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A CONTINUOUS REVIEW MODEL FOR THE REPARABLE ITEM INVENTORY SYSTEM

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A CONTINUOUS REVIEW MODEL FOR THE REPARABLE ITEM INVENTORY SYSTEM

by

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ABSTRACT

In general, a reparable inventory system deals with two classes of the same type of item. The first class consists of those items which have been procured from the manufacturer and have never been put to use. The second class consists of those items which have been used, have failed, have been repaired, and are ready to be used again. In addition to the two classes of items, the system also contains two distinct inventories. These are the ready-for-issue and the non-ready-for-issue inventories. The ready-for-issue inventory contains both classes of items in their usable state, and the non-ready-for-issue inventory contains only the latter in its failed but reparable state. This paper develops a quasi-probabilistic inventory model for the reparable inventory system based on the premise that the repair facility is the primary source of inventory, with procurements being made periodically only to supplement this primary source.

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1. INTRODUCTION

For the most part, the studies devoted to inventory systems have been directed toward the consumable systems. Recently, due to Defense Department concern over the economics of the systems, a considerable amount of interest has been focussed on studies of reparable systems. The studies to date on the reparable systems have been primarily along the lines of a deterministic system [1, 2], the interaction of the entities of the system [3], and glimpses of a probabilistic system [4].

The reparable inventory system differs from that of the classical textbook type in two major ways. First, the system contains two distinct inventories, one of which has as its members items that are in a ready-for-issue (RFI) state and the other containing items which are in a non-ready-for-issue (NRFI) state; i.e., the first contains items which are usable and the second contains items that must be repaired before they can be put to use. Second, the RFI inventory is made up of a mixture of new items and items that have been used, failed, repaired, and are ready to be used again.

The standard approach to the problem of how to run an inventory system has been to consider one in which there is a sole source of supply. In the reparable inventory system, this situation in general does not exist. Here the usable inventory may be thought of as being supplied by two separate processes which differ considerably from each other, the first being the manufacturer and the second being a repair facility. These processes may differ in their leadtimes and in the cost of producing a usable item. In general, the repair leadtime is shorter than the procurement leadtime, and the cost of repair is less than the cost of manufacturing. The arguments here tend to favor the repair facility as a primary source of inventory, but the savings in operating this facility must be large enough to warrant its existence. The criteria that an item must meet to mark it as reparable are threefold. First, the item must be physically capable of repair; second, the cost of accomplishing this repair must be considerably less than the purchase price of a new item; and third, the initial cost of the item must be large enough to warrant the cost associated with getting the carcasses back to the NRFI. Assuming that a repaired item is functionally identical to one which has never been used, it would seem that the repair facility is a more desirable supplier than the manufacturer and that the decision-maker should select only the facility to provide his inventory. This, in fact, would be the case if the system were one in which there were no losses and all carcasses were returned and met the criteria of being reparable. For example, if cost of repair is the criterion, then the cost of repairing some of the items might be considerably higher than the manufacturing cost. In this example, the rational thing for the decision-maker to do would be to discard those items that do not meet the criterion and replace them by procuring from the first supplier. Now assuming that the above is the case, then an examination of the system would indicate that a certain fraction (r) of the items demanded from the RFI inventory will eventually return to the RFI inventory.

Reference [1] deals with a deterministic model based on a "substitution" policy. This policy requires the repair facility to supply all the items demanded until the NRFI inventory drops to a level below that necessary for another repair batch induction. The next quantity that arrives at the RFI inventory is that which had previously been ordered through procurement. The demands are then satisfied with this procured quantity, while the NRFI inventory builds up and the cycle starts again. This paper will be devoted to this substitution policy, treating demand as a stochastic variable.

2. NOTATION AND ASSUMPTIONS

The "substitution" policy deals with a system that is self-sustaining to a certain point, at which time a procurement must arrive in order to compensate for the losses ((1 - r)%) to the system. Since the repair facility is the primary source of usable inventory, the model is developed to determine the most feasible batch size and the number of batches per cycle that should be supplied by this process. The model is developed with the measure of effectiveness being the cost of operating the system per year, using the following notation:

- Q_P - fixed procurement quantity, a decision variable;
- Q_R - fixed repair batch size, a decision variable;
- X - demand rate, units per unit time, a random variable with mean $\bar{X}$ and density $f(x, t)$ where t is a particular time period under consideration;
- r - recovery rate, a given constant;
- τ_P - procurement leadtime, a given constant;
- τ_R - repair leadtime, a given constant;
- A_P - fixed cost of placing a procurement order, per order, given;
- A_R - fixed cost associated with an induction at the repair facility, per induction, given;
- h_1 - RFI holding cost, dollars per unit per year, given;
- h_2 - NRFI holding cost, dollars per unit per year, given;
- n - number of inductions per cycle, a decision variable, $n = 0, 1, 2, \ldots$;
- Π - cost of incurring backorders, dollars per unit, given;
- T_R - expected time it takes for Q_R items to be demanded;

δ_P - procurement reorder level, based on inventory position, a decision variable;

δ_R - repair induction reorder level, based on inventory position, a decision variable;

T - system cycle time, a random variable, time between successive procurement quantity arrivals to RFI inventory;

TC_T - total cost of operating the system per cycle;

TC - total cost of operating the system per year.

The "real" world situation is a most difficult thing to express in terms of mathematical symbols. Here, as in most cases, assumptions have to be made in order to develop a model that, although only an approximation of the "real" world, gives an indication of how the "real" world behaves. The system considered in this paper is one which operates on a cyclic basis, where a cycle is the random amount of time (T) between the arrivals into the RFI inventory of procurement orders. The cycle will consist, therefore, of the arrival of one procurement and n repair inductions. As long as the distribution of the stochastic demand remains the same from one cycle to the next, an analysis of one cycle will describe how every other cycle behaves. The model presented considers a system whose inventory is made up of one type of item. The items that are repaired are considered equal in all respects to those procured. The model will be a continuous review-type based on an inventory position, (IP), where

$$IP = \text{inventory on hand} + \text{on order} - \text{backorders} ,$$

and the on-hand inventory is a non-negative quantity. The procurement and repair orders are placed when the IP drops to certain levels (the reorder levels). A fixed procurement order (Q_P) is placed the first time the IP falls to the procurement reorder level δ_P after an elapsed time of

$$\left(\frac{n - n_1 r}{r}\right) \frac{Q_R}{\bar{X}}$$

since the arrival of a procurement order, where

$$n_1 = \frac{T_R}{\tau_R} - \left[\frac{T_R}{\tau_R} \right] .$$

The $[Z]$ is the greatest integer in Z. The fixed repair orders (Q_R) are made the first time the IP falls to δ_R after the arrival of a repair induction. In order to simplify the problem somewhat, it is assumed that as a demand is made on RFI inventory, a carcass is returned to NRFI inventory of which r% are reparable. With this assumption, it is possible to determine the exact amount of NRFI inventory, given that we know the amount of RFI inventory. Since the system to be formulated is one in which there is one procurement per cycle and n repairs per cycle, the random cycle time (T) can be expressed in terms of the stochastic demand (X) and the quantity demanded per cycle,

$$T = \frac{Q_P + n Q_R}{X} \quad . \tag{1}$$

In the NRFI inventory, there will be exactly rXT items that enter the inventory each cycle and there will be $n Q_R$ items leaving. Hence, in steady state

$$r \bar{X} \bar{T} \doteq n Q_R$$

or using equation (1) and solving for Q_P yields

$$Q_P \doteq \frac{n(1 - r)\, Q_R}{r} \quad . \tag{2}$$

It is postulated that, on the average, at the time of the arrival of procurement orders and repair batches the net inventory will be at fixed positive buffer levels b_1 and b_2, respectively. The actual net inventory at times of order arrivals will, of course, fluctuate and, hence, the buffer levels are indicative of protection against stockouts.

