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An investigation of freezing of a liquid flowing over a flat plate.

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UNITED STATES
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THESIS

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ABSTRACT

The rate at which the solid phase develops in a fluid, moving with Blasius flow over a chilled flat plate is analyzed. The time dependent, local, solid layer thickness is determined as a function of the pertinent physical properties, the plate surface temperature and a one spatial dimension heat flux. Experimental equipment to measure the heat flux, transient ice growth and steady state ice profile is described. The spatial variation of the steady state ice profile is observed with respect to the streamwise coordinate.

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NOMENCLATURE

a_1	dimensionless Blasius stream function
C_1	constant of integration
h_L	latent heat of fusion
k_L	thermal conductivity of the liquid phase
k_S	thermal conductivity of the solid phase
P_R	Prandtl Number
q_L	heat flux of the liquid phase in the y direction
q_S	heat flux of the solid phase in the y direction
R_e	Reynolds Number
t	time
t_s	parameter with the dimension of time
T_f	fusion temperature of the solid phase
T_L	local temperature of the liquid phase
T_S	local temperature of the solid phase
T_w	temperature of the cold plate surface
T_∞	ambient liquid phase temperature
U_∞	free stream velocity in the x direction
x	dimensional position coordinate tangent to the plate in the direction of flow
y	dimensional position coordinate normal to the cold plate horizontal surface

Greek Symbols

η	transient solid phase thickness
η_s	steady state solid phase thickness
ν	kinematic viscosity
ξ	dimensionless thickness
ρ_L	mass density of the liquid phase
τ	dimensionless time

ACKNOWLEDGEMENT

The author wishes to express his appreciation to Professor M. D. Kelleher for his guidance, encouragement, criticism and unfailing good humor during the course of the investigation. A special note of appreciation is also given to Messers. J. Beck and K. Mothersell of the Mechanical Engineering Machine Shop, for their suggestions and efforts in the construction of the experimental equipment.

1. Introduction

In recent years there has been increased attention directed toward solving the problems of heat transfer through a moving fluid undergoing a phase change. The greater part of the work in this area is concentrated in the area of boiling; its various modes, bubble behavior and interactions. On the other hand, the rate of solid phase development in a fluid and the geometry of the steady state profile have been subjected to some new investigations. Much of the mounting interest in "phase-change" problems can be found in the aerospace and nuclear power industries. In the analysis of airfoils, consideration must be given to ice build-up. Proper design of space vehicles demands hydraulic systems and heat exchangers that are free of unwanted phase changes. Nuclear reactor engineers are studying the complexities associated with using liquid metals in coolant loops. Heat transfer with a phase change is, therefore, one of the problems that has become more acute as modern technology advances.

One of the earliest treatments of the problem of heat conduction through a solid in the presence of a change of phase was Stefan's [1]* classical paper on the formation of polar ice. He determined the position of the freezing line by formulating a one spatial dimension, heat conduction problem for a semi-infinite solid. Stefan assumed the water mass was at its solidification temperature and he obtained an equation for the position of the interface. This equation was, however, in a

*Numbers in brackets refer to references listed in the Bibliography complex mathematical form. A simplified version of that equation developed when the heat capacity of the ice was neglected. Stefan's approximate solution to the original equation stipulated a linear decrease in the solid phase temperature between the melting-point temperature at the interface and an assumed solid phase temperature of zero at the water-line of the ice mass. Neumann [2] provides an exact solution to the Stefan problem without restricting the liquid mass temperature to a constant or having to assume a temperature gradient in the solid phase. Even in a non-flow situation, the phase-change heat transfer problem becomes non-linear and requires special solutions that cannot be superposed. The "melting-freezing" problem in a fluid flow field is difficult to analyze because the spatial position and profile of the phase change boundary are not known a priori and therefore, must be found as part of the problem solution.

A review of recent literature discloses that mathematical analysis using approximate methods is employed in the works of Goodman [3,4,5], Biot [6,7], Libby and Chen [8], and Murray and Landis [9]. Goodman and Shea [3] presented an analysis of the melting problem in a finite slab. Cho and Sunderland [10] used Goodman's integral technique to find the temperature distribution in their work on melting finite slabs. Tien and Yen [11] applied Goodman's methods to obtain an approximate solution to a Stefan problem in which a semi-infinite slab melts at its lower face and the liquid phase is subjected to a negative temperature gradient. Assuming the liquid density is inversely proportional to the temperature, the system is seen to be inherently unstable and the heat transfer in the liquid is enhanced by natural convection.

Liquid solidification in a circular tube under steady state conditions was analyzed by Zerkle and Sunderland [12] and equations for the temperature, velocity and pressure distributions in a liquid flowing through a circular tube with solidification at the inner wall surface were derived. In the analysis it was assumed free convection within the liquid was negligible. The experimental data presented showed a significant variation between theoretical predictions and experimental results when convection exists. Transient freezing of a liquid flowing inside a circular tube was investigated analytically by Özisik and Mulligan [13]. They assumed the liquid flow characteristics inside the tube could be represented by a slug-flow velocity profile. Their other assumptions were similar to those of Zerkle and Sunderland [12]. Özisik and Mulligan obtained closer correlation to Zerkle and Sunderland's experimental data for steady-state solid-liquid profiles and total steady-state heat transfer rates than did Zerkle and Sunderland. The results of the transient part of Özisik and Mulligan's analysis was not correlated with experimental data because no published data existed for their situation. They limited the applicability of their results to small rates of change of thickness of the solid layer with time and with distance along the tube.

The freezing of fluids in forced flow over a flat plate has been analyzed mathematically by Beaubouef and Chapman [14]. Their exact numerical solution for solid phase thickness as a function of time includes the heat capacity of the ice layer. It compares closely with the approximate solution of Lapadula and Mueller [15] who used Biot's methods.

The unsteady formation of ice in a plane stagnation flow was studied by Yang [16]. The problem dealt with an instantaneously cooled plate normal to an isothermal flow of liquid. Short and long time solutions were obtained analytically for the ice thickness profile. Velocity profile distortions caused by the advance of the ice front were also discussed.

Published experimental results relevant to solidification on a flat surface geometry are somewhat limited. The work of Savino and Siegel [17, 18] provides much of the available experimental information. They did not consider the details of the fluid flow field in their analysis and correlations with experimental results. The heat capacities of the plate and the solidified material were accounted for in reference 18.

