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MODELING THE EFFECT OF INFORMATION ON CONFLICT OUTCOME

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THESIS

MODELING THE EFFECT OF INFORMATION ON CONFLICT OUTCOME

by

Kemal Tonguc

September 1979

Thesis Advisor: D. P. Gaver

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Modeling the Effect of Information on Conflict Outcome

by

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Submitted in partial fulfillment of the
requirements for the degree of

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I. INTRODUCTION

The fundamental role of ground-combat troops is to "shoot, move, and communicate". Consequently models of combat operations must in some manner represent the attendant processes of attrition, movement, and command, control and communications. One of the principal methodologies for assessing casualties in simulated combat engagements is that involving classical Lanchester equations and their elaborations, which recently have been surveyed by Taylor [1]. The Lanchester equations describe the changes in the opponent force sizes (e.g., numbers of tanks, ships, planes or men) in terms of those force sizes (and, if desired, compositions) and general weapon effectiveness (acquistion and firing rate, probability of kill). That is, the state and prospect of the combat at any time is summarized in terms of the force sizes alone: the state of the system is taken to be the vector [R(t), B(t)] where R(t) is just the number of Reds surviving at t and B(t) represents Blue survivors. Both R(t) and B(t) are commonly viewed as deterministic functions of time, but these functions can be regarded as approximating the mean values of random processes. Some stochastic versions of these functions are described in Lehuczky and Perla [2], Gye and Lewis [3]. Despite the simplicity of such formulations, striking and plausible qualitative results are sometimes obtainable; for

instance the so-called "square law" asserts the advantage of force concentration. That other physical parameters (e.g., speed of advance) may change matters is recognized in Bonder and Farrell [4].

Combat is a fantastically complex random process. Despite the complexity of the combat between two military forces, Lanchester-type models that we consider here are all deterministic in the sense that each of them will always yield the same output for a given set of input data. They are commonly used for computational reasons. The purpose of this paper is to point out that certain qualitative features of combat situations that seem to be only faintly and implicitly present in the present Lanchester formulations can be explicitly included to a suggestive degree. The specific reference here is to the influence of information upon combat progress. Moreover, the approach taken can probably be extended to remedy other modeling deficiencies.

It is widely recognized that information may have a decisive influence upon the progress of modern military combat. Present day capability to gather, collate or "fuse", and disseminate information about an opponent's - and own force's - location, movements, and even state of information, could certainly not have been visualized in Lanchester's day, or even later. It therefore seems imperative that the information states of the opposing forces be modeled so as to reflect the obvious leverage of information upon the outcome of physical combat, the result of which is the

attrition (or withdrawal, or redeployment, etc.) of Red and Blue forces.

The idea explored here is to expand the description of the state of the combat system in order to (i) recognize the effective differences in useful information possessed by members of the opposing forces, and (ii) to model the rate at which combat-effectiveness-enhancing information transfer occurs. The modeling technique used here resembles the classical Lanchesterian deterministic differential equation approach. It can be expanded in various stochastic direction if desired.

The technique and approach described here can be "made stochastic" in several ways, but no attempt is made to do so here. The emphasis is on the formulation of the equations to describe the phenomena of information transfer as well as physical attrition; in this paper, the interplay of these factors is investigated numerically and not analytically.

For some reason very little recognition seems to have been given to the similarity between military combat situations and models of human or animal population interaction, e.g., the competition and predator-prey models of mathematical population biology; as described in Bartlett [5], May [6], and Hassell [7]. Comparisons may be in order, and be profitable to one and all. Likewise the approach taken here to consider multi-stated dynamic processes has long been used in chemical reaction theory and lately in pharmacology,

where "compartment models" are standard concepts; as can be seen in Bischoff, Dedrick and Zaharko [8] and Gaver and Lehuczky [9]. Again it appears that interactions between investigators are timely, and may well be mutually advantageous and stimulating.

II. INFORMATION STATES

A. A SIMPLE EXAMPLE INVOLVING DEFENSE OF A STRONGHOLD

Suppose a force of size R attacks a bastion or stronghold defended by a force of size B. Assume that the loss of B is relatively small throughout the engagement, but the attacking R force suffers attrition from B. We shall allow this attrition to depend upon the number of B's that possess relevant information about R's, and consequently upon the change in that number.

It may be reasonable to assume that initially B does not know the precise location and status of the individual units of R. If so, it is appropriate to model R attrition as the result of area or unaimed fire by B:

$$\frac{dR(t)}{dt} = - \rho_u (R(t)/R) B \qquad (2.1)$$

which is of course easily solved with B constant:

$$R(t) = R \exp[-\rho_u (B/R) t] \qquad (2.2)$$

where $R = R(0)$

Note that the attrition thus predicted is sensitive to information available to B in at least two ways: First, equation (2.1) is based on general area fire by B; if proper

designation of individual R units could be achieved, then R might actually be diminished in accordance with aimed fire, i.e., modeled by

$$\frac{dR(t)}{dt} = -\rho_a B \ , \tag{2.3}$$

so

$$\begin{aligned} R(t) &= R - B \cdot \rho_a \cdot t, \quad 0 \leq t \leq \frac{R}{\rho_a B} \\ &= 0 \ , \qquad\qquad \frac{R}{\rho_a B} \leq t \end{aligned} \tag{2.4}$$

If the attrition parameter $\rho_a = \rho_u$ (it likely will not be) then the initial attrition rates are the same, but aimed fire is much more punishing to R as time advances, if the weapons and rate of fire are at all similar.

B. INFORMATION STATES

The affect of information upon Red attrition may be modeled as follows. Divide the Blue forces into two groups; (i) those in the unaimed fire information state, and (ii) those in the aimed fire state; all B's are in one state or the other. This affiliation is thought to be the result of possessing suitable information, and does not depend upon location (although terrain features may be important) or special equipment beyond what is needed to receive the information.

Let

$B_u(t)$ = number of Blues capable of executing unaimed or area fire at time t, and

$B_a(t)$ = number of Blues capable of executing aimed fire at t.

Hence we have

$$\frac{dR(t)}{dt} = -\rho_u(R(t)/R)B_u(t) - \rho_a B_a(t)$$

$$= -\rho_a B_a(t) - \rho_u \frac{R(t)}{R}(B - B_a(t)) \qquad (2.5)$$

assuming that B survives without attrition (at least initially), and that all B's are in action. If (2.5) is written as follows

$$\frac{dR(t)}{dt} + \rho_u \frac{R(t)}{R} B_u(t) = -\rho_a B_a(t) \qquad (2.6)$$

then it is easily integrable: apply the elementary integrating factor technique

$$\frac{d}{dt}[R \exp\left\{\int_o^t \rho_u B_u(z)dz\right\}] = -\rho_a B_a(t) \exp[\int_o^t \rho_u B_u(z)dz],$$

which leads to the formal solution

$$R(t) = R \exp[-\int_o^t R^{-1} \rho_u B_u(z)dz]$$

$$-\rho_a \int_o^t B_a(v) \exp[-\int_v^t R^{-1} \rho_u B_u(z)dz]dv, \qquad (2.7)$$

valid so long as the right-hand side is positive, and zero otherwise. Notice that if $B_u(t) \equiv B$, so no information is passed that allows conversion from unaimed to aimed fire, then (2.7) reduces to (2.1) for $B_a(t) = 0$. On the other hand, suppose $B_a(t) \equiv B$ and $B_u(t) = 0$, then (2.7) reduces to (2.3), the case of aimed fire, as is again proper. It is now of interest to trace the effect of some specific information flow mechanism upon Red survivorship. It turns out that this is best done numerically, for even the simple closed-form solution (2.7) is virtually uninterpretable, and matters rapidly deteriorate further when more complex models appear.

C. REPRESENTATIONS OF INFORMATION FLOW

In this section some possible representations for the change in the information states are presented. Note that no attempt is made to model the actual process of flow; the eventual impact upon R(t) of the rate or timing of transition from unaimed to aimed fire is all that will be investigated for the present.

1. Instantaneous Transition

Suppose

$$B_u(t) = B \qquad 0 \leq t \leq \bar{t}$$
$$B_a(t) = 0 \qquad (2.8)$$

while

$$B_u(t) = 0$$
$$B_a(t) = B\,, \qquad \bar{t} \leq t$$

In other words, all B forces receive and profit from the required information instantly at time $\bar{t}$ - possibly $\bar{t}$ is the time at which a reconnaissance effort is completed and the results disseminated. Note, too, that the charge could be the result of changed visibility for B, e.g., because of terrain changes (there is suddenly no cover) or because of weather effects, i.e., wind blowing away smoke used for camouflage.

It is easy to see from (2.7) that

$$R(t) = R \exp[-(\rho_u(B/R)t)] \qquad 0 \leq t \leq \bar{t} \qquad (2.9a)$$

$$= R \exp[-(\rho_u(B/R)t)] - \rho_a B(t-\bar{t}), \quad \bar{t} \leq t \qquad (2.9b)$$

where the last expression is replaced by zero when it becomes negative.

2. Gradual Transitions: First-Order Rate Process

Suppose we can describe the effect of information transfer as follows.

$$\frac{dB_a(t)}{dt} = kB_u(t) = k(B-B_a(t) \qquad (2.10)$$

or equivalently,

$$\frac{dB_u(t)}{dt} = -kB_u(t) \qquad (2.11)$$

Thus the rate of conversion to aimed fire is proportional to the number currently engaging in unaimed fire. Solutions are immediate:

$$B_u(t) = B(0)\ e^{-kt} \equiv Be^{-kt}$$
$$B_a(t) = B(1-e^{-kt}) \qquad (2.12)$$

This is a classical "learning curve" - the larger k, the more rapid is the learning -. Adoption of this model leads by specializing (2.7) to the expression ($R(0) \equiv R$

$$R(t) = R\ \exp[-\int_0^t \rho_u B e^{-kz} dz] - \rho_a B \int_0^t (1-e^{-kv}) \exp[-\rho_u B \int_v^t e^{-kz} dz] dv$$

Simplification gives

$$R(t) = R\ \exp[-\frac{\rho_u B}{k}(1-e^{-kt})] - \rho_a B \int_0^t (1-e^{-kv}) \exp(-\frac{\rho_u B}{k} \cdot e^{-kv} - e^{-kt}]) dv$$

$$= R\ \exp[-\frac{\rho_u B}{k}(1-e^{-kt})]$$

$$-\rho_a B\ \exp[\frac{\rho_u B}{k} e^{-kt}] \left\{ \int_0^t e^{-\alpha e^{-kv}} dv - \int_0^t e^{-\alpha e^{-kv}} e^{-kv} dv \right\} \qquad (2.13)$$

where $\alpha = \rho_u B/k$ for temporary convenience in the remaining integrals. Next reduce the integrals to the degree apparently possible:

$$\text{(i)} \quad \int_0^t e^{-\alpha e^{-kv}} dv = k^{-1} \int_{\alpha e^{-kt}}^{\alpha} e^{-x} \frac{dx}{x} = \frac{1}{k} [E_i(\alpha e^{-kt}) - E_i(\alpha)]$$

where $E_i(\cdot)$ is the exponential integral; see Abramowitz and Stegun [9], where tabulations and approximations are given.

$$\text{(ii)} \int_0^t e^{-\alpha e^{-kv}} e^{-kv} dv = (k\alpha)^{-1} \int_{\alpha e^{-kt}}^{t} e^{-x} dx = (k\alpha)^{-1}[e^{-\alpha e^{-kt}} - e^{-\alpha}]$$

Formula (2.13) expresses R(t) entirely in terms of tabulated functions, so numerical solutions are in hand, in principle. Alternatively, one could numerically solve (2.6) directly, using standard algorithms for solving ordinary first-order linear differential equations. Investigations of the sensitivity of the solutions to changes in parameters - particularly k, the "learning rate" - can then be straightforwardly carried out. Such numerical solutions are far more comprehensible than the formulae presented above.

3. Gradual Transition: Linear Increase

Let

$$\begin{aligned} B_a(t) &= kt , & 0 \leq kt \leq B \\ &= B , & kt \geq B \end{aligned} \qquad (2.14)$$

which can also be expressed in terms of a differential equation. This model might reflect the way in which information traverses a linear network, taking into account deterministic delays but no errors in the "pass-it-on" process. Now substitute into (2.7) to find R(t):

$$R(t) = R \exp\left(-\int_0^t \rho_u[B-kz]\right) - \rho_a \int_0^t (kv) \exp\left(-\int_v^t \rho_u[B-kz]\,dz\right) dv,$$

$$0 \leq t \leq B/k$$

$$= R \exp\left(-\int_0^{B/k} \rho_u[B-kz]\,dz\right) - \rho_a \left\{ \int_0^{B/k} kv \exp\left(-\int_v^{B/k} \rho_u[B-kz]\,dz\right) dv \right.$$

$$\left. + \int_{B/k}^t B \exp\left(-\int_v^{B/k} \rho_u[B-kz]\,dz\right) dv \right\}, \quad B/k < t \qquad (2.15)$$

with the usual proviso that $R(t) = 0$ if the right-hand side of the above expression becomes negative. Again everything can be integrated in tabulated form, although numerical solutions will probably be more useful.

III. COMBAT AND INFORMATION UNDER CONDITIONS OF MUTUAL ATTRITION

In the previous section we studied a model that illustrated the impact of information flow upon conflict. In this section the more conventional models that allow mutual attrition are re-examined, with the objective of tracing the effect of the compa-ative information-handling capabilities of the antagonists.

A. INFORMATION AND PHYSICAL STATES

The forces in conflict are classified as to whether they can accomplish unaimed or aimed fire (other classification may be more meaningful, and can probably be identified). That is

$B_u(t)$ = number of Blue forces in unaimed state at time t,

$B_a(t)$ = number of Blue forces in aimed state at time t;

$R_u(t)$ and $R_a(t)$ are defined analogously.

Now we will examine the cases that allow mutual attrition considering different possible representations of information flow.

1. Gradual Transition: First-Order Rate Process

Suppose we can describe the effect of information transfer as in (2.10) and (2.11). Then it can be re-written for both Blue and Red forces as follows.

$$\frac{dB_a(t)}{dt} = b_{ua}\, B_u(t) \qquad (3.1a)$$

$$\frac{dB_u(t)}{dt} = -b_{ua}\, B_u(t) \qquad (3.1b)$$

$$\frac{dR_a(t)}{dt} = r_{ua}\, R_u(t) \qquad (3.1c)$$

$$\frac{dR_u(t)}{dt} = -r_{ua}\, R_u(t) \qquad (3.1d)$$

The terms b_{ua} and r_{ua} may be thought of as representing a rate of information transfer causing a change from an unaimed to an aimed capability (or simply as learning rate). The above equations describe only changes in the number of combatants in each information state when there is no attrition, e.g., before a conflict actually starts.

Now consider the following representative set of four simultaneous differential equations suggested to describe the change in the state vector $\{R_u(t), R_a(t), B_u(t), B_a(t)\}$.

$$\frac{dR_u(t)}{dt} = -r_{ua}R_u(t) - \rho_{uu}(R_u(t)/R)B_u(t) - \rho_{au}B_a(t)\left(\frac{R_u(t)}{R_a(t)+R_u(t)}\right) \qquad (3.2a)$$

$$\frac{dR_a(t)}{dt} = r_{ua}R_u(t) - \rho_{ua}(R_a(t)/R)B_u(t) - \rho_{aa}B_a(t)\left(\frac{R_a(t)}{R_a(t)+R_u(t)}\right) \qquad (3.2b)$$

$$\frac{dB_u(t)}{dt} = -b_{ua}B_u(t) - \eta_{uu}(B_u(t)/B)R_u(t) - \eta_{au}R_a(t)\left(\frac{B_u(t)}{B_a(t)+B_u(t)}\right) \qquad (3.2c)$$

$$\frac{dB_a(t)}{dt} = b_{ua}B_u(t) - \eta_{ua}(B_a(t)/B)R_u(t) - \eta_{aa}R_a(t)\left(\frac{B_a(t)}{B_a(t)+B_u(t)}\right) \tag{3.2d}$$

The arguments used to derive these equations can be illustrated for, say, the first equation. They are analogous for those remaining.

(i) The term $- r_{ua}R_u(t)$ in (3.2a) - see also b_{ua} in (3.2c) - represents the rate at which forces capable of unaimed fire shift to aimed fire capability; with the rate of information transfer r_{ua}. The particular mathematical form is likely to be incorrect in detail; a more appropriate one can be derived by careful consideration of intelligence and reconnaissance activity and information dissemination.

It is the term, or its elaboration, that is effected by ADP equipment, communication systems, and the like. In a sense, the larger r_{ua}, the better is the C^2 capability of Red.

(ii) The term $- p_{uu}(R_u(t)/R)B_u(t)$ represents the attrition of Red unaimed forces by Blue unaimed. It can be regarded as the result of writing

$$-\rho_{uu}\left[\frac{R_a(t)+R_u(t)}{R}B_u(t)\right]\frac{R_u(t)}{R_a(t)+R_u(t)}$$

where the term $\left[\frac{R_a(t)+B_u(t)}{R}B_u(t)\right]$ is the classical unaimed fire term, with aimed and unaimed equally vulnerable, while $(R_u(t)/(R_a(t)+R_u(t))$ represents the probability that the

recipient of fire is actually an aimed Red element.

(iii) The term

$$-\rho_{au} B_a(t) \frac{R_u(t)}{R_a(t)+R_u(t)}$$

represents the attrition of Red unaimed forces by Blue aimed forces.

The parameters ρ_{uu}, ρ_{au}, ρ_{ua}, and ρ_{aa} represent physical attrition rates of Blue against Red, and the parameters η_{uu}, η_{au}, η_{ua}, and η_{aa} are the corresponding physical attrition rates for Red against Blue. Of course, all of these can be rendered time dependent, or otherwise altered as desired.

2. Instantaneous Transitions

Suppose all Blue forces receive and profit from the required information instantly at time t_b and all Red forces at time t_r. The number of combatants in the information states can be written as follows

$$B_u(t) = B(t) \qquad 0 \leq t \leq t_b$$
$$B_u(t) = 0 \qquad (3.3a)$$

while

$$B_u(t) = 0$$
$$B_a(t) = B(t) \qquad t_b \leq t$$

Similarly for Red forces;

$$R_u(t) = R(t) \qquad 0 \leq t \leq t_r$$
$$R_a(t) = 0 \qquad (3.3b)$$

while

$$R_u(t) = 0$$

$$R_a(t) = R(t) \qquad t_r \leq t$$

Suppose $t_b < t_r$; then three sets of differential equations can be written to describe the change in the state vector $\{R(t), B(t)\}$.

$$\frac{dB(t)}{dt} = -\eta_{uu}(B(t)/B)\ R(t) \qquad 0 \leq t \leq t_b \qquad (3.4a)$$

$$\frac{dR(t)}{dt} = -\rho_{uu}(R(t)/R)\ B(t)$$

and

$$\frac{dB(t)}{dt} = -\eta_{ua}(B(t)/B)\ R(t) \qquad t_b \leq t \leq t_r \qquad (3.4b)$$

$$\frac{dR(t)}{dt} = -\rho_{ua}\ B(t)$$

finally

$$\frac{dB(t)}{dt} = -\eta_{aa}\ R(t) \qquad t_r \leq t \qquad (3.4c)$$

$$\frac{dR(t)}{dt} = -\rho_{aa}\ B(t)$$

The arguments used to derive these equations are exactly the same as the arguments used for equations (3.2a, b,c,d). The above equations can be derived directly from equations (3.2a,b,c,d) by keeping the equations (3.3a,b) in mind.

IV. COMBAT WITH AND WITHOUT COORDINATION

In this section some simple models of combat that include a coordination effect are suggested. The method will be to study the effect of lack of coordination upon the attrition power of one force against another, and then compare this with the increased attrition power obtained under coordination - the latter being made possible by information flow.

A. MODEL 1: STATIC SALVO INTERCHANGE

Suppose a group of Blue forces confront one of Red forces. And suppose Blue wishes to attack Red, and does so without coordination, i.e., each B picks a member of R at random and fires at it once, independently of the behavior of the other Blues. For the moment assume that all Reds are equally likely to receive a Blue's fire. Also assume that the kill probability of B against R is unity; this is extreme but rather informative, and can later be relaxes.

Obviously the lack of coordination among Blues creates inefficiency: some Reds will receive two or more of Blue's missiles, while some therefore will receive none. As a measure of the effectiveness of such fire on the part of B, the expected number of Reds destroyed will be calculated.

