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Optimal mix of ammunition inventory and production capacity.

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Louis Baeriswyl

OPTIMAL MIX OF AMMUNITION INVENTORY AND PRODUCTION CAPACITY.

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by
Louis Baeriswyl, Jr.

APPROVAL SHEET

Title of Thesis: Optimal Mix of Ammunition Inventory and Production Capacity.

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ABSTRACT

Title of Thesis: Optimal Mix of Ammunition Inventory and Production Capacity.

Name of Candidate: Louis Baeriswyl, Jr.
Master of Arts, 1968

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Edward Morrison

What is the optimal mix of ammunition inventory and production capacity to meet mobilization requirements for a new ammunition item? This thesis presents an economic model to answer this question. Noteworthy features of this model include:

(1) The discounting of the cost stream associated with the optimal mix,

(2) An optimization that includes costs currently omitted in mobilization planning, namely, inventory storage and handling costs,

(3) The introduction of a quick reaction production capacity as a possible alternative to facilities with normal production lead times, and

(4) An extension of the mobilization planning horizon to the estimated end of a general war rather than only to that time when production output first equals expenditures.

The model is used to identify the optimal mix for a hypothetical 105mm high explosive howitzer round about to be

introduced into the inventory. Inputs consisted of hypothetical fixed and variable costs of production and storage, an ammunition expenditure profile, a fixed minimum ammunition level to meet the requirements for prepositioned ammunition and pipeline stocks, two production lead times, and a variety of general war scenarios.

The thesis shows that:

(1) The choice of the discounting interest rate significantly changes the optimal mix of ammunition inventory and production capacity; and

(2) The ability to swing into production quickly can justify high investment and maintenance costs for a quick reaction facility.

OPTIMAL MIX

OF

AMMUNITION INVENTORY AND PRODUCTION CAPACITY

by
Louis Baeriswyl, Jr.

Thesis submitted to the Faculty of the Graduate School of the University of Maryland in partial fulfillment of the requirements for the degree
of
Master of Arts
1968

PREFACE

Prior to initially searching for a suitable thesis subject, I decided to find and work on the solution of an actual defense problem, rather than a finger exercise. The problem had to allow me to demonstrate that I could apply new knowledge gained during the 13 month Defense Education Program course. The problem of determining the optimal mix of ammunition inventory and production capacity was undertaken because:

(1) It is a current defense problem of considerable importance involving substantial amounts of money.

(2) It is an economic problem that appeared to be amenable to solution by the techniques taught during the course.

(3) It is a problem that interested me because of my experience with ammunition expenditure rates in a Marine Corps study of ammunition logistics.

I initially expected to work on a problem that concerned my own service directly rather than on one in which the Marine Corps has only to establish its own end-product requirements. Had I undertaken a Marine Corps problem, I would have saved time by obtaining information in a familiar environment. But, fortunately, I had the generous assistance of U.S. Army personnel and Department of Defense civilians on this thesis problem.

The originality of the proposed solution stems in part from my not having been in close contact with the currently used planning technique. A fresh look was possible. At the same time, this lack of any real experience in the difficult work of ammunition procurement may have caused me to overlook important aspects of the problem. If there are such shortcomings, I naturally assume full responsibility for them.

It is hoped that this work will receive the careful attention of defense planners and that the proposed model or some variant of it will prove useful to them.

ACKNOWLEDGEMENTS

I am especially indebted to Dr. William Pettijohn, Special Assistant, Office of Assistant Secretary of Defense (SA) for suggesting this thesis subject and identifying information sources. Clopper Almon, Jr., Professor of Economics, University of Maryland first suggested that the trade-off between inventory and production capacity be based on a time phased situation. He made the major contribution by simplifying the model that is the heart of this thesis and suggested changes that strengthened and improved the text.

I am indebted to Dr. Edward S. Pearsall, Research Staff Member, Institute for Defense Analysis for identifying this thesis problem as one amenable to solution by linear programming and for initially suggesting the basic constraint equation used in the model. Mr. Edward Morrison, Research Staff Member, Institute for Defense Analysis made a valued contribution by his patient listening and useful comments throughout this undertaking.

I also gratefully express my indebtedness to Mr. Joseph T. Serevo, Programmer, Institute for Defense Analysis for programming assistance that far exceeded his regular responsibilities and the extra hours of work done in my behalf. I am particularly appreciative of the assistance of

my wife, Harriett, not only for her preparation and typing of this thesis throught its several drafts, but for her continuing encouragement. My family has made a real contribution to this year of schooling by their understanding, by freeing me of all the usual homeowner tasks and foregoing family activities that have allowed me to study and prepare this thesis.

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INTRODUCTION

A. Introduction.

When a war starts, the pressing demand for ammunition must be met initially from inventory. After some delay, production begins to meet the demand. The quicker production can start, the smaller inventory has to be. But a quick start requires that production capacity be kept ready. Both inventory and production readiness are expensive, but what is the optimal mix between the two?

This thesis presents an economic model for answering that question. The results of the model strongly suggest that we are presently spending more on inventory and less on capacity than we should. They also show that the choice of the discounting interest rate significantly changes the optimal mix of ammunition inventory and production capacity.

B. Organization of Thesis.

This thesis is divided into four chapters. The first is a brief introduction followed by the description of the evolved economic model. The third chapter shows and discusses the resulting optimal mixes obtained by applying input data for a hypothetical 105mm howitzer round and situation to the model. The final chapter contains the conclusions.

CHAPTER II

DESCRIPTION OF MODEL

A. Mathematical Model for Solution of Optimal Mix.

A mathematical model of simultaneous linear equations was built to identify the least-cost mix of ammunition inventory, I, and of the production capacity of two plants, P_1 and P_2. P_1 is a plant that can start production quickly; P_2 is a plant "in moth balls". The model solves for the optimal mix of inventory and production capacity for a new item of ammunition to meet general war requirements. The horizon of this model extends to the end of a hypothetical general war. It includes all ammunition costs for the war for this item. All solutions provide the same benefit, i.e., satisfy war requirements for this ammunition item, so that the least cost solution is optimal.

The model is explained in three steps. First, a simple model of an objective function - cost equation of the I, P_1 and P_2 mix - to be minimized and three constraint equations forms a departure point. Next, the model is refined by expanding the storage cost component in terms of the I, P_1 and P_2 variables. Finally, the cost equation is farther refined by discounting the cost stream associated with the mix. Specific values for coefficients are discussed and identified in Appendix 1. However, coefficients are defined as they are indtroduced.

Two examples are used to introduce the simple model and its first refinement. These are shown in the ammunition inventory profile in Figure 1. The abscissa of this figure is a time scale in months. Zero time is the time when the production of the initial inventory starts. A war is assumed to start at time t_0 and end at time t_3, so that the length of the war is t_3-t_0 months. The preceding period of peace is (t_0-0) or just t_0 months. The ammunition expenditure rate E and production capacities P_1 and P_2 are average rates that are constant with time. Either of the two plants can be in the optimal mix and produce ammunition at their rated capacity P_1 or P_2. The first plant can respond quickly to a production order and produces at full capacity P_1 starting at time t_1 only (t_1-t_0) months after the start of the war; the slower responding second plant starts producing at full capacity P_2 at time t_2 which is (t_2-t_0) months after the start of the war.

The amount of ammunition inventory at any time is shown by the ordinate of Figure 1, usually given in units of thousands of rounds.[1] In Example A, the initial inventory is I_A with the normal response production facility producing with an output rate of P_2. A minimum cost solution

1 Thousands of rounds is abbreviated as K rds subsequently in the text, tables and figures. Production capacity in thousands of rounds per month is abbreviated K rds/mo.

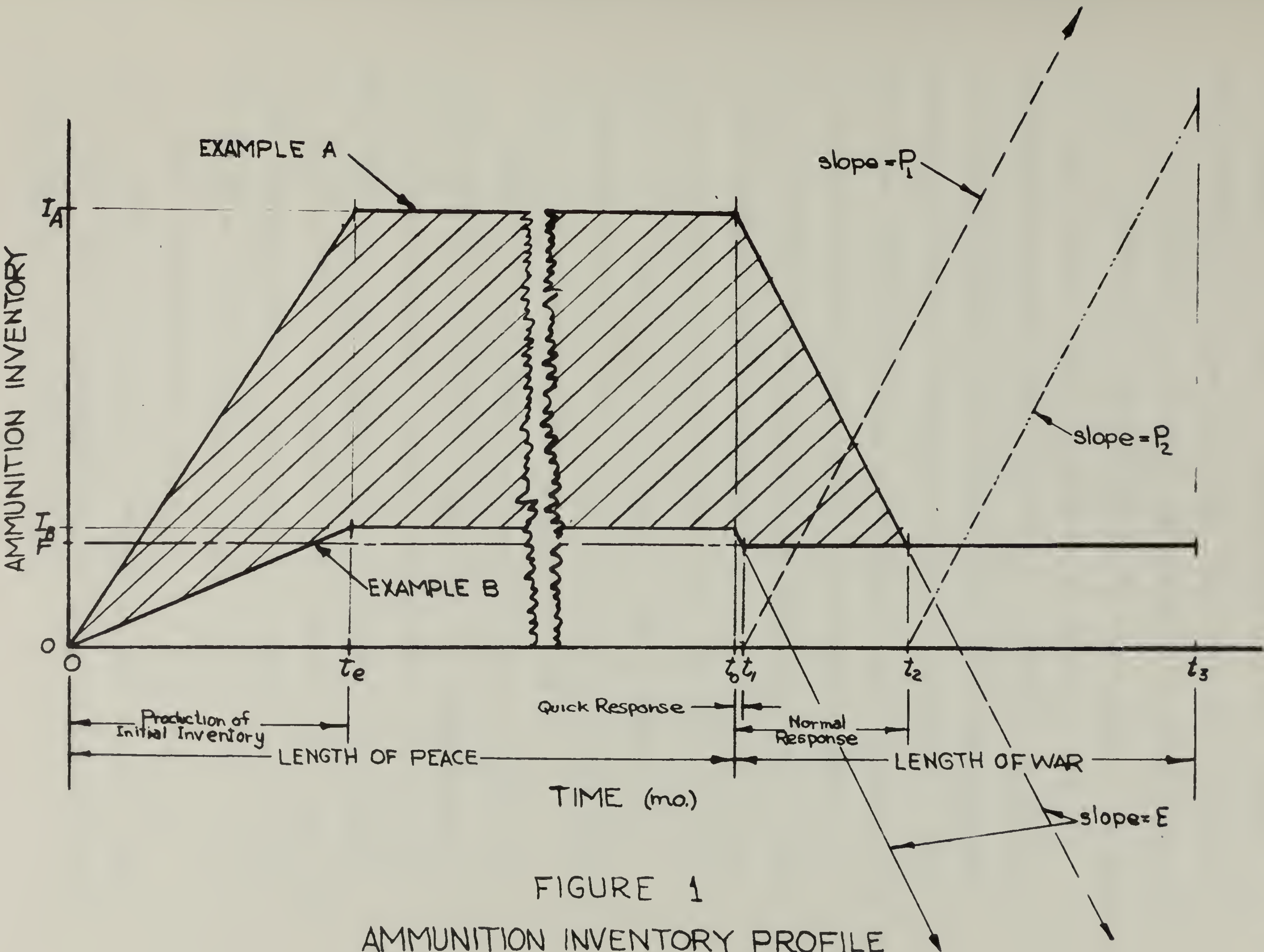

FIGURE 1

AMMUNITION INVENTORY PROFILE

for the mix of I_A and P_2 is illustrated by the constant inventory line from time t_e to t_0. The inventory is reduced from I_A at time t_0 to F at t_2 as a result of combat expenditures. The decline would continue if the normally responding plant did not start to produce at time t_2. The P_2 production capacity just equals the expenditure rate E, so that the inventory is not reduced below the fixed minimum ammunition level, F, during the balance of the war. Note that the initial inventory I_A was just large enough so that it was not reduced below F during the normal response of (t_2-t_0) months. A possible minimum cost solution for this simple example would be one where the production capacity P_2 just equals the expenditure rate E. The slope of the ammunition inventory line between t_0 and t_2 equals the expenditure rate E, which in this case equals the negative of the slope of the produced ammunition line P_2.

