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Empirical description of high wave number turbulence spectra

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EMPIRICAL DESCRIPTION OF HIGH WAVE NUMBER TURBULENCE SPECTRA

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THESIS

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ABSTRACT

The shapes of spectra obtained from measurements of ocean turbulence are examined. Kovasznay's equation for high wave number turbulence is used to describe the inertial through dissipation regions. Power spectra of velocity fluctuation recorded in the open sea are used to evaluate constants occurring in the Kovasznay relation. Semi-empirical forms of high wave number energy spectra are then displayed. These computed spectra should define a universal form to which mathematical models should adhere.

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I. INTRODUCTION

A. BACKGROUND

Turbulence occurring at small scales of the order of meters to centimeters are features that are of interest to engineers, oceanographers and meteorologists. Much of this interest lies in the transport properties of momentum, energy, and heat in turbulence even though the bulk of these fluxes take place over a wide range of scales. At larger scales such as encountered in geophysics the turbulence is governed by mean flow features and hence is influenced by boundary conditions and meteorological and oceanographic variables. At small scales, the turbulence becomes more ordered (in a stochastic sense) as the influence of the mean flow becomes less. Certain properties/features are common from one flow to another. At reasonably small scales turbulence is apparently governed by "universal" laws. A "universal" law implies that turbulence found in the ocean, atmosphere, or laboratory in gases, liquids, or plasmas will have the same behavior.

In 1941, A. N. Kolmogorov set up a mathematical framework for small scale turbulence. A statistical mathematician, Kolmogorov, nonetheless, based his theory on physical reasoning. As originally formulated, his theory applied to scales of motion which were isotropic. The theory is known as the Kolmogorov theory of isotropic turbulence. This theory predicted results which were subsequently experimentally verified. Specifically Kolmogorov predicted that there was a

range of scales where energy would fall off with wave number to the minus five-thirds power. This range where energy is transferred inertially is called the inertial subrange and describes scales of turbulence which are separate from the mean flow but not small enough to be affected by the viscosity. In this range

$$\emptyset(k) = K' \ \varepsilon^{2/3} \ k^{-5/3}(k) \tag{1}$$

where $\emptyset(k)$ is the one-dimensional energy spectrum $(\frac{cm}{sec})^2 \cdot$ (cm), K' is a "universal" constant, ε is the rate of dissipation of turbulent kinetic energy $(\frac{cm}{sec})^2 \cdot \frac{1}{sec}$, and k is the wave number $(\frac{1}{cm})$.

Recent experimental work has shown that this equation holds over a much wider range of scales than Kolmogorov anticipated. The law holds at large scales where the turbulence cannot possibly be isotropic. This is important to researchers in geophysical fluids because they are provided with a law which can be applied to a wide range of geophysical problems.

Turbulence in the atmosphere and in the ocean almost always exhibits an inertial subrange. If K' is known in the inertial subrange then ε may be determined by measuring $\emptyset(k)$.

The evaluation of ε is critical in oceanic turbulence and air-sea interaction since if ε is known, other parameters such as the drag coefficient, C_d and sensible heat flux may be estimated.

The "universal" law therefore easily allows the computation of ε, the paramount parameter of the theory, provided

that K' is computed accurately. In order to compute K' accurately, it is necessary to experimentally measure ε at scales where viscosity is taking energy from turbulence and transforming it from kinetic energy to heat energy. It is at such scales that most of the turbulent energy is dissipated. Viscosity becomes important at very small scales because viscous forces are effective only over short distances. Therefore, in order to measure ε accurately, measurement must be made at very small scales, scales smaller than encountered in the inertial subrange region; that is, beyond the inertial subrange. Such difficult measurements have been made over the past few years by Grant et al (1961), Pond et al (1966) and Nasmyth (1971), as a result of care in recording, analysis and improved sensor technology.

B. APPROACH

Theoretically there has not been another success like Kolmgorov's inertial subrange prediction. There is no single comparable expression for the region beyond the inertial subrange although several forms have been suggested. Because of the importance of ε to other turbulence investigations, further theoretical studies at all scales of turbulence will be hampered until this small scale region is understood.

This thesis, therefore, takes not a theoretical approach but an empirical approach. By using recently collected data, guideposts will be established for the theoretician by determining the constants of a semi-empirical equation of the curve

describing the shape of the spectrum at and beyond the inertial subrange region. Certain parameters will be evaluated and these evaluated parameters will form guideposts. Any theory, to be correct, should be able to reproduce the values or the empirical curve. Regression techniques will be used to determine the constants of a curve describing the shape of the spectrum of velocity fluctuations beyond the inertial subrange region.

C. KOVASZNAY'S EQUATION

1. Theoretical Background

There have been several proposals attempting to describe the transfer of energy across the spectrum (Hinze, 1959, p. 190 ff). In terms of the three-dimensional spectrum E(k), one can write

$$\frac{\partial E(k)}{\partial t} = T(k) - 2\nu k^2 \, E(k) \qquad (2)$$

where T(k) represents, at a given wave number, the energy change due to nonlinear interactions. It is the difference between energy increase and energy decrease. When this equation is averaged, phase information is lost, and as a result, there is not enough information available to solve it. The averaging, unfortunately, is absolutely necessary at some stage in the analysis. To work with the unaveraged equation means that one must know the response of the system to initial and subsequent perturbations. Because of the nature of turbulent flow, this response and the perturbations are essentially unknowable. Therefore, in order to solve the

averaged equation, information is put in the form of physically reasonable assumptions about $T(k)$. Such proposals are usually introduced in the integral form (2) namely

$$S(k') = \int_{k'}^{\infty} T(k)\, dk \qquad (3)$$

The physical significance of $S(k')$, which follows from the definition of $T(k)$, is that it is the total flux of energy across the wave number k'. Also following from the definition of $T(k)$ is the result

$$T(k) = 0 \qquad (4)$$

in the inertial subrange, because in the inertial subrange there is a local steady state. Furthermore, before the dissipation region is approached

$$S(k') = \varepsilon \qquad (5)$$

the rate of turbulent energy dissipation. It is in this setting that we introduce the proposal of Kovasznay.

2. Kovasznay's Proposal

The essence of Kovasznay's proposal is that $S(k')$ is determined by what goes on near k' only. In other words, he proposes that energy transfer is a local phenomenon, and is determined entirely by local parameters. From (5) and the inertial subrange law, then

$$S(k') \sim E(k')^{3/2}\, k'^{5/2} = \alpha E(k')^{3/2} \qquad (6)$$

where α is a constant. Then, from (2)

$$T(k) = \frac{\partial}{\partial k'} S(k') \Big|_{k'=k} = -\left\{\frac{3}{2}\alpha E^{1/2} \frac{\partial E}{\partial k}\, k^{5/2} + \frac{5}{2}\alpha E^{3/2} k^{3/2}\right\}$$

(7)

The negative sign is needed because k' is a lower limit; at the upper limit ($k = \infty$), $E(k)$ goes to zero very strongly and so gives no contribution to $T(k)$. In the inertial subrange

$$\frac{\partial E(k)}{\partial t} = 0 \tag{8}$$

so that

$$T(k) = 2\nu k^2 \, E(k) \tag{9}$$

From (7) and (9)

$$2\nu k^2 E(k) = -\frac{1}{2}\alpha\{3E^{1/2}\frac{dE}{dk} k^{5/2} + 5E^{3/2} k^{3/2}\} \tag{10}$$

or simplified,

$$\frac{dE}{dk} = -\frac{5}{3}\frac{E}{k} - \frac{4}{3}\frac{\nu}{\alpha}\sqrt{E/k} \tag{11}$$

For a solution one tries

$$E = A\emptyset k^{-5/3} \tag{12}$$

where A is a constant and $\emptyset$ is an unknown function. $A\, k^{-5/3}$ is chosen since a k dependence of this nature is desired, at least in the inertial subrange. When (12) is substituted in (11)

$$\frac{dE}{dk} = -5/3\, A\emptyset k^{-8/3} + Ak^{-5/3}\frac{d\emptyset}{dk} \tag{13}$$

Notice that

$$-\frac{5}{3}\frac{E}{k} = = -\frac{5}{3} A\emptyset k^{-8/3} \tag{14}$$

relates the first terms of (13) and (11). For the second term of (13)

$$Ak^{-5/3}\frac{d\emptyset}{dk} = -\frac{4}{3}\frac{\nu}{\alpha}(\frac{E}{k})^{1/2}$$

$$= -\frac{4}{3}\frac{\nu}{\alpha}\sqrt{A}\, k^{-4/3}\sqrt{\emptyset} \tag{15}$$

or

$$\phi^{-1/2}\, d\phi = -\frac{4}{3}\ \frac{\nu}{\alpha}\ \frac{\sqrt{A}}{A}\ k^{1/3}\, dk \qquad (16)$$

Upon integrating

$$\phi^{1/2} = -\frac{\nu\sqrt{A}}{2\alpha A}\ k^{4/3} + \text{constant} \qquad (17)$$

Now at low wave number, from (5)

$$\varepsilon = \alpha E^{3/2}\ k^{5/2} \qquad (18)$$

so that

$$\varepsilon = \alpha(Ak^{-5/3}\ \phi)^{3/2}\ k^{5/2}$$

$$= \alpha A^{3/2}\ \phi^{3/2} \qquad (19)$$

Thus, at low wave number,

$$\phi^{3/2} = \frac{\varepsilon}{\alpha A^{3/2}}$$

or

$$\phi^{1/2} = \frac{\varepsilon^{1/3}}{\alpha^{1/3}A^{1/2}} \qquad (20)$$

which is the constant of integration occurring in (17).

Therefore

$$\phi^{1/2} = -\frac{\nu}{2\alpha}\ \frac{\sqrt{A}}{A}\ k^{4/3} + \frac{\nu k_s^{4/3}}{\alpha^{1/3}\sqrt{A}} \qquad (21)$$

If we introduce the Kolmogorov wave number $k_s = (\varepsilon/\nu^3)^{1/4}$, then

$$\phi^{1/2} = -\frac{\nu}{2\alpha}\ \frac{\sqrt{A}}{A}\ k^{4/3} + \frac{\nu k_s^{4/3}}{\alpha^{1/3}\sqrt{A}}$$

or

$$\phi = \left(\frac{\varepsilon}{\alpha}\right)^{2/3}\ \frac{1}{A}\ \left[1 - \frac{1}{2\alpha^{2/3}}\ \left(\frac{k}{k_s}\right)^{4/3}\right]^2 \qquad (22)$$

Replacing (22) in (12)

$$E(k) = \left(\frac{\varepsilon}{\alpha}\right)^{2/3} k^{-5/3}\ \left[1 - \frac{1}{2\alpha^{2/3}}\ \left(\frac{k}{k_s}\right)^{4/3}\right]^2 \qquad (23)$$

which is Kovasznay's formula. It is to be noted this derivation leads to an equation which differs from the equation described in Hinze (1959).