1. The Expected Number of Red Forces Destroyed

This is a classical "occupancy problem" and can be neatly solved by use of indicator functions. If X_R is the random variable denoting the number of Reds hit by B missiles after one B salvo, note that

$$X_R = 1_1 + 1_2 + \text{---} 1_R \tag{4.1}$$

where the indicator

$$1_j = \begin{cases} 1 & \text{if j'th R hit by B} \\ 0 & \text{otherwise} \end{cases}$$

Now

$$E[X_R] = \sum_{j=1}^{R} E[1_j] \tag{4.2}$$

Since each 1_j has the same marginal distribution, we need only calculate that the probability that all B shots are directed elsewhere is $[(R-1)/R]^B$ and so

$$P(1_j = 0) = (1 - \frac{1}{R})^B \tag{4.3}$$

while

$$P(1_j = 1) = 1-P(1_j = 0) = 1-(1 - \frac{1}{R})^B$$

and therefore it follows that the expected number of Reds hit under uncoordinated attack is

$$E[X_R] = R[1 - (1 - \frac{1}{R})^B] \qquad (4.4)$$

Calculation of the variance and distribution is also possible, but is more complicated. It is also possible to derive a formula for the situation in which the probability of a B killing each R depends upon which R is fired upon. That is, suppose each B picks the j'th R with probability r_j. Then the probability that no B picks the j'th R is $(1-r_j)^B$, and finally

$$E[X_R] = \sum_{j=1}^{R} [1 - (1-r_j)^B] \qquad (4.5)$$

It is even possible to calculate the expected number of Reds destroyed if the probability that the i'th B picks the j'th R independently is r_{ij}. For then the probability that the j'th R is not picked is

$$(1-r_{1j})(1-r_{2j})\text{----}(1-r_{Bj}) = \prod_{i=1}^{B} (1-r_{ij})$$

and, adding up over the j Reds we find

$$E[X_R] = \sum_{j=1}^{R} 1 - \prod_{i=1}^{B} (1-r_{ij})] \qquad (4.6)$$

For the moment we stick with the simple model (4.4) for discussion.

It is instructive to look at the ratio

$$\frac{E[X_R]}{R} = \text{Expected fraction of Reds hit}$$

as the latter depends upon the (fixed) ratio of B to R: $B/R = \beta$. Thus from (4.4)

$$\frac{E[X_R]}{R} = [1-(1 - \frac{1}{R})^{\beta R}] \to 1-e^{-\beta} \qquad (4.7)$$

if B (and R) become large. This is very simple and handy and leads to an immediate assessment of the effect of coordination, for by our assumptions if B (= βR in number) fires in a coordinated fashion at R, i.e., each B has only one R target, then $X_R \equiv E[X_R] = \beta R$, provided $\beta \leq 1$ ($B \leq R$) while $X_R \equiv E[X_R] = R$ if $B \geq 1$ ($B > R$). If we assess the advantage of coordination by

$$A(\beta) = \frac{E[X_R] \text{ Under coordination}}{E[X_R] \text{ Without coordination}} = \begin{cases} \frac{\beta}{1-e^{-\beta}}, & \beta \leq 1 \\ \frac{1}{1-e^{-\beta}}, & \beta > 1 \end{cases} \qquad (4.8)$$

Here is a sketchy numerical table to illustrate the gain from coordination at constant B-to-R ratio (β) when B and R are large

β	$A(\beta)$
0.2	1.10
0.4	1.21
0.6	1.33
0.8	1.45
1.0	1.58
1.2	1.43
1.4	1.33
1.6	1.25

In this simple model coordination pays off most when the forces are about equally numerous: if B is much smaller than R then the chances of random overlap are small and so coordination is not required, while if B greatly outnumbers R coordination will again not be required to assure coverage.

2. The Expected Number of Blue Forces Destroyed

The above model merely calculates the effect of a single B action against R. If we assume that R fires simultaneously at B then the corresponding expected number of Blues hit is, by symmetry,

$$E[X_B] = B[1-(1 - \frac{1}{B})^R]_j \qquad (4.9)$$

this comes from (4.4)

B. MODEL 2: DYNAMIC SALVO INTERCHANGE

Suppose the two forces A and B are now imagined to interchange fire steadily. Let B(t) and R(t) denote the (expected) numbers of each surviving at time t. Assume that the expected number of Reds actually targeted by Blues can be calculated by use of formula (4.4) with R(t) = R and B(t) = B, and the same for Blues targeted by Reds using (4.9). Let P_R be the effective kill probability per unit time of Reds against Blues and P_B be the corresponding quantity for B. Note that the attrition aspect of our model is very much simplified, and we are approximating expectations. Refinements can, and should, be made. However, forging ahead we are led to write down these differential equations to describe the mutual attrition of the B and R forces when both behave in an uncoordinated manner:

$$\frac{dR(t)}{dt} = - P_R R(t) \left[1-\left(1 - \frac{1}{R(t)}\right)^{B(t)}\right] \qquad (4.10a)$$

and

$$\frac{dB(t)}{dt} = - P_B B(t) \left[1-\left(1 - \frac{1}{B(t)}\right)^{R(t)}\right] \qquad (4.10b)$$

These equations are highly non-linear, and there seems to be no easy solution; numerical methods must be used. Division of one by another gives an implicit relationship between R and B

$$\frac{dR}{dB} = - \frac{P_R}{P_B} \frac{R}{B} \left[\frac{1-(1-1/R)^B}{1-(1-1/B)^R}\right]$$

again a solution in simple form is not in evidence.

V. THE EFFECT OF INFORMATION IN COMBAT WITH AND WITHOUT COORDINATION

Recall that the forces in conflict are classified as to whether they can accomplish unaimed or aimed fire. Those were

$B_u(t)$=number of Blue forces in unaimed state at time t,

$B_a(t)$=number of Blue forces in aimed state at time t.

We should consider coordination only for aimed forces; since the group of forces in the unaimed fire information state have no certain idea about the location of opponents, they cannot naturally be considered as coordinated or uncoordinated forces. So, obviously we can only talk about coordination of the group of forces that are in the aimed fire information state.

A. THE EFFECT OF INFORMATION IN COMBAT WITHOUT COORDINATION UNDER CONDITIONS OF MUTUAL ATTRITION

Suppose both Blue-aimed forces and Red-aimed forces attack without coordination. Also suppose the transition from unaimed to aimed fire can be modeled as gradual transition according to a first-order rate process. Then consider the following representative set of four simultaneous differential equations suggested to describe the change in the state vector $\{R_u(t), R_a(t), B_u(t), B_a(t)\}$:

$$\frac{dR_u(t)}{dt} = -r_{ua}R_u(t) - \rho_{uu}(R_u(t)/R)B_u(t) - \rho_{au}R_u(t)[1-(1 - \frac{1}{R_a(t)+R_u(t)})^{B_a(t)}] \quad (5.1a)$$

$$\frac{dR_a(t)}{dt} = r_{ua}R_u(t) - \rho_{ua}(R_a(t)/R)B_u(t) - \rho_{aa}R_a(t)[1-(1 - \frac{1}{R_a(t)+R_u(t)})^{B_a(t)}] \quad (5.1b)$$

$$\frac{dB_u(t)}{dt} = -b_{ua}B_u(t) - \eta_{uu}(B_u(t)/B)R_u(t) - \eta_{au}B_u(t)[1-(1 - \frac{1}{B_a(t)+B_u(t)})^{R_a(t)}] \quad (5.1c)$$

$$\frac{dB_a(t)}{dt} = b_{ua}B_u(t) - \eta_{ua}(B_a(t)/B)R_u(t) - \eta_{aa}B_a(t)[1-(1 - \frac{1}{B_a(t)+B_u(t)})^{R_a(t)}] \quad (5.1d)$$

The parameters used in the aimed parts of the equations correspond to the effective kill probabilities per unit time. That is,

ρ_{au} and ρ_{aa} correspond to P_{r_j}

η_{au} and η_{aa} correspond to P_B.

B. THE EFFECT OF INFORMATION IN COMBAT WITH AND WITHOUT COORDINATION UNDER CONDITIONS OF MUTUAL ATTRITION

Suppose Blue-aimed forces attack with coordination, while Red-aimed forces have no coordination in their attack. Again considering a gradual transition from unaimed to aimed fire, the set of simultaneous differential equations would be as follows:

$$\frac{dR_u(t)}{dt} = -r_{ua}R_u(t) - \rho_{uu}(R_u(t)/R)B_u(t) - \rho_{au}B_a(t)\left(\frac{R_u(t)}{R_a(t)+R_u(t)}\right) \qquad (5.2a)$$

$$\frac{dR_a(t)}{dt} = r_{ua}R_u(t) - \rho_{ua}(R_a(t)/R)B_u(t) - \rho_{aa}B_a(t)\left(\frac{R_a(t)}{R_a(t)+R_u(t)}\right) \qquad (5.2b)$$

$$\frac{dB_u(t)}{dt} = -b_{ua}B_u(t) - \eta_{uu}(B_u(t)/B)R_u(t) - \eta_{au}B_u(t)\left[1-\left(1-\frac{1}{B_a(t)+B_u(t)}\right)^{R_a(t)}\right] \qquad (5.2c)$$

$$\frac{dB_a(t)}{dt} = b_{ua}B_u(t) - \eta_{ua}(B_a(t)/B)R_u(t) - \eta_{aa}B_a(t)\left[1-\left(1-\frac{1}{B_a(t)+B_u(t)}\right)^{R_a(t)}\right] \qquad (5.2d)$$

VI. NUMERICAL EXAMINATION OF MUTUAL ATTRITION

In this section the models that are presented before are explored numerically. Our numerical results suggest that the interplay between the physical (e.g., exchange rate) parameters and the information transfer parameters can indeed lead to quite interesting combat outcomes. In all cases initial forces are the same. In later investigations this will be changed (Appendix A).

A. COMBAT OUTCOMES UNDER CONDITIONS OF MUTUAL ATTRITION WITH COORDINATED-AIMED FIRE

1. Information Transfer With Gradual Transitions (First-Order Rate Process)

Here the equations (3.2a,b,c,d) are re-examined numerically

Exhibit 1: $r_{ua} = 0.5$, $b_{ua} = 1.5$, $\rho_{uu} = \rho_{ua} = 0.01$

$\rho_{au} = \rho_{aa} = 0.1$, $\eta_{uu} = \eta_{ua} = 0.02$, $\eta_{ua} = \eta_{aa} = 0.2$

The Blue information handling capability or transfer rate, b_{ua}, is greater than (three times) that of Red, r_{ua}, but the physical attrition rates of Blue by Red are sufficiently high to overcome this advantage; Red wins, and is never behind.

Exhibit 2: $r_{ua} = 0.5$, $b_{ua} = 1.5$, $\rho_{uu} = \rho_{ua} = 0.01$

$\rho_{au} = \rho_{aa} = 0.1$, $\eta_{uu} = \eta_{ua} = 0.015$, $\eta_{au} = \eta_{aa} = 0.15$

Here Blue's information handling capability again exceeds Red's by a factor of three, but Red's physical superiority is not so great. Thus for a time Blue survivors exceed Red. However, eventually Red wins if the fight goes on long enough. Note that if the fight is terminated at a break point of, say, 20% loss, the number of Blue survivors would exceed the number of Red surviving.

Exhibit 3: $r_{ua} = 0.5$, $b_{ua} = 1.5$, $\rho_{uu} = \rho_{ua} = 0.015$ $\rho_{au} = \rho_{aa} = 0.1$, $\eta_{uu} = \eta_{ua} = 0.01$, $\eta_{au} = \eta_{aa} = 0.1$

Again Blue surpasses Red in information handling. And Blue exceeds Red's effectiveness during the unaimed phase, but not during the aimed phase. Blue enjoys a longer, but still temporary, advantage, eventually losing in a fight to the finish. Nevertheless, at a reasonable break point level Blue would win. This would not happen were Red to possess a sufficient "information edge".

Exhibit 4: $r_{ua} = 0.5$, $b_{ua} = 1.0$, $\rho_{uu} = \rho_{ua} = 0.01$ $\rho_{au} = \rho_{aa} = 0.1$, $\eta_{uu} = \eta_{ua} = 0.01$, $\eta_{au} = \eta_{aa} = 0.12$

In this example, Blue's information-handling advantage is decisive, being 20 times that of Red. Contrast this to Red's physical equality in the Unaimed State, and actual superiority in the Aimed State. Despite this, it appears that Blue surpasses Red from the start. This effect must be the result of superior information handling as portrayed by our model.

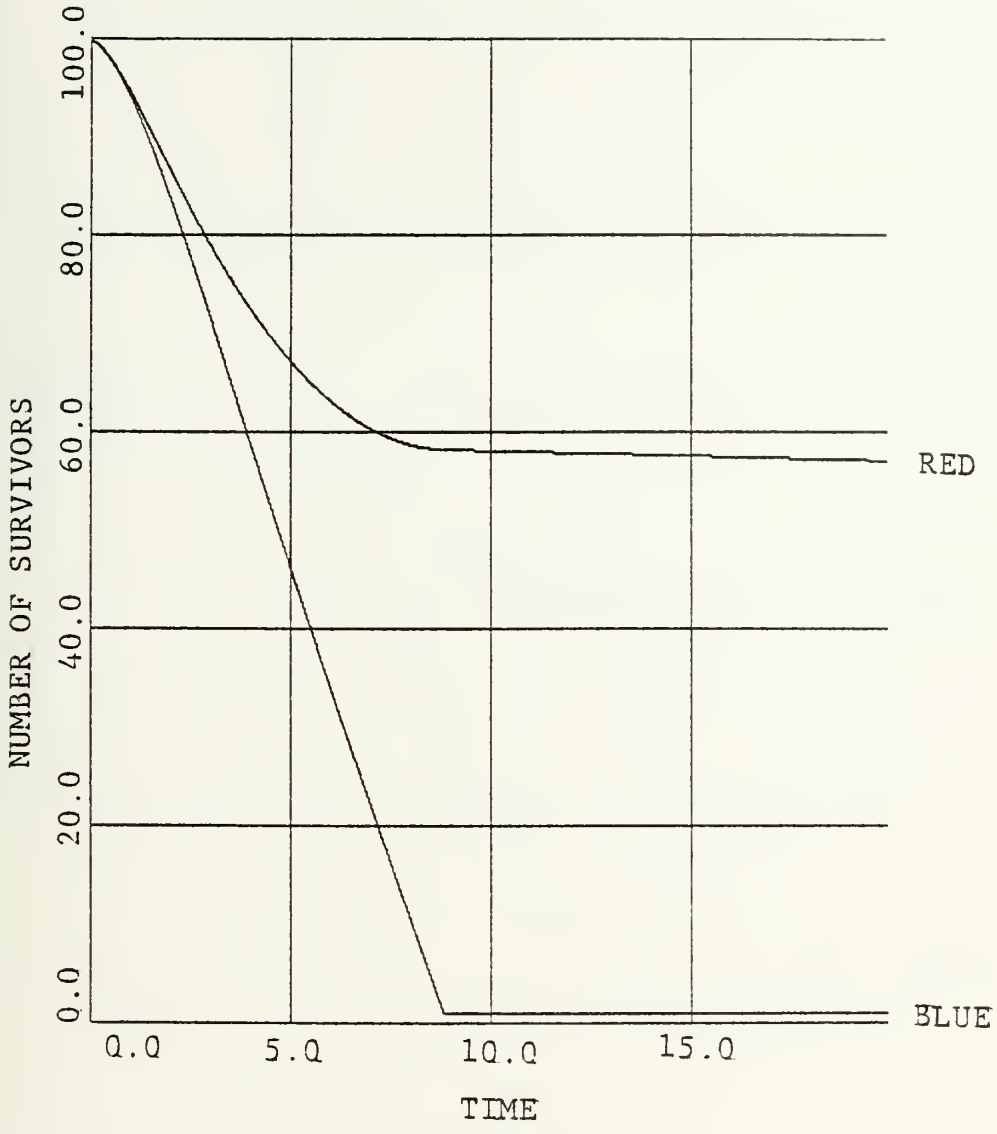


Exhibit 1.

$b_{ua} = 1.5$ $\quad r_{ua} = 0.5$

$\rho_{uu} = \rho_{ua} = 0.01$ $\quad \eta_{uu} = \eta_{ua} = 0.02$

$\rho_{au} = \rho_{aa} = 0.1$ $\quad \eta_{au} = \eta_{aa} = 0.2$

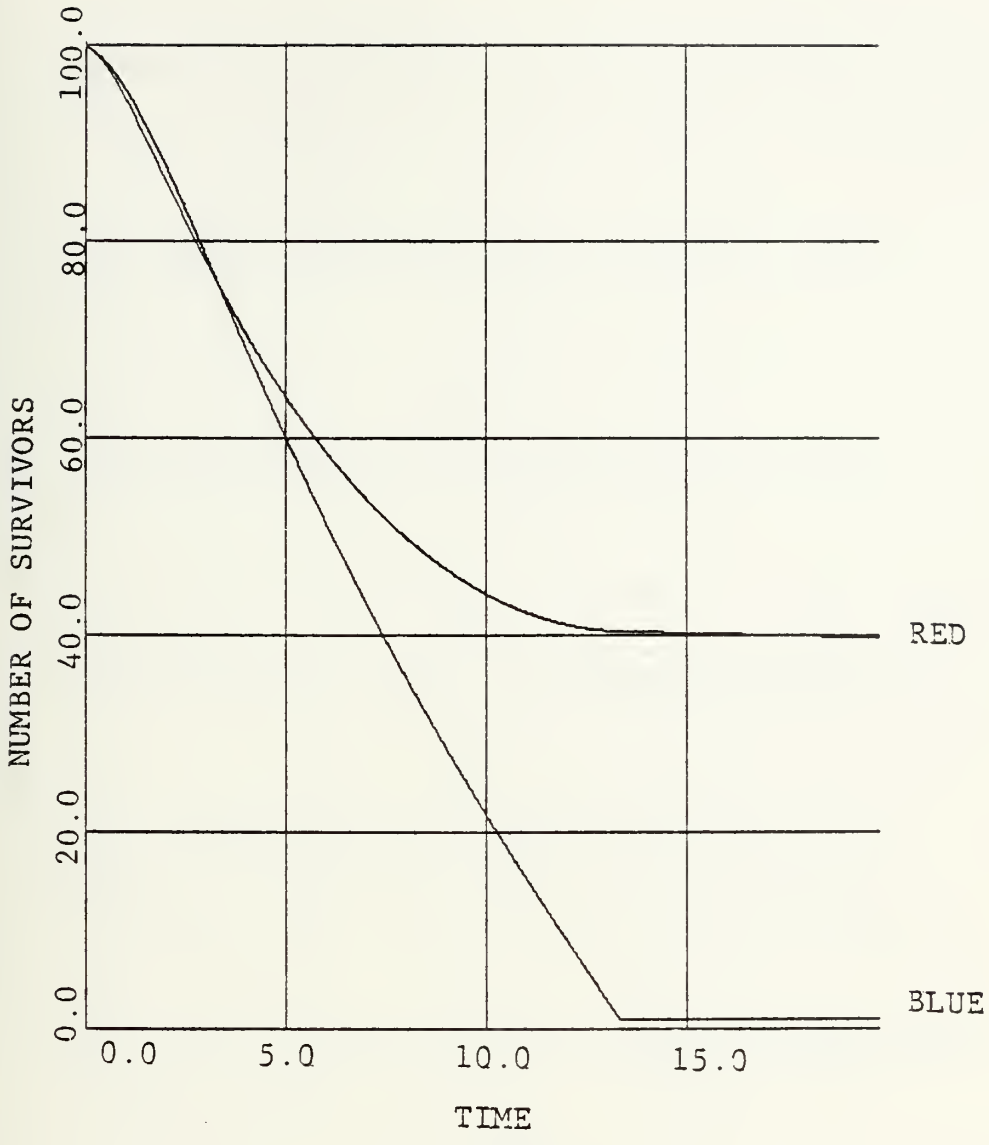


Exhibit 2.