This thesis, therefore, was undertaken to compare the experimental findings of Siegel and Savino using a plate of similar geometry, but larger in size. Instrumentation was provided to gain information on transient solidification in the flow field as a function of time and also as a function of the streamwise spatial coordinate. The capability of direct measurement of the thickness contour of the steady state ice profile was included in the experimental equipment design. Further, a different analysis was developed for predicting the transient thickness of the solidified layer.

2. Theoretical Analysis

The problem being considered is the freezing of a fluid in steady, laminar, forced flow over a flat plate that has been suddenly cooled.

The basic assumptions used are similar to those stated in references 14, 17 and 18. They are:

a. The thickness of the deposited layer, η , is such that heat conduction in the solid layer is considered one-dimensional.

b. Blasius flow is assumed so that the fluid boundary layer is not affected by the solid phase geometry.

c. The temperature of the cold plate, T_w, and the solid-fluid interface temperature, T_f, are considered constant.

d. The physical properties of both the liquid and solid phases are constant.

e. The heat capacity of the ice layer is negligible.

The one-dimensional model and some of the notation employed are shown in Figure 1. A warm liquid, moving with Blasius flow, passes over a flat plate. The plate is chilled and freezing is assumed to begin (t=0) as the plate's upper surface reaches the temperature of solidification of the liquid. During the freezing process the latent heat of fusion, h_L, is released at the liquid-solid interface. The solid phase develops until it reaches a steady-state thickness, η_s . It is assumed that this thickness will never become so great as to disturb the Blasius flow.

Neglecting the heat capacity of the ice, the one - dimensional heat flux for the solid phase is:

$$q_s = k_s \frac{T_f - T_w}{\eta} \tag{1}$$

where the solid phase thickness η , is a function of time and the streamwise spatial coordinate, x.

An interface energy balance yields:

$$q_s = k_L \left(\frac{\partial T_L}{\partial y}\right)_{y=\eta} + \rho_L h_L \left(\frac{\partial \eta}{\partial t}\right) \tag{2}$$

For Blasius flow the temperature gradient is given by reference 19 as:

$$\left(\frac{\partial T_L}{\partial y}\right)_{y=\eta} = \sqrt{\frac{U_\infty}{\nu x}} \; a_1 \left(T_\infty - T_f\right) \tag{3}$$

where $a_1 = 0.332 \, P_R^{1/3}$

Substituting equation (3) into equation (2) and solving for the freezing rate

$$\left(\frac{\partial \eta}{\partial t}\right) = \frac{k_s}{\rho_L h_L} \; \frac{T_f - T_W}{\eta} \; - \; \frac{K_L}{\rho_L h_L} \sqrt{\frac{U_\infty}{\nu x}} \, (.332) \, P_R^{1/3} \left(T_\infty - T_f\right) \tag{4}$$

For steady state conditions

$$\left(\frac{\partial \eta}{\partial t}\right)_{\eta = \eta_s} = 0$$

Then equation (4) can be solved for the steady state thickness

$$\eta_s = \frac{k_s}{K_L} \; \frac{\left(T_f - T_W\right)}{\left(T_\infty - T_f\right) \sqrt{\frac{U_\infty}{\nu x}} \, (.332) \, P_R^{1/3}}$$

Substituting in the Reynold's number

$$Re_x = \frac{U_\infty x}{\nu}$$

get

$$\eta_s = \frac{k_s}{K_L} \; \frac{\left(T_f - T_W\right) x}{\left(T_\infty - T_f\right) (.332) \, Re_x^{1/2} \, P_R^{1/3}} \tag{5}$$

Equation (4) can then be transformed into non-dimensional form by dividing by η_s :

$$\frac{1}{\eta_s}\left(\frac{\partial \eta}{\partial t}\right) = \frac{\partial}{\partial t}\left(\frac{\eta}{\eta_s}\right) = \frac{k_s}{\rho_L h_L}\left(\frac{T_f - T_w}{\eta_s^2}\right)\frac{\eta_s}{\eta} - \frac{k_L}{\rho_L h_L}\,\frac{Re_x^{1/2}}{x}\,\frac{(.332)P_R^{1/3}}{\eta_s}\left(T_\infty - T_f\right)$$

$$= \frac{k_s}{\rho_L h_L}\left(\frac{T_f - T_w}{\eta_s^2}\right)\left[\frac{\eta_s}{\eta} - \frac{K_L}{K_s}\left(\frac{T_\infty - T_f}{T_f - T_w}\right)\frac{(.332)Re_x^{1/2} P_R^{1/3}}{x}\,\eta_s\right]$$

$$\frac{\partial}{\partial t}\left(\frac{\eta}{\eta_s}\right) = \frac{k_s}{\rho_L h_L}\left(\frac{T_f - T_w}{\eta_s^2}\right)\left[\frac{\eta_s}{\eta} - 1\right] \tag{6}$$

Defining a parameter whose dimension is time as

$$t_s \equiv \frac{\rho_L h_L}{k_s}\left(\frac{\eta_s^2}{T_f - T_w}\right) \quad ;$$

non-dimensional time becomes

$$\tau = \frac{t}{t_s}$$

and non-dimensional thickness is

$$\xi = \frac{\eta}{\eta_s}$$

Therefore, equation (6) becomes

$$\frac{d}{d\tau}(\xi) = \frac{1}{\xi} - 1$$

$$= \frac{1-\xi}{\xi}$$

or

$$\frac{\xi\, d\xi}{1-\xi} = d\tau \tag{7}$$

Integrating equation (7), we get

$$(1-\xi) - \ln(1-\xi) = \tau + C_1 \tag{8}$$

To evaluate the constant of integration the following initial condition is used:

at $\tau = 0, \xi = 0$

Then from equation (8) get

$$C_1 = 1$$

and

$$\tau + 1 = (1 - \xi) - \ln(1 - \xi)$$

or

$$\tau = -[\xi + \ln(1 - \xi) \qquad (9)$$

Figure 2 is a graph of equation (9).

The above solution can be shown to be the limiting case for a solid phase heat capacity equal to zero in the analyses of Beaubouef and Chapman [14] and Savino and Siegel [17].

This analysis is unique in its straightforward derivation of the closed form solution as compared with the equations given in references 14 and 17. The above solution also has a distinct advantage in its ease of computation.