The appealing aspect of Kovasznay's result is that it fits observations well and is relatively easy to use. However, at high wave numbers unrealistic asymptotic regions appear. Through the use of well defined spectra, the value of α will be evaluated and some precision attached to the wave number at which Kovasznay's equation is no longer valid.

3. Application

If a consistent value of α is found, then from (23) it is possible to estimate k_s and the shape of the spectrum beyond k_s. Inertial subrange measurements also allow an estimate of k_s from

$$E(k) = K' \varepsilon^{2/3} k^{-5/3} \qquad (24)$$

since K' is now reasonably well established. Various reasons exist for wanting to know the form of the velocity spectrum beyond k_s, but a recent one concerns laser propagation in the free atmosphere and over waves. The smallest scales of motion become very important in this application.

II. DESCRIPTION OF DATA

A. DATA COLLECTION

Six sets of data were used to obtain the spectral descriptions of oceanic turbulence. These data were obtained by Defense Research Establishment Pacific and analyzed by Nasmyth (1970). The data used are from his runs. Sections 1 through 5 follow closely the description given by Nasmyth (1970) in his doctoral thesis.

1. Area

These data were the results of two seagoing expeditions carried out in November 1967 and January-February 1969 in deep water off the west coast of Vancouver Island, British Columbia. Previously the measurements of small scale turbulence in the open sea by the Defense Research Establishment Pacific, Defense Research Board of Canada were made in 1962 by a submarine with hull mounted sensors. This limited the depth of measurement to about 100 meters. The sensor response permitted observations of scales down to 2 or 3 millimeters defined as "well down into the range of energy dissipation by viscous effects or up into the dissipation range of wave numbers." The data of 1967 and 1969 were obtained by a towed body permitting observations to depths exceeding 300 meters.

The measurements are influenced by processes occurring in the upper 330 meters of the ocean and environmental aspects of the winter months. The measurements were more or less randomly scattered over a ten thousand square kilometer area,

part way down the continental slope with ocean depths varying from 1000 meters but usually over 2000 meters. The results should not be considered representative of the world oceans, but the dynamic features of the area are certainly like those in many other parts of the ocean.

2. Sensors

The sensors were mounted on the submerged body which towed from the surface ship by a servo controlled winch that compensated for ship motion to an accuracy of about 30 centimeters in body depth in normal sea states. The winch could maintain the towed body at any desired depth down to 330 meters. In the "cycling" mode, the winch may be programmed to vary depth at a constant rate over a predetermined depth interval (typically 6-30 meters) so that at a constant towing speed the body follows a saw tooth path through the water.

The velocity probe consists of an active element of evaporated platinum film on a conical glass tip. Electrical connections to the film are made through leads embedded in the glass stem. It functions like a hot wire anemometer. Since the maximum dimension of the film is approximately 0.5 millimeters, a resolution of features as small as 2 millimeters is possible. A useful frequency response up to 1000 Hz is possible.

The velocity probe is extremely sensitive to vibration and extensive precautions were taken to eliminate interference. Nonetheless, vibration remains the limiting factor

in the sensitivity of the velocity system through a midrange of the spectrum from about 1 Hz to 20 Hz.

Another major difficulty in the use of this type of probe in an oceanic environment is planktonic fouling. This is the reason all successful measurements in offshore waters have been made in the winter months when the plankton concentration is the least. When a plankton particle contacts the velocity probe, the effect is to momentarily increase the effective thickness of the boundary layer over the film. The record then shows a sharp transient indicating an apparent momentary decrease in velocity.

If the plankton adhere only briefly, there is no problem, but if one is stuck, then the probe must be cleaned. This is accomplished by a system that spews a high velocity reverse flow over the tip of the probe without reducing towing speed or otherwise interrupting the experiment. The velocity probe outputs are watched and the unfouling can be accomplished in a few seconds by manual control from the ship.

Under favorable conditions during the winter, a plankton particle may be contacted every few seconds, but it will only be necessary to wash the probe two or three times an hour. Therefore, it is possible to obtain records up to 30 minutes in length with only minor contamination which has negligible effect on the spectral content of the velocity signal.

3. Errors

The possible contamination of the velocity spectra by planktonic fouling has been discussed. The contamination of the spectra by temperature is not so easily determined. The magnitude of possible errors was estimated by indirect methods as no direct measurements of the dynamic response of the velocity probe to fluctuations in temperature are available. Without going into detail, I report that Nasmyth concludes that the velocity signals are at least predominantly real velocity without any major temperature component. One qualitative argument is that the power spectrum generated by the velocity signals do not exhibit any of the unique characteristics of the spectra of temperature fluctuations in turbulent flow [Grant, Hughes, Vogel, and Moilliet (1968)] and the conclusion is that the signal must be predominantly due to velocity fluctuations.

4. Spectra

The power spectra have been computed from a number of samples of velocity fluctuations recorded during the November 1967 and January-February 1969 cruises. For satisfactory spectral analysis, it is necessary to work with reasonably uniform samples of as long a duration as possible. Besides the problems already discussed, there are other interferences that are frequently encountered in the data collection. Turbulence occurs in patches within layers. Since there is no way of predicting in advance what the signal levels in the next patch will be, many good samples are lost because of

inappropriate gain settings. When probe washes are necessary it may ruin an otherwise useful section of record. Any appreciable change in ship's speed during recording will distort the resulting spectrum. The point is that if one is able to get suitable samples of any length, the sample is valuable.

During November 1967, 32 hours of recording during two weeks at sea were accomplished. During January-February 1969, 35 hours of recording during three weeks at sea were made. From these raw data samples, there are 20 or 25 samples varying in length from 10 seconds to two minutes from which meaningful spectra were derived. In the cycling mode sample, times are short because of changing turbulence layers. In the constant depth mode sample, lengths are longer and several spectra of 52 seconds duration or longer were obtained. Wherever possible, noise is subtracted from the original spectrum resulting in a noise-free spectrum.

5. <u>Recording</u>

Components of circuitry which must be located close to the sensors are enclosed in pressure cases in the towed body. From there, information from all sensors is fed up the armored multi-conductor towing cable for further processing and recording on the ship. The lower 100 meters of cable are faired to reduced vibration.

On board ship the signal from each thermistor is divided into two channels. One is amplified and recorded directly and the other is electronically differentiated with

respect to time before recording. This method gives an acceptable signal-noise ratio at the higher frequencies otherwise unobtainable on a single channel. The signal from the velocity probe is divided into three channels for similar reasons. The first channel or "A" channel is recorded broadband with no filtering. "B" channel is effectively differentiated up to about 10 Hz and cuts off at about 20 Hz. "C" channel differentiates up to 1000 Hz and then cuts off. The signal from the temperature probe is treated similarly except that there is a low-frequency cutoff on "A" at about 0.01 Hz and "B" extends up to about 100 Hz.

Signal channels from all sensors together with coded timing signals and a voice commentary are recorded on two seven channel magnetic tapes with multiplexing as required. At the same time, selected channels are monitored on oscilloscopes as an aid in selecting appropriate gain settings and 12 channels are recorded on two paper charts giving a permanent visual record. This served as a planning aid during the experiment and for later use during analysis.

B. DATA USED

Data sets 4, 5, and 6 consisted of 45 to 47 data points; whereas, data sets 7, 8 and 9 consisted of 174 to 176 data points. These six sets of data were plotted on log-log graph paper in normalized form as log $\frac{\emptyset(k)}{(\varepsilon\nu^5)^{1/4}}$ vs log $(\frac{k}{k_s})$, (Figures 1-6). The data are presented in this form to make comparison of spectra easier. Normalized plots should lie on top of each

other. The plots covered 8 to 11 decades along the $\emptyset(k)$, or energy axis and 4 to 6 decades along the k, or wave number axis. This remarkably large range of values makes them ideally suited for examining details of the high wave number region of the turbulent energy spectrum and testing Kovasznay's equation.

The data points clearly indicated the expected -5/3 slope inertial subrange region and the "tailing off" in the low energy, high wave number region beyond the inertial subrange. Several points were discarded by this author because of obvious inconsistencies. Only runs 4, 5, and 6 had no points discarded. Run 7 had eleven points discarded; 8 had ten points discarded and run 9 had nineteen points. Because of the similarity of these data sets, it was considered necessary only to use data set 4 for analysis.

III. ANALYSIS AND APPROACHES

A. APPROACHES

1. Linear Regression

Since a direct substitution of α for (23) is difficult, the equation was expressed in a form which was amenable to regression techniques. Despite several attempts, consistent results could not be obtained and this method of approach to the problem was discarded.

2. Inertial Subrange

An alternate approach was to consider only the ISR. In the inertial subrange the one-dimensional energy spectrum and the three-dimensional energy spectrum have the same power law behavior. The Kovasznay equation then may be reduced to

$$\dot{\emptyset}(k) \simeq \alpha^{-2/3}\, \varepsilon^{2/3}\, k^{-5/3} \tag{25}$$

[where $\emptyset(k)$ is the one-dimensional energy equation] since clearly these terms will dominate. This reduced form allows observed $\emptyset(k)$ and k to be used in this region to evaluate α. Since

$$\emptyset(k) = K'\, \varepsilon^{2/3}\, k^{-5/3} \tag{26}$$

then it is natural to associate $\alpha^{-2/3}$ with K'. Since K' $\simeq$ 0.5, then an α value of 1.59 can be expected.

3. Exponential Curve

Beyond the inertial subrange, $\emptyset(k)$ falls off exponentially with k. A desirable equation would be one that collapsed to Equation (26) in the inertial subrange but

generated an exponential decay at high wave numbers. Further, such an equation should be expressed in terms of meaningful physical quantities. This is the goal of the theoretician if not this thesis. The object here is simply to provide empirical clues. Consequently, an exponential regression analysis was attempted.

B. ANALYSIS

1. Linear Regression

Equation (23) was taken and rewritten in the form

$$\emptyset(k)=\alpha^{-2/3}\varepsilon^{2/3}k^{-5/3}-\alpha^{-4/3}\varepsilon^{2/3}k_s^{-4/3}k^{-1/3}+.25\alpha^{-6/3}\varepsilon^{2/3}k_s^{-8/3}k \tag{27}$$

Simplified into an equation suitable for regression techniques this becomes

$$Y = A_1k^{-5/3} + A_2k^{-1/3} + A_3k \tag{28}$$

where A_1, A_2 and A_3 are given below.

