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EXPERIMENTAL INVESTIGATION OF THE PARAMETERS EFFECTING THE PRESSURE PROFILES OF FINITE AMPLITUDE STANDING WAVES IN AIR-FILLED TUBES

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ABSTRACT

The finite-amplitude standing wave behavior of air within several tubes was experimentally investigated near standard conditions of temperature and pressure for frequencies below 3000 cps. The sound pressure levels obtained were large enough to generate shock waves of 0.10 atmospheres. The air in the tube was driven by means of a vibrating piston. The motion of the piston was measured by an accelerometer while the pressure at the rigid end was measured with a condenser microphone. It was found that the wave forms for different tubes would be the same if the following quantities were made equal: 1) the ratio of the driver acceleration to the acoustic attenuation constant, and 2) the phase angle between the acceleration and the pressure at infinitesimal amplitudes. Values of the attenuation constant were determined by several different methods including measuring the decay of pressure after clamping the driver piston and by determining the ratio of acceleration to pressure. Observed attenuation constants were of the order of 10^{-4} cm^{-1}. The resonant frequency for infinitesimal amplitude was observed to be the frequency maximizing both the average rectified pressure and the shock strength only for the stronger shocks. However, for weak shocks the shock strength became a maximum at lower and lower frequencies as the pressure decreased. This tendency, as indicated by the amount of second harmonic distortion, continued in the weak finite amplitude region.

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1. Introduction.

The purpose of this research is to investigate the parameters that effect the behavior of high intensity standing waves in air contained in tubes. In many practical instances the sound pressure levels within cavities are found to be of such large magnitude that infinitesimal amplitude theory no longer applies.

The investigations reported here were carried out in air-filled tubes each terminated with a rigid boundary at one end and driven by a vibrating piston at the other.

The acoustic pressure at the fixed end (x = L where L is the tube length) predicated by the theory of infinitesimal amplitude waves with no dissipation is given by the equation:

$$P = \frac{A\rho_0 \cos \omega t}{k \sin kL}$$

Where
A = acceleration amplitude of the driver piston at x = 0
k = wave number ω/c
c = local speed of sound
ρ_0 = density of the gas

The resonant frequencies are given by $kL = n\pi$, and at resonance the predicated pressures are infinite.

If dissipative terms are introduced into the acoustic wave equation, then the pressure at x = L becomes:

$$P = \frac{A\rho_0}{k}\left[\frac{1}{(\alpha L \cos kL)^2 + \sin^2 kL}\right]^{\frac{1}{2}} \cos \omega t$$

Where α is the attenuation constant depending on viscosity, heat conduction, molecular relaxation of the gas, and the wall effects.

This solution is still not complete: although it eliminates the singularities at resonance, it is still a linear theory predicting continuous pressure-time behavior near resonance. This is an inadequate description since experimenters have observed that shock waves are present in the tube near resonant frequencies.

In the region between infinitesimal amplitude waves and shock waves several things are happening. As the amplitude increases the sinusoidal pressure wave begins to distort as a result of increased inportance of non-linear effects in the gas. These non-linearities generate harmonics of the fundamental which lead to an apparent distortion of the observed waveform. As the strength of the fundamental is increased, the distortion grows until a shock wave finally developes.

Most investigators of finite amplitude effects have confined their studies to traveling waves (2, 5, 8, 19, 20). There have, however, been a few investigators of finite amplitude standing waves (1, 4, 17).

In this study a tube terminated at one end by a rigid piston, and driven at the other by an oscillating piston was used to investigate finite amplitude standing waves. The use of a piston as a sound source in a tube is no new concept (11, 12, 18). How-

ever, most pistons in the past have been driven by some sort of reciprocating mechanical driver. The space program has created the need for vibration exciters to test missile parts, and out of this need has come the machine used in this experiment as a piston driver. Reichwein (13) first used a driver of this type with a capability of 16 g acceleration to drive the piston in a tube. The tube was terminated by a plug containing a condenser microphone to monitor the pressure at the terminal end. He investigated threshold behavior of periodic shock waves in resonating gas columns and reached the conclusion that in a given tube the waveform was a function only of the acceleration of the driver piston and the "distance" off resonance as measured by the phase angle between the shock front and the piston acceleration.

Sanders[1] later observed that the waveform depended markedly on the "tightness" of the tube enclosure. He found that if there were leaks at either end of the tube shock waves would not develop until the acceleration was much higher than for a "tight" tube, and that the waveform in a rectangular cross section tube was different from that in a round cross section tube. It was

[1]Sanders, J. V., private communication.

postulated that the tube attenuation was an additional parameter affecting shock wave development.

The attenuation of infinitesimal sound in a tube has been treated by a number of people both theoretically and experimentally (6, 7, 13, 16, 21). It turns out that the attenuation constant in a tube is proportional to the square root of frequency due to the wall effects, plus other terms attributable to free space attenuation, etc. In this experiment tube effects are about four orders of magnitude greater than the other effects. All of the investigators have been able to verify theory to reasonable accuracy, but all have measured an attenuation constant higher than the theoretical one. Different explanations have been advanced for the discrepancies, but none have been proven.

The experimental set-up used in the present experiment is essentially the same as that used by Reichwein. However, the 16 g vibration exciter was replaced by one of 50 g capability. The original experimental set-up was further modified by the addition of a water jacket on the tube to stabilize tube temperature. This arrangement enabled the experimenters to study standing waves from the infinitesimal amplitude region to the finite amplitude shock region without any equipment changes.

2. Equipment Description

Oscillator The Hewlett-Packard Model 205 AG Audio Signal

[illegible]

Generator was found to be a very stable oscillator. In the frequency range of 20-20,000 cps there is a drift of two per cent or less, and if allowed to warm up for thirty minutes the drift was less than 0.01 cycle in the frequency range of interest.

Piston driver The MB Model C10 VB Vibration Exciter was selected for the wide range of accelerations attainable. The electrically driven piston is preferable to a mechanical or electro-mechanical device because the acceleration of the electrical driver can easily be controlled and there is enough power to drive a 0.5 lb load with 50 g acceleration or a 30 lb load with 25 g acceleration. The dynamic range for the loads imposed was from a maximum of 50 g acceleration to a minimum of 0.5 g where 60 cps noise made the signal to noise ratio unfavorable.

Microphone The Brüel and Kjær Condenser Microphone Type 4136 was used for the pressure pickup. Since the microphone was embedded in the rigid end of the tube it was necessary to use a microphone of small diameter so as to produce as small perturbation of the boundary conditions as possible. For this reason the 0.25 in microphone was selected. In addition, for the investigation of finite amplitude waves a microphone was needed with a relatively short rise time. The rise time of this microphone is 4,5 micro seconds and the frequency response is essentially flat from 25 cps to 80,000 cps.

Frequency counter Hewlett-Packard Electronic Counter Model 522B was used for frequency determination. The time interval unit on a ten period average was used for frequencies below 1000 cps and the frequency unit on a ten second count was used for frequencies above 1000 cps so that a 0.1 cps accuracy in frequency determination could be obtained.

Accelerometer Endevco Model 2215 Accelerometer was bolted to the driver face plate. This accelometer had been cross calibrated by Sanders yielding a sensitivity of 4.82 rms mv/peak g.

Oscilloscope A Tektronix Type 565 Dual Beam-Dual Time Base Oscilloscope was used with plug in units 2A60 and 3A72. With these plug in units three traces could be viewed simultaneously. The dual time base permitted the expansion of any portion of the waveform, and in particular the shock front for detailed inspection. The delay feature incorporated in this scope also allowed phase between two traces to be read directly from a dial.

Tube enclosure Several tube arrangements were used to investigate a set of different boundary conditions. The basic arrangement is shown in Fig. 1-1 and is considered to be the most practical set-up since the tube length can be readily varied. The tube was of aluminium, length 5 ft. 11 in., wall thickness of 0.20 in., and an inside diameter of 1.75 in. Early in the

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experiment it was found that driving the piston at large amplitudes caused the temperature of the air in the tube to increase thereby changing the resonance frequency. Therefore the tube was fitted with a double wall so that water could be circulated between the walls to stabilize the temperature. This water-jacket kept the temperature in the tube constant at 14.5°C by absorbing the heat from the inside of the tube and also shielding the inside from the heat produced by the electronic equipment in the room.

The microphone was mounted in a movable piston eight inches long with its head flush with the face of the piston. The microphone piston and the driver piston were placed in the tube with "O" rings and vacuum grease to produce an air tight seal. The driver piston was connected to the face of the driver plate by means of a short threaded shaft.

3.1 Theory of Infinitesimal Amplitude Plane Waves in a Dissipative Fluid

The standard form of the linearized plane wave equation using particle displacement ξ as the acoustic variable is $\frac{\partial^2 \xi}{\partial t^2} = c^2 \frac{\partial^2 \xi}{\partial x^2}$. A solution of this equation is

$$\underset{\sim}{\xi} = \underset{\sim}{B}\, e^{j(\omega t - Kx)} + \underset{\sim}{D}\, e^{j(\omega t + Kx)}$$

where $\underset{\sim}{B}$ is the complex displacement amplitude of a plane wave of frequency ω and wave length constant K, traveling in the positive x direction with velocity c; and $\underset{\sim}{D}$ is the amplitude of a similar wave traveling in the negative x direction. The subscript tilde will be used to denote complex quantities.

This solution and the wave equation for which it is a solution are based on the assumption that there are no acoustical energy losses, either in the medium or at the boundaries of the medium. Appropriate mechanisms have been developed to deal with these losses, some of which will be discussed in section 3.2. Assuming that these losses exist and that they are a function of the distance traveled by the plane wave through the medium, it is possible to modify the plane wave equation to account for these losses. If the expression $K = \omega/c$ is modified to $\underset{\sim}{K} = \omega/\underset{\sim}{c}$ where $\underset{\sim}{c}$ is a complex velocity and $\underset{\sim}{K}$ is considered a complex wave length constant with real part K and imaginary part $-j\alpha$ then it can be shown that the plane wave equation may be re-written as $\frac{\partial^2 \xi}{\partial t^2} = \underset{\sim}{c} \frac{\partial^2 \xi}{\partial x^2}$

with a solution $\underset{\sim}{\xi} = \underset{\sim}{B} e^{j(\omega t - \underset{\sim}{k} x)} + \underset{\sim}{D} e^{j(\omega t + \underset{\sim}{k} x)}$

which may also be written $\underset{\sim}{\xi} = \underset{\sim}{B} e^{-\alpha x} e^{j(\omega t - k x)} + \underset{\sim}{D} e^{\alpha x} e^{j(\omega t + k x)}$

This solution is similar to that for the unattenuated wave except that the displacement amplitude is attenuated in accordance with the term $e^{\pm \alpha x}$. The acceleration of the particles for infinitesimal waves may be found by taking the second derivative of particle displacement with respect to time, $\underset{\sim}{a} = \frac{\partial^2 \underset{\sim}{\xi}}{\partial t^2} = -\omega^2 \underset{\sim}{\xi}$

The pressure of the wave can be expressed as

$$\underset{\sim}{p} = -\rho_0 \underset{\sim}{c}^2 \frac{\partial \underset{\sim}{\xi}}{\partial x}$$

$$= \frac{-\rho_0 \omega^2}{(k - j\alpha)^2} \left[-(jk + \alpha) \underset{\sim}{B} e^{-\alpha x} e^{j(\omega t - kx)} + (jk + \alpha) \underset{\sim}{D} e^{\alpha x} e^{j(\omega t + kx)} \right]$$

$$= \frac{j\rho_0 \omega^2}{k - j\alpha} \left[\underset{\sim}{B} e^{-\alpha x} e^{j(\omega t - kx)} - \underset{\sim}{D} e^{\alpha x} e^{j(\omega t + kx)} \right]$$

Let us now consider a plane wave of this form being propagated in a tube closed at one end and driven at the other by a piston moving with an acceleration $Ae^{j\omega t}$ where A is the real acceleration amplitude. We take the piston rest position to be at x = 0 and the rigid end to be at x = L.

Two boundary conditions are imposed: that the particles be stationary at x = L, and that they be moving with acceleration

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$Ae^{j\omega t}$ at x = 0.

At x = 0 $$A e^{j\omega t} = -\omega^2 \left[\underset{\sim}{B}\, e^{j\omega t} + \underset{\sim}{D}\, e^{j\omega t} \right]$$

At x = L $$0 = -\omega^2 \left[\underset{\sim}{B}\, e^{-\alpha L} e^{-jKL} + \underset{\sim}{D}\, e^{\alpha L} e^{jKL} \right] e^{j\omega t}$$

Solving for B and D in terms of A

$$\underset{\sim}{B} = \frac{-A\, e^{2\alpha L} e^{2jKL}}{\omega^2 \left(e^{2\alpha L} e^{2jKL} - 1 \right)}$$

$$\underset{\sim}{D} = \frac{A}{\omega^2 \left(e^{2\alpha L} e^{2jKL} - 1 \right)}$$

When these values of $\underset{\sim}{B}$ and $\underset{\sim}{D}$ are substituted in the pressure equation, pressure becomes

$$\underset{\sim}{p} = \frac{-jA\rho_0}{K - j\alpha} \left[\frac{e^{2\alpha L} e^{2jKL} e^{-\alpha x} e^{-jKx} + e^{\alpha x} e^{jKx}}{e^{2\alpha L} e^{2jKL} - 1} \right] e^{j\omega t}$$

Evaluating this expression at x = L

$$\underset{\sim}{p}_{x=L} = \frac{-jA\rho_0}{K(1 - j\frac{\alpha}{K})} \left[\frac{2\, e^{\alpha L} e^{jKL}}{e^{2\alpha L} e^{2jKL} - 1} \right] e^{j\omega t} = \frac{-jA\rho_0}{K(1 - j\frac{\alpha}{K})} \left[\frac{e^{j\omega t}}{\sinh \alpha L \cos KL + j \cosh \alpha L \sin KL} \right]$$

The amplitude of the pressure at the rigid end, $P \triangleq \left| \underset{\sim}{p} \right|_{x=L}$ is

$$P = \frac{A\rho_0}{K\left[1 + \left(\frac{\alpha}{K}\right)^2\right]^{\frac{1}{2}}} \left[\frac{1}{\sinh^2 \alpha L \cos^2 KL + \cosh^2 \alpha L \sin^2 KL} \right]^{\frac{1}{2}}$$

For the range of experimental values investigated α is of order 10^{-4}, K of order 10^{-2}, and L of order 10^{2}. Since $KL \approx 1$ and $\alpha L \ll KL$, the approximations can be made that $\sinh \alpha L = \alpha L$, $\cosh \alpha L = 1$. Using this and the fact that $\alpha/K \ll 1$,

$$P = \frac{\rho_0 A}{K} \left[\frac{1}{(\alpha L \cos KL)^2 + \sin^2 KL} \right]^{\frac{1}{2}}$$

This equation for P is correct to first order in αL. Finally $\tilde{p}$ may be expressed as $Pe^{j(\omega t+\theta)}$ where θ is

$$\theta = \frac{-\pi}{2} + \tan^{-1}\frac{\alpha}{K} - \tan^{-1}(\coth \alpha L \tan KL)$$

To investigate the acoustic behavior of the system as the frequency or length is varied, the derivative of P with respect to either variable is taken and set equal to zero.