Example B illustrates the same situation, except that a quick response facility has replaced the normally responding plant as a possible minimum cost solution. This quick reaction plant, with capacity P_1 also equal to the expenditure rate E, starts producing (t_2-t_1) months sooner than the normal response plant. The result is a reduction in inventory between the two examples, namely, I_A-I_B.

From this example, we see the quick response plant makes a smaller inventory (I_B) adequate. The reduced inventory costs may well outweigh the high costs of providing and maintaining the quick response plant.

B. Simple Inventory and Production Capacity Model with Three Constraint Equations.

The simple model includes an objective function that is a cost equation for the inventory and production capacity. Three constraint equations require that the initial inventory plus produced output be equal to or greater than the fixed minimum ammunition level plus ammunition expended to each of three times, i.e., when the quick and normal production facilities each start producing, and when the war ends. There are also three non-negative side conditions for the initial inventory and the two production capacities.

The three constraint equations are:

$$(1) \quad I \geq F + E(t_1 - t_0)$$

$$(2) \quad I + (t_2 - t_1)P_1 \geq F + E(t_2 - t_0)$$

$$(3) \quad I + (t_3 - t_1)P_1 + (t_3 - t_2)P_2 = F + E(t_3 - t_0),$$

with side conditions:

$$I \geq 0 \ , \ P_1 \geq 0 \ , \ P_2 \geq 0 \ ;$$

where:

I - Initial ammunition inventory level in 1000 rounds.

P_1 - Ammunition production capacity (rate) for quick reaction facility in 1000 rounds per month commencing at t_1.

P_2 - Ammunition production capacity (rate) for normal reaction facility in 1000 rounds per month commencing at t_2.

F - Fixed minimum ammunition level in 1000 rounds.

E - Estimate of mean combat expenditure rate of item of ammunition in 1000 rounds per month.

t - Time in months with zero reference at time that initial inventory is started.

t_e - Time it takes to produce initial inventory.

t_0 - Time ammunition expenditure is initiated; start of war.

t_1 - Time quick reaction production facility starts producing.

t_2 - Time normal reaction production facility starts producing.

t_3 - Time ammunition expenditure is terminated; end of war.

The three constraints are basically the same. The left hand side represents available ammunition, while the right hand side identifies the requirements for ammunition. In general, the initial inventory plus any quantity produced to the time considered, i.e., $(t-t_1)P_1$, must be equal to or greater than the fixed minimum ammunition level plus the combat expenditures to that same time, i.e., $E(t-t_0)$. The first constraint requires that the initial inventory be greater than or equal to the fixed minimum ammunition level plus the ammunition expended from the start of the war until the quick reaction production facility (P_1) starts producing at t_1, i.e., plus $E(t_1-t_0)$. The second and third constraints specify this same relationship at times t_2 and t_3, except that the third constraint equation is an equality as no excess inventory is wanted above F at time t_3. These constraints can be visualized by examining the ordinates of Figure 1 at times t_1, t_2 and t_3.

The constraints can be simplified by substituting:[2]

(3') $P_1 + P_2 = E$

for

(3) $I+(t_3-t_1)P_1 + (t_3-t_2)P_2 = F + E(t_3-t_0)$.

The below cost equation, appropriate to the development of the simple model at this point, is composed of three major cost elements. The first element is associated with the procurement and storage costs of the initial ammunition inventory. The last two elements relate to the fixed and variable costs of the two types of production facilities. The cost components of the latter two elements are similar.

$$C = \left[a_p I + a_r I + a_s \sum_{\phi}(\Delta t_\phi)\bar{I}_\phi\right] + \left[a_1 + a_{1t}(t_1-0) + b_1(t_3-t_1)\right]P_1 + \left[a_2 + a_{2t}(t_2-0) + b_2(t_3-t_2)\right]P_2 ,$$

where:

C - Total cost of satisfying ammunition requirements for a general war in dollars.

a_p- Initial inventory procurement cost coefficient in dollars per 1000 rounds.

---

2 The equality is demonstrated by:

(3') $P_1 + P_2 = E$

(3") $t_3P_1 + t_3P_2 = t_3E$

(2) $I + (t_2-t_1)P_1 \geq F + E(t_2-t_0)$

Adding (3") and (2):

$I + (t_3-t_1)P_1 + t_2P_1 + t_3P_2 \geq F + E(t_3-t_0) + E(t_2)$

$I + (t_3-t_1)P_1 + t_3P_2 - t_2(E-P_1) \geq F + E(t_3-t_0)$

$I + (t_3-t_1)P_1 + (t_3-t_2)P_2 \geq F + E(t_3-t_0)$ which can be changed to an equality to preclude an excess of inventory at t_3.

a_r - Receiving costs coefficient for the one time handling of the initial inventory when placed in and removed from storage in dollar per 1000 rounds.

a_s - Storage costs coefficient associated with initial inventory in dollars per 1000 rounds - months.

a_1 - Fixed cost coefficient of building the quick reaction capacity, i.e., for plant, equipment and long lead time components, in dollars per 1000 rounds per month production capacity.

a_{1t} - Fixed cost coefficient of maintaining the quick reaction facility that is a function of time prior to the start of production, e.g., plant and equipment maintenance costs, long lead time component storage costs, employee training costs, in dollars per 1000 rounds.

b_1 - Variable cost coefficient of production from quick reaction production facility in dollars per 1000 rounds.

a_2 - Fixed cost coefficient of building the normal reaction capacity in dollars per 1000 rounds per month production capacity.

a_{2t} - Fixed cost coefficient of maintaining the normal reaction facility that is a function of time prior to the start of production in dollars per 1000 rounds.

b_2 - Variable cost coefficient of production from normal reaction production facility in dollars per 1000 rounds.

$\bar{I}_\phi$ - Average inventory during time phase ϕ in 1000 rounds.

Δt_ϕ - Time period for storage phase ϕ in months.

The three cost elements of the above cost equation, i.e., those related to I, P_1 and P_2, will be examined in turn.

The inventory cost element, (Cost A) I, consists of three components: (1) the cost of procuring the inventory, (2) a handling cost, and (3) a storage cost. The cost of procuring the initial inventory is the unit cost of the ammunition times the size of the inventory, namely, a_pI.

The initial inventory is produced in the same plant which is in the optimal mix and subsequently provides the mobilization base. As there is ordinarily no urgency to the production of the initial inventory, compared to meeting immediate combat requirements, the cost of the initial inventory should be based upon the lower costs of the normal response plant rather than the quick response plant. The cost coefficients of production facility with capacity P_2 are applicable to the production of the initial inventory. Even if the quick response plant produces the initial ammunition inventory, its extra personnel training costs, and long lead time ammunition component costs to provide the quick response are not applicable charges against the initial inventory. Thus, the initial inventory procurement costs is based on b_2 even though the inventory may be produced by either type plant. Thus, b_2 is substituted for a_p in the cost equation (in Cost A).

When ammunition is placed in a storage site, there is a one time handling cost which is not incurred when ammunition is moved directly to a port of embarkation, and thence to a combat area. This cost is proportional to the amount of ammunition stored, i.e., the inventory. The extra handling cost equals a one time handling cost coefficient (a_r) times the size of the inventory (I).

While the inventory is stored, costs are incurred which are a function of both the storage period and the size of

the stored inventory. These storage costs cover the operation and maintenance of the storage site, e.g., security service, fire protection, and maintenance of storage bunkers. The total storage cost is equal to a composite storage cost coefficient, a_s, times the number of months the ammunition inventory is in storage, Δt_ϕ, times the average number of rounds stored furing that period, $\bar{I}_\phi$. The storage cost will be further explained and expanded in the next refinement of the model. At this point, the costs directly related to the inventory will be represented as:[3]

$$(\text{Cost A})\ I = \left[(b_2+a_r)I + a_s \sum_\phi (\Delta t_\phi)\bar{I}_\phi\right]$$

The costs associated with both type production facilities have the same form. There are three components in both of these cost elements. The first component, $a_1(P_1)$ is the fixed cost of the production facility including the fixed costs for plant, equipment, tools, and the procurement cost of any long lead time ammunition components that are required to permit a quick response to a production order. This fixed cost is divided by the capacity of the plant so that the dimension of a_1 is dollars per 1000 rounds of output per month. The second cost component, $a_{1t}(t_1-0)P_1$, is the maintenance cost of retaining the quick

3 The expression (Cost A) is not correct in detail as $\bar{I}_\phi$ is a function of the variables I, P_1 and P_2. This expression is used in the simple model only to permit the early visualization of the problem in the linear programming tableau.

production facility in a high state of readiness. Such costs are a function of the length of time the plant is held in readiness and the size of the plant, i.e., production capacity. These costs include the normally required maintenance of the production facility and the extra training of the labor force necessary to provide the quick reaction capability. The third cost component is the variable cost of producing ammunition, $b_1(t_3-t_1)P_1$, from the highly responsive plant during the hypothetical war. The coefficient b_1 has dimensions of dollars per 1000 rounds. The cost of the quick reaction capacity is represented as:

$$(\text{Cost B})P_1 = \left[a_1+a_{1t}(t_1-0)+b_1(t_3-t_1)\right]P_1$$

The cost components of the production capacity of the slower reacting plant, i.e., the normal response facility with capacity P_2, are of the same type as those for P_1. However, the time phasing of these latter costs and values for some individual coefficients are different. The costs of the normal response plant can be represented as:

$$(\text{Cost C})P_2 = \left[a_2+a_{2t}(t_2-0)+b_2(t_3-t_2)\right]P_2$$

The linear programming tableau for the simple model is:

$$
\begin{aligned}
(\text{Cost A})I + (\text{Cost B})P_1 + (\text{Cost C})P_2 &= \text{COST} \\
I &\geq F + E(t_1-t_0) \\
I + (t_2-t_1)P_1 &\geq F + E(t_2-t_0) \\
P_1 + P_2 &= E
\end{aligned}
$$

and $I \geq 0;\ P_1 \geq 0;\ P_2 \geq 0.$

C. Refinement of Simple Model by Expansion of Storage Cost Component.

The storage cost component, $a_s \sum_{\phi}(\Delta t_{\phi})\overline{I}_{\phi}$, must be expanded to represent the cost for each of the ϕ storage phases. Since storage costs are a function of the amount of inventory stored, it is necessary to identify the mean inventory of each phase, $\overline{I}_{\phi}$, in terms of the variables of the model. The five mean inventory levels are obtained by averaging the inventory at the beginning and end of each phase. These beginning and end point inventories are illustrated in the inventory profile of Figure 2.