The regression analysis program appeared to be very sensitive to bad data. According to the Kovasznay equation, A_1 and A_3 should be > 0 and $A_2 < 0$. Furthermore, $|A_1| > |A_2| > |A_3|$. No such consistency was found for the six sets of data when using the Regre Program. Of the six sets, there were in fact three pairs. Each pair corresponded to spectra from a single time series of turbulence data. The difference is that one set of the pair contained four times as many points as the other. Even in the paired data sets, results proved inconsistent. Further, the poor regeneration of the

curve using the A_1, A_2, and A_3 computed by Regre caused this method to be discarded.

2. <u>Substitution</u>

Equation (28) was used where

$$A_1 = (\varepsilon/\alpha)^{2/3} \qquad A_2 = \frac{-\varepsilon^{2/3}}{(\alpha k_s)^{4/3}} \qquad A_3 = \frac{.25\varepsilon^{2/3}}{\alpha^2 k_s^{8/3}}$$

A direct substitution of $\alpha = 1.6$ was used. The values of A_1, A_2, and A_3 were computed. The values are presented in Table I. These values give a regeneration of the curve and is an alternative but simpler way of expressing Kovasznay's equation. A comparison of the curve generated with these values of A_1, A_2, and A_3 and actual data is shown in Figure 7. The agreement is good in the inertial subrange.

3. <u>Gompertz Curve</u>

The Gompertz curve $y = e^{(\alpha+\beta\rho^x)}$ was used because it was readily available. It is unsatisfactory, however, because even if solved and a good curve fit obtained, the quantities α, β, ρ, and x do not lend themselves to physical interpretation. There are no dimensions on the right hand side. Another curve should be used but the form is not immediately obvious and is perhaps worthy of a separate research investigation.

IV. CONCLUSIONS

Since approaches III-A-1 and III-A-3 were unsuccessful, only the result of Approach III-A-2 will be discussed.

As mentioned in III-A-2, an α value of 1.59 was expected. A spread of α from 1.8 to more than 6.0 was obtained from the real data in the manner described below.

A plot was made of α versus wave number (Figure 8). A minimum occurs in the vicinity of $k \simeq 0.5$. Beyond this wave number α, values continue to increase with increasing wave number.

Various values of α obtained using the second method above were substituted in the full Kovasznay equation and the resulting curves compared with actual data (Figure 9). Of the several curves generated, low values of α generated better fits, the best fit occurring for the lowest α, namely $\alpha = 1.8$. This is also closest to the expected value of $\alpha \simeq 1.6$. The fit of curves with $\alpha = 1.6$ and $\alpha = 1.8$ were very similar although the lower α value curve dropped off more steeply. Another feature of interest is that in the inertial subrange, $\emptyset(k)$ values are lower for higher α (since α is raised to a negative power) but beyond the inertial subrange (where other terms dominate) the opposite is true. The crossover point occurs around $k \simeq 20$. The significance of this, if any, is not clear.

The limit of the Kovasznay result appears to be at a wave number $k \simeq 50$. Beyond this wave number, the values of energy increase with increasing wave number, a phenomenon that was expected. This phenomenon obviously is inconsistent with observed results and indeed would be catastrophic theoretically. However, practically it is not important since present technology does not allow measurements to be made at such high wave numbers.

In summary, as α increases, the fitted curve approximates the actual curve with decreasing accuracy. The best α occurs at the minimum in the α versus k (Figure 8) and is approximately where $k < 0.5$. The reconstructed curve agrees well up to $k \simeq 2.0$. For $k > 2.0$, $\emptyset(k)$ in actual data falls off more rapidly than $\emptyset(k)$ computed. Good agreement in the ISR is expected and observed. The dissipation range was not expected to agree. Figure 9 displays the quantitative results showing existing agreement.

TABLE I

A_1	$= 0.3753922$
A_2	$= -0.0056287$
A_3	$= 0.0000211$

APPENDIX A

Velocity Spectrum Data Set 4

k	Ø(k)	k	Ø(k)
.2266+01	.4364-01	.1813+01	.8377-01
.4533+01	.8166-02	.2538+01	.4508-01
.6800+01	.2103-02	.3264+01	.2399-01
.9066+01	.5977-03	.3989+01	.1258-01
.1133+02	.2055-03	.1360+00	.7338+01
.1360+02	.7826-04	.2040+00	.3987+01
.1586+02	.3360-04	.2720+00	.2511+01
.2040+02	.7937-05	.3400+00	.1899+01
.2493+02	.2734-05	.4760+00	.1163+01
.2946+02	.1177-05	.6120+00	.6428+00
.9070+00	.2542+00	.7480+00	.4442+00
.1813+01	.8550-01	.8840+00	.3645+00
.2720+01	.3687-01	.1020+00	.2606+00
.3626+01	.1682-01	.0544+00	.3321+02
.4533+01	.8331-02	.0680+00	.2045+02
.5440+01	.4598-02	.0816+00	.1726+02
.6346+01	.2607-02	.0952+00	.1360+02
.8160+01	.8691-03	.1224+00	.9453+01
.9973+01	.3628-03	.1496+00	.6308+01
.1178+02	.1596-03	.1768+00	.6341+01
.3630+00	.1615+01	.2040+00	.4608+01
.7250+00	.3830+00	.2312+00	.3297+01
.1088+01	.1911+00	.3268+00	.1945+01
.1450+01	.1229+00		

APPENDIX B

Velocity Spectrum Data Set 5

k	Ø(k)	k	Ø(k)
.4327+01	.1264-01	.2423+01	.6419-01
.6490+01	.3475-02	.3115+01	.3671-01
.8658+01	.1007-02	.3808+01	.2015-01
.1081+02	.3460-02	.1298+00	.9699+01
.1298+02	.1313-03	.1947+00	.5546+01
.1514+02	.5580-04	.2596+00	.3611+01
.1730+02	.2539-04	.3245+00	.2408+01
.1947+02	.1225-04	.4543+00	.1268+01
.2380+02	.3536-05	.5841+00	.7726+00
.2812+02	.1302-05	.7140+00	.5618+00
.1730+01	.1174+00	.8438+00	.4313+00
.2596+01	.5373-01	.9736+00	.3431+00
.3461+01	.2606-01	.5192-01	.4939+02
.4327+01	.1346-01	.6490-01	.3352+02
.5192+01	.7589-02	.7789-01	.2398+02
.6058+01	.4347-02	.9087-01	.1841+02
.7789+01	.1497-02	.1168+00	.1176+02
.9520+01	.6147-03	.1428+00	.8556+01
.1125+02	.2664-03	.1687+00	.7468+01
.6923+00	.4671+00	.1947+00	.5946+01
.1038+01	.2467+00	.2206+00	.5076+01
.1384+01	.1682+00	.3115+00	.2790+01
.1730+01	.1168+00		

APPENDIX C
Velocity Spectrum Data Set 6

k	Ø(k)	k	Ø(k)
.4501+01	.1518-01	.2268+01	.8186-01
.6076+01	.4249-02	.2916+01	.4336-01
.8102+01	.1259-02	.3564+01	.2392-01
.1012+02	.4279-03	.1215+00	.1328+02
.1215+02	.1634-03	.1822+00	.7371+01
.1417+02	.7195-04	.2430+00	.4531+01
.1620+02	.3287-04	.3038+00	.3101+01
.1822+02	.1597-04	.4253+00	.1678+01
.2228+02	.4921-05	.5468+00	.1075+01
.2633+02	.2059-05	.6684+00	.6782+01
.1620+01	.1547+00	.7899+00	.5269+01
.2430+01	.6958-01	.9104+00	.4223+01
.3240+01	.3245-01	.4861-01	.4705+02
.4051+01	.1676-01	.6076-01	.3137+02
.4861+01	.9774-02	.7291-01	.2536+02
.5671+01	.5748-02	.8507-01	.2171+02
.7291+01	.2027-02	.1093+00	.1421+02
.8912+01	.8019-03	.1336+00	.1054+02
.1053+02	.3577-03	.1579+00	.6388+01
.6481+00	.5820+00	.1882+00	.6400+01
.9722+00	.3049+00	.2066+00	.4356+01
.1296+01	.2094+00	.2916+00	.2565+01
.1696+01	.1449+00		

APPENDIX D

Velocity Spectrum Data Set 7

k	Ø(k)	k	Ø(k)
.2163+01	.6141-01	.3808+01	.2015-01
.4327+01	.1264-01	.4154+01	.1511-01
.6490+01	.3475-02	.4500+01	.1183-01
.8654+01	.1007-02	.4846+01	.9733-02
.1081+02	.3460-03	.5192+01	.7448-02
.1298+02	.1313-03	.3461+00	.2533+01
.1514+02	.5580-04	.6923+00	.5638+00
.1730+02	.2539-04	.1038+01	.2937+00
.1947+02	.1225-04	.1384+01	.1935+00
.2163+02	.6396-05	.1730+01	.1317+00
.2380+02	.3536-05	.2077+01	.9446-01
.2596+02	.2066-05	.2423+01	.6852-01
.2812+02	.1302-05	.2769+01	.4991-01
.3029+02	.8278-06	.3115+01	.3887-01
.3245+02	.5645-06	.3461+01	.2913-01
.3461+02	.4053-06	.3808+01	.2096-01
.3678+02	.3089-06	.4154+01	.1532-01
.3894+02	.2456-06	.6490-01	.1722+02
.8554+00	.3147+00	.1298+00	.9699+01
.1730+01	.7174+00	.1947+00	.5546+01
.2596+01	.5373-01	.2596+00	.3611+01
.3461+01	.2606-01	.3245+00	.2404+01
.4327+01	.1346-01	.3894+00	.1659+01
.5192+01	.7589-02	.4543+00	.1268+01
.6058+01	.4347-02	.5192+00	.1015+01
.6923+01	.2526-02	.5841+00	.7726+00
.7789+01	.1497-02	.6490+00	.6376+00
.8654+01	.9408-03	.7140+00	.5618+00
.9520+01	.6147-03	.7789+00	.4778+00
.1038+02	.4032-03	.8438+00	.4313+00
.1125+02	.2664-03	.9087+00	.4004+00
.1211+02	.1798-03	.9736+00	.3431+00
.1298+02	.1237-03	.1038+01	.2916+00
.1384+02	.7531-04	.1103+01	.2697+00
.3461+00	.1936+01	.1163+01	.2479+00
.6923+00	.4671+00	.1298+01	.2317+00
.1038+01	.2467+00	.1428+01	.1788+00
.1384+01	.1682+00	.1557+01	.1693+00
.1730+01	.1168+00	.1687+01	.1395+00
.2077+01	.8659-01	.1817+01	.1176+00
.2423+01	.6419-01	.1947+01	.1150+00
.2769+01	.4715-01	.2077+01	.4810-01
.3115+01	.3671-01	.1298-01	.1287+04
.3461+01	.2789-01	.2596-01	.2217+03

