

Calhoun: The NPS Institutional Archive DSpace Repository

# Development and study of a two-dimensional dynamic model of a towed buoy 

Bell, John Franklin

Monterey, California. U.S. Naval Postgraduate School
https://hdl.handle.net/10945/11995

This publication is a work of the U.S. Government as defined in Title 17, United States Code, Section 101. Copyright protection is not available for this work in the United States.

Downloaded from NPS Archive: Calhoun

BELL, J.

```
DEVELOPMENT AND STUDY OF A TWO-
DIMENSIONAL DYNAMIC MODEL OF A
    TOWED BUOY
    by
```

    John Franklin Bell
    DUDLEY KNOX LIBRARY
 MONTEREY, C

# United States <br> Naval Postgraduate School 



## THESIS

DEVELOPMENT AND STUDY OF
A TWO-DIMENSIONAL DY'NAMIC MODEL OF A TOWED BUOY
by

John Franklin Bell

December 1969

This document has been approved for public release and sale; its distribution is unlimited.

Lubrary
U.S. linval Postgraduate Schood

Konterey, California 93940

```
                    Development and Study of
            A Two-Dimensional Dynamic Model
                    Of a Towed Buoy
                    by
                    John Franklin Bell
Lieutenant (junior grade), United States Navy
    B.S., United States Naval Academy, }196
            Submitted in partial fulfillment of the
                    requirements for the degree of
                    MASTER OF SCIENCE IN ELECTRICAL ENGINEERING
                    from the
NAVAL POSTGRADUATE SCHOOL
            December 1969
```


## ABSTRACT

A two-dimensional dynamic model of a towed buoy designed to remain submerged just below the water surface is developed. The study includes the model development, the analysis of certain model parameters, the development of a tension force, and the range of stability of the model to initial condition perturbations.

## TABLE OF CONTENTS

I. INTRODUCTION ..... 11
II. EQUATIONS OF MOTION AND PROBLEM SETUP ..... 14
A. BODY COEFFICIENTS ..... 16
B. FIN SERVO LOOP ..... 20
III. THE BUOY MODEL ..... 25
IV. STUDY OF THE BUOY MODEL ..... 36
A. SERVO INPUT COEFFICIENTS STUDY ..... 36
B. TENSION FORCE FORMULATION ..... 44
C. STUDY OF INITIAL CONDITION PERTURBATIONS ..... 46
V. CONCLUSIONS AND RECOMMENDATIONS ..... 49
APPENDIX A RESPONSES OF THE BUOY TO INITIAL CONDITION PERTURBATIONS ..... 50
APPENDIX B SYMBOLS AND ABBREVIATIONS USED IN THE COMPUTER PROGRAMS ..... 75
COMPUTER PROGRAM INITIAL BUOY MODEL ..... 79
COMPUTER PROGRAM STUDY OF $\mathrm{T}=1940 .+\mathrm{L} * \mathrm{SC}$ ..... 82
BIBLIOGRAPHY ..... 85
INITIAL DISTRIBUTION LIST ..... 86
FORM DD 1473 ..... 87
 jov Destauchaigieoc duvak pope-ctCCN AJ , reviarmoll

## LIST OF TABLES

I. COEFFICIENT OF LIFT ..... 17
II. COEFFICIENT OF DRAG ..... 18
III. COEFFICIENT OF MOMENT ..... 19
IV. MAPPING OF STABLE COMBINATIONS OF ADE AND ADR ..... 37
V. STABILITY RANGE OF INITIAL CONDITIONS ..... 48


## LIST OF FIGURES

1. SUBMARINE COMMUNICATIONS BUOY ..... 12
2. THE BUOY ..... 15
3. CURVES AND DATA POINTS FOR $C_{L}$ ..... 21
4. CURVES AND DATA POINTS FOR $C_{D}$ ..... 22
5. CURVES AND DATA POINTS FOR $\mathrm{C}_{\mathrm{M}}$ ..... 23
6. THE FIN SERVO CONTROL SYSTEM ..... 24
7. THE FIN SERVO LOOP ..... 25
8. THE K/s ${ }^{2}$ SERVO SYSTEM ..... 25
9. STEP RES PONSES OF TWO $\mathrm{K} / \mathrm{s}^{2}$ SYSTEMS ..... 27
10. THE BUOY ..... $3)$
11. THE MODEL BLOCK DIAGRAM ..... 31
12. RESPONSE OF $\mathrm{ADE}=0.50$ AND $\mathrm{ADR}=0.3$ ..... 38
13. RESPONSE OF $\operatorname{ADE}=1.6$ AND $\mathrm{ADR}=0.09$ ..... 39
14. RESPONSE OF ADE=1.2 AND ADR=0.3 ..... 40
15. RESPONSE OF $\mathrm{ADE}=1.2 \mathrm{AND} \operatorname{ADR}=0.07$ ..... 41
16. RESPONSE OF $\operatorname{ADE}=2.5$ AND $A D R=0.3$ ..... 42
17. THE NEW TENSION FORCE FORMULATION ..... 45


F force, in pounds
m mass, in slugs
a acceleration, in $\mathrm{ft} / \mathrm{sec}^{2}$
M moment, in ft•pounds
$I_{y} \quad$ mass moment of inertia about $Y$ axis, in slug $\cdot \mathrm{ft}^{2}$
6 angular acceleration in $\mathrm{rad} / \mathrm{sec}^{2}$
D drag, in pounds
$C_{L} \quad$ coefficient of lift
$C_{D} \quad$ coefficient of drag
$C_{\text {DA }}$ coefficient of drag of the antenna
$\mathrm{C}_{\mathrm{M}} \quad$ coefficient of the pitching moment about the tow point
$\rho \quad$ mass density of water, approximately equal to 1.99
S characteristic area of the buoy
$\overline{\mathrm{C}} \quad$ mean chord length of the buoy
b span of the buoy
T tension, in pounds
L lift, in pounds

## ACKNOWLEDGEMENT

The author would like to give his special thanks to Dr. George J. Thaler, who provided the inspiration and guidance for this study; to Mr. Tom Gibbons and Mr. Don Gray of NSRDC, who provided ideas and data on the buoy; to Kelly, and to Doris and her midnight shift, at the computer center for their effort and patience; and to Kihi for her love, patience, and understanding.

## I. INTRODUCTION

The purpose of this investigation is to attempt to mathematically model and computer-simulate the two-dimensional motion of a submarinetowed buoy. This has a very specific military application in submarine communications systems; the basic system configuration as a submarine communication link is illustrated in Fig. 1. It is desired that the buoy stay underwater at all times, and remain within $\pm 1 \mathrm{ft}$. of four feet below the water surface; this gives the buoy an operating range from three to five feet below the surface. An additional system requirement is that the buoy system stay within its allowable operating zone while following waves up to sea-state four.

Many investigators, such as Lacey, have dealt with ship-towed or airplane-towed devices, where the buoy is the "bottom-following" type. Pode and other early investigators studied the steady-state configuration of cables in a steady uniform stream; and more recently Schram developed, by the method of characteristics, a two-dimensional dynamic cable-buoy model. However, he treated the buoy as a vector force and did not go into the actual buoy dynamics.

