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# Managing New and Remanufactured Products

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We study a firm that makes new products in the first period and uses returned cores to offer remanufactured products, along with new products, in future periods. We introduce the monopoly environment in two-period and multiperiod scenarios to identify thresholds in remanufacturing operations. Next, we focus our attention on the duopoly environment where an independent operator (IO) may intercept cores of products made by the original equipment manufacturer (OEM) to sell remanufactured products in future periods. We characterize the production quantities associated with self-selection and explore the effect of various parameters in the Nash equilibrium. Among other results, we find that if remanufacturing is very profitable, the original-equipment manufacturer may forgo some of the first-period margin by lowering the price and selling additional units to increase the number of cores available for remanufacturing in future periods. Further, as the threat of competition increases, the OEM is more likely to completely utilize all available cores, offering the remanufactured products at a lower price.

*Key words:* remanufacturing; duopoly; self-selection; product-line pricing

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## 1. Introduction

Remanufacturing is a process in which used products are disassembled and its parts are repaired and used in the production of new products. A successful remanufacturing operation often adopts high-quality standards. It allows offering products that enhance brand equity and keep customers loyal. Often, the company expands its market coverage by offering remanufactured products at a low price, side by side with the new products. The benefits of remanufacturing are even greater when the remanufactured product is indistinguishable from the new product. The disposable camera is a good example of a product where the firm benefits from the cost reduction, while charging full price for the remanufactured product.

We analyze a model where the remanufactured and the original products are indistinguishable to the customer. We analyze two-period and multiperiod scenarios where the manufacturer only produces the new product in the first period, but has the option of making new and remanufactured products in subsequent periods. Pricing decisions impact the dynamics across periods in such cases. For example, if the price is high in the first period, profits in the first period might increase, but the number of reusable products available in the second period decreases, thereby reducing second-period profit potential. However, if the price is

low in the first period, initial profits might decrease, but the firm has an abundant supply of cores for remanufacturing in future periods. First, we derive the optimal quantities and prices, and characterize the optimality conditions for a monopolist that offers either both product types or just new products in this market. Next, we consider a competitive setting where a competitor may intercept the returning cores and exercise the remanufacturing profit opportunity. This resembles the case where local film-processing centers intercept single-use cameras and remanufacture them to sell under their brand. In this setting, we characterize the strategic regions of operation. We provide analytical insights for all cases.

The rest of the paper is organized as follows. Section 2 presents the related literature. Section 3 analyzes the monopolistic setting. Section 4 analyzes the impact of competition, and we conclude in §5.

## 2. Related Research

There is general agreement that remanufacturing is environmentally efficient and profitable (Ferrer and Whybark 2000, 2002; Guide 2000; Larson et al. 2000). Ayres et al. (1997) evaluate the economics of remanufacturing and point out some problematic issues specific to remanufacturing that have a substantial impact on profitability. Ferrer and Ayres (2000)

analyze the macroeconomic impact of remanufacturing and confirm the intuition that it promotes demand for labor and reduces the consumption of raw materials.

Remanufactured products can play an important role in increasing profits, resulting from reduced production costs. McConocha and Speh (1991) argue that remanufacturing creates important benefits: (1) savings in labor, materials, and energy costs, (2) shorter production lead times, (3) balanced production lines, (4) new market development opportunities, and (5) a positive, socially concerned image. Even though profitable and environmentally efficient in many cases, remanufacturing presents great challenges for research and practice. Many of the trade-offs require interdisciplinary analysis. Guide et al. (2000) enumerate several complicating characteristics in remanufacturing that relate to process uncertainty. These issues include the timing and volume of product returns, yield estimation, balancing demand with core returns, and managing reverse logistics (Ferrer and Whybark 2001; Guide 1997, 2000; Kekre et al. 2003; Richter and Sombrutzki 2000; Teunter et al. 2000; van der Laan and Salomon 1999). Fleischmann et al. (1997) review quantitative models. Corbett and Kleindorfer (2001) introduce the special issue on manufacturing and ecologistics in POM.

Our research extends the perspectives regarding the production of durable and quasi-durable goods with monopoly in the first period and remanufacturing competition in subsequent periods. Discussions of other secondary distribution/segmentation channels are found in Purohit and Staelin (1994). They provide different policies to increase the total manufacturer's profit in a two-period model that compares buyback and lease schemes. In related research, Hendel and Lizzeri (1999) propose a model where consumers have heterogeneous valuations for quality so that used-good markets play a locative role to address the interference introduced by the first market on the secondary market of a monopolist company. Heese et al. (2005) determine a manufacturer's incentive mechanism to develop a smoothly functioning secondary market based on a buyback contract with predetermined delivery. Other related two-period models include Van Ackere and Reyniers (1993, 1995), Levinthal and Purohit (1989), and Purohit (1992).

Ferrer (1996) modeled the profit of a monopolist that has the opportunity to market remanufactured and new products in a steady-state environment. The firm offers a remanufactured and an all-new product in a given market, maximizing profits with an appropriate pricing scheme. The research shows three profitable strategies, which depend on the operating parameters: to make (1) both product types, (2) just

the new product, or (3) just the remanufactured product. In the last alternative, the firm indicates that the product is remanufactured, even though many individual units might be new. Other research has addressed the marketability of remanufactured products. Debo et al. (2005) extend the latter model to an infinite-horizon setting and determine the product technology a priori to maximize the profitability of the market segmentation. Majumder and Groenevelt (2001) describe a two-period model where the original-equipment manufacturer (OEM) may choose to remanufacture or not in the second period. The reverse logistics process is based on the "shell allocation mechanism" observed in the respective market. Four of these mechanisms are considered: whether each of the players (the OEM and the independent operator) can or cannot use the cores that are not utilized by the other company. They prove the existence of a unique pure-strategy Nash equilibrium as the solution of the competitive environment in each of the four allocation mechanisms studied.

Our research differs from the above three papers, and in some cases extends them in the following manner. As opposed to Ferrer (1996) and Debo et al. (2005), our attention is focused on optimal equilibrium solutions in the duopoly case. Our article extends Majumder and Groenevelt (2001) in that we also consider a multiperiod setting where the independent operator (IO) competes in the second and subsequent periods. In our core collection process, neither company can use the cores that are not used by its competitor (a situation similar to the third shell allocation mechanism introduced by Majumder and Groenevelt). Our demand function differs from theirs because it ensures self-selection: Customers show higher preference for OEM's product than for IO's product, a behavior quantified in Lemma 1. Secondly, we extend the analysis to obtain closed-form solutions for prices and quantities in the Nash equilibrium and characterize the optimal solution region that previously was numerically explored.

### 3. Monopoly: Making Remanufactured and All-New Products

We introduce a monopolist that makes new products in the first period and has the opportunity to make new and remanufactured products in future periods. We assume that customers cannot distinguish between the two products, allowing the firm to charge the same price for both of them. We consider three approaches to this question. We start with a two-period deterministic model to keep the analysis simple and obtain sharper insights. Then, we extend the analysis to multiperiod and infinite planning horizons.

