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AN APPLICATION OF STATIC MARGINAL ANALYSIS
IN THE GENERATION OF A U. S. NAVY
REPAIR MATERIAL REQUIREMENTS LIST

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THESIS

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by

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and

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I. BACKGROUND

The Naval Air Systems Command Instruction 4700.5B of April 30, 1975 is the most recent in a series of instructions defining policy and prescribing procedures for supply support in commercial rework of aeronautical weapon systems and aircraft engines. The implementation of this series of instructions is through the Single Supply Support Control Point (SSSCP) concept. This concept involves an organization, referred to as the SSSCP, which is charged with two objectives of interest to this thesis: first, to achieve dollar savings by providing available government furnished material (GFM) to the commercial contractor for the support of a rework program and secondly, to minimize the rework turnaround time by reducing the overall supply response time through dedicated single point management.

Upon award of a commercial rework contract, an initial supply of available GFM is provided the contractor. The quantity of material provided is determined using a Repair Material Requirements List (RMRL). The RMRL is used by the SSSCP and the contractor as a guide for positioning and requisitioning GFM, respectively, to support an initial 90 day rework production schedule of end items. Timely receipt of this material insures support for the end items first inducted for rework and allows for an orderly implementation of follow-on material support procedures.

Before the development of the RMRL in the early sixties, a contractor was provided 100% of requirements of each line item for each end item to be reworked in the first 90 days of the contract. As an example, if the end item contained ten units of line item Y and 36 end items were to be reworked in the first 90 days, then 36 x 10 or 360 units of issue of item Y would be provided. During the contract performance phase, the contractor was charged to maintain a moving average of the usage rate of each line item and to use this information to order the expected demand for the next increment of end items to be reworked under the contract. The information gathered was subsequently formalized into the current Usage and Assets Report which gives the number of end items reworked and quantities of each line item used since the time of the last report and the quantity of each line item on hand at the time of the report.

By accumulating these recordes over several contracts the SSSCP was able to devise a replacement factor for each line item, according to the following formula:

$$R_i = \frac{U_i}{Q_i \cdot N_c} \quad ; \quad i = 1,2,...,n ,$$

where R_i = the replacement factor for the i^{th} line item.

U_i = the total number of line item i used over the several contracts.

Q_i = the quantity of line item i required for each end item.

N_c = the total number of end items requiring item i completed over the several contracts.

n = the total number of different line items applicable to the particular end item.

The resultant R_i is expressed as a percentage and rounded to the nearest integer value. Items with historical usages too low to produce a R_i of 1% or greater after rounding are not included in the RMRL. The combination of the quantity required per end item and the historical demand resulting in such a low R_i, apparently does not warrant the inclusion of these items in an initial inventory.

The replacement factors that are 1% or greater after rounding become the key elements in the generation of the RMRL. As presently structured, the RMRL is a computer-based listing giving National Item Identification Number (NIIN)/Manufacture Part Number, nomenclature, unit of issue, number of units of issue required per end item (Q_i), replacement factor (R_i), gross requirement (explained below), unit of issue cost, cost of the gross requirement and total cost for the RMRL. The gross requirement (G_i) is the quantity to initially be shipped to the contractor. It is determined from the quantity required (Q_i) per end item and the replacement factor (R_i), as follows:

$$G_i = \frac{R_i}{100} \times Q_i \times N \quad ; \quad i = 1,2,\ldots,n$$

where R_i is expressed as a percentage

N = the estimated number of end items to be reworked during the initial 90 days.

n = the number of different line items on the particular RMRL.

It should be noted that G_i is rounded to the next higher integer value and that G_i is never less than one.

The SSSCP, through the RMRL, will provide a contractor with the quantities calculated according to the above formulae as material for initial support. These quantities are the nearest integer value above the mean historical usage as long as the replacement factor, after rounding, is at least 1%. The occasional demand for an item not provided via the RMRL is satisfied by the follow-on material support procedures instituted at the time of contract award.

In an earlier time when there was much less concern over the allocation of limited budgets, the RMRL would not have been required. By providing 100% of engineering requirements, the disruption and cost associated with a stockout and with an order placement could be kept to a minimum during the first 90 days. Of course the amount of funds required to provide inventory storage, protection and control would be high and excessive funds would be spent shipping the very low usage material to one contractor after another until they are finally incorporated in the project or discarded due to wear and tear.

Today, however, with the multitude of military programs vying for a limited budget, a continuing search for cost-saving efficiencies is being carried out at all levels. The RMRL is an example of just such an efficiency, for it provides a much more realistic level of inventory (the expected demand for 90 days) than was provided prior to the implementation. However, further improvements appear possible for the RMRL generation technique. This thesis will present two improvements and will illustrate their potential benefits through an example.

II. NATURE OF THE PROBLEM

A. PROBLEM STATEMENT

One of the several problems associated with providing an initial inventory is the lack of knowledge concerning the underlying demand generation probability distribution function. This lack of knowledge usually leads to the use of an assumed distribution or to an inventory based on expected values such as the present RMRL. Another problem is in the choice of an optimization element, which could be any of a number of measures of effectiveness, e.g. number of orders or number of stockouts. The number of stockouts, or more correctly the cost of stockouts, is an appropriate optimization element since programs have experienced severe disruption and added costs due to unexpected stockoutages. As an extreme example of the size of the cost danger inherent in stockoutages, consider case number 15272 before the Armed Services Board of Contract Appeals, dated 18 March 1974 [1]. This dispute involved, among other things, the late and defective delivery of GFM to an aircraft rework contractor. The initial contract price was $4,164,326.84 with an ultimate claim settlement of $2,246,764.00 over and above the initial contract price. While the case was very involved and contained many claims and counterclaims, it demonstrates the potential cost impact of stockoutages.

The problem to be addressed in this thesis can be stated as follows:

Given a probability distribution of demand, develop a RMRL generation technique that minimizes the total expected cost of stockouts over all items during the initial contract period, subject to a budget constraint.

If s_i represents the number of units of item i to be stocked initially, then the problem can be stated mathematically as:

Find the value of $s_i \geq 0$, $i = 1,2,\ldots,n$, which

$$\text{minimizes} \quad \sum_{i=1}^{n} \pi_i \sum_{x=s_i}^{\infty} (x - s_i)\, p_i(x)$$

$$\text{subject to} \quad \sum_{i=1}^{n} c_i s_i \leq C$$

where n = the number of different line items

c_i = the unit cost for the i^{th} item

x = the demand for a line item

$p_i(x)$ = the probability that x units of line item i will be demanded

π_i = the weight (penalty cost or essentiality) of a stockout for item i.

B. SOLVING THE PROBLEM

The problem stated in the preceding section can be solved by the use of the Lagrange multiplier approach if demand x is a continuous random variable with density function $f_i(x)$. It should be noted that the budget constraint is binding, i.e. the constraint is active at the minimization point.

For the objective function,

$$\text{minimize} \quad \sum_{i=1}^{n} \pi_i \int_{s_i}^{\infty} (x - s_i)\, f_i(x)\, dx$$

the Lagrangian function would be

$$L = \sum_{i=1}^{n} \pi_i \int_{s_i}^{\infty} (x - s_i)\, f_i(x)\, dx + \theta\left(\sum_{i=1}^{n} c_i s_i - C\right) \qquad (1)$$

where the Lagrange multiplier is θ.