3. Experimental Equipment and Procedure

The experimental equipment shown schematically in Figure 3 utilized a variable speed, axial flow pump to control the flow of ordinary tap water over the test plate. The pump took suction from a reservoir. Re-circulation of a large water volume during each test run did not change the temperature of the liquid because of the large thermal capacity of the water mass. From the pump, the water passed through an A.S.M.E. calibrated orifice, a diffuser and a series of flow straighteners consisting of a metal honeycomb and wire screens. The test section of the water channel was nominally one foot square in cross-section. From the channel, the liquid flowed into a back-water tank which was used as an aid to flow control. The water passed from the tank, through a control valve and a rotometer, back to the reservoir. The flow straighteners in the channel upstream of the test section provided a laminar, uniform water velocity profile across the plate surface. The test section itself was a clear plastic rectangular channel (see Figure 4). The support frame for the test section was adjustable so as to provide a zero pressure gradient in the flow field along the bottom of the channel.

The refrigerant was re-circulated through a closed loop by a one-half horsepower commercial compressor unit. The boiling chamber was soldered to the bottom of the test plate. Freon flow was regulated through a manually controlled expansion valve. The temperature of the refrigerant was adjusted in each test run by control of the pressure in the boiling chamber.

The plate was machined from oxygen free, high capacity, pure copper

12 inches long, 6 inches wide and 2 inches thick. Copper was selected as the test material because of its extremely high thermal conductivity. This property, acting in conjunction with the plate thickness was deemed sufficient to insure a constant plate temperature at its top surface. The upper surface of the test plate was machined flat and highly polished to minimize the possibility of any flow disturbances across it.

The test plate and refrigerant boiling chamber were supported by a frame designed to reduce conduction heat losses to a minimum and to provide position adjustment such that the top surface of the plate lay in the plane of the channel bottom (see Figure 4). A rubber-base, waterproof sealant of low thermal conductivity was used to fill the gap between the plate edge and the plastic of the channel bottom. All other exposed sides of the plate and the boiling chamber were covered with a styrofoam insulating material.

Instrumentation of the test plate consisted of nine stainless steel sheathed, ceramic insulated, ungrounded, copper-constantan thermocouples. The outside diameter of the sheathing was 1/16 inch and the thermocouple wires were number 30 AWG, with a nominal diameter of 10 mills. The location of the thermocouples is indicated in Figure 5. This particular arrangement provided a temperature profile across the central area of the top horizontal surface of the plate. Five evenly spaced points across the thickness of the plate near the vertical centerline of the mass of the plate were selected to provide heat flux information. These thermocouple positions were purposely staggered slightly off the plate centerlines to reduce mutual interference and minimize discontinuities in the heat flow.

Benefiting from the experience of Savino and Siegel [17], a thermocouple probe (Figure 6) was used to measure the ice layer thickness. Its construction was similar to that described in reference 17. The probe consisted of butt welded copper-constantan wires of .003 inch diameter. The wires were supported by two sewing needles mounted at 90° angles to the bottom of a ¼-inch stainless steel rod. The needle mounting was designed to reduce flow disturbance around the thermocouple junction. The rod extended up to the top of the channel where it was attached to a micrometer head through a sleeve and cross-bar device. The micrometer spindle was mounted in a ball bearing attached to the cross channel carriage, through which the sleeve and probe traveled. The entire device enabled the micrometer head to raise and lower the thermocouple while maintaining a preset alignment of the needles with respect to the fluid flow. The micrometer head was attached to a two dimensional traversing carriage. The carriage frame was mounted on the test section with leveling screws. Thus, the thermocouple junction enjoyed movement in three degrees of freedom and could be accurately positioned at any specified height above the plate as well as at any position over the horizontal area of the test plate. Machinist scales and indicators were attached to the traversing carriage to position the thermocouple at various pre-selected points on the plate's horizontal surface during successive test runs. As Savino and Siegel [17] noted: The thermocouple and its needle supports were sufficiently small so that no local distortions of the interface were observed when the thermocouple was placed in contact with it.

The reference junction for all thermocouples was boiling nitrogen at atmospheric pressure. All test data was observed away from the edges of the plate where the heat transfer was more closely one-dimensional (a basic assumption in the theoretical analysis). A procedure described by Savino and Siegel [17] was used to measure instantaneous thickness during a test run. The thermocouple junction was set 0.005 inch above the plate, and the probe temperature was monitored using a tape output recorder. As the ice layer developed, the probe temperature decreased and at 32°F. the ice layer contacted the probe. The thermocouple junction was then raised 0.005 inch rapidly so that it would not freeze in the ice. The recorder then registered a step increase in the fluid temperature and the cycle was repeated. The recorder output indicated a series of steps and the chart recorded the time increment corresponding to each 0.005 inch of growth of the ice layer.

Direct measurement of the thickness of the ice contour was undertaken through use of the two dimensional traversing carriage. A length of drill rod was substituted for the thermocouple probe. The lower end of the rod was shaped into a semi-sphere. The machinist scales and indicators, mounted on the carriage frame in the direction of flow and normal to it, provided a means to position the depth indicating rod over any predetermined plate position. The micrometer head was set at zero when the rod rested on the plate surface. In this configuration the traversing carriage indicated the ice thickness at selected uniform intervals over the horizontal surface of the layer.

4. Discussion of Theoretical and Experimental Results

The theoretical analysis assumed that, at zero time, the plate surface over which the water was flowing was at the freezing temperature and ice began to form. Under experimental conditions, ice formed rapidly and evenly over the horizontal surface of the plate. Five seconds elapsed between formation of the initial patch of ice on the upstream edge until the plate was completely covered. Ice formed along the direction of flow. This fact, coupled with the observation that the refrigerant entered the boiling chamber at the upstream edge of the plate, makes apparent the existence of a layer of subcooled water in contact with the plate prior to the initial ice nucleation. Positive verification of this phenomenon was not obtained because the thermocouple recorders available for use, at the time of the experiment, did not possess the required sensitivity. Electromotive forces of the order of 5 millivolts were encountered when liquid nitrogen was used as a thermocouple reference junction. A full scale value of 5 millivolts on a recorder strip chart resulted in poor temperature discrimination near the freezing level. Transient temperature data from the plate and the probe did not provide a valid comparison for the theoretical analysis because of the same recorder limitations.

Meaningful data was hampered by particles of rust and debris that deposited on the bottom of the test section. The sources of this foreign matter and the oil droplets in the water was traced to the piping system and to leakage of lubricating oil from the axial flow pump respectively.