$b_{ua} = 0.5$ $\quad r_{ua} = 0.5$

$\rho_{uu} = \rho_{ua} = 0.01$ $\quad \eta_{uu} = \eta_{ua} = 0.015$

$\rho_{au} = \rho_{aa} = 0.1$ $\quad \eta_{au} = \eta_{aa} = 0.15$

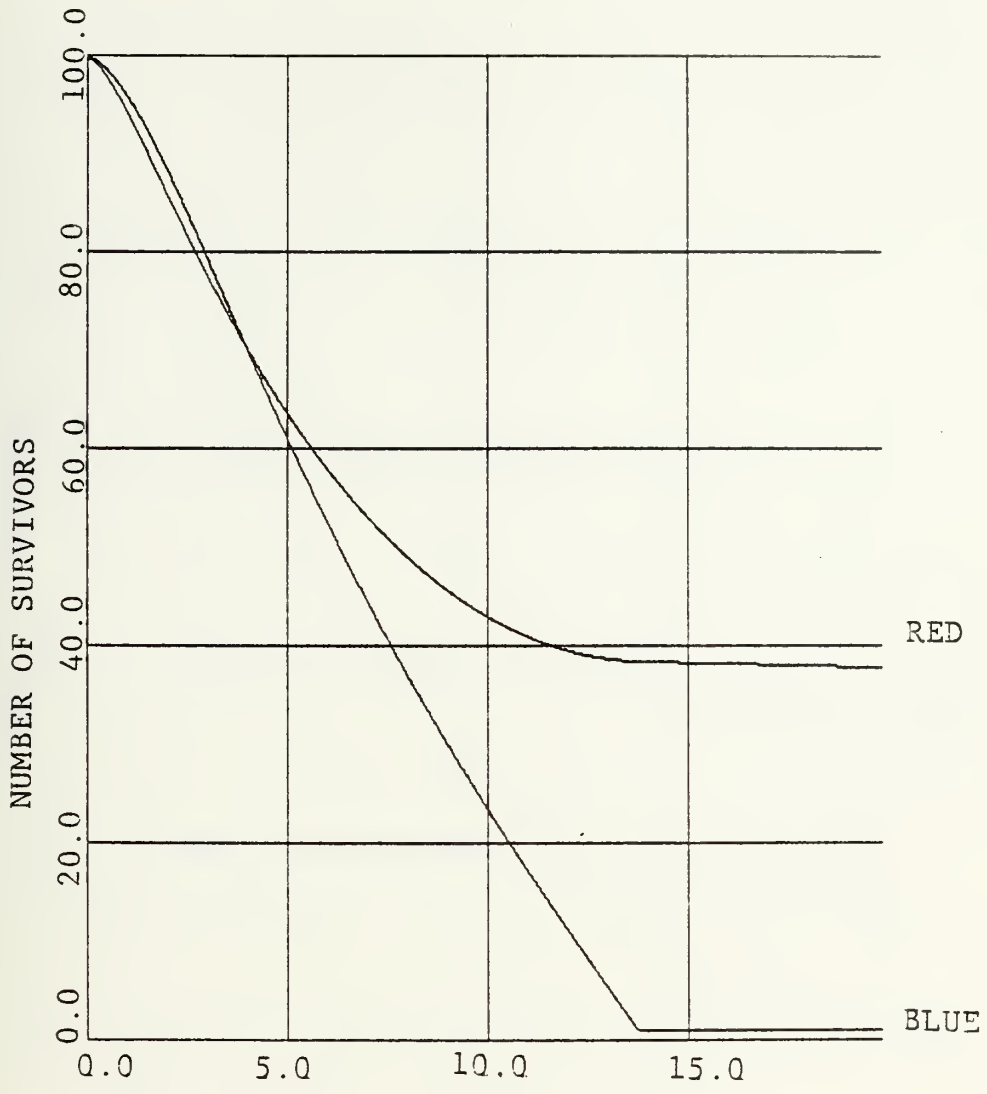


Exhibit 3.

$b_{ua} = 1.5$ $\quad r_{ua} = 0.5$

$\rho_{uu} = \rho_{ua} = 0.015$ $\quad \eta_{uu} = \eta_{ua} = 0.01$

$\rho_{au} = \rho_{aa} = 0.1$ $\quad \eta_{au} = \eta_{aa} = 0.15$

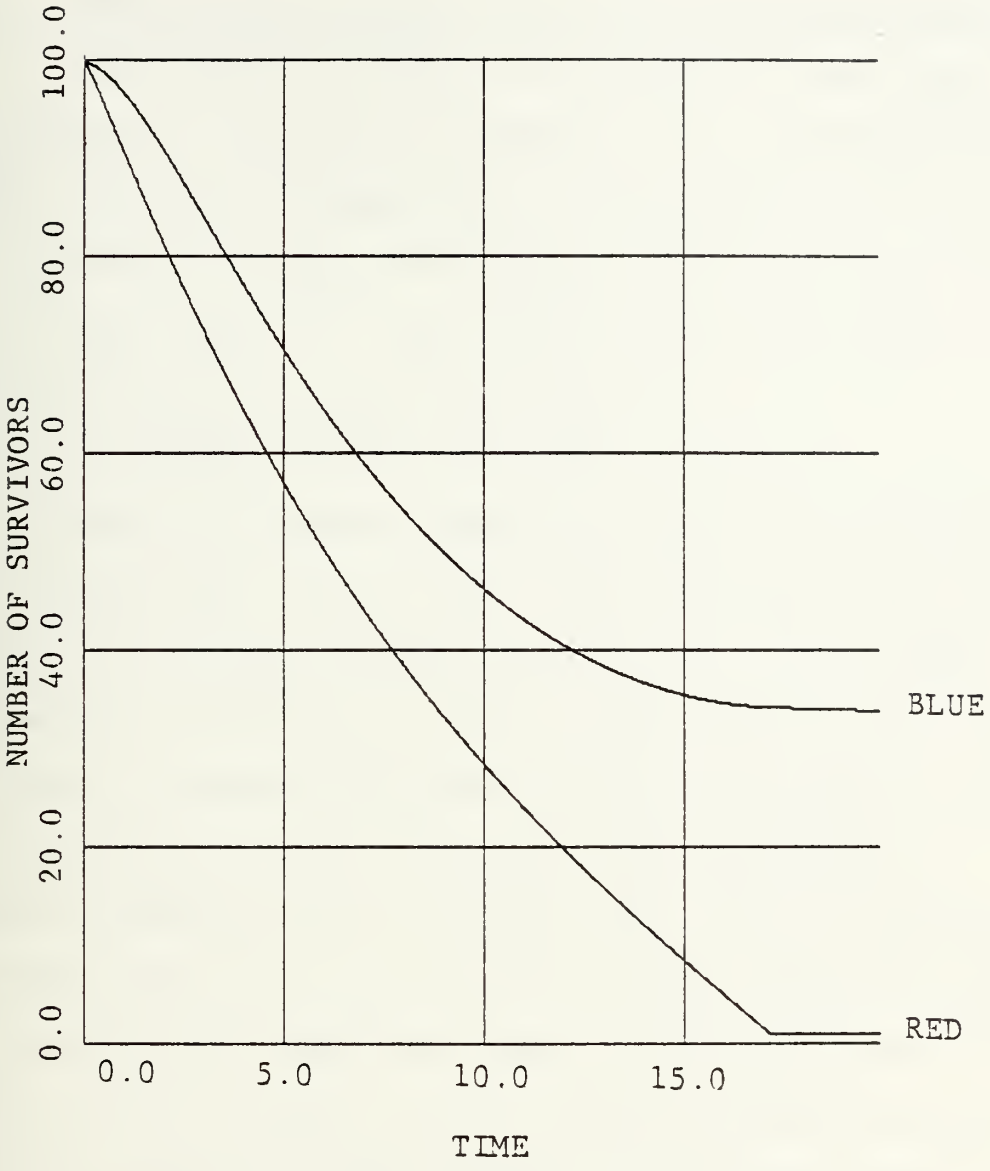


Exhibit 4.

$b_{ua} = 10$ $\quad r_{ua} = 0.5$

$\rho_{uu} = \rho_{ua} = 0.01$ $\quad \eta_{uu} = \eta_{ua} = 0.01$

$\rho_{au} = \rho_{aa} = 0.1$ $\quad \eta_{au} = \eta_{aa} = 0.12$

2. Information Transfer with Instantaneous Transition

Some graphical outputs are presented here using same numerical values as in Figures 1,2,3,4 correspondingly as the result of re-examination of the equations (3.4a,b,c).

Exhibit 5: $r_{ua} = 0.5$, $b_{ua} = 1.5$, $\rho_{uu} = \rho_{ua} = 0.01$

$\rho_{au} = \rho_{aa} = 0.1$, $\eta_{uu} = \eta_{ua} = 0.02$, $\eta_{au} = \eta_{aa} = 0.2$

$t_b = \frac{1}{b_{ua}} = 0.667$, $t_r = \frac{1}{r_{ua}} = 2$

Blue's information handling capability is greater than (three times) that of Red. In other words Blue forces receive the required information to convert the unaimed fire to aimed fire instantly at time t_b which is one-third of the time t_r at which Red forces receive the required information instantly. But on the other hand the physical attrition rates of Blue by Red are high. At the beginning (when both sides are in the unaimed state) Red stays ahead till the time t_b. Then Blue dominates Red with aimed fire. At time t_r Red converts his fire to aimed phase and wins eventually using its effectiveness in the aimed phase.

Exhibit 6: $r_{ua} = 0.5$, $b_{ua} = 1.5$, $\rho_{uu} = \rho_{ua} = 0.01$

$\rho_{au} = \rho_{aa} = 0.1$, $\eta_{uu} = \eta_{ua} = 0.015$, $\eta_{au} = \eta_{aa} = 0.15$

$t_b = \frac{1}{b_{ua}} = 0.667$, $t_r = \frac{1}{r_{ua}} = 2$

Again Blue's information handling capability is as three times fast as Red's. But Red's physical superiority is smaller. As the result of this, it takes time for Red to catch up to Blue and win the fight. But if the fight is

terminated at a break point of 40% loss, the number of Blue survivors would exceed the number of Reds surviving.

Exhibit 7: $r_{ua} = 0.5$, $b_{ua} = 1.5$, $\rho_{uu} = \rho_{ua} = 0.015$

$\rho_{au} = \rho_{aa} = 0.1$, $\eta_{uu} = \eta_{ua} = 0.01$, $\eta_{au} = \eta_{aa} = 0.15$

$t_b = \frac{1}{b_{ua}} = 0.667$, $t_r = \frac{1}{r_{ua}} = 2$

Blue is better again in information handling. Blue also exceeds Red's effectiveness during the unaimed phase, but not during the aimed phase. Blue stays ahead for a longer time, but Red eventually wins the fight. At any break point level before 50% loss, Blue would win.

Exhibit 8: $r_{ua} = 0.5$, $b_{ua} = 10$, $\rho_{uu} = \rho_{ua} = 0.01$

$\rho_{au} = \rho_{aa} = 0.1$, $\eta_{uu} = \eta_{ua} = 0.01$, $\eta_{au} = \eta_{aa} = 0.12$

$t_b = \frac{1}{b_{ua}} = 0.1$, $t_r = \frac{1}{r_{ua}} = 2$

Here Blue is capable of information handling much better than Red is, being 20 times faster than Red. On the contrary Red exceeds Blue's effectiveness during the aimed phase. They are equally effective during the unaimed phase. Despite this Blue surpasses Red from the start, as the result of superior information handling.

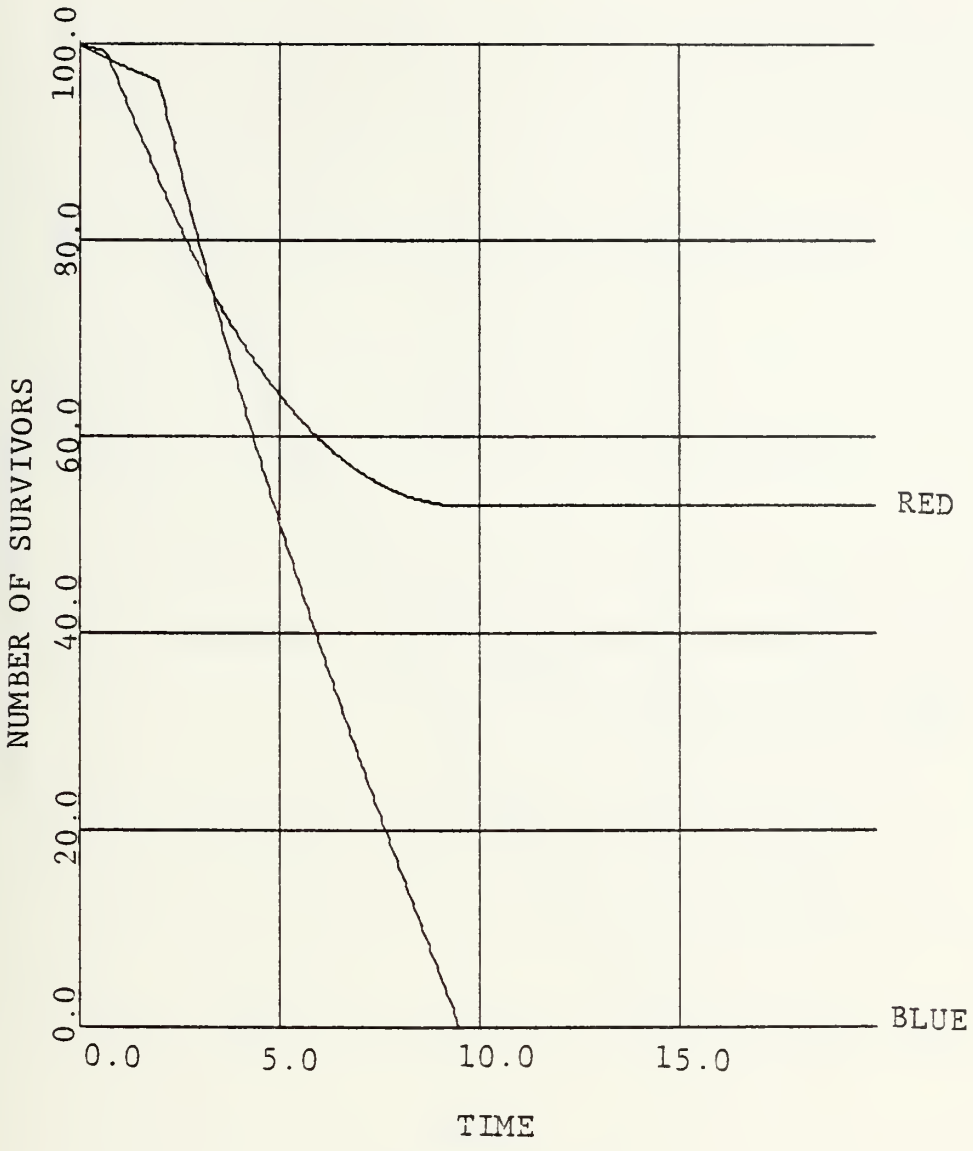


Exhibit 5.

$b_{ua} = 1.5$ $\quad r_{ua} = 0.5$

$\rho_{uu} = \rho_{ua} = 0.01$ $\quad \eta_{uu} = \eta_{ua} = 0.02$

$\rho_{au} = \rho_{aa} = 0.1$ $\quad \eta_{au} = \eta_{aa} = 0.2$

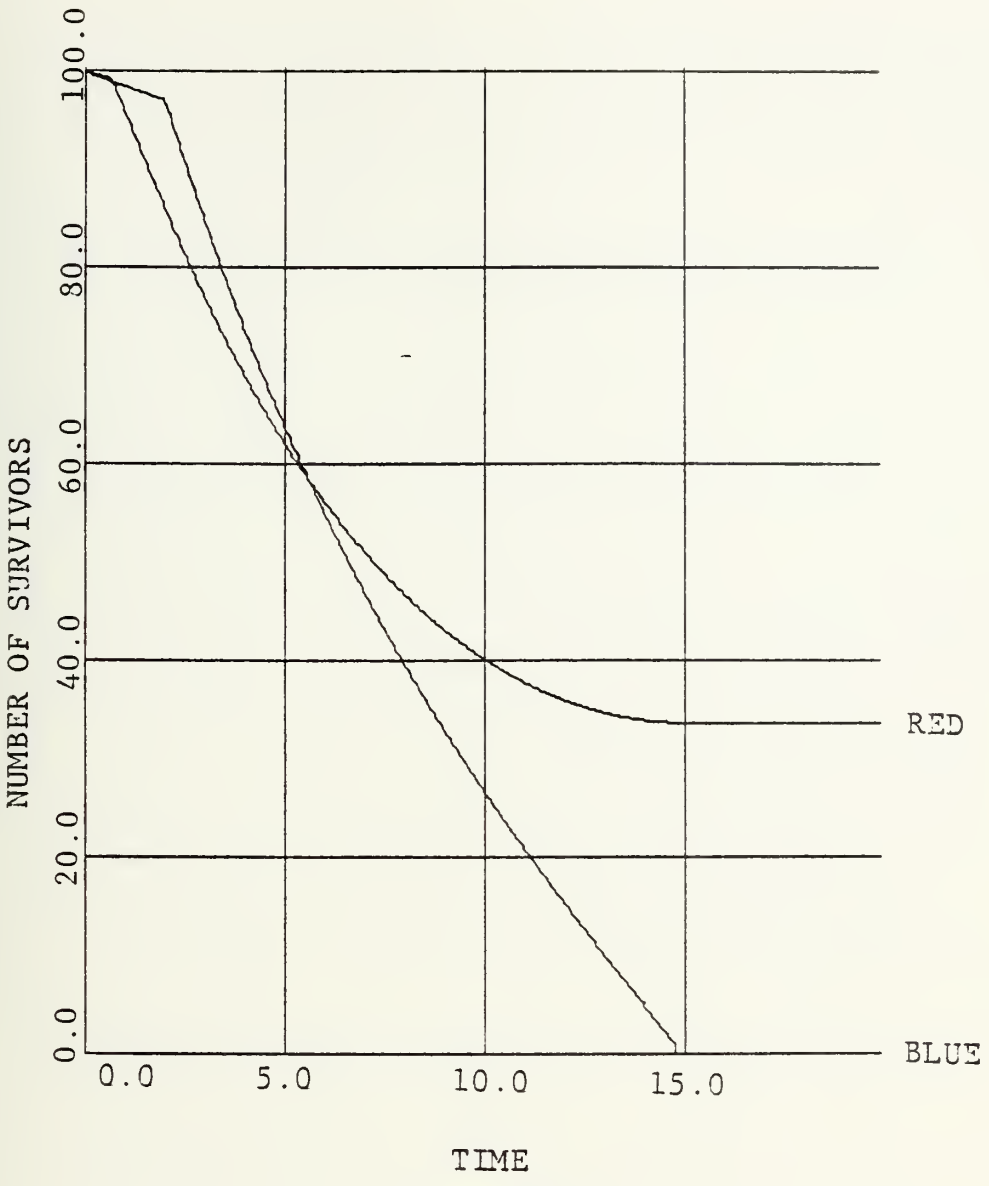


Exhibit 6.

$b_{ua} = 1.5$ $\quad r_{ua} = 0.5$

$\rho_{uu} = \rho_{ua} = 0.01$ $\quad \eta_{uu} = \eta_{ua} = 0.015$

$\rho_{au} = \rho_{aa} = 0.1$ $\quad \eta_{au} = \eta_{aa} = 0.15$

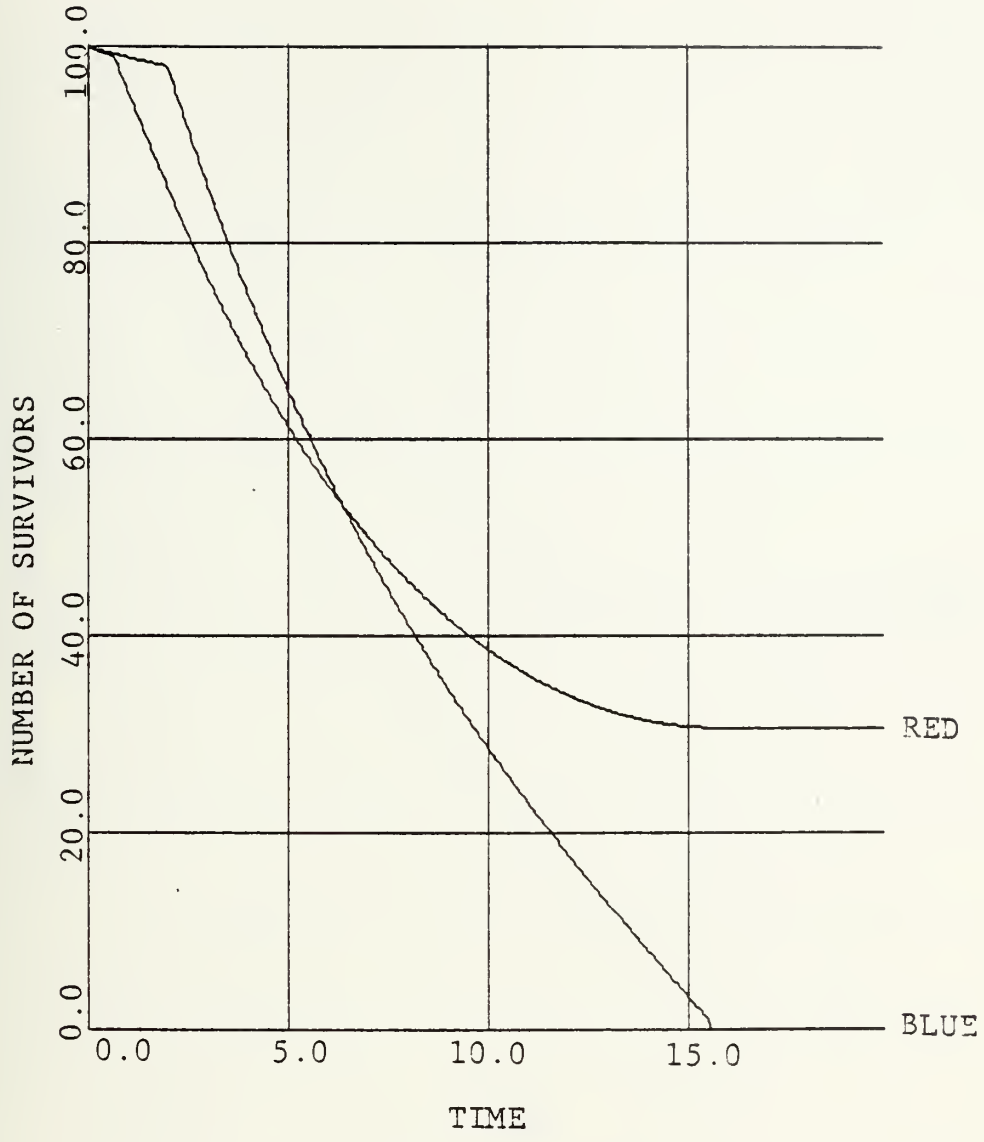


Exhibit 7.