k	Ø(k)	k	Ø(k)
.3894-01	.7043+02	-.1187+01	.1525+01
.5192-01	.5097+02	-.1108+01	.1379+01
.6490-01	.3334+02	-.1041+01	.1265+01
.7789-01	.2404+02	-.9835+00	.1190+01
.9087-01	.1796+02	-.9324+00	.1070+01
.1038+00	.1425+02	-.8866+00	.9655+00
.1168+00	.1067+02	-.8452+00	.9322+00
.1298+00	.8304+01	-.8074+00	.9046+00
.1428+00	.7412+01	-.7727+00	.8732+00
.1557+00	.7218+01	-.7405+00	.8174+00
.1687+00	.6513+01	-.7105+00	.7742+00
.1817+00	.5709+01	-.6825+00	.7500+00
.1947+00	.5139+01	-.6562+00	.7055+00
.2077+00	.4900+01	-.6313+00	.6036+00
.2206+00	.4386+01	-.5856+00	.6258+00
.2336+00	.3600+01	-.5442+00	.5153+00
.2596+00	.3771+01	-.5064+00	.4456+00
.2856+00	.3081+01	-.4716+00	.3482+00
.3115+00	.2636+01	-.4395+00	.2460+00
.3375+00	.2099+01	-.4095+00	.2155+00
.3634+00	.1674+01	-.3815+00	-.1714+00
.3894+00	.1615+01	-.1886+01	.3109+01
.4154+00	.1615+01	-.1585+01	.2345+01
.1298-01	.5325+03	-.1409+01	.1847+01
.2596-01	.1245+03	-.1284+01	.1707+01
.3894-01	.6853+02	-.1187+01	.1523+01
.5192-01	.4939+02	-.1108+01	.1381+01
.6490-01	.3352+02	-.1041+01	.1254+01
.7789-01	.2398+02	-.9835+00	.1153+01
.9087-01	.1841+02	-.9324+00	.1028+01
.1038+00	.1550+02	-.8866+00	.9193+00
.1168+00	.1176+02	-.8452+00	.8699+00
.1298+00	.9237+01	-.8074+00	.8584+00
.1428+00	.8556+01	-.7727+00	.8137+00
.1557+00	.8029+01	-.7405+00	.7566+00
.1687+00	.7468+01	-.7105+00	.7108+00
.1817+00	.6568+01	-.6825+00	.6902+00
.1947+00	.5946+01	-.6562+00	.6421+00
.2077+00	.5624+01	-.6313+00	.5563+00
.2206+00	.5076+01	-.5856+00	.5764+00
.2336+00	.4014+01	-.5442+00	.4887+00
.2596+00	.4225+01	-.5064+00	.4209+00
.2856+00	.3275+01	-.4716+00	.3220+00
.3115+00	.2790+01	-.4395+00	.2237+00
.3375+00	.2229+01	-.4095+00	.2082+00
.3634+00	.1762+01	-.3815+00	-.1000+02
.3894+00	.1642+01	.1081-02	.9721+04
.4154+00	.6738+00	.2163-02	.6339+04
-.1886+01	.2726+01	.3245-02	.4608+04
-.1585+01	.2095+01	.4327-02	.3343+04
-.1409+01	.1835+01	.5409-02	.2598+04
-.1284+01	.1693+01	.6490-02	.1879+04

k	Ø(k)
.7572-02	.1271+04
.8654-02	.1207+04
.9736-02	.7168+03
.1081-01	.3742+03
.1190-01	.5903+03
.1298-01	.5717+03
.1406-01	.3760+03
.1514-01	.3715+03
.1622-01	.3424+03
.1730-01	.3385+03
.1839-01	.6875+03
.1947-01	.3491+03
.2163-01	.1060+03
.2380-01	.9513+02
.2596-01	.9774+02
.2812-01	.1222+03
.3029-01	.6649+02
.3245-01	.5879+02
.3461-01	.5220+02
.3678-01	.5370+03
.3894-01	.8703+02
.4110-01	.3341+03
.4327-01	.4864+02
.4760-01	.2585+02
.5192-01	.4629+02
.5625-01	.5433+02
.6058-01	.4407+02
.6490-01	.3166+02
.6923-01	.3758+02
.7356-01	.2539+02
.7789-01	.2104+02
.8221-01	.2558+02
.8654-01	.2025+02

APPENDIX E
Velocity Spectrum Data Set 8

k	Ø(k)	k	Ø(k)
.2025+01	.7442-01	.3889+01	.1863-01
.4051+01	.1518-01	.4213+01	.1431-01
.6076+01	.4249-02	.4537+01	.1190-01
.8102+01	.1259-02	.4861+01	.9555-02
.1012+02	.4279-03	.6076-01	.2395+02
.1215+02	.1634-03	.1215+00	.1328+02
.1417+02	.7145-04	.1822+00	.7371+01
.1620+02	.3287-04	.2430+00	.4531+01
.1822+02	.1597-04	.3038+00	.3101+01
.2025+02	.8187-05	.3645+00	.2290+01
.2228+02	.4921-05	.4253+00	.1678+01
.2430+02	.2787-05	.4861+00	.1334+01
.2636+02	.2059-05	.5468+00	.1075+01
.2635+02	.1159-05	.6076+00	.8430+00
.3038+02	.9579-06	.6684+00	.6782+00
.3240+02	.5871-06	.7291+00	.5776+00
.3443+02	.5260-06	.7899+00	.5269+00
.3645+02	.3692-06	.8507+00	.4803+00
.8102+00	.4143+00	.9114+00	.4223+00
.1620+01	.1547+00	.9722+00	.3648+00
.2430+01	.6958-01	.1033+01	.3426+00
.3240+01	.3245-01	.1093+01	.3213+00
.4051+01	.1676-01	.1215+01	.2948+00
.4861+01	.9774-02	.1336+01	.2458+00
.5671+01	.5748-02	.1458+01	.2127+00
.6481+01	.3409-02	.1579+01	.1674+00
.7291+01	.2027-02	.1701+01	.1740+00
.8102+01	.1251-02	.1822+01	.1280+00
.8912+01	.8019-03	.1944+01	.1301+00
.9722+01	.5324-03	.3240+00	.3383+01
.1053+02	.3577-03	.6481+00	.7058+00
.1134+02	.2383-03	.9722+00	.3638+00
.1215+02	.1624-03	.1296+01	.2421+00
.1296+02	.9926-04	.1620+01	.1847+00
.3240+00	.2457+01	.1944+01	.1180+00
.6481+00	.5820+00	.2268+01	.8795-01
.9722+00	.3049+00	.2592+01	.6393-01
.1296+01	.2094+00	.2916+01	.4561-01
.1620+01	.1449+00	.3240+01	.3265-01
.1944+01	.1076+00	.3634+01	.2467-01
.2268+01	.8186-01	.3889+01	.1870-01
.2592+01	.6005-01	.1215-01	.6753+03
.2916+01	.4336-01	.2430-01	.1545+03
.3240+01	.3140-01	.3645-01	.7864+02
.3504+01	.2392-01	.4861-01	.4705+02

k	∅(k)	k	∅(k)
.6076-01	.3137+02	.3402+00	.2195+01
.7291-01	.2536+02	.3645+00	.1592+01
.8507-01	.2171+02	.3889+00	.1764+01
.9722-01	.1786+02	.1012-02	.2081+05
.1093+00	.1421+02	.2025-02	.1359+05
.1215+00	.1235+02	.3038-02	.8952+04
.1336+00	.1054+02	.4051-02	.5749+04
.1458+00	.8478+01	.5063-02	.5161+04
.1579+00	.7336+01	.6076-02	.4043+04
.1701+00	.7237+01	.7089-02	.2057+04
.1822+00	.7342+01	.8102-02	.1260+04
.1944+00	.5975+01	.9114-02	.8931+03
.2086+00	.4964+01	.1012-01	.5926+03
.2187+00	.4904+01	.1114-01	.4505+03
.2430+00	.4309+01	.1215-01	.4141+03
.2916+00	.2570+01	.1316-01	.3837+03
.3159+00	.2240+01	.1417-01	.2777+03
.3402+00	.2224+01	.1519-01	.2459+03
.3645+00	.1627+01	.1620-01	.2012+03
.3889+00	.1782+01	.1721-01	.1791+03
.1215-01	.2404+04	.1822-01	.2685+03
.2430-01	.2778+03	.2025-01	.1747+03
.3645-01	.7597+02	.2228-01	.1572+03
.4861-01	.4960+02	.2430-01	.1538+03
.6076-01	.2984+02	.2633-01	.1735+03
.7291-01	.2551+02	.2835-01	.9218+02
.8507-01	.2115+02	.3038-01	.1327+03
.9722-01	.1725+02	.3240-01	.7887+02
.1093+00	.1304+02	.3443-01	.1136+03
.1215+00	.1155+02	.3645-01	.8374+02
.1336+00	.9539+01	.3848-01	.7422+02
.1458+00	.7767+01	.4051-01	.5865+02
.1579+00	.6388+01	.4456-01	.2869+02
.1701+00	.6289+01	.4861-01	.3091+02
.1822+00	.6400+01	.5266-01	.2391+02
.1944+00	.5371+01	.5671-01	.4713+02
.2066+00	.4356+01	.6076-01	.3175+02
.2187+00	.4443+01	.6481-01	.1761+02
.2430+00	.3855+01	.6886-01	.3178+02
.2673+00	.2962+01	.7291-01	.1023+02
.2916+00	.2565+01	.7697-01	.2331+02
.3159+00	.2171+01	.8102-01	.2476+02