**Notation.** Variables, parameters, and demand function.

- $Q$  Size of the potential market, constant every period.
- $p_{i,j}$   $q_{i,j}$  Price charged ( $p$ ) and quantity demanded ( $q$ ) for product  $j$  in period  $i$ . The subscripts are  $i = 1, 2, \dots$ , and  $j = N$  (new),  $R$  (remanufactured), or  $A$  (all OEM products made in the period). Therefore,  $q_{iA} = q_{iN} + q_{iR}$ . In the first period, only the new product is made, and  $j$  is omitted. The demand function is assumed to be linearly decreasing in price.
- $c$   $s$  Marginal cost to make a new product ( $c$ ) and cost savings per remanufactured product ( $s$ ). The cost to make a remanufactured product is  $c - s$ .
- $\gamma$  Core collection yield, defined as the fraction of new products made in period  $i$  that is available for remanufacturing in period  $i + 1$ . Therefore,  $q_{iR} \leq \gamma(q_{i-1,A})$ .
- $\beta$  Discount factor per period ( $\beta \leq 1$ ).

Two of these parameters, the remanufacturing savings ( $s$ ) and the collection yield ( $\gamma$ ), characterize the firm's ability to perform key activities in the remanufacturing process. The cost parameter ( $c$ ) characterizes the firm's ability to manufacture the product. We select the remanufacturing savings ( $s$ ) as the key parameter to define the strategy space in each scenario that we analyze, because it seems to be the parameter over which managers can have the greatest impact through strategic management of the facility's resources.

### 3.1. Two-Period Monopoly Model

The manufacturer chooses the prices and quantities that maximize profits. In the first period, only the new product is offered. In the second period, the firm decides which products are offered. We formulate the problem as follows:

$$\begin{aligned} & \text{Max}_{p_1, p_2, q_{2R}} (p_1 - c)q_1 + \beta[(p_2 - c)q_{2N} + (p_2 - c + s)q_{2R}] \\ & \text{subject to } \gamma q_1 \geq q_{2R} \\ & \quad Q - p_2 \geq q_{2R} \\ & \quad Q - p_2 = q_{2N} + q_{2R} \\ & \quad q_1 = Q - p_1. \end{aligned}$$

This optimization leads to the following result.

**THEOREM 1. (EXISTENCE OF A REMANUFACTURING THRESHOLD).** *There is a threshold savings value  $s^* = ((1 - \gamma)(Q - c))/(\beta\gamma^2)$  such that, if  $s < s^*$ , the manufacturer makes both remanufactured and new products in the second period, and the optimal prices are*

$$p_1 = \frac{Q + c - \beta s \gamma}{2} \quad \text{and} \quad p_2 = \frac{Q + c}{2}.$$

If  $s \geq s^*$ , the new products are not made in the second period, and the optimal prices are

$$\begin{aligned} p_1 &= \frac{Q(1 - \beta\gamma + 2\beta\gamma^2) + c(1 + \beta\gamma) - s\beta\gamma}{2(1 + \beta\gamma^2)} \quad \text{and} \\ p_2 &= \frac{Q(2 - \gamma + \beta\gamma^2) + c(\gamma + \beta\gamma^2) - s\beta\gamma^2}{2(1 + \beta\gamma^2)}. \end{aligned}$$

The optimal policy is continuous at the remanufacturing threshold  $s^*$ .

**PROOF.** See the appendix (available as an online supplement at <http://mansci.pubs.informs.org/ecompanion.html>).

This theorem indicates when the monopolist makes new products in the second period. This happens whenever the remanufacturing savings is low relative to the threshold value. Figure 1 illustrates the behavior, showing how prices and profit change with the cost savings parameter.

**COROLLARY 1.1.**

- (i) The threshold savings level  $s^*$  is decreasing in  $c$ ,  $\beta$ ,  $\gamma$ ;
- (ii) If all cores are collected, the threshold savings level is zero (i.e.,  $\gamma = 1 \rightarrow s^* = 0$ ). Consequently, the new product is not made in Period 2, and the prices are given by

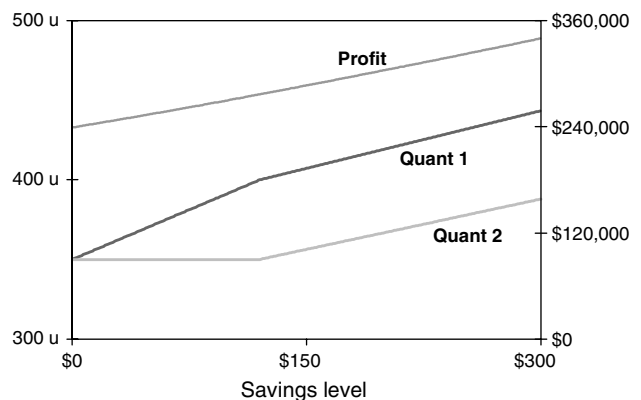
$$p_1 = p_2 = \frac{Q + c}{2} - \frac{s\beta}{2(1 + \beta)}.$$

This corollary implies that as the number of cores available increases, there is less need to price-discriminate across periods. Here is another result:

**COROLLARY 1.2.**

- (i) In either period, prices are equal to or lower than if the firm did not remanufacture.
- (ii) In the second period, price is equal to or higher than in the first period.

**Figure 1** A Remanufacturing Monopolist  $Q = 1,000$ ,  $c = \$350$ ,  $\beta = 0.95$ ,  $\gamma = 7/8$ ,  $s^* = \$112$



Part (ii) is surprising because there are remanufactured products in the second-period mix, yet price is higher than in Period 1, when all products are new. The rationale is that the monopolist is concerned with building a customer base that buys the products in Period 1 *and* provides cores that are essential for the low cost production in Period 2. The monopolist skims the market with the low cost that can only occur in Period 2. It happens that production drops in Period 2, particularly if no new products are made then. Because the firm serves a smaller number of customers, and these customers do not distinguish remanufactured from new products, the monopolist can afford to raise prices.

### 3.2. Monopoly Model with Multiperiod Horizon

Consider a planning horizon longer than two periods. The monopolist tries to maximize total profit by choosing the prices and quantities in each of  $M$  periods, constrained by the number of cores available for remanufacturing. The model becomes:

$$\begin{aligned} \text{Max}_{p_k, q_{k,R}} & (p_1 - c)q_1 + \sum_{k=2}^M \beta^{k-1} ((p_k - c)q_{k,N} + (p_k - c + s)q_{k,R}) \\ \text{s.t.} & \quad \gamma(Q - p_{k-1}) \geq q_{k,R}, \quad k = 2, \dots, M \\ & \quad Q - p_k \geq q_{k,R}, \quad k = 2, \dots, M \\ & \quad q_{k,N} + q_{k,R} = Q - p_k, \quad k = 2, \dots, M \\ & \quad q_1 = Q - p_1. \end{aligned}$$

The first constraint indicates that, in any given period, the number of remanufactured products is limited by the number of cores collected from previous period sales. The solution in this optimization depends on the relative values of parameters  $Q$ ,  $c$ ,  $s$ , and  $\gamma$ . Different parameter values may cause a constraint to bind or not. The following result is consistent with Theorem 1.