$b_{ua} = 1.5$ $\quad r_{ua} = 0.5$

$\rho_{uu} = \rho_{ua} = 0.015$ $\quad \eta_{uu} = \eta_{ua} = 0.01$

$\rho_{au} = \rho_{aa} = 0.1$ $\quad \eta_{au} = \eta_{aa} = 0.15$

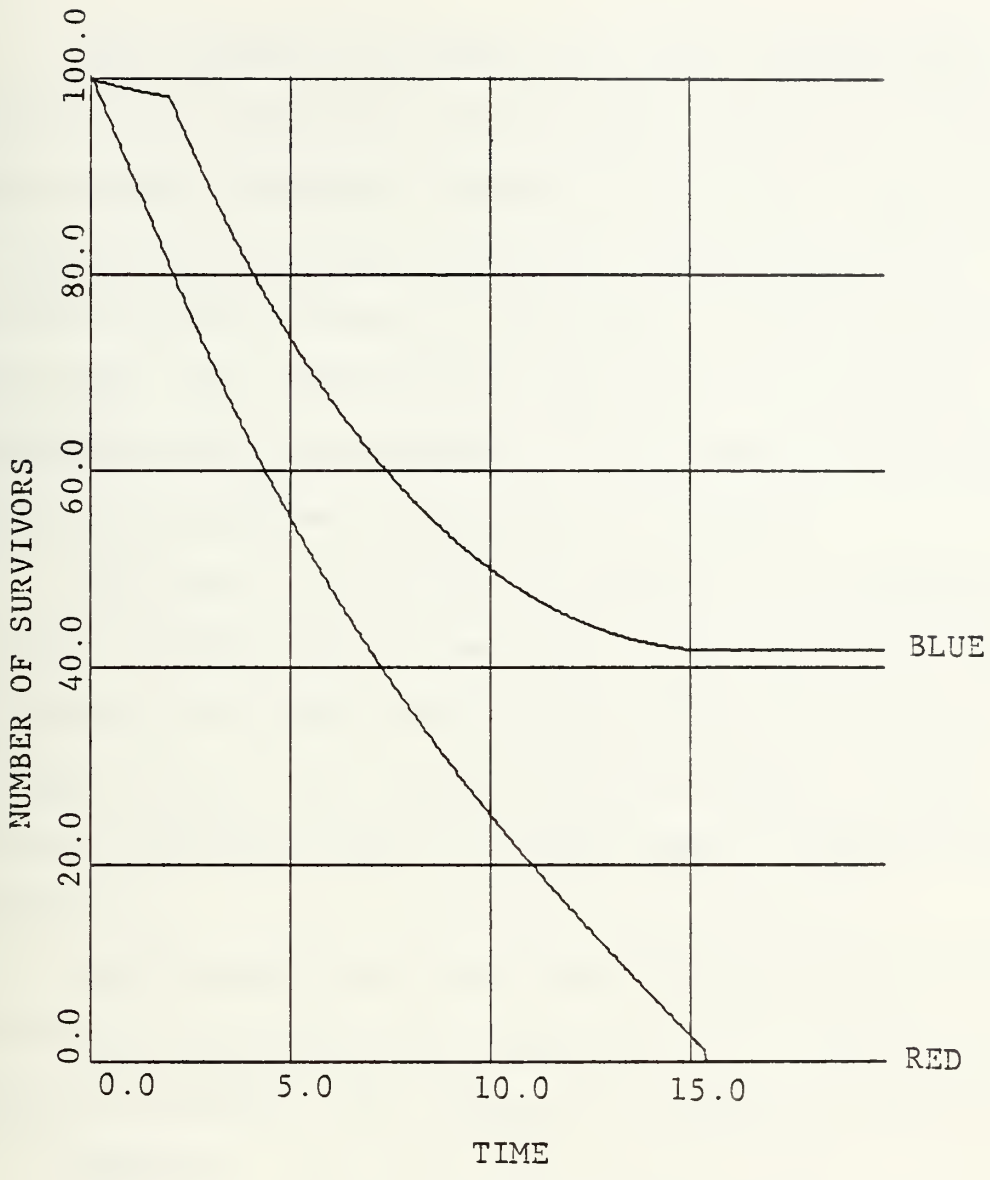


Exhibit 8.

$b_{ua} = 10$ $\quad r_{ua} = 0.5$

$\rho_{uu} = \rho_{ua} = 0.01$ $\quad \eta_{uu} = \eta_{ua} = 0.01$

$\rho_{au} = \rho_{aa} = 0.1$ $\quad \eta_{au} = \eta_{aa} = 0.12$

If we compare the graphs for the instantaneous transition cases with the corresponding exhibits (1,2,3,4) for the gradual transition cases it can be said that the side that has the smaller attrition rates against the other side (which is Blue in our cases) is better-off in case in which the instantaneous transition is in effect. That is Blue forces stay ahead a little longer in the case in which instantaneous transition is in effect.

This must be the result of making the transformation from unaimed fire to aimed fire instantaneously rather than making it in some period of time gradually, so that Blue aimed forces have a greater effect against all Red forces at transition time t_b for the case of instantaneous transition.

The following free-hand graphs display the relationship between survivors when a) gradual to b) instantaneous information transfer with comparable parameters otherwise.

Exhibit 9: $r_{ua} = 0.5$, $b_{ua} = 1.5$, $\rho_{uu} = \rho_{ua} = 0.01$
$\rho_{au} = \rho_{aa} = 0.1$, $\eta_{uu} = \eta_{ua} = 0.02$, $\eta_{au} = \eta_{aa} = 0.2$
$B(0) = 100$, $R(0) = 100$

Here it can be seen that for the instantaneous transition case Blue stays ahead a short period of time, while Blue is never ahead in the gradual transition case.

Exhibit 10: $r_{ua} = 0.5$, $b_{ua} = 1.5$, $\rho_{uu} = \rho_{ua} = 0.01$
$\rho_{au} = \rho_{aa} = 0.1$, $\eta_{uu} = \eta_{ua} = 0.015$, $\eta_{au} = \eta_{aa} = 0.15$
$R(0) = 100$, $B(0) = 100$

Again the period of time that Blue stays ahead is longer for the instantaneous transition case.

Exhibit 11: $r_{ua} = 0.5$, $b_{ua} = 1.5$, $\rho_{uu} = \rho_{ua} = 0.015$
$\rho_{au} = \rho_{aa} = 0.1$, $\eta_{uu} = \eta_{ua} = 0.01$, $\eta_{au} = \eta_{aa} = 0.15$
$B(0) = 100$, $R(0) = 100$

Here, again Blue is better-off in the instantaneous transition case.

Exhibit 12: $r_{ua} = 0.5$, $b_{ua} = 10$, $\rho_{uu} = \rho_{ua} = 0.01$
$\rho_{au} = \rho_{aa} = 0.1$, $\eta_{uu} = \eta_{ua} = 0.01$, $\eta_{au} = \eta_{aa} = 0.12$
$B(0) = 100$, $R(0) = 100$

Here Blue is the winning side. And at the end of combat the number of Blue survivors are larger in number in the case of instantaneous transition.

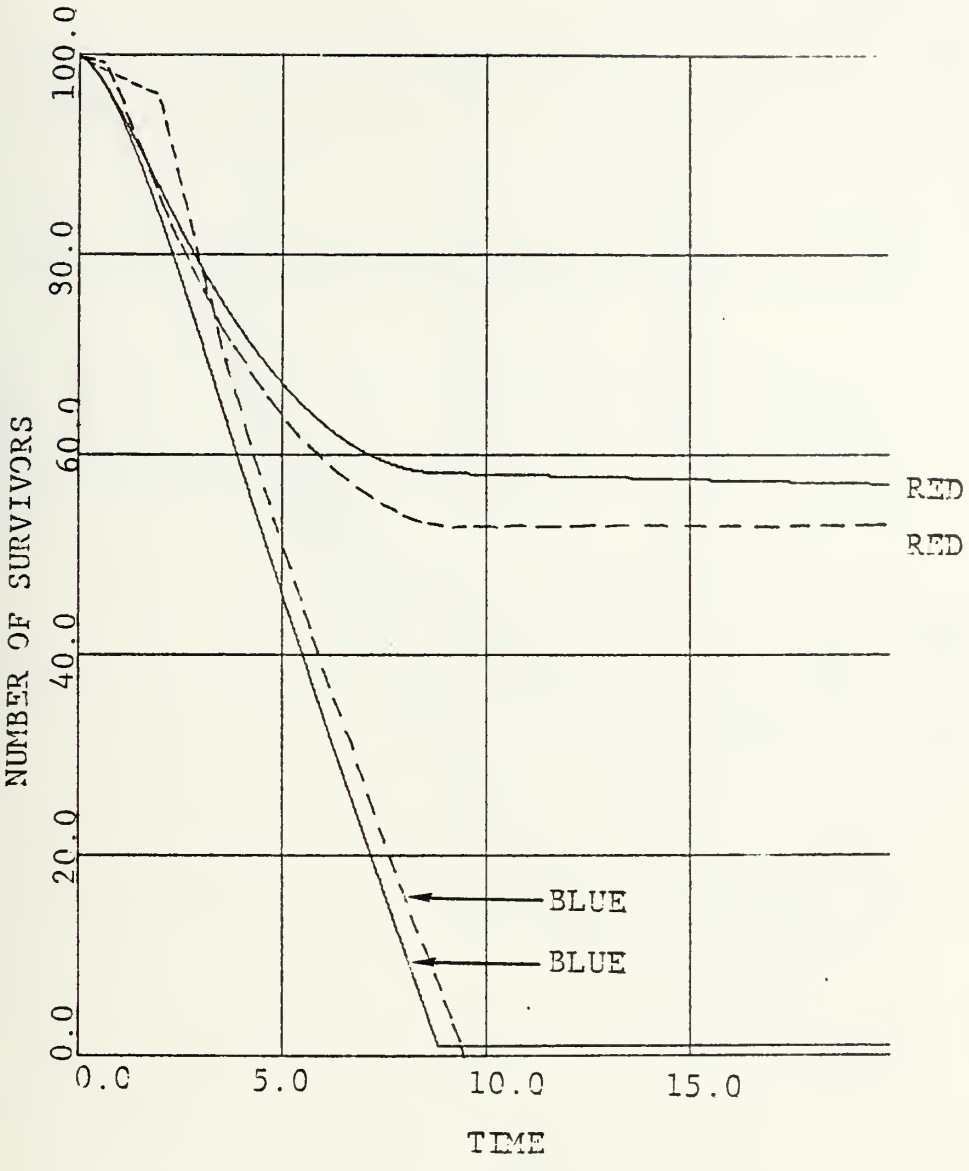
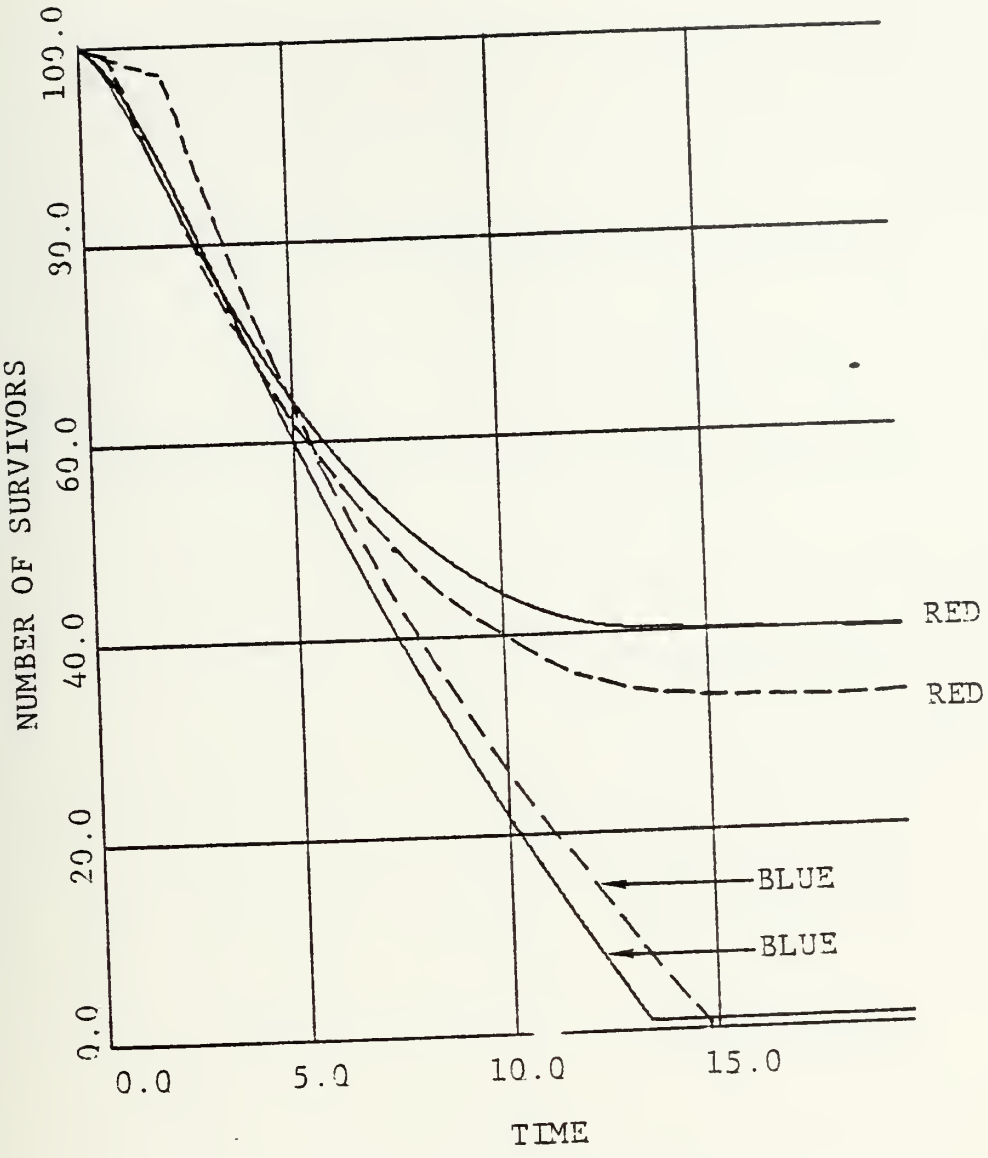


Exhibit 9.

$b_{ua} = 1.5$ $\quad r_{ua} = 0.5$

$\rho_{uu} = \rho_{ua} = 0.01$ $\quad \eta_{uu} = \eta_{ua} = 0.02$

$\rho_{au} = \rho_{aa} = 0.1$ $\quad \eta_{au} = \eta_{aa} = 0.2$



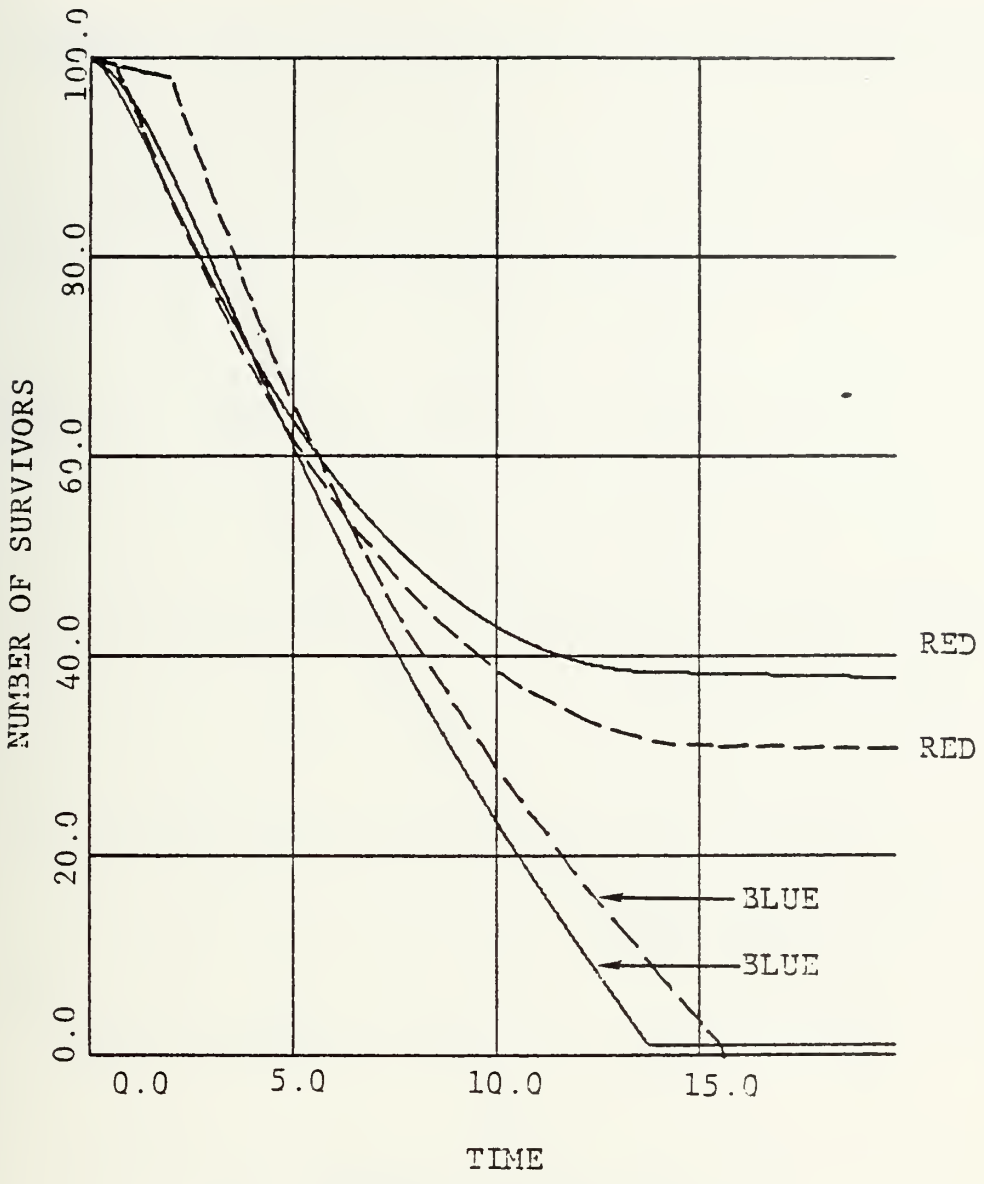
——— Gradual transition
---- Instantaneous transition

Exhibit 10.

$b_{ua} = 1.5$ $\quad r_{ua} = 0.5$

$\rho_{uu} = \rho_{ua} = 0.01$ $\quad \eta_{uu} = \eta_{ua} = 0.015$

$\rho_{au} = \rho_{aa} = 0.1$ $\quad \eta_{au} = \eta_{aa} = 0.15$



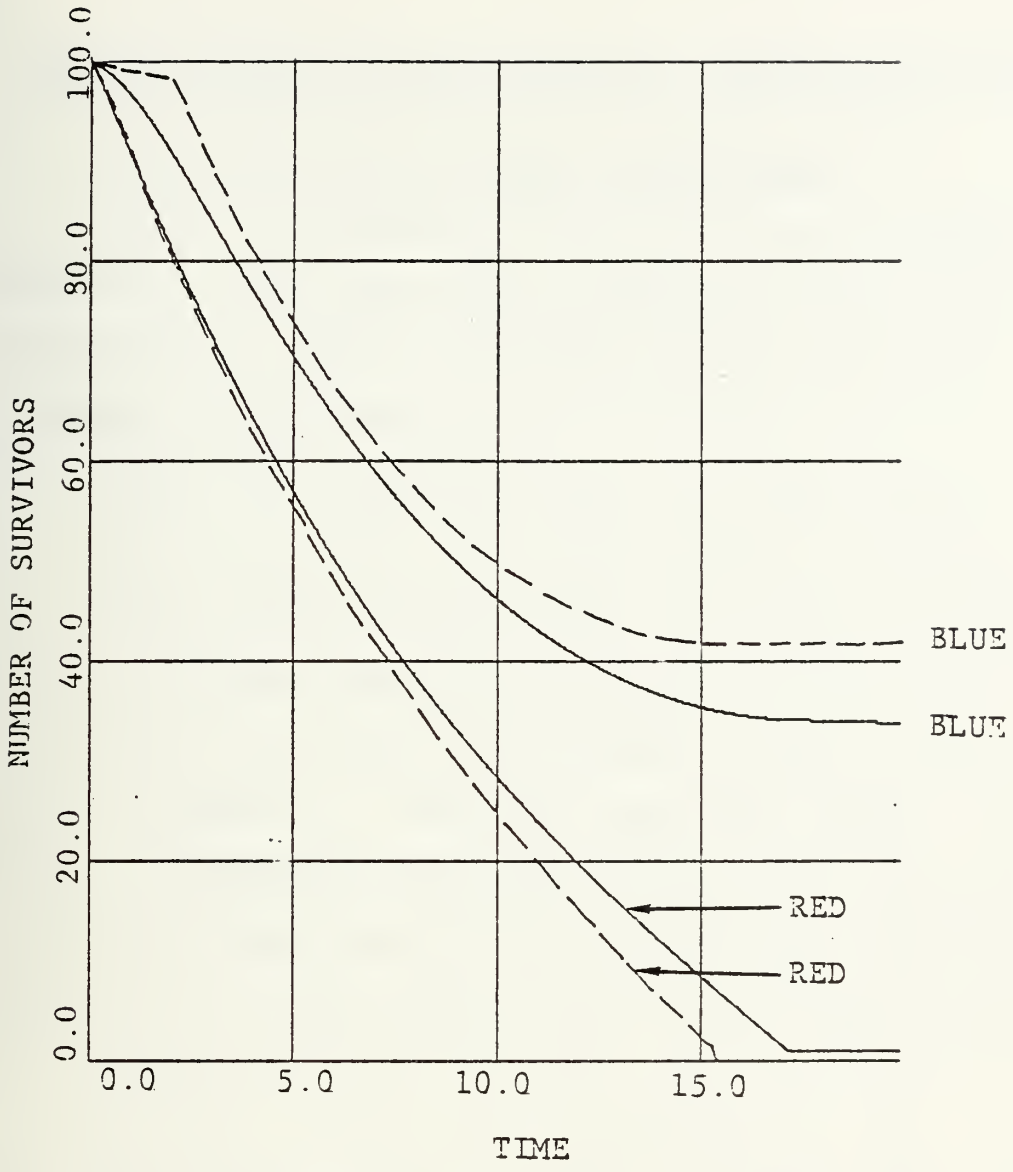
—— Gradual transition
---- Instantaneous transition

Exhibit 11.