For $P = \frac{\rho A}{K}\left[(\alpha L \cos KL)^2 + \sin^2 KL\right]^{-\frac{1}{2}}$

setting $\frac{\partial P}{\partial L} = 0$ leads to

$$\cos KL\left[-K(\alpha L)^2 \sin KL + \alpha^2 L \cos KL + K \sin KL\right] = 0$$

When cos KL = 0, P is a minimum. The requirement for cos KL = 0 is that $KL = (m-\frac{1}{2})\pi$ or $L = \frac{(m-\frac{1}{2})\pi}{K}$

$$P_{min} = \frac{\rho_0 A}{K} \quad or \quad \frac{\rho_0 A L}{(m-\frac{1}{2})\pi}$$

This leads to the conclusion that the same value of minimum pressure will be observed at lengths of the tube corresponding to $\frac{(m-\frac{1}{2})\pi}{K}$ where m is an integer greater than or equal to one. The maximum value of pressure occurs when $KL = \frac{(\alpha L)^2}{KL[1-(\alpha L)^2]}$, that is $KL = n\pi$ to first order in αL. The value of P_{max} is $\frac{\rho_0 A}{\alpha K L}$

Taking the partial derivative of P with respect to frequency and setting it equal to zero leads to a determination of the resonances and antiresonances of the system, but the result is more complex since the parameter α is also a function of

frequency but not of length.

Fig. 3-1 illustrates the dependence of P on length of the tube at a fixed frequency, and Fig. 3-2 illustrates the dependence of P on frequency at a fixed length. The frequency dependence of α is taken into consideration in Fig. 3.2.

3.2 Attenuation of Sound Waves in Tubes

Acoustic waves being transmitted in a medium must undergo losses of acoustic energy. These losses are of two types, losses due to dissipation in the medium itself and losses associated with conditions at the boundary of the medium. Losses in the medium may be viscous losses, heat losses and losses due to molecular exchange of energy. For dry air at standard conditions these losses in the medium may be represented by an attenuation coefficient α of the approximate value $2 \times 10^{-13} f^2$ where f is the frequency of the acoustic wave in cycles per second and the units of α are cm^{-1}. For the size of tube and frequency range dealt with in this experiment these losses in the medium are several orders of magnitude smaller than the losses due to boundary conditions and may be safely ignored.

Losses associated with boundary conditions may arise from the direct absorption of acoustic energy by the walls. The requirement of zero particle velocity at the walls leads to a

velocity gradient from the center to the walls and thus to dissipative forces which are related to the shear viscosity of the medium. In addition, since temperature is essentially constant at the walls, there is a tendency for heat to be conducted to the walls from the medium during a condensation and from walls to medium during rarefaction, resulting in a dissipation of acoustic energy.

The theory of attenuation in a tube of circular cross section has been developed by Helmholtz, Kirchoff and Lord Rayleigh [14]. A simplified exposition of this theory may be found in "Fundamentals of Acoustics" by Kinsler and Frey [9]. The attenuation in a tube due to boundary effects is shown to be $\alpha = \frac{1}{ac}\sqrt{\frac{\mu_e \omega}{2\rho}}$ where a is the radius of the tube, μ_e is an effective coefficient of shear viscosity, ω is radian frequency, c is free field speed of propagation in the medium, and ρ is density of the medium. The speed of propagation in a tube is slower than in an infinite medium and is given by the expression $c' = c\left(1 - \frac{1}{2a}\sqrt{\frac{2\mu_e}{\rho\omega}}\right)$. The use of an effective shear viscosity rather than an actual coefficient of shear viscosity is a technique developed by Kirchoff and later Rayleigh to take into account the effect of heat conduction to the walls. The relationship between μ_e and μ is $\mu_e = \mu\left[1 + \frac{(\gamma-1)K}{\gamma^{\frac{1}{2}} C_p \mu}\right]^2$ where k is the thermal

conductivity of the gas, γ is the ratio of specific heats, and c_p is the specific heat at constant pressure. Investigators have stated that in a tube of rectangular cross section the attenuation due to wall effects is directly proportional to the ratio of the perimeter to the cross sectional area, in place of the reciprocal of the radius in the case of the cylindrical tube [4,16].

The derivation of the theory for attenuation in a rectangular tube is complicated by the fact that it must be accomplished using rectangular coordinates, and an extra dimension must be considered which was not necessary in the cylindrical tube due to its circular symmetry. The simplest means of approach to the rectangular case is to consider the case of a plane wave being propagated between two parallel flat plates in a direction x parallel to the plates. Consider an element of volume between these plates of length dx in the direction x, of unit width in the direction parallel to the plates and perpendicular to the direction of propagation, and thickness dy in the direction perpendicular to the plates. This element, of volume dxdy, has exerted on it a force due to the gradient of pressure in the x direction of magnitude $-\frac{\partial p}{\partial x} dx dy$. In addition, due to the boundary condition of zero particle velocity at the walls there will be a particle velocity gradient in the y direction. Since the viscous force on a surface of area dx is $\mu dx \frac{\partial u}{\partial y}$

where μ is the coefficient of shear viscosity, the net force due to the viscous forces on the element of volume will be

$\frac{-\partial}{\partial y}\left[-\mu dx \frac{\partial u}{\partial y}\right] dy$. If these forces are equated to the mass of the medium contained in the volume element multiplied by the acceleration $\frac{\partial u}{\partial t}$, a partial differential equation in u is obtained as follows

$$\frac{-\partial p}{\partial x} = -\mu \frac{\partial^2 u}{\partial y^2} + \rho \frac{\partial u}{\partial t}$$

If the motion is assumed to be sinusoidal and of frequency ω then p may be expressed as $P e^{j(\omega t - Kx)}$ and u as $\underset{\sim}{U} e^{j(\omega t - Kx)}$ so that $\frac{\partial u}{\partial t}$ may be replaced by $j\omega u$.

Then $$-\frac{\partial P}{\partial x} = \left[j\rho\omega - \mu \frac{\partial^2}{\partial y^2}\right] \underset{\sim}{U}$$

or $$\frac{-1}{\mu} \frac{\partial P}{\partial x} = \left[\frac{\partial^2}{\partial y^2} + k^2\right] \underset{\sim}{U}$$

where $K^2 = \frac{-j\rho\omega}{\mu}$ and $K = (1-j)\sqrt{\frac{\rho\omega}{2\mu}}$

A solution of the equation is $\underset{\sim}{U} = \frac{1}{\mu K^2} \frac{\partial P}{\partial x} + \underset{\sim}{A}\, e^{jKy}$

Applying the boundary condition that $\underset{\sim}{U} = 0$ at $a/2$ and $-a/2$ where a is the distance between the plates

$$\underset{\sim}{A} = \frac{-1}{\mu K^2} \frac{\partial P}{\partial x} e^{-\frac{jKa}{2}}$$

$$\underset{\sim}{U} = \frac{1}{\mu K^2} \frac{\partial P}{\partial x} \left[1 - e^{j\left(Ky - \frac{Ka}{2}\right)}\right]$$

The average value of $\underset{\sim}{U}$ over the distance a, since $\underset{\sim}{U}$ is symmetrical about y = 0 may be found by integration as follows

$$\underset{\sim}{U} = \frac{2}{a}\int_0^{\frac{a}{2}} \underset{\sim}{U}\, dy = \frac{2}{a}\int_0^{\frac{a}{2}} \frac{1}{\mu K^2}\frac{\partial P}{\partial x}\left(1 - e^{j\left(Ky - \frac{Ka}{2}\right)}\right) dy$$

$$\overline{\underset{\sim}{U}} = \frac{1}{\mu K^2} \frac{\partial P}{\partial x} \left[1 - \frac{2}{jKa}\left(1 - e^{-\frac{jKa}{2}}\right)\right]$$

Since $K = (1-j)\sqrt{\frac{\rho\omega}{2\mu}}$

$$e^{-\frac{jKa}{2}} = e^{-\sqrt{\frac{\rho\omega}{2\mu}}\,a}\; e^{-j\sqrt{\frac{\rho\omega}{2\mu}}\,a}$$

For $a\sqrt{\frac{\rho\omega}{2\mu}} \gg 10$ which is the case for the experimental situation, $e^{-\frac{jKa}{2}} \ll 1$ and can be neglected.

$\bar{U}$ then is approximately $\frac{1}{\mu K^2}\frac{\partial P}{\partial X}\left(1-\frac{2}{jKa}\right)$

Solving for $\frac{-\partial P}{\partial X}$, $\frac{-\partial P}{\partial X} = -\mu K^2\left(1-\frac{2}{jKa}\right)^{-1}$

which is approximately

$$\frac{-\partial P}{\partial X} = -\mu K^2\left(1+\frac{2}{jKa}\right) \quad \text{or} \quad -\mu K^2\left(1-\frac{2j}{Ka}\right)$$

Since $K^2 = -\frac{j\rho\omega}{\mu}$ and $K = (1-j)\sqrt{\frac{\rho\omega}{2\mu}}$

$$-\frac{\partial P}{\partial X} = \left[j\rho\omega + \frac{\sqrt{2\mu\rho\omega}}{a}(1+j)\right]\bar{U}$$

From this point the derivation follows exactly that of the cylindrical tube. The reactance term $j\rho\omega$ is the normal reactance term associated with $\rho\frac{\partial u}{\partial t}$ and the effect of viscosity is to introduce an additional reactance term $j\rho\omega\left(\frac{1}{a}\sqrt{\frac{2\mu}{\rho\omega}}\right)$ so that the effective density of the viscous medium is $\rho' = \rho\left(1+\frac{1}{a}\sqrt{\frac{2\mu}{\rho\omega}}\right)$ The effect of this effective density on the medium leads to the expressions for c' and α previously given for the cylindrical tube. The conclusion is that the attenuation of a plane wave between two parallel plates is inversely proportional to the distance between the plates, just as it is inversely proportional to the

radius in the cylindrical tube. Generalizing to the case of a rectangular tube, it is apparent that the attenuation due to the two sets of parallel plates making up the walls should be additive if corner corrections are negligible. The attenuation should then be proportional to $\left(\frac{1}{a} + \frac{1}{b}\right)$ where a and be are the dimensions of the cross section of the tube. $\left(\frac{1}{a} + \frac{1}{b}\right)$ can be expressed as $\left(\frac{a+b}{ab}\right)$ which is one half the ratio of the perimeter to the cross sectional area. This relationship agrees with the statement previously made that attenuation is directly proportional to the ratio of perimeter to area. (Fig. 3-10) shows the comparison of α obtained using this theoretical development with the α actually measured in the rectangular tube.)

In the special case of standing waves in a cylindrical tube Parker [13] states that the attenuation due to the wall effects should be increased to take into consideration the losses at a rigid end. His relationship is $\alpha_T = \alpha_w \left(1 + R/L\right)$ where R is the tube radius and L is the tube length. This additional attenuation is of the order of three per cent for the tube size used in the experiment.

3.3 Investigation of Attenuation in Tubes

The investigation of attenuation in the infinitesimal amplitude region carried out in this study consisted of measuring attenuation by five methods suggested by the theory

previously developed. These methods were then checked for consistency and finally compared with the attenuation predicted by theory.

The first method of measuring attenuation involved measuring phase changes of pressure with respect to acceleration by varying frequency in the vicinity of resonance. In the expression

$$\tilde{p} = P e^{j(\omega t + \theta)}$$

where

$$\theta = -\frac{\pi}{2} + \tan^{-1}\frac{\alpha}{k} - \tan^{-1}(\coth \alpha L \tan kL)$$

θ represents the phase angle between the acceleration of the piston and the pressure at the rigid end of the tube. If the same simplifying assumptions made previously are applied to the equation for θ it may be solved for α with the result

$$\alpha = \frac{-\pi \Delta f}{f_o L \cot \theta}$$

where f_o is the frequency for which θ is $\frac{\pi}{2}$ and Δf is the change in frequency from f_o necessary to obtain some phase angle other than $\frac{\pi}{2}$. The method becomes more accurate as larger values of Δf are obtained, since Δf is a small quantity of the order of a few cycles per second. The experimental procedure used was to vary the frequency about the center frequency f_o, read values of θ on a dual beam oscilloscope, calculate values of attenuation for each set of readings and compare consistency. The accelerometer and microphone voltages were displayed as vertical inputs to a dual beam, time shared CRO. The Lissajous technique was also tried but discarded since it introduced additional labor

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into the method without increasing accuracy. Phase meters were also tried but the relatively low input impedances of these meters significantly loaded the circuits and introduced phase changes. Throughout the experiment an attempt was made to keep electronic circuitry to a minimum of complexity, since amplifiers and filters introduced phase changes which were both amplitude and frequency dependent in an unpredictable manner. In the early stages of the experiment it was discovered that the value of f_o was extremely sensitive to temperature changes in the medium, a one degree centigrade change corresponding to a 0.34 cyle per second change in f_o at 200 cps. Since Δf was of the order of one cps, this was a significant source of error. The problem was eliminated by water jacketing the tube to stabilize the temperature, after which drift in the value of f_o became negligible. Fig. 3-3.