The steady state ice layer was nominally .1 inch thick over the surface of the plate. Edge effects were confined to a 1/8 inch perimeter around the downstream edge and the two edges parallel to the flow. Streaks in the ice layer extended from the upstream edge for ½ inch in the flow direction. The streaks were caused by small irregularities in the sealant applied at the edge and to debris on the bottom of the test section. Localized depressions in the ice layer thickness occurred around the sheathed thermocouples imbedded in the plate's surface. These irregularities became less pronounced as the ice thickness increased. In all other areas of the plate the ice was clear and its surface was very smooth.

While measuring the thickness of the steady state ice layer, the rod was observed to cause depressions in the ice layer. If the rod was left in the proximity of the layer it would distort the flow to the extent that ridges would develop on the surface of the ice.

Much time was expended in obtaining the optimum refrigerant charge for the system. Changes in flow velocity had great effect on the cooling rate produced by the freon.

5. Conclusions

The object of the theoretical analysis was to develop a means for predicting the transient growth of the solid phase of a liquid flowing over a chilled flat plate. The object was achieved by making two basic assumptions: (1) the liquid moved over the plate with Blasius flow and (2) conduction in the solid was one dimensional and the heat capacity of the ice was neglected.

The layer of subcooled water in contact with the plate prior to ice nucleation must be accounted for to make experimental data correspond with the analysis.

The recording equipment, in its present configuration, will not provide usable data on transient temperatures in the plate. The thermocouple probe record did not provide an accurate indication of solid phase development. The rod used to measure the steady state ice layer thickness distorted the flow and it conducted heat toward the ice surface.

An acceptable steady state ice layer thickness can be obtained by increasing the capacity of the refrigerant boiling chamber. In its present design, almost half of the area of the chamber that is in contact with the plate is used for baffles and flow straighteners. More flexibility for the refrigerant system to meet a wide spectrum of cooling rates would be a desirable feature in experimental equipment of this kind.

The difference in conductivity of the bulky thermocouples imbedded in the surface of the plate and the solder that surrounds them cause noticeable distortions in the temperature distribution and the ice layer.

6. Recommendations for Future Work

Install a filter bed between the pump and the test section to remove the suspended dirt, rust particles and oil seepage from the pump lubricating system.

Obtain Blasius flow over a greater range of water channel velocities by modifications in the design and combinations of the metal honeycomb and wire screens used to straighten the flow.

A table of test section flow velocities versus pump speed, pump control board settings and depth of water in the channel would aid investigators who plan to use the channel for further experiments.

Use a multi-channel thermocouple strip recorder or a digital voltmeter coupled with an automatic sequential scanning switch to obtain transient temperatures.

Investigate the feasibility of using other than boiling liquid nitrogen as a reference junction. For example, an ice bath may work with a digital voltmeter or liquid oxygen may provide a more stable reference.

Increase the volume of the refrigerant boiling chamber and reduce the thickness of the baffles. Install a receiver in the refrigeration unit to increase the volume of freon in the system. This in turn would improve the flexibility of the system to respond to variations in plate cooling rates imposed by an investigator.

Reduce the size of the thermocouples on the surface of the plate. The difference in conductivity of the sheathing and copper affect the shape of the ice layer. Small diameter wire thermocouples would require more shallow grooves and less solder. They have the added advantage of a faster time response than the sheathed type.

Insulate the tip of the rod used for steady state profile measurement, with a material of low thermal conductivity. Streamline both the rod and the probe shafts to reduce the flow disturbances.

Obtain experimental data holding one of the following variables constant while measuring the other two:

a. water flow rate

b. refrigerant flow rate

c. position on the plate horizontal surface.