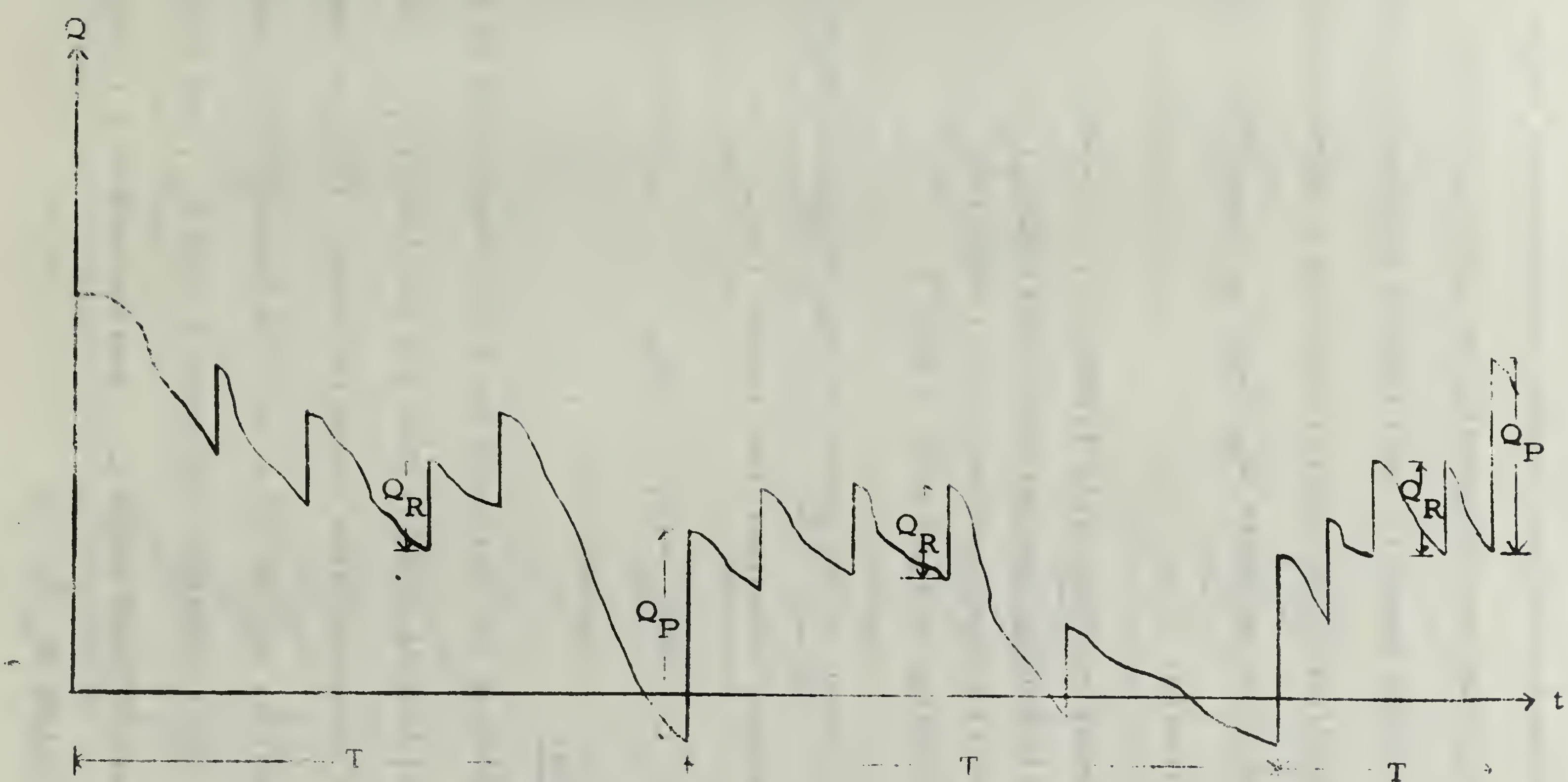
Every time an order is placed for either a procurement or a repair induction there is associated a cost assumed to be independent of the number of items either repaired or procured. These costs are those necessary to support the personnel and equipment involved in placing orders. The cost (C) of the items themselves is considered to be independent of the quantity procured. The RFI holding cost (h_1) is assumed to be equal to the cost of an item (C) times an inventory carrying charge (I), i.e.,

$$h_1 = IC \text{ in dollars per unit year .}$$

Since the NRFI inventory is made up of just carcasses, the holding cost (h_2) is considerably less than h_1. It is assumed that h_2 is equal to the cost of an item (C), minus the cost of repair (C_R), all times the inventory carrying charge (I), i.e.,

$$h_2 = I(C - C_R), \text{ in dollars per unit year .}$$

The shortage cost, or cost of incurring backorders, is assumed to be a function of the number of items backordered. The cost per backorder (Π) is an intangible type of cost and in most cases should be assigned by the decision-maker. The value of Π has a direct effect on the desirability of incurring backorders, i.e., large values of Π make backorders extremely undesirable. The value assigned to Π is, in most cases, much greater than the value associated with holding either RFI or NRFI inventory. This being the case, the encountering of backorders will be the exception rather than the rule. Having made this assumption, an expected value formulation, using the expected values as parameters, can be used to determine the cost of holding both RFI and NRFI inventory; but an exact procedure will have to be employed to determine the expected number of backorders. Figure 1 gives an indication as to how the system really behaves; and Figures 2, 3, and 4 depict the expected value formulation.



Stochastic Demand Representation of the RFI Inventory
Depicting Only the Net Inventory

3. THE MODEL

3.1 Introduction

With the preceding assumptions in hand, an expression will be developed for the expected cost of operating the system per cycle. This cost, divided by the cycle time (T), will yield an expression for the expected cost of operating the system per unit time in terms of known constants and the decision variables Q_P, Q_R, δ_R, δ_P, and n. The cost per cycle is given by:

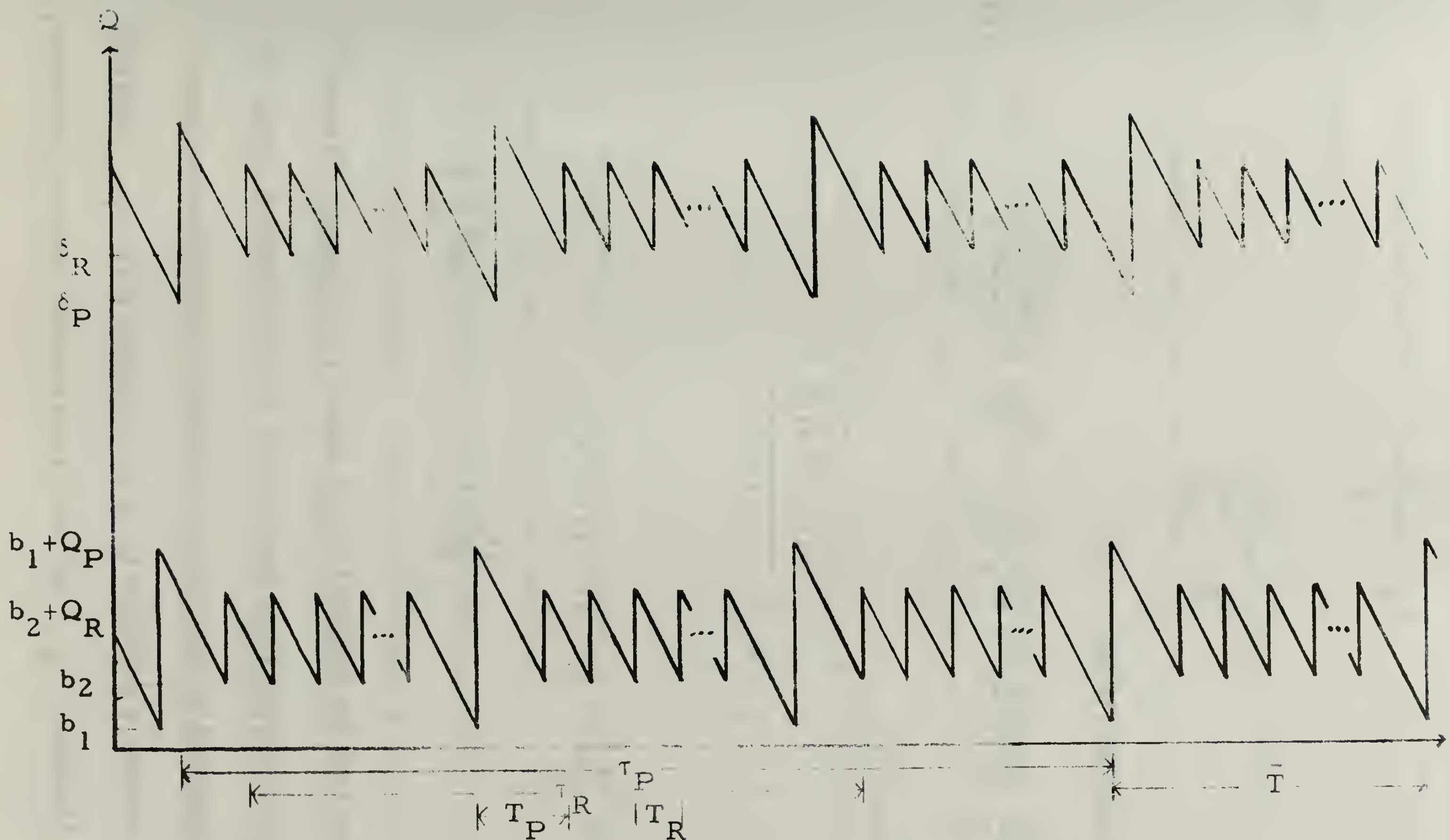
$$
\begin{aligned}
TC_T = \; & \text{order cost per cycle } (ORD_T) \; + \\
& \text{RFI holding cost per cycle } (RFI\ HOL_T) \; + \\
& \text{NRFI holding cost per cycle } (NRFI\ HOL_T) \; + \\
& \text{shortage cost per cycle } (SHG_T) \; .
\end{aligned}
$$

The order cost will simply be the cost of placing one procurement order plus the cost of having n inductions from repair, i.e.,

$$ORD_T = A_P + n A_R \; . \tag{3}$$

3.2 RFI Holding Cost

The expected holding cost per cycle for RFI inventory will be given by the product of the holding cost per unit, per unit time (h_1), and the area under the RFI inventory curve during the cycle. The area under the curve, see Figure 2, will be the area of the rectangle with sides b_1 and T_P, the area of the rectangle with sides b_2 and nT_R, the area of the triangle with base T_P and height Q_P, and the area of n triangles with base T_R and height Q_R.



Expected Value Representation of the RFI Inventory
Depicting Both Net Inventory and Inventory Position

FIGURE 2

$$\text{RFI HOL}_T = h_1 \left\{ b_1 T_P + nb_2 T_R + \frac{Q_P T_P}{2} + \frac{n Q_R T_R}{2} \right\} . \tag{4}$$

The time T_P is the expected time it takes for Q_P items to be demanded; hence,

$$T_P = \frac{Q_P}{\bar{X}}$$

and (5)

$$T_R = \frac{Q_R}{\bar{X}} \quad .$$

Substituting (5) into (4) yields

$$\text{RFI HOL}_T = \frac{h_1}{\bar{X}} \left\{ b_1 Q_P + nb_2 Q_R + \frac{Q_P^2}{2} + \frac{nQ_R^2}{2} \right\} . \tag{6}$$

Now using equation (2),

$$Q_P \doteq \frac{n(1 - r) Q_R}{r} ,$$

(4) becomes

$$\text{RFI HOL}_T = \frac{h_1 Q_R n}{r \bar{X}} \left\{ b_1 (1 - r) + r b_2 + \frac{Q_R}{2r} \left[r^2 + n(1 - r)^2 \right] \right\} . \tag{7}$$

The buffer b_1 can now be determined by again referring to Figure 2 and noting the inventory position just before a procurement order is placed and what happens to the system during the procurement leadtime τ_P . The inventory position is at the trigger level δ_P . During the procurement leadtime τ_P , all the procurement orders previously placed will have arrived and all the outstanding orders for repair inductions will also have arrived. Some nK (K is to be determined) repair inductions whose orders have not yet been placed will also arrive. The quantity that leaves the system during this leadtime will be the mean procurement leadtime demand $\bar{X}\tau_P = Z_P$. Hence, the expected buffer will be