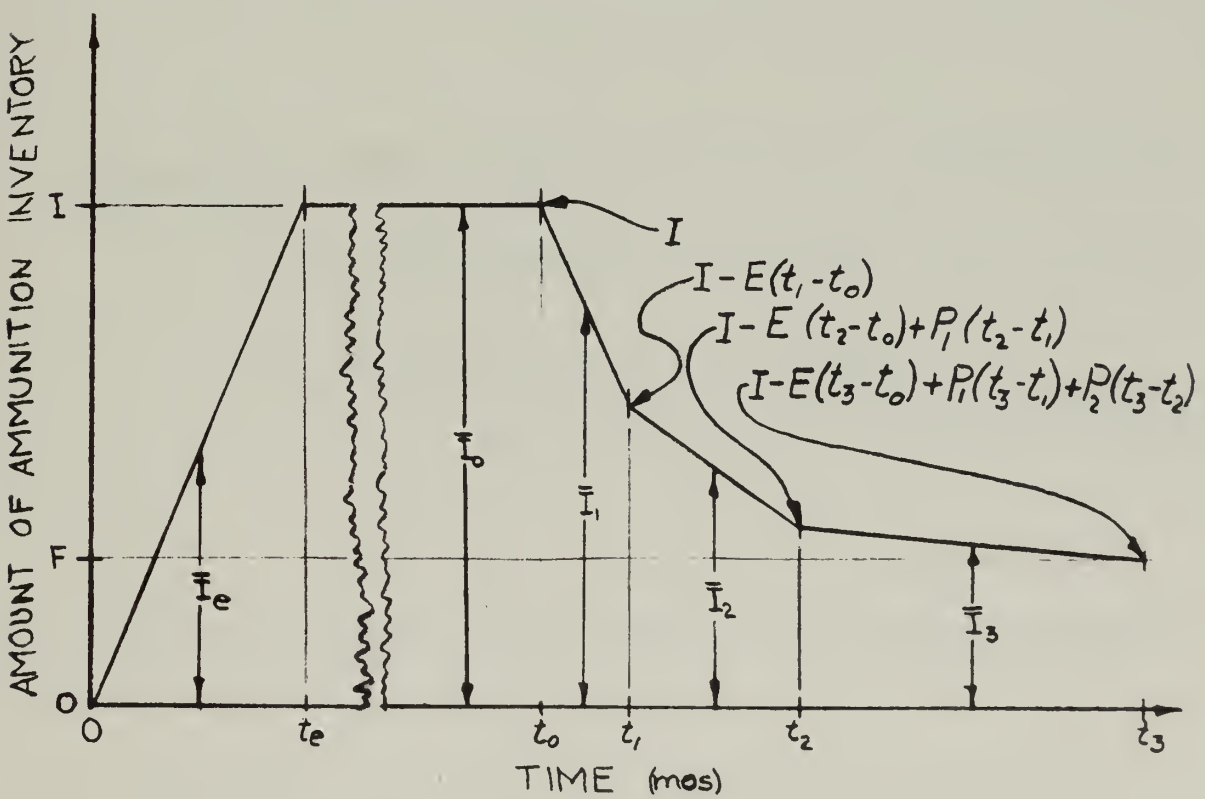

FIGURE 2
AVERAGE AMMUNITION INVENTORY

The average inventory for each phase is identified as $\bar{I}_\phi$. These averages can be quantified in terms of the initial inventory and the amount of ammunition expended and produced. Thus:

$$\bar{I}_e = I/2$$

$$\bar{I}_0 = I$$

$$\bar{I}_1 = I - (E/2)(t_1-t_0)$$

$$\bar{I}_2 = I - (E/2)\left[(t_1-t_0)+(t_2-t_0)\right] + (P_1/2)(t_2-t_1)$$

$$\bar{I}_3 = I - (E/2)\left[(t_2-t_0)+(t_3-t_0)\right] + (P_1/2)\left[(t_2-t_1)+(t_3-t_1)\right] + (P_2/2)(t_3-t_2)$$

The storage cost can then be written as:

$$a_s \sum_\phi (\Delta t_\phi)\bar{I}_\phi = a_s \Big\{ (t_e-0)\left[\frac{I}{2}\right]+(t_0-t_e)I+(t_1-t_0)\left[I-(E/2)(t_1-t_0)\right]$$
$$+(t_2-t_1)\left[I-(E/2)[(t_1-t_0)+(t_2-t_0)] + (P_1/2)(t_2-t_1)\right]$$
$$+(t_3-t_2)\left[I-(E/2)[(t_2-t_0)+(t_3-t_0)] +(P_1/2)[(t_2-t_1)+(t_3-t_1)]+(P_2/2)(t_3-t_2)\right]\Big\}$$

The revised cost equation[4] and constraints provides the basis for the undiscounted model shown below.

---

4 The terms could have been separated by variables I, P_1, P_2 and a constant. This was not done as the discounting of the cost stream (done in the next paragraph) requires that each cost component be related to its time of occurrence.

$$COST = (a_\pi + \ell_2)I + a_I\Big\{(t_e - 0)\Big(\frac{I}{2}\Big) + (t_0 - t_e)I + (t_1 - t_0)\Big[I - \frac{E}{2}(t_1 - t_0)\Big]$$

$$+ (t_2 - t_1)\Big[I - \frac{E}{2}\Big((t_1 - t_0) + (t_2 - t_0)\Big) + \frac{P_1}{2}(t_2 - t_1)\Big] + (t_3 - t_2)\Big[I - \frac{E}{2}\Big((t_2 - t_0) + (t_3 - t_0)\Big)$$

$$+ \frac{P_1}{2}\Big((t_2 - t_1) + (t_3 - t_1)\Big) + \frac{P_2}{2}(t_3 - t_2)\Big]\Big\} + \Big[a_1 + a_{1t}(t_1 - 0) + \ell_1(t_3 - t_1)\Big]P_1$$

$$+ \Big[a_2 + a_{2t}(t_2 - 0) + \ell_2(t_3 - t_2)\Big]P_2$$

(1) $I \geq F + E(t_1 - t_0)$

(2) $I + (t_2 - t_1)P_1 \geq F + E(t_2 - t_0)$

(3) $+P_1 \quad +P_2 = E$

And: $I \geq 0$, $P_1 \geq 0$, $P_2 \geq 0$.

D. Refinement of Model by Discounting the Cost Stream.

The various components of the evolved cost equation are incurred at different times. Investment costs are discounted from the start of the month during which they are incurred. Similarly, recurring costs, e.g., inventory storage costs, and plant maintenance costs, were discounted monthly at the start of each month using an annual interest rate of r. Four selected annual interest rates (r) that the government now uses or perhaps ought to use for investment decisions were used to determine how much these rates affected the optimal mix.

The magnitude and time phasing of costs of an optimal mix including a normal or quick response facility are different. Figure 3 illustrates this difference by the cost profiles for an I and P_2, and I and P_1 mix by the upper and lower set of two profiles, respectively. The time of the occurrence of critical events is shown on the abscissa of the bottom profile. The abscissas are coincident but not to scale; nor are the ordinates to scale. The cost profiles illustrate the time phasing of costs associated with the two possible optimal mix examples in Figure 1.

The choice of discounting interest rate can alter the optimal mix. Consider the illustration of the I and P_1 mix cost profiles with large expenditures deferred until the war starts, while the I and P_2 mix incurs large costs for a larger inventory early in the period of peace. We might expect the I and P_2 mix to be optimal at a low discounting interest rate. At some increased interest rate the deferred large expenditures associated with the I and P_1 mix would be so heavily discounted that this latter mix could become optimal.

Does such a change of the optimal mix from a normal to a quick response plant occur at a discounting interest rate within the range of those used by government decision makers? That this happens is shown in the next chapter on results.