**THEOREM 2.** *Let the remanufacturing savings satisfy*

$$s \leq s^* = \frac{(1 - \gamma)(Q - c)}{\beta\gamma^2}.$$

*Hence, if the planning horizon is  $M$  periods, it is optimal to adopt a constant price policy in every period except the last. Production of new products should be sufficient to replace the cores that are lost at the end of the previous period, and total production in periods 1,  $\dots$ ,  $M - 1$  remains constant. The prices and quantities are:*

$$\begin{aligned} \text{Period 1: } & p_1 = \frac{1}{2}(Q + c - s\beta\gamma) \text{ and } q_1 = \frac{1}{2}(Q - c + s\beta\gamma); \\ \text{Periods } k = 2, \dots, M - 1: & p_k = \frac{1}{2}(Q + c - s\beta\gamma), q_{k,R} = \\ & \frac{1}{2}\gamma(Q - c + s\beta\gamma) \text{ and } q_{k,N} = \frac{1}{2}(1 - \gamma)(Q - c + s\beta\gamma); \\ \text{Period } M: & p_M = \frac{1}{2}(Q + c), q_{M,R} = \frac{1}{2}\gamma(Q - c + s\beta\gamma) \text{ and} \\ & q_{M,N} = \frac{1}{2}((1 - \gamma)(Q - c) + s\beta\gamma^2). \end{aligned}$$

**PROOF.** See the appendix.

This theorem indicates that it is optimal to remanufacture all cores collected, as long as the remanufacturing savings are smaller than the threshold ( $0 < s \leq s^*$ ). It also shows that the firm should adopt higher prices in the last period, skimming the entire consumer surplus. However, if  $s > s^*$ , the policy in Theorem 2 cannot be applied because it would violate one or more nonnegativity constraints. Instead, the monopolist makes more new products in early periods and more remanufactured products in later periods. As the remanufacturing savings parameter approaches  $c$ , the monopolist may offer only remanufactured products in the final periods of the planning horizon.

We illustrate this theorem with the five-period monopoly in Table 1. Each block corresponds to one variable of the optimal policy in Periods 1 through 5. Notice the policy in the column  $s = 1$ . In it,  $p_1 = p_2 = p_3 = p_4 = 7.185$  and  $p_5 = 7.5$ . Also,  $q_{1,R} = 0$ ,  $q_{2,R} = q_{3,R} = q_{4,R} = q_{5,R} = 1.9705$ . Moreover,  $q_{1,N} = 2.815$ ,  $q_{2,N} = q_{3,N} = q_{4,N} = 0.8445$ , and  $q_{5,N} = 0.5295$ .

The theorem can only be used for  $s \leq s^*$ ; otherwise, it violates the nonnegativity constraint for  $q_{5,N}$ . However, we obtained numerically the optimal policy when  $s \in (s^*, c]$ , shown in *italics*. Notice that in this range the remanufacturing savings is so great that the firm has no incentive to produce new products in the final period. Rather, the firm offers very low prices in Periods 1–3 to increase the quantities available for remanufacturing in Periods 2–5. Towards the end of the planning horizon, prices increase because the manufacturer does not plan to make new products in the final period. The larger is the remanufacturing savings, the more the firm sacrifices profit in Period 1 for a greater reward in future periods.

### 3.3. Monopoly Model with Infinite Horizon

The infinite-horizon model is a good approximation of long planning horizons, because of the exponentially decreasing value of future earnings. In the absence of any perturbation, such as the end of the planning horizon, we force the use of the same policy in every period after the ramp-up in Period 1: The monopolist makes just the new product in the Period 1, and Period 2 is the first of an infinite stream of identical periods. The profit generated by the identical periods is a discounted multiple of the profit in Period 2. Hence, we can simplify the notation and say that for any period  $i > 2$ ,  $p_{i,j} = p_{2,j} = p_2$ , and  $q_{i,j} = q_{2,j} = q_j$ . Here is the model:

$$\begin{aligned} \text{Max}_{p_1, p_2, q_R} & (p_1 - c)q_1 + \frac{\beta}{1 - \beta} [(p_2 - c)q_N + (p_2 - c + s)q_R] \\ \text{subject to} & \quad \gamma(Q - p_1) \geq q_R \\ & \quad \gamma(Q - p_2) \geq q_R \end{aligned}$$

**Table 1** A Five-Period Remanufacturing Monopolist  $Q = 10, c = 5, \beta = 0.9, \gamma = 0.7, s^* = 3.4014$

Remanufacturing savings level (s)											
0	0.5	1	1.5	2	2.5	3	3.4014	3.5	4	4.5	5
Prices each period ( $p_1, p_2, p_3, p_4, p_5$ )											
7.5	7.3425	7.185	7.0275	6.87	6.7125	6.555	6.4286	6.3975	6.240	6.0825	5.925
7.5	7.3425	7.185	7.0275	6.87	6.7125	6.555	6.4286	6.3975	6.240	6.0825	5.925
7.5	7.3425	7.185	7.0275	6.87	6.7125	6.555	6.4286	6.3975	6.240	6.0825	5.925
7.5	7.3425	7.185	7.0275	6.87	6.7125	6.555	6.4286	6.407	6.2977	6.1884	6.079
7.5	7.5	7.5	7.5	7.5	7.5	7.5	7.5	7.4849	7.4084	7.3319	7.2554
Total quantity each period											
2.5	2.6575	2.815	2.9725	3.13	3.2875	3.445	3.5714	3.6025	3.760	3.9175	4.075
2.5	2.6575	2.815	2.9725	3.13	3.2875	3.445	3.5714	3.6025	3.760	3.9175	4.075
2.5	2.6575	2.815	2.9725	3.13	3.2875	3.445	3.5714	3.6025	3.760	3.9175	4.075
2.5	2.6575	2.815	2.9725	3.13	3.2875	3.445	3.5714	3.593	3.7023	3.8116	3.921
2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5151	2.5916	2.6681	2.7446
Remanufactured quantity each period ( $q_{1,R}, q_{2,R}, q_{3,R}, q_{4,R}, q_{5,R}$ )											
0	0	0	0	0	0	0	0	0	0	0	0
1.75	1.8603	1.9705	2.0808	2.191	2.3013	2.4115	2.5	2.522	2.632	2.7423	2.8525
1.75	1.8603	1.9705	2.0808	2.191	2.3013	2.4115	2.5	2.522	2.632	2.7423	2.8525
1.75	1.8603	1.9705	2.0808	2.191	2.3013	2.4115	2.5	2.522	2.632	2.7423	2.8525
1.75	1.8603	1.9705	2.0808	2.191	2.3013	2.4115	2.5	2.5151	2.5916	2.6681	2.7446
New quantity each period ( $q_{1,N}, q_{2,N}, q_{3,N}, q_{4,N}, q_{5,N}$ )											
2.5	2.6575	2.815	2.9725	3.13	3.2875	3.445	3.5714	3.6025	3.760	3.9175	4.0750
0.75	0.7973	0.8445	0.8918	0.939	0.9863	1.0335	1.0714	1.081	1.128	1.1753	1.2225
0.75	0.7973	0.8445	0.8918	0.939	0.9863	1.0335	1.0714	1.081	1.128	1.1752	1.2225
0.75	0.7973	0.8445	0.8918	0.939	0.9863	1.0335	1.0714	1.071	1.070	1.0693	1.0684
0.75	0.6398	0.5295	0.4193	0.309	0.1988	0.0885	0	0	0	0	0
Profit each period											
6.25	6.2252	6.1508	6.0267	5.853	5.6298	5.3570	5.1020	5.0345	4.662	4.241	3.7694
6.25	7.1553	8.1213	9.1479	10.235	11.3830	12.5915	13.6054	13.861	15.190	16.581	18.032
6.25	7.1553	8.1213	9.1479	10.235	11.3830	12.5915	13.6054	13.861	15.190	16.581	18.032
6.25	7.1553	8.1213	9.1479	10.235	11.3830	12.5915	13.6054	13.8815	15.333	16.870	18.4936
6.25	7.1801	8.2205	9.3711	10.632	12.0031	13.4845	14.7534	15.0526	16.608	18.228	19.9133
Total profit											
25.594	28.388	31.352	34.487	37.792	41.268	44.915	47.965	48.732	52.712	56.852	61.151