$$b_1 = \delta_P - Z_P + nK\,Q_R \quad . \tag{8}$$

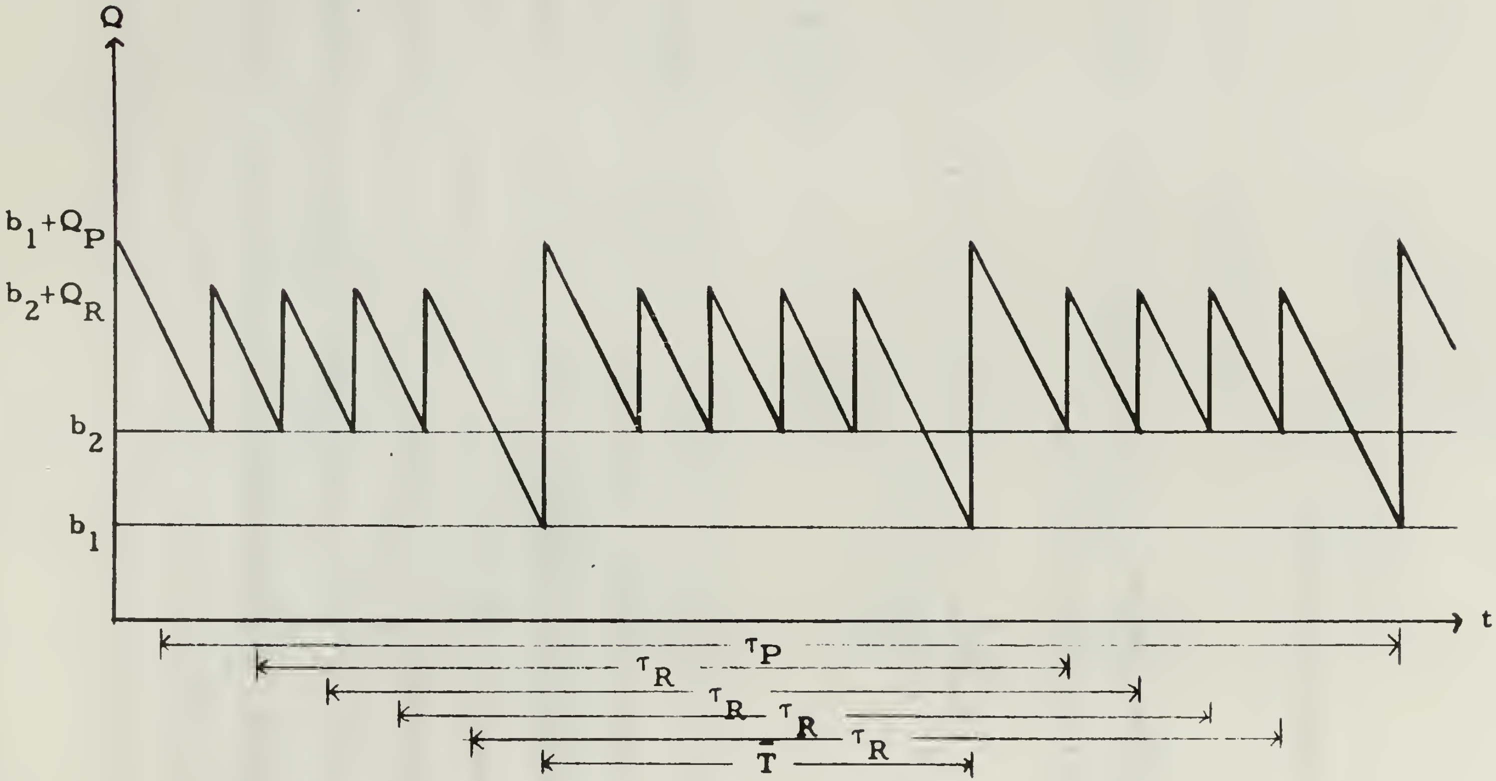
By similar analysis, the buffer b_2 is

$$b_2 = \delta_R - Z_R - K\,Q_P \quad , \tag{9}$$

where $K\,Q_P$ items previously ordered will not arrive during τ_R. Substituting (8) and (9) into (7) yields

$$\text{RFI HOL}_T = \frac{h_1 Q_R n}{r\bar{X}} \left\{ (\delta_P - Z_P)(1-r) + (\delta_R - Z_R) r + \frac{Q_R}{2r}\left[r^2 + n(1-r)^2 \right] \right\} \quad . \tag{10}$$

In general, K will be a function of the number of cycles of random length T that occur during the leadtimes. Figure 3, for example, indicates that there are slightly less than three cycles occurring during the repair leadtime. Here it is obvious K is equal to one. If τ_P remains the same and τ_R happens to be one cycle shorter, then in this example $K = 2 - 0 = 2$, which implies that eight repair inductions are ordered and arrive during the procurement leadtime. For the general case,



Expected Value Representation of RFI Inventory Depicting Relationships Between $\bar{T}$, τ_R, and τ_P

FIGURE 3

$$K = \left[\frac{\tau_P}{\bar{T}}\right] - \left[\frac{\tau_R}{\bar{T}}\right] = \left[\frac{Z_P\, r}{n Q_R}\right] - \left[\frac{Z_R\, r}{n Q_R}\right] , \qquad (11)$$

where $[Z]$ indicates the greatest integer in Z .

3.3 NRFI Holding Cost

The NRFI holding cost will be obtained in a manner similar to that of the RFI holding cost. In this case, the area in question is depicted in Figure 4. This area will be the areas of the rectangle with sides b_3 and T, the triangle with base T and height nQ_R, and $\frac{n(n-1)}{2}$ parallelograms of horizontal length T_R and height Q_R. The NRFI holding cost will then be

$$\text{NRFI HOL}_T = h_2 \left\{ b_2 \bar{T} + \frac{n}{2} Q_R \bar{T} - \frac{n(n-1)}{2\bar{X}} Q_R^{\,2} \right\} . \qquad (12)$$

During the repair leadtime, the NRFI inventory must supply the RFI inventory with $nQ_R \left(\frac{\tau_R}{\bar{T}}\right)$ items. In steady state, the level of the buffer b_3 is the lowest possible level of the NRFI inventory, i.e.,

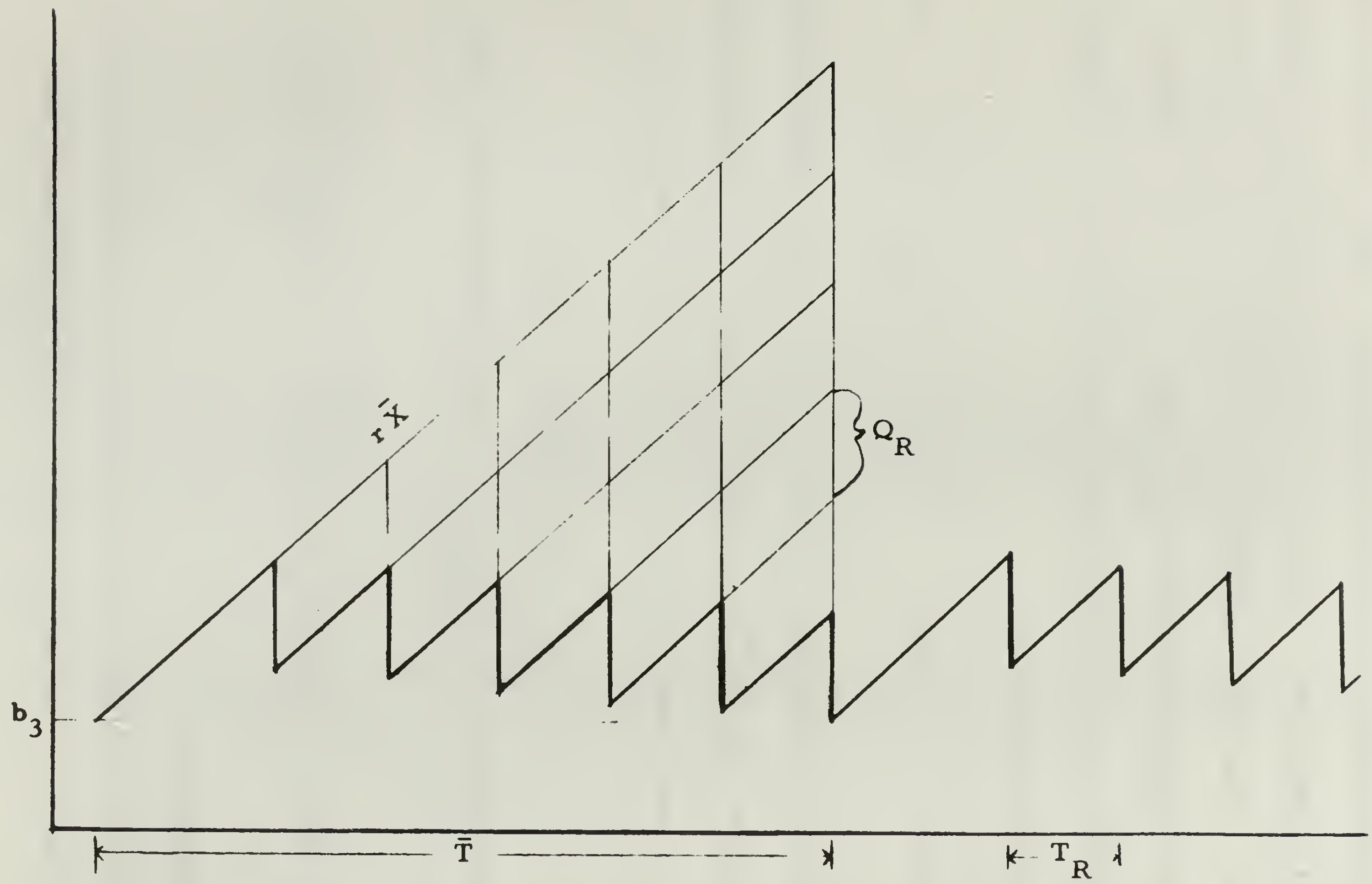
$$b_3 = nQ_R \left(\frac{\tau_R}{\bar{T}}\right) = r Z_R . \qquad (13)$$

Substituting equation (13) into (12) the NRFI holding cost per cycle becomes

$$\text{NRFI HOL}_T = h_2 \left\{ r Z_R \bar{T} + \frac{nQ_R \bar{T}}{2} - \frac{n(n-1)\, Q_R^{\,2}}{2\bar{X}} \right\} . \qquad (14)$$

3.4 Shortage Cost

Departing from an expected value analysis, an expression for the shortage cost per cycle is obtained by examining Figures 1 and 2 .