Then, for $s_i^* \geq 0$, where s_i^* is the optimal value of s_i, the value of s_i^* can be obtained from the calculus. The necessary conditions are given by equations (2) and (3)

$$\boxed{\frac{dL}{ds_i} = 0 = -\pi_i F_i(s_i) + \theta c_i \;;} \qquad i = 1,2,\ldots,n \qquad (2)$$

$$\frac{dL}{d\theta} = 0 \quad \text{or} \quad \sum_{i=1}^{n} c_i s_i = C \quad ; \quad s_i \geq 0 \qquad (3)$$

Equation (2) can be rewritten as

$$F_i(s_i) = \frac{\theta c_i}{\pi_i} \quad ; \quad i = 1,2,\ldots,n \qquad (4)$$

The usual computation procedure is to select a θ, compute s_i from (4), and then compute the left-hand side of equation (3), which we will denote by $\hat{C}$. If $\hat{C} > C$, a larger value of θ is selected and s_i is recomputed, if $\hat{C} < C$, a smaller value of θ is selected and s_i is recomputed. If $\hat{C} = C$ the current set of s_i values are optimal.

In the present case, however, the above technique based on differentiation cannot be used since s_i is restricted to integer values and the demand distribution is discrete for low demand items.

This problem can be resolved by the well-known method of finite differences [2]. In this method the differential equation is replaced by a difference equation because the continuous region in which the solution is desired has been replaced by a set of discrete points. If the number of items stocked is changed from $s_i - 1$ to s_i, the reduction in expected stockout cost is $\pi_i P_i(s_i)$, where $P_i(s_i) = \sum_{x=s_i+1}^{\infty} p(x)$, the complementary cumulative of the demand distribution for item i. The amount of the budget consumed by adding this unit is c_i. However, the finite differences method requires a comparison of all possible combinations of s_i values in order to determine the optimal combination of line items. Although theoretically possible, a heuristic process based on marginal analysis provides a much more intuitively appealing solution technique. The theory of marginal analysis has been used in inventory theory in numerous papers (see for example [3], [4], [5], [6], [7] and [8]).

In essence the theory states that an efficient mix of productive inputs is the mix for which the "marginal product equals marginal costs." In the present case of the generation of a RMRL, this means that the composition should be such that the inclusion of an additional unit of an item is solely

dependent on the decrease in expected stockout cost per budget dollar consumed. Thus the expected stockout cost reduction per unit increase in budget consumed is

$$\frac{\pi_i P_i(s_i)}{c_i}$$

The marginal analysis procedure progressively assigns a unit to the inventory of that item which yields the greatest reduction in expected stockout cost per unit increase in budget usage.

The first step is to set all $s_i = 0$ and compute

$$\max_i \left\{ \frac{\pi_i}{c_i} P_i(s_i + 1) \right\} = \max_i \left\{ \frac{\pi_i}{c_i} P_i(1) \right\} \tag{5}$$

If the maximum is taken on for item j, set $s_j = 1$ and deduct the unit price for unit j from the budget. The second step is then to compute

$$\max \left\{ \max_{i \neq j} \left\{ \frac{\pi_i}{c_i} P_i(1) \right\} , \frac{\pi_j}{c_j} P_j(2) \right\} \tag{6}$$

The next unit of inventory is assigned to the index where the maximum is taken on, and the unit price for the item included is deducted from the budget. This process will continue according to the general formula

$$\max \left\{ \max_{i \neq j} \left\{ \frac{\pi_i}{c_i} P_i(s_i) \right\} , \frac{\pi_j}{c_j} P_j(s_j + 1) \right\} \tag{7}$$

until adding an additional unit of item i would exceed the budget constraint.

It should be noted, however, that the method described does not insure optimality [3]. Specifically the method may stop too soon. If the item i selected from the marginal analysis has a c_i value greater than the remaining budget, the procedure terminates even though some other item j may have a c_j value less than the remaining budget. An obvious improvement in this area could be the inclusion of a subroutine that would select from remaining items the best one from those having c_j's smaller than the remaining budget.

The following example illustrates the procedure for $n = 2$. Assume that for item no. 1 and 2, the unit prices are \$500 and \$800 and the stockout costs are \$100 and \$200. Further assume that demands are Poisson distributed, and that the mean historical usages are 3 and 2 units respectively. Finally assume a budget constraint C of \$7,000. Then the optimal mix of units is 6 units for s_1 and 5 units for s_2, and the expected cost of stockoutage is \$1.51 (\$0.67 for s_1 and \$0.84 for s_2).

Table I presents the computations using equations (5), (6) and (7). In the table, x is the number of units stocked, $P_i(x)$ is the probability of using at least x+1 units and $\frac{\pi_i}{c_i} P_i$ is the reduction in expected stockoutage cost at the present stock level x. The inclusion number gives the sequence of inclusion of items 1 and 2. For example, item

TABLE I

x	$P_1(x)$	$\frac{\pi_1}{c_1} P_1$	INCLUSION NUMBER	$P_2(x)$	$\frac{\pi_2}{c_2} P_2$	INCLUSION NUMBER
0	.95021	.19004	2	.86464	.21616	1
1	.80085	.16017	3	.60399	.15099	4
2	.57681	.11536	5	.32332	.08083	6
3	.35277	.07055	7	.14288	.03572	9
4	.18474	.03694	8	.05265	.01316	11
5	.08392	.01678	10	.01656	.00414	13
6	.03351	.00670	12	.00453	.00113	15
7	.01190	.00238	14	.00110	.00027	17
8	.00380	.00076	16	.00024	.00006	19
9	.00110	.00022	18	.00005	.00001	21
10	.00029	.00006	20	.00001	.00000	
11	.00007	.00001	22			
12	.00002	.00000				

no. 2 gets one unit of inventory first because $\frac{\pi_2 P_2(1)}{c_2}$ is greater than $\frac{\pi_1 P_1(1)}{c_1}$. Then item no. 1 gets one unit since $\frac{\pi_2 P_2(2)}{c_2}$ is smaller than $\frac{\pi_1 P_1(1)}{c_1}$.

The above information is graphically shown in Figure 1. The solution from marginal analysis is optimal in this case as the budget is completely consumed.

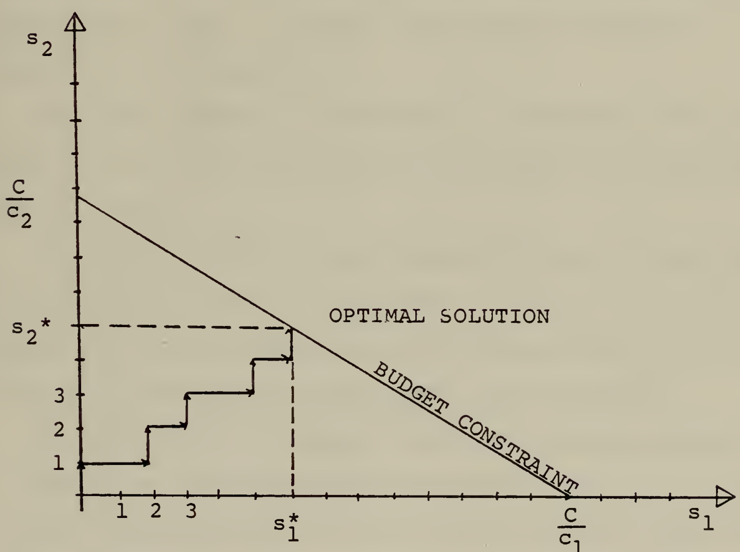

Figure 1

In the example above and in the formulae generated it was assumed that the cost of a stockoutage for any item i was known. However, this is not usually the case in a realistic situation. One approach is to use the π_i's to reflect an item's essentiality. The problem of essentiality of items over and above that indicated by their stockage history and/or cost has been addressed by several authors (see for example [9]) and will not be dealt with in this thesis. Instead of an actual stockout cost figure for each of the line items, it is assumed that the stockout cost is the same for all items and, hence, instead of minimizing the expected cost of outages, the expected number of stockouts is minimized.