Steady-state studies do not show the response of the buoy and cable to perturbations such as sea state. It is this response that will prove most useful in the study of buoy and cable parameters. This study has been primarily concerned with developing the dynamic model of the buoy; and the tension force generated by the cable has been loosely approximated.


The model is of a buoy presently being developed for the U. S. Navy; thus, the dimensions and other data were obtained from an existing prototype.

This study develops the two-dimensional model around two basic equations; force equals mass times acceleration, and the moment equals the mass moment of inertia times the angular acceleration. The model block diagram was built around these two equations, several basic hydrodynamic relationships, and the wind-tunnel and dimensional data supplied on the buoy. Once the block diagram was formulated, it was then ready to be computerized. The model was simulated using the IBM 360-67 at the Naval Postgraduate School, and DSL (Digital Simulation Language); DSL is a very simple, but versatile simulation language developed by IBM. Once the model was computerized, the lengthy and time-consuming task of optimizing certain parameters and finding a realistic formulation of the tension force followed. The determination of the tension force and its coupling with the buoy has been one of the most difficult problems in the formulation of the buoy-cable model. This has been a problem in all buoy-cable models where the dynamic response is of interest; this is mainly due to the fact that it is not exactly known how the forces generated by the buoy and cable react on and with each other. The development of the model and the results of the simulation are presented.

## II. EQUATIONS OF MOTION AND PROBLEM SETUP

The purpose of the buoy is to remain four feet below the water surface so that a protruding antenna may maintain two-way communications; when the buoy is operating in high seas it must follow the sea surface. The body, as shown in Fig. 2., is basically a fat wing that produces dynamic lift according to its angle of attack. This angle of attack may be changed by torque produced by the fin. Besides the dynamic lift, produced by the body angle of attack, there is the static lift of positive buoyancy. The buoy model is based on the two basic equations:

$$
\begin{gathered}
\mathrm{F}=\mathrm{ma} \\
\mathrm{M}=\mathrm{I}_{\mathrm{y}} \ddot{\theta}
\end{gathered}
$$

The first equation is used to describe movement in the horizontal and vertical planes, the X and Z planes respectively. The second is used to describe the rotation of the body about the $Y$ axis; these axis are shown in Fig. 2. The three hydrodynamic forces of lift, drag, and moment are defined as follows:

$$
\begin{aligned}
& L=\frac{1}{2} C_{L} \rho V^{2} S \\
& D=\frac{1}{2} C_{D} \rho V^{2} S \\
& M=\frac{1}{2} C_{M} \rho V^{2} S \bar{C}
\end{aligned}
$$

Along with the tension force produced by the cable and the buoyancy of the body, these threc equations describe all the forces acting on the

body. Before going into the complete buoy block diagram, the body coefficients will be put into a more convenient form, and the fin servo loop, which controls the buoy attitude, will be examined.

## A. BODY COEFFICIENTS

In both aerodynamics and hydrodynamics, there are various dynamic coefficients; these are determined by the shape of the body involved. There are tables of coefficients for certain common wing shapes. However, when an odd body configuration is being studied, it may not be possible to apply the standard shapes in order to derive overall coefficients for the body. In this case the coefficients of the body must be determined by wind-tunnel tests.

This was the case with the system being studied; the body is of non-standard shape, but the fin is the standard NACA 0012 shape. The three coefficients, $C_{L}, C_{D}$, and $C_{M}$, were measured at various body and fin angles. The coefficient of lift and the coefficient of moment include the effect of the positive buoyancy force of 490 pounds. The effect of the antenna drag is not included in any of the coefficients. The values of these coefficients are shown in Tables I-III as a function of body angle of attack and fin angle of attack. It was decided that for computer implementation, these coefficients would be easier to handle if they were in equation form. This was done by applying a least-squares fit twice to each data set; the resulting equations may be found in Subroutine Tink of the digital program of the model. The resultant curves and

FIN ANGLE (Deg.)

|  | -10 | -7.5 | -5. | -2.5 | 0.0 | 2.5 | 5. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -12 | -.054 | -.039 | -.027 | -.016 | -.003 | .0064 | .0211 |
| -9 | -.001 | .0110 | .0253 | .0353 | .0477 | .0589 | .0756 |
| -6 | .0585 | .0647 | .0776 | .0901 | .1020 | .1139 | .1306 |
| -3 | .1102 | .1214 | .1351 | .1461 | .1593 | .1723 | .1876 |
| BODY <br> ANGLE <br> (Deg.) 3 | .1676 | .1792 | .1929 | .2035 | .2166 | .2294 | .2454 |
| 6 | .3044 | .3163 | .3285 | .3414 | .3547 | .3678 | .3828 |
| 9 | .3834 | .3946 | .4075 | .4195 | .4333 | .4461 | .4593 |
| 12 | .4654 | .4774 | .4911 | .5025 | .5155 | .5298 | .5386 |

Table I

FIN ANGLE (Deg.)

|  | -10 | -7.5 | -5 | -2.5 | 0.0 | 2.5 | 5. |
| ---: | :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| -12 | .0415 | .0374 | .0364 | .0331 | .0327 | .0307 | .0320 |
| -9 | .0345 | .0314 | .0310 | .0284 | .0283 | .0271 | .0288 |
| -6 | .0319 | .0302 | .0298 | .0279 | .0287 | .0284 | .0306 |
| -3 | .0347 | .0334 | .0336 | .0322 | .0343 | .0347 | .0377 |
| BODY <br> ANGLE <br> (Deg.) | .0514 | .0523 | .0404 | .0415 | .0404 | .0430 | .0444 |
| 6 | .0696 | .0703 | .0721 | .0730 | .0769 | .0804 | .0844 |
| 9 | .0956 | .0972 | .0998 | .1007 | .1048 | .1087 | .1143 |

COEFFICIENT OF DRAG

Table II

## FIN ANGLE (Deg.)

|  | -10 | -7.5 | -5 | -2.5 | 0.0 | 2.5 | 5. |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| -6 | .0981 | .0908 | .0834 | .0772 | .0707 | .0639 | .0554 |
| -3 | .0794 | .0723 | .0651 | .0593 | .0528 | .0452 | .0370 |
| 0 | .0585 | .0512 | .0439 | .0382 | .0314 | .0237 | .0158 |
| BODY 3 | .0539 | .0289 | .0214 | .0156 | .0086 | .0010 | -.007 |
| ANGLE <br> (Deg.) 6 | .0106 | .0036 | -.003 | -.009 | -.017 | -.024 | -.032 |
| 9 | -.016 | -.023 | -.030 | -.036 | -.044 | -.051 | -.058 |
| 12 | -.044 | -.050 | -.057 | -.064 | -.071 | -.078 | -.084 |

COEFFICIENT OF MOMENT

Table III
data points for constant body angle of attack and varying fin angle of attack are shown in Figs. 3-5.