The expression $P_{max} = \frac{\rho_o A}{\alpha K L}$ from which $\alpha = \frac{\rho_o A}{K L P_{max}}$ provides a simple and rapid means of determining α. Since KL is approximately $n\pi$ the parameters to be measured are acceleration and pressure, ρ_o being obtained from a handbook as a function of temperature. Assuming proper calibration of both accelerometer and microphone, this method should produce accurate results. Since KL may be made any one of many values of n within the frequency range of the equipment,

[illegible]

values of α for many different frequencies may be obtained without varying the length of the tube. Fig. 3-4.

The α obtained by measuring P and A should be the same at all the modes n obtained by varying length and holding frequency fixed. The product KL P_{max} should be a constant for all n for a given A. The experimental results of plotting nP_{max} as ordinate against n for a fixed value of A was not the expected straight line but a curve which asymptotically approached a horizontal line for high values of n. Fig. 3-7. The explanation of this phenomenon comes from a consideration of the losses making up α . The theoretical value of α was derived assuming a boundary condition of a rigid end at x = L. If there are leaks around the gaskets there will be significant increases in the value of attenuation measured. By using a single parameter α the losses at the ends have been treated as though they were distributed over the whole length of the tube. A more appropriate treatment would be to treat attenuation of pressure amplitude for a wave traveling down a tube and striking an end as attenuation of the form $P_E = P_0 e^{-\alpha(X_E - X_0) - \alpha_E}$ where $e^{-\alpha_E}$ is the loss in amplitude due to the end. If the length of the tube is increased the attenuation due to the end does not increase but the attenuation due to the walls is increased. As tube length becomes greater the losses at the end become less significant. Thus at higher values of n the end losses will be

proportionally less significant and the measured values of α should approach asymptotically the value of α for ideal end conditions. Fig. 3-8.

By comparing the value of α for n = 1 with the value of for a higher value of n at the same frequency an indication of the tightness of the system is obtained. As a result of these comparisons a greatly improved rigid end piston was designed and installed. With this piston the system became so leakproof that it was difficult to change the tube length. It took several minutes in each case for pressure equilibrium to be established after a change in tube length.

The expression $P_{min} = \frac{P_o A}{K}$ leads to the conclusion that P_{min} is a function of frequency but not of length, that is, for a given value of frequency the values of P_{min} obtained by varying L through different values of $KL = (m-1/2)\pi$ will all be the same. This conclusion was verified within the limits of experimental accuracy.

If the expressions for P_{max} and P_{min} are combined, the result $\alpha = \frac{1}{L}\frac{P_{min}}{P_{max}}$ is obtained, where L is the length of the tube at which P_{max} was obtained. Since α may be found using a ratio of pressures this method of determining α is considerably less sensitive to calibration errors. One objection to the method is that at the small values of P_{min} involved, system noise becomes significant.

An additional method of determining α is to determine the frequency or length change necessary to reduce the value of P to 0.707 of its maximum value, that is to reduce it be three decibels. In this "bandwidth" method

$$\left(\frac{P_{max}}{P_{half\ power}}\right)^2 = 2 = \frac{(\alpha L \cos KL)^2 + \sin^2 KL}{(\alpha L)^2}$$

In the vicinity of resonance KL may be assumed to be some small value $\Delta(KL)$ away from $n\pi$ and α may be assumed constant for the small range of frequencies involved. $\mathrm{Sin}^2 KL$ may be approximated by $[\Delta(KL)]^2$ or, with L held constant, by $L^2(\Delta K)^2$. $\mathrm{Cos}^2 KL$ may be approximated by 1. Then

$$2 = \frac{(\alpha L)^2 + L^2(\Delta K)^2}{(\alpha L)^2}$$

$$\alpha = \Delta K$$

In practice it will be simpler to measure the change in frequency between down 3 db points on both sides of the resonant peak. In terms of frequency $\alpha = \frac{\pi \Delta f}{c}$ where Δf is defined as the frequency change necessary to go from one of the half-power points to the other, and c is the speed of propagation in the tube. This relationship was used by Parker [13] to determine attenuation in a tube filled with oxygen.

The bandwidth method is a simple method with the advantage of employing a ratio of pressures, thus eliminating calibration

errors. Since the Δ f in the above expression is only of the order of a few cycles per second, and the determination of the driver piston frequency was only to the nearest tenth of a cycle per second, this method was not particularly consistent. A more accurately controllable system and a more accurate frequency counter would improve this method considerably. Obtaining the half-power points accurately on a voltmeter is also difficult and not consistent.

A method of measuring α independent of the previous methods is to measure the attenuation of the plane wave in the tube after the driving piston is stopped. When the power to the piston is cut off the piston stops in one or two cycles, leaving the plane wave to decay in a tube with rigid stationary ends at $x = 0$ and $x = L$. A picture of this exponential decay can be taken with a Polaroid camera mounted on the oscilloscope with the sweep time base adjusted to display 3 or 4 time constants of the decay. (Figure 3-5). If the magnitude of the pressure is plotted against time on semi-log paper a straight line is obtained, from which a decay constant is calculated. (Figure 3-9). This decay constant is in units of sec^{-1} but it may be converted to α in units of cm^{-1} by dividing it by the speed of propagation in the tube.

The objections to this method are first that it is only appropriate for $KL = 1\pi$, since for higher values of n beats occur

which make analysis of the exponential decay more difficult. (Figure 3-5, 3-6). Second, it is slower. Third, non-linearities in the display system of the oscilloscope become significant. Fourth, the different end condition at x = 0 may result in a slightly different value of α than existed when the piston was moving. Even with all these objections, this method proved quite consistent and on a par with the other methods.

The investigation of attenuation in the acoustic region in a tube of rectangular cross section was carried out in a similar fashion as in the round tube. The equipment used made it impossible to vary the length of the tube more than a few centimeters. All methods of determining α could be used with the exception of the method involving the ratio P_{min}/P_{max}. The results of the four methods of obtaining attenuation used on the rectangular tube were consistent and in agreement with theory. Figure 3-10. Table 1 illustrates the comparison of theoretical values of attenuation for both the cylindrical and rectangular tubes with the attenuation measured by the five methods discussed. These results illustrate the consistency that can be obtained by making measurements under identical conditions.

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4. Investigation of Finite Amplitude Waves in Tubes

As discussed in the introduction, when a piston is sinusoidally driven with large amplitude in a rigid ended tube, the pressure at the end of the tube is not sinusoidal when the frequency is near resonance. It is observed that the pressure wave at the rigid end of the tube contains a discontinuity. A pressure discontinuity of the type observed is characteristic of a shock wave.

Saenger and Hudson (1960) developed a finite amplitude wave theory based on the assumption that the solution to the problem is the sum of a continuous part, which is calculated as a power series, and a discontinuous part, which satisfies the shock conditions. This theory is that used by Reichwein (1962) in his investigation of threshold behavior of shock waves in a tube using similar equipment to ours. Saenger and Hudson also did some experimental work and reported fair agreement with their solution based on a minimal number of observations. Two other theoreticians in the field of finite amplitude waves are Betchov (1958) who also constructed his solution from an assumed sum of a continuous part, and a discontinuous part, and Chester (1962) who refines a collection of the previous theories.

Effects of attenuation on waveform Figure 4-1 is a montage of oscillograms illustrating the growth of shock waves as a function of piston acceleration, and phase angle away from resonance in two tubes of different cross sections. The resonance frequency in both tubes was 89 cps, and the phase angle between pressure at the rigid end and acceleration of the piston was measured in the infinitesimal amplitude region with pressure having the leading phase. The normalizing factor A_o was 81.3 m/sec^2 in the round cross section tube and 93.8 m/sec^2 in the rectangular cross section tube, and was the acceleration necessary to produce an undistorted wave of 1.57×10^{-3} bars rms pressure.

It is clear from these oscillograms that there is an intermediate region between the infinitesimal amplitude and shock regions. This intermediate region is characterized by an increasing distortion of the infinitesimal amplitude wave as the acceleration of the driver is increased until a discontinunity appears and will be called the weak finite amplitude region. Saenger and Hudson predict a sudden appearance of the shock wave which was not observed. Instead there is such a gradual growth of distortion that there was considerable disagreement between observers as to when a shock was actually present or to when the wave was merely badly distorted. From the rise times of the measuring instruments it was calculated that the rise time of the fully

developed shock wave was less than 4 micro seconds. Saenger and Hudson further predicted that a shock could not exist below a piston acceleration which is ten times larger than that observed at 100 cps.

The fact that it was necessary to water-jacket the tube in order to keep the resonant frequency the same for high amplitude work yielded some experimental evidence that the air temperature in the tube was rising as predicted by Saenger and Hudson. The tube resonance conditions were not observed to change when infinitesimal amplitude waves were present in the tube.

With the available frequency counter, frequency could not be determined with an accuracy greater than 0.1 cps. This presented difficulties when it was required to reproduce measurements at a given frequency. Phase angle, which can be measured to 1.0°, is a more sensitive parameter than frequency as can be seen on the montage where two cps correspond to approximately 60° phase angle near resonance. Phase angles were adjusted by keeping the tube length constant and changing the frequency of the driver until the desired phase angle was observed. It was necessary, however, to measure phase angles in the infinitesimal amplitude region because, if the upgoing crossing of the axis is chosen as the point at which to measure phase, this point is not fixed as the amplitude of the piston is increased to the point of pressure wave

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[illegible]

[illegible]

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distortion; i.e., the weak finite amplitude wave phase angle must be defined in a different manner than the infinitesimal amplitude phase angle and will not be defined in this report. The definition of phase angle used in the shock region is the phase angle between the shock front and the upgoing axis crossing of the piston acceleration.

Data for the montage were limited to 60° off resonance by the acceleration capability of our equipment since observations that were made at phase angles further from resonance revealed no shock wave development at the maximum acceleration of the equipment. If more powerful equipment had been available shock waves might have been observed at frequencies further from resonance, but just how far off resonance shock waves will develop cannot be reckoned. From the shape of the montage it might be expected that if unlimited acceleration was available shock waves could be produced for any frequency away from resonance; however, the above mentioned theories predict the shock condition only at resonance and frequencies near resonance.

The similarity of the waveforms in the two tubes is remarkable since the tube cross sections were so different. The reason for this similarity lies in the process of normalizing the acceleration. In the infinitesimal amplitude region pressure is proportional to the acceleration of the driver and inversely proportional to the

attenuation constant. (Refer to section 3.3) If the acceleration required to produce a given infinitesimal amplitude pressure is taken as a normalizing factor in one tube, and the acceleration required to produce the same pressure in a second tube is taken as the normalizing factor in that tube, then the effect of the different attenuations in the two tubes is negated by this normalization process. Even though the attenuation measurements made in the infinitesimal amplitude region do not apply in the finite amplitude region it is remarkable that the wave forms in the different tubes are similar in both regions when the accelerations are normalized in the infinitesimal amplitude region. Had the acceleration itself been used as a parameter (as predicted by Reichwein) the wave-forms would not have been in such close agreement. The attenuations within the tubes used were nearly the same, but tubes with quite different attenuations can be expected to produce identical waveforms if the accelerations are thus normalized.

<u>Weak finite amplitude region</u> It was observed that visual distortion of the pressure wave appeared when the magnitude of the second harmonic was 10 per cent that of the fundamental. A wave analyzer was used to measure the second harmonic as the acceleration of the piston was increased until the magnitude of the second harmonic was 10 per cent that of the fundamental.

These measurements were made in a round cross section tube and in a rectangular cross section tube and the results are plotted in Fig. 4-2 and 4-3. The same procedure was repeated for 20 per cent second harmonic and for the threshold of shock. No quantative data were plotted for the threshold of shock, however, since the point at which the shock appears fully developed depends on the opinion of the observer. It appeared as though the threshold of shock was near the point of 30 per cent second harmonic, but data taken for this region are not presented since the definition of shock becomes more obscure as frequency is taken away from resonance. The wave analyzer used introduced a phase change which was a function of frequency and amplitude which precluded determination of the phase relationship of the second harmonic to the fundamental. It is interesting to note that for a given acceleration or pressure the maximum distortion occurs at a noticably lower frequency than the infinitesimal amplitude resonant frequency.

Shock region Reichwein investigated the parameter defined by Saenger and Hudson to be the phase angle between the maximum displacement of the piston out of the tube and the arrival of the shock wave at the piston. He measured velocity of the piston whereas we monitored piston acceleration resulting in

[illegible]

[illegible]

our phase angle parameter being 90° out of phase with his Δ parameter. Reichwein drove his system at maximum amplitude, set his desired Δmax by varying frequency or length of the tube, and then reduced the amplitude down by steps until the amplitudes of the piston acceleration became so low that shocks could no longer be observed. The value of Δmax for which no change in Δ occurred as amplitude decreased was not $\Delta max = 0$ as might be expected but was some value in the vicinity of $\Delta max = -10^\circ$ dependent on the resonant frequency. He further predicted that if amplitudes of acceleration were increased above his maximum available acceleration, that Δ would remain constant.

We repeated the same procedure using a maximum acceleration which was approximately three times that of Reichwein. The curves obtained, and presented as Figure 4-4, have the same shape as those presented by Reichwein. The ordinate of this graph is not Δ, but the phase angle between the shock at the rigid end and the upward going axis crossing of the piston acceleration. This phase angle is equal to $\Delta + 90^\circ$. For accelerations up to 200 m/sec^2 (the maximum used by Reichwein) our results were in agreement with his, and as the acceleration was increased further the predicted trend was followed until about 400 m/sec^2. For accelerations above 400 m/sec^2, Reichwein's prediction that phase angle would remain constant could not be

verified due to piston acceleration distortion problems.