$$q_N + q_R = Q - p_2$$

$$q_1 = Q - p_1.$$

The Lagrangean is concave, leading to the following result.

**THEOREM 3.** *If the product is offered for a large number of periods, it is optimal to adopt the same price in the first period as in the following periods. Moreover, production of new products should be sufficient to replace the cores that are lost in the process, such that total production remains constant. The prices and quantities are*

$$p_i = \frac{Q + c - s\beta\gamma}{2}$$

$$q_1 = q_A = \frac{Q - c + s\beta\gamma}{2}, \quad q_R = \gamma q_A \quad \text{and} \quad q_N = (1 - \gamma)q_A.$$

**PROOF.** See the appendix.

This theorem consolidates the results in the previous sections in a very intuitive manner. Because

the profit function does not incur fixed costs each period—a reasonable assumption in a dedicated production line—there is no incentive to store cores beyond the current period. Once a core is collected, it is remanufactured immediately or it is discarded.

## 4. Duopoly

Manufacturers of single-use cameras have been facing an unusual type of competition. Large local labs have instituted their own used-camera collection systems to feed parallel remanufacturing processes. They insert a new film in the camera, replace some critical parts, place their own private label, and sell them in supermarkets and drugstores. Manufacturers of printer cartridges face a very similar experience. In this section, we model this type of competition. In the first period, the original-equipment manufacturer (OEM) sells the new product. At the end of the first period, some units return to the OEM, and an independent operator (IO) intercepts the remainder. The OEM makes  $q_1 = (Q - p_1)$

in Period 1, knowing that it will face competition in the future. The IO responds with a lower price, knowing that, other things equal, customers prefer the OEM brand. The following lemma identifies the demand as a function of the prices of two vertically differentiated products.

**LEMMA 1.** *Suppose that two competing products, high (H) and low (L), are offered in the market. Let the variable  $z$  characterize the consumers according to their valuation of the high product. Let  $\alpha \in (0, 1)$  indicate the customer's tolerance for the low product. Large values of  $\alpha$  indicate that customers accept the low product better than if  $\alpha$  is small. We describe the utility that a consumer of type  $z$  enjoys when buying each product as  $U_H(z) = z - p_H$  (high) or  $U_L(z) = \alpha z - p_L$  (low). Let there be  $Q$  potential consumers uniformly distributed from 0 to  $Q$ , according to their valuation of the high product. If  $\alpha \in [p_L/p_H; 1 - (p_H - p_L)/Q]$ , the number of consumers buying each product type is:*

$$q_H(p_H, p_L) = \frac{(1 - \alpha)Q - p_H + p_L}{1 - \alpha}; \quad (1)$$

$$q_L(p_H, p_L) = \frac{\alpha p_H - p_L}{\alpha(1 - \alpha)}. \quad (2)$$

If  $\alpha < p_L/p_H$ ,  $q_L = 0$  and  $q_H = Q - p_H$ . If  $\alpha > 1 - (p_H - p_L)/Q$ ,  $q_L = Q - p_L/\alpha$  and  $q_H = 0$ .

**PROOF.** See the appendix.

Equations (1) and (2) are similar to the demand functions in a Bertrand duopoly: For each player, the demand increases when the competitor's price increases or when the player's own price decreases. However, Lemma 1 provides exact coefficients for the Bertrand demand function, identifying how customers select vertically differentiated products. Using this result, we can express the quantities sold as functions of the behavior-inducing prices. Let the OEM face competition in period  $k$ . We extend the notation in the previous section using  $p_{k,I}$  and  $q_{k,I}$  as the price and quantity sold by an independent operator (IO). Because each firm would only consider prices that would lead to nonnegative demand, we may disregard prices that would lead to  $\alpha \notin [p_{k,A}/p_{k,I}; 1 - (p_{k,A} - p_{k,I})/Q]$ . Hence, based on Lemma 1, we have:

$$q_{k,A}(p_{k,A}, p_{k,I}) = \frac{(1 - \alpha)Q - p_{k,A} + p_{k,I}}{1 - \alpha}; \quad (3)$$

$$q_{k,I}(p_{k,A}, p_{k,I}) = \frac{\alpha p_{k,A} - p_{k,I}}{\alpha(1 - \alpha)}. \quad (4)$$

#### 4.1. Two-Period Duopoly Model

Consider that the OEM makes only new products in Period 1, but it may sell either new products, remanufactured products (constrained by product returns), or both in Period 2. All units made by the IO are remanufactured. The remanufacturing savings are  $s_A$  for the

OEM and  $s_I$  for the IO. Also, the OEM collects a fraction  $\gamma$  of all units made in Period 1, and the IO collects the remainder. The OEM's objective function is:

$$\text{Max}_{p_1, p_{2A}, q_{2R}} (p_1 - c)q_1 + \beta[(p_{2A} - c)q_{2N} + (p_{2A} - c + s_A)q_{2R}]$$

$$\text{subject to } \gamma q_1 \geq q_{2R}$$

$$q_{2A} \geq q_{2R}$$

$$q_{2N} + q_{2R} = q_{2A}$$

$$q_1 = Q - p_1.$$

If we replace  $q_{2A}$  by its expression in (3), the Lagrangean of the OEM's objective function is

$$\begin{aligned} L_A = & (p_1 - c)(Q - p_1) \\ & + \beta \left( (p_{2A} - c) \frac{(1 - \alpha)Q - p_{2A} + p_{2I}}{1 - \alpha} + s_A q_{2R} \right) \\ & + \lambda_A (\gamma(Q - p_1) - q_{2R}) \\ & + \mu_A \left( \frac{(1 - \alpha)Q - p_{2A} + p_{2I}}{1 - \alpha} - q_{2R} \right). \end{aligned}$$

