SUBMARINE IN SIX
*DEGREES OF FREEDOM
*
TITLE COMPENSATED NONLINEAR SIX DEGREE OF FREEDOM SUBMARINE SIMULATION
PARAM KH = 1.00
PARAM K1 = 0.015
PARAM K2 = 2.0
PARAM UC = 18.69
*ARAM ORYAW=1.5726
PARAM ORYAW=0.2618
*PARAM ZOR=10.
PARAM POR=0.0
*
*BALLAST TANKS CONTAINS FOR DIFFERENT AXIAL SPEEDS
*
*FOR 5 KTS
*ARAM AT = -0.800E-5
*ARAM FT = 0.800E-5
*ARAM AU = 1.400E-5
*FOR 6 KTS
*ARAM AT = -1.030E-5
*ARAM FT = 1.030E-5
*ARAM AU = 2.500E-5
*FOR 8 KTS
*ARAM AT = -1.85E-5
*ARAM FT = 1.85E-5
*ARAM AU = 4.5E-5
*FOR 9 KTS
PARAM AT = -2.35E-5
PARAM FT = 2.35E-5
PARAM AU = 5.7E-5
*FOR 12KTS
*ARAM AT = -4.138E-5
*ARAM FT = 4.138E-5
*ARAM AU = 9.77E-5
*FOR 18KTS
*ARAM AT = -8.400E-5
*ARAM FT = 8.400E-5
*ARAM AU = 1.80E-4
*FOR 25KTS
*ARAM AT = -9.080E-5
*ARAM FT = 9.080E-5
*ARAM AU = 2.100E-4
*
*PRECALCULATED COFACTORS
*
PARAM DEL=.18901E-16,    COFAA=.212502E-14, COFAB = 0.0,    COFAC = 0.0
PARAM COFAD = 0.0,       COFAE = 0.0,       COFAF = 0.0,    COFBA = 0.0
PARAM COFBB=.153152E-14, COFBC=0.0,    COFBD=-.186106E-10,  COFBE = 0.0
```

```
PARAM COFBF=.17543E-12,  COFCA=0.0,    COFCB = 0.0,    COFCC =.11665E-14
PARAM COFCD = 0.0,       COFCE=-.999506E-13,COFCF = 0.0,    COFDA = 0.0
PARAM COFDB=-.905797E-16,COFDC=0.0,    COFDD=.294191E-11,    COFDE = 0.0
PARAM COFDF=-.224359E-13,COFEA=0.0,    COFEB = 0.0,    COFEC=-.58035E-18
PARAM COFED = 0.0,       COFEE=.19562E-13,  COFEF = 0.0,    COFFA = 0.0
PARAM COFFB=.162929E-17, COFFC=0.0,    COFFD=-.318591E-13,    COFFE = 0.0
PARAM COFFF =.179521E-13
*
*HYDRODYNAMIC COEFFICIENTS AND SUBMARINE CHARACTERISTICS
*
PARAM LC = 415.0, ML = .0087445, A1 = -0.001,  A2 = -.00095, A3 = .00195
PARAM IX=7.3114E-6,IY=5.6867E-4,  IZ = 5.6867E-4
PARAM XUDOT =-.00015, XVR = .011,  XWQ = -.0075, XVV = .0065, XWW =.002
PARAM XDRDR =-.0028, XDSDS=-.0025,XDBDB=-.0026, XQQ =-.0002, XRR=-.00009
PARAM XRP   = .00025
PARAM YVDOT =-.011,   YWP = .0075,  YV =-.021,  Y1V1V=-.06,    YR = .003
PARAM YV1R1 =-.0073,   YP =-.0007,YRDOT=.00009, YPDOT=-.0003, YDR =.0062
PARAM YPQ   =.0002,   YWV =-.065
PARAM ZWDOT =-.0075,  ZVP =-.007,   ZS =-.0001,   ZW =-.011, ZW1W1=-.03
PARAM ZVV   =.065,      ZQ =-.0045,ZW1Q1=-.006,   ZVR =-.008, ZRR =-.0015
PARAM ZDS   =-.005,   ZDB =-.0025,ZQDOT=-.0002, Z1W1 = 0.0,  ZWW = 0.0
PARAM ZRP   =-.0009
PARAM KPDOT =-3.E-6,  KQR =-.0001,KRDOT=-7.E-6, K1P1P=-8.E-7, KV =-.0007
PARAM K1V1V =-.0009,   KP =-3.5E-5, KR =-4.E-5, KVDOT=-.00025,KVW=.0035
PARAM KDR   =7.E-5,   KWP = 2.5E-4
PARAM MQDOT =-.0004,  MRP =.00015,  MS =4.E-5,     MW =.003, M1W1W=-.005
PARAM MVV   =.015,      MQ =-.0025,M1W1Q=-.002,   MVR =-.004, MRR=-.00055
PARAM MWDOT =-.0002,  MDS =-.0025, MDB =.0005,  M1W1 = 0.0,  MVP =.0009
PARAM NRDOT =-5.E-4,  NPQ =-4.E-4,NPDOT=-7.E-6,   NV =-.0075,N1V1V=.014
PARAM NR    =-.003, N1V1R =-.0045,  NP =-2.E-6,NVDOT =.0003, NDR =-.003
PARAM NWV   =.015,    NWP =-.0002
PARAM BZB   =1.011413E-3
INCON YADOT = 0.0, RODOT = 0.0, PIDOT = 0.0
INCON DS = 0.0 , DB = 0.0, DR =0.0
CONTRL FINTIM=360.,DELT=.01,DELS=.5
PRINT 1.,YAWDEG,ROLDEG,DEPTH,PITDEG,U
*
INITIAL
      LC2 = LC**2
      IZX = IZ-IX
      IYX = IY-IX
      IZY = IZ-IY
*
*ERROR LIMIT CALCULATION
*
DYNAMIC
      IF(U.LT.15.) LIMVER=35.