The next example is presented to illustrate the problem when the cost of a stockoutage is assumed to be the same for all items.

The data in Table II were obtained from an actual RMRL for the R3350 engine. In the RMRL 14 items had a historical demand of .02 per engine, but the RMRL provided the contractor with one of each of the line items for 38 engines.

The left side of Table II shows the Poisson probabilities of demand for items that have an average demand of .02 per engine. Unfortunately, an item with an average demand of .02 part per engine does not experience exactly .02 demands per engine. Some engines have no demand, some have a demand of one or two. Each of the individual 14 items having an average demand rate of .02 per engine is subject to these

TABLE II

SPARES WITH DEMANDS OF .02 PER ENGINE

DEMAND FOR A SINGLE ITEM		EXPECTED SUPPLY RESULTS FOR 14 LINE ITEMS			
POSSIBLE DEMANDS	PROBABILITY		SURPLUS	CONSUMPTION	SHORTAGE
0	.98022				
1	.01965	STOCK ZERO	0	0	.2772
2	.00013				
3	.00000				
4	-	STOCK ONE OF EACH	13.7228	.2772	.0028
5	-				
6	-				
7	-				
8	-				

same probabilities. The items are assumed to be independent of each other in their demand distributions. The right hand part of Table II shows the expected supply results from all 14 items combined on one engine if the items are not stocked at all, and if the stock level is one of each.

As can be seen the expected number of stockouts on one engine in the case where zero is stocked is as low as .2772, indicating that none of the items should be stocked initially but should be ordered if needed when a stockout occurs. However, since an inventory consists of several items with

different historical mean demands, and since the initial inventory is subject to a cost constraint, we have to consider both the probability that a item will be used and the cost which must be paid in order to include it in the inventory. The composition of the initial inventory can then be arranged according to static marginal analysis so as to obtain the maximum protection from the available budget.

The cost and demand data for four items were obtained from the earlier mentioned RMRL, in order to show the composition of an inventory when cost as well as demand history were taken into consideration. Table III shows this data.

TABLE III

DEMAND-COST DATA FOR SAMPLE PROBLEM

ITEM	AVERAGE DEMAND PER ENGINE	UNIT COST $
1	.02	.08
2	.02	225.00
3	.30	.05
4	.30	19.00

TABLE IV

MARGINAL PROTECTION

X	REDUCTION IN EXPECTED NUMBER OF STOCKOUTS			
	ITEM 1	ITEM 2	ITEM 3	ITEM 4
1	.0198	.0198	.25918	.25918
2	.0001	.0001	.03693	.03693
3	-	-	.00359	.00359
4	-	-	.00026	.00026
5	-	-	.00001	.00001

Table IV shows the marginal protection for each of the four units in Table III. The marginal protection measures the additional value or "product" provided by each additional unit. The first column indicates the number of units of the particular item being considered. In the second column the first value is 0.0198 and is the reduction in the expected number of stockouts resulting from stocking one unit instead of none for item 1. The second value is 0.0001 and represents the reduction in the expected number of stockouts by stocking two units instead of one.

As can be seen, the first unit of item 1 provides as much protection as the first unit of item 2, as do the first units of items 3 and 4. The first units of items 3 and 4 provide better protection than those of 1 and 2.

Consider now the marginal protection per dollar. To allow for the cost effect, the marginal protection from each unit of each item is divided by the unit cost as shown in Table V, giving the marginal protection per dollar unit cost.

TABLE V

MARGINAL PROTECTION PER $ UNIT COST

UNIT NUMBER	ITEM 1 (0.08)	ITEM 2 (225.00)	ITEM 3 (0.05)	ITEM 4 (19.00)
1	.2475	.0001	5.1836	.0136
2	.0012	.0000	.7386	.0019
3	-	-	.0718	.0002
4	-	-	.0052	.0000
5	-	-	.0005	-

Once the marginal protection per dollar unit cost has been computed the process of selecting the units to go into the initial inventory is identical to that of the preceding example. All of the units are arranged in descending value of marginal protection per dollar as shown in Table VI.

Assuming a budget constraint of $265.00 the initial inventory would consist of the various numbers of units of the four items shown in column 3 in Table VI. Note that the budget is not exhausted and hence the process is not optimal in this example.

TABLE VI

LINE ITEMS ORDERED ON
MARGINAL PROTECTION PER $

RANK	MARGINAL PROTECTION PER $	ITEM AND UNIT #	ITEM COST	TOTAL COST
1	5.1836	3-1	.05	.05
2	.7386	3-2	.05	.10
3	.2475	1-1	.08	.18
4	.0718	3-3	.05	.23
5	.0136	4-1	19.00	19.23
6	.0052	3-4	.05	19.28
7	.0019	4-2	19.00	38.28
8	.0012	1-2	.08	38.36
9	.0005	3-5	.05	38.41
10	.0002	4-3	19.00	47.41
11	.0001	2-1	225.00	262.41

Inherent in the static marginal analysis theory are several assumptions. In the case of determining initial inventory levels these are:

1. It is possible to make all adjustments in the initial inventory prior to the period in which the inventory is to be used.
2. Subsequent adjustments during the period of use are not allowed.
3. The demand for the items in the inventory is independent of the quantities stocked.
4. There are no possibilities of substituting one item for another.
5. There is no discontinuity in the possible inventory quantities.
6. There is only one scarce resource which limits the size of the inventory.

The ability to make all adjustments prior to the period under question appears to be true in the generation of a RMRL. but it is not necessary to do so since reordering is allowed and will become a necessity during the contract period. On the other hand, the initial RMRL is the basis for the contractor's future ordering policy, i.e. he is allowed to order the difference between the RMRL quantity and what he has on hand plus eventual backorders in the initial contract period and beyond until the number of end items left to rework is less than the number used for generating the RMRL. Then the

reorder policy changes in a way such that the reorder quantity limit becomes the expected usage per line item times the number of end items left to do. This change is intended to reduce the ending inventory in the contractor's warehouse which will have to be returned to the SSSCP. Since the RMRL is the basis for future orders, an initial RMRL which minimizes the number of stockouts in the initial period should help to do so in the remainder of the program and hence benefit the total program.

It is true that the demands for any line item are independent of the quantity stocked since the rework is done according to a specification.

There is no possibility of substituting one line item for another since each line item has an individual National Item Identification Number indicating that it is truly unique.