The Lagrangean is concave with respect to all three decisions variables. Hence, the KKT optimization conditions are:

$$\frac{\partial L_A}{\partial p_1} = Q - 2p_1 + c - \gamma \lambda_A = 0;$$

$$\frac{\partial L_A}{\partial p_{2A}} = \frac{\beta}{1 - \alpha} ((1 - \alpha)Q - 2p_{2A} + p_{2I} + c) - \frac{\mu_A}{1 - \alpha} = 0;$$

$$\frac{\partial L_A}{\partial q_{2R}} = s_A \beta - \lambda_A - \mu_A = 0;$$

$$\lambda_A (\gamma(Q - p_1) - q_{2R}) = 0;$$

$$\mu_A \left( \frac{(1 - \alpha)Q - p_{2A} + p_{2I}}{1 - \alpha} - q_{2R} \right) = 0;$$

$$\lambda_A \geq 0 \quad \text{and} \quad \mu_A \geq 0.$$

We evaluate the IO response to the OEM actions. The independent operator is constrained by its ability to intercept cores that the OEM would collect otherwise. Being rational, the IO attempts to maximize its profit, subject to the number of cores intercepted, as follows:

$$\text{Max}_{p_{2I}} (p_{2I} - c + s_I)q_{2I}$$

$$\text{subject to } (1 - \gamma)q_1 \geq q_{2I};$$

$$q_{2I} \geq 0.$$

The first constraint restricts the number of cores that the IO can remanufacture. Once we replace  $q_{2I}$  by its expression in (4), the Lagrangean of this objective function becomes

$$\begin{aligned} L_I = & (p_{2I} - c + s_I) \frac{\alpha p_{2A} - p_{2I}}{\alpha(1 - \alpha)} \\ & + \lambda_I \left( (1 - \gamma)(Q - p_1) - \frac{\alpha p_{2A} - p_{2I}}{\alpha(1 - \alpha)} \right) + \mu_I \frac{\alpha p_{2A} - p_{2I}}{\alpha(1 - \alpha)}. \end{aligned}$$

This Lagrangean is also concave. The KKT optimization conditions are:

$$\begin{aligned} \frac{\partial L_I}{\partial p_{2I}} &= \frac{\alpha p_{2A} - 2p_{2I} + c - s_I + \lambda_I}{\alpha(1 - \alpha)} = 0; \\ \lambda_I \left( (1 - \gamma)(Q - p_1) - \frac{\alpha p_{2A} - p_{2I}}{\alpha(1 - \alpha)} \right) &= 0; \\ \mu_I \frac{\alpha p_{2A} - p_{2I}}{\alpha(1 - \alpha)} &= 0; \\ \lambda_I \geq 0 \quad \text{and} \quad \mu_I &\geq 0. \end{aligned}$$

These conditions lead to the theorem that describes their optimal policies.

**THEOREM 4. (CHARACTERIZATION OF THE COMPETITIVE SPACE).** *The OEM remanufactures all cores that it collects. Moreover, seven linear relations between  $s_A$  and  $s_I$  (see Table 2 and Figure 2) characterize the optimal policies for the OEM and the IO:*

A. *If  $s_A < \sigma_{AB}$  and  $s_I < \sigma_{AC}$ , the OEM makes new products in Period 2, and the IO does not make any product.*

B. *If  $\sigma_{AB} < s_A$  and  $s_I < \sigma_{BD} + m_{BD}s_A$ , the OEM does not make new products in Period 2, and the IO does not make any product.*

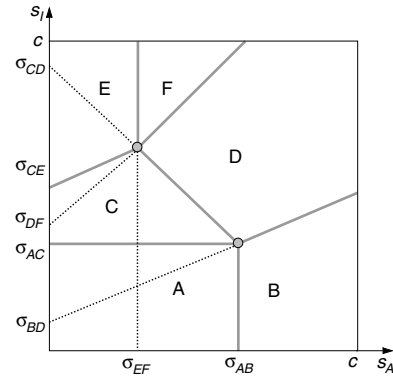
C. *If  $\sigma_{AC} < s_I < \min(\sigma_{CE} + m_{CE}s_A, \sigma_{CD} + m_{CD}s_A)$ , the OEM makes new products in Period 2, and the IO remanufactures some of the intercepted cores.*

D. *If  $\max(\sigma_{BD} + m_{BD}s_A, \sigma_{CD} + m_{CD}s_A) < s_I < \sigma_{DF} + m_{DF}s_A$ , the OEM does not make new products in Period 2, and the IO remanufactures some of the intercepted cores.*

**Table 2** Critical Values in the Remanufacturing Duopoly

Intercept and slope	Expression
$\sigma_{AB}$	$\frac{(Q - c)(2 - \gamma)(2 - \alpha)}{(2 - \alpha)\beta\gamma^2}$
$\sigma_{AC}$	$\frac{(1 - \alpha)(2c - \alpha Q)}{(2 - \alpha)}$
$\sigma_{BD}$	$\frac{c(2(1 + \beta\gamma^2) - \alpha\gamma(1 + 2\beta\gamma)) - \alpha Q(2 - \gamma + (1 - \alpha)\beta\gamma^2)}{2 + (2 - \alpha)\beta\gamma^2}$
$\sigma_{CD}$	$\frac{(1 - \alpha)[Q(4 - (4 - \alpha)\gamma) - c(2 - (4 - \alpha)\gamma)]}{2}$
$\sigma_{CE}$	$\frac{(1 - \alpha)[\alpha Q(2 - 4\gamma) - (\alpha Q + c(4 - \alpha))(1 - \gamma) - 4c]}{2(2 - \alpha)}$
$\sigma_{DF}$	$\frac{(1 - \alpha)[2c(1 + \beta\gamma^2) + (Q(2 - 3\gamma) - c(1 - \gamma))\alpha\beta\gamma - (Q - c)\alpha\gamma] - (Q - c)\alpha^2}{2 + (\alpha + 2\gamma(1 - \alpha))\beta\gamma}$
$\sigma_{EF}$	$\frac{(Q - c)(2 - \alpha - 2\gamma + 2\alpha\gamma)}{(\alpha + 2\gamma - 2\alpha\gamma)\beta\gamma}$
$m_{BD}$	$\frac{\alpha\beta\gamma^2}{2 + (2 - \alpha)\beta\gamma^2}$
$m_{CD}$	$\frac{-(4 - \alpha)(1 - \alpha)\beta\gamma^2}{2}$
$m_{CE}$	$\frac{(4 - \alpha)(1 - \alpha)(1 - \gamma)\alpha\beta\gamma}{2(2 - \alpha)}$
$m_{DF}$	$\frac{(2 - \alpha - \gamma + \alpha\gamma)\alpha\beta\gamma}{2(1 + \beta\gamma^2) - (1 - 2\gamma)\alpha\beta\gamma}$

**Figure 2** Two-Period Remanufacturing Duopoly



E. *If  $\sigma_{CE} + m_{CE}s_A < s_I$  and  $s_A < \sigma_{EF}$ , both the OEM and the IO remanufacture all cores that they collect, while the OEM makes additional new products in Period 2.*

F. *If  $\sigma_{EF} < s_A$  and  $\sigma_{DF} + m_{DF}s_A < s_I$ , both the OEM and the IO remanufacture all cores that they collect, and no new product is made in Period 2.*

**PROOF.** See the appendix.

The savings parameters  $s_A$  and  $s_I$  define the six scenarios in the second period: (1) for low values of  $s_A$ , the OEM makes both products; (2) for high values of  $s_A$ , the OEM makes just the remanufactured product. Moreover, (1) for low values of  $s_I$ , the imitator (IO) does not make any product; (2) for intermediate values of  $s_I$ , the IO remanufactures only part of the intercepted cores, and the remainder is wasted; (3) for high value of  $s_I$ , the IO remanufactures all intercepted cores. Here are two examples.