$b_{ua} = 1.5$ $\quad r_{ua} = 0.5$

$\rho_{uu} = \rho_{ua} = 0.015$ $\quad \eta_{uu} = \eta_{ua} = 0.01$

$\rho_{au} = \rho_{aa} = 0.1$ $\quad \eta_{au} = \eta_{aa} = 0.15$



—— Gradual transition
---- Instantaneous transition

Exhibit 12.

$b_{ua} = 10$ $\quad r_{ua} = 0.5$

$\rho_{uu} = \rho_{ua} = 0.01$ $\quad \eta_{uu} = \eta_{ua} = 0.01$

$\rho_{au} = \rho_{aa} = 0.1$ $\quad \eta_{au} = \eta_{aa} = 0.12$

B. COMBAT OUTCOMES UNDER CONDITIONS OF MUTUAL ATTRITION WITH AND WITHOUT COORDINATION OF AIMED FORCES

1. Both Side Attack without Coordination

Here, the equations (5.1a,b,c,d) are re-examined numerically. Again same set of numerical values are used correspondingly (exhibit 13 through exhibit 16) to be able to compare to each other.

Exhibit 13: $r_{ua} = 0.5$, $b_{ua} = 1.5$, $\rho_{uu} = \rho_{ua} = 0.01$

$\rho_{au} = \rho_{aa} = 0.1$, $\eta_{uu} = \eta_{ua} = 0.02$, $\eta_{au} = \eta_{aa} = 0.2$

Blue's information handling capability or transfer rate b_{ua} is three times greater than that of Red, r_{ua}. And again physical attrition rates of Blue by Red are high as they are in exhibit 1. Red wins, and never behind. If exhibit 1 is compared with exhibit 13 it can be seen that in exhibit 13 combat takes twice as much time. This must be the result of being uncoordinated in the case of exhibit 13.

Exhibit 14: $r_{ua} = 0.5$, $b_{ua} = 1.5$, $\rho_{uu} = \rho_{ua} = 0.01$

$\rho_{au} = \rho_{aa} = 0.1$, $\eta_{uu} = \eta_{ua} = 0.015$, $\eta_{au} = \eta_{aa} = 0.15$

Again, Blue's information-handling capability exceeds Red's by a factor three. Red's physical superiority is little less than previous case. But this does not make Blue to stay ahead even for a little while as contrary to the case in exhibit 2. The explanation for this could be again being not coordinated enough to take advantage of opponent's smaller superiority.

Exhibit 15: $r_{ua} = 0.5$, $b_{ua} = 1.5$, $\rho_{uu} = \rho_{ua} = 0.015$

$\rho_{au} = \rho_{aa} = 0.1$, $\eta_{uu} = \eta_{ua} = 0.01$, $\eta_{au} = \eta_{aa} = 0.15$

Here Blue is better than Red in information handling again. And Blue exceeds Red's effectiveness during the unaimed case, but Red is more effective than Blue in aimed case. Blue stays ahead for a little period of time, eventually losing in a fight to the finish. Even at a break point level of more than 10% loss, Blue wouldn't win.

Exhibit 16: $r_{ua} = 0.5$, $b_{ua} = 10$, $\rho_{uu} = \rho_{ua} = 0.01$,

$\rho_{au} = \rho_{aa} = 0.1$, $\eta_{uu} = \eta_{ua} = 0.01$, $\eta_{au} = \eta_{aa} = 0.12$

Here Blue's information-handling advantage is high, being 20 times that of Red. Contrast to this to Red's physical equality in the Unaimed state. Blue stays ahead for a longer time using the information-handling advantage but still Blue is not capable enough in information-handling to be able to win fight at the end. If the fight is terminated at a break point of 40% loss, Blue would win.

Exhibit 17: $r_{ua} = 0.5$, $b_{ua} = 90$, $\rho_{uu} = \rho_{ua} = 0.01$

$\rho_{au} = \rho_{aa} = 0.1$, $\eta_{uu} = \eta_{ua} = 0.01$, $\eta_{au} = \eta_{aa} = 0.109$

In this example, Blue's information-handling is decisive, being 180 times that of Red. And Red is more effective only in Aimed state. So finally Blue surpasses Red from the start with the advantage of information handling.

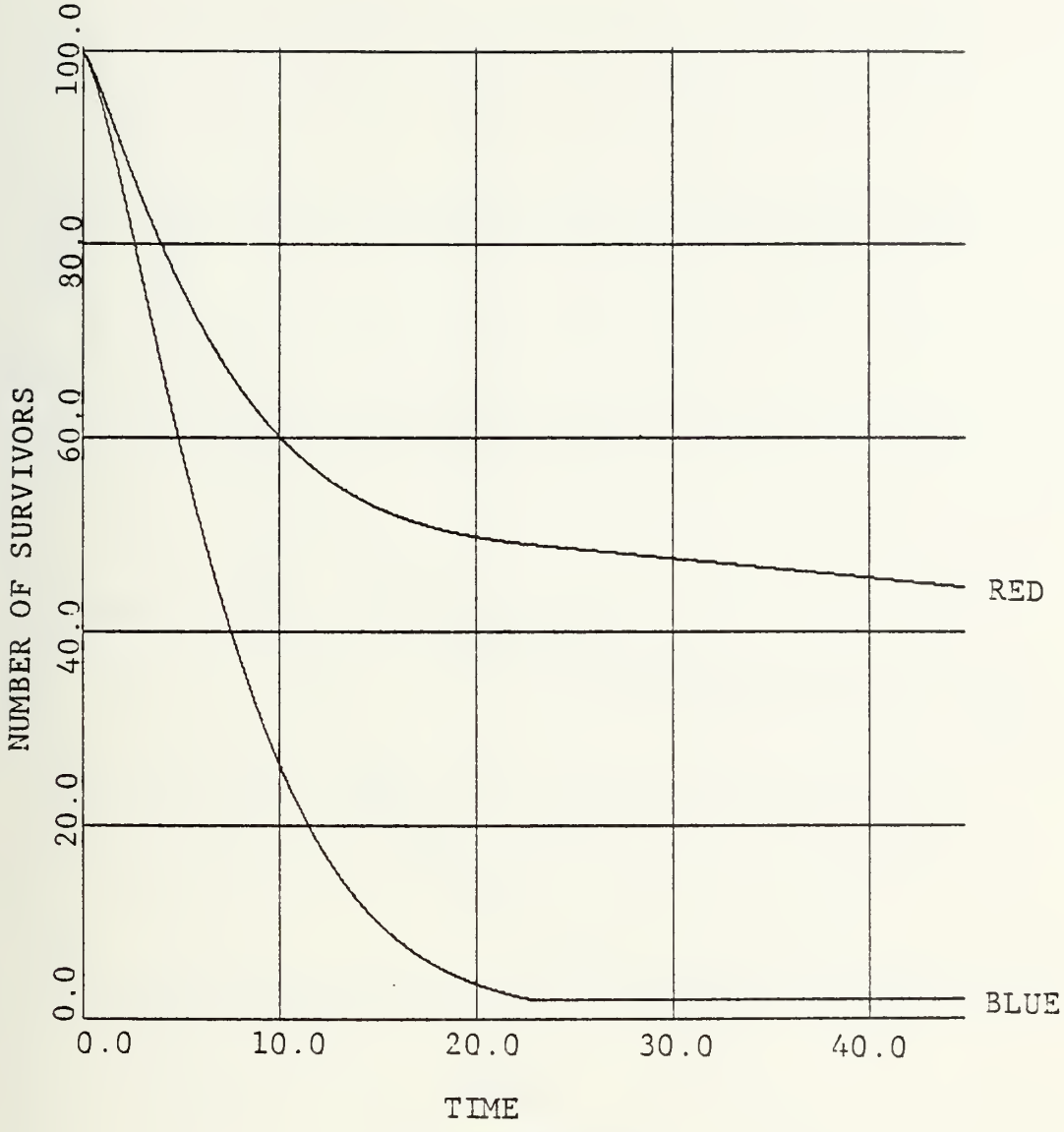


Exhibit 13.

$b_{ua} = 1.5$ $\quad r_{ua} = 0.5$

$\rho_{uu} = \rho_{ua} = 0.01$ $\quad \eta_{uu} = \eta_{ua} = 0.02$

$\rho_{au} = \rho_{aa} = 0.1$ $\quad \eta_{au} = \eta_{aa} = 0.2$

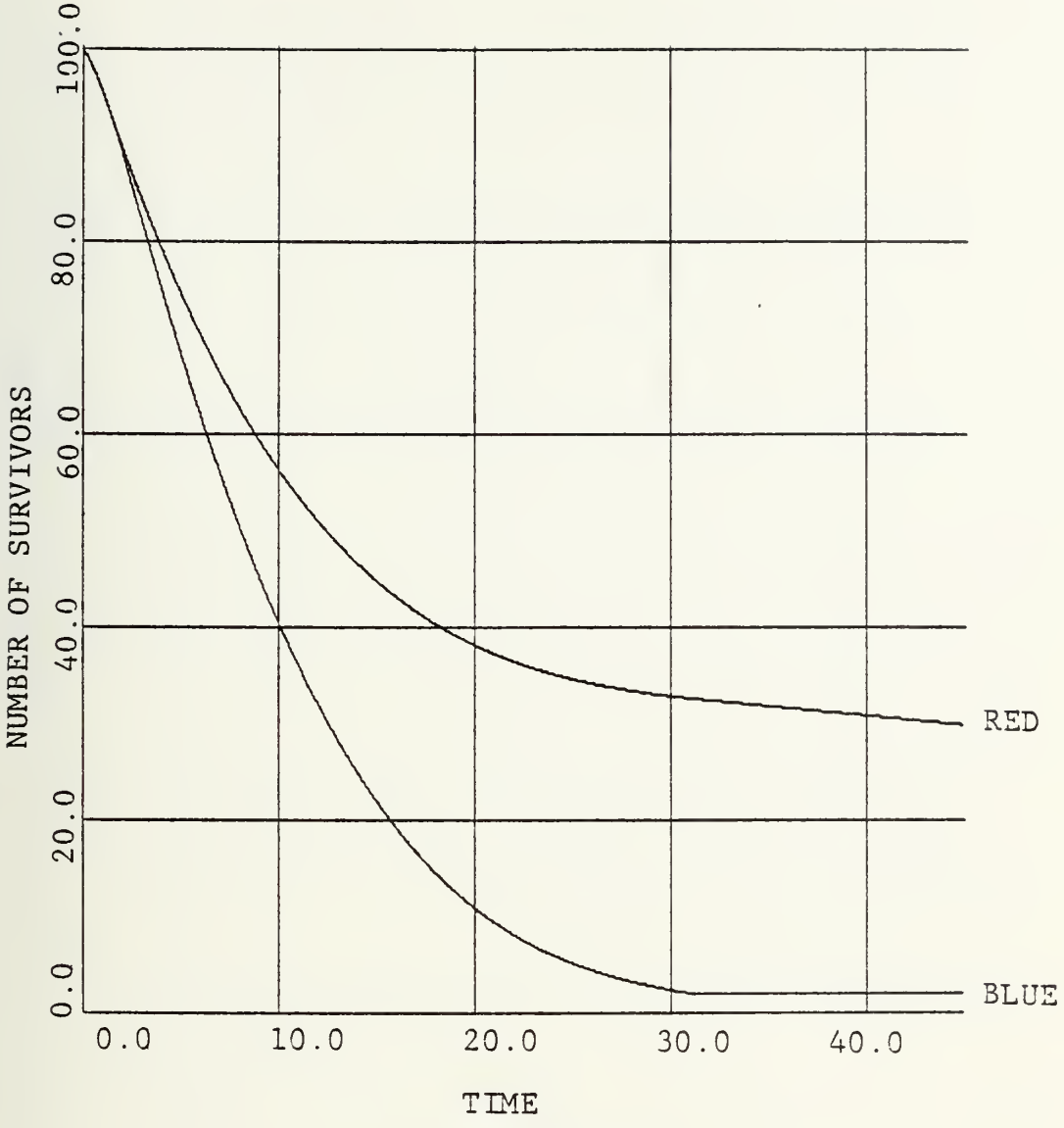


Exhibit 14.

$b_{ua} = 1.5$ $\quad r_{ua} = 0.5$

$\rho_{uu} = \rho_{ua} = 0.01$ $\quad \eta_{uu} = \eta_{ua} = 0.015$

$\rho_{au} = \rho_{aa} = 0.1$ $\quad \eta_{au} = \eta_{aa} = 0.15$

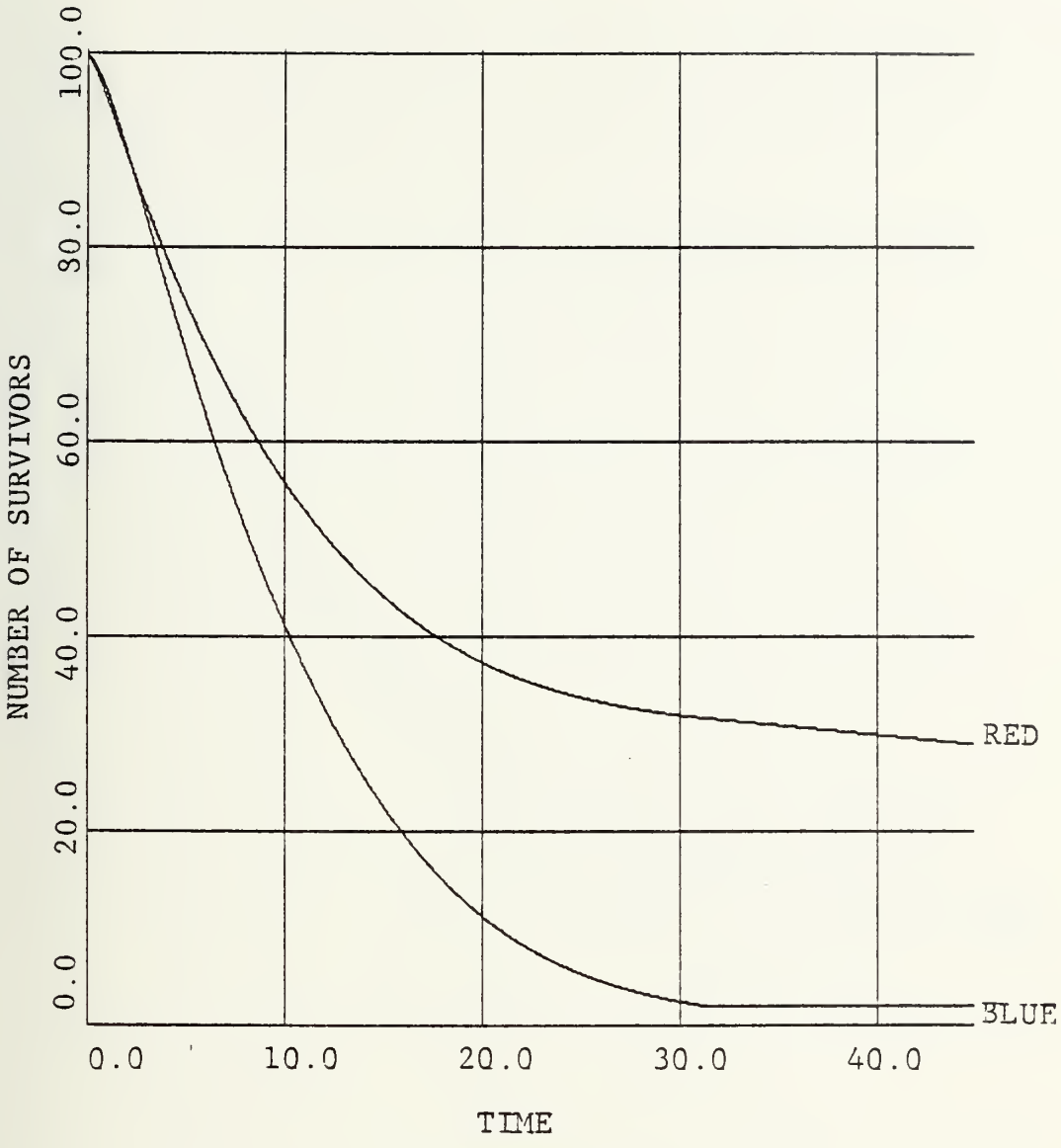


Exhibit 15.

$b_{ua} = 1.5$ $\quad r_{ua} = 0.5$

$\rho_{uu} = \rho_{ua} = 0.015$ $\quad \eta_{uu} = \eta_{ua} = 0.01$

$\rho_{au} = \rho_{aa} = 0.1$ $\quad \eta_{au} = \eta_{aa} = 0.15$

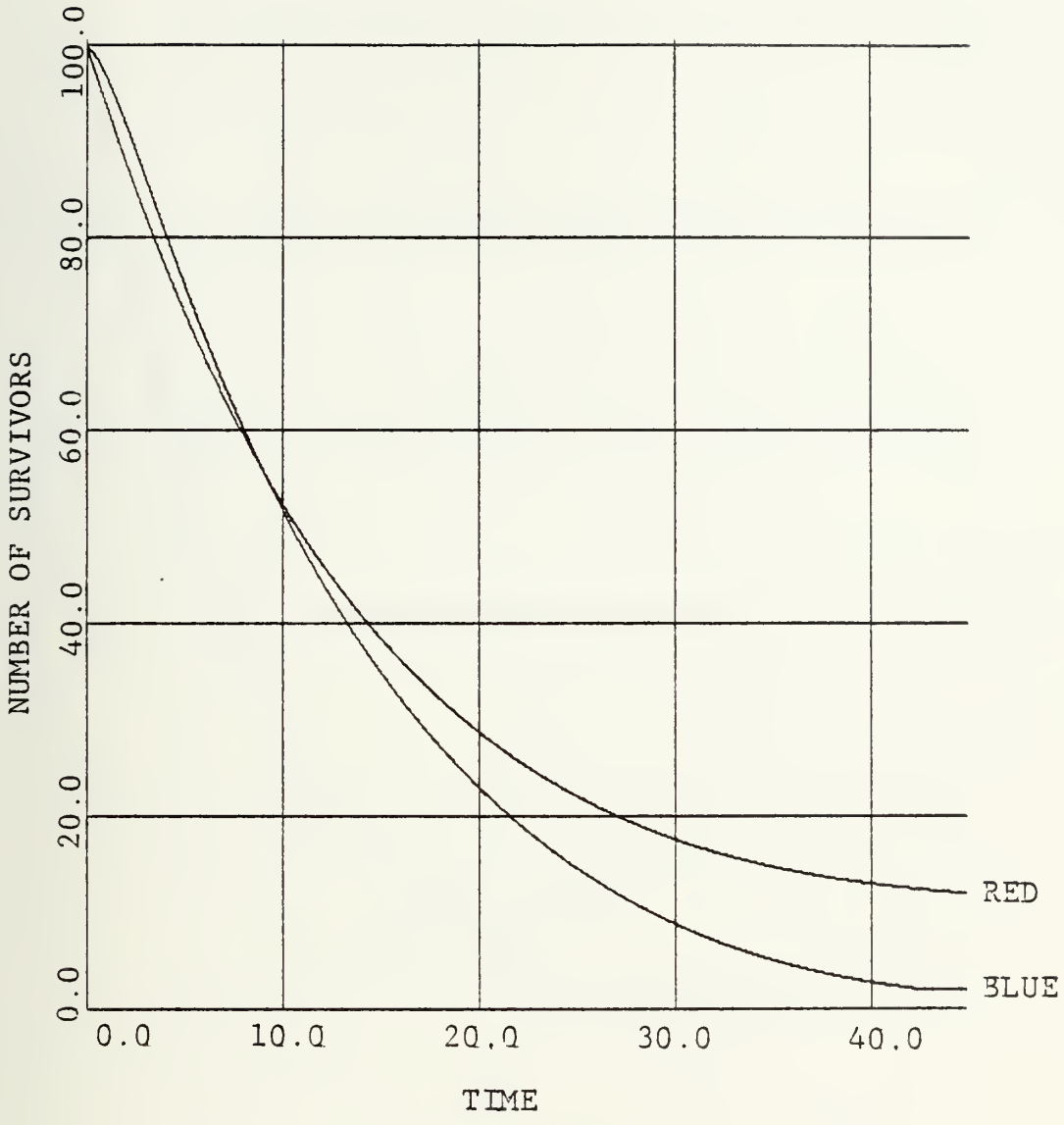


Exhibit 16.