## B. FIN SERVO LOOP

The diagram of the fin and the servo-control system is shown in Fig. 6. The purpose of the fin is to orient the body so that it will stay four feet below the water surface. The fine produces a moment on the buoy which causes the body angle of attack to increase or decrease. This increase or decrease in body angle of attack causes the body lift to increase or decrease and the body rises or descends as is necessary.

The fin actuator loop is driven by a Size 23, Kearfott motor. The loop is designed to have a rise time of $0.04 \mathrm{sec} . ;$ this actuator loop is shown in Fig. 7. The servo loop used in the buoy model is considered to be a $\mathrm{K} / \mathrm{s}^{2}$ plant with tachometer and unity feedback; this is shown in Fig. 8. The system transfer function:

$$
\begin{array}{r}
\frac{G}{l+G H}=\frac{K_{A} / s^{2}}{1+K_{A} / s^{2}\left(K_{t} s+1\right)} \\
\frac{K_{A}}{s^{2}+K_{A} K_{t} s+K_{A}}
\end{array}
$$

and the characteristic equation:

$$
s^{2}+K_{A} K_{t}+K_{A}=0
$$

Assuming $K_{A}=1000$

$$
w_{n}^{2}=K_{A}=1000
$$



CURVES AND DATA POINTS FOR $C_{L}$
Figure 3


Figure 4


CURVES AND DATA POINTS FOR $\mathrm{C}_{\mathrm{M}}$
Figure 5

Figure 6


THE FIN SERVO LOOP
Figure 7


THE K/S² SERVO SYSTEM
Figure 8

$$
\begin{aligned}
& 2 \xi w_{n}=K_{A} K_{t} \\
& K_{t}=\frac{2 \xi w_{n}}{1000} .
\end{aligned}
$$

Taking a value of $\xi=0.7$ :

$$
\begin{aligned}
\mathrm{K}_{\mathrm{t}} & =2(0.7)(31.62) / 1000 \\
& =.0443 .
\end{aligned}
$$

With $K_{A}$ and $K_{t}$ set at the above values, the rise time of the servo loop is about 0.1 sec . This is not as fast as desired, but by changing the two gains the desired rise time may be obtained. Two different responses of the fin servo loop are shown in Fig. 9; both have an input step of 0.174533 rad . ( 10 degrees). The first set of gains is used in the model simulation. This is because it is doubtful that the inputs into the servo loop will require that the fin move that fast; if so, then the buoy has probably reached the limits of its surface-following capability.

The inputs into the fin servo loop are shown in Fig. 6. The fin servo loop that was simulated above is composed of the summer, servo amplifier, motor and gears, linkage to the fin, and the unity and tachometer feedback from the fin to the summer. The other three inputs will come from gages and electronic sensors in the body. These inputs are the depth error, the depth rate, and the body pitch rate. In the model simulation, there are three constants (ADE, ADR, and APR) which multiply these thrce inputs. The values of these three constants are discussed later in the study.



STEP RESPONSES OF TWO K/S ${ }^{2}$ SYSTEMS
Figure 9

## III. THE BUOY MODEL

In this development of the buoy model, the effect of water currents, the forces generated by the heaving water surface, and the free-surface effect are not taken into account. Currents can be considered neglible compared to the heaving forces the buoy might experience. The freesurface effect is experienced by the buoy because it is a body moving at a depth comparable to its length; a more detailed and complicated model would have to include this force. The force generated by the heaving water surface would not affect the buoy as much as might be expected. This is because the body, as long, as its vertical freedom is not restricted, will act like a particle in the water and will move up and down with the surface. Thus, the heaving forces will actually help maintain the buoy near the water surface, which is what is desired. If the buoy were being towed directly above its towpoint, it would have little or no vertical freedom; however, if the buoy is well behind the towpoint, then the cable will provide little restriction to the buoy's vertical freedom. Tests on the actual buoy have shown that with 475 ft . of cable and the two point at "a depth of $240 \mathrm{ft} .$, the buoy can heave up to 20 ft . with the waves without the cable decreasing the buoy's vertical freedom of motion by more than 10 percent. ${ }^{1}$

[^0]The buoy model will now be developed; the buoy diagram is again produced with some specific dimensions (Fig. 10). Besides the diagrammed dimensions, the buoy has the following additional characteristics:

$$
\begin{aligned}
& \mathrm{S}=14.25 \mathrm{ft}^{2} \\
& \overline{\mathrm{C}}=6.0 \mathrm{ft} \\
& \mathrm{~b}=2.375 \mathrm{ft} \\
& \text { mass }=70 \mathrm{slugs} \\
& \text { mass moment of inertia }=975 \mathrm{slug} \cdot \mathrm{ft}^{2} \\
& \text { volume }=17.5 \mathrm{ft}^{3}
\end{aligned}
$$

The block diagram of the dynamic model is shown in Fig. 11. As was previously stated, the buoy model is based on the summation of forces in the X and Z planes, and about the Y axis. The summed forces in the $X$ and $Z$ directions are divided by the mass of the buoy, and the moments about the $Y$ axis are summed and divided by the mass moment of inertia. This yields the acceleration in the horizontal and vertical directions, and the angular acceleration about the Y axis. The accelerations are each integrated to give the velocities, and these, when integrated, give the horizontal and vertical position and the angular orientation.

In this first model, the tension force is generated as a function of horizontal velocity and body angle of attack. The tension function was formed in the same manner as the coefficient equations, and was based on predicted values of the tension in the cable. This tension function may also be found in Subroutine Tink of the computer program.




The velocities and positions are used to generate the forces and moments that are summed in the basic equations. The forces and moments on a body are relative to the velocity vector of the medium, and not to the local horizontal. Thus, the sum of the squares of the horizontal and vertical velocities yields the relative velocity squared, V2. The arc tangent of the vertical velocity over the horizontal velocity is taken to give the angle of the velocity vector, VAN, with respect to the true horizontal. The angular orientation of the body (angle of attack), Al, is summed with VAN to produce the angle of attack of the body with respect to the velocity vector, $A R ; A R$ is in radians. The fin angle of attack, $F A$, is also summed with VAN to yield the fin angle of attack relative to the velocity vector, $B R$. These two angles of attack, $A R$ and $B R$, are multiplied by a conversion factor of $57.29578 \mathrm{deg} / \mathrm{rad}$ to yield the angles of attack, $A$ and $B$, in degrees.

The two angles, A and B, are the arguments which determine the coefficient values of body lift, drag, and moment. The coefficients of lift and drag are both multiplied times V2 and a constant to produce the body lift, BL, and the body drag, BD. The constant is $\frac{\frac{1}{2}}{\rho} \rho \mathrm{~S}$, and is equal to 14.17875 .

The moment coefficient is multiplied times V2 and a constant, $\frac{1}{2} p S \bar{C}$; this constant is equal to 84.0725 . This body moment, YCM, is summed with the moments produced by the antenna, to yield the total moment, YM, acting on the body about the towpoint.