*Case 1.*  $Q = 1,000$ ,  $c = \$200$ ,  $\alpha = 0.9$ ,  $\beta = 0.75$ , and  $\gamma = 0.3$ . The manufacturing cost is low, customer's tolerance for private labels is high,  $\beta$  is low, and the IO intercepts most of the cores, leaving only 30% to the OEM. In this case,  $\sigma_{AB}$  and  $\sigma_{EF}$  are greater than  $c$  and  $\sigma_{AC} < 0$ . Hence, only scenarios C and E are present. Because  $\sigma_{CE} = 26$  and  $m_{CE} \cong 0$ , the IO remanufactures all intercepted cores when  $s_I > \$26$ . On the other hand, the OEM is so short on cores that it must build new products to meet demand in the second period, regardless of its remanufacturing savings. This outcome fits the intuition nicely, because the market accepts private labels well, and the IO captures most cores at the end of the first period.

*Case 2.*  $Q = 1,000$ ,  $c = \$800$ ,  $\alpha = 0.3$ ,  $\beta = 0.95$ , and  $\gamma = 0.5$ . Here, the manufacturing cost is high, customer's tolerance for private labels is very low,  $\beta$  is high, and the OEM collects one-half of all cores. The critical values are  $\sigma_{AB} = 570$ ,  $\sigma_{AC} = 535$ ,  $\sigma_{BD} = 518$ ,  $\sigma_{CE} = 558$ ,  $\sigma_{CD} = 710$ ,  $\sigma_{DF} = 548$ , and  $\sigma_{EF} = 421$ , implying that all scenarios are found in this case. The manufacturing cost is very high, leading to high price and low volume in the first period. Hence, scenarios A and



B prevail most of the time, unless the IO has very high remanufacturing savings.

**COROLLARY 4.1.**

(i) *Shorter cycle time (which translates into higher  $\beta$ ) leads to lower prices in both periods.*

(ii) *Lower manufacturing cost (lower  $c$ ) or higher remanufacturing savings (higher  $s_A$  and  $s_I$ ) reduce prices in both periods.*

(iii) *Higher customer tolerance (high  $\alpha$ ) reduces OEM price in Period 2.*

**COROLLARY 4.2.**

(i) *Higher fraction of cores returning to the OEM (higher  $\gamma$ ) reduces the number of new products in the second period.*

(ii) *Higher manufacturing costs (high  $c$ ) reduce the number of units remanufactured by the independent operator (IO) in the second period.*

Notice that the OEM is a de facto monopolist in scenarios A and B. As such,  $\sigma_{AB}$  and  $\sigma_{EF}$  play a role similar to the critical value  $s^*$  that we identified in §3. Because there is a *threat* that an imitator might introduce an alternative product in Period 2, the OEM adapts. The threat, driven by the customer’s tolerance for the alternative product, leads to  $s^* < \sigma_{AB}$ . If  $\alpha = 0$ , the threat disappears, and  $s^* = \sigma_{AB} = \sigma_{EF}$ . However, if  $s_I$  or  $\alpha$  is sufficiently high, the monopolist is less likely to prevent competitive entry without hurting its own profitability. This leads to Theorem 5:

**THEOREM 5. (MONOPOLIST BEHAVIOR IN THE PRESENCE OF COMPETITIVE THREAT).**

(i) *If  $s_A \leq s^*$ , the OEM charges the same price in Period 1, under competitive threat or not. However, the price in Period 2 is lower when there is competitive threat than when there is not.*

(ii) *If  $s^* < s_A \leq \sigma_{AB}$ , or if  $\sigma_{AB} < s_A$ , and the OEM is under competitive threat, the price is lower in both periods than when there is no threat.*

**PROOF.** See the appendix.

**4.2. Multiperiod Duopoly Model**

Now, we consider a multiperiod planning horizon. The OEM initializes the market making new products in Period 1 and faces competition from an independent operator in the periods that follow. We assume that unused cores left at the end of the period do not carry over to the next period. (The rationale is that, because the remanufactured product costs less to make, the firm has no incentive to delay the revenue that a core can generate.) Hence, the OEM’s objective function is:

$$\begin{aligned} \text{Max}_{p_1, p_{k,A}, q_{k,R}} & (p_1 - c)q_1 + \sum_{k=2}^M \beta^{k-1} ((p_{k,A} - c)q_{k,N} \\ & + (p_{k,A} - c + s_A)q_{k,R}) \end{aligned}$$

$$\begin{aligned} \text{subject to} \quad & \gamma(q_{k-1,A} + q_{k-1,I}) \geq q_{k,R}, \quad k = 3, \dots, M \\ & q_{k,A} \geq q_{k,R}, \quad k = 2, \dots, M \\ & \gamma q_1 \geq q_{2,R} \\ & q_{k,N} + q_{k,R} = q_{k,A}, \quad k = 2, \dots, M \\ & q_1 = Q - p_1. \end{aligned}$$

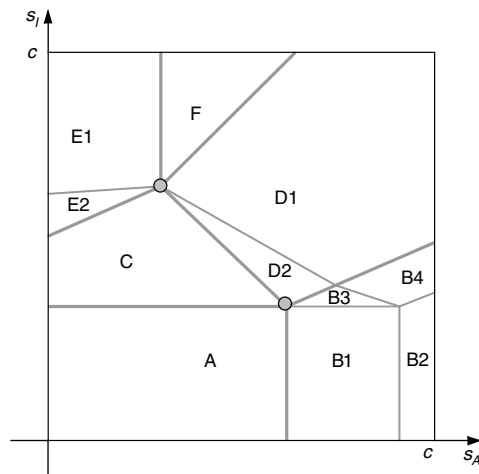
Likewise, the IO’s objective function is:

$$\begin{aligned} \text{Max}_{p_{k,I}} & \sum_{k=2}^M \beta^{k-1} (p_{k,I} - c + s_I)q_{k,I} \\ \text{subject to} \quad & (1 - \gamma)(q_{k-1,A} + q_{k-1,I}) \geq q_{k,I}, \quad k = 3, \dots, M \\ & q_{k,I} \geq 0, \quad k = 2, \dots, M \\ & (1 - \gamma)(Q - p_1) \geq q_{2,I}. \end{aligned}$$

There are several Nash equilibria defining the optimal policies that maximize these firms’ profits. Each equilibrium is the unique solution for a given set of parameters, which we characterize in terms of the remanufacturing savings pair  $(s_A, s_I)$ . If the planning horizon has only three periods ( $M = 3$ ), the analysis shows that the competitive space will look like Figure 3, where each region represents a type of equilibrium. Notice the similarity with Figure 2. Scenarios A, C, and F have the same boundaries in both figures: When parameters  $s_A$  and  $s_I$  fall in any of these areas, the firms adopt the same policies in Periods 2 and 3 as they would do in the last period of a two-period planning horizon. The same similarity exists between scenarios B2, D1, and E1 of the three-period horizon and scenarios B, D, and E of the two-period planning horizon. However, there are differences:

- Scenarios B1, B3, and D2: The OEM makes new products in the last period of the planning horizon, but not in Period 2.