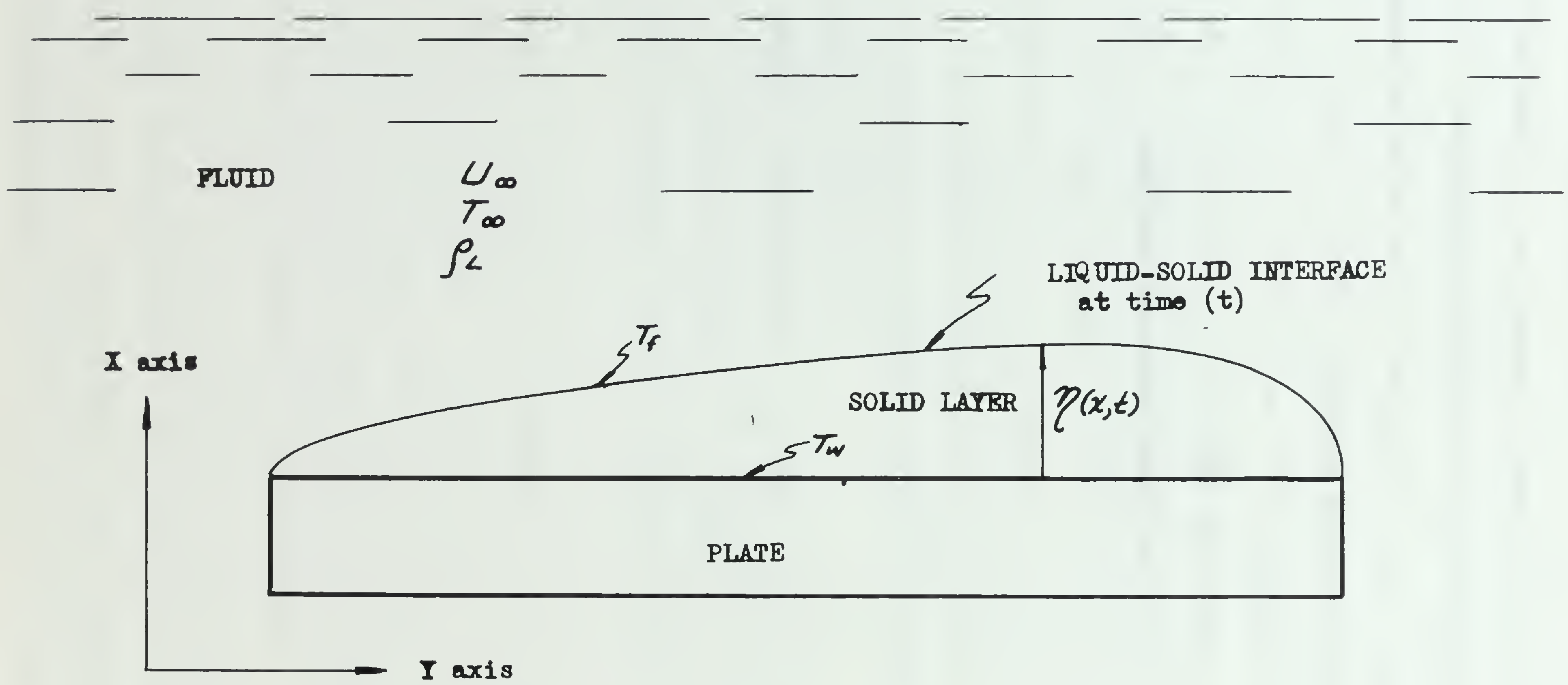


FIGURE 1. ONE DIMENSIONAL MODEL OF THE TRANSIENT SOLIDIFICATION PROBLEM

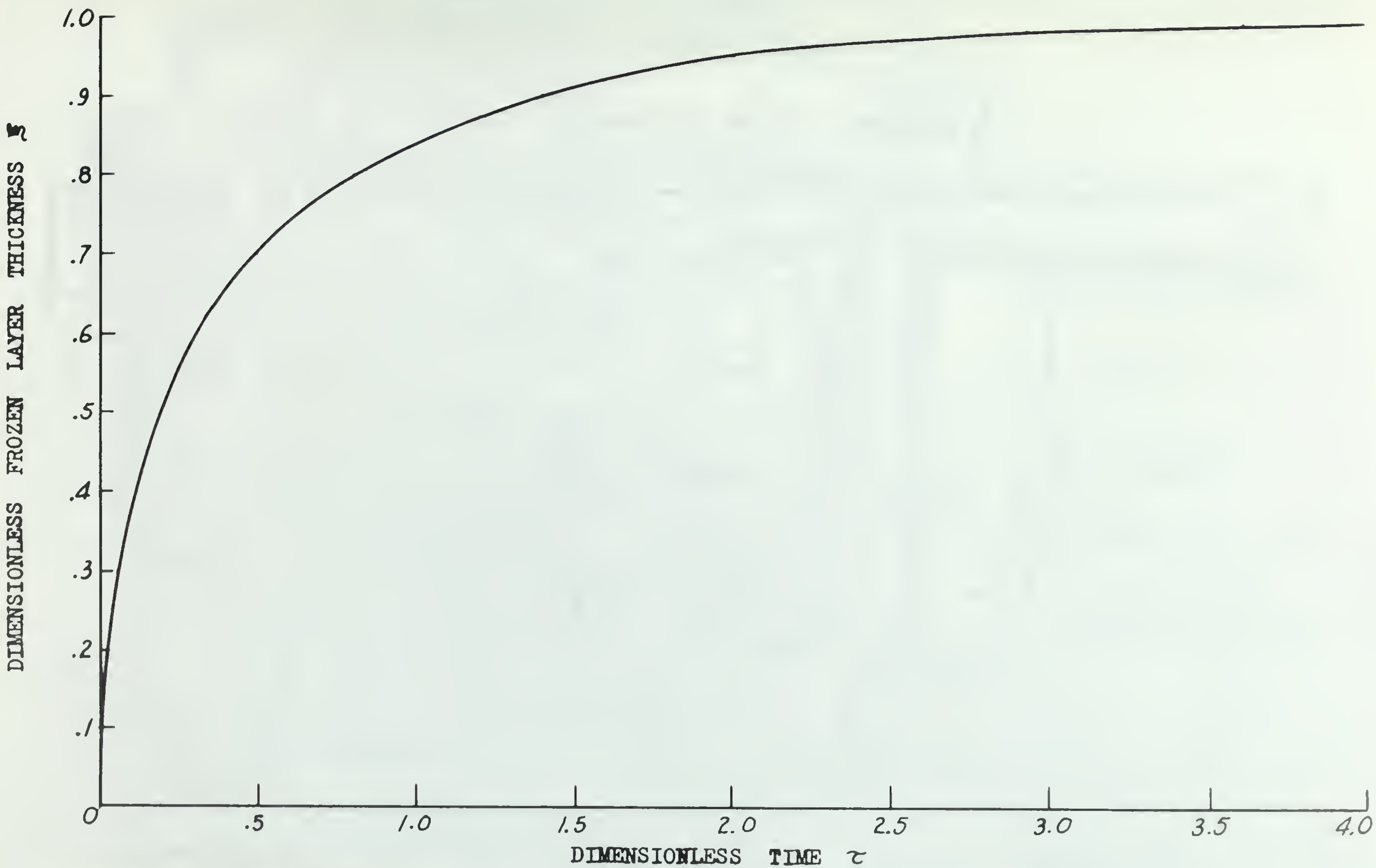