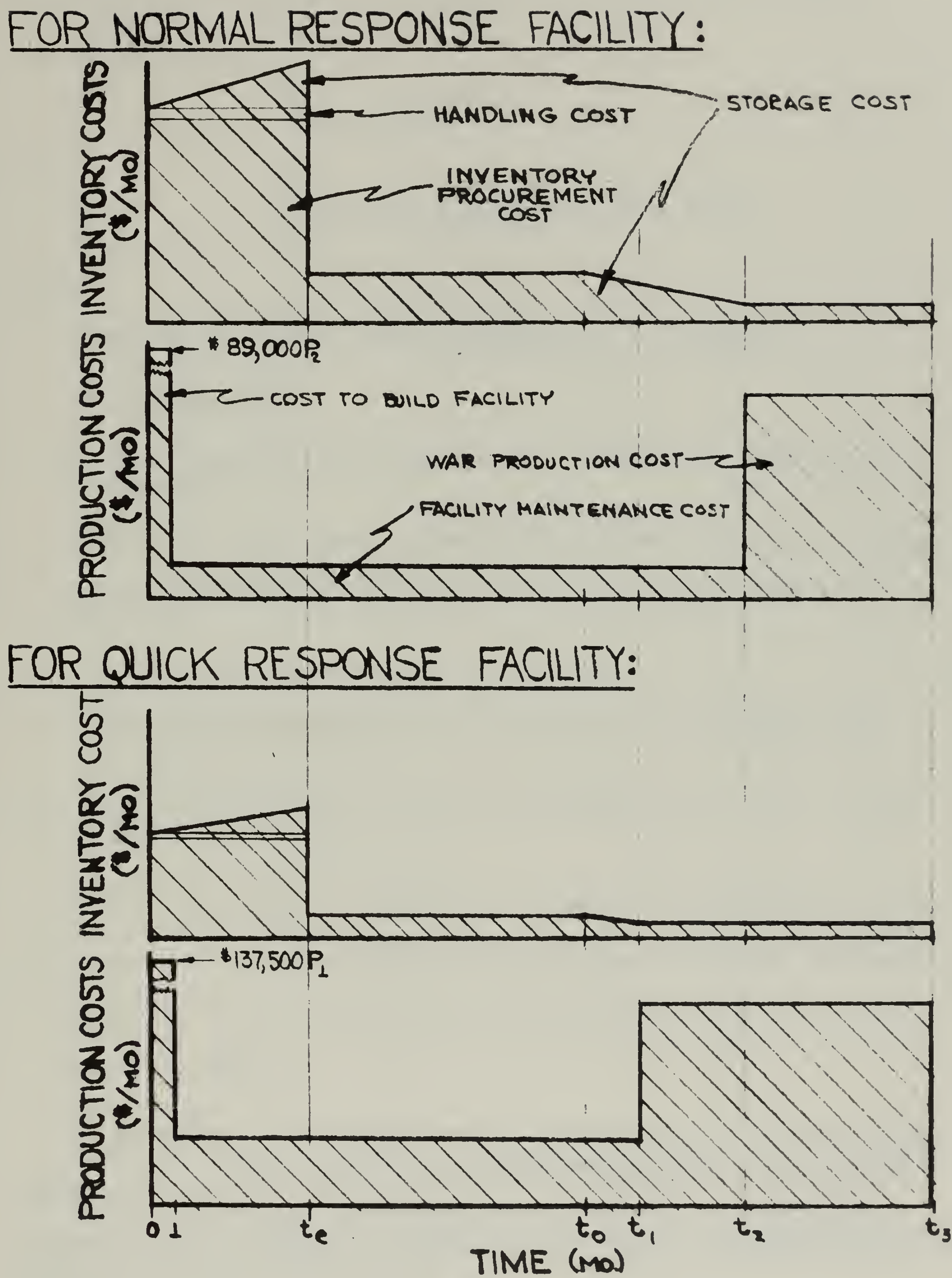

FIGURE 3
COST PROFILE OF I & P MIX

Two simple mathematical expressions are used to discount the cost equation:[5]

$$P = P_m (d)^m$$

and:

$$S_{om} = A_m \left[\frac{1-d^m}{1-d} \right], \quad \text{for } |d|<1.$$

where:

d - Monthly discount rate expressed as a decimal;[6]

$$d = \left(\frac{1}{1+(r/12)} \right)$$

r - Annual interest rate used for discounting expressed as a decimal.

m - Period of time over which discounting occurs in months.

P_m - Cost incurred "m" months from the present time in dollars.

P - Discounted present cost in dollars.

A_m - Cost incurred monthly at the start of each month expressed in dollars.

S_{om} - Discounted present cost, in dollars, of a sum of equal monthly costs, A_m.

These two basic discounting formulas are applied to the previous cost equation and the resulting terms are collected by variables I, P_1, and P_2 and remaining constants as shown on the next page.

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5 Excellent treatments on discounting cost streams are found in: (1) Abert, James G., Structuring Cost Effectiveness Analysis, Logistic Review & Military Logistics Journal, Vol.II, No. 7, 1966, pp 26-28, (2) Niskanen, A Suggested Treatment of Time-Distributed Expenditures in Defense Systems Analysis, Internal Note N-396 (R), Institute for Defense Analysis, 17 October 1966, p. 4.

6 Baumol, William J., Economic Theory and Operations Analysis, Second Edition, pp. 422 & 423.

The refined cost equation of the model prior to discounting is:

$$COST = \left[\frac{(a_r + b_2)(t_e - 0)}{(t_e - 0)}\right] I + a_\Delta \left\{ (t_e - 0)\left(\frac{I}{2}\right) + (t_o - t_e) I + (t_1 - t_o)\left[I - \frac{E}{2}(t_1 - t_o)\right] \right.$$

$$+ (t_2 - t_1)\left[I - \frac{E}{2}\left((t_1 - t_o) + (t_2 - t_o)\right) + \frac{P_1}{2}(t_2 - t_1)\right] + (t_3 - t_2)\left[I - \frac{E}{2}\left((t_2 - t_o) + (t_3 - t_o)\right) + \frac{P_1}{2}\left((t_2 - t_1) + (t_3 - t_1)\right) + \frac{P_2}{2}(t_3 - t_2)\right] \Bigg\}$$

$$+ \left[a_1 + a_{1t}(t_1) + b_1(t_3 - t_1)\right] P_1 + \left[a_2 + a_{2t}(t_2) + b_2(t_3 - t_2)\right] P_2$$

After discounting it becomes:

$$COST_{DISC'T} = \left(\frac{1}{t_e}\right)\left[(a_r + b_2)\left(\frac{1 - d^{(t_e - 0)}}{1 - d}\right)\right](I) + a_\Delta \left\{ \left[\frac{1 - d^{(t_e - 0)}}{1 - d}\right](1)\left(\frac{I}{2}\right) + \left[\frac{1 - d^{(t_o - t_e)}}{1 - d}\right](d)^{t_e}(I) + \left[\frac{1 - d^{(t_1 - t_o)}}{1 - d}\right](d)^{t_o}\left[I - \frac{E}{2}(t_1 - t_o)\right] \right.$$

$$+ \left[\frac{1 - d^{(t_2 - t_1)}}{1 - d}\right](d)^{t_1}\left[I - \frac{E}{2}\left((t_1 - t_o) + (t_2 - t_o)\right) + \frac{P_1}{2}(t_2 - t_1)\right] + \left[\frac{1 - d^{(t_3 - t_2)}}{1 - d}\right](d)^{t_2}\left[I - \frac{E}{2}\left((t_2 - t_o) + (t_3 - t_o)\right) + \frac{P_1}{2}\left((t_2 - t_1) + (t_3 - t_1)\right)\right.$$

$$\left.\left. + \frac{P_2}{2}(t_3 - t_2)\right]\right\} + \left\{a_1 + a_{1t}\left[\frac{1 - d^{(t_1)}}{1 - d}\right](1) + b_1\left[\frac{1 - d^{(t_3 - t_1)}}{1 - d}\right](d)^{t_1}\right\} P_1 + \left\{a_2 + a_{2t}\left[\frac{1 - d^{(t_2)}}{1 - d}\right](1) + b_2\left[\frac{1 - d^{(t_3 - t_2)}}{1 - d}\right](d)^{t_2}\right\} P_2$$

Then collecting terms by the variables I, P_1 and P_2

$$COST_{DIS'T} =$$

$$\left\{\left(\frac{1}{t_e}\right)\left\{(a_n + b_2)\left[\frac{1-d^{t_e}}{1-d}\right]\right\} + \left(\frac{a_\Delta}{1-d}\right)\left[\left(\frac{1}{2}\right)(1-d^{t_e}) + (1-d^{(t_o-t_e)})(d)^{t_e} + (1-d^{(t_1-t_o)})(d)^{t_o} + (1-d^{(t_2-t_1)})(d)^{t_1} + (1-d^{(t_3-t_2)})(d)^{t_2}\right]\right\} I$$

$$+ \left\{\left(\frac{1}{1-d}\right)\left[a_1(1-d) + a_{1t}(1-d^{(t_1)}) + b_1(1-d^{(t_3-t_1)})(d)^{t_1} + \frac{a_\Delta}{2}\left[(t_2-t_1)(1-d^{(t_2-t_1)})(d)^{t_1} + \left((t_2-t_1)+(t_3-t_1)\right)(1-d^{(t_3-t_2)})(d)^{t_2}\right]\right]\right\} P_1$$

$$+ \left\{\left(\frac{1}{1-d}\right)\left[a_2(1-d) + a_{2t}(1-d^{(t_2)}) + b_2(1-d^{(t_3-t_2)})(d)^{t_2} + \frac{a_\Delta}{2}\left[(t_3-t_2)(1-d^{(t_3-t_2)})(d)^{t_2}\right]\right]\right\} P_2$$

$$+ \left\{\left(\frac{-a_\Delta}{1-d}\right)\left(\frac{E}{2}\right)\left[(t_1-t_o)(1-d^{(t_1-t_o)})(d)^{t_o} + \left((t_1-t_o)+(t_2-t_o)\right)(1-d^{(t_3-t_1)})(d)^{t_1} + \left((t_2-t_1)+(t_3-t_1)\right)(1-d^{(t_3-t_2)})(d)^{t_2}\right]\right\}$$

The discounted cost equation in abbreivated form is:

$$COST_D = (\text{Cost A}')\ I + (\text{Cost B}')\ P_1 + (\text{Cost C}')\ P_2 + (\text{Cost K}')$$

Appendix 2 contains a discount table as a function of time and selected interest rates used with this model.

E. Linear Program Tableau.

The linear program tableau of the discounted model is shown below.

$$(\text{Cost } A')I+(\text{Cost } B')P_1+(\text{Cost } C')P_2+(\text{Cost } K')= \text{COST}_D$$

$$I \geq F+E(t_1-t_0)$$

$$I + (t_2-t_1)P_1 \geq F+E(t_2-t_0)$$

$$P_1 + P_2 = E$$

and $I\geq 0,\ P_1\geq 0,\ P_2\geq 0.$

F. Inputs.

The inputs used with the model are discussed and specified in Appendix 1 which is summarized in Table 1 on the next page.

TABLE 1

SUMMARY TABULATION OF INPUTS USED WITH ECONOMIC MODEL

Discounting interest rate:

r = 0.001, 0.04, 0.10 and 0.15.

Ammunition expenditure rate:

E = 750,000 rounds per month.

Fixed Minimum Ammunition Level:

F = 4,000,000 rounds.

Length of war:

$t_3 - t_0$ = 36 months.

Length of preceding period of peace:

t_0 = 60, 120 and 240 months.

Response time of production facilities, for quick-reaction plant:

$t_1 - t_0$ = 0.25 month,

and, normal reaction plant:

$t_2 - t_0$ = 6 months.

Length of time to produce the initial inventory:

t_e = 12 months.