**Figure 3** Three-Period Remanufacturing Duopoly



- Scenarios B3 and B4: The IO remanufactures some cores collected in the last period of the planning horizon, but none in Period 2.

- Scenario E2: The IO remanufactures all cores collected in Period 2, but only part of the cores in the last period of the planning horizon.

If  $M > 3$ , (i.e., longer planning horizon) scenarios B, D, and E have additional partitions to accommodate end-gaming strategies that are appropriate with extreme values of  $s_A$  and  $s_I$ . For example, if the planning horizon is four periods, scenario E in the two-period planning horizon is partitioned in three scenarios: (1) IO remanufactures all cores in Periods 2, 3, and 4; (2) IO remanufactures all cores in Periods 2 and 3, but not all in Period 4; and (3) IO remanufactures all cores in Period 2, but not all in Period 3 and 4. The three scenarios give continuity to scenario C in which the IO remanufactures only part of the cores collected.

Scenario C is perhaps the most interesting because it represents a quite realistic range of remanufacturing savings for both companies. In this scenario, the capacity constraints are nonbinding, leading to this result applicable to any planning horizon:

**THEOREM 6. (COMPETITIVE BEHAVIOR IN A MULTI-PERIOD PLANNING HORIZON).** *If the firms have a multiperiod planning horizon, and their savings parameter  $s_A$  and  $s_I$  satisfy the inequality  $\sigma_{AC} < s_I < \min(\sigma_{CE} + m_{CE} s_A, \sigma_{CD} + m_{CD} s_A)$ , (see Table 2) then we have:*

(i) Both firms remanufacture in every period past Period 1.

(ii) The OEM maintains a constant production volume by making a sufficient number of new units to replace those intercepted by the IO each period.

(iii) The OEM adopts a constant price policy in Periods 2 to  $M$ , remanufacturing all cores that it collects each period.

(iv) The IO adopts a constant price policy in Periods 2 to  $M$ , remanufacturing only part of the cores intercepted each period.

The prices and quantities are:

$$q_1 = \frac{Q - c + s_A \beta \gamma}{2}, \quad p_1 = \frac{Q + c - s_A \beta \gamma}{2}.$$

$$p_{k,A} = \frac{2(1 - \alpha)Q + 3c - s_I}{4 - \alpha},$$

$$q_{k,A} = q_{k,N} + q_{k,R} = \frac{(1 - \alpha)(2Q - c) - s_I}{(1 - \alpha)(4 - \alpha)}. \quad \text{and}$$

$$q_{k,R} = \gamma(q_{k-1,A} + q_{k-1,I}), \quad k = 2, \dots, M.$$

$$p_{k,I} = \frac{\alpha(1 - \alpha)Q + (2 + \alpha)c - 2s_I}{4 - \alpha},$$

$$q_{k,I} = \frac{(1 - \alpha)(\alpha Q - 2c) + (2 - \alpha)s_I}{(1 - \alpha)(4 - \alpha)\alpha}, \quad k = 2, \dots, M.$$

PROOF. See the appendix.

This theorem is quite powerful for two reasons. First, because both firms adopt a constant policy regardless of the duration of the planning horizon, it remains valid even if each firm plans its strategy with a different horizon in mind. Even if the OEM and the IO have different planning horizons, each firm would independently select constant policies as if they planned to leave the market at the same time. Second, although the theorem does not address the complete range of values of  $s_A$  and  $s_I$ , its range has significant managerial interest, because each firm is capable of significant (while still realistic) remanufacturing savings.

### 4.3. Duopoly Model with Infinite Horizon

If the planning horizon is very long, we can approximate the problem with an infinite-horizon model as we did in §3.3, because the value of future earnings is exponentially decreasing. We model Period 1 as the initialization period, followed by an infinite stream of identical periods, when both firms adopt constant policies. Also, both firms collect as many cores as possible, regardless of its origin. Hence, we simplify our notation: if  $i > 2$ ,  $p_{ij} = p_{2j} = p_j$ , and  $q_{ij} = q_{2j} = q_j$ . The OEM has this objective function:

$$\text{Max}_{p_1, p_A, q_R} (p_1 - c)q_1 + \frac{\beta}{1 - \beta} ((p_A - c)q_N + (p_A - c + s_A)q_R)$$

subject to

$$\gamma(q_A + q_I) \geq q_R$$

$$q_A \geq q_R$$

$$\gamma q_1 \geq q_R$$

$$q_N + q_R = q_A$$

$$q_1 = Q - p_1.$$

Likewise, the objective function of the IO is

$$\text{Max}_{q_I} \frac{\beta}{1 - \beta} (p_I - c + s_I)q_I$$

subject to

$$(1 - \gamma)(q_A + q_I) \geq q_I$$

$$(1 - \gamma)q_1 \geq q_I$$

$$q_I \geq 0.$$

The KKT conditions generate the system of equations that leads to the following theorem:

**THEOREM 7.** *The OEM remanufactures all cores collected in Period 2. Moreover, seven linear relations between  $s_A$  and  $s_I$  characterize the optimal policies for the OEM and the IO in an infinite planning horizon (see Table 3 and*

**Table 3** Critical Values in the Duopoly with Infinite Planning Horizon

Intercept or slope	Expression
$\sigma_{RS}$	$\frac{(Q-c)\alpha(1-\beta)}{(2-\alpha)\beta\gamma}$
$\sigma_{TR}$	$-\frac{Q\alpha(2+\alpha)-c(4-2\alpha+\alpha^2)}{4}$
$\sigma_{RU}$	$\frac{(2c-Q\alpha)(1-\alpha)}{2-\alpha}$
$\sigma_{UV}$	$\frac{(1-\alpha)(\alpha Q(2-4\gamma-(1-\gamma)\alpha)-c(4-\alpha)(1-\gamma)\alpha+4c)}{2(2-\alpha)}$
$\sigma_{TW}$	$\frac{(1-\alpha)(\alpha Q(2-3\gamma)+c(2\gamma-(1-\gamma)\alpha))}{2\gamma(1-\alpha)+\alpha}$
$\sigma_{VW}$	$\frac{(Q-c)(1-\beta)(2-\alpha-2\gamma(1-\alpha))}{\beta\gamma(\alpha+2\gamma(1-\alpha))}$
$m_1$	$\frac{(4-\alpha)\alpha\beta\gamma}{4(1-\beta)}$
$m_2$	$\frac{(1-\alpha)(4-\alpha)(1-\gamma)\alpha\beta\gamma}{2(2-\alpha)(1-\beta)}$

Figure 4), as follows:

R. If  $s_I < \sigma_{RU}$  and  $s_A < \sigma_{RS}$ , the OEM uses only some of the cores collected in future periods, and the IO does not participate.