There is no definite information on the antenna, but it is assumed that it will only contribute a drag force; this drag force will contribute to the total drag and moment on the body. The antenna coefficient of drag is assumed to remain constant at a value of 0.50 . Thus, the antenna drag is dependent on a constant times the frontal surface area times the relative velocity squared. Assuming a frontal width of 2 inches, the drag force of the antenna will then vary with the length of antenna under water, and V2. The length of antenna will be approximated as the depth of the body below the surface. The constant in this case is $\frac{1}{2}(1 / 6) \rho C_{D A^{\prime}}$ which equals 0.04146 . The antenna drag is multiplied by the sine and cosine of VAN, SV, and CV, to give two moment components. These times their respective moment arms yield YADS and YADC.

The body lift and drag, and antenna drag are multiplied by SV and CV, and the resulting six values are summed into the horizontal and vertical forces. In this first realization of the model, these horizontal and vertical forces are used before being summed with the horizontal and vertical components of the tension force, to determine the angle FE with which the tension force acts on the body. This means that the tension force acts exactly opposite to the vector of body forces; this simplifying assumption is used in the initial stages of the model development.

The depth error and depth rate inputs into the fin servo loop are taken from the vertical position and velocity, Z and Zl , respectively. These are multiplicd by a constant, 0.443 , which converts ft and $\mathrm{ft} / \mathrm{sec}$ into psi/ft and psi sec/ft. These are multiplied by gain constants ADE and

ADR before being summed with the body pitch rate, Yl, times its gain constant, APR. These three inputs are fed into the fin servo loop. The orientation of the fin causes a torque on the body which either increases or decreases the body angle of attack; this enables the body to rise or descend.

This is the initial dynamic model; it changes as more insight and knowledge are gained on the model. The first study with the model deals with the three coefficients ADE, ADR, and APR.

## IV. STUDY OF THE BUOY MODEL

Once the model was digitally programmed and the initial programming mistakes ironed out, the first step was to study possible values of the fin servo input coefficients. The first computer program realization is shown in the program titled "INITIAL BUOY MODEL". The buoy is initiated at a depth, ZI , of 4.0 ft , a speed, XlI of $20.2536 \mathrm{ft} / \mathrm{sec}$ ( 12 knots ), body angle of attack, AlI, of 0.10472 rad ( 6 degrees), and a fin angle of attack, FAI, of 0.02 rad ( 1.15 degrees). The tension force and the coefficients of lift, drag, and moment are called out of Subroutine Tink, which is in double-precision format.

## A. SERVO INPUT COEFFICIENTS STUDY

The first attempt in this study was to see if the buoy would stabilize with only one input into the fin servo loop; either the depth error, the depth rate, or the body pitch rate. This proved fruitless, so different combinations of $A D E$ and $A D R$ were tried, and it was found that these two, with APR set equal to zero, would stabilize the system. A full range of values of $A D E$ and $A D R$ were tried to see which combinations would bring the buoy to a steady-state position. A mapping of these values is shown in Table IV; the value given for each combination is the depth at which the buoy steadied out. The responses for several of these combinations are shown in Figs. 12-16. It appears from the depth values that the tension force is too strong, since in every case the buoy steadies out deeper than the desired depth of 4.0 ft . This became obvious when the

|  |  |  |  |  |  |  | ADE |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.3 | 0.5 | 0.7 | 0.9 | 1.1 | 1.2 | 1.3 | 1.4 | 1.6 | 1.8 | 2.0 | 2.5 | 3.0 |
| 0.05 |  |  |  |  |  | 4.599 | 4.547 | 4.511 |  |  |  |  |  |
| 0.06 |  |  |  |  | 4.656 | 4.587 | 4.539 | 4.502 |  |  |  |  |  |
| 0.07 |  |  |  |  | 4.637 | 4.579 | 4.533 | 4.496 |  |  |  |  |  |
| 0.08 |  |  |  |  | 4.627 | 4.572 | 4.527 | 4.492 | 4.454 | 4.388 |  |  |  |
| 0.09 |  |  |  |  | 4.691 | 4.567 | 4.524 | 4.488 | 4.433 | 4.385 |  |  |  |
| 0.10 |  |  |  | 4.778 | 4.614 | 4.563 | 4.520 | 4.485 | 4.427 | 4.383 |  |  |  |
| 0.20 |  | 5.292 | 4.922 | 4.719 | 4.591 | 4.543 | 4.502 | 4.467 | 4.411 | 4.367 | 4.332 |  |  |
| 0.30 | 6.101 | 5.270 | 4.911 | 4.712 | 4.584 | 4.536 | 4.496 | 4.462 | 4.405 | 4.362 | 4.327 | 4.265 | 4.255 |
| 0.40 |  | 5.268 | 4.909 | 4.710 | 4.583 | 4.536 | 4.495 | 4.461 | 4.404 | 4.360 | 4.325 | 4.263 | 4.227 |
| 0.50 |  |  | 4.909 | 4.711 | 4.593 |  |  |  |  |  |  |  |  |
| 0.60 |  |  |  | 4.712 | 4.586 |  |  |  |  |  |  |  |  |
| 0.70 |  |  |  |  | 4.537 | 4.496 | 4.469 |  |  |  |  |  |  |
|  |  |  | PPING | OF STA | BLE CO | MBINAT | IONS OF | F ADE | AND ADR |  |  |  |  |
|  |  |  |  |  | Table I |  |  |  |  |  |  |  |  |

$$
\stackrel{\sim}{\underset{\sim}{\sim}}
$$





Figure 14

steady-state body angle of attack and fin angle of attack were checked; both angles were large and showed that the model was working properly, but that the buoy was fighting against an unrealistic tension force formulation.

It was decided from the mapping of the two coefficients, $A D E$ and ADR, that three sets would be used in further studies of the buoy model. The first pair was chosen on the basis that it was on the intersection of the values of $A D E$ and $A D R$ which had the widest range of stable values, and that this set was towards the center of the mapping. This was $A D E-1.2$ and $A D R=0.30$. It was then decided that the other two pairs would vary in one coefficient away from this point. These other two pairs are $A D E=1.2$ and $A D R=0.07$, and $A D E=2.5$ and $A D R=0.30$. The responses with these three pairs of coefficients are shown in Figs. 14-16.