Expected Value Representation of the NRFI Inventory

FIGURE 4

Here backorders can be incurred at $(n + 1)$ different times during the cycle. There is a possibility of stock-out just before the procurement quantity Q_P arrives and just before the arrival of each of the n repair batches. The expected number of shortages, E, per cycle can be obtained by examining what takes place during the leadtimes. Just before a repair order is placed, the IP is at a level δ_R. As discussed previously, at the end of the repair leadtime all the items previously ordered will have arrived with the exception of KQ_P. The number of items short will be zero as long as the leadtime demand is less than $\delta_R' - KQ_P$; hence, the expected number of items short of the end of n repair leadtimes is

$$n \int_{\delta_R - KQ_P}^{\infty} (x - \delta_R + KQ_P)\, f(x; \tau_R)\, dx \quad .$$

Using a similar analysis on the procurement leadtime, the expected number of items short of the end of this period is

$$\int_{\delta_P + nKQ_P}^{\infty} (x - \delta_P - nKQ_P)\, f(x; \tau_P)\, dx \quad ,$$

where there are nKQ_R items that are ordered and come in during the leadtime. The expected number of items short per cycle will be

$$E = \int_{\delta_P + nKQ_R}^{\infty} (x - \delta_P - nKQ_R)\, f(x; \tau_P)\, dx$$

$$+ n \int_{\delta_R - KQ_P}^{\infty} (x - \delta_R + KQ_P)\, f(x; \tau_R)\, dx \quad , \qquad (15)$$

where x is a dummy variable indicating leadtime demand and K is given by equation (11). The expected cost of backorders per cycle is simply

$$SHC_T = \Pi E \quad . \tag{16}$$

3.5 Total Cost

The total cost per cycle is given by the expression

$$TC_T = ORD_T + RFI\ HOL_T + NRFI\ HOL_T + SHC_T \quad .$$

Since

$$T = \frac{(Q_P + nQ_R)}{X} \doteq \frac{nQ_R}{r\bar{X}} \quad ,$$

the total cost per year will be

$$TC = \frac{r\bar{X}}{nQ_R} \left\{ ORD_T + RFI\ HOL_T + NRFI\ HOL_T + SHC_T \right\} . \tag{17}$$

Substituting equations (3), (10), (14), (15), and (16) into (17), the total cost per year becomes

$$TC = \frac{(A_P + nA_R)}{nQ_R} \bar{X} r + h_1 \Big\{ (\delta_P - Z_P)(1 - r) + (\delta_R - Z_R) r$$

$$+ \frac{Q_R}{2r} \left[r^2 + n(1 - r)^2 \right] \Big\}$$

$$+ h_2 \left\{ r Z_R + \frac{Q_R}{2} \left[n - r(n - 1) \right] \right\}$$

$$+ \frac{\Pi \bar{X} r}{n Q_R} \Big\{ \int_{\delta_P + nKQ_R}^{\infty} (x - \delta_P - nKQ_R) \; f(x; \tau_P) \, dx$$

$$+ n \int_{\delta_R - \frac{n(1 - r)}{r} KQ_R}^{\infty} \left(x - \delta_R + \frac{n(1 - r)}{r} KQ_R\right)$$

$$\cdot f(x; \tau_R) \, dx \Big\} \; . \tag{18}$$

Before attempting to minimize (18) with respect to the decision variables, a value for K must be determined. From equation (11), K is given as

$$K = \left[\frac{Z_P r}{n Q_R} \right] - \left[\frac{Z_R r}{n Q_R} \right]$$

which can be rewritten as

$$K = \left(\frac{Z_P r}{n Q_R} - D_1 \right) - \left(\frac{Z_R r}{n Q_R} - D_2 \right)$$