Resonance in the finite amplitude region The definition of resonance in the infinitesimal amplitude region is that frequency at which the maximum rms pressure is obtained for a constant piston acceleration, and it has been shown that this occurs when the phase angle between pressure and acceleration is 90°. At the frequency of infinitesimal amplitude resonance, for all but weak shocks, the phase angle between the shock and the acceleration is 90°. If the piston acceleration is held constant and the frequency changed, it was observed that both the shock strength and the average rectified voltage (as measured on a VTVM) decreased. This is in complete harmony with the definition of resonance in the infinitesimal amplitude region. If the shock strength is plotted for a constant value of acceleration, then the maxima of the curves obtained for different values of acceleration lie at the infinitesimal amplitude resonance frequency for high accelerations as mentioned above, and move to progressively lower frequencies as the piston acceleration is decreased.

For the weak finite amplitude region where there are no shocks, Fig. 4-2 leads us to define resonance as that frequency for which the per cent of second harmonic is a maximum for constant piston acceleration. With this definition the tendency

for the resonant frequency to become lower as the acceleration is decreased continues to follow the same pattern as in the weak shock region. Unfortunately data were not collected to indicate whether or not this definition is consistent with the maximization of the average rectified voltage. The general trend for the resonant frequency to move to lower frequencies appears to continue as the infintesimal amplitude region is approached, however, no data were taken to follow this resonant frequency drift into the infinitesimal amplitude region.

CONCLUSIONS

The most significant result of this study is the establishment of the ratio of acceleration to attenuation constant as a similarity parameter for waveforms in tubes of different cross-sectional area and shape. Once this ratio is known in two dissimliar tubes, their waveforms will be found to be the same for any value of acceleration as long as this ratio and the phase angle off resonance are the same. This similarity parameter is predicted theoretically for infinitesimal amplitude standing waves, but it is shown that the same parameter is applicable for weak finite amplitude and shock waves. The similarity parameter was investigated for only two types of tubes, but there is no reason to expect different behavior for different shapes of tubes with radically different attenuations assuming that the systems are "tight" in all cases.

In order to determine this similarity parameter, five methods of measuring attenuation in the infinitesimal amplitude region have been developed and have been demonstrated to be consistent with each other and with theory. The values of attenuation constants obtained agreed with the results of other experimenters. These methods can be effectively adapted to the measurement of attenuation in a variety of gases under a wide range of pressures and temperatures.

In the region of weak finite amplitude waves the resonant frequency may be defined as the frequency causing greatest harmonic distortion for a given acceleration. This resonant frequency is observed to be lower than the resonant frequency for infinitesimal amplitude waves. This difference between the two resonant frequencies decreases as acceleration is increased. These observations are consistent with the results in the shock region where resonance is defined as the frequency for maximum shock strength. As the acceleration is increased in the shock region, the maximum shock strength occurs closer and closer to the resonant frequency of infinitesimal amplitude waves.

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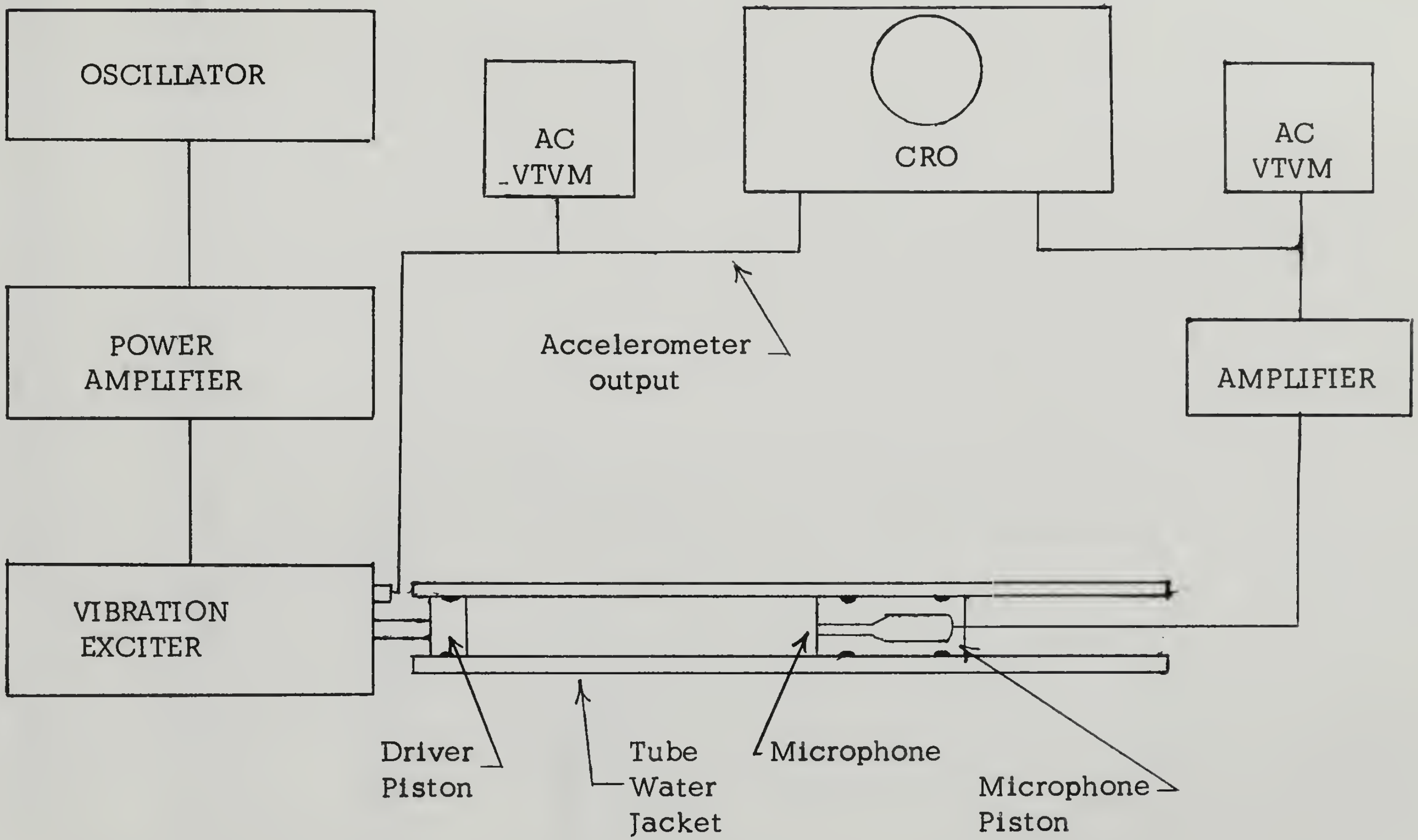


Figure 1.1 EQUIPMENT SETUP

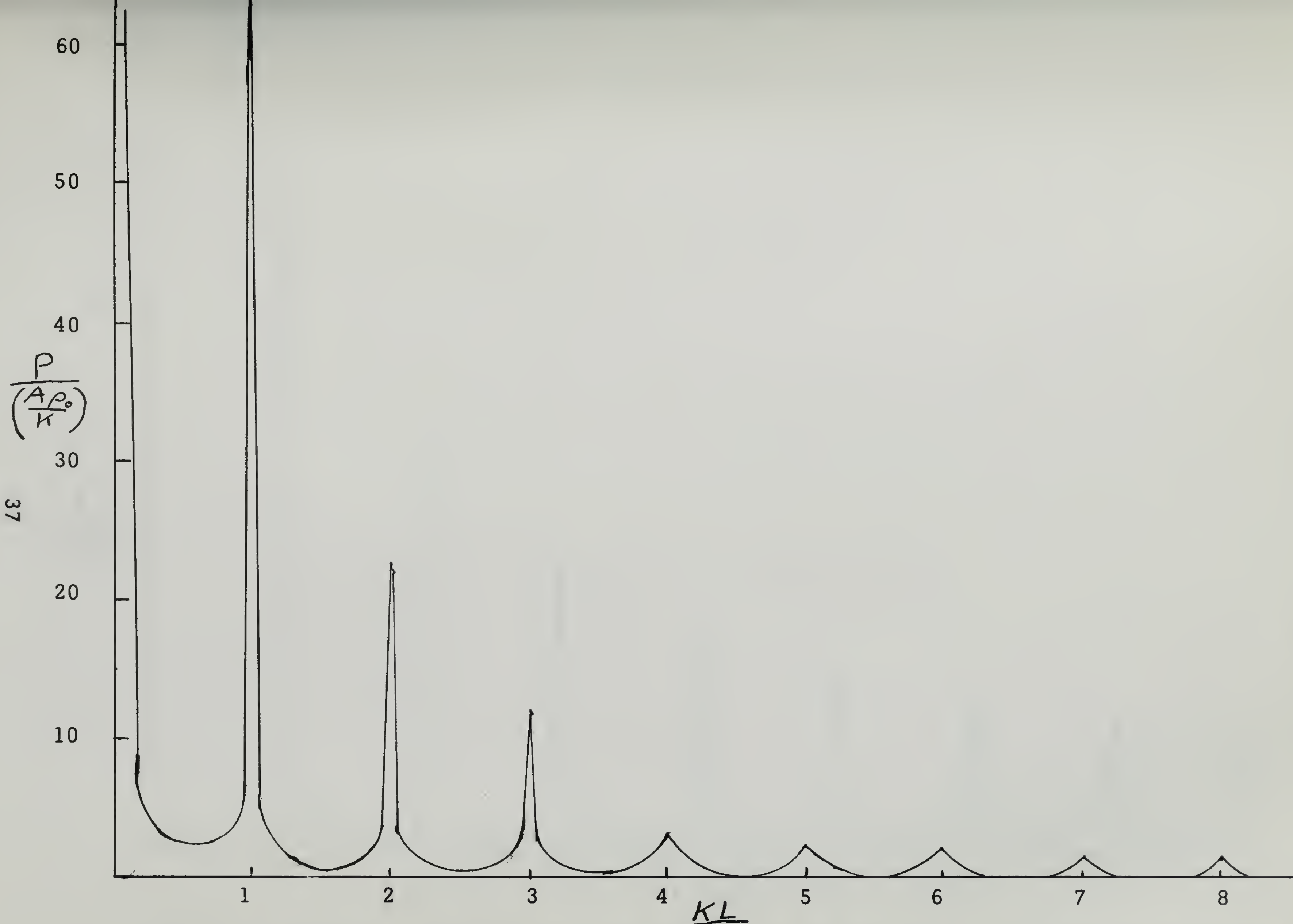


Figure 3-1. Variation of pressure magnitude at the rigid end as frequency is varied, maintaining tube length fixed.

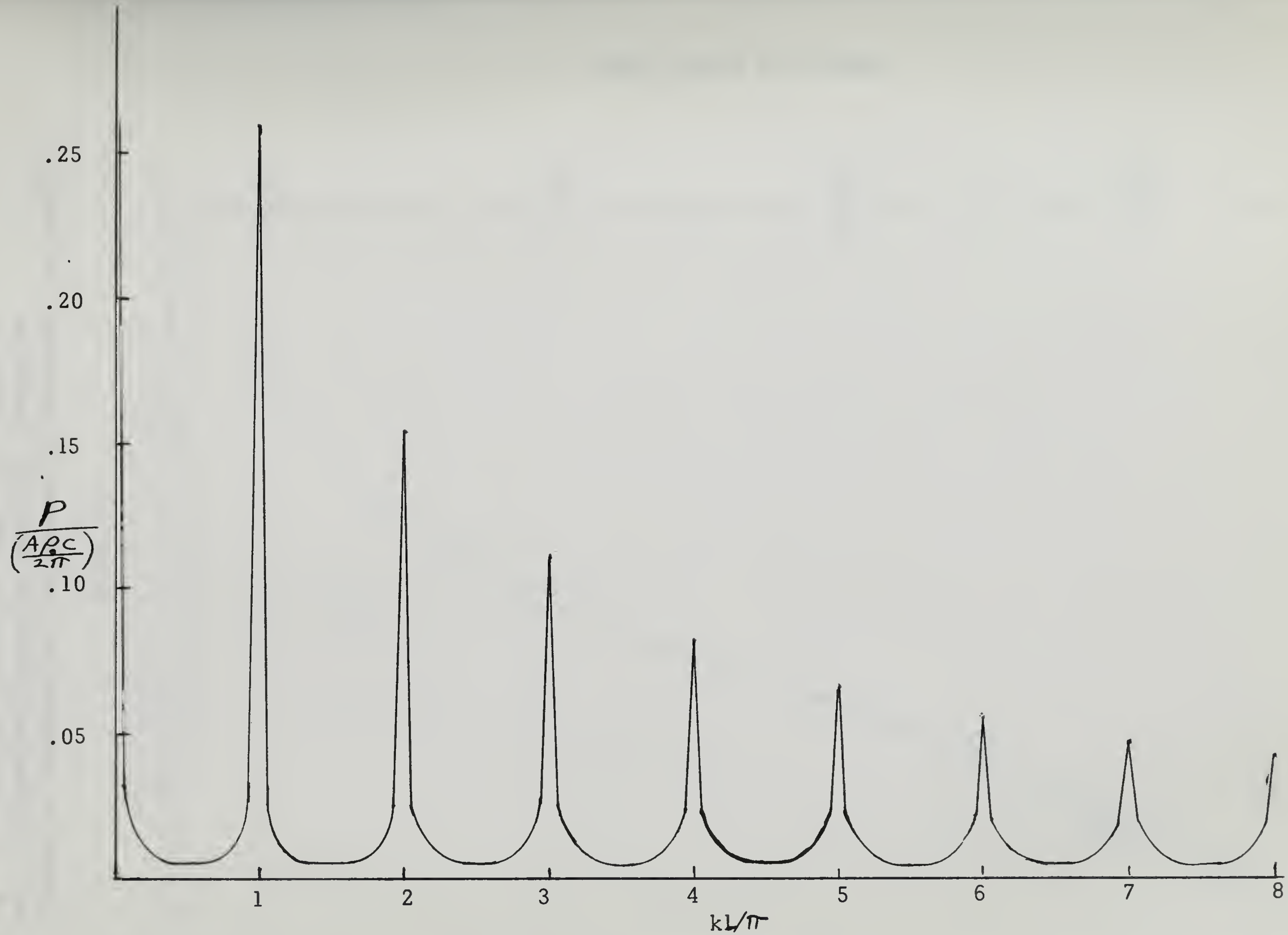


Figure 3-2. Variation of normalized pressure magnitude at the rigid end as the length of the tube is varied, with frequency fixed at 200 cps.