The coefficient APR remained to be studied. When this coefficient was given a value larger than zero there was no appreciable change in the response of the buoy. To check the range of possible values of APR, the tension force, $T$, was changed so that it was equal to the square root of the sum of the squares of F1 and F2. Thus, the buoy would stay at four feet below the surface and the effect of APR could be seen in the response of the buoy's angle of attack. Both negative and positive values of APR were tried, and negative values immediately drove the buoy model unstable. Positive values of APR, up to 7.0 , were tried with the three pairs of depth error and depth rate coefficients. At this upper value of
7.0, the body started to oscillate away from stability about the Y axis; it was apparent that this high value of gain was approaching the stability limit of the buoy model. Based on this, a median value of 3.0 was chosen for APR. The three final sets of coefficients are as follows:

| ADE | ADR | APR |
| ---: | ---: | ---: |
| 1.2 | 0.07 | 3.0 |
| 1.2 | 0.30 | 3.0 |
| 2.5 | 0.30 | 3.0 |

## B. TENSION FORCE FORMULATION

The next step in this model study was to formulate a new, and hopefully more realistic, tension force, rather than using the predicted tension data supplied with the buoy. It was decided that this tension function would be a constant plus or minus the distance away from the equilibrium point times what will be called a spring constant, SC. The basis for this formulation is shown in Fig. 17. First it is assumed that the last portion of the cable connected to the buoy, LO, is straight, and that of the total length of the cable this section is the most dynamic. By computing the increase or decrease in ZO and XO , a new cable length, LN, may be calculated. If LN is larger than LO then the tension increases; if it is smaller then the tension decreases. The difference in length (LN-LO) is L and this is multiplied times the spring constant, SC , which is studied for several different values later on. The new tension force now takes the form:

$$
\mathrm{T}=1940+\mathrm{L} \cdot \mathrm{SC}
$$



THE NEW TENSION FORCE FORMULATION
Figure 17

The value 1940 is the steady-state tension value that the model settled to in the earlier parameter studies; the angle FSS is the steadystate value of FE . This new tension force was implemented into the buoy model. The buoy will now be initiated at a depth of zero feet; this is possible because no heaving force or surface effect is taken into account in the model; other minor changes are made to take this new initial condition into account. The new computerized model program is titled "STUDY OF $T=1940+\mathrm{L} * S C$ ". The body angle is now being initiated at 4.8 degrees and the initial fin angle is 0.0 degrees; these two values are also the steady-state values for these angles found earlier. The next step in the formulation of this tension force was to find an appropriate value for the spring constant. The program was run with the three sets of coefficients and the new initial conditions, while SC was varied from 2000. to 0.0 in steps of 250 . The two sets of coefficients with $\mathrm{ADE}=1.2$ were stable for all values down to $\mathrm{SC}=250$, while the set with $\mathrm{ADE}=2.5$ was stable down to $S C=1750$. This third set $(A D E=2.5, A D R=0.30$, and $A P R=3.0)$ was dropped from further investigation since this was believed to be an unrealistic value for the spring constant.
C. STUDY OF INITAL CONDITION PERTURBATIONS

This final form of the model with the new tension force formulation was next examined to see how stable the model was under various initial condition perturbations. It was decided to perturb the initial vertical velocity, the body angle of attack, and the fin angle of attack. Three different spring constants were used with the two remaining sets of
coefficients; these were $\mathrm{SC}=500, \mathrm{SC}=1000$, and $\mathrm{SC}=1500$. It was also decided that two different values for the approximation of the dynamic length of the cable would be used; these were $\mathrm{ZO}=100$ and $\mathrm{ZO}=50$. ( ZO simply because it was used to initiate the model rather than LO). The final results were tabulated, and are shown in Table $V$; the initial angles of attack (AlI and FAI) are in degrees, and the initial vertical velocity (Z1I) is in $\mathrm{ft} / \mathrm{sec}$.

The responses of the buoy model to some of these initial condition perturbations are shown in Appendix A. The twenty-four plots include all the maximum perturbations listed for $S C=500$ and $S C=1500$. These plots do not really tell much about the buoy model. They do show that the buoy will settle to steady state fairly rapidly, and that the buoy will not deviate very far from its desired depth. Plots with the same pair of coefficients (ADE and ADR), with the same dynamic length of cable, and with the same type of inital condition perturbation, but with different values of spring constant, seem to be very similar. The responses with the larger spring constant $(S C=1500)$ seem to settle more quickly, even though they are oscillating through a wider range of depths. All the responses to the fin angle perturbations are remarkably similar. This might be because the fin is quickly driven back to the desired stability position; and since all the fin angle perturbations are the same pair of values, this similarity in responses is not surprising. The perturbing of the initial conditions for the new tension force realization was the point at which this study ended.

| $\mathrm{ADE}=1.2$ | $\mathrm{ADE}=1.2$ | $\mathrm{ADE}=1.2$ | $\mathrm{ADE}=1.2$ |
| :--- | :--- | :--- | :--- |
| $\mathrm{ADR}=0.07$ | $\mathrm{ADR}=0.3$ | $\mathrm{ADR}=0.07$ | $\mathrm{ADR}=3.0$ |
| $\mathrm{APR}=3$. | $\mathrm{APR}=3$. | $\mathrm{APR}=3$. | $\mathrm{APR}=3$. |

$S C=500$

| AlI | $-2.1 \rightarrow+0.9$ | $-1.8 \rightarrow+0.9$ | $-1.7 \rightarrow+1$. | $-1.5 \rightarrow+0.9$ |
| :--- | :--- | :--- | :--- | :--- |
| AlI | $+2.5 \rightarrow+8.0$ | $+3.0 \rightarrow+7.5$ | $+2.0 \rightarrow+7$. | $+2.5 \rightarrow+6.5$ |
| FAI | $-17.5 \rightarrow+15.0$ | $-17.5 \rightarrow+15.0$ | $-17.5 \rightarrow+15.0$ | $-17.5 \rightarrow+15.0$ |

$S C=1000$

| ZlI | $-4.8 \rightarrow+2.8$ | $-2.9 \rightarrow+2.6$ | $-4.3 \rightarrow+2.5$ | $-2.8 \rightarrow+2.2$ |
| :--- | ---: | ---: | ---: | ---: |
| AlI | $-0.5 \rightarrow+9.5$ | $0.0 \rightarrow+9.0$ | $-0.5 \rightarrow+8.0$ | $0.0 \rightarrow+8.0$ |
| FAI | $-17.5 \rightarrow 15.0$ | $-17.5 \rightarrow+15.0$ | $-17.5 \rightarrow+15.0$ | $-17.5 \rightarrow+15.0$ |

$S C=1500$

| ZlI | $-4.8 \rightarrow+3.1$ | $-2.7 \rightarrow+3.3$ | $-4.8 \rightarrow+3.0$ | $-2.7 \rightarrow+2.9$ |
| :--- | :--- | :--- | :--- | :--- |
| AlI | $-2.0 \rightarrow+10.0$ | $-1.0 \rightarrow+9.5$ | $-1.5 \rightarrow+9.0$ | $-1.0 \rightarrow+8.5$ |
| FAI | $-17.5 \rightarrow+15.0$ | $-17.5 \rightarrow+15.0$ | $-17.5 \rightarrow+15.0$ | $-17.5 \rightarrow+15.0$ |

STABILITY RANGE OF INITIAL CONDITIONS

Table V

## V. CONCLUSIONS AND RECOMMENDATIONS

The study presented here is only a beginning to this problem. There are many facets and additions to this problem that remain to be studied. First of all, the dynamic characteristics and wave-following capabilities of this model were never investigated. Other additions could be made to the model to take into account the heaving forces generated by the waves and the free-surface effect which will definitely affect the motion and capabilities of the buoy. There remains much to be studied in the area of the dynamic cable and the coupling of the tension force to the buoy model. Other tension force realizations might be tried with several spring constants in a single cable length; or the cable might be approximated by a series of straight lengths, each with its own spring constant. The possibilities of adding a damping coefficient into the tension force realization need to be investigated; the possibilities are almost endles. Other methods of simulating the buoy need to be tried, especially that of using a hybrid computer (an analog computer integrated with a digital computer); hybrid computers are more suited to this type of real-time simulation work. A dynamic model that allowed for all the possible dynamic forces and all the possible variations in buoys and cables, would be invaluable in the design and engineering of towed buoys.