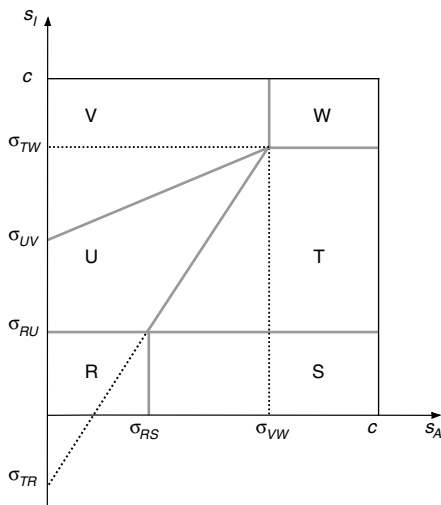
S. If  $s_I < \sigma_{RU}$  and  $\sigma_{RS} < s_A$ , the OEM uses all cores collected every period, and the IO does not participate.

T. If  $\sigma_{RU} < s_I < \sigma_{TW}$  and  $s_I < \sigma_{TR} + m_1 s_A$ , the OEM uses all cores collected every period, and the IO remanufactures only part of the cores collected every period.

U. If  $\sigma_{RU} < s_I$  and  $\sigma_{TR} + m_1 s_A < s_I < \sigma_{UV} + m_2 s_A$ , the OEM uses only part of the cores collected in future periods, and the IO remanufactures only part of the cores collected every period.

V. If  $\sigma_{UV} + m_2 s_A < s_I$  and  $s_A < \sigma_{VW}$ , both firms use all cores collected in the second period, but only part of the cores collected in future periods.

**Figure 4** Remanufacturing Duopoly with Infinite Planning Horizon



W. If  $\sigma_{TW} < s_I$  and  $\sigma_{VW} < s_A$ , both firms remanufacture all cores in every period.

PROOF. See the appendix.

The theorem characterizes each player’s strategy based on one’s own remanufacturing savings, the other firm’s savings, and the original manufacturing cost. The OEM always remanufactures all the cores that it collects from first-period sales. As Figure 4 suggests, the IO fully participates if its savings level is high compared to that of the OEM. Likewise, the OEM’s participation increases if the IO cannot generate significant remanufacturing savings. If the  $s_A \gg s_I$ , the OEM might be alone in this market.

The IO usually remanufactures only part of the cores that it collects (scenarios U and T) unless it generates extremely high remanufacturing savings, or customers have high tolerance for private labels, as the following cases illustrate:

Case 3.  $Q = 1,000$ ,  $c = \$200$ ,  $\alpha = 0.8$ ,  $\beta = 0.925$ , and  $\gamma = 1/3$ . The manufacturing cost is low, customer’s tolerance for generic brand is very high, and the OEM collects one-third of all cores (the IO collects the remainder). Consequently, the IO always participates in this market ( $\sigma_{RU} < 0$  and scenarios R and S vanish). The OEM remanufactures all cores if the IO savings are lower than  $2.6s_A - \$408$  (scenario T). Finally, if the IO generates savings greater than  $0.58s_A + \$47$ , it remanufactures every core that it collects (scenario V). Because  $\sigma_{VW} > c$ , scenario W also vanishes.

Case 4.  $Q = 1,000$ ,  $c = \$400$ ,  $\alpha = 0.30$ ,  $\beta = 0.925$ , and  $\gamma = 1/3$ . The manufacturing cost is high, customer’s tolerance is very low, and the OEM collects one-third of all cores. The IO does not participate in this market unless it generates savings greater than half of the original manufacturing costs ( $\sigma_{RU} = 206$ ). The OEM remanufactures all cores that it collects whenever it generates savings greater than  $\$26$  and the IO generates savings lower than  $1.14s_A - \$177$  (scenarios S and T). If the IO generates savings greater than  $0.63s_A + \$297$ , it remanufactures every core that it collects (scenario V). Because  $\sigma_{TW} > c$ , scenario W vanishes again.

Notice that the two cases differ just on the customers’ tolerance for generic brand and on the marginal cost of the new product. Yet, while in the first case the IO can profitably remanufacture most or all cores that it intercepts, in the second case, the IO has to generate very high savings in its remanufacturing process to have a chance to participate in this market, despite intercepting 2/3 of all cores each period.

### 5. Conclusions and Future Research

Many companies have organized their product line based on remanufacturing capabilities, for example,

manufacturers of printer cartridges, single-use cameras, tires, hospital beds, medical equipment, military equipment, and many other products. Often, they operate in a monopoly or quasi-monopoly environment. However, competition may occur if an independent operator intercepts some of the cores, which is common in industries that have an abundant return flow of recoverable cores. This paper analyzes the new and the remanufactured product in the monopoly and duopoly scenarios, and identifies insights that help managers of remanufacturing operations to find effective policies for their product lines.

For the original-equipment manufacturer, an overarching observation is that, as the marginal cost of remanufacturing decreases, the value of making new products in the first period increases, and the value of making new products in future periods decreases. In other words, if remanufacturing is very profitable, the firm forgoes some of the first-period margin by making additional units in the first period to increase the number of cores available for remanufacturing later. *This behavior does not change, whether the OEM is a monopolist or not, operating with any planning horizon.* We considered the assumption that an independent operator (IO) has access to a fraction of the cores at the end of each period. Considering the discount that the market requires from generic brands, if the remanufacturing savings is not sufficiently high, the IO does not have any share of the market. The reason is the following: Once the price of OEM is set, the IO can only hope to attract customers that consider the OEM's price too high. However, to reach those customers, the IO would have to set price below marginal cost, which is not reasonable. As the savings increase, the IO is able to reach some customers in the market. Gradually, the savings may be so high that the IO remanufactures all cores that it collects.

In addition to the two-period model, we also analyze multiperiod and infinite planning horizons. Both models show that the optimal strategies obtained from the two-period model are quite relevant in longer planning horizons: The optimal policy in the last period is similar, both in two-period and in multiperiod planning horizons.

This paper takes an important step towards understanding the transition in the life of the product from an all-new production to a mixed product line in which remanufactured and new products coexist. As this area of research expands, it is important to understand the complete life cycle of the remanufactured product line. We have yet to learn the impact of stochastic remanufacturing yield on multiperiod competition. That is the topic of our future research.

An online supplement to this paper is available on the *Management Science* website (<http://mansci.pubs.informs.org/ecompanion.html>).

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