$b_{ua} = 10$ $\quad r_{ua} = 0.5$

$\rho_{uu} = \rho_{ua} = 0.01$ $\quad \eta_{uu} = \eta_{ua} = 0.01$

$\rho_{au} = \rho_{aa} = 0.1$ $\quad \eta_{au} = \eta_{aa} = 0.12$

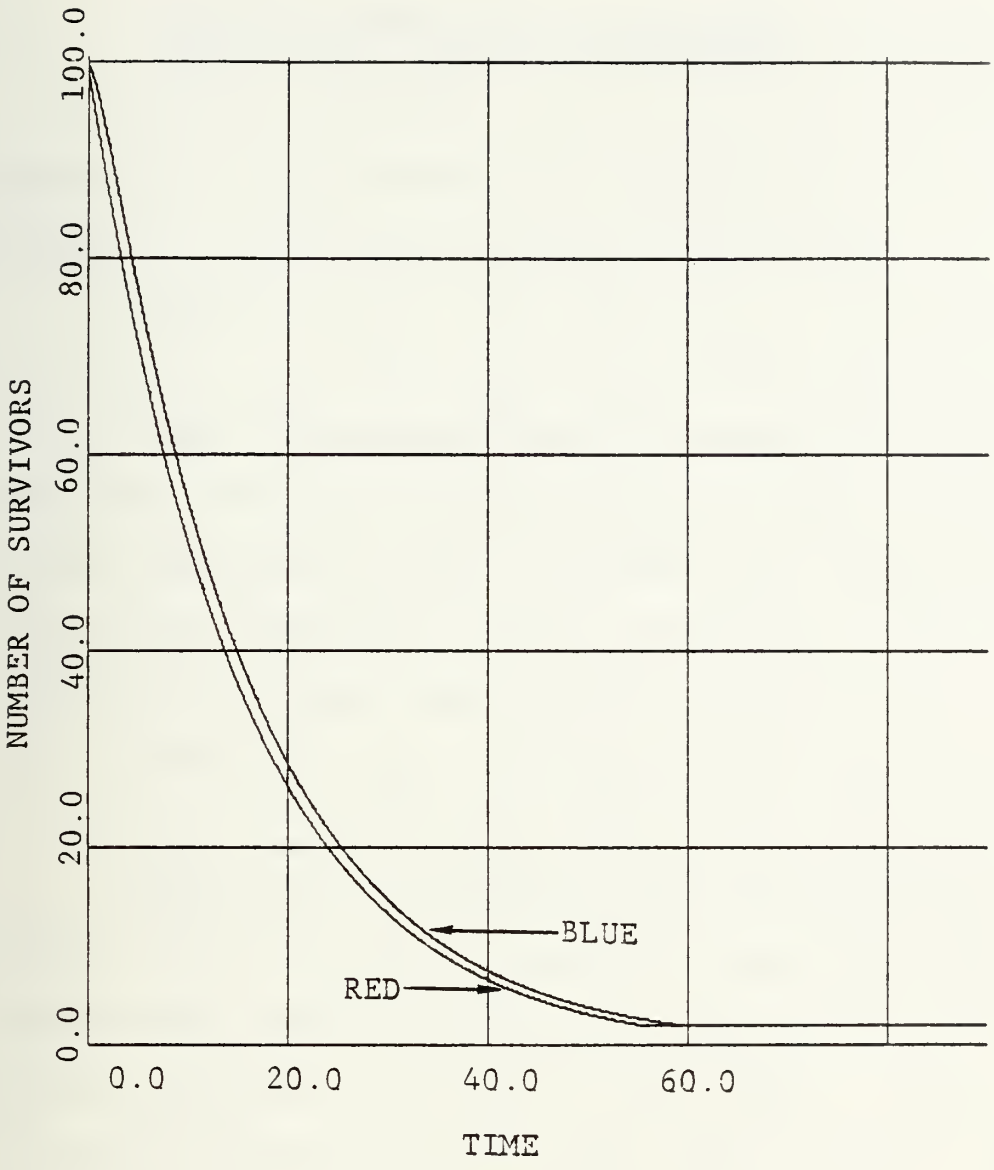


Exhibit 17.

$b_{ua} = 90$ $\quad r_{ua} = 0.5$

$\rho_{uu} = \rho_{ua} = 0.01$ $\quad \eta_{uu} = \eta_{ua} = 0.01$

$\rho_{au} = \rho_{aa} = 0.1$ $\quad \eta_{au} = \eta_{aa} = 0.109$

2. One Side Attacks With Coordination, the Other Side Attacks Without Coordination

Here the equations (5.2a,b,c,d) are re-examined numerically. Blue attacks with coordination of his aimed forces, but Red has uncoordinated-aimed forces.

Exhibit 18: $r_{ua} = 0.5$, $b_{ua} = 1.5$, $\rho_{uu} = \rho_{ua} = 0.01$
$\rho_{au} = \rho_{aa} = 0.1$, $\eta_{uu} = \eta_{ua} = 0.02$, $\eta_{au} = \eta_{aa} = 0.2$

Blue's information handling capability is greater than that of Red, but physical attrition rates of Blue by Red are sufficiently high to overcome this advantage and Blue's other advantage of being coordinated. Red wins, even though stays at same level of survivors with Blue.

Exhibit 19: $r_{ua} = 0.5$, $b_{ua} = 1.5$, $\rho_{uu} = \rho_{ua} = 0.01$
$\rho_{au} = \rho_{aa} = 0.1$, $\eta_{uu} = \eta_{ua} = 0.1$, $\eta_{au} = \eta_{aa} = 0.13$

Again Blue's information handling capability is greater than that of Red. But Red is much more effective than previous case in unaimed state, and little less more effective than previous case in aimed phase. As the result of this Red stays ahead for a long time in spite of Blue's coordination and information-handling capability. But when Blue converts all his fire unaimed phase to aimed phase Red starts to stay behind and loses the fight at the end. If the fight is terminated at a break point of 60% loss, the number of Red survivors would exceed the number of Reds surviving.

Exhibit 20: $r_{ua} = 0.5$, $b_{ua} = 1.5$, $\rho_{uu} = \rho_{ua} = 0.01$
$\rho_{au} = \rho_{aa} = 0.1$, $\eta_{uu} = \eta_{ua} = 0.015$, $\eta_{au} = \eta_{aa} = 0.15$

Blue's information-handling capability again exceeds Red's by a factor of three, but Red's physical superiority is not high enough. So Blue surpasses Red from the start with the advantage of better information-handling and having coordinated-aimed forces.

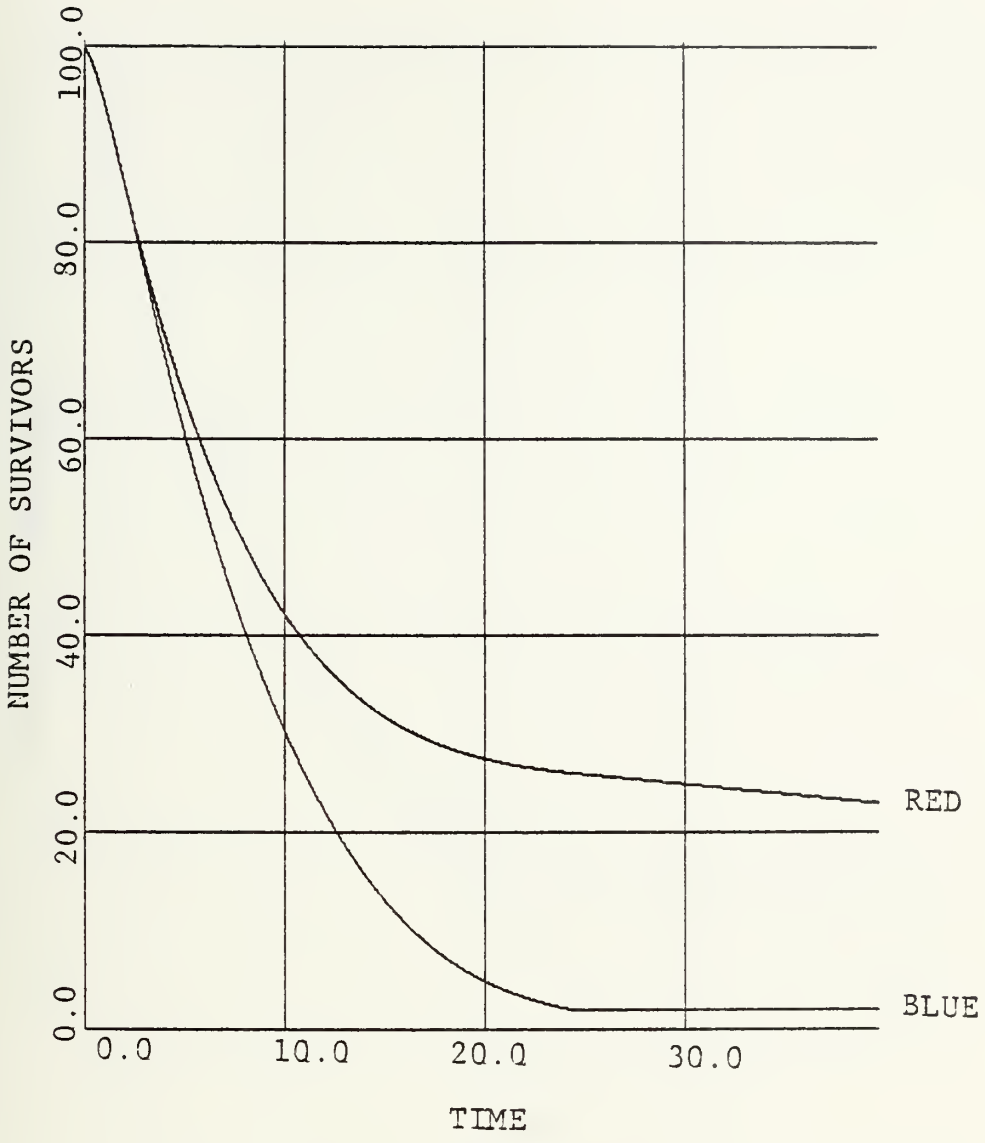


Exhibit 18.

$b_{ua} = 1.5$ $\quad r_{ua} = 0.5$

$\rho_{uu} = \rho_{ua} = 0.01$ $\quad \eta_{uu} = \eta_{ua} = 0.02$

$\rho_{au} = \rho_{aa} = 0.1$ $\quad \eta_{au} = \eta_{aa} = 0.2$

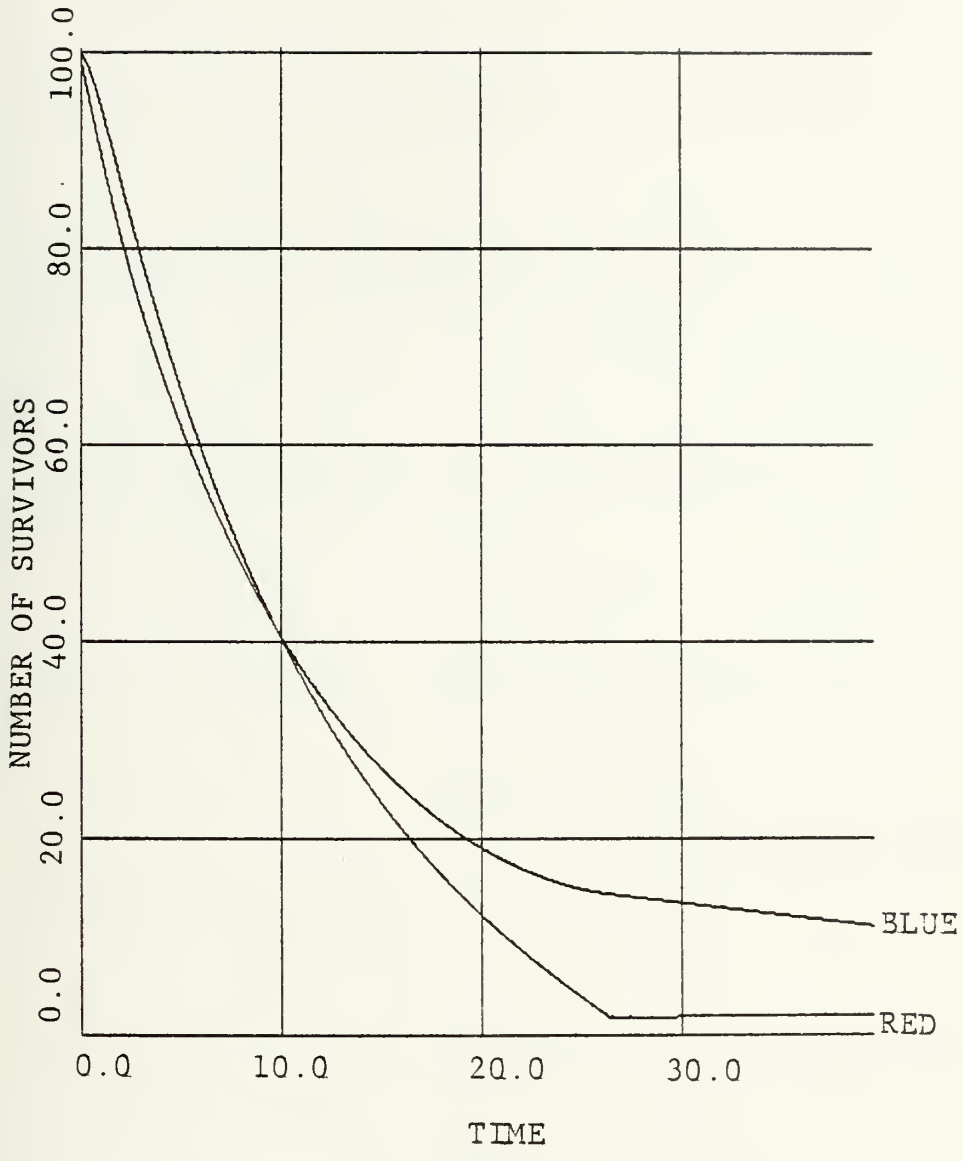


Exhibit 19.

$b_{ua} = 1.5$ $\quad r_{ua} = 0.5$

$\rho_{uu} = \rho_{ua} = 0.01$ $\quad \eta_{uu} = \eta_{ua} = 0.1$

$\rho_{au} = \rho_{aa} = 0.1$ $\quad \eta_{au} = \eta_{aa} = 0.13$

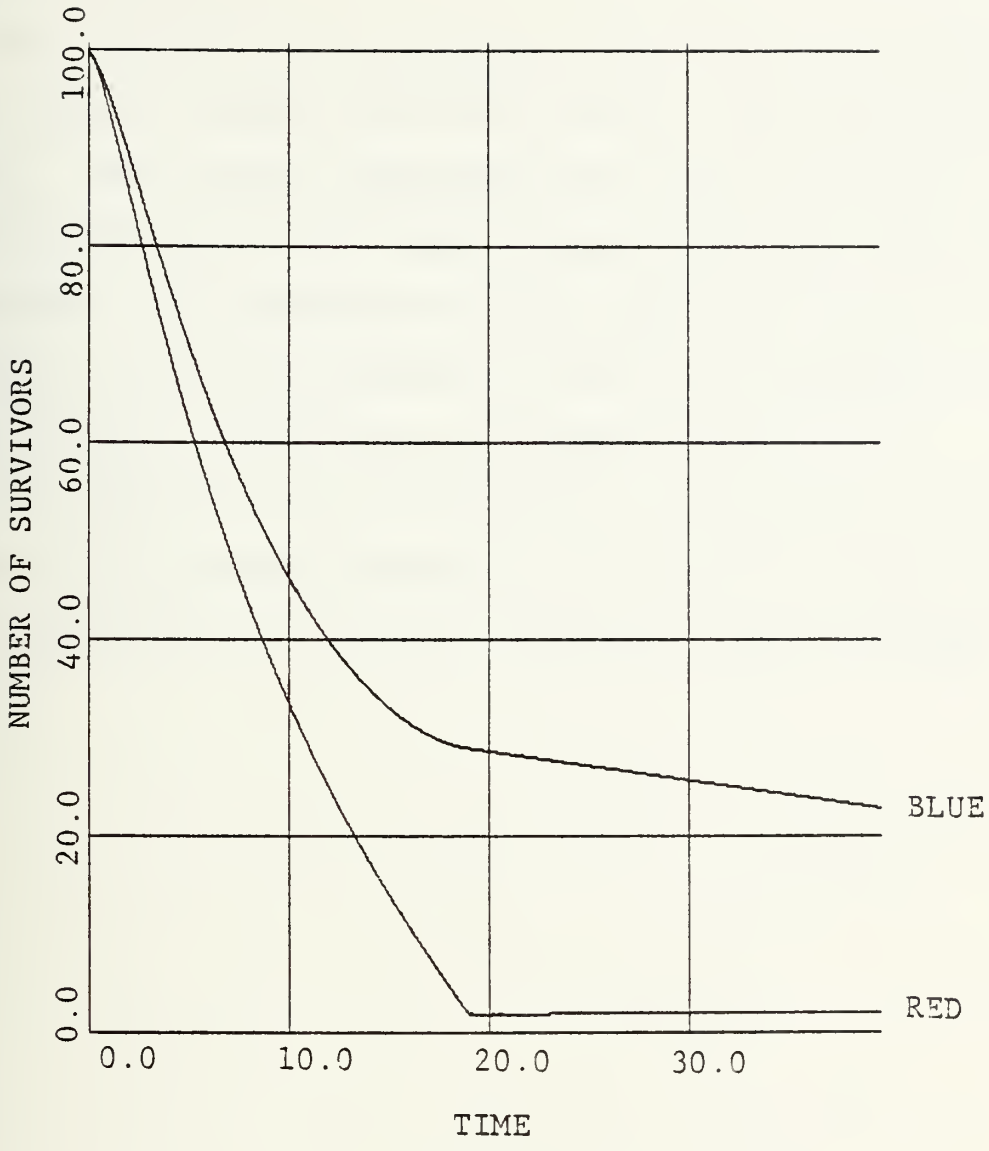


Exhibit 20.

$b_{ua} = 1.5$ $\quad r_{ua} = 0.5$

$\rho_{uu} = \rho_{ua} = 0.01$ $\quad \eta_{uu} = \eta_{ua} = 0.015$

$\rho_{au} = \rho_{aa} = 0.1$ $\quad \eta_{au} = \eta_{aa} = 0.15$

C. CONCLUSIONS

The influence of information transfer and coordination upon combat progress have been studied, along Lanchesterian lines. Some possible representations for the change in the information states have been proposed and their consequences explored. The conventional states have been allowed mutual attrition, including the effect of information transfer and coordination, are examined numerically.

Our numerical results showed that the different combinations of the physical parameters and the information transfer parameters can decisively influence combat outcomes.

APPENDIX A

In this section, graphs for situations in which Blue and Red do not have the same initial force sizes are displayed for some of the cases (using same physical attrition and information transformation rules) as those we examined before.

A. COMBAT UNDER CONDITIONS OF MUTUAL ATTRITION WITH COORDINATED-AIMED FORCES

1. Information Transfer With Gradual Transitions; First-Order Rate Process

Here the equations (3.2a,b,c,d) are re-examined numerically with different initial force sizes.

Exhibit 21: $r_{ua} = 0.5$, $b_{ua} = 1.5$, $\rho_{uu} = \rho_{ua} = 0.01$
$\rho_{au} = \rho_{aa} = 0.1$, $\eta_{uu} = \eta_{ua} = 0.02$, $\eta_{au} = \eta_{aa} = 0.2$
$B(0) = 120$, $R(0) = 100$

If we compare this with Exhibit 1 it can be said that the 20% increase in Blue initial forces puts Blue into the position of winning if the fight is terminated at a break point level of less than 50% loss for Red.

Exhibit 22: $r_{ua} = 0.5$, $b_{ua} = 1.5$, $\rho_{uu} = \rho_{ua} = 0.01$
$\rho_{au} = \rho_{aa} = 0.1$, $\eta_{uu} = \eta_{ua} = 0.015$, $\eta_{au} = \eta_{aa} = 0.15$.
$B(0) = 120$, $R(0) = 100$

Again by comparison with corresponding graph in Exhibit 2 it is that a 20% increase in Blue initial forces makes Blue victorious at the end.