Cost coefficients:

a_1 =	\$137,500.	/K rds/mo
a_{1t} =	\$244.	/K rds
b_1 =	\$25,000.	/K rds
a_2 =	\$89,000.	/K rds/mo
a_{2t} =	\$149.30	/K rds
b_2 =	\$25,000.	/K rds
a_r =	\$200.	/K rds
a_s =	\$5.60	/K rds-mo

CHAPTER III

RESULTS

Although the model has been shown in the linear programming tableau, the optimal mix can be determined graphically using the discounted cost equation, three constraints, and side conditions, i.e.,

$$COST_D = (\text{Cost } A')I + (\text{Cost } B')P_1 + (\text{Cost } C')P_2 + (\text{Cost } K')$$

$$(1) \quad I \geq 4{,}187.5 \quad ,$$

$$(2) \quad I + 5.75\, P_1 \geq 8{,}500 \quad ,$$

$$(3') \quad P_1 + P_2 = 750. \quad ,$$

and $I \geq 0,\ P_1 \geq 0,\ P_2 \geq 0.$

Before solving for the variables, the model will be further simplified by eliminating one variable, P_2, and reducing the model to two variables. Solving the third constraint for P_2,

$$P_2 = 750 - P_1 \quad ,$$

and substituting this into the cost equation gives:

$$COST_D = (\text{Cost } A')I+(\text{Cost } B')P_1+(\text{Cost } C')(750-P_1)+(\text{Cost } K').$$

The model then reduces to:

$$I + \left[\frac{(\text{Cost } B') - (\text{Cost } C')}{(\text{Cost } A')}\right]P_1 = \text{minimum}$$

$$(1) \quad I \geq 4{,}187.5$$

$$(2) \quad I + 5.75\ P_1 \geq 8{,}500.$$

and: $I \geq 0,\ P_1 \geq 0.$

The impact of a change in the discounting interest rate on the optimal mix can be illustrated by plotting the two constraints in Figure 4. The first is a horizonal line with I = 4,187.5 K rds; the second is a straight line from I = 8,500 K rds on the inventory axis (point B) to 1.48 million rounds per month on the P_1 axis. The intersection of these two constraints is labeled point A. The binding constraints and side condition, $P_1 \geq 0$, are shown by the red line segments on Figure 4.

The I and P_1 mix constant cost lines for a discounting interest rate of 0.1, 4.0, 10 and 15 percent and $t_0 = 60$ months situation are drawn in Figure 4 from data in Table 2. With all other inputs fixed, the slope of the I and P_1 mix constant cost lines increase as the discounting interest rate increases. Since the slope of the constraint line A - B is -5.75, the constant cost lines with a slope greater than -5.75 are tangent to the feasible set at point A. Those constant cost lines with a slope less than -5.75 are tangent to the feasible set at point B.

The constant cost lines for a discounting interest rate of 10 and 15% (tangent at point A) identify an optimal mix of 4,187,500 rounds of inventory and a quick reaction plant with a capacity of 750,000 rounds per month. If the chosen discounting interest rate were 0.1 or 4.0%, the constant cost line for the I and P_1 mix would be tangent at point B. This identifies:

P_1 = 0 K rds/mo and

I = 8,500 K rds as a minimum cost mix. But since

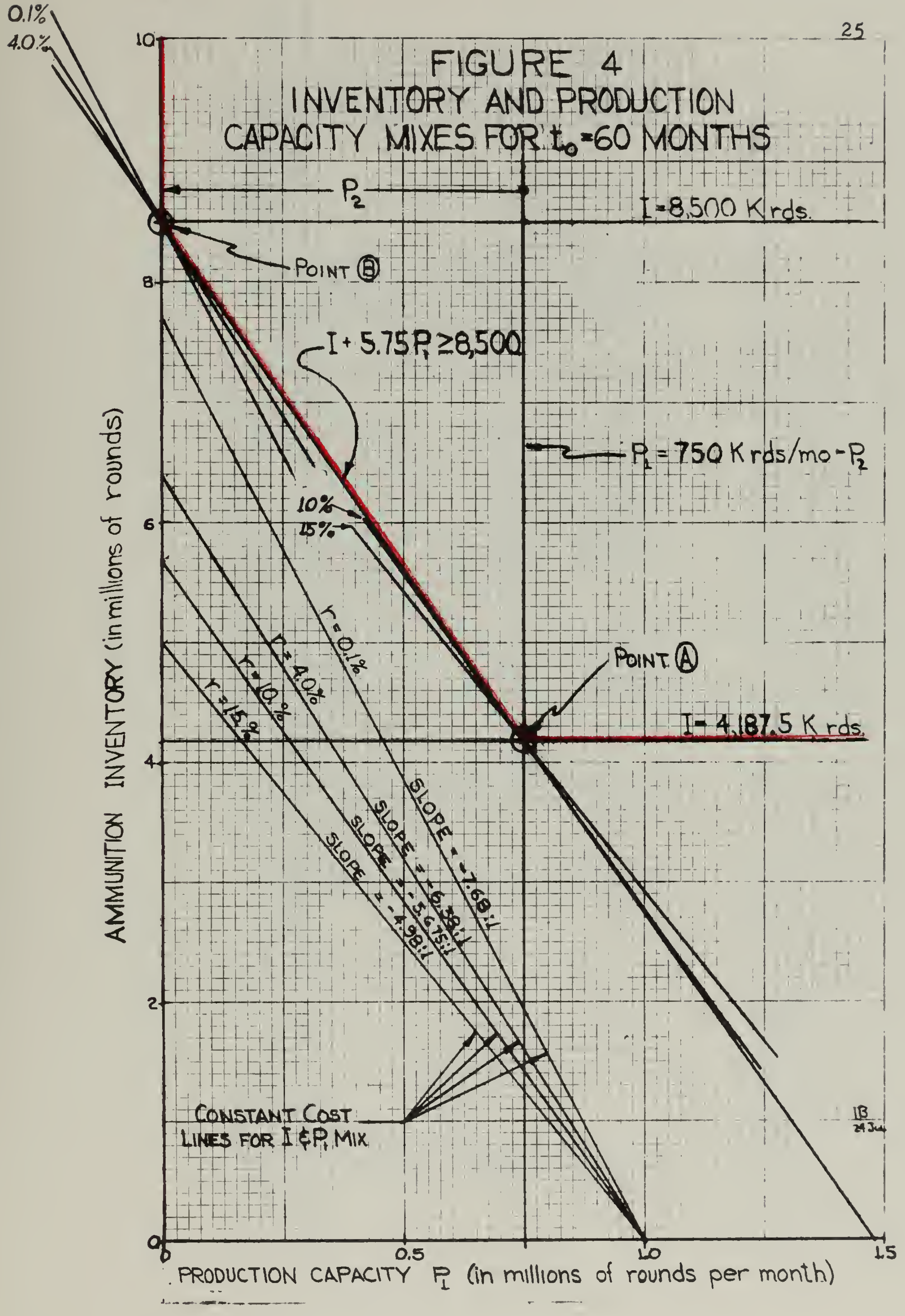
FIGURE 4
INVENTORY AND PRODUCTION
CAPACITY MIXES FOR t₀ =60 MONTHS
0.1%
4.0%
P2
I=8,500 K rds.
POINT B
I + 5.75 P1 ≥ 8,500
P1 = 750 K rds/mo - P2
10%
15%
r = 0.1%
r = 4.0%
r = 10%
r = 15%
POINT A
I = 4,187.5 K rds.
SLOPE = -7.68:1
SLOPE = -6.38:1
SLOPE = -5.675:1
SLOPE = -4.98:1
CONSTANT COST
LINES FOR I & P1 MIX
AMMUNITION INVENTORY (in millions of rounds)
PRODUCTION CAPACITY P1 (in millions of rounds per month)
0
0.5
1.0
1.5
2
4
6
8
10

TABLE 2

DISCOUNTED COST FOR INVENTORY & PRODUCTION CAPACITY MIXES

Discounting Interest Rate (r in %)	Length of Peace (t_0 in mo)	Cost A' in $\$10^6$/K rds	Cost B' in $\$10^6$/K rds/mo	Cost C' in $\$10^6$/K rds/mo	Cost K' in $\$10^6$	Discounted Cost[1] in \$Million for I&P_2 Mix	Discounted Cost[1] in \$Million for I&P_1 Mix
0.1	60	25.7	1043.7	846.3	-2.6	850.4	388.0
4.		25.2	844.2	673.7	-2.0	717.0	736.6
10.		24.4	621.9	483.4	-1.3	568.2	565.9
15.		23.8	492.7	374.2	-1.0	481.0	461.6
0.1	120	26.0	1053.7	851.4	-2.6	856.4	896.4
4.[2]		—	—	—	—	—	—
10.		24.5	443.5	335.8	-0.8	458.7	433.7
15.		23.9	316.4	230.7	-0.4	375.6	337.0
0.1	240	26.7	1073.8	861.7	-2.6	869.4	915.2
4.		25.7	558.8	430.5	-1.1	539.4	525.6
10.		24.6	269.1	191.5	-0.3	352.3	304.2
15.		23.9	193.1	130.2	-0.1	300.7	244.9

1 The lower of the two discounted costs identifies the optimal mix which is shown in the block.

2 Calculations for r= 4% and t_0=120 months were not made.

$P_2 = 750 - P_1$, the optimal mix for these two lower discounting interest rates is a significantly larger inventory of 8.5 million rounds and a normal response plant (P_2) with a capacity of 750,000 rounds per month.

The impact of a change in the discounting interest rate is shown by the counterclockwise rotation of the constant cost lines (i.e., increased slope) as the interest rate is increased. If the slope of the constant cost line is less than the slope of the constraint line $\hat{A}$ - B, the optimal mix is:

P_2 = 750,000 rounds per month, and
I = 8,500,000 rounds;

if greater than the slope of the constraint line $\hat{A}$ - $\bar{B}$, the optimal mix includes a quick reaction facility and smaller inventory of:

P_1 = 750,000 rounds per month, and
I = 4,187,500 rounds.

The Cost A', B', C' and K' coefficients and the resulting discounted costs for both type mixes are shown in Table 2 for four interest rates and three lengths of peace.

Table 2 also shows that as the discounting interest rate increases for each of the three values of t_0, the I and P_1 mix replaces the I and P_2 mix as the optimal mix. Even for the short 60 month period of peace, a change in the discounting interest rate from 4% to 10% causes the optimal mix to change from a large inventory and normal response plant to a significantly smaller inventory and quick response

plant. This indicates that the high investment and maintenance cost of the quick reaction facility have been justified by the reduced total cost resulting from reduced inventories.

The optimal mix changes at a lower discounting interest rate when the period of peace preceding the war is lengthened. Figure 5 and the data of Table 2 illustrates the impact of the same four discounting interest rates on the optimal mix when the period of peace is lengthened from 60 to 240 months. The optimal mix changes from an I and P_2 mix to a smaller inventory and quick reaction production plant (P_1) mix for a discounting interest rate between 0.1 and 4.0%.

The value of t_2-t_1 (the difference in the production lead time of the normal and quick reaction plants) is critical for determining which type mix will be optimal. The production lead times have been examined in a coarse way with only two alternatives used in the model, i.e., P_1 or P_2. A finer grain examination of this critical parameter and its associated cost seems warranted as a future effort.