or

$$K = \frac{(Z_P - Z_R) r}{n Q_R} - (D_1 - D_2) \tag{19}$$

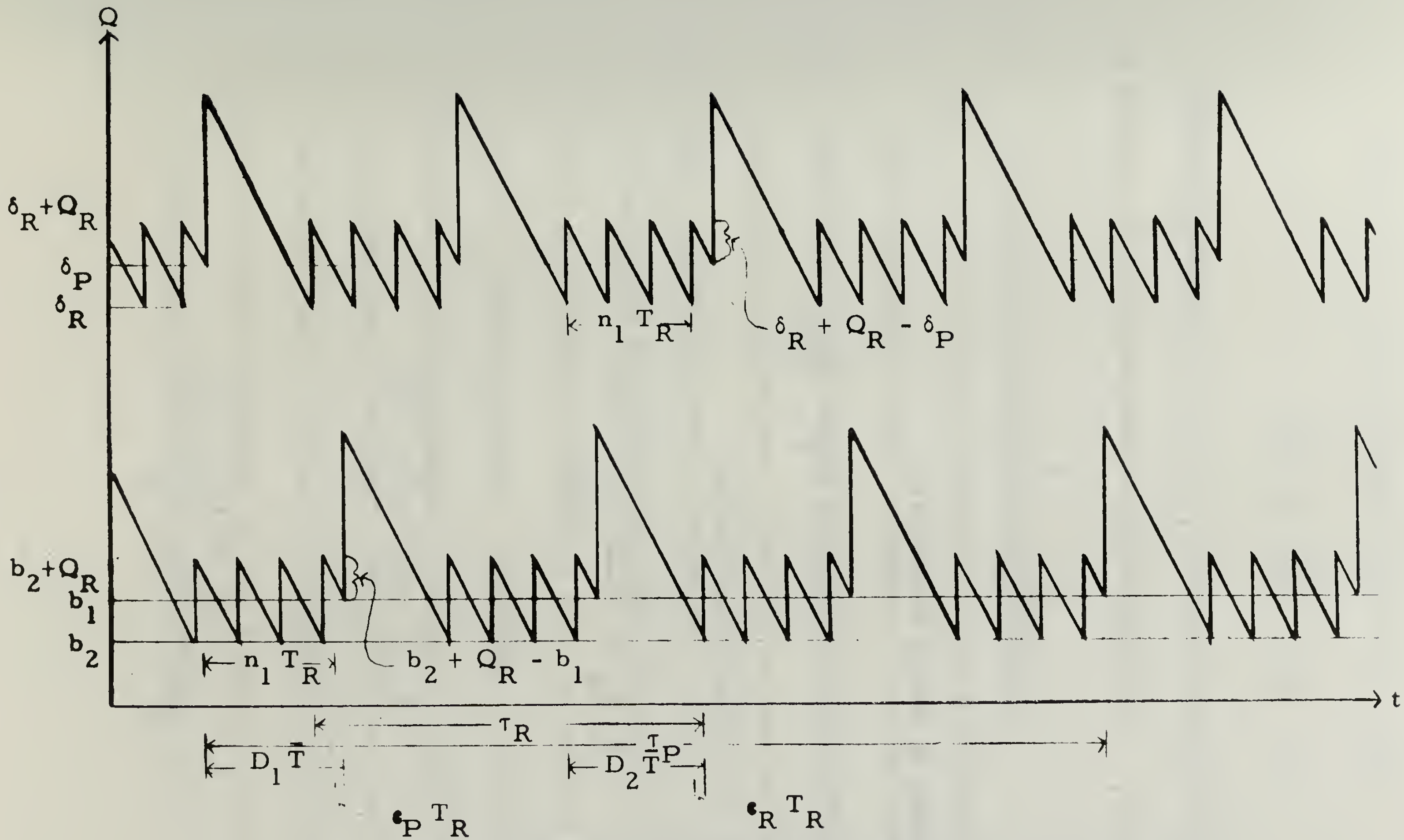
where

$$0 \le D_i < 1 , \quad \text{for} \quad i = 1 , 2 .$$

Substituting (19) into (18), the total cost per year becomes

$$
\begin{aligned}
TC = {} & \frac{(A_P + nA_R)}{nQ_R} r\bar{X} + h_1 \Big\{ (\delta_P - Z_P)(1 - r) + (\delta_R - Z_R) r \\
& + \frac{Q_R}{2r} \left[r^2 + n(1 - r)^2 \right] \Big\} \\
& + h_2 \Big\{ r Z_R + \frac{Q_R}{2} \left[n - r(n - 1) \right] \\
& + \frac{\Pi r \bar{X}}{nQ_R} \Bigg\{ \int_{\delta_R + r(Z_P - Z_R) - nQ_R(D_1 - D_2)}^{\infty} (x - \delta_P - r(Z_P - Z_R) + nQ_R(D_1 - D_2)) \\
& \qquad \cdot f(x; \tau_P)\, dx \\
& + n \int_{\delta_R - (1 - r)(Z_P - Z_R) + \frac{n(1 - r)}{r} Q_R(D_1 - D_2)}^{\infty} \Big(x - \delta_R + (1 - r)(Z_P - Z_R) - \frac{n(1 - r)}{r} \\
& \qquad Q_R(D_1 - D_2)\Big) f(x; \tau_R)\, dx \Bigg\} .
\end{aligned}
\tag{20}
$$

Referring to Figure 5, a more exact analysis can be made on the values of D_1 and D_2. It is noted that D_1 is the time between placing of a procurement order and the arrival of the next procurement, which had previously been ordered. Noting the net inventory, it is apparent



Expected Value Representation of RFI Inventory
Depicting Relationships Between D_1 and D_2

FIGURE 5

that during this same amount of time some n_1, ($n_1 \leq n$), inductions arrive from the NRFI inventory. Hence, D_1 can be written as

$$D_1 \bar{T} = n_1 T_R + \epsilon_P T_R$$

or

$$D_1 = \frac{n_1 r}{n} + \frac{\epsilon_P r}{n} ,$$

where ϵ_P is that fraction between $n_1 T_R$ and $(n_1 + 1) T_R$ which is also included in D_1. A similar analysis indicates that D_2 is given by

$$D_2 = \frac{n_1 r}{n} + \frac{\epsilon_R r}{n} .$$

Hence, the fraction D can be written as

$$D = D_1 - D_2 = \frac{r}{n} (\epsilon_P - \epsilon_R) .$$

From Figure 5, it is readily apparent that

$$\epsilon_P T_R < \frac{b_2 + Q_R - b_1}{\bar{X}}$$

or

$$\epsilon_P < \frac{b_2 - b_1 + Q_R}{Q_R} .$$

Also,

$$\epsilon_R < \frac{\delta_R + Q_R - \delta_P}{Q_R} .$$

Since ϵ_P and ϵ_R are both positive, then

$$\epsilon_P - \epsilon_R < \left| \frac{b_2 - b_1 - \delta_R + \delta_P}{Q_R} \right|$$

where $|Z|$ is the absolute value of Z. But,

$$b_1 - b_2 \approx \delta_P - \delta_R \ .$$

Consequently,

$$\epsilon_P - \epsilon_R \approx 0 \ .$$

3.6 <u>Determination of Decision Variables</u>

Now, in order to operate the system economically, values of the decision variables must be obtained which minimize the total cost per unit time. These values will be obtained by setting the first order conditions equal to zero. This will be done for the decision variables Q_R, δ_P, and δ_R only; the integer n will be determined by a stepping process, and Q_P follows Q_R through equation (2). This stepping process will start by setting $n = 1$, for which a value of Q_R, δ_R, δ_P, and TC will be computed. The value of n will be increased by one, and the procedure repeated. The process will continue until the minimum cost is obtained. The first order conditions are obtained by taking the partial derivatives of the total cost equation (20) with respect to the decision variables Q_R, δ_P, and δ_R. First the partial derivative of total cost per year with respect to Q_R is

$$\frac{\partial TC}{\partial Q_R} = - \frac{(A_P + n A_R)}{n Q_R^2} r \bar{X} + \frac{h_1}{2r} \left\{ r^2 + n(1-r)^2 \right\}$$

$$+ \frac{h_2}{2} \left\{ n - r(n-1) \right\}$$

$$- \frac{\Pi r \bar{X}}{n Q_R^2} \left\{ \int_{\delta_P + r(Z_P - Z_R) - nQ_R(D_1 - D_2)}^{\infty} (x - \delta_P - r(Z_P - Z_R)) \, f(x; \tau_P) \, dx \right.$$

$$\left. + \; n \int_{\delta_R - (1-r)(Z_P - Z_R) + \frac{n(1-r)}{r} Q_R (D_1 - D_2)}^{\infty} (x - \delta_R + (1-r)(Z_P - Z_R)) \, f(x; \tau_R) \, dx \right\} . \tag{21}$$

It becomes quickly apparent that equation (21) cannot be solved explicitly for either Q_R, δ_P, or δ_R ; but an expression can be obtained for Q_R in terms of Q_R, δ_P, and δ_R. Setting equation (21) equal to zero, it may be written as

$$Q_R = \left[\left\{ \frac{\frac{2r^2 \bar{X}}{n}}{h_1 (r^2 + n(1-r)^2) + h_2 (rn - r^2 (n-1))} \right\} \left\{ A_P + n A_R \right. \right.$$

$$+ \Pi \left(\int_{\delta_P + r(Z_P - Z_R) - nQ_R(D_1 - D_2)}^{\infty} (x - \delta_P - r(Z_P - Z_R)) \, f(x; \tau_P) \, dx \right.$$

$$+ n \int_{\delta_R - (1-r)(Z_P - Z_R) + \frac{n(1-r)}{r} Q_R (D_1 - D_2)}^{\infty} (x - \delta_R + (1-r)(Z_P - Z_R))$$

$$\left. \left. \left. \cdot f(x; \tau_R) \, dx \right) \right\} \right]^{\frac{1}{2}} . \quad (22)$$

Next, the partial derivative of total cost per unit time with respect to δ_R is

$$\frac{\partial TC}{\partial \delta_R} = h_1 r - \frac{\Pi r \bar{X}}{Q_R} \left[1 - F(\delta_R - (1-r)(Z_P - Z_R) \right.$$

$$\left. + \frac{n(1-r)}{r} Q_R (D_1 - D_2); \tau_R) \right] \quad (23)$$

where $F(x;t)$ is the cumulative distribution function of demand during time t. Setting (22) equal to zero, it can be written as

$$\frac{h_1 Q_R}{\Pi \bar{X}} = \Big[1 - F(\delta_R - (1 - r)(Z_P - Z_R) + \frac{n(1 - r)}{r} Q_R (D_1 - D_2); \tau_R) \Big] . \qquad (24)$$