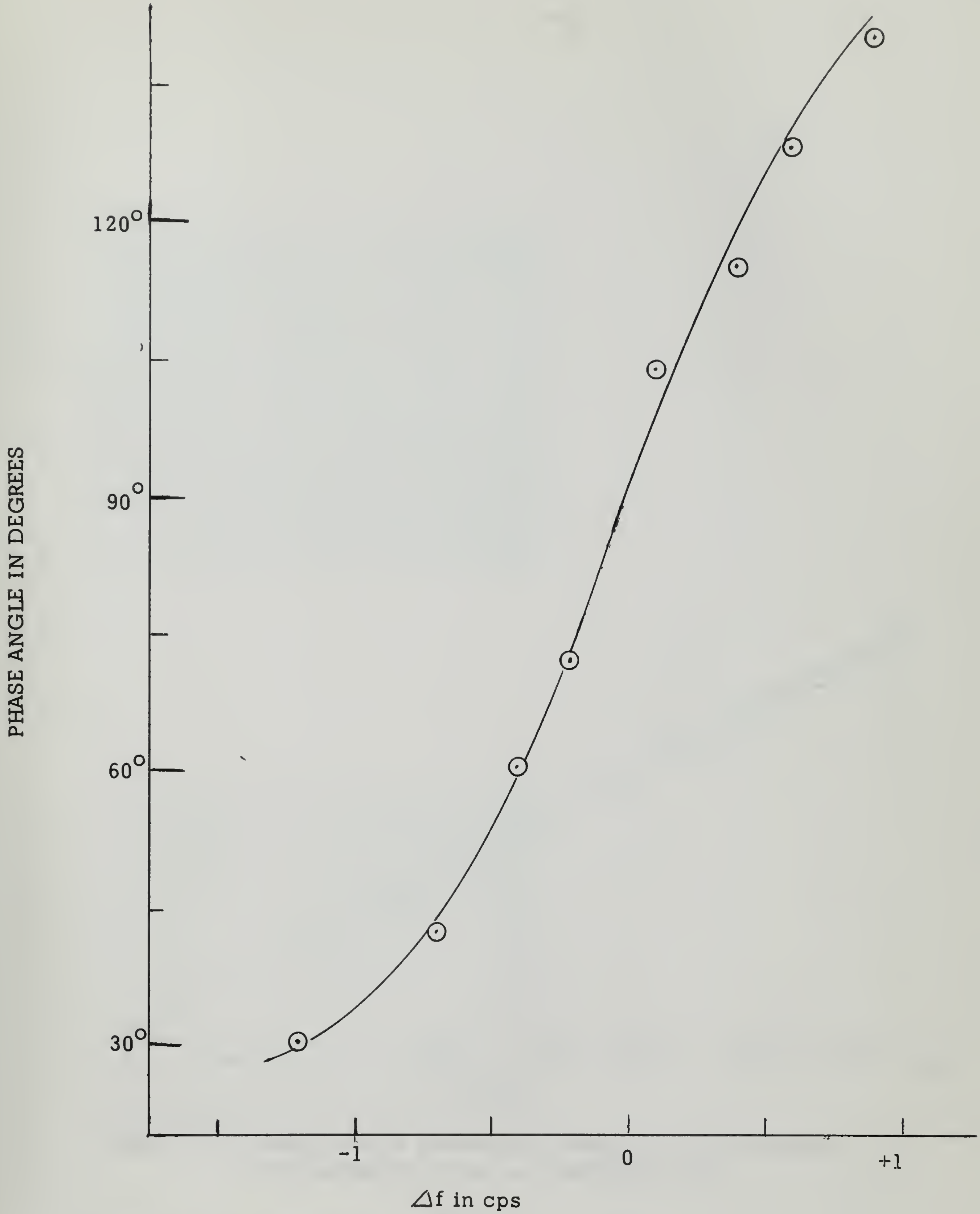


Figure 3-3. Variation of phase angle θ as frequency is varied by an amount Δf about the center frequency f_o. The smooth curve represents theoretical variation of θ with Δf.

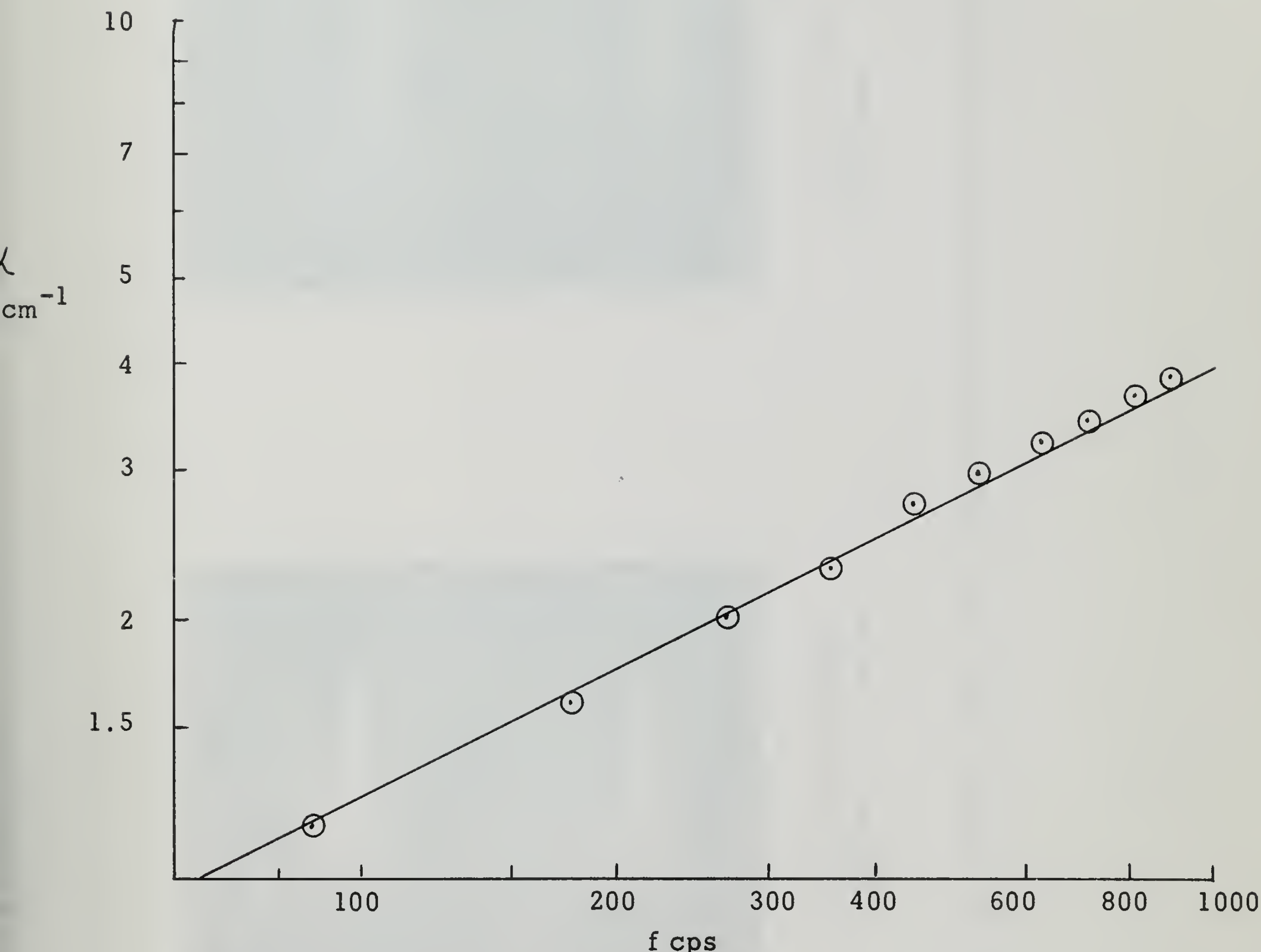
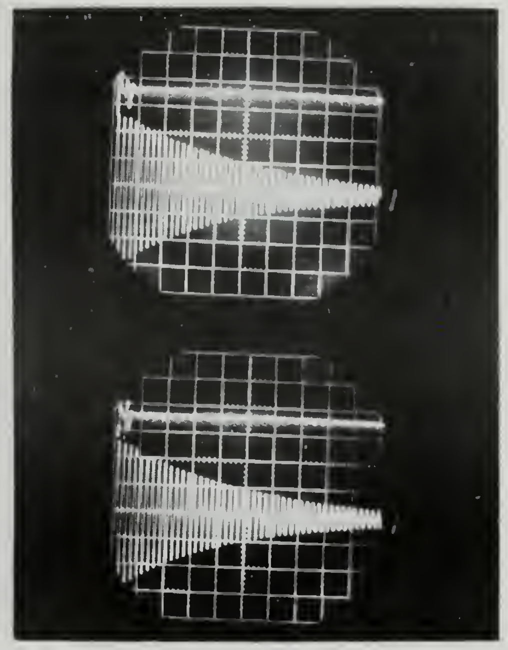
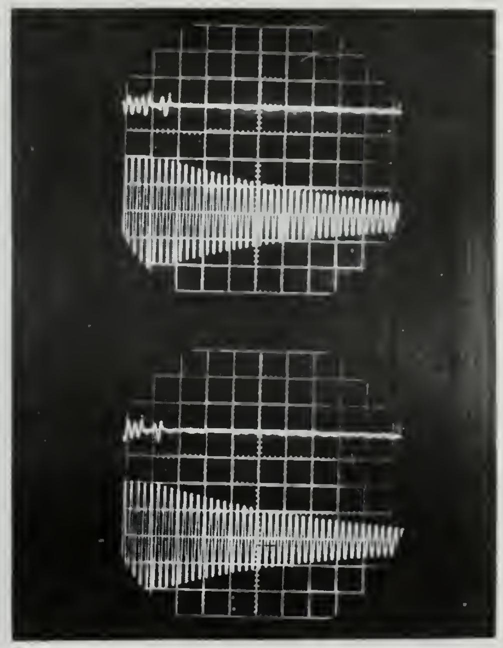


Figure 3-4. Comparison of attenuation measured by A/nP method with theoretical value in cylindrical tube.

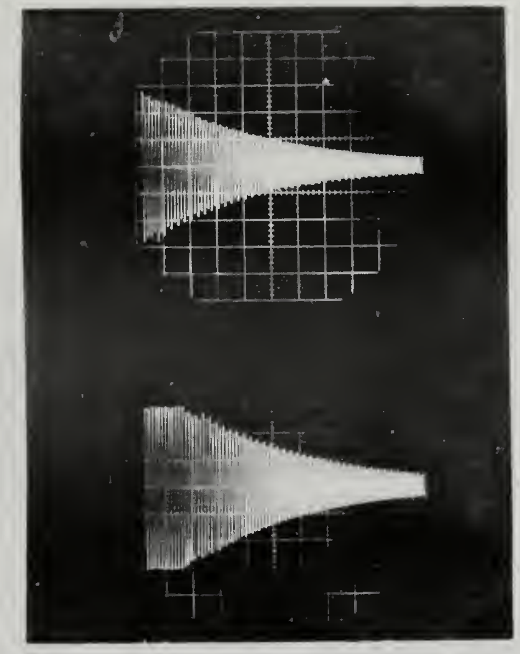


Fundamental Mode
89.0 cps $kl = \pi$



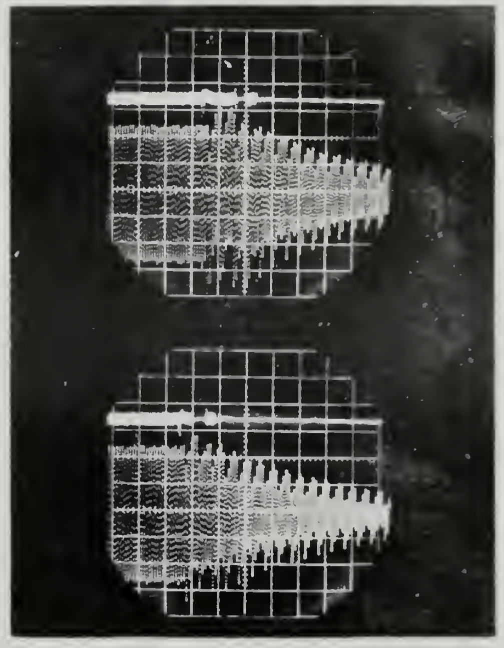
Fundamental Mode
200.0 cps $kl = \pi$

Figure 3.5 Exponential decay of plane acoustic waves in a tube closed at both ends.



Fundamental Mode

200.0 cps $kl = 2\pi$



Fundamental Mode

100.0 cps $kl = 5\pi$

Figure 3.6 Exponential decay of plane acoustic waves in a tube closed at both ends.