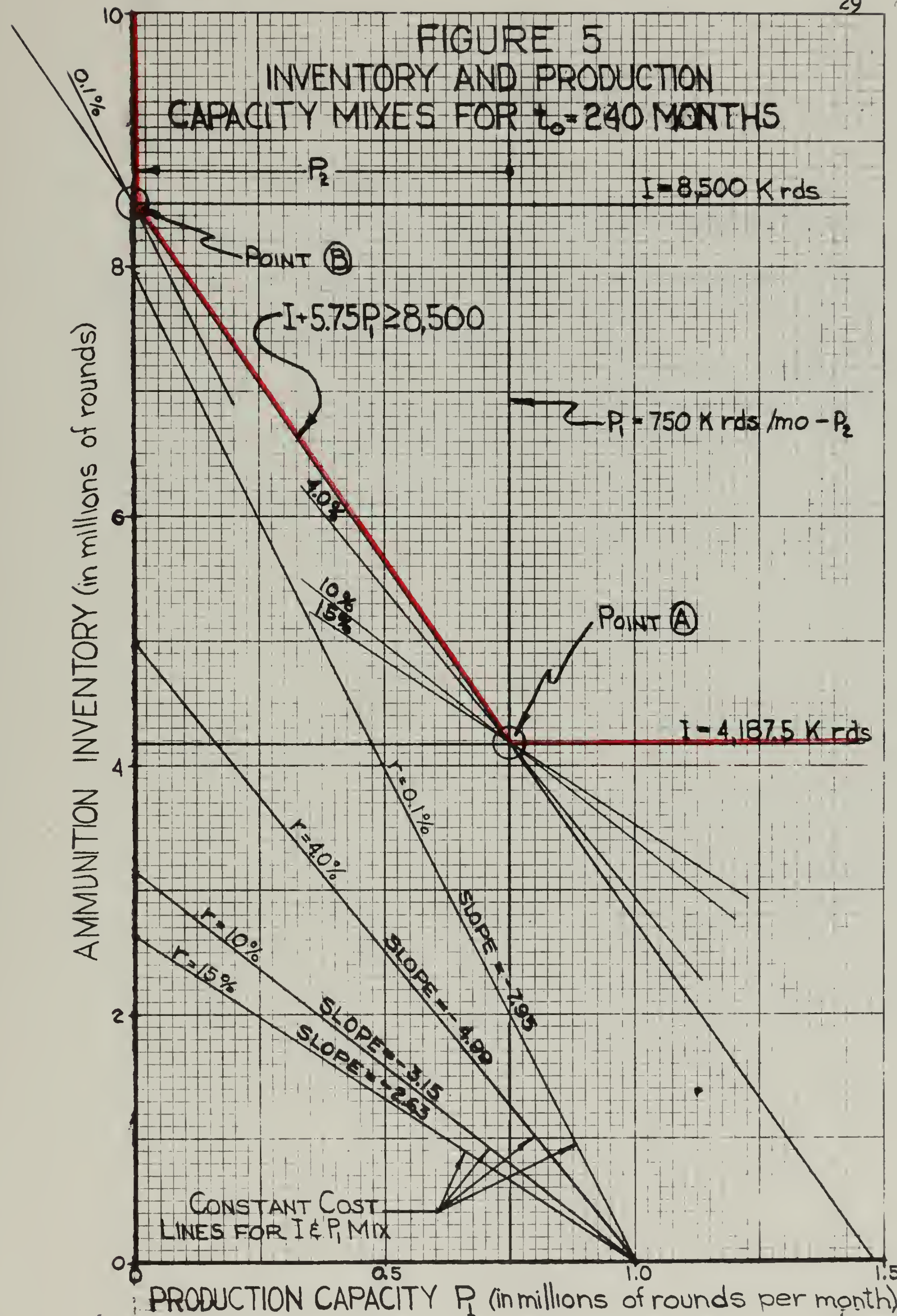
FIGURE 5
INVENTORY AND PRODUCTION
CAPACITY MIXES FOR t_0 = 240 MONTHS
P2
I = 8,500 K rds
POINT (B)
I + 5.75P1 ≥ 8,500
P1 = 750 K rds /mo − P2
POINT (A)
I = 4,187.5 K rds
0.1%
4.0%
10%
15%
r = 0.1%
r = 40%
r = 10%
r = 15%
SLOPE = −7.95
SLOPE = −4.99
SLOPE = −3.15
SLOPE = −2.63
CONSTANT COST
LINES FOR I & P1 MIX
AMMUNITION INVENTORY (in millions of rounds)
PRODUCTION CAPACITY P1 (in millions of rounds per month)
0
2
4
6
8
10
0
0.5
1.0
1.5

CHAPTER IV

CONCLUSIONS

A. It is concluded that:

1. The choice of the discounting interest rate significantly changes the optimal mix of ammunition inventory and production capacity; and

2. The ability to swing into production quickly can justify high investment and maintenance costs for a quick reaction facility.

APPENDIX 1

INPUT DATA

A. Unclassified and Hypothetical Data.

The model built for this thesis is in a general form applicable to any new item of ammunition. Specific input data is needed. The 105mm high explosive howitzer round is used as an illustrative example as it is so common that combat expenditures, production techniques and production costs can be accepted more readily than if the ammunition item were strange. Also, since the 105mm howitzer round has been in the inventory for a long time, there is available more unclassified information than would otherwise be the case. All of the input data used is unclassified. Where the actual data is classified, clearly hypothetical situations have been posed. For example, the scenario for general war includes the commitment of U.S. forces to combat. This commitment is arbitrarily based on phasing 20 divisions into combat over an eight month period to determine an average expenditure rate. All inputs for which references are not cited are hypothetical. The cost parameters are either drawn from unclassified government sources or are hypothetical. The solutions provided by the model for the optimal inventory and production capacity mixes are thus also hypothetical.

B. Type of Parameters.

It is useful to regard the input parameters as either technical or subjective parameters, for the purpose of discussing them. Technical parameters are those which are based on relatively certain economic or engineering information. These parameters can be quantified with a small variance and are not a matter of individual judgements and interpretations. Examples of technical parameters used in the model are all cost coefficients for the investment in inventory and production capacity, and storage, handling, maintenance and employee training costs. The responsiveness, i.e., production lead time, of a proposed production facility is a technical parameter that can be identified. The appropriate discounting interest rate for government use is actually a technical parameter that can be identified from information on the opportunity cost of capital in the private sector.[1] This thesis reviews the extent to which departments and agencies of the Federal government discounted cost streams and what interest rate was used for making investment decisions. The range of values used might lead to the impression that the discounting interest rate is a subjective judgement. Rather, its wide range stems from a lack of a coherent, consistent policy to set an appropriate rate.

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1 U.S. Congress, Joint Economic Committee, The Planning-Programming-Budgeting System: Progress and Potential (86-7410), p.6.

Subjective parameters are based almost entirely on the informed judgement of knowledgeable decision makers, or are such that an arbitrary value will suffice. Subjective parameters are such that a wide range of values would be proposed for a specific situation. Decision makers may be expected to have widely different views of what a future general war will be like because of the many related imponderables. The variance in scenarios, for example, generates a wide range of values for ammunition expenditure rates and fixed minimum ammunition levels to provide for initial equipping of combat units, prepositioned stocks, pipeline stockage and initial combat expenditures.

When the next war will occur and how long it will last are also subjective judgements which the decision maker must render to realistically interpret mobilization requirements. These estimates are important because of the effect they have on discounting costs that are deferred for a long period and as the length of the war bears directly upon the amount of ammunition expended.

There is a final subjective parameter, i.e., the length of time during which the initial inventory is produced. The size of the inventory and the assumed minimum time to produce it identifies the initial production capacity. This capacity is provided by using the mobilization base at some fraction of its full output rate. Within this output of three eight hour shifts a day, seven days a week, there is a wide choice of lesser outputs that can

produce economically, e.g., one eight hour shift, five days a week.

Even fine judgement, substantial knowledge and perfect information of current conditions can only lead to plans that anticipate, not predict, what might be needed to meet mobilization requirements. The imponderables of the problem and tenuous character of some inputs should make it clear that any plan may prove to be wide of the mark.

C. Selection of Interest Rate for Discounting Government Investment Decision.

The technique of discounting a stream of returns or costs is widely accepted. Most basic economics texts include a section on discounting and explain its impact on making investment decisions.[2,3] It is generally concluded that "opportunity" costs should be the basis for determining the discounting interest rate.[4] It appears that the use of

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2 Samuelson, Paul A., Economics: An Introductory Analysis, Sixth Edition, pp. 584-588.

3 Baumol, William J., Economic Theory and Operations Analysis, pp. 422-470.

4 Interest and discount rate are often used interchangeably to mean the annual interest rate at which future returns or costs are discounted. However, the discount rate is uniformly defined as: $D=1/(1+R)$, where D is the discount rate and R is the annual interest rate. This incorrect interchangeable usage is usually understood by the context of its use. Unfortunately, some reference to a "higher" or "lower" discount rate rather than interest rate can and does lead to confusion. Baumol[3] after carefully defining discount rate in the usual way (p.422), proceeds to misuse it in the following chapter (p.440). To avoid possible confusion, reference is only made to the "interest rate" used for discounting which is called the "discounting interest rate" throughout this text.

discounting for investment decisions is relatively new. A cursory examination of the literature reveals numerous articles published in economic journals in the late 1950's and early 1960's.[5,6,7] Several authors have objectively examined the question of what discounting interest rate is appropriate for the Federal government in making investment decisions. The more sophisticated authors have concluded that none of the three traditional devices used for setting the level of the discounting interest rate are appropriate. The rate of return on the marginal private investment, the national time preference, and the interest rate at which the government can borrow long-term funds were each examined and discarded by E. B. Berman.

Hitch and McKean discuss discounting and concluded that, "the rate the government had to pay to borrow funds . . . on the order of 3 percent , . . . is a suitable <u>minimum</u> rate".[8] The Joint Economic Committee of the U.S. Congress has taken a specific interest in what discounting

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5 Hirshleifer, J., <u>On the Theory of Optimal Investment Decisions</u>. The Journal of Political Economy, Vol. No. 4, August, 1958, pp. 329-352.

6 Dickson, R. Russell, Jr., <u>How Discounted-Cash-Flow Analysis Reshapes Capital Programs</u>, Business Horizons, Vol. 3, No. 1, Spring 1960, pp. 85-90.

7. Berman, E.B., <u>The Normative Interest Rate</u>, P -1796. The RAND Corp, all.

8 Hitch, Charles J. & Roland N. McKean, <u>Economics of Defense in the Nuclear Age</u>, p. 210.

interest rate should be applied to the government's Planning-Programming-Budgeting System.[9] In testimony before Congress,[10] Baumol, Stockfisch and others confirmed that opportunity cost in the private sector should be the basis for the discounting interest rate used by the Federal government. Dr. Stockfisch's investigation indicated that the average rate of return on "earnings assets" before taxes for regulated and non-regulated industries was 10% and 15% respectively, with a weighted average based on business investments on plant and equipment by year to be 13.7% before taxes.

As a result of its hearings in 1967 and 1968, the Joint Economic Committee identified the opportunity cost of capital in the private sector based on "earnings assets" as the most relevant basis for the discounting interest rate. It further identified the wide range of discounting interest rates currently used in various departments and agencies of the Federal government. A survey of the various departments and agencies showed that there was no central guidance on discounting. Of 29 departments and agencies queried, 10 reported that they not only did not currently apply discounting to their investment decisions, but did not plan to do so in the future. The Department of Defense, however,

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9. U. S. Congress, Joint Economic Committee, _The Planning-Programming-Budgeting System: Progress and Potentials_, December 1967, pp. 5-8.