Exhibit 23: $r_{ua} = 0.5$, $b_{ua} = 1.5$, $\rho_{uu} = \rho_{ua} = 0.015$
$\rho_{au} = \rho_{aa} = 0.1$, $\eta_{uu} = \eta_{ua} = 0.01$, $\eta_{au} = \eta_{aa} = 0.15$
B(0)= 120, R(0)= 100

Here again a 20% increase in initial Blue force makes Blue the winner of the combat.

Exhibit 24: $r_{ua} = 0.5$, $b_{ua} = 10$, $\rho_{uu} = \rho_{ua} = 0.01$
$\rho_{au} = \rho_{aa} = 0.1$, $\eta_{uu} = \eta_{ua} = 0.01$, $\eta_{au} = \eta_{aa} = 0.12$
B(0)= 100, R(0)= 120

If we make the comparison with the corresponding graph in Exhibit 4 it can be said that Red has to have not only physical superiority in the aimed phase but also 20% more initial force than Blue to win the combat at the end.

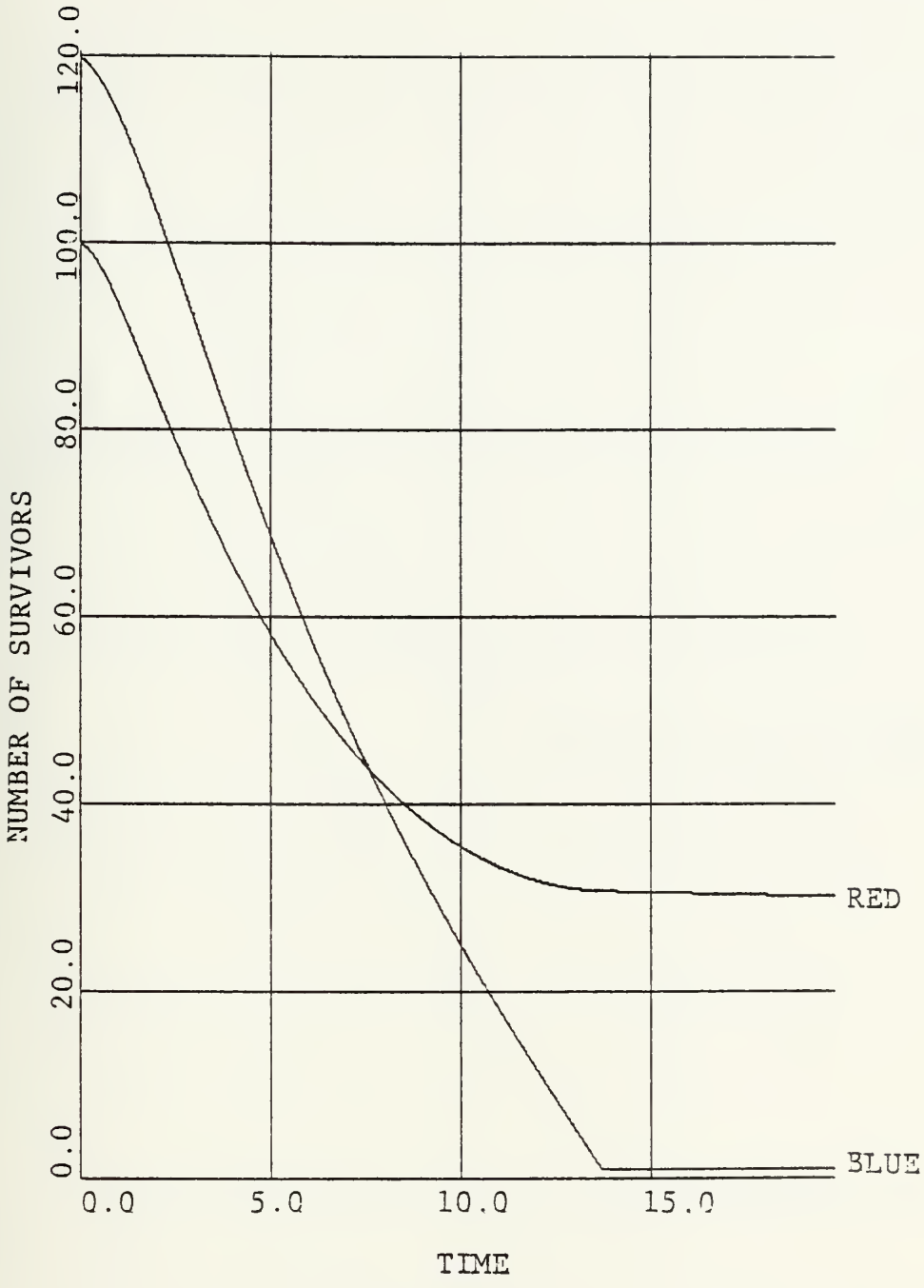


Exhibit 21.

$b_{ua} = 1.5$ $\quad r_{ua} = 0.5$

$\rho_{uu} = \rho_{ua} = 0.01$ $\quad \eta_{uu} = \eta_{ua} = 0.02$

$\rho_{au} = \rho_{aa} = 0.1$ $\quad \eta_{au} = \eta_{aa} = 0.2$

$B(0) = 120$ $\quad R(0) = 100$

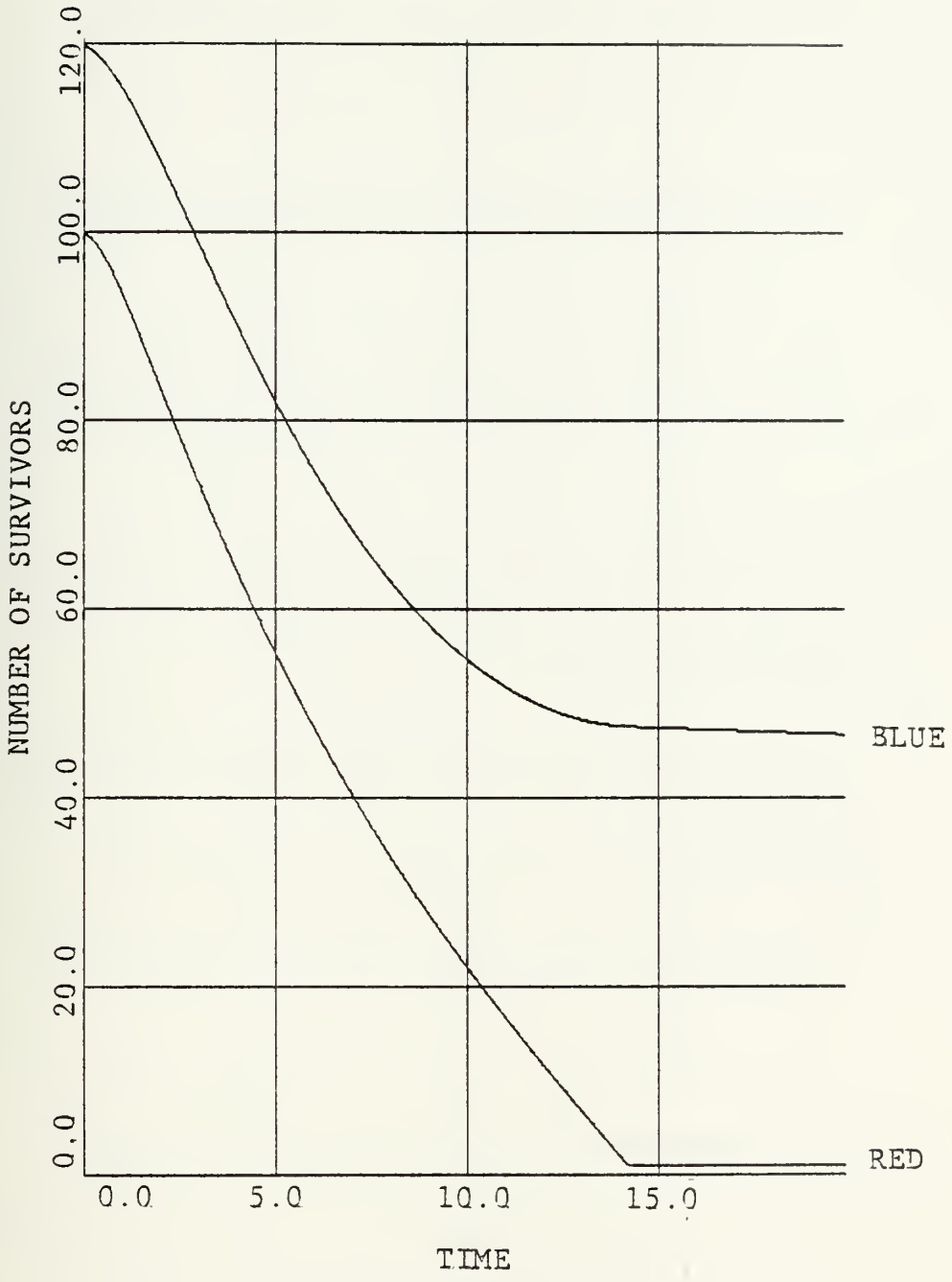


Exhibit 22.

$b_{ua} = 1.5$ $\quad r_{ua} = 0.5$

$\rho_{uu} = \rho_{ua} = 0.01$ $\quad \eta_{uu} = \eta_{ua} = 0.015$

$\rho_{au} = \rho_{aa} = 0.1$ $\quad \eta_{au} = \eta_{aa} = 0.15$

$B(0) = 120$ $\quad R(0) = 100$

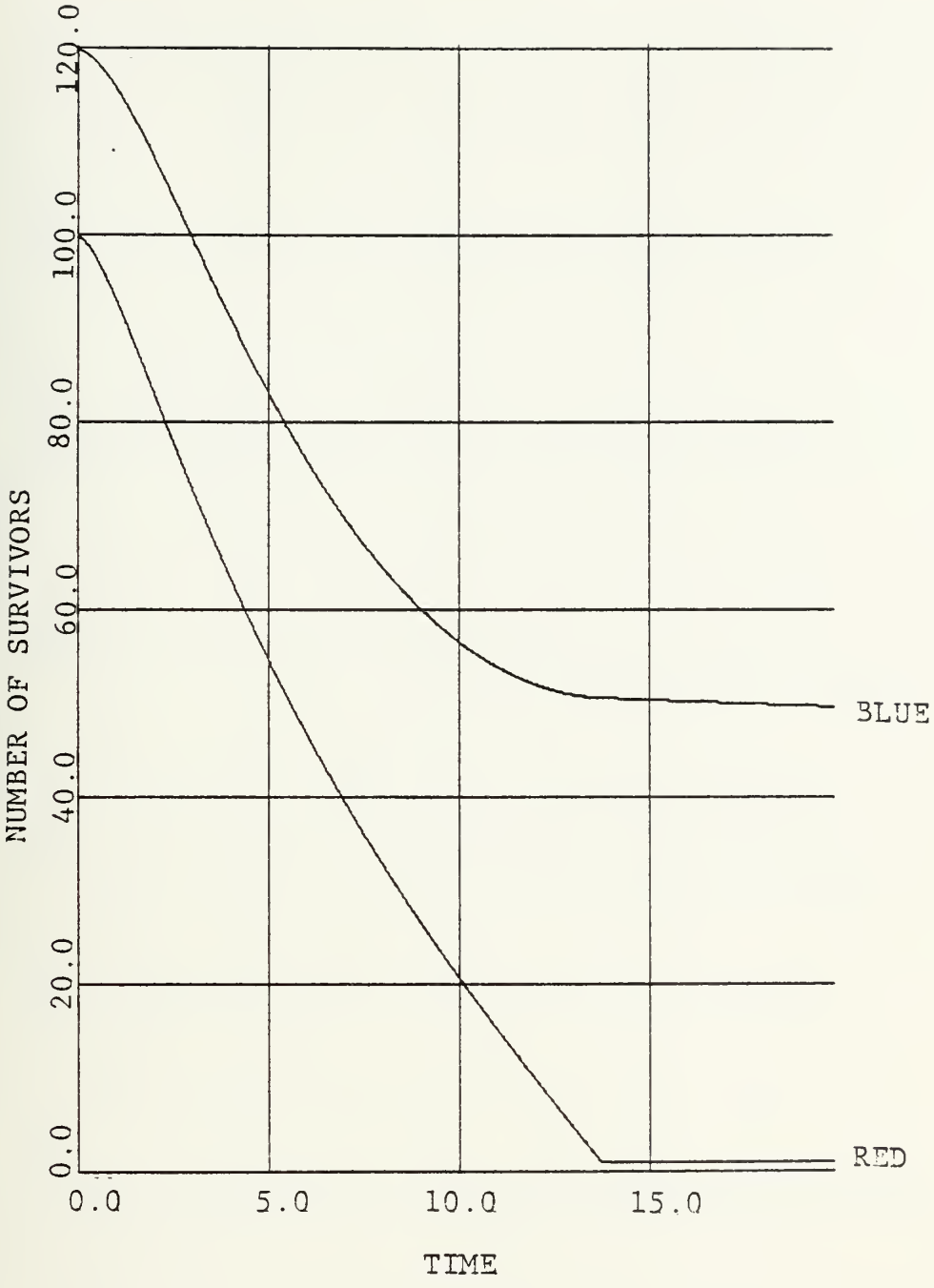


Exhibit 23.

$b_{ua} = 1.5$ $\quad r_{ua} = 0.5$

$\rho_{uu} = \rho_{ua} = 0.015$ $\quad \eta_{uu} = \eta_{ua} = 0.01$

$\rho_{au} = \rho_{aa} = 0.1$ $\quad \eta_{au} = \eta_{aa} = 0.15$

$B(0) = 120$ $\quad R(0) = 100$

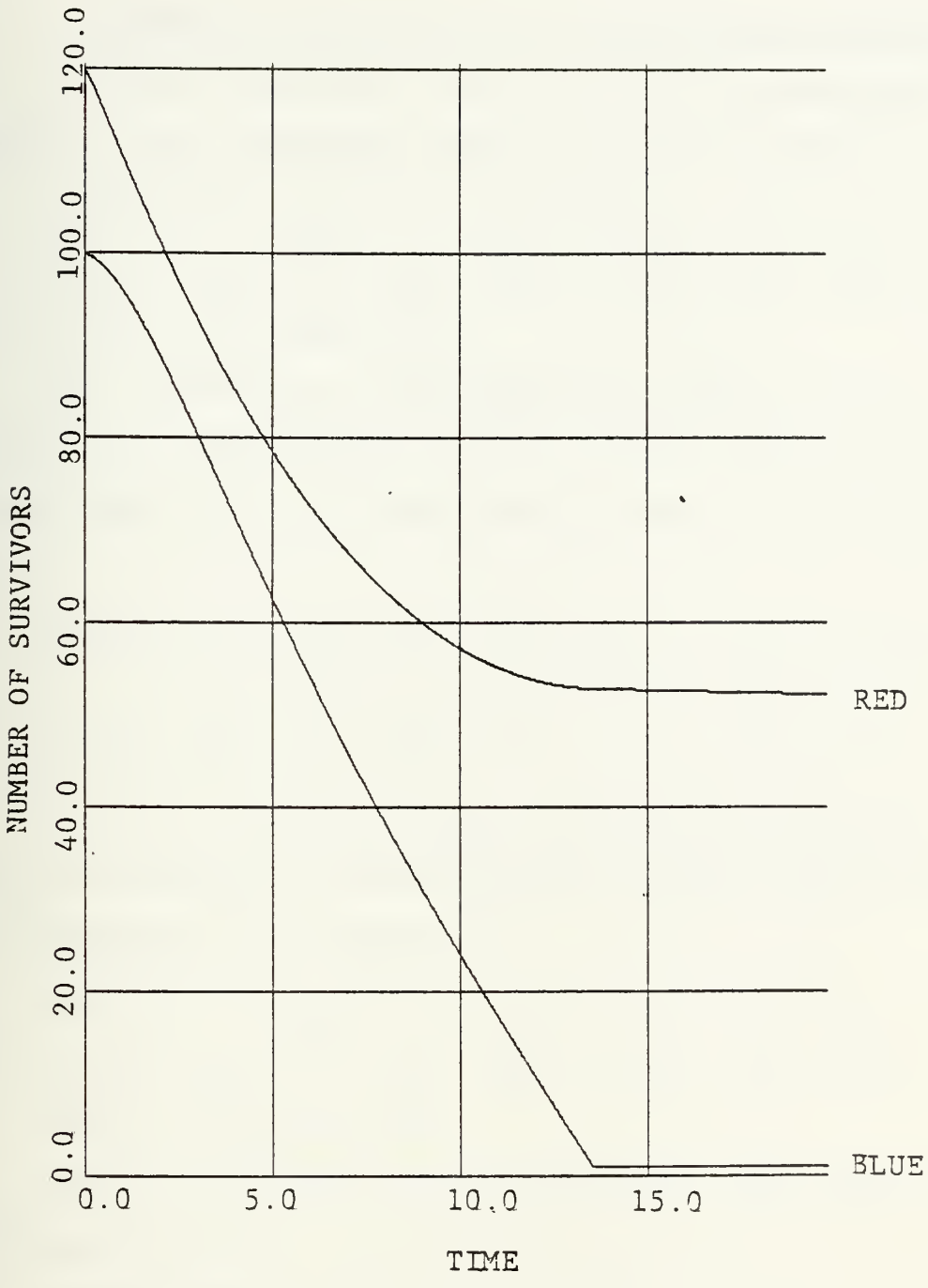


Exhibit 24.

$b_{ua} = 10$ $r_{ua} = 0.5$

$\rho_{uu} = \rho_{ua} = 0.01$ $\eta_{uu} = \eta_{ua} = 0.01$

$\rho_{au} = \rho_{aa} = 0.1$ $\eta_{au} = \eta_{aa} = 0.12$

$B(0) = 100$ $R(0) = 120$

2. Information Transfer With Instantaneous Transition

Here the equations (3.4a,b,c,d) are re-examined numerically with different numbers of initial forces.

Exhibit 25: $r_{ua} = 0.5$, $b_{ua} = 1.5$, $\rho_{uu} = \rho_{ua} = 0.01$
$\rho_{au} = \rho_{aa} = 0.1$, $\eta_{uu} = \eta_{ua} = 0.02$, $\eta_{uu} = \eta_{ua} = 0.2$
$B(0) = 120$, $R(0) = 100$

By comparison of this graph with Exhibit 5 it appears that a 20% increase in Blue initial force results in a Red force loss at a break point level of less than 70% loss.

Exhibit 26: $r_{ua} = 0.5$, $b_{ua} = 1.5$, $\rho_{uu} = \rho_{ua} = 0.01$
$\rho_{au} = \rho_{aa} = 0.1$, $\eta_{uu} = \eta_{ua} = 0.015$, $\eta_{au} = \eta_{aa} = 0.15$
$B(0) = 120$, $R(0) = 100$

It can be seen here that Blue wins the combat by using the advantage of having 20% more initial force than Red by comparison with Exhibit 6.

Exhibit 27: $r_{ua} = 0.5$, $b_{ua} = 1.5$, $\rho_{uu} = \rho_{ua} = 0.015$
$\rho_{au} = \rho_{aa} = 0.1$, $\eta_{uu} = \eta_{ua} = 0.01$, $\eta_{au} = \eta_{aa} = 0.15$
$B(0) = 120$, $R(0) = 100$

Again increasing Blue initial force size makes Blue victorious over Red.

Exhibit 28: $r_{ua} = 0.5$, $b_{ua} = 10$, $\rho_{uu} = \rho_{ua} = 0.01$
$\rho_{au} = \rho_{aa} = 0.1$, $\eta_{uu} = \eta_{ua} = 0.01$, $\eta_{au} = \eta_{aa} = 0.12$
$B(0) = 100$, $R(0) = 120$

The same effect of initial forces can be seen here by comparison with Exhibit 8.

As we mentioned earlier, by comparison of these graphs with the graphs for the case of gradual transitions as first-order rate process that we examined previously, we can say that Blue stays ahead longer in the case of instantaneous transitions by using the advantage of having all his survivors in aimed-fire phase of time t_b.