Finally, the partial derivative of total cost per unit time with respect to δ_P is

$$\frac{\partial TC}{\partial \delta_P} = h_1 (1 - r) - \frac{\Pi r \bar{X}}{n Q_R} \Big[1 - F(\delta_P + r(Z_P - Z_R) - n Q_R (D_1 - D_2); \tau_P) \Big] . \qquad (25)$$

Here again there is no way of expressing any of the decision variables explicitly in terms of the other, but an expression can be obtained which will yield a solution through iterations with equations (22) and (24). Setting (25) equal to zero, it may be written as

$$\frac{h_1 Q_R n (1 - r)}{\Pi r \bar{X}} = \Big[1 - F(\delta_P + r(Z_P - Z_R) - n Q_R (D_1 - D_2); \tau_P) \Big] . \qquad (26)$$

If the decision variables Q_P had been used instead of Q_R, equations (22), (24), and (26) would have been

$$Q_P = \left[\left\{ \frac{2\,(1-r)^2\, n\,\bar{X}}{h_1\,(r^2 + n(1-r)^2) + h_2\,(rn - r^2\,(n-1))} \right\} \left\{ A_P + n\,A_R \right. \right.$$

$$+ \; \Pi \left(\int_{\delta_R + r\,(Z_P - Z_R) - \frac{r}{1-r} Q_P\,(D_1 - D_2)}^{\infty} (x - \delta_P - r\,(Z_P - Z_R))\; f\,(x\,;\,\tau_P)\,dx \right. \tag{27}$$

$$\left. \left. \left. + \; n \int_{\delta_P - (1-r)\,(Z_P - Z_R) + Q_P\,(D_1 - D_2)}^{\infty} (x - \delta_P + (1-r)\,(Z_P - Z_R))\; f\,(x\,;\,\tau_R)\,dx \right) \right\} \right]^{\frac{1}{2}}$$

and

$$\frac{h_1\, r\, Q_P}{n\,(1-r)\,\Pi\,\bar{X}} = \Big[1 - F\,(\delta_R - (1-r)\,(Z_P - Z_R) + Q_P\,(D_1 - D_2)\,;\,\tau_R) \Big] \tag{28}$$

and

$$\frac{h_1\, Q_P}{\Pi\,\bar{X}} = \Big[1 - F\,(\delta_P + r\,(Z_P - Z_R) - \frac{r}{1-r}\, Q_P\,(D_1 - D_2)\,;\,\tau_P) \Big]. \tag{29}$$

As a sidelight, it might be of interest to note that if $r = 0$, above equations would become

$$Q_P = \left[\frac{2\bar{X}}{h_1} \left(A_P + \Pi \int_{\delta_P}^{\infty} (x - \delta_P)\, f(x;\tau_P)\, dx\right) \right]^{1/2}$$

and

$$\frac{h_1 Q_P}{\Pi \bar{X}} = \left[1 - F(\delta_P ; \tau_P) \right]$$

which are identical to the results obtained in consumable inventory theory [5] using a similar approach.

3.7 Procedures to Obtain Solutions

There are several methods of obtaining solutions to the above equations. One which makes no assumptions on the fraction D is as follows:

1. Set $n = 1$.
2. Solve for an initial Q_R using the equation

$$Q_R = \frac{r}{n(1 - r)} \sqrt{\frac{2 A_P \bar{X}}{h_1}} \, .$$

3. Solve for $D = (D_1 - D_2)$ using the Q_R from step 2 as follows:

$$D = \frac{(Z_P - Z_R)\, r}{n\, Q_R} - \left(\left[\frac{Z_P\, r}{n Q_R} \right] - \left[\frac{Z_R\, r}{n Q_R} \right] \right)$$

where [Z] is the greatest integer in Z .

4. Substitute this value of D along with the values of Q_R and n into equations (24) and (26) to determine δ_R and δ_P .

5. Using the determined values of Q_R, δ_P, δ_R, D, and n in the right-hand side of (22), determine a new Q_R.

6. Continue steps 3, 4, and 5 until the three equations converge. If they do not converge, assume a value for D and repeat steps 3 and 5 until they do. Since the value of D is very small, a good value to assume would be zero.

7. Compute the total cost per unit time with the resulting values of Q_R, δ_R, δ_P, n, and K.

8. Increase n by one and repeat steps 2 through 8 until the minimum cost is determined. Select those values of Q_R, δ_R, δ_P, and n which give the minimum cost and compute Q_P from equation (2).

A second procedure would be to set $D = 0$ and repeat procedure one, keeping D at zero. This procedure should be used only in the event that the results of procedure one prove to be infeasible.

4. A NUMERICAL EXAMPLE

Since this paper is an extension of the substitution policy, the following example is the same as that presented in reference [1]. The values of the parameters were given as follows: $A_P = \$750$, $A_R = \$100$, $r = 0.9$, $\bar{X} = 1{,}000$ units per year, $h_1 = \$200$ per unit year, $h_2 = \$20$ per unit year, $\tau_P = 1.0$ years, $\tau_R = 0.25$ years. The value of h_1 was based on a unit cost (C) of \$1,000 per unit, which implies a carrying charge (I) of 0.2 per year, and the cost of repair of \$900 per unit. Here a shortage cost per unit (Π) is set equal to \$1,000, and the demand on the RFI inventory is assumed to have a normal distribution with mean and variance both equal to $\bar{X}t$. The first procedure set forth at the end of the previous section was followed but the results, listed in Table 1, obtained for most of the steps of the integer n yielded values of the fraction D that were outside the permissible range set forth in the previous section. The value of the fraction was set equal to zero, and the procedure was repeated keeping D at zero. The results, listed in Table 2, of this second procedure were almost identical to those obtained in the first except that the reorder points changed considerably. With the fraction outside its permissible range, the values obtained for the reorder points were impossible; i.e., once the IP got down to the δ_R level, it would be impossible for it to get back up to the δ_P level. For example, the trigger levels obtained for $n = 17$ were $\delta_R = 356$, $\delta_P = 465$, and the repair quantity was 36 units. Once the IP got down to $\delta_R = 356$, the highest it could ever be after this would be $\delta_R + Q_R = 392$. Since $\delta_R + Q_R$ is less than δ_P , a procurement order would never again be placed and the net inventory (NI) would tend toward negative infinity. The results for D = 0, with rounding off, were: 17 lot-size inductions from the repair facility of 35 units each; a procurement quantity of 67 units; an average cycle length of 0.67 years; reorder points, based on IP, of $\delta_R = 363$ and $\delta_P = 394$; all at an expected total cost per year of TC = $18,831.67 . It must be mentioned that this total cost does not include the cost of items nor the cost of repairing items. The results obtained are in excellent agreement with those in the example of reference [1], which are: 19 inductions of 30 units each; a procurement of 63 items; and a cycle (deterministic) time of 0.63 years. An examination of Tables 1 and 2 indicates that the buffers b_1 and b_2 are approximately the same for both procedures. With the buffers the same, it is expected that the cost should be the same; consequently, by setting the fraction D equal to zero, the order quantities and costs do not change but trigger levels that are feasible, are obtained. Again, an examination of the values in Tables 1 and 2 quickly reveals that the total cost of operating the system is quite insensitive to the order quantities and the number of inductions in the neighborhood of the optimum. These results are consistent with those obtained in consumable inventory theory [5] in that a plot of total cost versus procurement quantity is quite flat in the neighborhood of the optimum.

Even as complicated as the equations appear, they are quite easily solved with the aid of a computer. The previous example was programed in FORTRAN 60 and run on a CDC 1604, with a total run time of slightly less than two minutes. The results of the iterations for $n = 13$ and $D = 0$ are listed in Table 3. As can be seen from this table, the equations converge quite rapidly, which was the case for all steps of n from one through 25.