10. U.S. Congress, Joint Economic Committee, Hearing before Sub-Committee on Economy in Government on September 14, 19, 20 & 21, _1967, The Planning-Programming-Budgeting System: Progress and Potentials_, pp. 129-179.

does regularly use discounting in arriving at investment decisions. Since the Department of Defense is among the largest investors, it is probably correct to estimate that one-half of the Federal government's major investment decisions are discounted. The Department of Defense frequently uses 10% as a discounting interest rate. Water improvement projects have for a number of years been discounted at an interest rate equal to that at which the Federal government borrows money on fifteen year bonds. In the past, such bonds have averaged 3 1/8% to 3½%. An interest rate of 4% has been widely used in discounting investment projects for water conservation and land reclamation, and river and harbor projects. On the other hand, the Labor Department in certain educational and training programs (an investment in people) uses a one year horizon as an alternate means of discounting, which is a discounting interest rate of 100%.

Congressional interest has been focused on discounting by President Johnson's directive of August 25, 1965 establishing Planning, Programing and Budgeting System techniques for all Federal agencies. The Joint Economic Committee, in its report, concluded that a "discount rate . . . based on opportunity cost in the private sector . . . , and that the discounting interest rate should be at least 10%[11] . . . , for Federal investment decisions.

11 Same as 9, pp. 5 & 6.

To illustrate the impact of discounting on the optimal mix, four discounting interest rates, i.e., 0.1, 4.0, 10, and 15 percent, were used as inputs in each of 3 situations. These values were used to illustrate: (1) the no discounting case, approximated by 0.1 percent[12]; (2) a value typically used in many of the departments and agencies of the Federal government; (3) the lower limit that the Joint Economic Committee, U.S. Congress feels is appropriate is 10%; and (4) the only upper limit knowledgeably suggested which is 15%.[13] The mechanics of discounting used in the model precludes the use of a zero interest rate. A zero discounting interest rate was approximated in the model by the use of 1/10th of 1%.

D. Ammunition Expenditure Rate.

The 105mm high explosive howitzer round was used as an example in this thesis. It has been selected because it is the most common U.S. artillery round and because of certain desirable unclassified data that is available. The latter

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12 This proxy was required as $|d| < 1$ (p.18) is not satisfied when the discounting interest rate is zero. The divergence between zero and 0.1% (and the other used values) discounting interest rates is shown in the table of Appendix 2. I actually rediscovered this limitation of the summing technique used when the test computer runs rejected zero as an input for the discounting interest rate.

13 Stockfisch, J.A., <u>The Interest Rate Applicable to Government Investment Projects</u>, prepared statement to the Joint Economic Committee on September 20, 1967.

permits the hypotheses of this thesis to be illustrated with a "real example". The model may be applied to current mobilization planning for any new round of ammunition by introducing the classified ammunition production capacity profile, ammunition expenditure profile and current cost data.

The U.S. Army's individual weapon expenditure rates are classified, while those of the U.S. Marine Corps are unclassified. The Marine Corps uses an expenditure rate of 40.6 and 33.7 rounds per day per 105mm howitzer for planning for the assault and extended operations ashore phases, respectively.[14] U.S. Army historical publications on World War II and Korea give an excellent insight on all types of ammunition expenditures. For example, in the European theater between September 1944 and May 1945, expenditure rates for the 105mm howitzer averaged 27.0 rounds per tube per day.[15] Peak expenditures, of course, exceeded the average rates with a high monthly expenditure rate of 40.8 rounds per tube per day for this same caliber in July of 1944.

It should be recognized that no tabulation of combat expenditures serves as a reliable guide to actual ammunition

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14 Marine Corps Order 8010.1A Ch. 2; Subject: Class V Logistical Procedure, p. 38, Table I.

15 Ruppenthal, Roland G., Logistical Support of the Armies, Vol. II: September 1944 - May 1945, p. 267.

requirements. Ammunition of one type or another is usually in short supply and is often rationed almost from the beginning of an operation. This was the case within the first week of the invasion of France because insufficient ammunition was being unloaded on the continent, and because of a shortage of inland transportation.[16] In Europe ammunition rationing became the rule rather than the exception.

Further, average monthly expenditure rates will have great variability depending on: (1) the organization of the friendly and enemy forces, (2) the relative strength and the missions of the opposing forces, (3) the immediate terrain, (4) the weather, and even (5) the season of the year. For example, two opposing forces on the same terrain, in the same strength, and with all factors of the situation unchanged, except the season of the year, will expend different amounts of ammunition due solely to the differences in season. The fewer hours of daylight in the winter reduces the number of firing missions as fewer targets are observed, while more unobserved harrassing and interdiction missions would be fired. Also, the longer hours of darkness would entail the expenditure of more illuminating ammunition. The tenuous nature of ammunition expenditure rates becomes clear, when the least important of several variables can make a significant difference in average monthly expenditure rates.

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16 Ruppenthal, Roland G., Logistical Support of the Armies, Vol. I: May 1941 - September 1944, pp. 525 & 528.

As inputs for the model, the time phased expenditure rates have been approximated by a single (average) expenditure rate, i.e., one straight line. Actually, the ammunition expenditure profile could be represented by a discontinuous curve of straight line segments. The model can accommodate such a representation at the expense of some added complexity. Instead of the right hand side of the first three constraint equations being, in part, $E\,(\Delta t)$, it would be necessary to substitute $E_1(\Delta t_1)+E_2(\Delta t_2)$, where the two different expenditure rates apply during time period Δt_1, and Δt_2, respectively. This complexity is not warranted when one considers the tenuous nature of the average ammunition expenditure rate.[17] Expenditure rates come from scenarios to support national objectives and strategic plans which, of necessity, are based upon assumptions that would be difficult to judge even if they did not continue to change.

> It is difficult for persons not familiar with the task to appreciate the problems inherent in such activities: the imponderables as to the enemy's intentions and capabilities; the inability to anticipate the area of probable employment of forces and the nature of military tasks in war; the limitations imposed by budgets and politics; the complexities inherent in alliances; the appropriate types and levels of equipment and standards of support for units and individuals; the availability of time for production after the onset of an emergency; and the time needed to move units and equipment to the theaters of combat. For all the striking advances in scientific analysis and in the use of electronic devices, planning judgment will always bulk large in the requirements equation.[18]

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17 Archer, Harry Clarke, The Computation of Military Materiel Requirements, pp. 72-73, 192.

18 Yoshpe, Harry B., Requirements: Matching Needs with Resources, p. 115.

As inputs for the model, the 105mm howitzer expenditure rate was assumed to be 50 rounds per tube per day under conditions of intense combat and 25 rounds per tube per day under conditions of sustained combat.

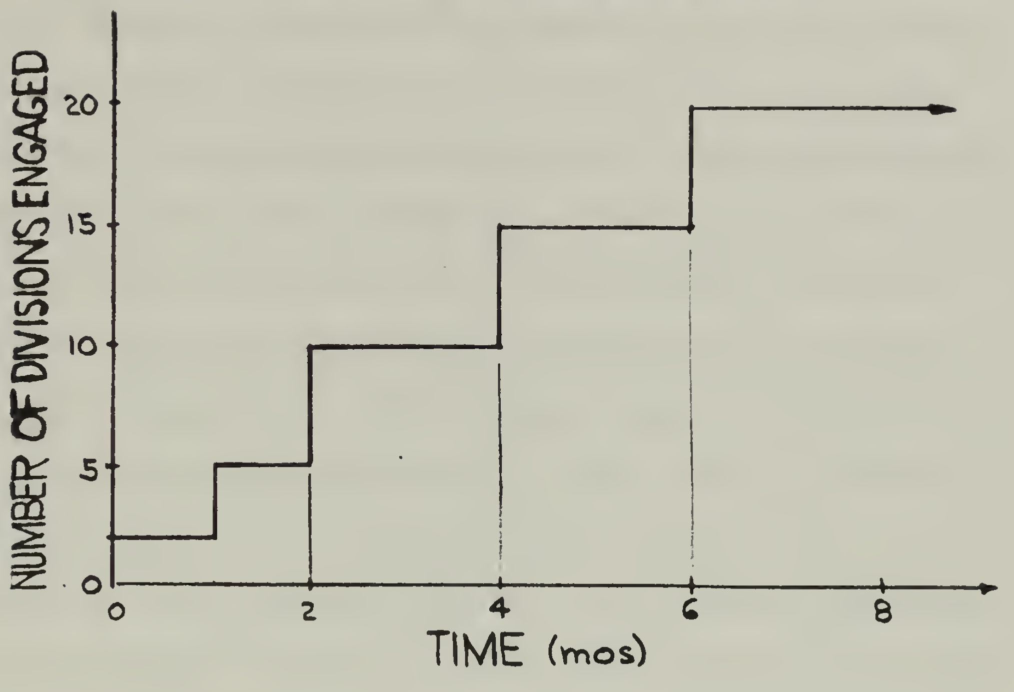
Figure 6 illustrates a hypothetical Profile of Engaged Divisions showing the commitment of 20 U.S. divisions to combat over an eight month period. It is further assumed that a division committed to combat initially expend ammunition at intense combat rates for 15 days and thereafter at sustained rates. The number of 105mm howitzers assigned per division depends on the type of division, e.g., infantry, mechanized-infantry, airborne, Marine. In the latter, there are 76 of these howitzers. An input of 75 105mm howitzers per division was used with the model.

These assumptions result in the hypothetical expenditure of a total of 6,020,000 rounds in eight months, with the calculation for each month being shown below Figure 6. This averages to 753,000 rounds per month, which was rounded off to 750,000 rounds per month as the expenditure rate used in this thesis. While this is a gross estimate, it certainly should be acceptable within the broad range of uncertainties connected with the general war scenario and estimated expenditure rates per howitzer per day.

E. Fixed Minimum Ammunition Level.

The fixed minimum ammunition level, F, is a subjective parameter. The selected value of F directly affects the size of the inventory, but has no effect on P_1 or P_2. The

FIGURE 6
PROFILE OF ENGAGED DIVISIONS



The ammunition expended each month can be calculated as shown.