FIGURE 2. GRAPH OF THE ANALYTICAL SOLUTION

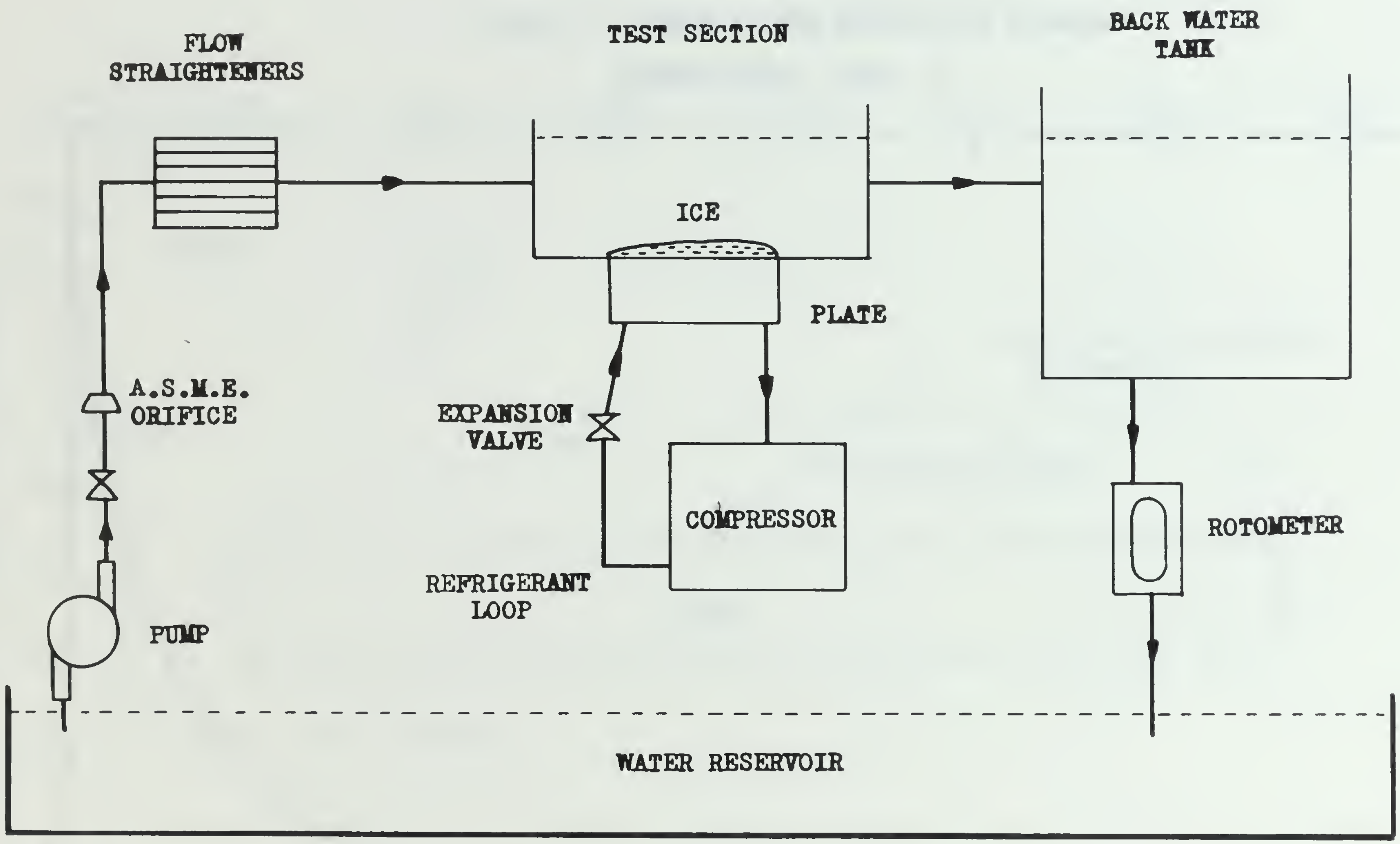


FIGURE 3. SCHEMATIC OF THE TEST EQUIPMENT

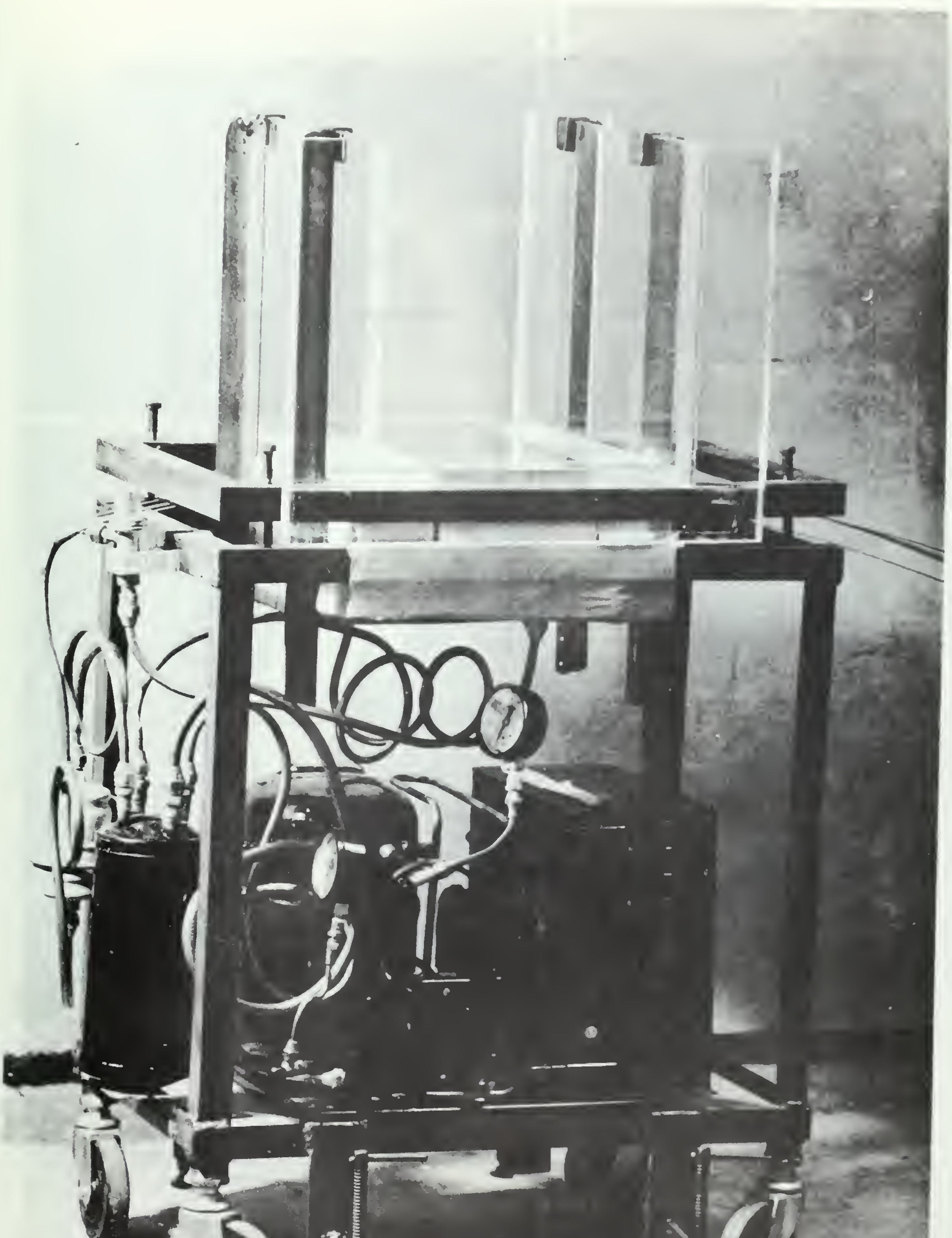