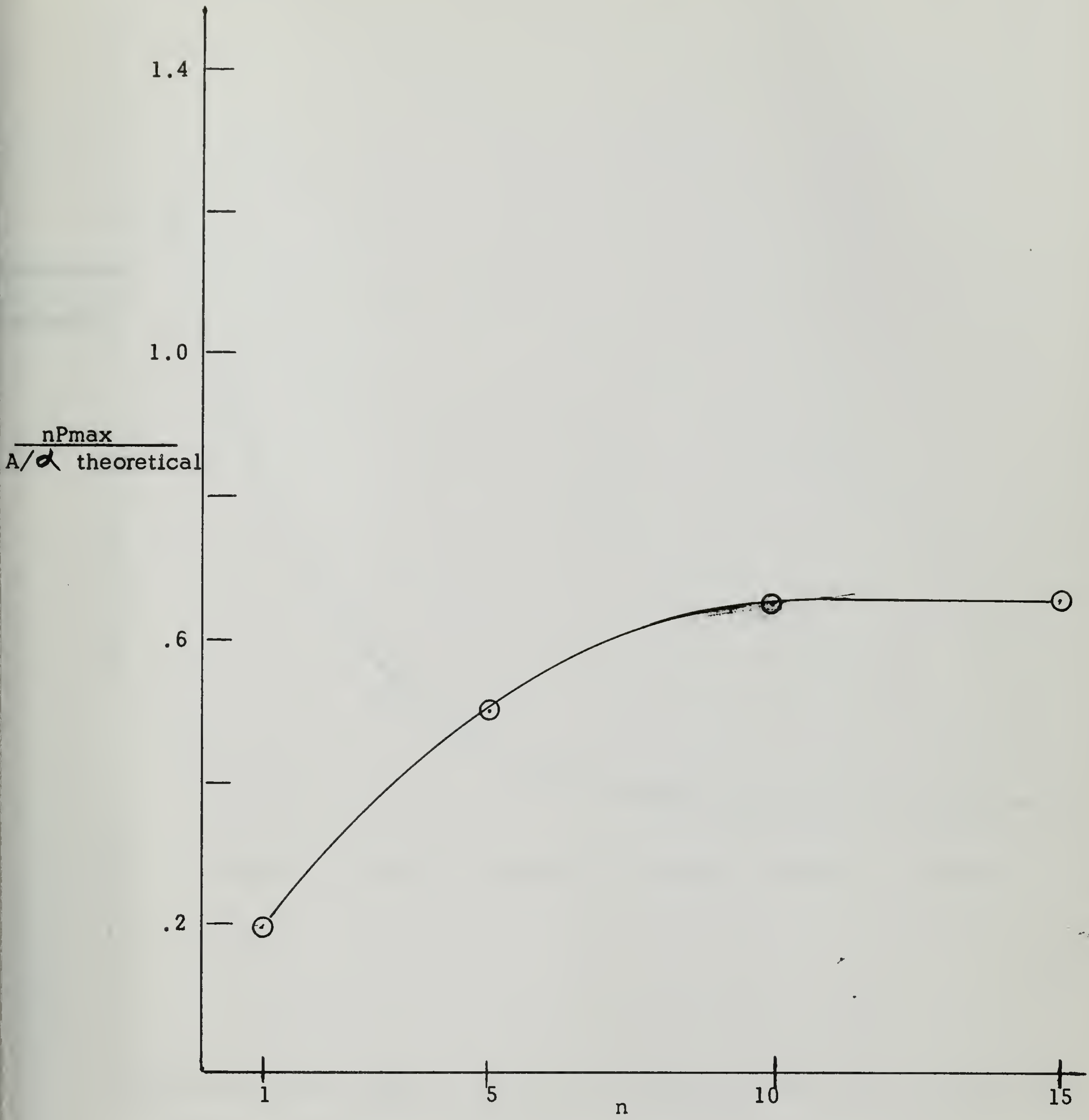


Figure 3-7. Asymptotic approach of nPmax to a constant for high values of n.

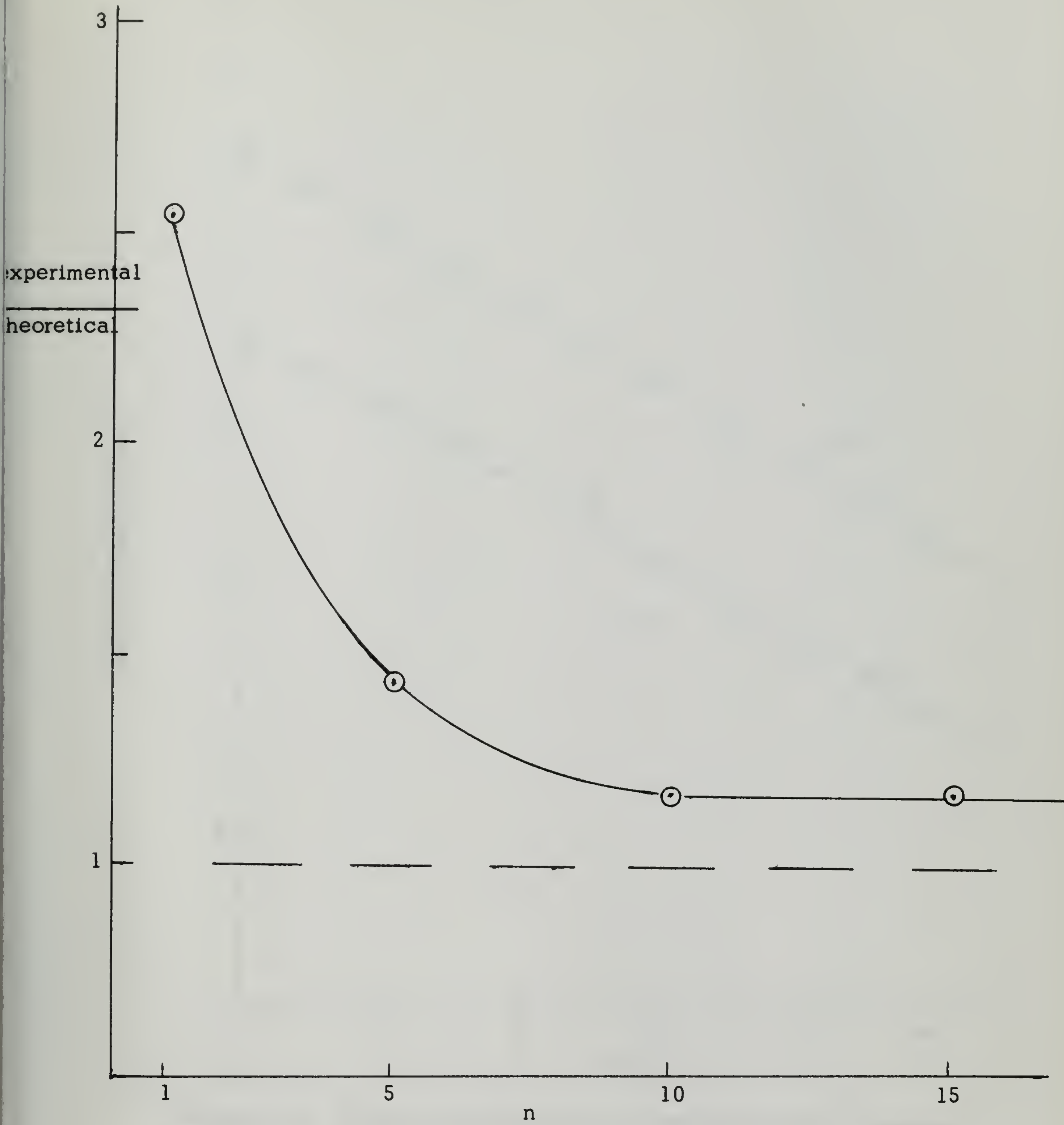


Figure 3-8. Asymptotic approach of experimental value of attenuation to theoretical value for high values of n.

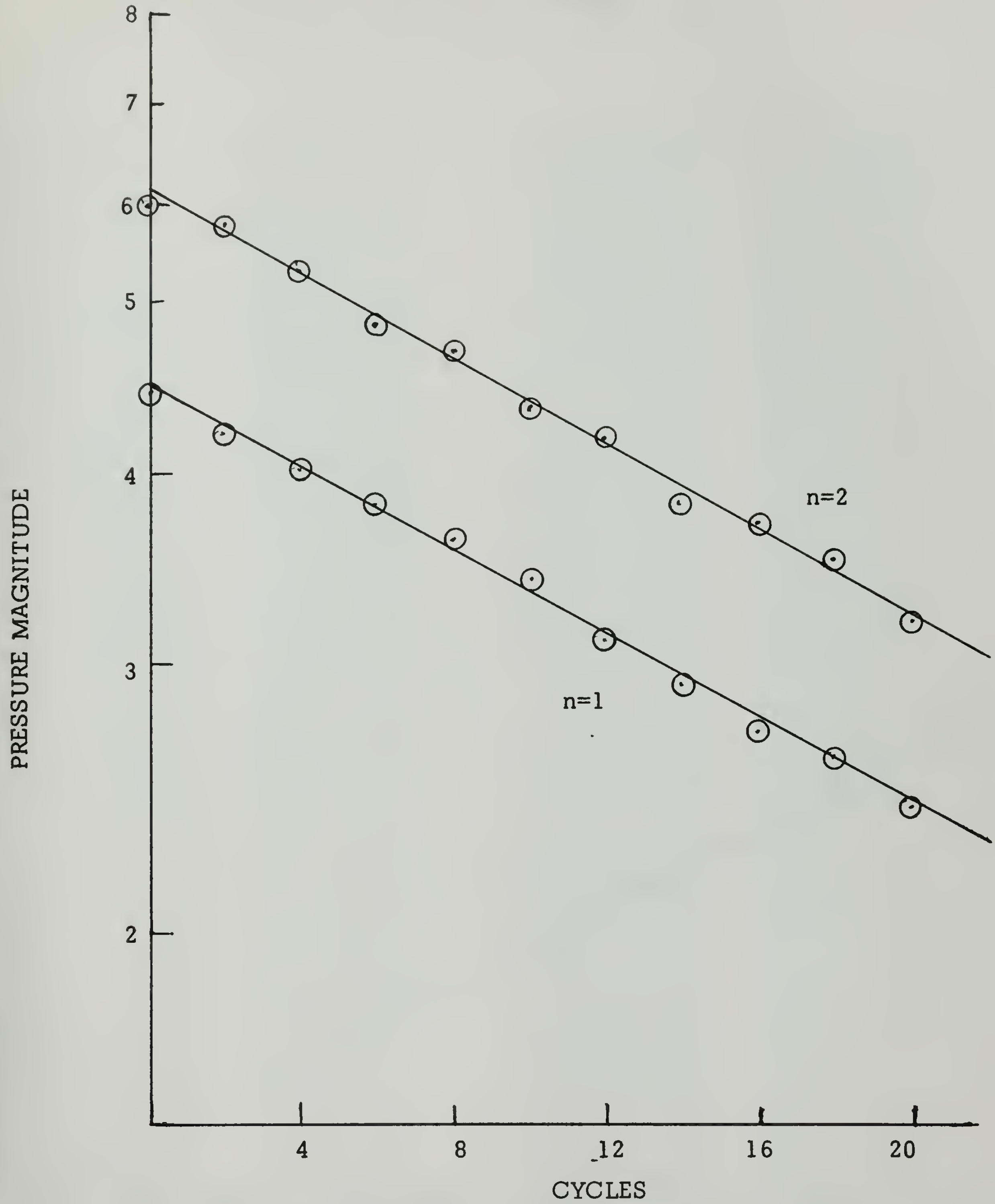


Figure 3-9. Typical semi-log plot of exponential decay of pressure, illustrating attenuation by reverberation technique.