APPENDIX F
Velocity Spectrum Data Set 9

k	∅(k)	k	∅(k)
.2266+01	.4364-01	.4352+01	.9581-02
.4533+01	.8166-02	.4714+01	.7259-02
.6800+01	.2103-02	.5077+01	.5953-02
.9066+01	.5977-03	.5440+01	.4404-02
.1133+02	.2055-03	.3626+00	.2190+01
.1360+02	.7826-04	.7253+00	.4729+00
.1586+02	.3360-04	.1088+01	.2351+00
.1813+02	.1567-04	.1450+01	.1502+00
.2040+02	.7937-05	.1813+01	.1030+00
.2266+02	.4434-05	.2176+01	.7319-01
.2493+02	.2734-05	.2538+01	.5268-01
.2720+02	.1797-05	.2901+01	.3860-01
.2946+02	.1177-05	.3264+01	.2789-01
.3173+02	.8137-06	.3626+01	.1921-01
.3400+02	.5987-06	.3989+01	.1437-01
.3626+02	.4416-06	.4352+01	.1086-01
.3853+02	.3509-06	.2833-01	.1086-01
.4080+02	.2922-06	.5666-01	.1086-01
.9066+00	.2452+00	.6800-01	.1357+02
.1813+01	.8550-01	.1360+00	.7338+01
.2720+01	.3687-01	.2040+00	.3987+01
.3626+01	.1682-01	.2720+00	.2511+01
.4533+01	.8331-02	.3400+00	.1899+01
.5440+01	.4598-02	.4080+00	.1524+01
.6346+01	.2607-02	.4760+00	.1163+01
.7253+01	.1492-02	.5440+00	.8491+00
.8160+01	.8691-03	.6120+00	.6428+00
.9066+01	.5539-03	.6800+00	.5384+00
.9975+01	.3528-03	.7480+00	.4442+00
.1089+02	.2381-03	.8160+00	.3873+00
.1178+02	.1596-03	.8840+00	.3645+00
.1269+02	.1055-03	.9520+00	.3071+00
.1360+02	.7012-04	.1020+01	.2606+00
.1450+02	.4286-04	.1088+01	.2463+00
.3626+00	.1615+01	.1156+01	.2169+00
.7253+00	.3830+00	.1224+01	.1714+00
.1038+01	.1911+00	.1360+01	.1668+00
.1450+01	.1229+00	.1496+01	.1471+00
.1813+01	.8377-01	.1632+01	.1150+00
.2176+01	.6140-01	.1768+01	.1036+00
.2538+01	.4508-01	.1904+01	.9932-01
.2901+01	.3290-01	.2040+01	.9028-01
.3264+01	.2399-01	.2176+01	.6476-01
.3826+01	.1692-01	.1360-01	.5393+03
.3989+01	.1258-01	.2720-01	.1416+03

k	$\phi(k)$	k	$\phi(k)$
.4080-01	.6473+02	.3536+00	.1470+01
.5440-01	.3321+02	.3808+00	.2018+01
.6800-01	.2045+02	.4080+00	.1462+01
.8160-01	.1726+02	.4352+00	.1413+01
.9520-01	.1360+02	.1133-02	.5631+04
.1088+00	.1119+02	.2266-02	.3842+04
.1224+00	.9453+01	.3400-02	.2843+04
.1360+00	.7766+01	.4533-02	.2069+04
.1496+00	.6308+01	.5666-02	.1862+04
.1832+00	.5954+01	.6800-02	.1248+04
.1763+00	.6341+01	.7933-02	.7569+03
.1904+00	.5562+01	.9066-02	.7216+03
.2040+00	.4608+01	.1020-01	.5015+03
.2176+00	.3920+01	.1133-01	.2628+03
.2312+00	.3297+01	.1246-01	.1317+03
.2448+00	.2555+01	.1360-01	.1596+03
.2720+00	.2475+01	.1473-01	.4404+03
.2992+00	.2507+01	.1586-01	.6856+03
.3264+00	.1945+01	.1700-01	.7990+03
.3536+00	.1544+01	.1813-01	.6287+03
.3808+00	.2011+01	.1926-01	.2860+03
.4080+00	.1421+01	.2040-01	.1644+03
.4352+00	.1376+01	.2266-01	.1902+03
.1360-01	.9109+03	.2493-01	.2100+03
.2720-01	.2283+03	.2720-01	.1512+03
.4080-01	.7399+02	.2946-01	.1310+03
.5440-01	.3535+02	.3173-01	.1027+03
.6800-01	.2012+02	.3400-01	.7257+02
.8160-01	.1635+02	.3626-01	.5452+02
.9520-01	.1399+02	.3853-01	.5672+02
.1088+00	.1113+02	.4080-01	.6259+02
.1224+00	.9165+01	.4306-01	.6576+02
.1360+00	.7516+01	.4533-01	.5946+02
.1496+00	.6018+01	.4986-01	.6716+02
.1632+00	.5731+01	.5440-01	.1814+02
.1768+00	.6061+01	.5893-01	.2297+02
.1904+00	.5376+01	.6346-01	.1206+02
.2040+00	.4234+01	.6800-01	.1747+02
.2176+00	.3527+01	.7253-01	.2328+02
.2312+00	.2933+01	.7703-01	.2080+02
.2448+00	.2334+01	.8160-01	.1705+02
.2720+00	.2147+01	.8613-01	.1192+02
.2992+00	.2400+01	.9066-01	.2965+02
.3264+00	.1889+01		

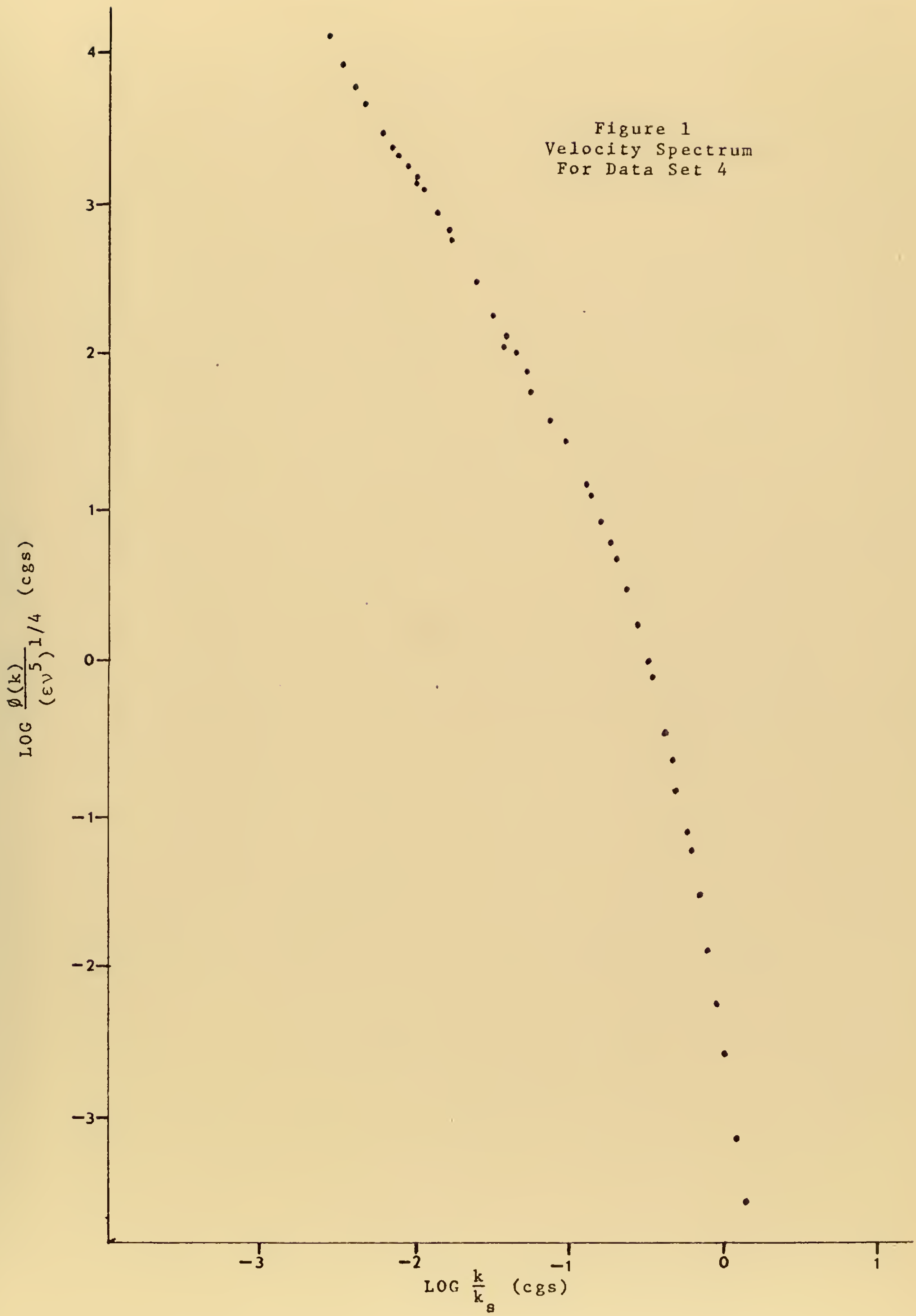


Figure 1
Velocity Spectrum
For Data Set 4
4
3
2
1
0
−1
−2
−3
LOG $\frac{\phi(k)}{(\varepsilon\nu^5)^{1/4}}$ (cgs)
−3
−2
−1
0
1
LOG $\frac{k}{k_s}$ (cgs)