There is some discontinuity for approximately 3% of the line items. By discontinuity is meant that the unit of issue is other than one, e.g. some washers come in boxes with a hundred items while the demands will be unity. However this is not believed to cause serious problems because of the relative low frequency of occurrence and is hence ignored.

The primary scarce resource in this problem is money; a budget constraint for the initial procurement predominates. All other usual limitations such as warehouse space (which is rented by the SSSCP at the contractors facility) could be transformed to a measure in monetary values.

From the above discussion it is obvious that the problem under study does not satisfy all of the underlying assumptions appropriate to the application of a static marginal analysis. However, the use of static marginal analysis, as an aid to managerial judgment may offer the possibility of reaching better decisions than could be reached by the use of the present system.

III. THE DATA BASE

In order to provide a comparison between present procedures and the proposed solution technique a current RMRL was obtained from SSSCP, Norfolk. Of the listings in existence, a RMRL for a pending contract for the overhaul of the R3350 aircraft engine appeared to be most appropriate. It was of a small size as such listings go (i.e. 2106 line items), the actual RMRL was immediately available, the majority of the nomenclature used was recognizable by the authors and two other matching pieces of background data could be provided. Copies of all the requisitions from a just completed contract for the overhaul of 167 R3350 engines[1] were provided by SSSCP for the analysis. A microfilm cartridge containing the actual quantity usage of line items applicable to the same contract was available and obtained on loan.

Initially the entire RMRL was to be included in the proposed comparison. As mentioned, it was relatively small (as compared to a complete aircraft RMRL with thousands of line items) and the inclusion of the entire RMRL would have lent credance to any findings. However, even with only 2106

[1]For background information, the R3350 is a large reciprocating, radial, aircooled aircraft engine which is on the P2 and C118 series aircraft of the U.S. Navy, U.S. Air Force and the military forces of several foreign nations.

items, the estimated computer memory requirement for the generation and simulation programs (to be described later) was 1,750,000 storage locations. A program calling for such a large memory allocation would have required esoteric programming and information management techniques which are beyond the capabilities of the authors. It should be noted however that the major portion of the memory requirement is involved in the simulation routines and that a usable program to generate a RMRL of 2106 line items under the techniques of static marginal analysis as in Chapter II would require only approximately 80,000 storage locations. For these reasons the concept of including the entire population was given over to one of sampling.

An initial review of the data suggested that a stratified sample could produce statistical benefits while reducing the sampling requirement. There appeared to be naturally occurring breaks when transaction value (unit cost times usage) was considered as a stratification medium. However, the data items required for the stratification analysis were distributed throughout the three different data documents, none of which were in machine readable form. Any attempt to stratify around these naturally occurring breaks would have required the transcription into machine readable form of the entire population just to determine the naturally occurring breaks upon which to stratify. The sheer magnitude of the task precluded this procedure.

A random sample of 200 line items was drawn from the combined data contained in the approximately 3800 IBM requisition cards and the RMRL. The requisitions were included in the population base to enable a sampling of those items which, because of very low demand, had not been included in the RMRL. The sample size of 200 was chosen based on an intuitive tradeoff between the computer requirements of a larger sample and the desire to include some of the non-RMRL items. The resultant sample of 200 did contain four such items. Although all four were requisitioned, only two showed usages according to the microfilm cartridge. There were therefore, 196 or 9.31% out of a possible 2106 line items applicable to the RMRL. The RMRL assigned cost associated with these 196 items at gross requirement levels (see Chapter I) represented 12.12% of the total of $1,216,401.20. This difference in percentages can be attributed, at least in part, to the wide divergence in the natural strata mentioned earlier. There were only nine very high transaction valued items in the data base of 2106 items and the sample drawing process chose a seemingly disproportionate number (two out of 196).

Two other significant problems noted with the data base were:

1. The requisitions cards obtained were copies of the originals and as such did not contain disposition information. This information was hand written on the original cards and

fell into one of three categories. The requisition could be filled as requested; it could be cancelled; or it could be modified to ship either more or less than requested.

2. The RMRL obtained had been recently generated for another contract yet to be let with a proposed induction of 38 engines in the first 90 days, while the completed contract operated with an RMRL generated for 36 engines.

Since requisition information was not available the requisition cards were useful only as an indication of the contractors order timing policy. As for the difference in the base for the RMRL, assurances were given by SSSCP, Norfolk that the replacement factors had not been updated so that a 36 engine RMRL could be devised from the data contained in the data base RMRL.

A RMRL based on current Navy procedures was produced for the 200 items in the random sample by employing the formulae given in Chapter I and the associated data from the data base (see Appendix A). The quantities provided were those of a RMRL for 36 engines for the 196 line items applicable to the original RMRL. The remaining four items were included but each at a zero level since they had not been included on the original RMRL. This listing, even though it contains items at a zero level which an actual RMRL would not, as explained in Chapter I, will be referred to as the STANDARD RMRL throughout the remainder of this thesis. The total value of the items provided by this STANDARD RMRL was $138,062.63.

IV. MODELS AND PROCEDURES

A. MODELS

This chapter describes the models developed and implemented in FORTRAN IV on the IBM 360/67 digital computer at the U.S. Naval Postgraduate School, Monterey, California.

The two primary models are:

1. A queuing model designed to simulate the flow of engines through a rework production facility.
2. A model to implement the marginal analysis concept in the generation of a revised RMRL.

The queuing model was developed to allow the simulation of the repair of the 167 engines in the data base contract, to see if the change in the RMRL generation technique did, in fact, influence the number of stockouts.

The first decision in the implementation of the second model was to determine which probability distribution provided an appropriate description of the random demand. The Poisson distribution was found to be the best based on tests to be described in Chapter V.

Having selected the probability distribution, the coding of a model to generate a RMRL based on the static marginal analysis concept was straight forward and follows the description given in Chapter II. The program flow involves the calculation of the probability of no stockout given that one more unit is added for each of the 200 line items,

dividing each by the appropriate unit price, searching the list thus produced to find the largest ratio and adding one unit to the inventory of the item with the highest ratio. One unit of issue price for the item selected is then subtracted from the starting budget. The new "probability" of that line item is then recalculated and divided by unit price. This process is continued until the inclusion of one more unit overdraws the budget.

Two major computer limitations caused deviations from the above straight forward implementation. First, when the model accumulates sufficient numbers of a given line item, the incremental protection obtained by adding one more item is very small and hence, severe rounding errors can occur even though double precision is used. This causes erroneous selection of the largest ratio when the ratios are all very small. Secondly, the FORTRAN compiler disallows the calculation of powers of e outside the range -180.218 to +174.673. Subsequently this was further reduced to ± 150.00 in the marginal analysis program.

The first problem was circumvented by incorporating into the model a test on the level of each line item after a new addition. When the level accumulated had reached a preset number of standard deviations (or "sigmas") (determined under Poisson demand as the square root of the mean) to the right of the mean, that line item was excluded from further consideration in the RMRL generation. This stopping point was denoted as a number of "sigmas" and was used as a specified

parameter on each simulation run (e.g., a 3 sigma run). The problems associated with the inclusion of such an artificial cut-off parameter will be discussed in Chapter V.