      IF(U.GE.15.) LIMVER = 25
      IF(U.GE.25.) LIMVER = 15.
      IF(U.LT.20.3) LIMHOR=0.070
      IF(U.GE.20.3) LIMHOR=0.050
      IF(U.GE.30.4) LIMHOR=0.035
      HER = ORYAW - YAW
      ZER = ZOR - DEPTH
      PER = POR - PITCH
      ZERR= LIMIT(-LIMVER,LIMVER,ZER)
```

```
      HERR= LIMIT(-LIMHOR,LIMHOR,HER)
*
DERIVATIVE
*
*PRECALCULATION FOR EQUATIONS OF MOTION
*
      PA1 = XDRDR*U*U*DR*DR/LC
      PA2 = XDSDS*U*U*DS*DS/LC
      PA3 = XDBDB*U*U*DB*DB/LC
      PB  = YDR*U*U*DR/LC
      PC2 = ZDS*U*U*DS/LC
      PC3 = ZDB*U*U*DB/LC
      PD  = KDR*U*U*DR/LC2
      PE2 = MDS*U*U*DS/LC2
      PE3 = MDB*U*U*DB/LC2
      PF  = NDR*U*U*DR/LC2
      PA  = PA1 + PA2 + PA3
      PC  = PC2 + PC3
      PE  = PE2 + PE3
      ABV = ABS(V)
      ABW = ABS(W)
      ABP = ABS(P)
      ABQ = ABS(Q)
      ABR = ABS(R)
      VVWW= V*V + W*W
      AVW = SQRT(VVWW)
      ABWP=FCNSW(W,-1.,0.,1.)
      ABVP=FCNSW(V,-1.,0.,1.)
      SA1 =+LC*(XQQ*Q**2 + XRR*R**2 + XRP*R*P)
      SA2 =+(ML*V*R + XVR*V*R + XWQ*W*Q -ML*W*Q)
      SA3 =+(XVV*V**2 + XWW*W**2)/LC - SIN(PITCH)*(AT+FT+AU)
      SA4 =+(A1*U**2 + A2*U*UC + A3*UC**2)/LC
      SB1 =+LC*YPQ*P*Q
      SB2 =+(YWP*W*P + YV1R1*ABR*AVW*ABVP +ML*W*P - ML*U*R)
      SB3 =+(YWV*W*V + Y1V1V*AVW*V)/LC + SIN(ROLL)*COS(PITCH)*(AT+FT+AU)
      SB4 =(YR*R +YP*P +YV*V/LC)*U
      SC1 = LC*R*(ZRR*R + ZRP*P)
      SC2 =+(ZVP*V*P + ZVR*V*R + ZW1Q1*ABQ*AVW*ABWP + ML*U*Q - ML*P*V)
      SC3 =+(ZWW*W**2 + ZVV*V**2 + ZW1W1*W*AVW + U*Z1W1*ABW + U*U*ZS)/LC
      SC4 = ZQ*U*Q + ZW*U*W/LC + COS(PITCH)*COS(ROLL)*(AT+FT+AU)
      SD1 =+(KQR*Q*R + K1P1P*ABP*P) - IZY*Q*R
      SD2 = (KWP*W*P-BZB*SIN(ROLL)*COS(PITCH))/LC
      SD3 =+(K1V1V*V*AVW + KVW*V*W + KS*U**2)/LC2
      SD4 = ((KP*P + KR*R)/LC + KV*V/LC2)*U
      SE1 = (MRP*P + MRR*R + IZX*P)*R
      SE2 = ((MVR*R + MVP*P)*V + M1W1Q*AVW*Q - BZB*SIN(PITCH))/LC
      SE3 =(MVV*V**2 + MWW*W**2 + M1W1W*AVW*W +M1W1*U*AVW + U**2*MS)/LC2
      SE4 = MQ*U*Q/LC + (MW*U*W -(175.5*FT-219.5*AT)*COS(PITCH)*...