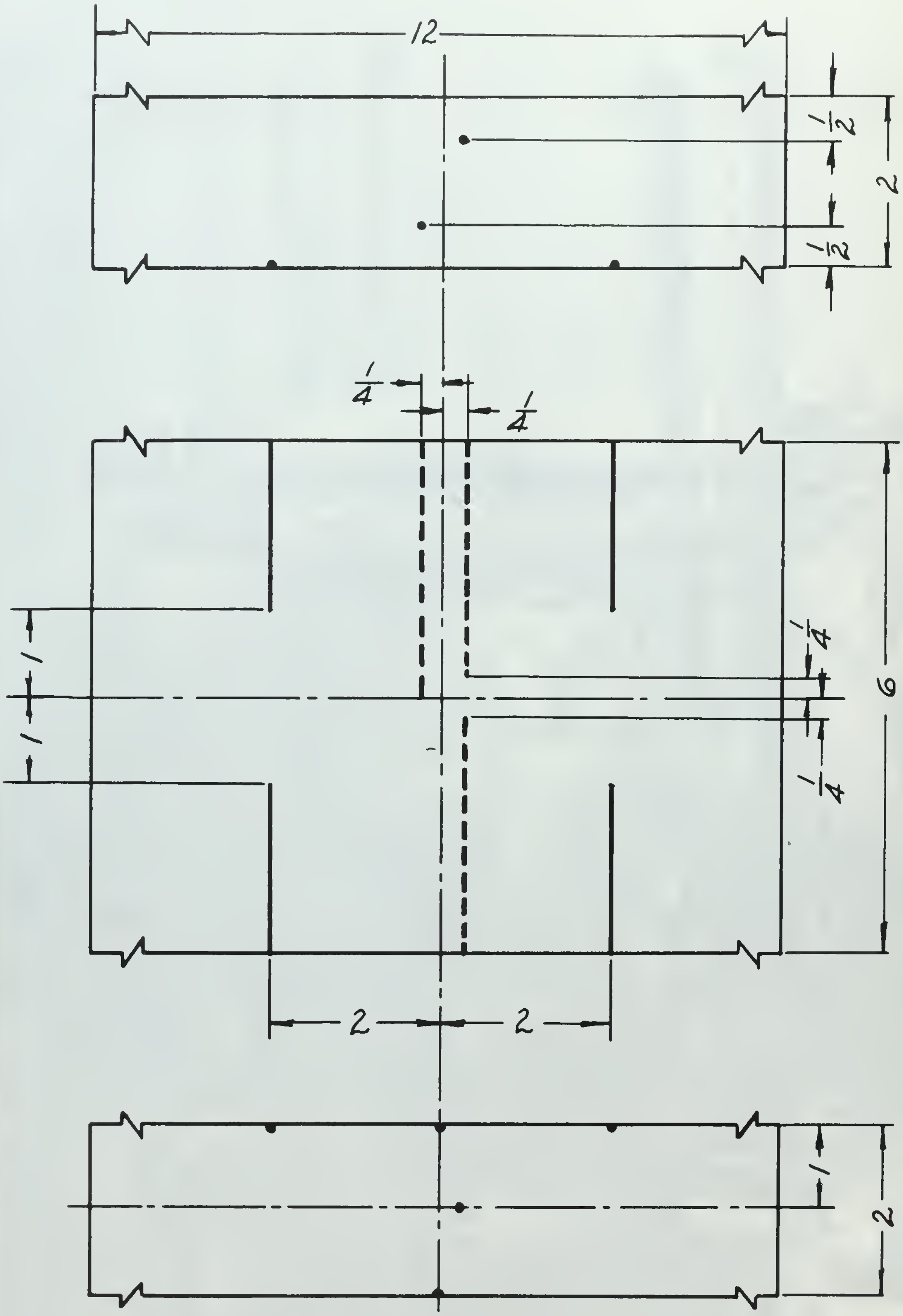


FIGURE 4. TEST SECTION



NOTE: ALL THERMOCOUPLE LOCATIONS AND PLATE DIMENSIONS
ARE IN INCHES.