The second problem impacts those line items having mean demands per engine above approximately 4.16 since 4.16 x 36 $\simeq$ 150.00 and because the "powers of e calculation" is required in obtaining the Poisson probabilities. There were 23 such line items in the random sample of 200. These 23 items were subsequently excluded from the subroutine used to generate the marginal analysis (denoted RMRLLO) and were instead input to a special routine, labeled RMRLHI. This latter routine accumulated a number of units of each line item which was equal to the number of items originally determined by the STANDARD RMRL, i.e. the historical mean demand. Providing the inventories of the high mean items at their STANDARD RMRL quantities restricts the comparisons available from the simulation to those effects assignable to the RMRL generation technique on the low mean items. Inclusion of the higher mean items in the marginal analysis could be accomplished by use of the standardized normal approximation to the Poisson distribution. This approximation was not implemented due to time restrictions and will be investigated in detail in a follow-on research effort of one of the authors.

The model develops a RMRL from the input of a starting budget (normally equal to that which is required to support

a STANDARD RMRL) and the standard deviation bound or "sigma". This bound is the same for all items in RMRLLO. All 23 items which are inputs to RMRLHI are calculated into inventory first, the amount of money required for this volume of high demand items is then subtracted from the budget and the remainder of the budget is transferred to RMRLLO to generate the quantities of the remaining 177 line items.

The model developed to simulate engine flow through a production facility assumed the following factors:

1. The delivery of replacement parts was assured, with fixed delay times of 15 days for a routine order, and 2 days for an emergency order (placed as soon as a stockout occurs).
2. The time required to completely rework an engine was a function of the number of items to be replaced on the engine.
3. The initial RMRL quantities were all delivered prior to the induction of the first engine.
4. The demand for replacement parts could be viewed as a random process.
5. The order quantity policy implemented was as specified by SSSCP, Norfolk.
6. The reorder level of stocks was set to be the level expected to be demanded on eight engines.

Although the time required in actual practice to rework an engine was not known, the aggregate of approximately 380

days for 167 engine was known. Therefore several functional relationships were tested to determine which would provide this total time, yet would spread the engines out in time depending on a relative small change in the number of demands from one engine to the next. The one chosen assumed the following form

$$T = k \cdot e^{N}$$

T = the rework time for one engine

N = the number of items demanded to be replaced on a particular engine, divided by 100.

k = a constant.

In the model, T was then rounded to the next higher integer value. During the simulation runs the simulated total rework time for 167 engines fluctuated between 366 and 379 days depending upon the number of stockouts. The above functional relationship was not selected as a representation of reality, only as a means of producing a reasonable spread of the engines through the production facility and to approximate the total aggregate time used.

While it is true that an engine is overhauled according to specification, it is not true that a predetermined number of items are used on each engine other than a certain minimum amount of work associated with tear-down and build-up. Hence,

the demand for items to be replaced on each engine can be regarded as generated by a random process, in this case by the Poisson distribution as explained in Chapter V.

The order quantity policy implemented on the data-base contract is as described in Chapter II.

The actual reorder point was unknown. The reorder point for each line item was therefore set to be the expected demand of any item for eight engines. This level was chosen since the lead time was 15 days during which time approximately 8 engines would go through the production facility.

With the above factors specified, the simulation program was coded in discrete time flow fashion. Each day was searched for an engine just completing overhaul, if one was found, an engine waiting for overhaul was inducted. Demands were then generated for each of the two hundred line items, the quantities required were drawn from stock, and the time required to complete the engine was calculated. A computer flag was placed forward in time the number of days equal to the above time calculation.

The process was allowed to continue until the 167 engines were completed.

B. PROCEDURES

The general computer procedures followed are summarized in Figure 2 and consisted of the input of the two hundred line item data cards (see Appendix A), the programs described above and commented upon in Appendix B, and various initialization

cards, into the U.S. Naval Postgraduate School's IBM 360/67 digital computer. The resultant printed material is summarized in Tables VII through X in Chapter V.

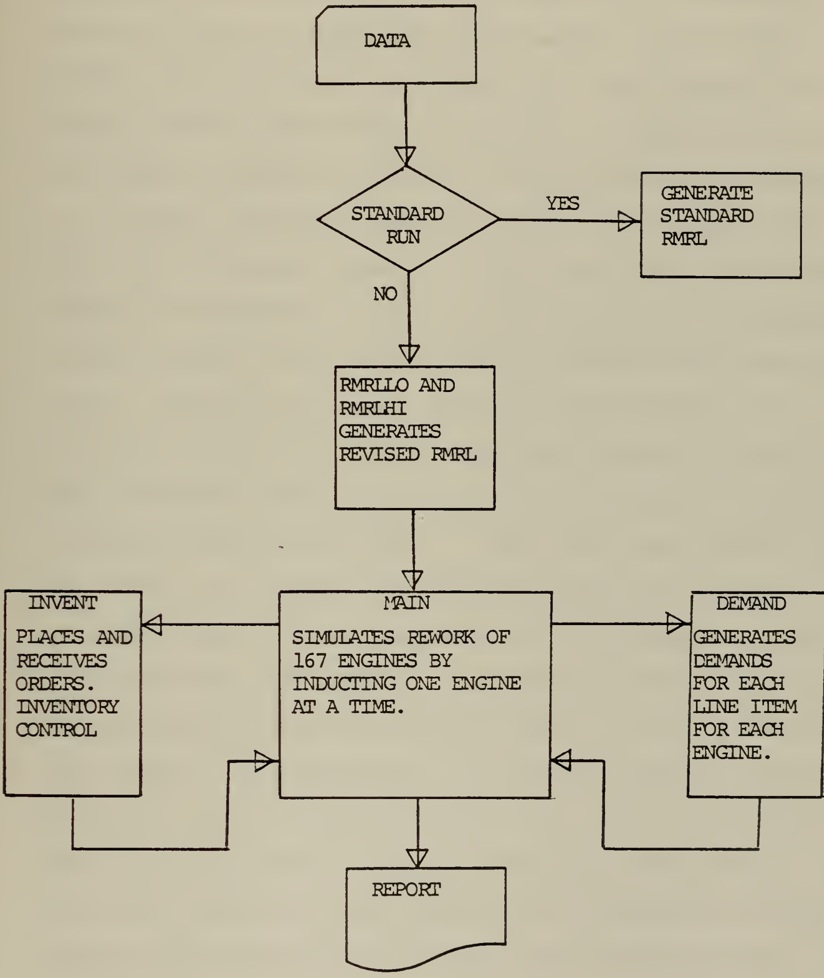

FIGURE 2

V. EVALUATION OF RESULTS

The first decision in the implementation of marginal analysis was to determine which probability distribution should be used to develop the incremental probabilities needed for equations (4), (5) and (6). The decision rule chosen was to accept that distribution of those tested which, when used to generate demands in the simulation program for 167 engines, produced usages which most closely matched the actual contract usage. To that end, Table VII was produced by performing a least squares linear fit of the predicted contract usage versus actual contract usage with the aid of the Computer Center Library program BMD02R. A separate predicted contract usage was obtained for each of four different demand probability distributions: Poisson, Uniform, Binomial and Normal. These four were chosen, with the exception of Normal, because all are of the single parameter type and that parameter can be determined from the historical mean demand. The standard deviation assumed in the Normal distribution was $\sqrt{\mu}$ where μ is the historical mean demand. The selection of $\sqrt{\mu}$ as the standard deviation was arbitrary and driven primarily by a lack of detailed data as to the actual variance involved. The standard deviation could have been specified as a parametric input, however time constraints precluded this investigation.