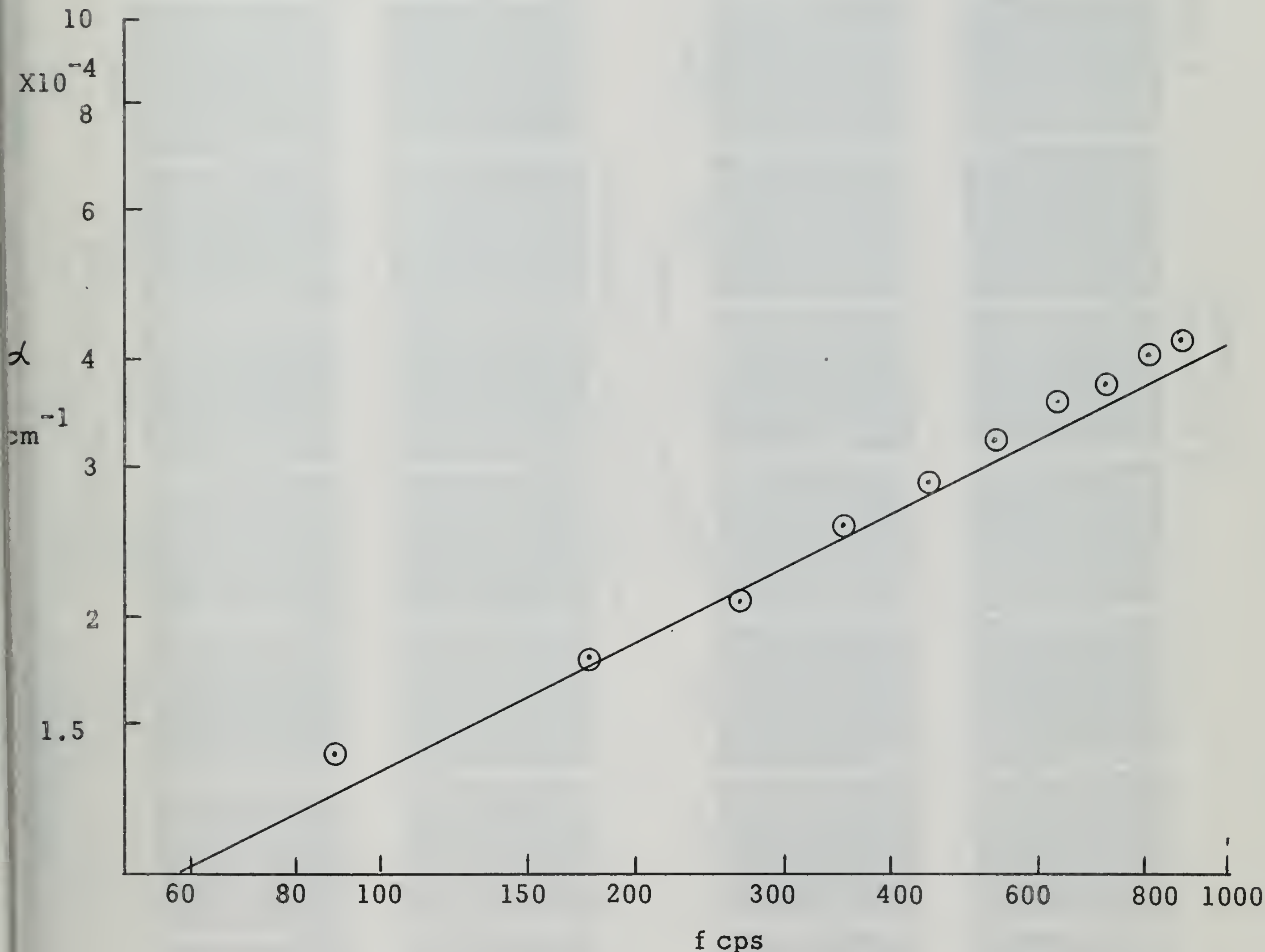
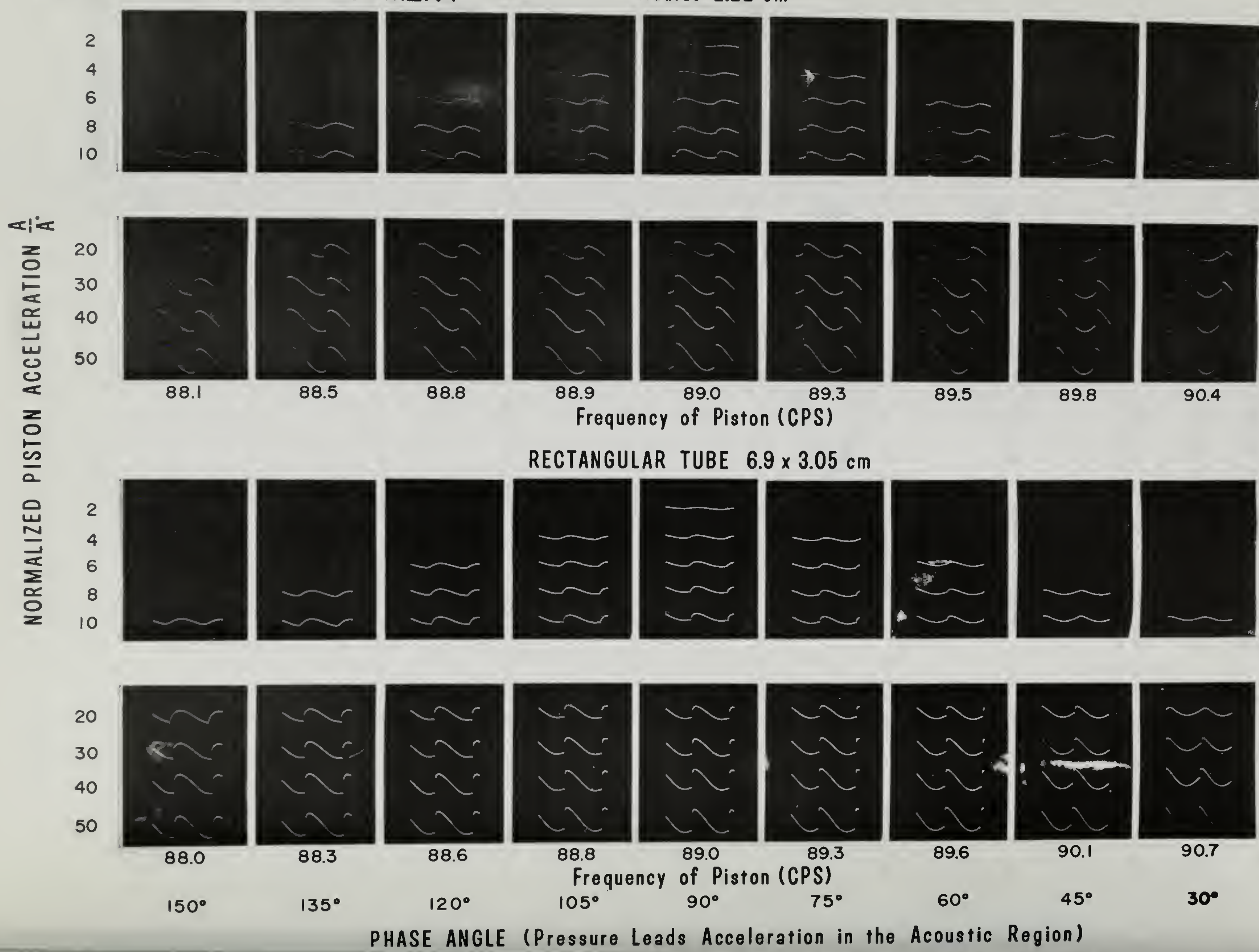


Figure 3-10. Comparison of attenuation measured by method with theoretical value in rectangular tube.

Figure 4.1

SHOCK WAVE DEVELOPMENT



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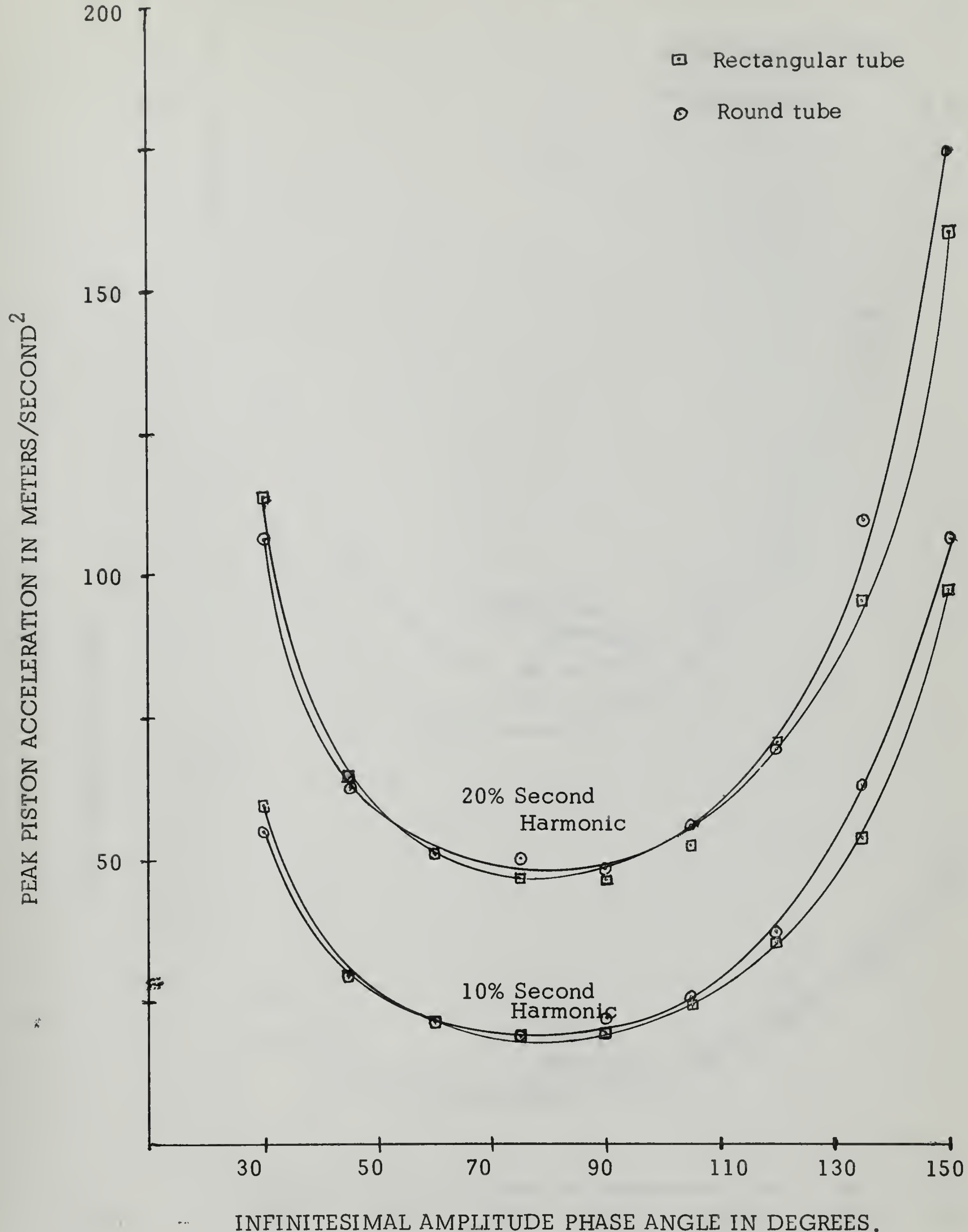

Figure 4-2. Second harmonic distortion of weak finite amplitude standing waves

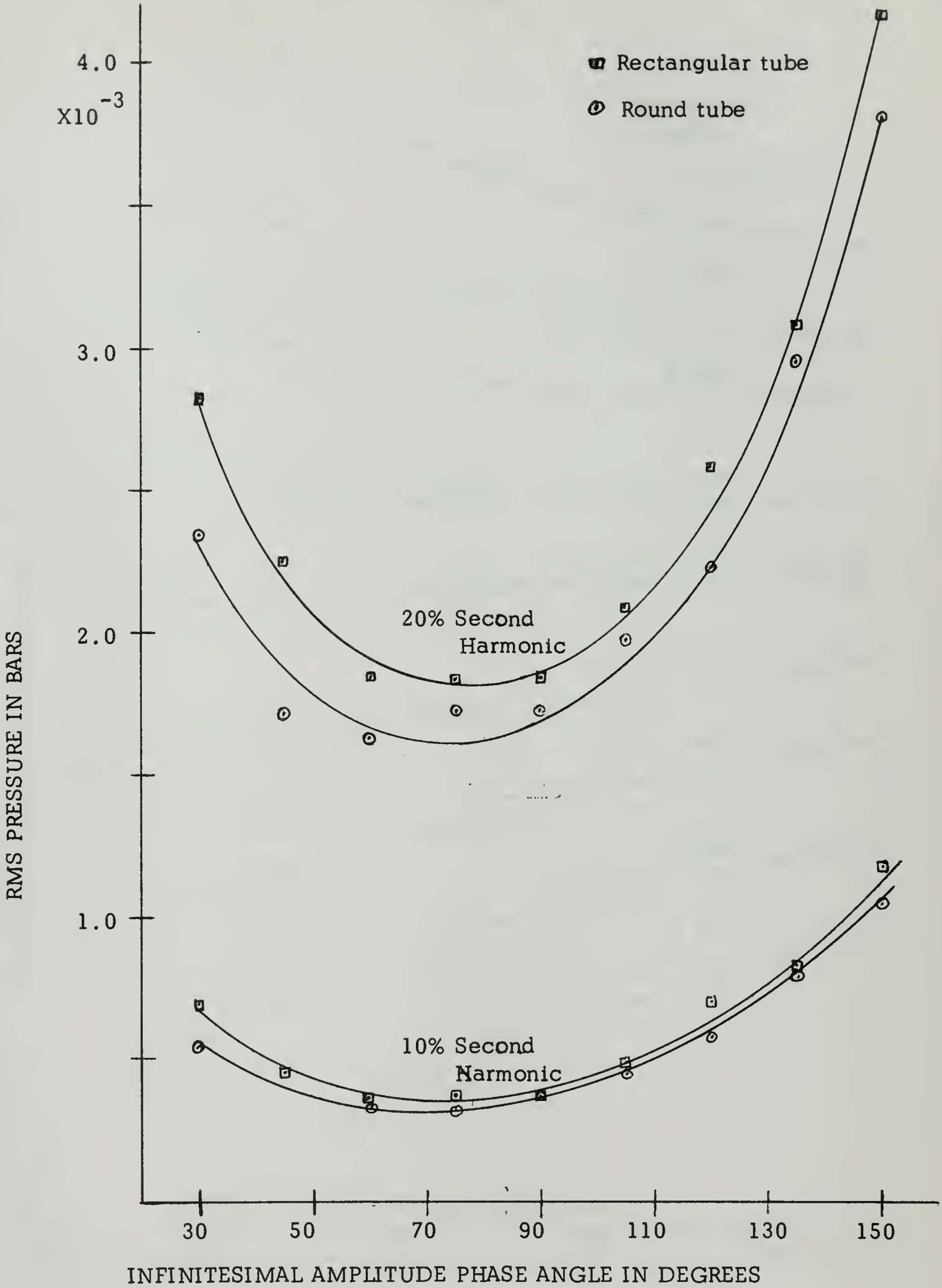

Figure 4-3. Second harmonic distortion of weak amplitude standing waves.