In order to get some indication as to how the repair leadtime affects the total operating cost of the system per year, the above example was run again on the computer with τ_R set equal to 0.5 years vice 0.25 years. The results of this run are listed in Table 4. As can be seen by comparing Tables 2 and 4, the total cost of maintaining the system can be reduced for this example by almost $8,000 per year by decreasing the repair leadtime from 0.5 years to 0.25 years.

n	Q_R	Q_P	δ_R	δ_P	b_1	b_2	D	TC	T
1	93	10	355	438	91	32	0.24	22897.08	0.103
2	70	16	362	382	86	34	-0.20	20943.00	0.156
3	61	20	346	538	87	35	0.70	20156.25	0.201
4	55	24	359	423	83	36	0.07	19727.19	0.244
5	50	28	372	316	79	37	-0.35	19459.28	0.283
6	48	32	383	212	77	37	-0.66	19278.91	0.321
7	46	36	358	435	77	37	0.10	19151.79	0.357
8	44	39	365	370	73	37	-0.08	19059.61	0.392
9	43	43	372	308	75	37	-0.24	18991.66	0.426
10	41	46	380	247	75	38	-0.37	18941.25	0.459
11	40	49	386	188	75	38	-0.48	18903.92	0.492
12	39	52	340	601	75	37	0.43	18876.64	0.523
13	38	55	343	573	74	37	0.35	18857.23	0.554
14	38	58	346	545	74	37	0.28	18844.08	0.585
15	37	61	349	518	72	37	0.22	18836.02	0.614
16	36	64	353	491	68	39	0.17	18832.12	0.643
17	36	67	356	465	67	39	0.12	18831.67	0.672
18	35	70	358	439	70	38	0.07	18834.12	0.700
19	34	73	361	414	69	38	0.03	18839.00	0.728
20	34	75	364	389	70	38	-0.01	18845.95	0.755
21	33	78	367	365	68	39	-0.04	18854.68	0.781
22	33	81	370	341	67	38	-0.07	18864.93	0.805
23	33	83	372	317	67	39	-0.10	18876.50	0.833
24	32	86	375	294	69	39	-0.13	18889.21	0.859
25	32	88	378	271	65	40	-0.15	18902.92	0.884

Results of Numerical Example Using Procedure 1

TABLE 1

n	Q_R	Q_P	δ_R	δ_P	b_1	b_2	TC	T
1	93	10	357	415	90	32	22897.08	0.103
2	70	16	359	411	86	34	20943.00	0.156
3	61	20	360	408	83	35	20156.25	0.201
4	55	24	361	406	81	36	19727.19	0.244
5	51	28	362	405	80	37	19459.28	0.283
6	48	32	362	403	78	37	19278.92	0.321
7	46	36	362	402	77	37	19151.79	0.357
8	44	39	362	401	76	37	19059.61	0.392
9	43	43	363	400	75	38	18991.66	0.426
10	41	46	363	400	75	38	18941.25	0.459
11	40	49	363	399	74	38	18903.92	0.492
12	39	52	363	398	73	38	18876.64	0.523
13	38	55	363	397	72	38	18857.22	0.554
14	38	58	363	397	72	38	18844.08	0.585
15	37	61	364	396	71	39	18836.02	0.614
16	36	64	364	396	71	39	18832.12	0.643
17	36	67	364	395	70	39	18831.67	0.672
18	35	70	364	394	69	39	18834.12	0.700
19	34	73	364	394	69	39	18839.00	0.728
20	34	75	364	393	68	39	18845.95	0.754
21	33	78	364	393	68	39	18854.68	0.781
22	33	81	364	393	68	39	18864.93	0.807
23	33	83	364	392	67	39	18876.50	0.833
24	32	86	364	392	67	39	18889.21	0.859
25	32	88	364	392	67	39	18902.92	0.884

Results of Numerical Example Using Procedure 2

TABLE 2

n	Q_R	Q_P	δ_R	δ_P	TC
13	100	144	357	385	25479.71
13	46	67	362	395	19636.90
13	39	57	363	397	18957.23
13	39	56	363	397	18870.31
13	38	55	363	397	18858.94
13	38	55	363	397	18857.45
13	38	55	363	397	18857.26
13	38	55	363	397	18857.23
13	38	55	363	397	18857.23
13	38	55	363	397	18857.23
13	38	55	363	397	18857.23
13	38	55	363	397	18857.23

Representative Example of the Number of Iterations
Required to Determine Minimum Cost

TABLE 3

n	Q_R	Q_P	δ_R	δ_P	TC
1	96	11	596	640	30475
2	73	16	599	636	28627
3	63	21	600	633	27896
4	57	25	601	631	27503
5	53	29	602	630	27261
6	50	33	602	628	27101
7	48	37	602	627	26991
8	46	41	603	626	26913
9	45	45	603	625	26857
10	43	48	603	624	26818
11	42	51	603	623	26790
12	41	55	604	622	26772
13	40	58	604	622	26761
14	39	61	604	621	26755
15	39	64	604	621	26755
16	38	67	604	620	26757
17	37	70	604	619	26763
18	37	73	605	619	26772
19	36	76	605	618	26782
20	36	79	605	618	26795
21	35	82	605	618	26809
22	35	85	605	617	26824
23	34	87	605	617	26841
24	34	90	605	616	26858
25	33	92	605	616	26876

Results of Numerical Example Using Procedure 2
with the Repair Leadtime Equal to
One-Half Year Vice One-Fourth Year

TABLE 4

5. CONCLUSION

In addition to the basic assumptions listed previously, the model was developed with one particular type of reparable item in mind. This type of item can be classified as being essential with a high demand rate, which usually implies that backorders are highly undesirable. Items that are inexpensive, in general, do not meet the criteria of being reparable; hence, one more characteristic of the type of item in mind is that it is relatively expensive. Items that cannot be handled by the model are those which permit backorders to occur quite frequently. This class consists of those items which are quite expensive with a low demand rate. To handle this type of item, a model would have to be developed using an exact treatment of both the holding and shortage costs. Another type of item that could not be handled with the model would be one in which the distribution of demand changes considerably from one cycle to the next. An example of this type would be an item which was either being phased in or being phased out of the inventory system.

If the assumption dealing with the return of carcasses to NRFI is beyond all realm of possibility, then the repair leadtime would have to be a random variable. This random variable would be a constant δ_R as long as there are enough items in NRFI inventory to accommodate a repair induction at the time a repair order is placed. If there are not enough items in NRFI inventory at the time an induction order is placed, then the leadtime would be increased a random amount of time τ . This would be the time necessary to accumulate enough carcasses to satisfy the lot size restriction of Q_R items. In this case, the relationship between Q_P and Q_R would not exist, and the distribution of demand during the repair leadtime would be a function of two random variables. Even with the above type of items excluded, there are still a considerable number of items that are of the category specified by the assumptions, and the model presented should result in values of the decision variables that are fairly accurate.

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13. ABSTRACT

In general, a reparable inventory system deals with two classes of the same type of item. The first class consists of those items which have been procured from the manufacturer and have never been put to use. The second class consists of those items which have been used, have failed, have been repaired, and are ready to be used again. In addition to the two classes of items, the system also contains two distinct inventories. These are the ready-for-issue and the non-ready-for-issue inventories. The ready-for-issue inventory contains both classes of items in their usable state, and the non-ready-for-issue inventory contains only the latter in its failed but reparable state. This paper develops a quasi-probabilistic inventory model for the reparable inventory system based on the premise that the repair facility is the primary source of inventory, with procurements being made periodically only to supplement this primary source.

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Reparable						
Non-ready-for-issue						
Ready-for-issue						
Inventory						