      COS(ROLL))/LC2
      SF1 = (NPQ-IYX)*P*Q
      SF2 =+(NWP*W*P + N1V1R*AVW*R)/LC
      SF3 = (NWV*W + N1V1V*AVW)*V/LC2
      SF4 = (NP*P+NR*R)*U/LC+(NV*U*V+(175.5*FT-219.5*AT)*COS(PITCH)*...
      SIN(ROLL))/LC2
      SA  = SA1 + SA2 + SA3 + SA4
      SB  = SB1 + SB2 + SB3 + SB4
```

```
      SC  = SC1 + SC2 + SC3 + SC4
      SD  = SD1 + SD2 + SD3 + SD4
      SE  = SE1 + SE2 + SE3 + SE4
      SF  = SF1 + SF2 + SF3 + SF4
      ZA  = SA + PA
      ZB  = SB + PB
      ZC  = SC + PC
      ZD  = SD + PD
      ZE  = SE + PE
      ZF  = SF + PF
*
*EQUATIONS OF MOTION
*
      UDOT =(COFAA*ZA+COFAB*ZB+COFAC*ZC+COFAD*ZD+COFAE*ZE+COFAF*ZF)/DEL
      VDOT =(COFBA*ZA+COFBB*ZB+COFBC*ZC+COFBD*ZD+COFBE*ZE+COFBF*ZF)/DEL
      WDOT =(COFCA*ZA+COFCB*ZB+COFCC*ZC+COFCD*ZD+COFCE*ZE+COFCF*ZF)/DEL
      PDOT =(COFDA*ZA+COFDB*ZB+COFDC*ZC+COFDD*ZD+COFDE*ZE+COFDF*ZF)/DEL
      QDOT =(COFEA*ZA+COFEB*ZB+COFEC*ZC+COFED*ZD+COFEE*ZE+COFEF*ZF)/DEL
      RDOT =(COFFA*ZA+COFFB*ZB+COFFC*ZC+COFFD*ZD+COFFE*ZE+COFFF*ZF)/DEL
*
*AUXILARY EQUATIONS
*
      ZODOT =-U*SIN(PITCH)+V*COS(PITCH)*SIN(ROLL)+W*COS(PITCH)*COS(ROLL)
      PIDOT = Q*COS(ROLL)-R*SIN(ROLL)
      YADOT = (R*COS(ROLL)+Q*SIN(ROLL))/COS(PITCH)
      RODOT = P+YADOT*SIN(PITCH)
      U = INTGRL(UC,UDOT)
      V = INTGRL(0.,VDOT)
      W = INTGRL(0.,WDOT)
      P = INTGRL(0.,PDOT)
      Q = INTGRL(0.,QDOT)
      R = INTGRL(0.,RDOT)
      DEPTH = INTGRL(0.0,ZODOT)
      ROLL  = INTGRL(0.0,RODOT)
      PITCH = INTGRL(0.0,PIDOT)
      YAW   = INTGRL(0.,YADOT)
*COMPENSATOR GC11
      C11 = -K1*ZERR
      C12 = LEDLAG(0.,1.0,0.1,C11)
      DB  = REALPL(0.,.667,C12)
*
*COMPENSATOR GC22
      C21 = -K2*PER
      DS  = REALPL(0.,.667,C21)
*
*COMPENSATOR GC
      C1 = -KH*HERR
      C2 = LEDLAG(0.,100.,10.,C1)
      DR  = REALPL(0.,.667,C2)
*
      DBDEG = DB*57.296
      DSDEG = DS*57.296
      DRDEG = DR*57.296
      PITDEG= PITCH*57.296
      ROLDEG= ROLL*57.296
```

```
      YAWDEG= YAW*57.296
SAVE   0.1,V,ZDOT,DEPTH,PITDEG,ROLDEG,YAWDEG,DRDEG,DSDEG,DBDEG
GRAPH(DE=TEK618)TIME,ROLLDEG,YAWDEG,V
LABEL  15 DEGREE COURSE CHANGE  U=10 KTS.
*GRAPH(DE=TEK618)TIME,PITDEG,DEPTH,ZDOT
*LABEL  10 FEET DEPTH CHANGE  U=10 KTS.
```

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