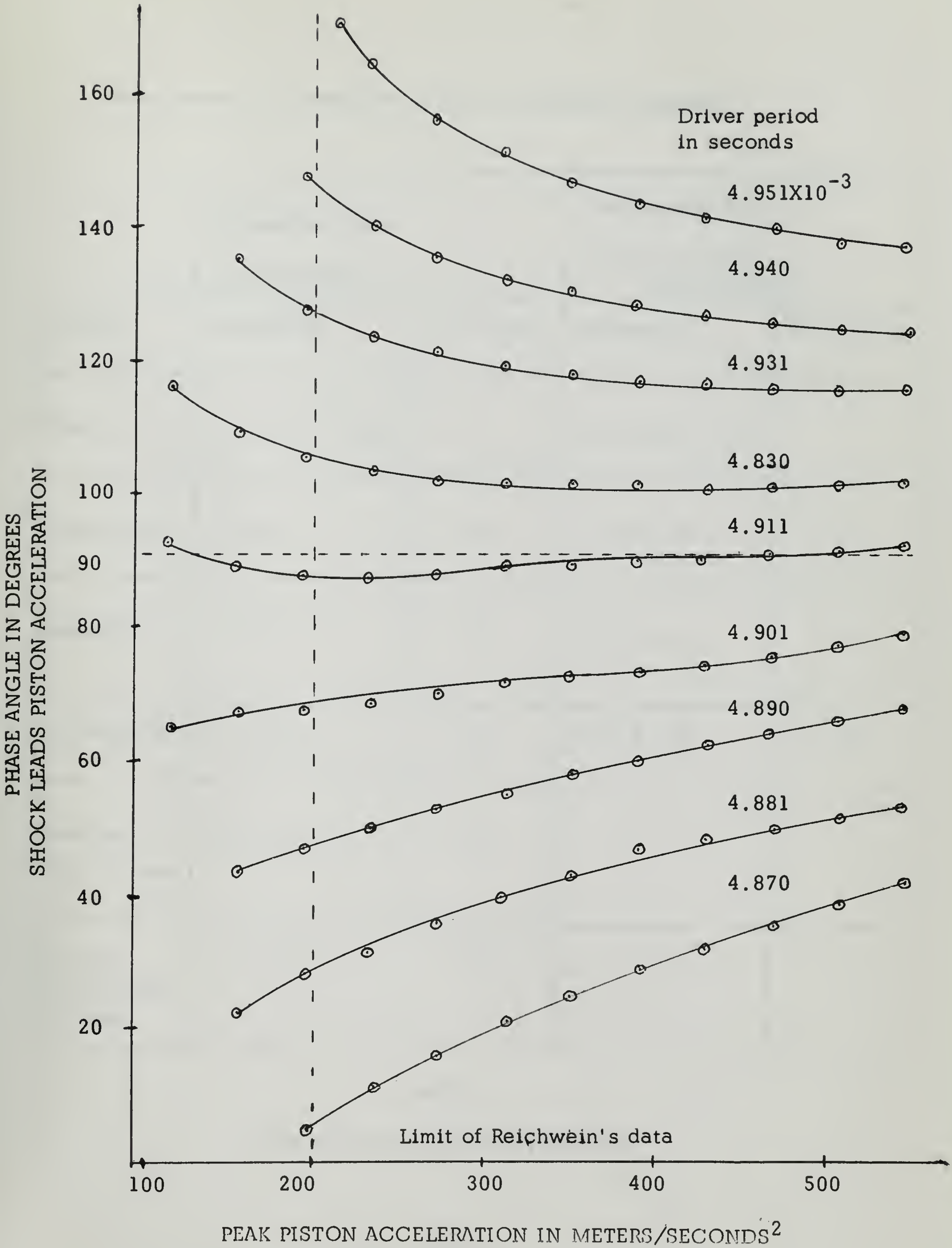

Figure 4-4. Behavior of strong shocks in standing waves.

Determination of Attenuation at 89 cps Resonant Frequency

	Cylindrical Tube (2.22 cm radius)		Rectangular Tube (3.05 x 6.90 cm Cross section)	
Method	Attenuation	α/α theor.	Attenuation	α/α theor.
Theory	1.17×10^{-4} cm^{-1}		1.23×10^{-4} cm^{-1}	
Phase Angle	1.34 $\pm$ .20	1.15	1.56 $\pm$ .20	1.26
Bandwidth	1.28 $\pm$ .15	1.09	1.37 $\pm$ .15	1.11
Reverberation	1.29 $\pm$ .10	1.10	1.45 $\pm$ .10	1.18
Ratio of P_{min} to P_{max}	1.76 $\pm$ .50	1.51		
Ratio of Acceleration to P_{max}	1.17 $\pm$ .20	1.00	1.37 $\pm$ .20	1.11

Table 1

(See Appendix II for error analysis)

BIBLIOGRAPHY

1. Betchov, R., Nonlinear Oscillations of a Column of Gas. Phys. Fluids, Vol 1, p. 205, 1958.

2. Blackstock, D. T., Propagation of Plane Sound Waves of Finite Amplitude in Nondissipative Fluids. J. Acoust. Soc. Am., Vol 34, p. 9, 1962.

3. Bogert, B. P., Classical Viscosity in Tubes and Cavities of Large Dimensions. J. Acoust. Soc. Am., Vol 22, p. 432, 1950.

4. Chester, W., Resonant Oscillations in Closed Tubes. California Institute of Technology, Pasadena, California.

5. Cook, B. D., New Procedure for Computing Finite-Amplitude Distortion. J. Acoust. Soc. Am., Vol 34, p. 941, 1962.

6. Fay, R. D., Attenuation of Sound in Tubes. J. Acoust. Soc. Am., Vol 12, p. 62, 1940.

7. Harris, C. M., Absorption of Sound in Air in the Audio Frequency Range. J. Acoust. Soc. Am., Vol 34, p. 11, 1963.

8. Keller, J. B., Finite Amplitude Sound Waves. J. Acoust. Soc. Am., Vol 25, p. 212, 1953.

9. Kinsler, L. E. and Frey, A. R., Fundamentals of Acoustics, John Wiley & Sons, Inc., 1950.

10. Lester, W. W., On the Theory of the Propagation of Plane, Finite Amplitude Waves in a Dissipative Fluid. J. Acoust. Soc. Am., Vol 33, p. 1196, 1961

11. Lettau, El, Messungen and Gasschwingungen Grosser Amplitude in Rohrleitungen. Deut. Kraftfhrtforschl, Heft 39, p. 17, 1939.

12. Mayer-Schuchard, Cl, Schwingungen von Luftsaülen mit Grosser Amplitude. Forschungsheft 376, p. 13, 1936.

13. Parker, J. G., Effect of Several Light Molecules on the Vibrational Relaxation Time of Oxygen. J. Chem. Phys., Vol 34, p. 1763, 1961.

14. Lord Rayleigh, Theory of Sound, Vol 2, Macmillan and Co., Ltd., London, 1929.

15. Reichwein, F. E., A Preliminary Investigation into Threshold Behavior of Periodic Shock Waves in Resonating Gas Columns. Thesis, United States Naval Postgraduate School, 1962.

16. Rogers, R., The Attenuation of Sound in Tubes. J. Acoust. Soc. Am., Vol 11, p. 480, 1939.

17. Saenger, R. A. and Hudson, G. E., Periodic Shock Waves in Resonating Gas Columns. J. Acoust. Soc. Am., Vol 32, p. 961, 1960.

18. Schmidt, El, Schwingungen von Gassaülen mit Grosser Amplitude in Rohrlitungen. er. Deut. Ingl, Vol 79. p. 671, 1935.

19. Thuras, A. L., Jenkins, R. T., and O'Neil, H. T., Extraneous Frequencies Generated in Air Carrying Intense Sound Waves. J. Acoust. Soc. Am., Vol 6, p. 173. 1935.

20. Werth, G. C., and Delsasso, L. P., Attenuation of Repeated Shock Waves in Tubes. J. Acoust. Soc. Am., Vol 26, p. 59, 1954.

21. Weston, D. E., The Theory of the Propagation of Plane Sound Waves in Tubes. Phys. Soc. of London, Vol 66B, p. 695, 1953.

22. White, J. E., A Method for Measuring Source Impedance and Tube Attenuation. J. Acoust. Soc. Am., Vol 22, p. 565, 1950.

APPENDIX I

THEORY OF STANDING WAVES IN A CLOSED CYLINDER DRIVEN SINUSOIDALLY

The values of attenuation measured in the round tube were found to be highly dependent on the tightness of the system. An excellent way to minimize leakage and at the same time to minimize alignment problems would be to seal both ends of the tube rigidly and to shake not just one piston but the whole system. In this case the only possible source of leaks would be a small gasket where the probe microphone would be inserted.

Assuming again a dissipative plane wave and applying the boundary conditions $\underset{\sim}{a} = Ae^{j\omega t}$ at x = 0 and x = L we obtain

$$A = -\omega^2(\underset{\sim}{B} + \underset{\sim}{D})$$

$$A = -\omega^2(\underset{\sim}{B}e^{-\alpha L}e^{-jKL} + \underset{\sim}{D}e^{\alpha L}e^{jKL})$$

from which

$$\underset{\sim}{B} = \frac{A}{\omega^2}\left[\frac{1 - e^{\alpha L}e^{jKL}}{e^{\alpha L}e^{jKL} - e^{-\alpha L}e^{-jKL}}\right]$$

$$\underset{\sim}{D} = \frac{-A}{\omega^2}\left[\frac{1 - e^{-\alpha L}e^{-jKL}}{e^{\alpha L}e^{jKL} - e^{-\alpha L}e^{-jKL}}\right]$$

Applying these results to the expression for pressure previously obtained in section (3.I) and evaluating at x = 0 and x = L

$$\underset{\sim}{p}_{x=L} = \frac{j\rho A}{K - j\alpha}\left[\frac{e^{\alpha L}e^{jKL} + e^{-\alpha L}e^{-jKL} - 2}{e^{\alpha L}e^{jKL} - e^{-\alpha L}e^{-jKL}}\right]e^{j\omega t}$$

$$\underset{\sim}{p}_{x=0} = \underset{\sim}{p}_{x=L}\, e^{j\pi}$$

Rewriting $p_{x=L}$ in terms of transcendental functions

$$p_{x=L} = \frac{-j\rho_0 A}{K - j\alpha}\left[\frac{1-(\cosh\alpha L\cos KL + j\sinh\alpha L\sin KL)}{\sinh\alpha L\cos KL + j\cosh\alpha L\sin KL}\right]e^{j\omega t}$$

$$P = \frac{A\rho_0}{K\left[1+\left(\frac{\alpha}{K}\right)^2\right]^{\frac{1}{2}}}\left[\frac{(1-\cosh\alpha L\cos KL)^2+(\sinh\alpha L\sin KL)^2}{(\sinh\alpha L\cos KL)^2+(\cosh\alpha L\sin KL)^2}\right]^{\frac{1}{2}}$$

Again making the assumptions previously made,

$$P = \frac{A\rho_0}{K}\left[\frac{(1-\cos KL)^2+(\alpha L\sin KL)^2}{(\alpha L\cos KL)^2+\sin^2 KL}\right]^{\frac{1}{2}}$$

This expression approaches a maximum for values of KL = $(2n-1)\pi$ and a minimum for KL = $2n\pi$

Evaluating P at KL = $(2n-1)\pi$

$$P = \frac{2A\rho_0}{\alpha KL}$$

$$\alpha = \frac{2A\rho_0}{P(2n-1)\pi}$$

This expression should provide an experimental method of obtaining α for n different values of KL. The coefficient may also be found by the bandwidth method. The same result, $\alpha = \Delta K$ holds for this system as for the system with only one end driven.

The advantage of having a very tight system, characteristic of this system, was offset by several disadvantages. First, the

system was weight limited, which limited the size of tube to be shaken and eliminated the possibility of water jacketing the tube. Second, the length of the tube could not be varied. Third, and most serious, the microphone mounting could not be satisfactorily isolated from the vibration of the system. The microphone was extremely sensitive to vibration and the sensitivity was a very critical and unpredictable function of frequency.

The prediction that pressure would be at a maximum at odd integral values of n was verified. The prediction that pressure would be at a minimum for even integral values of n was not verified, since relatively weak but still discernable resonance peaks were found there. These weak peaks are probably the result of harmonics in the driver input and do not invalidate the theory. The only value of α which was closely consistent with theory was that obtained from the reverberation technique, which indicated that a tight system had been obtained, but not significantly tighter than the system with only the driver piston moving. Under the circumstances, further investigations of this technique did not seem warranted.

APPENDIX II

ERROR ANALYSIS FOR ATTENUATION MEASUREMENTS

1. Phase Angle Method. Phase angle measurements may be in error by several degrees due to drift in the vertical position of the oscilloscope trace. Frequency measurements may be in error by a maximum of 0.2 cps. Although determination of α from a single measurement of θ and Δf may vary by as much as 50% from the average value, averaging six readings equally spaced about the center frequency should reduce the error to less than 20%.

2. Ratio of Acceleration to Pressure Method. This method is dependent on the accuracy of accelerometer and microphone. Assuming a calibration uncertainty of 1 db in the microphone and 0.5 db in the accelerometer, the overall systematic error is estimated at 15%.

3. Bandwidth Method. As in the phase angle method the major source of error is in the determination of frequency, which should not be in error by more than 0.2 cps. Since the bandwidth was about 1.5 cps maximum error should be approximately 15% for frequencies around 100 cps.

4. Ratio of P_{min} to P_{max} Method. System noise due to 60 cps hum increased the value of P_{min} by an estimated 20%. Although this method is dependent on a ratio of pressures which should make it independent of calibration errors, the large difference between the

two pressures makes it necessary to read the corresponding voltages on different scales of the VTVM, thus introducing a possible error in voltmeter calibration.

5. Reverberation Method. Since the driver piston is stopped for this method the boundary condition for this end is not the same as for the other methods, so the actual attenuation is not necessarily the same. The spread of data points on a semi-log graph for a single determination of α is approximately 3% but separate determinations of α vary as much as 10%, apparently due to changing end conditions.

An additional problem in comparing these methods for consistency is that it is extremely difficult to duplicate end conditions for a series of attenuation determination, even with a well-designed rigid-end piston. A certain determination of α may be entirely valid, but if the microphone piston is withdrawn from the tube and re-inserted the attenuation may change due to the piston being in a slightly different position.

It should be kept in mind that the method of determining α which gives results closest to the theoretical value is not necessarily the best method. Investigations of attenuation have generally indicated values of attenuation 12 to 18% higher than the theoretical prediction.