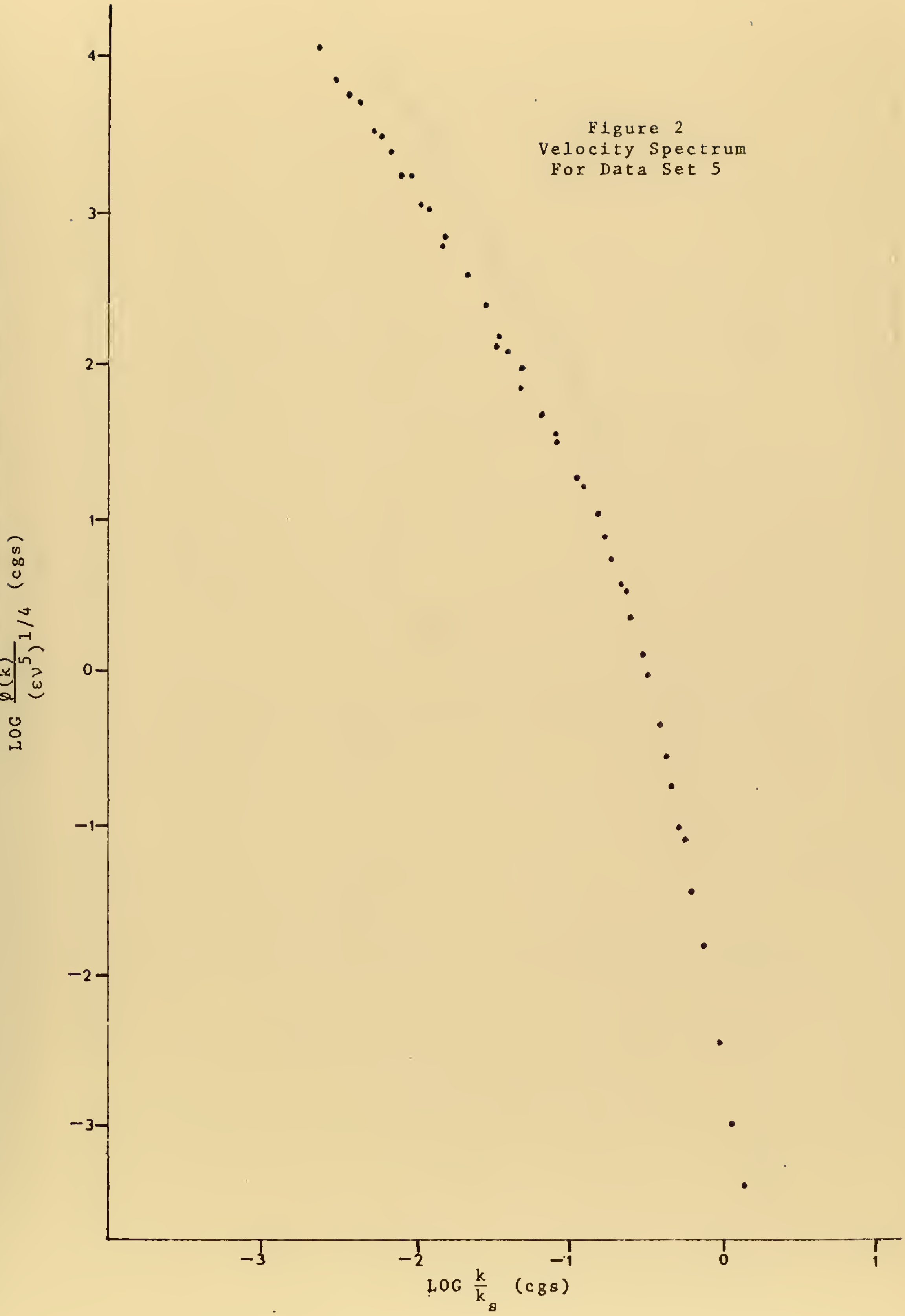


Figure 2
Velocity Spectrum
For Data Set 5

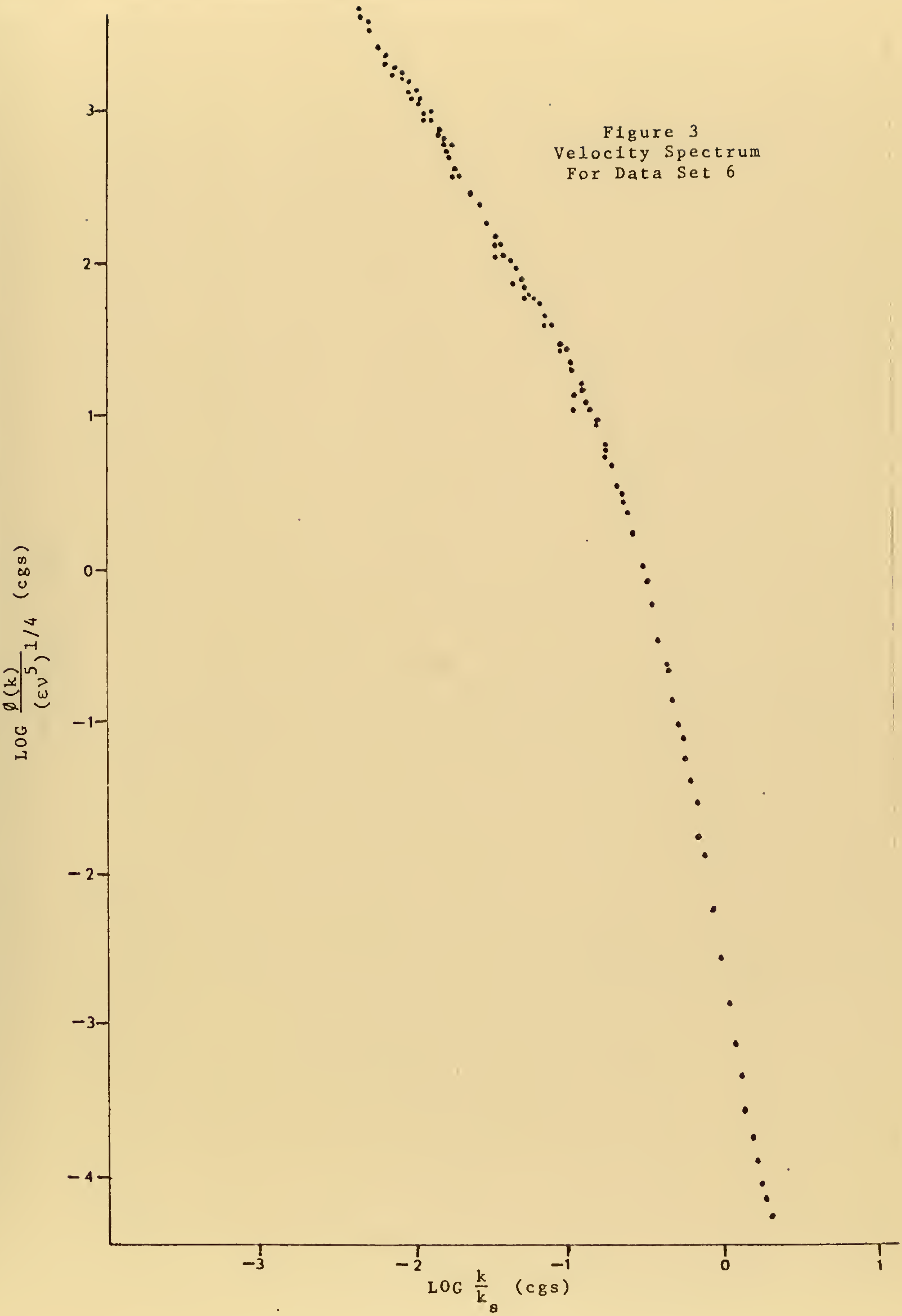


Figure 3
Velocity Spectrum
For Data Set 6
3
2
1
0
-1
-2
-3
-4
-3
-2
-1
0
1
LOG $\frac{\emptyset(k)}{(\varepsilon\nu^5)^{1/4}}$ (cgs)
LOG $\frac{k}{k_s}$ (cgs)

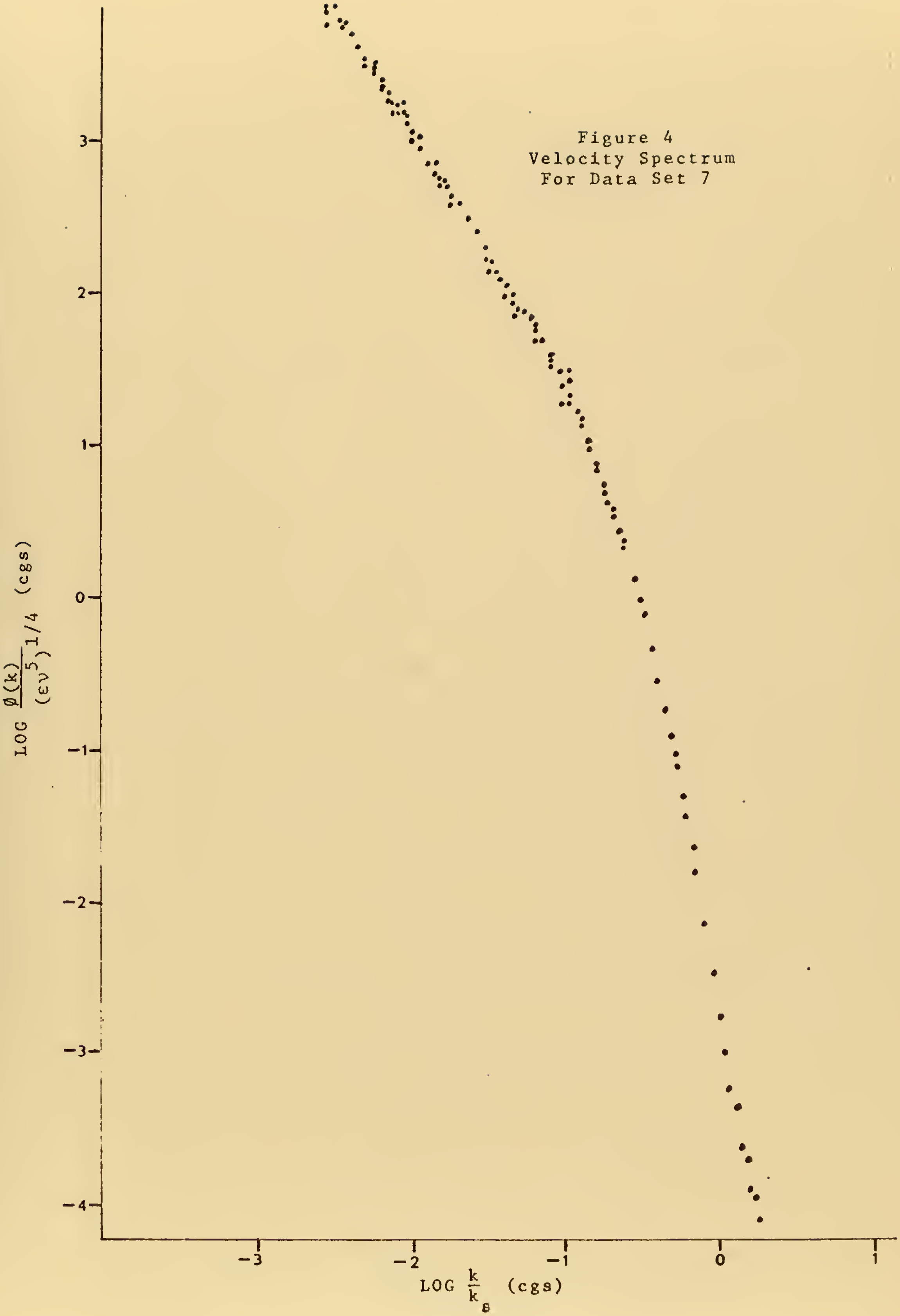


Figure 4
Velocity Spectrum
For Data Set 7
LOG $\frac{\phi(k)}{(\epsilon\nu^5)^{1/4}}$ (cgs)
3
2
1
0
-1
-2
-3
-4
-3
-2
-1
0
1
LOG $\frac{k}{k_s}$ (cgs)