It should be noted that the demand for each of the 200 line items was assumed to be driven by the same probability distribution function. No attempt was made to mix two or more distributions within a single simulation run (e.g. Poisson for low demand items and Normal for high demand items). The possibility exists that such mixing and/or a standard deviation other than $\sqrt{\mu}$ might have produced a closer match to actual contract usage.

The BMDO2R program allows a forced zero intercept, which was activated. A perfect match to actual contract usage would therefore be a line with slope of +1.0 and with the sum of the errors squared equal to zero.

Four measures were chosen to be used as criteria to decide the most closely matched distribution. Two of these were the just mentioned slope (b) and the sum of the errors squared. Further, the following measure was calculated:

$$S = \sum_i (x_i - y_i)^2 c_i$$

where

x_i = the actual contract usage of line item i.

y_i = the predicted contract usage of line item i.

c_i = the unit price of line item i.

This measure, a weighted sum of the errors squared was calculated to provide a mechanism for emphasizing when any

distribution failed to match the actual usage in the high cost items. This rationale follows from the fact that the static marginal analysis technique gives up high cost/low demand items for low cost/high demand items. Therefore good predictions in the high cost region are of particular importance. In addition the contract cost for the total usage of all 200 line items was included as a final measure.

TABLE VII

DEMAND GENERATION

VS

ACTUAL CONTRACT USAGE

DEMAND DISTRIBUTION USED	SLOPE b	SUM OF THE ERRORS SQUARED	S	CONTRACT COST
ACTUAL CONTRACT	NA	NA	NA	617,849.31
POISSON	.99704	155,606	$.219 \times 10^7$	643,869.61
UNIFORM	1.00529	1,093,724	1.221×10^7	404,136.31
BINOMIAL	1.00122	62,993,488	6.422×10^7	632,674.98
NORMAL	1.01221	371,014	$.524.10^7$	436,914.81

As can be seen from Table VII the slopes (b) for all the demand distributions tested were very close to +1.0 with the Binomial being the closest followed by the Poisson. The sum of the errors squared for Poisson is by far the smallest (the jump to the next lowest is plus 138%). As for the S

value the Poisson is again the best demand generating distribution, indicating that this distribution most closely matches the entire population, which is confirmed by the total cost prediction in the last column. This is in spite of the better total cost prediction by Binomial because the sum of the errors squared and the S value for this distribution indicate large deviations from the actual usages. Therefore, considering all the above measures the Poisson distribution was chosen as the demand generating function in this thesis.

Once the demand generation technique was decided upon, the next decision to be made was which of the "sigma" parameter values would produce the lowest number of stockouts. From initial investigations it appeared that for the static marginal analysis to produce benefits in the generation of a RMRL by way of reduced stockouts, the sigma value should be as high as possible. This is because the higher the value of sigma the further the marginal analysis technique is allowed to continue prior to individual line item truncation created by additional units of a given line item not being included in inventory when the inventory level of that item reaches the mean plus the specified "sigma" value. In the present case, at a level of six "sigmas" and higher no truncation appeared to occur before the marginal analysis computations were terminated. Table VIII presents the results of a study of the influence of the value of "sigma" on the marginal analysis.

TABLE VIII

REVISED RMRL GENERATION WITH
VARIED SIGMA VALUE FOR RMRLLO

	# OF ORDERS	# OF STOCKOUTS	RESIDUAL VALUE
STANDARD	2510	165	$20,406.85
REVISED			
0 sigma	2561	187	$20,338.24
1 sigma	2432	171	20,253.62
2 sigma	2329	153	20,258.58
3 sigma	2279	141	20,272.05
4 sigma	2262	146	20,281.18
5 sigma	2262	140	20,283.81
$\geq$6 sigma	2254	139	20,282.89

Even though the lowest number of stockouts obtained during these simulation runs occurred at 6 sigma or higher it should be noted that rounding errors, as mentioned in Chapter IV, may be preventing an even lower number of stockouts. In addition, it should be noted that the number of orders is also lowest for the six sigma value or greater. This can be considered as a side benefit of the marginal analysis technique since the number of orders is reduced over STANDARD a greater percentage than that of the stockout reduction. The residual value (the dollar value of the inventory remaining in the contractor's facility upon

contract completion) is included to show that the reduction of stockouts was not the result of a greatly enlarged inventory being carried.

Table VIII shows a 16% reduction in the number of stockouts and a 10% reduction in the number of orders from generating the RMRL by static marginal analysis for the low mean demand items versus the present system of providing the mean historical demand. It should be noted that the maximum quantity allowed under the contractor's order policy implemented for Table VIII was based on the STANDARD RMRL even though a REVISED RMRL was initially issued. In order to see the effect of this restriction on the order quantity limits Table IX was produced by changing the contractor's order quantity limit to that provided by the REVISED RMRL.

TABLE IX

IMPACT OF REVISED RMRL GENERATION
AND REVISED ORDER POLICY ON SAMPLE DATA

ITEM			RMRL GENERATION METHOD								
			(1) STANDARD			(2) REVISED			(3) REVISED WITH REVISED ORDER POLICY		
NIIN	μ	c_i	Q	S	O	Q	S	O	Q	S	O
2095394	.07	.01	3	2	10	12	2	6	12	0	2
3128836	.03	129.00	1	1	3	2	1	2	2	0	2
1006170	2.04	.86	72	3	30	99	0	28	99	0	21
3065839	2.76	18.18	99	3	27	114	1	24	114	0	20
6514692	.01	1770.00	1	1	1	0	2	2	0	2	2

This table shows a selected sample from RMRLLO of one line item drawn from each of the possible combinations of relatively high and low mean and high and low cost line items. In addition, one very low mean/very high cost item is included. The left hand portion of the table gives the identification information, (NIIN), mean historical demand (μ), and unit price (c_i). The right hand portion is broken into three parts, each giving the quantity initially provided (Q), the number of stockouts (S) and the number of orders (O) resulting from each of the three methods used in the simulation. As can be seen by comparing the columns numbered (1) and (2), the static marginal analysis technique used in column (2) normally provides a larger initial quantity resulting in fewer stockouts and fewer orders. The exception shown in the fifth row is a prime example of how marginal analysis trades off a low mean/high cost item but pays a penalty by increasing the number of stockouts and orders for that item. Column (3) shows the further improvement of allowing the REVISED RMRL quantities rather than the STANDARD quantities to be used as subsequent ordering limits. For the runs labeled "REVISED with REVISED ORDERING", the ordering policy was therefore "order up to the REVISED RMRL quantity less any on-hand plus on-order minus backorders". Note that the number of stockouts and/or the number of orders is further reduced except, again, for the fifth row item.

The full and aggregated effect of implementing the revised ordering limits is shown in Table X. This table

repeats, for comparison purposes, the STANDARD and the REVISED "six sigma" runs first shown in Table VIII. In addition, the third row displays the REVISED six sigma run with the above described REVISED ORDERING policy implemented. All three of these runs were at the full initial budget of $138,062.63. The last five rows of Table X show the effect on the number of stockouts and orders caused by an incremental reduction of the initial budget for the REVISED RMRL with REVISED ORDERING policy.