## APPENDIX A

## RESPONSES OF THE BUOY TO INITIAL CONDITION PERTURBATIONS

The following 24 plots are the responses of the buoy to the maximum initial condition perturbations listed in Table $V$ for $S C=500$ and $\mathrm{SC}=1500$. The plots are of depth versus time. The x scale for all the plots is $5 \mathrm{sec} / \mathrm{inch}$; and the y scales are in ft/inch.






















|  |  |  |  |  |  |  | ADE $=1.2$ <br> ADR 0.3 <br> APR=3.0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.06 |  |  |  |  |  |  |  |




## APPENDIX B

SYMBOLS AND ABBREVIATIONS USED IN THE COMPUTER PROGRAMS

21 Z2

FV summation of vertical forces
Fl summation of the vertical forces less the vertical tension component

Xl horizontal velocity in ft/sec. (initial condition XlI)
horizontal acceleration in $\mathrm{ft} / \mathrm{sec}^{2}$
FH summation of the horizontal forces
F2 summation of the horizontal forces less the horizontal tension force

FF the absolute value of Fl
FG the absolute value of F 2

FE the angle with which the tension force acts on the body in radians

SFE the sine of FE
CFE the cosine of $F E$
TSF the vertical component of the tension force
TCF the horizontal component of the tension force
C horizontal velocity in knots
VAN the angle of the velocity vector relative to the horizontal in radians
vertical position in ft . (initial condition ZT )
vertical velocity in $\mathrm{ft} / \mathrm{sec}$. (initial condition ZlI )
vertical acceleration in $\mathrm{ft} / \mathrm{sec}^{2}$
summation of vertical forces
summation of the vertical forces less the vertical tension
horizontal velocity in $\mathrm{ft} / \mathrm{sec}$. (initial condition X1I)
summation of the horizontal forces
summation of the horizontal forces less the horizontal
tension force
the absolute value of Fl
the absolute value of F2
the angle with which the tension force acts on the body
madians
the cosine of FE
the horizontal component of the tension force
horizontal velocity inknots
VAN the angle of the velocity vector relative to the horizontal
in radians
2 the magnitude of the velocity vector squared

Al the body angle of attack relative to the horizontal in radians (initial condition AlI)

Yl the body pitch rate about the $Y$ axis in radians/sec (initial condition YlI)

Y2 angular acceleration of the body about the $Y$ axis in radians/second ${ }^{2}$

YM summation of the moments on the body about the Yaxis
AR the body angle of attack relative to the velocity vector in radians

AR in degrees
FA the fin angle of attack relative to the horizontal in radians (initial condition FAI)

FB used in the fin servo loop
FC used in the fin servo loop
FD summation of the inputs and the feedback in the fin servo loop

BR the fin angle of attack relative to the velocity vector in radians

B $\quad B R$ in degrees
CL coefficient of lift of the body
CD coefficient of drag of the body
CM coefficient of moment of the body
T the tension force
CV the cosine of VAN
SV the sine of VAN
ZAB the absolute value of $z$
$A D$ the antenna drag
BL the body lift
BD the body drag
YCM the moment on the body about the Y axis
YADS the moment on the body due to the vertical component of
the antenna drag
YADC the moment on the body due to the horizontal component
of the antenna drag
ZD

TFS
the tangent of FSS

XO the horizontal approximation to the dynamic length of the cable; see Fig. 17

ZN the new computed value of ZO
XR the relative movement out of steady state in the X direction

X the change in position out of the steady state in the X direction

XN the new computed value of XO
LN the new computed length of the dynamic length of the cable

L the change in length of the cable
FE as stated previously, but computed differently

＊＊ $Z=I N T G R L(Z I, Z 1)$
$Z 1=I N T G R L(Z I I, Z 2)$
$Z 2=F V / 70$.
$F V=F 1+T S F$
$F I=-B L * C V-B D * S V-A D * S V$
$H C R I Z O N T A L$ FORCE，ACCELERATION，AND VELOCITY
$\times 1=I N T G R L(\times 1 I, \times 2)$
$\times 2=F H / 70$.
$F H=F 2+T C F$
$F 2=B L * S V-B D * C V-A D * C V$ $Z=I N T G R L(Z I, Z 1)$
$Z 1=I N T G R L(Z I I, Z 2)$
$Z 2=F V / 70$ ．
$F V=F 1+T S F$
$F I=-B L * C V-B D * S V-A D * S V$
$H C R I Z O N T A L$ FORCE，ACCELERATION，AND VELOCITY
$\times I=I N T G R L(\times 1 I, \times 2)$
$\times 2=F H / 70 *$
$F H=F 2+T C F$
$F 2=B L * S V-B D * C V-A D * C V$
 $Z=I N T G R L(Z I, Z 1)$
$21=I N T G R L(Z I I, Z 2)$
$Z 2=F V 170$ ．
$F V=F 1+T S F$
$F I=-B L * C V-B D * S V-A D * S V$
$H C R I Z O N T A L$ FORCE，ACCELERATION，AND VELOCITY
$\times 1=I N T G R L(\times 1 I, \times 2)$
$\times 2=F H / 70$
$F H=F 2+T C F$
$F 2=B L * S V-B D * C V-A D * C V$ $Z=I N T G R L(z I, Z 1)$
$Z 1=I N T G R L(Z I I, Z 2)$
$Z 2=F V / 70$.
$F V=F 1+T S F$
$F 1=-B L * C V-B D * S V-A D * S V$
$H C R I Z O N T A L$ FORCE，ACCELERATION，AND VELOCITY
$\times 1=I N T G R L(\times 1 I, \times 2)$
$\times 2=F H / 70$ O．
$F H=F 2+T C F$
$F 2=B L * S V-B D * C V-A D * C V$
CALCULATION OF THE aNGLE fe
F，FGI
AND POSITION
ACCELERATION，
 ORIZONTAL AND VERTICAL COMPONENTS DF TFNSIUN
ームムい
 N $F F=A B S(F 1)$
angle and magnitude squared
21＊＊2 $-$
－×zマ




－
ANGULAR MOMENT，ACCELERATION，VELOCITY，AND POSITION ABOUT Y AXIS
$A 1=I N T G R L(A 1 I, Y 1)$
$Y 1=1$ NTGRL
$Y$ YII，Y2）
$Y M=Y M / 975$ S
FIN SERVO LDOP

COMPUTATION OF ANGLES OF ATtACK RELATIVE TO THE VELOCITY VECTOR
 CALLING THE BODY COEFFICIENTS AND TENSION FORCE OUT OF THE SUBROUTINE
COMP CM， $\mathrm{C}=\mathrm{T}$ INK（A，B，C）
OMPUTING ANTENNA DRAG CALLING THE BODY COEFFICIENTS AND TENSION FORCE OUT OF THE SUBROUTINE
CL，CDCM，$=$ INK IA，B，C）