FIGURE 5. TEST PLATE INSTRUMENTATION

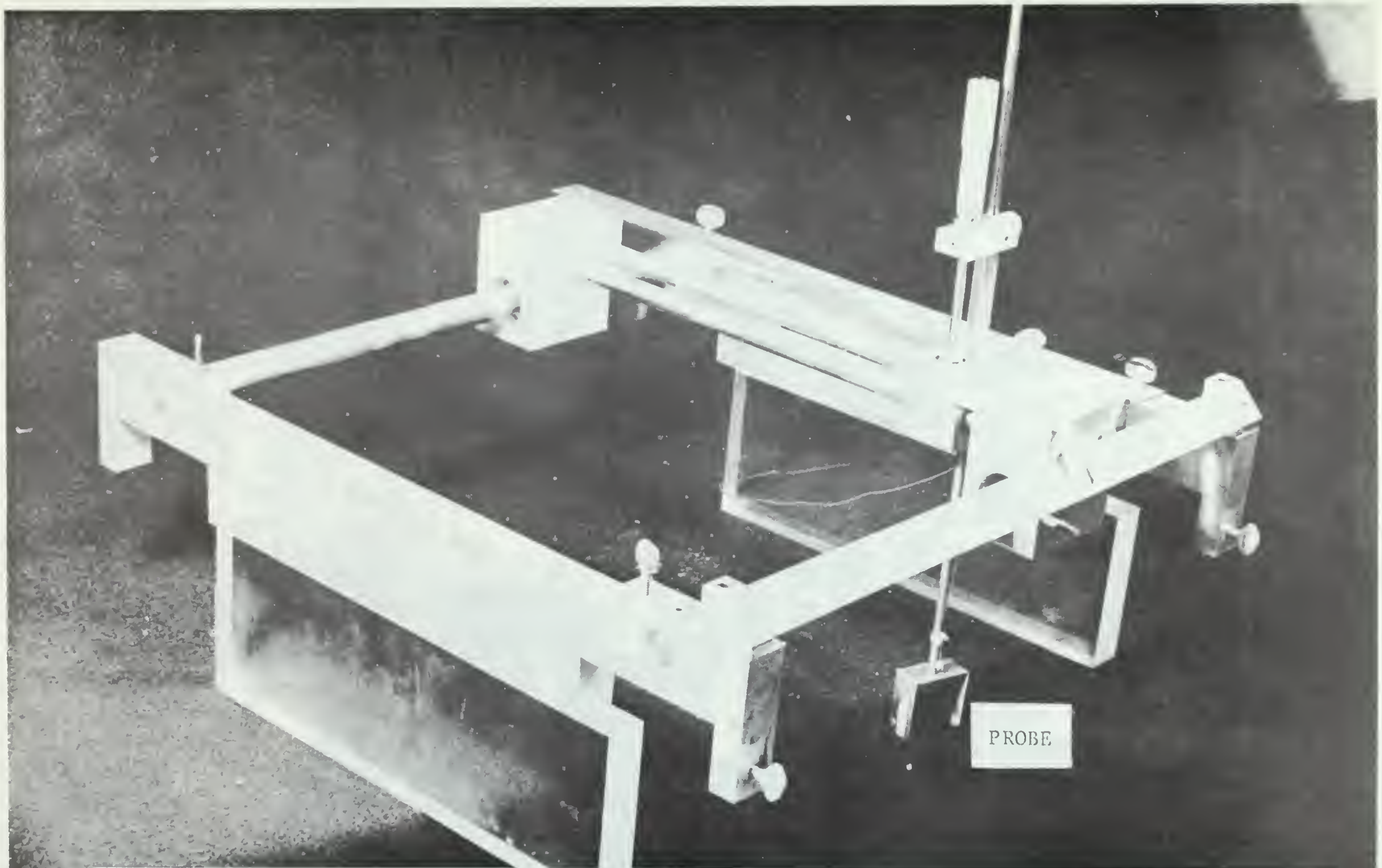


FIGURE 6. DETAIL OF THE THERMOCOUPLE PROBE AND TRAVERSING CARRIAGE

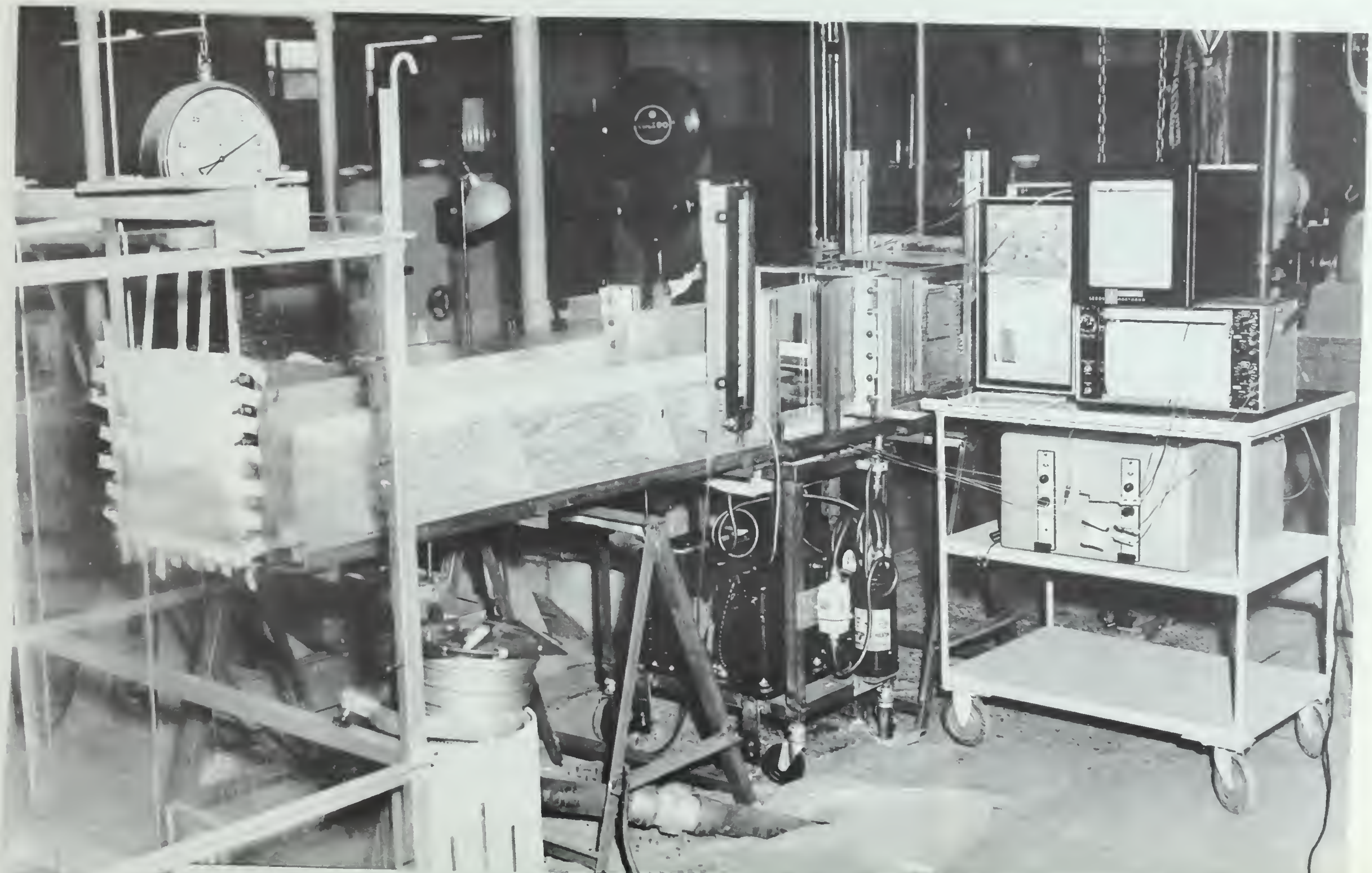


FIGURE 7. VIEW OF THE ENTIRE APPARATUS

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ABSTRACT

The rate at which the solid phase develops in a fluid, moving with Blasius flow over a chilled flat plate is analyzed. The time dependent, local, solid layer thickness is determined as a function of the pertinent physical properties, the plate surface temperature and a one spatial dimension heat flux. Experimental equipment to measure the heat flux, transient ice growth and steady state ice profile is described. The spatial variation of the steady state ice profile is observed with respect to the streamwise coordinate.

14 KEY WORDS	LINK A		LINK B		LINK C	
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Ice Formation						
Solidification						
Liquid-Solid Phase Change						
Ice on Flat Plate						
Freezing of a Moving Liquid						