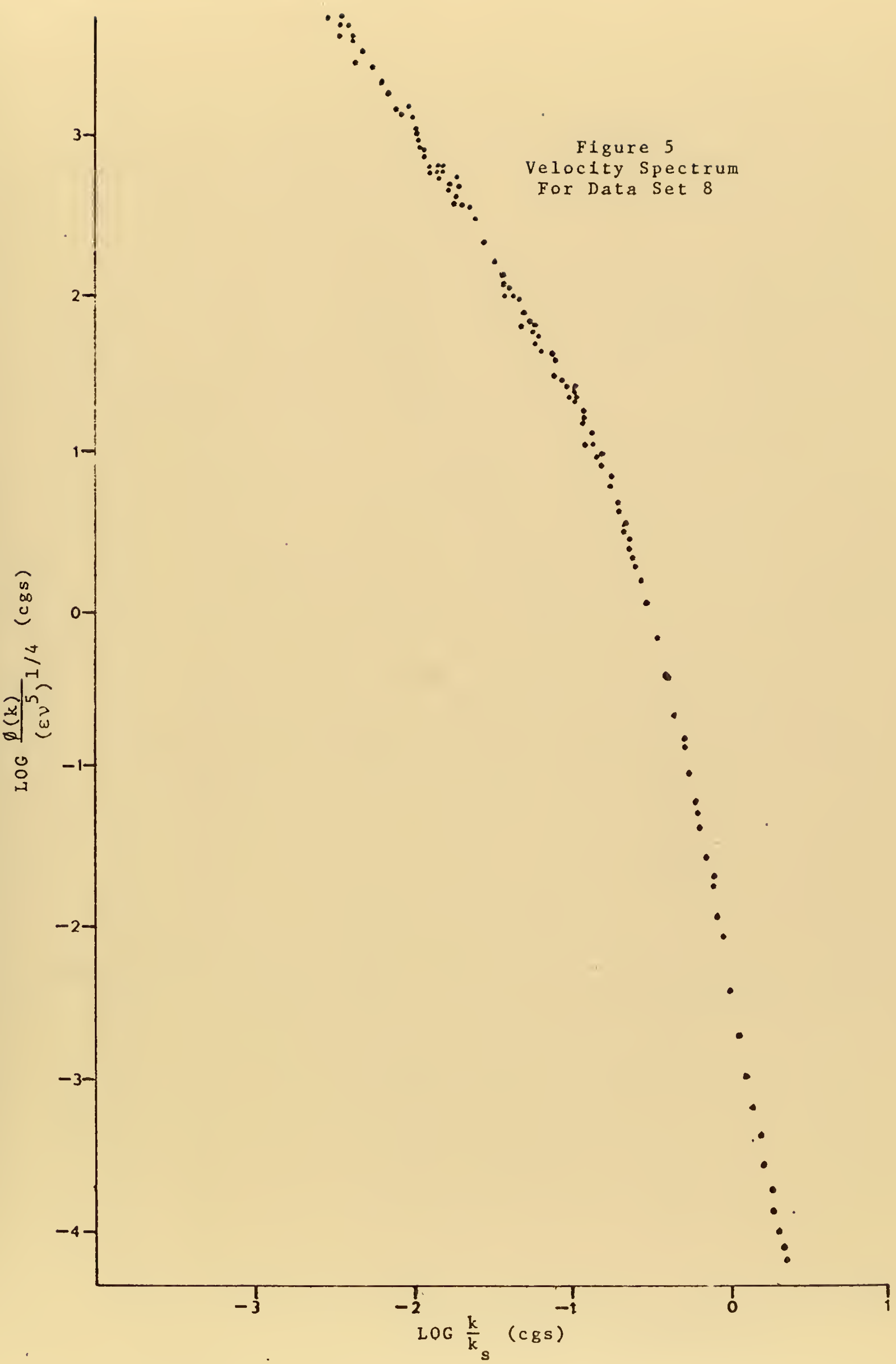


Figure 5
Velocity Spectrum
For Data Set 8
LOG $\frac{\phi(k)}{(\varepsilon\nu^5)^{1/4}}$ (cgs)
3
2
1
0
−1
−2
−3
−4
−3
−2
−1
0
1
LOG $\frac{k}{k_s}$ (cgs)

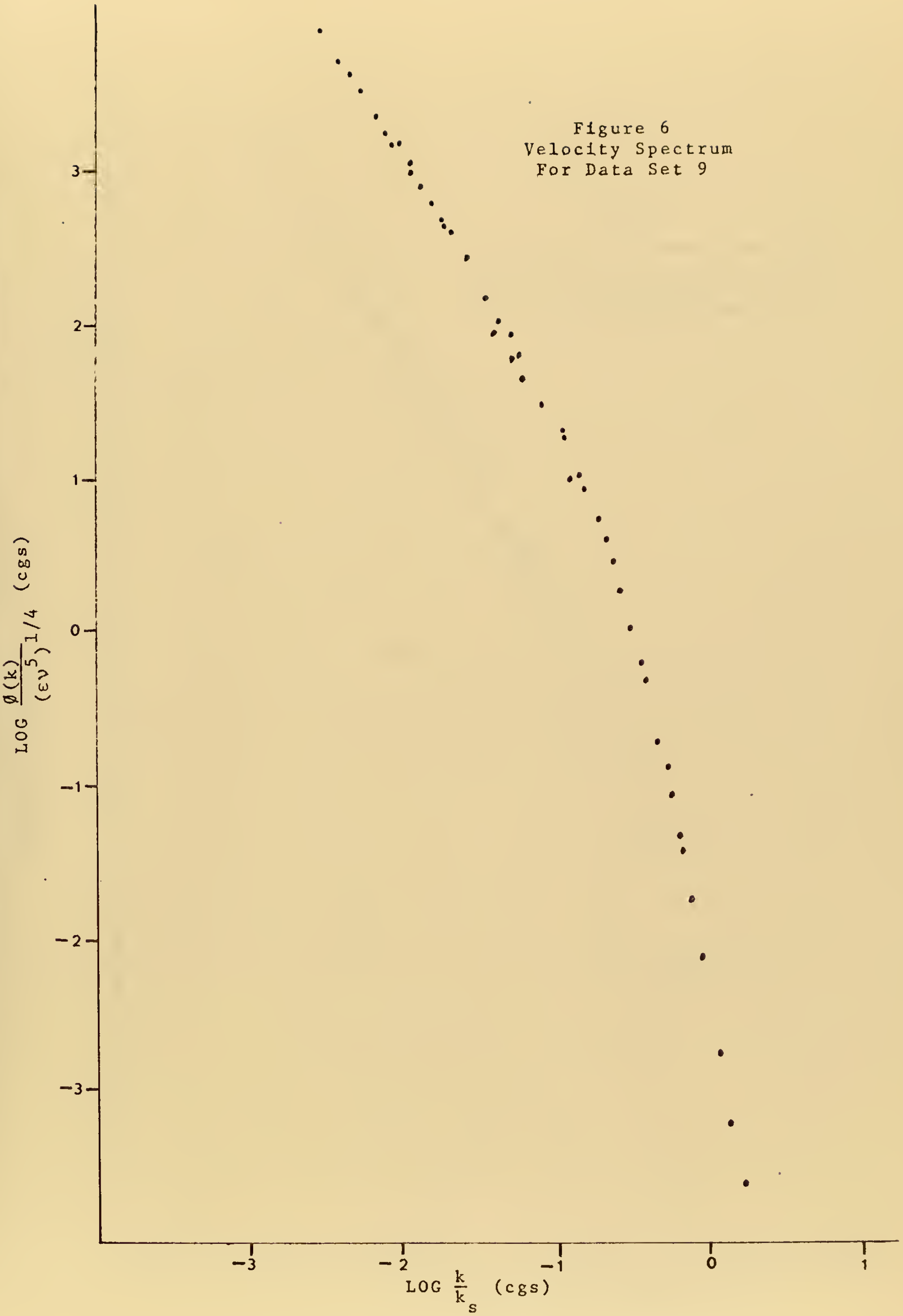


Figure 6
Velocity Spectrum
For Data Set 9
3
2
1
0
-1
-2
-3
LOG $\frac{\phi(k)}{(\varepsilon\nu^5)^{1/4}}$ (cgs)
-3
-2
-1
0
1
LOG $\frac{k}{k_s}$ (cgs)