ONG


$A D=2 A B * \cdot 146 * V 2$ MOMENT
 －5

＊
＊
＊＊
来事
率
率必
＊＊＊PROGRAM CONTROL CARDS
RINT，1， $2,21, F V, \times 1, F H, T, A, B$
AN ADDIT IONAL SET OF PARAMETERS TO BE STUDIED
NOT

|  |  | H | $\cdots 000$ | －40れてく | InN |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $N$ | レ゚レ゚ーロ＊＊ | $00 \cdot 1 N 0$ | 1＊1＊1＊ | 00 |
|  | ＋ | $0 \cdot+\sim-N$ | ON－＋＋ | －00－0－ | N1 |
|  | N | $1 N+0000$ | $10+1000$ | －0N000 | － 0 |
|  | ＊ | $0+* m * 1$ | －ナめーシN | の＊がち＊ | $1 \sim$ |
|  | ＊ | Noina＊ | NO＊－600 |  | mo＊ |
|  | ＜ | m＊OU4t＋ | NMO000－4 | ＋ 0 － | ＊ |
|  | ＊ | －＊ 1 － $\mathrm{H}_{\text {\％}}$ | NOONI | NHN＊ | ＊ $0^{0}$ |
|  | 5 | －40－min＊ | N．100\％ | mnonom | くの＊ |
|  | $\bigcirc$ | O＊ 0 －－ | N＋ロN』 | MーMーい！na | ＊ 0 |
|  | 1 | aganiol | N10．NE！ | －1－1－1 | O－4 |
|  | 0 | － 0 O 1 oun | ONMNN＋ | जGーロNO | －＊＊ |
|  | $\infty$ | 1 Nu－áo | －＊1 小f | $+\infty 1++a$ | $\sim \infty$ |
|  | ） | の－0＊－－ | ＋＊Oいな＊ | NONON－ | －10 |
|  |  | ＋ H m＊a－1 | 100か＊＊ | 长 1 \＃$m$＊ 0 | $\cdots<1$ |
|  |  | NTO＜a＋0 | N N＊ 000 | ＊ 0 ＊ | －＊0 |
|  | $m<$ | 4＊ $0^{*}$＊ 0 Nm | ＊ | cmostcor | HON |
|  | O＊ | ＊ 10000 N | － 0 －${ }^{\text {a }}$ |  | mon |
|  | － | －くN＋0＊＊－ | $\infty 11 \sim \infty$ | $0+\infty 000$ | man |
|  |  |  | ＊ 0 ＋0） 1 | － $0 \cdot \cdots$ | － 0 ＋ |
|  | m1 | $1+\infty 00+\ldots m$ | LTM100 | 1ヵ1－1． | $\rightarrow \mathrm{mm}$ |
|  |  | －0．1法r－1 | ONOOIL | $01010+$ | $+-1$ |
|  | $<\infty$ | O1mon＊om | $1-1 \sim 0$ |  | Nunin |
|  | ＊ | －0＋10＊ 1 | －00－50 | に，○＊＋＊ |  |
|  | N－ | －ainonaca | －TanNo | ○＊${ }^{\text {\％＊}}$－ | ＊$+\infty$ |
|  |  |  | comone | No00000 | ＜－1 |
|  |  | Ln＊mr－oun | monoron | －＊ 0 ＊ | ＋＋＋ |
|  | C－ |  | N－NOON | サーナーがN | OO＊ |
|  | M－ | －10＊$-10 \%$ | ntm 00 | ナームーい | $00 \%$ |
|  |  | －000NOOa | $a+\sigma \cdot N=1$ | －1－1－1 | N•《 |
|  | N0 | oino ithan | のmNJ•1 | NOーロN0 | －10\％ |
|  | 01 | 1－1 ominoo＊ | ＊ 1 －${ }^{\text {－}}$ | $+01 \sim+a$ | MNN |
|  | N | 4NO－＊ | tooml＊ | －1－000N | －No |
|  | N＊ | ＊＋－＋＋0 100 | $101 \%$－ | ＊- ＊ 0 ＊ | $1-1$ |
|  | O＊ | $4<m \infty<5+0$ | $\infty 1-4+\infty$ | comrar | $10+0$ |
|  |  |  | ＊ 0 ＋ | ONOMOO | $\infty$ ¢ |
|  | N＊ | mo＊0ヵ＊ | M上く＊＊ $0^{*}$ | 101min | － |
|  | $N+$ N | －Om40－ON＊ | Om＊ 0 ＊ 0 | corcmoin | －\＄ |
|  | －0 | 1－4＊ 1 000＊ | $10-0<1$ | a．s．0． | ＋ $4 m$ |
|  |  | On－0＋10e | －m小1＊0 | NJMNMー | 4 |
|  |  | $0 \cdot-$－ndo | moto－s | $0 N 10+m 1$ | －ms |
|  |  | －1N1极 | Mr＊ 0 ON | AtmJmt | 00 |
|  | No | OV100＊－NT | $\infty \bigcirc \infty$ |  | C1N |
|  | inis | のサ小 | じ・＊の＊aか | Or＊＊＊ | $00+$ |
|  | in |  | $\rightarrow 0 \infty$－ | ーかNの・か | anm |
| Uミ | No | に！ | n＋00n | －＊＊－＋ | N」＊ |
|  | $\rightarrow+0$ | O＜minhr＊o | rmioor | NOMO1－ | －0， |
|  | － | 人•N•1－4＜n | －000－4． | $1-+-\infty$ | ナNく |
|  | － | －mNm－0 ott | NIN•1NO | N1N10 | －$\infty$＊ |
|  |  | $\cdots+0 \cdot+N m \infty N$ | $1000+$ | ＋00001 | Nino |
|  | N1 | ininnaloo | N＝O1 mN | $1-100 N$ | ono |
| $\underline{L}$ | $1+m$ | nool＊mil | OmNNO＊ | Noroantr | $1 \cdot \uparrow$ |
|  | $\infty$ | \％rar hoo． | $10 \times$ m＊ | かのOJNo | Orn |
|  | － | － $0^{\circ} \times 1$ 1－10 | $0 \infty \mathrm{~m}$－$-\infty$（ | NTOUNNO | $0+m$ |
|  | $0<$ | （No＊nool | NJOCOL |  | Noth |
|  | Ox | NO\＆On ORN | UN．\＃MNT | － | $0 * 1$ |
| Wの | $\checkmark$ | へmt＊－Lna＊ | － | －OTomaran | $0 \% 0$ |
|  | DC |  | J－10．1 | $m \cdot 0 \cdot 0$. | 0＜u |
| E | $=$ | OO－1 100 m | NnN1 yo |  | い＊ |
|  | Un | － 1 trinot | N1＊O100 | －＋＋＋－ 1 |  |
| $\infty$ | 0 | UNO－HA＋＋N | ーーシ ールへ | amrim Im | 00 |
|  | O－ | ＋＋－－N－ | $\bullet+\infty m * 0$ |  | N |
| $\times$ | － 0 | Otmome 11 | umm＊ | －+ ＋＋＋ | － |
| $\infty<$ | 11 |  | リれトすが， | リ以さがの | 1 |
| ）u | －10 |  | $\rightarrow * \cup \infty+$－ | N＊＊＊＊ | 110 |
|  | U0 | U40mot\＃N | いがトひの。 | ＋uató | 上r |
|  |  | －nmy or | $\rightarrow$ Nmy | －nNmun |  |