TABLE X

SIMULATION OF 167 ENGINES USING STANDARD RMRL, REVISED RMRL AND REVISED RMRL WITH REVISED ORDERING

RUN	INITIAL BUDGET	# OF ORDERS	# OF STOCKOUTS	RESIDUAL VALUE
STANDARD	$138,062.63	2510	165	$20,406.85
REVISED 6 sigma	138,062.63	2254	139	20,282.89
REVISED	138,062.63	1841	99	24,795.09
6 sigma	125,000.00	1921	102	21,975.40
with	115,000.00	2031	132	21,561.76
REVISED	105,000.00	2128	152	17,452.42
ORDERING	95,000.00	2333	174	23,603.58
	85,000.00	2604	223	21,545.05

As can be seen, upon comparing the first three rows, the REVISED RMRL with the REVISED ORDERING policy gives the greatest reduction in number of stockout (40%) and in number of orders (26%).

A minor "penalty" is paid, however, in the form of a slightly increased residual inventory due to the change in the ordering policy. Further, it can be seen in the five last rows that, if the level of stockouts produced by the present system is acceptable (i.e. by the STANDARD run), an initial budget reduction of approximately $38,000 is possible. It should also be noted that, at this reduced budget level, the number of orders are still reduced by approximately 14%.

The five last rows show that the ending inventory is highly sensitive to the composition of the initial RMRL quantity and hence to the ordering quantity limit. This fluctuation can further be attributed to a few very high cost items and to the interactions involved in the initial budget and the reorder point. However, any savings in the initial budget are only temporary since the total usage over the contract will be the same regardless of when the material is delivered. But the process allows for the opportunity of temporary reallocation of the initial savings into other programs.

VI. CONCLUSIONS AND RECOMMENDATIONS

A. SUMMARY AND CONCLUSIONS

A revised RMRL generation technique was developed employing static marginal analysis and was shown to greatly reduce the number of stockouts as compared to that which would be expected by use of the present system based on mean historical demand. As a consequence the number of orders were also lower than the number of orders in the present system partly because of fewer back-orders and partly because of the change in the composition of the initial RMRL and hence in the maximum quantities allowed on a contractor's subsequent orders.

The use of marginal analysis requires that the demand generating probability distribution be known. The Poisson distribution appears to match most closely the actual demands for spare parts of the particular contract used as a comparison. Hence the Poisson distribution was used in the REVISED RMRL generation and was further assumed to apply to all line items in the sample.

Even though the number of stockouts was reduced, no general claim to optimality can be made. This because of the implementation of static marginal analysis (as described in the previous chapters) does not assure budget exhaustion. In the present case, however, only a little more than one dollar out of $138,062.63 was not consumed in the REVISED RMRL

generation technique. In addition, in the example, the higher mean demand items were excluded from the marginal analysis process. Inclusion of these items in the marginal analysis can be expected to give an even further reduction in the number of stockouts and orders.

B. RECOMMENDATIONS

It is recommended that the use of static marginal analysis be given serious consideration as a technique for generating Repair Material Requirements Lists in the U.S. Navy. Prior to such implementation, further investigations into the detailed historical demand data should be performed in order to derive the most appropriate demand probability distribution or mix of distributions.

The technical computer problem resulting in the exclusion of the high mean items in the marginal analysis process will be investigated by one of the authors in a follow-on effort. This investigation will attempt inclusion by use of the standardized normal approximation to the Poisson distribution. This author will also attempt to determine the improvements to be gained by the incorporation of a subroutine to exhaust the initial budget after the marginal analysis process has terminated.

In order to insure that the marginal analysis techniques developed in this thesis do not produce impractical results due to item essentiality, a technique for incorporating such essentiality data must be investigated. One way would be

to obtain a managerial judgment on the RMRL quantities of essential items. These quantities could be input as starting inventory, the associated costs subtracted from the initial budget and the remainder of the budget distributed according to static marginal analysis. Another and a more quantitative approach would be to obtain either actual or estimated cost figures for stockouts for each of the line items (the π_i values discussed in Chapter II). The equations from Chapter II could then be used in the marginal analysis and the essentiality problem incorporated directly.

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APPENDIX A

LIST OF SAMPLE DATA

NIIN	QTY PER ENG	RPL FCT	UNIT PRICE	CONTRACT USAGE
7047523	0	0	20.00	0
1749497	0	0	2.04	35
916593	0	0	69.00	0
184096	1	1	334.00	2
242896	2	10	16.75	32
316599	4	1	40.00	6
330368	20	12	35.30	407
379260	9	1	0.08	10
379363	1	7	5.21	11
379423	2	13	0.10	45
379691	2	15	0.05	50
458865	2	5	0.69	17
489131	45	3	62.40	190
1006170	2	102	0.86	340
1416693	1	6	0.05	10
1711509	1	7	2.94	12
2062981	1	3	0.30	5
2076434	18	5	0.42	157
2095394	1	7	0.01	12
2105349	9	41	0.06	617
2131789	1	4	54.05	6
2131813	3	1	107.80	7
2173185	1	32	12.18	53
2250470	2	51	74.00	172
2440514	5	2	0.10	15
2537554	1	16	0.36	27
2750475	7	8	1.05	97
2750632	18	9	0.05	263
2762769	18	1	0.05	32
2906984	1	17	0.57	28
2913285	1	16	0.06	26
2913291	4	18	0.06	123
2913303	2	10	0.04	34
2923120	2	4	0.20	12
2978384	6	11	1.32	112
2986868	2	1	0.14	4
3036014	1	43	2.70	71
3036123	1	2	37.43	3
3037779	1	44	0.09	73
3049019	1	37	35.00	61
3065839	4	69	18.18	461
3075570	2	4	3.33	15
3102870	1	1	88.75	2
3108941	9	14	9.60	208
3108946	8	48	2.21	637
3109004	1	13	86.19	21
3109005	6	12	85.12	120
3128836	1	3	129.00	5
3133636	7	5	0.47	61
3133653	1	6	36.84	10

NIIN	QTY PER ENG	RPL FCT	UNIT PRICE	CONTRACT USAGE
3133661	8	7	0.16	100
3133672	84	2	15.44	243
3144629	3	51	0.22	256
3144651	7	3	0.12	30
3144661	9	16	0.34	245
3148139	18	10	1.40	302
3236729	1	7	12.00	12
3236730	1	7	5.20	11
3260802	1	14	136.21	24
3266635	1	28	29.00	46
3266649	1	1	6.05	1
3266652	1	12	9.10	20
3266657	1	5	100.00	8
3306478	16	14	10.07	376
3306479	16	9	21.97	237
3320476	1	7	3.00	12
3320477	2	6	21.50	20
3320485	1	5	77.04	8
3354807	4	3	40.00	17
3357073	22	1	0.27	20
3421180	1	38	0.72	63
3441409	3	6	36.50	30
3459562	1	5	6.13	8
3596844	1	1	0.74	2
3963912	8	48	0.11	646
4335422	2	65	3.60	218
4423415	2	7	4.58	23
4451522	1	3	129.96	5
4460530	1	4	66.00	6
4788907	1	34	0.49	56
4789077	2	8	0.96	26
4848265	2	1	0.08	2
5058611	150	1	2.99	140
5058634	3	22	854.00	108
5063334	18	1	138.20	24
5085494	6	5	19.00	55
5126425	1	20	29.61	33
5129631	1	7	1.23	11
5129635	1	3	37.21	5
5129694	2	60	0.01	199
5129707	1	2	48.50	3
5129730	2	89	0.03	298
5129739	1	2	225.00	3
5129777	1	15	111.00	25
5129790	1	13	7.39	22
5150800	1	8	2.88	14
5155558	2	56	0.14	187
5163785	2	10	4.90	35
5164844	1	29	54.63	48
5224835	2	34	9.60	113