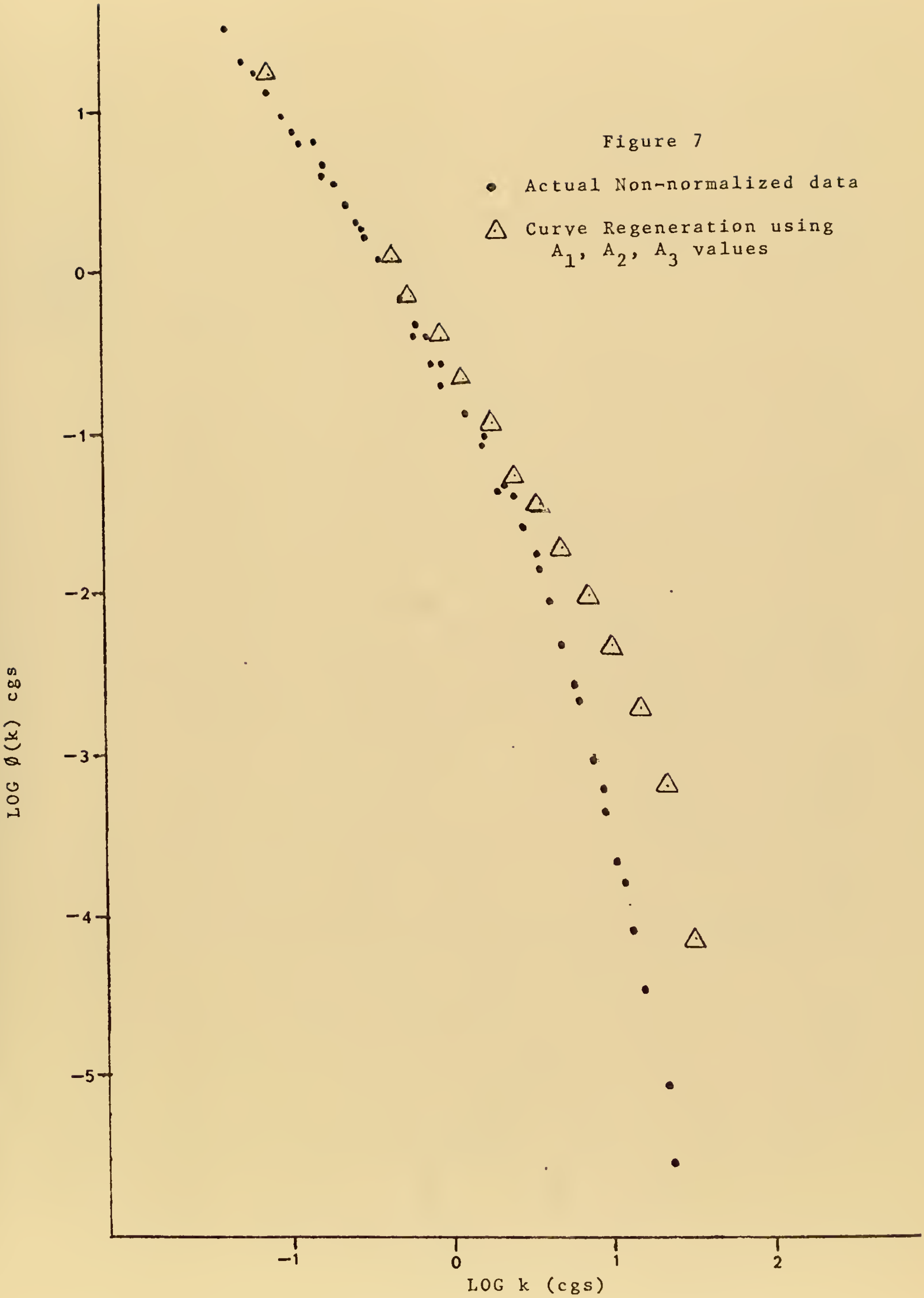
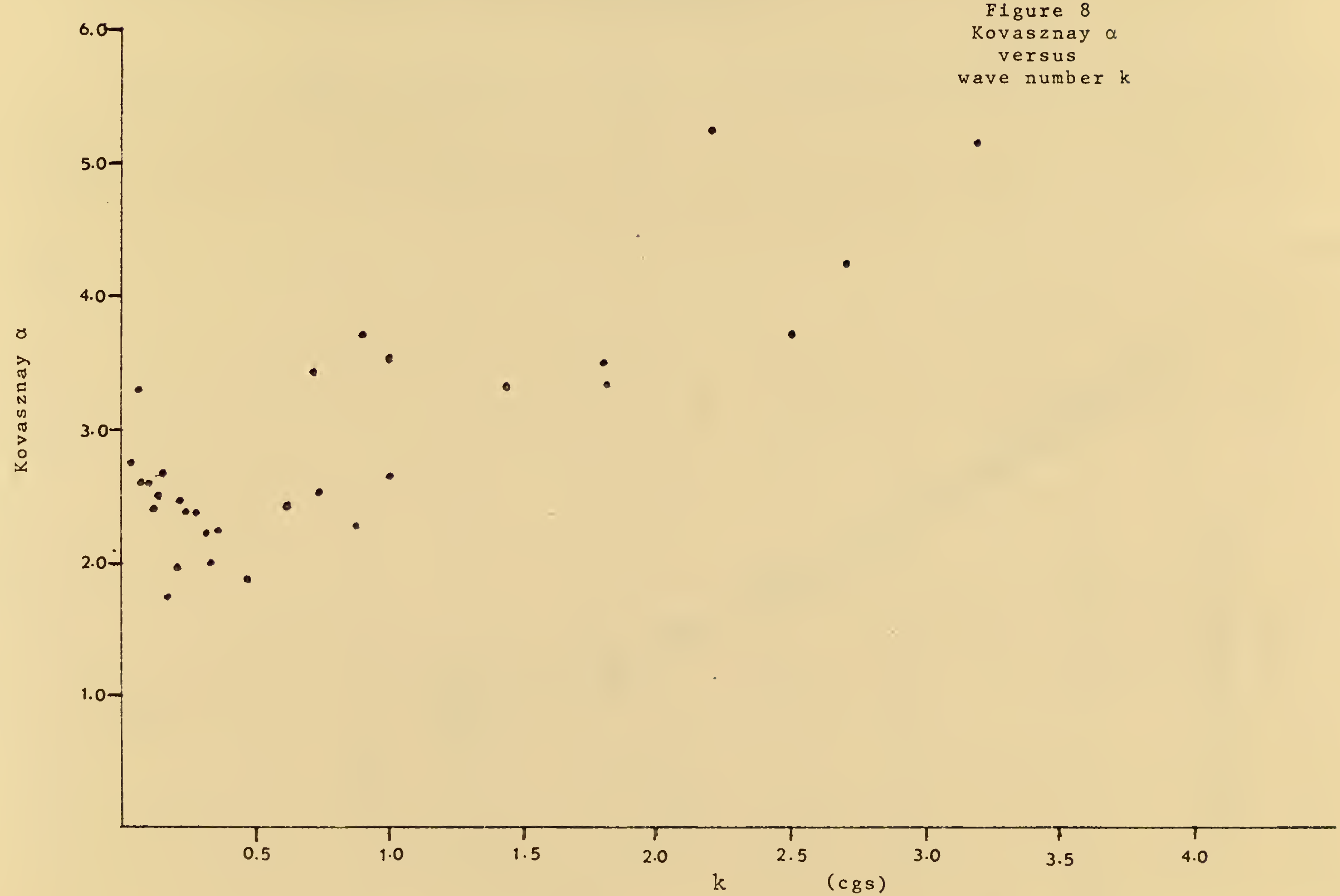


Figure 7
Actual Non-normalized data
Curve Regeneration using A_1, A_2, A_3 values
LOG $\emptyset$(k) cgs
LOG k (cgs)
1
0
−1
−2
−3
−4
−5
−1
0
1
2

Figure 8
Kovasznay α
versus
wave number k



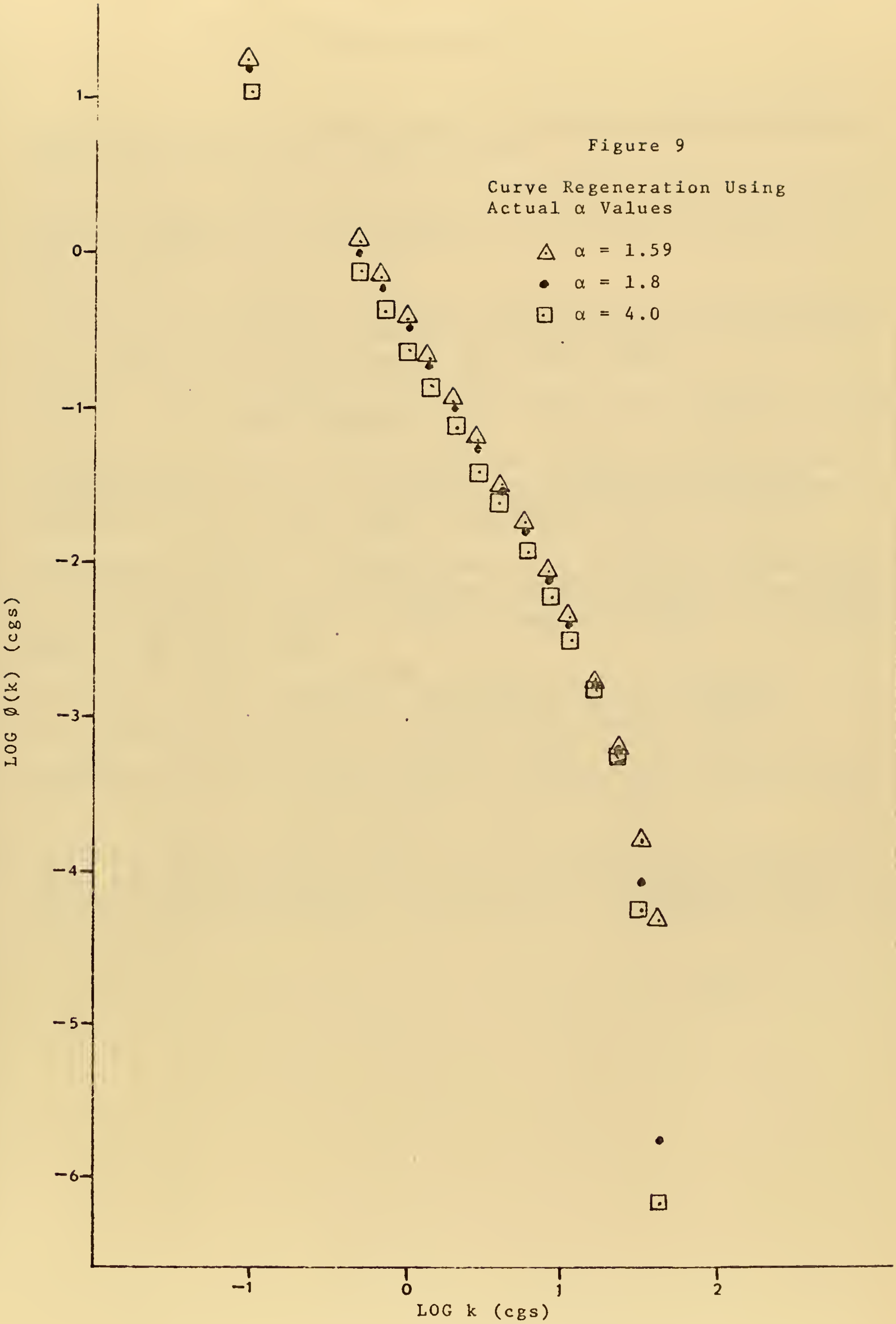


Figure 9
Curve Regeneration Using
Actual α Values
α = 1.59
α = 1.8
α = 4.0
LOG Ø(k) (cgs)
LOG k (cgs)
1
0
−1
−2
−3
−4
−5
−6
−1
0
1
2

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13. ABSTRACT

The shapes of spectra obtained from measurements of ocean turbulence are examined. Kovasznay's equation for high wave number turbulence is used to describe the inertial through dissipation regions. Power spectra of velocity fluctuation recorded in the open sea are used to evaluate constants occurring in the Kovasznay relation. Semi-empirical forms of high wave number energy spectra are then displayed. These computed spectra should define a universal form to which mathematical models should adhere.

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Power spectra						
Wave number						