| TITLE | STUDY OF $T=$ 1940. $+\mathrm{L} * \mathrm{SC}$ |  |
| :---: | :---: | :---: |
| *** | PARAMETERS TO BE STUDIED AND INITIAL CONDITIONS |  |
| PARAM | $A D E=1.2, A D R=.30, A P R=3 ., F S S=1.3307, Z 0=100 ., S C=500$. |  |
| INCON |  |  |
| *** | VERTICAL FCRCE, ACCELERATION, VELOCITY, AND POSITION | POSITION |
|  | $\begin{aligned} & Z=I N T G R L(Z I, Z 1) \\ & Z 1=I N T G R L(Z I I, Z 2) \\ & Z 2=F V / 70 \\ & F V=F I+T S F \\ & F I=-B L * C V-B D * S V-A D * S V \end{aligned}$ |  |
| *** | HCRILONTAL FURCE, ACCELERATION, AND VELOCITY |  |
|  | $\begin{aligned} & \times 1=I N T G R L(X 1 I, \times 2) \\ & \times 2=F H / 7 C \\ & F H=F 2+T C F \\ & F 2=B L * S V-B D * C V-A D * C V \end{aligned}$ |  |
| *** | $\begin{aligned} & \text { VELOCITY ANGLE AND MAGNITUDE SQUARED } \\ & \text { VAN ATAN2 } Z 1, X 11 \\ & \text { VZ }=Z 1 * * 2+X I * * 2 \\ & \text { SV }=\text { SIN(VAN) } \\ & C V=C O S(V A N) \end{aligned}$ |  |
| *** | ANGULAR MOMENT, ACCELERATION, VELOCITY, AND POSITION ABOUT Y | Y AXIS |
|  | $\begin{aligned} & A 1=I N T G R L(A 1 I, Y 1) \\ & Y 1=I N T G R L(Y I I, Y 2) \\ & Y 2=Y M / 975 \\ & Y M=Y C M+Y A D C-Y A D S \end{aligned}$ |  |
| *** | FIN SERVO LOOP |  |
|  | $\begin{aligned} & F A=I N T G R L(F A I, F B) \\ & F B=I N T G R L(F B I, F C) \\ & F C=F D * 1 C O O \\ & F D=-.0443 * F B-F A+D E+D R+Y P R \end{aligned}$ |  |
| * | CCMPUTATICN OF ANGLES OF ATTACK RELATIVE TO THE VELOCITY V |  |



CALCULATION OF the angle fe
FSSI $\qquad$
TENSION
LOOP


1. David Taylor Model Basin Report 2531, A Fortran IV Program for the Calculation of the Equilibrium Configuration of a Flexible Cable in a Uniform Stream, E. Cuthill, February 1968.
2. David Taylor Model Basin Report 687, Tables for Computing the Equilibrium Configuration of a Flexible Cable in a Uniform Stream, L. Pode, March 1951.
3. Naval Research Establishment, Dartmouth, Nova Scotia, Steady State Theory of Towing Cables, M. C. Eames, October, 1966.
4. Naval Ship Research and Development Center Report 2510, Standard Equations of Motion for Submarine Simulation, M. Gertler and G. R. Hagen, June 1967.
5. Naval Ship Research and Development Center, Preliminary WindTunnel Data for a Submarine Towed Communications (STC) Body, May 1968.
6. Naval Ship Research and Development Center, Sea Surface Followage by Towed Body, H. Soulant, March 1969.
7. Schram, J. W., A Three-Dimensional Analysis of a Towed System, Ph. D. Dissertation, Rutgers University, 1968.
8. USNIMDL Report 219, Dynamics of Towed Underwater Vehicles, A. G. Strandhagen, 1963.
9. Whicker, L. F., The Oscillatory Motion of Cable-Towed Bodies, Ph. D. Dissertation, University of California, Berkeley, 1957.

No. Copies

1. Defense Documentation Center ..... 20
Cameron Station
Alexandria, Virginia 22314
2. Library, Code 0212 ..... 2
Naval Postgraduate School
Monterey, Ca lifornia 93940
3. Commander, Naval Ordnance Systems Command ..... 1
Department of the Navy
Washington, D. C. 20360
4. Dr. George J. Thaler, Code 52 Tr ..... 5
Department of Electrical Engineering Naval Postgraduate School
Monterey, California 93940
5. LT(jg) John F. Bell, USN ..... 1
Apt. 911, Warwick Towers
1131 University Blvd. West
Silver Spring, Maryland 20902
6. Mr. Tom Gibbons, Code 549 ..... 1
Naval Ship Research and Development Center Washington, D. C. 20007
7. Dr. Lloyd F. Bell, Senior Scientist ..... 1
Tetra Tech, Inc.
1911 Fort Meyer Dr.
Arlington, Virginia 22209
8. Mr. Ben G. Conklin ..... 1
2060 Alewa Dr.
Honolulu, Hawaii 96817

# Naval Postgraduate School <br> Monterey, California 93940 

Unclassified
2b. GROUP

3 REPORT TITLE
Development and Study of A Two-Dimensional Dynamic Model of a Towed Buoy

4 DESCRIPTIVE NOTES (TYpo of roport and.inclusive dates)
Master's Thesis; December 1969
5. AUTHORISI (First name, middle inllial. leat name)

John Franklin Bell

| 6. REPORT DATE <br> December 1969 | 7a. TOTAL NO. OFPAGES 85 <br> 7b. NO. OF REFS 9 |
| :---: | :---: |
| 6a. CONTRACT OR GRANT NO <br> b. PROJECTNO | Qa. ORIGINATOR'S REPORT NUMBER(S) |
| c. | 9b. OTHER REPORT NO(5) (Any other numbera that may bo eesigned this report) |
| d. |  |

10. DISTRIBUTION STATEMENT

This document has been approved for public release and sale; its distribution is unlimited.
11. SUPPLEMENTARYNOTES
2. SPONSORING MILITARY ACTIVITY

Naval Postgraduate School
Monterey, California 93940
13 ABSTRACT

A two-dimensional dynamic model of a towed buoy designed to remain submerged just below the water surface is developed. The study includes the model development, the analysis of certain model parameters, the development of a tension force, and the range of stability of the model to initial condition perturbations.

thes B36 175
Development and study of a two-dimension


32768001034804
DUDLEY KNOX LIBRARY


[^0]:    ${ }^{1}$ Soulant, H. Sea Surface Followage by Towed Buoy (Preliminary Study), p. 11, March 1969.