Month	Calculation	Expended
1	(2)(75)[15(50) + 15(25)]	~169,000
2	(2)(75)(30)(25) + 3(75)[15(50)+15(25)]	~365,000
3	(5)(75)(30)(25) + 5(75)[15(50)+15(25)]	~703,000
4	(10)(75)(30)(25)	~563,000
5	(10)(75)(30)(25) + (5)(75)[15(50)+15(25)]	~984,000
6	(15)(75)(30)(25)	~844,000
7	(15)(75)(30)(25) + (5)(75)[15(50)+15(25)]	~1,266,000
8	(20)(75)(30)(25)	~1,126,000
	Total:	6,020,000

combat scenario for the conceived general war will have a major impact on prepositioned ammunition and on pipeline stocks. These two elements of the fixed minimum ammunition level have received considerable attention because of the size of investment involved. Most work in this area has concentrated on the management aspects of the ammunition inventory. The size and location of prepositioned stocks, and the size of the inventory to fill the pipeline are both critically dependent on available transportation at the time of the emergency.[19] The enormous size of the pipeline, i.e., quantity of ammunition to fill this need is better appreciated by the . . . "estimate that by the end of the war (World War II) only one half of the 21 million tons of ammunition produced in the U.S. had been sent overseas and less than one fourth actually had been expended".[20] While not a typical premobilization situation, it does help give perspective to the problems of selecting a value for F.

A wide range of values for F are possible, dependent as it is upon the imponderables of the expected combat theater, the theater's distance from the continental United States, and the ability of the United States to deliver ammunition and support obscure future combat operations. Alternate means of transportation, e.g., air lift and sea lift, and the availability of an adequate port and airfield all bear on this problem.

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19 Yoshpe, Harry B., National Security Management, Requirements: Matching Needs with Resources, pp. 66-67.

20 Smith, R. Elberton, United States Army in War II, The Army and Economic Mobilization, p. 207.

A single value for F was established for use in the model on the following basis. Prepositioned ammunition was set (hypothetical) at 120 days support for 10 divisions, with the initial 15 days of support at intense expenditure rates (50 rounds/howitzer/day) and the balance at normal (sustained) rates (25 rounds/howitzer/day). The authorized pipeline is assumed to be 90 days long and to be able to support 10 divisions at sustained rate. Therefore,

$$F = \text{Prepositioned Ammunition} + \text{Pipeline Stocks}$$

$$= \left[(10)(75)(15(50) + 105(25))\right] + \left[(10)(75)(90)(25)\right]$$

$$= 2,531,250 + 1,687,500) \text{ rounds}$$

$$F = 4,218,750 \text{ rounds.}$$

This was rounded to F = 4,000 K rds[21] for use as an input to the model. While this 10 division pipeline doesn't match the 20 divisions assumed to be committed by D+6 months, it does meet the time requirements of the 10 divisions committed at D+90 days and provides a measure of safety in meeting the estimated total expenditure of 6,020,000 rounds in the first eight months of combat.

F. Length of War and Preceding Period of Peace.

A 36 month long war and three preceding periods of peace were used as inputs to the model. The three selected values for the preceding period of peace are 60, 120 and

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21 Thousands of rounds is abbreviated as K rds and production capacity in thousands of rounds per month is abbreviated as K rds/mo.

240 months. The length of the peace, (t_0-0), is critical to the selection of the optimal mix through the impact of discounting the deferred war time production costs. However, (t_0-0) has no effect on the size of P_1 or P_2.

G. Responsiveness of Production Facilities.

The quick and normal response facilities have assigned response times of 0.25 and 6 months, respectively. These two selected values are more than an order of magnitude apart. These response times apply to plants with unique characteristics that are apparent from the discussion of their respective cost coefficients.

H. Time It Takes to Produce the Initial Inventory.

The time period during which the initial inventory is produced (t_e), the size of the initial inventory (I) and the production capacity used (P_0) are related by:

$$I = (t_e)P_0$$

For any given inventory, a larger P_0 is required as the time taken to produce the inventory is reduced. Unless war is imminent, t_e is extended as long as possible so that a part of the production base is kept in a high state of readiness by being used. Efficient production will impose some practical minimum output rate (P_0). This used capacity will not exceed that of the mobilization base, P_1 or P_2, unless the initial inventory is required in a relatively short period of time.

As an input to this model, 12 months has been selected for t_e. This value seems more acceptable when war is not

expected for five years, rather than in 10 or 20 years. A larger t_e would be more appropriate to a situation with the latter two longer periods of peace.

I. Cost Inputs.

It was assumed that a new Army Ammunition Plant for the production (i.e., loading, assembly, and packing) of 105mm howitzer ammunition could be obtained for $56,000,000 for all required construction (buildings, roads, and loading docks), equipment and tools. Such a plant is further assumed to have a capacity of 625,000 105mm howitzer rounds per month (using three 8 hour shifts seven days a week). If maintained as a mobilization base, it could produce at full capacity with a production lead time of 6 months.

Therefore, the fixed cost coefficient for a normal reaction capacity is:

$$a_2 = \frac{\$56,000,000}{625\text{ K rds/mo}} \approx \$89,000/\text{K rds/mo.}$$

This normal response facility, when held in readiness but not producing, will incur monthly costs for maintenance of plant and leasing the property. These recurring fixed costs are assumed to be 2 percent of the replacement value of the plant, equipment, tools and any required long lead time components as an annual cost. Thus, the value for the fixed cost coefficient that is a function of time for the normal reaction facility is:

$$a_{2t} = \frac{(\$56,000,000)(.02/\text{yr})}{(625\text{ K rds/mo})(12\text{ mo/yr})}$$

$$a_{2t} = \$149.3/\text{Krds}$$

The variable cost of producing from the normal response facilities is assumed to be \$25,000/K rds. This unit variable cost is already in the form of the variable cost coefficient. Thus,

$$b_2 = \$25{,}000/\text{K rds}$$

The costs related to the quick response facility (those with subscript 1 on the coefficients) are the same type as those for the slower responding plant. Recall that we are again considering a new plant of the latest technology in an industry not particularly suited for automation. Thus, the improved responsiveness is achieved via: (1) the stocking of long lead time components for the 105mm howitzer round, including necessary packing materials, and (2) the contractural arrangement for having a trained and experienced labor force available so that production at near full capacity could be initiated in one week.

For a gross calculation of the investment in long lead time components, it was estimated that these components would be valued at \$8000 for each thousand finished rounds, i.e., at a little less than one third of the variable cost. If it is further assumed that long lead time components to meet six months of full capacity output will be procured, the added investment cost is:

Added Investment Cost = (\$8000/K rds)(625 K rds/mo)(6 mo.)

= \$30,000,000.

Thus,

$$a_1 = \frac{\$56,000,000+\$30,000,000}{625 \text{ K rds/mo}} = \$137,500/\text{K rds/mo.}$$

Several assumptions had to be made to estimate the personnel training costs to achieve a quick response. It was assumed that:

(1) A contract could be negotiated with a company, located in the same community as this Army Ammunition Plant, to cross-train the necessary fraction of their personnel for loading, assembly and packing operations in the quick response plant and to operate that plant,

(2) A labor force of 2,500 persons is required and,

(3) A period of 40 hours of cross-training is required by each member of the labor force to achieve the desired quick response.

Also, bearing on the cross-training costs are the current average hourly earnings and labor separation rate for ammunition workers. Gross hourly earnings of \$3.30/hour and a separation rate of 2.9 percent/month were used.[22] Thus, annual training costs were calculated to be:

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22 Bureau of Labor Statistic, U.S. Department of Labor, Employment and Earnings and Monthly Report on the Labor Force, July & September 1967 & March 1968, pp. 66 & 111, 90 & 115, respectively.

Annual training costs = (2,500 men)(40 hr/3yr)($3.30 man hr)

= $110,000/yr.

$$\text{Monthly training cost per K rds/mo.} = \frac{(\$110,000/\text{yr})}{(12\ \text{mo/yr})(625\ \text{K rds/mo})}$$

$$= \$14.67/\text{K rds} \approx \$15./\text{K rds}$$

Annual maintenance costs are higher on the quick response facility than the normal one because of the additional $30,000,000 investment in long lead time components. These components incur monthly storage costs which are aggregated under the previously used 2 percent of the present value as an annual maintenance cost. Thus, the monthly maintenance cost per K rds/mo. is:

(.02/yr)($86,000,000)/(625 K rds/mo)(12 mo/yr) = $229./K rds

Then:

a_{it} = $229./K rds + $15/K rds = $244/K rds.

The variable cost of production for the quick and normal reaction facilities are the same since the material and labor costs are assumed to be equal. Thus,

$b_1 = b_2$ = $25,000/K rds.

Storage cost data was obtained from the Army Ammunition and Procurement Supply Agency at Joliet, Illinois. Annual ammunition storage costs were given as $2.24 per ton - year with a separate one time handling (receiving) cost of $6.67/ton. The complete 105mm howitzer round (packed two to a box) weighs 60 pounds per round.

Therefore, the monthly storage cost coefficient and receiv-

ing cost coefficient are:

$$a_s = (\$2.24/\text{ton-yr})(60\ \text{lb/rd})(1\ \text{ton}/2000\ \text{lb}) \times(1000\ \text{rds/K rd})(1\ \text{yr}/12\ \text{mo})$$
$$= \$5.60/\text{K rds-mo.}$$

and

$$a_r = (\$6.67/\text{ton})(60\ \text{lb/rd})(1\ \text{ton}/2000\ \text{lb})(1000\ \text{rd/K rd})$$
$$= \$200./\text{K rds.}$$

The cost coefficients applied to the model are:

$$a_1 = \$137,500.\ \ /\text{K rds/mo}$$
$$a_{1t} = \$244.\ \ /\text{K rds}$$
$$b_1 = \$25,000.\ \ /\text{K rds}$$
$$a_2 = \$89,000.\ \ /\text{K rds/mo}$$
$$a_{2t} = \$149.30\ \ /\text{K rds}$$
$$b_2 = \$25,000.\ \ /\text{K rds}$$
$$a_r = \$200.\ \ /\text{K rds}$$
$$a_s = \$5.60\ \ /\text{K rds-mo}$$

APPENDIX 2

PRESENT VALUE OF $100. DISCOUNTED MONTHLY AS A FUNCTION OF ANNUAL INTEREST RATE AND TIME IN MONTHS

Time in months	Discounting Interest Rate in Percent			
	0.1	4	10	15
1	$99.99	$99.67	$99.17	$98.17
2	.98	99.34	98.35	97.55
3	.98	99.01	97.54	96.34
4	.97	98.68	96.73	95.15
5	.96	98.35	95.94	93.98
6	.95	98.02	95.14	92.82
7	.94	97.70	94.36	91.67
8	.93	97.37	93.58	90.54
9	.93	97.05	92.80	89.42
10	.92	96.73	92.04	88.32
11	.91	96.41	91.28	87.23
12	.90	96.09	90.52	86.15
13	.89	95.77	89.77	85.09
14	.88	95.45	89.03	84.04
15	.88	95.13	88.30	83.00
24	.80	92.32	81.94	74.22
36	.70	88.71	74.17	63.94
48	.60	85.24	67.14	55.09
60	.50	81.90	60.78	47.46
90	.25	74.12	47.38	32.69
120	.01	67.08	36.94	22.52
180	98.51	54.94	22.45	10.69
240	98.02	44.99	13.65	5.07

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