NIIN	QTY PER ENG	RPL FCT	UNIT PRICE	CONTRACT USAGE
5255110	6	6	0.06	63
5277488	2	78	0.15	260
5285651	32	10	0.12	510
5285683	1	10	0.98	17
5516876	4	7	2.21	49
5555751	1	25	14.00	42
5668943	1	4	592.00	6
5676397	6	7	0.97	70
5727165	1	33	6.76	55
5739655	3	5	218.00	25
5796324	2	23	1.48	76
5804444	2	73	1.53	245
5849563	4	4	0.02	30
5901802	150	1	9.20	150
5918215	1	8	849.00	14
5941171	1	5	42.43	8
5995989	5	44	0.06	368
5996406	2	71	0.14	238
6023691	2	13	144.00	42
6048493	8	5	0.25	68
6048494	1	6	0.25	10
6058293	1	2	78.09	3
6066965	2	7	170.10	22
6138001	168	2	8.40	588
6182527	1	14	0.21	24
6233794	1	11	10.61	19
6250754	1	4	399.91	7
6322052	1	5	0.02	9
6384070	6	1	1.10	9
6501192	1	23	50.56	39
6501194	1	2	15.50	4
6514692	1	1	1770.00	1
6547284	4	4	42.37	24
6547287	4	7	55.00	46
6563168	3	60	0.01	303
6598523	1	2	563.00	3
6621800	6	25	10.00	249
6622281	8	1	12.00	15
6622476	1	35	0.50	58
6736677	1	2	950.00	4
6807297	5	13	0.03	110
6807628	24	2	0.59	98
6969469	1	3	60.00	13
6969477	1	1	8.08	2
6974802	4	2	58.06	12
7047531	1	3	176.00	5
7161469	1	23	29.50	39
7162944	1	2	292.52	3
7162955	9	2	530.00	24
7172218	1	2	146.26	4

NIIN	QTY PER ENG	RPL FCT	UNIT PRICE	CONTRACT USAGE
7172223	4	35	0.10	235
7172404	4	2	40.19	11
7194426	2	30	72.57	99
7197729	8	30	1.25	401
7204894	1	15	18.98	25
7303275	1	8	99.00	14
7575069	1	6	0.24	10
7974052	27	1	0.08	50
8032651	4	61	0.22	410
8117017	1	23	10.50	38
8301942	1	3	35.00	5
8303008	1	2	26.80	4
8303010	1	2	48.50	3
8303012	1	4	61.41	7
8303040	1	7	57.83	11
8846264	2	10	775.22	32
8991790	1	1	0.50	1
9038282	1	10	0.03	16
9152018	2	1	3.50	3
9317218	2	3	36.10	10
9500039	118	1	3.68	102
9631387	3	13	161.95	65
9631388	3	4	152.00	22
9670092	1	17	13.00	29
9773423	6	3	17.50	32
9782993	9	1	8.45	17
1187490	9	75	0.10	1132
1476306	24	21	0.10	833
1984735	218	31	0.23	11329
2105221	36	82	0.08	4944
2686041	16	45	0.07	1200
3036651	36	12	0.69	727
3133641	9	48	0.15	726
3439277	72	92	0.05	11074
4790482	6	111	0.28	1111
5159073	6	85	0.16	855
5309323	72	22	0.09	2589
5513093	35	17	0.37	977
5804634	102	12	120.00	1961
5961865	54	17	0.17	1510
5966095	26	48	0.23	2078
6061829	205	4	1.12	1495
6118234	300	4	12.16	1922
6527000	52	28	0.02	2431
6621790	36	54	0.53	3258
6724938	18	60	0.04	1804
7220101	18	50	0.04	1491
8641347	150	9	6.52	2285
9086292	15	33	0.17	820
2973756	0	0	0.69	8001

APPENDIX B

COMPUTER PROGRAMS

The simulation model and the RMRL generation models were combined into one computer program consisting of the following primary routines:

1. MAIN PROGRAM

The main program initializes the various arrays and reads in an initialization card which specifies the probability distribution to be used in the random demand process whether the STANDARD RMRL or REVISED RMRL (generated by marginal analysis) is to be used, the initial budget and the sigma parameter. The two hundred line-item data cards are then read and the STANDARD RMRL is generated, if so specified; otherwise the two routines RMRLHI and RMRLLO are called and the REVISED RMRL is generated at the specified sigma level. The production process is started by the induction of the first five engines. After startup the heart of the program (a 380 day counter loop) is entered. Each day, this loop searches for an incoming order to be placed in stock and searches for a completed engine. If an order is found waiting, subroutine INVENT is called. If an engine is found, a new engine is inducted, subroutine DEMAND is then called, the quantities of each line item demanded is accumulated, subroutine INVENT is called to handle the demand, the completion time is calculated and a flag is

placed forward in time. After the 167 engines are complete, the orders yet to be received are cleared up and subroutine REPORT is called.

2. RMRLHI Subroutine

RMRLHI generates the portion of the REVISED RMRL for high mean demand items. In particular, those line items with engine means greater than 4.16 units, as explained earlier in Chapter IV, are handled.

3. RMRLLO Subroutine

RMRLLO generates the portion of the REVISED RMRL for low mean demand items using the concept of static marginal analysis as explained in Chapter II.

4. DEMAND Subroutine

DEMAND generates two hundred pseudo random numbers each time it is called with a probability distribution as specified from the initialization card.

5. INVENT Subroutine

INVENT, when called as a result of demands being generated, subtracts the usage from inventory stock and if necessary places an order for each line item affected. The quantity ordered is the original RMRL quantity less any units on-hand plus presently on-order minus any units on back-order. Backorders accumulate when demands exceed inventory stock. At the time of a stockout, the number of

stockouts is increased by one, one day is added to engine completion time as a delay penalty, an emergency order is placed and the amount of the outage is placed into a backorder file.

The mechanism of order placement is the forward storage in time, in a 380 x 200 storage location array, of the quantity ordered. Each order is placed forward an appropriate number of days depending on whether it is an emergency order or a routine order. Each order is stored according to NIIN (along the 200 dimension) and day of expected arrival (along the 380 dimension). The main program then searches this array each new day for incoming orders.

When INVENT is called because of an incoming order, the order is added to existing stock line item by line item, after first subtracting any appropriate backorder.

6. REPORT Subroutine

After all engines are complete and all incoming orders have been handled, REPORT is called to print the results of the simulation. For each of the 200 hundred National Item Identification Numbered line items, the ending inventory, its value, the quantity ordered, the number of such orders, the cost of the units used and the number of stockouts applicable to that line item are printed. In addition, grand totals for the ending inventory value, the number of orders, the usage cost and the number of stockouts are printed.

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