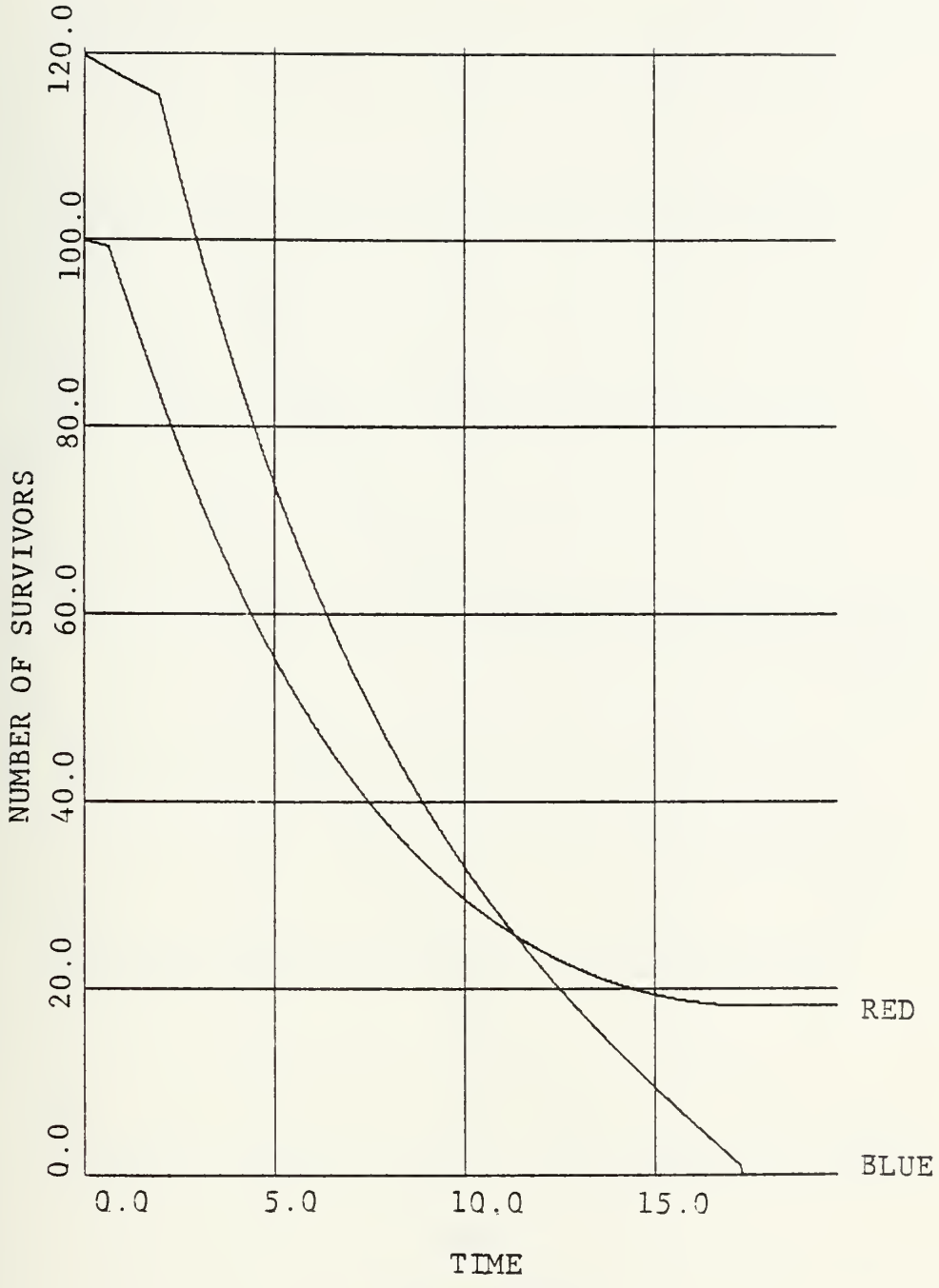


Exhibit 25.

$b_{ua} = 1.5$ $\quad r_{ua} = 0.5$

$\rho_{uu} = \rho_{ua} = 0.01$ $\quad \eta_{uu} = \eta_{ua} = 0.02$

$\rho_{au} = \rho_{aa} = 0.1$ $\quad \eta_{au} = \eta_{aa} = 0.2$

$B(0) = 120$ $\quad R(0) = 100$

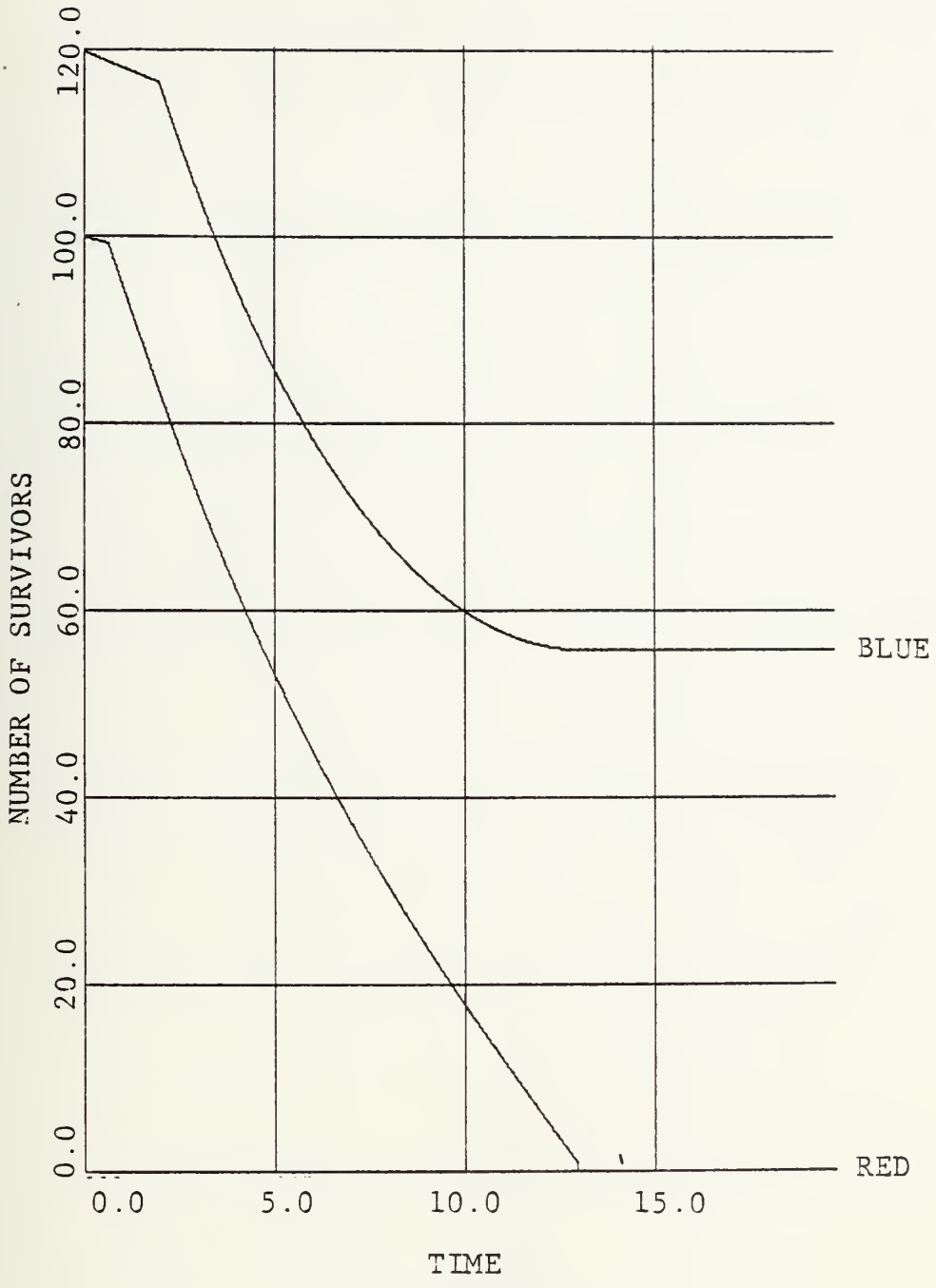


Exhibit 26.

$b_{ua} = 1.5$ $\quad r_{ua} = 0.5$

$\rho_{uu} = \rho_{ua} = 0.01$ $\quad \eta_{uu} = \eta_{ua} = 0.015$

$\rho_{au} = \rho_{aa} = 0.1$ $\quad \eta_{au} = \eta_{aa} = 0.15$

B(0)= 120 $\quad$ R(0)= 100

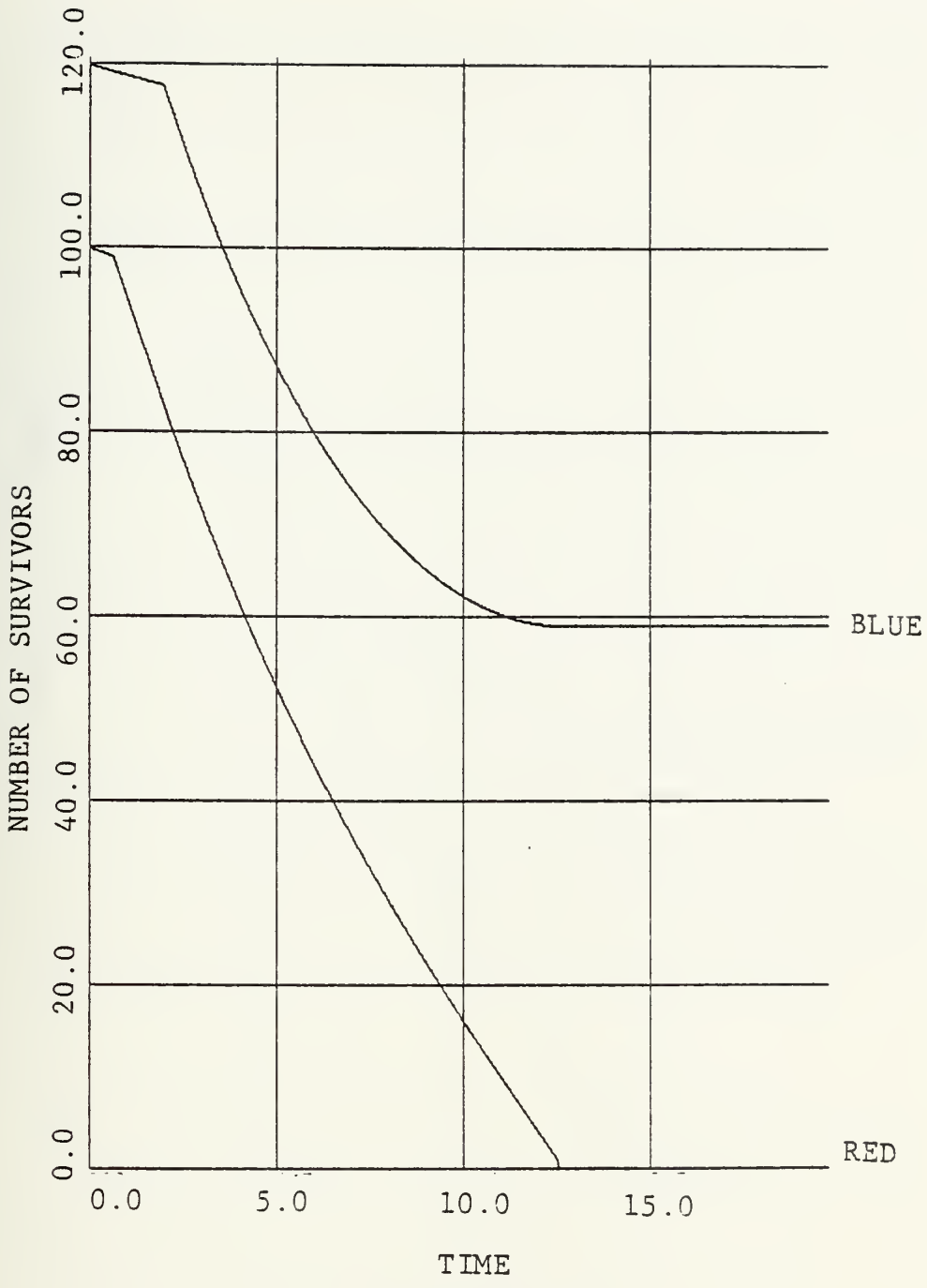


Exhibit 27.

$b_{ua} = 1.5$ $\quad r_{ua} = 0.5$

$\rho_{uu} = \rho_{ua} = 0.015$ $\quad \eta_{uu} = \eta_{ua} = 0.01$

$\rho_{au} = \rho_{aa} = 0.1$ $\quad \eta_{au} = \eta_{aa} = 0.15$

$B(0) = 120$ $\quad R(0) = 100$

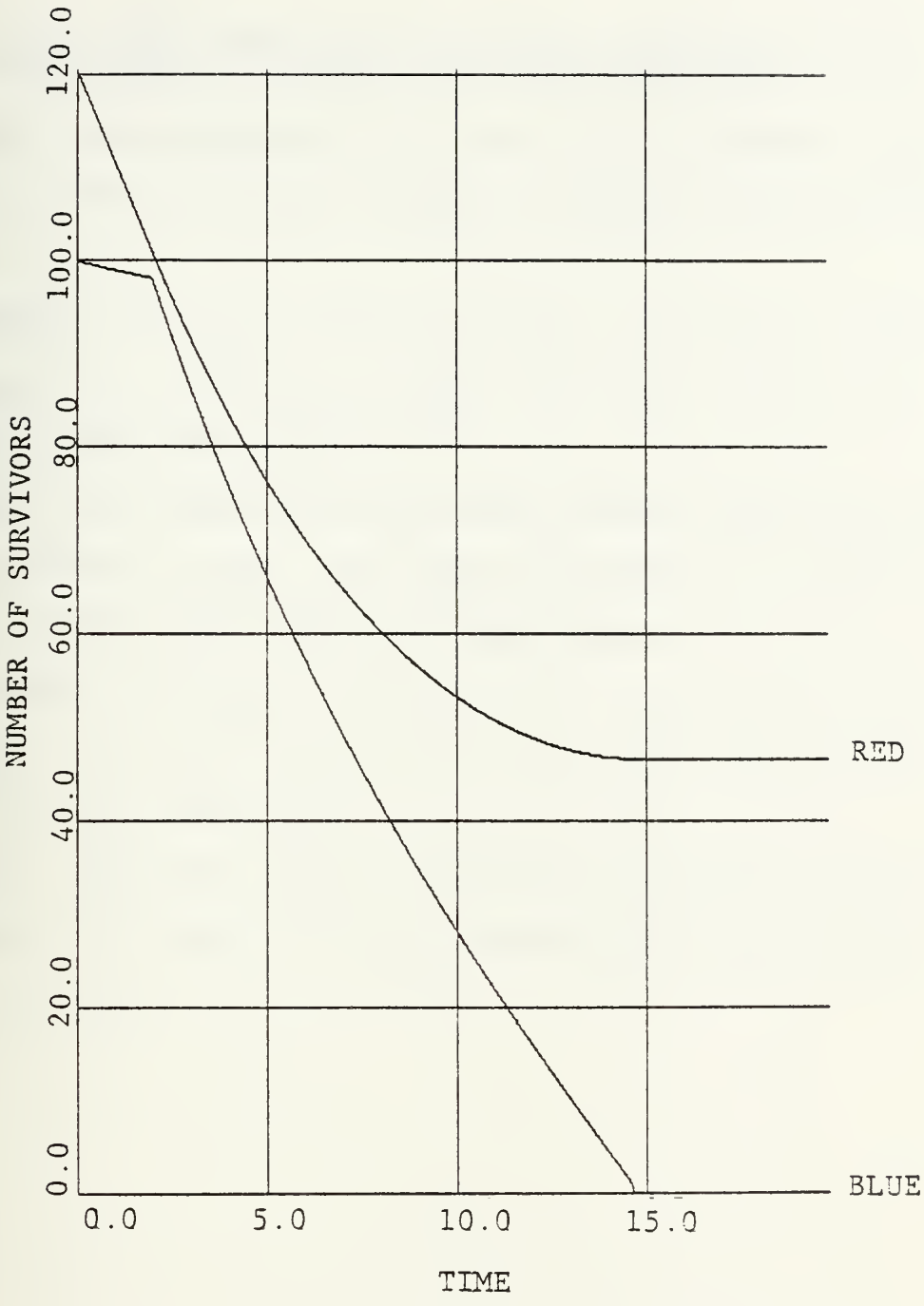


Exhibit 28.

$b_{ua} = 10$ $r_{ua} = 0.5$

$\rho_{uu} = \rho_{ua} = 0.01$ $\eta_{uu} = \eta_{ua} = 0.01$

$\rho_{au} = \rho_{aa} = 0.1$ $\eta_{au} = \eta_{aa} = 0.12$

$B(0) = 100$ $R(0) = 120$

B. COMBAT UNDER CONDITIONS OF MUTUAL ATTRITION WITHOUT COORDINATION OF AIMED FORCES FOR BOTH SIDES

Here the equations(5.1a,b,c,d) are re-examined numerically by using different number of initial forces for one side.

Exhibit 29: $r_{ua} = 0.5$, $b_{ua} = 1.5$, $\rho_{uu} = \rho_{ua} = 0.01$
$\rho_{au} = \rho_{aa} = 0.1$, $\eta_{uu} = \eta_{ua} = 0.02$, $\eta_{au} = \eta_{aa} = 0.2$
$B(0) = 150$, $R(0) = 100$

Here Blue starts with 50% more initial forces than Red. As the result of this, Blue wins the combat at a break point level of less than 70% loss of Red forces.

Exhibit 30: $r_{ua} = 0.5$, $b_{ua} = 1.5$, $\rho_{uu} = \rho_{ua} = 0.015$
$\rho_{au} = \rho_{aa} = 0.1$, $\eta_{uu} = \eta_{ua} = 0.01$, $\eta_{au} = \eta_{aa} = 0.15$
$B(0) = 120$, $R(0) = 100$

Here again the effect of increase of initial forces can be seen in the case of having uncoordinated-aimed forces for both sides.

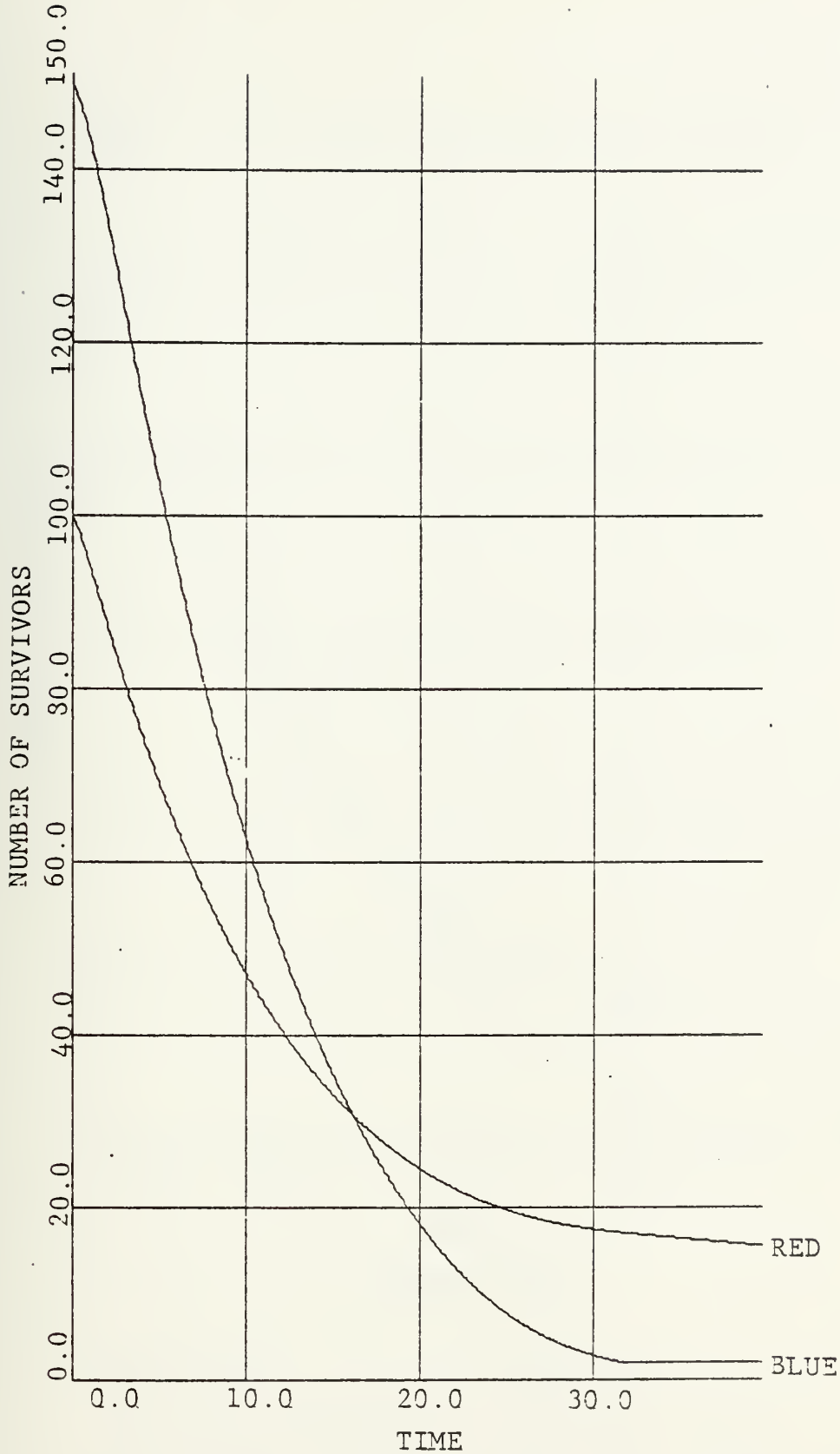


Exhibit 29.

$r_{ua} = 0.5$, $b_{ua} = 1.5$, $\rho_{uu} = \rho_{ua} = 0.01$, $\rho_{au} = \rho_{aa} = 0.1$, $\eta_{uu} = \eta_{ua} = 0.02$, $\eta_{au} = \eta_{aa} = 0.2$, B(0)= 150, R(0)= 100

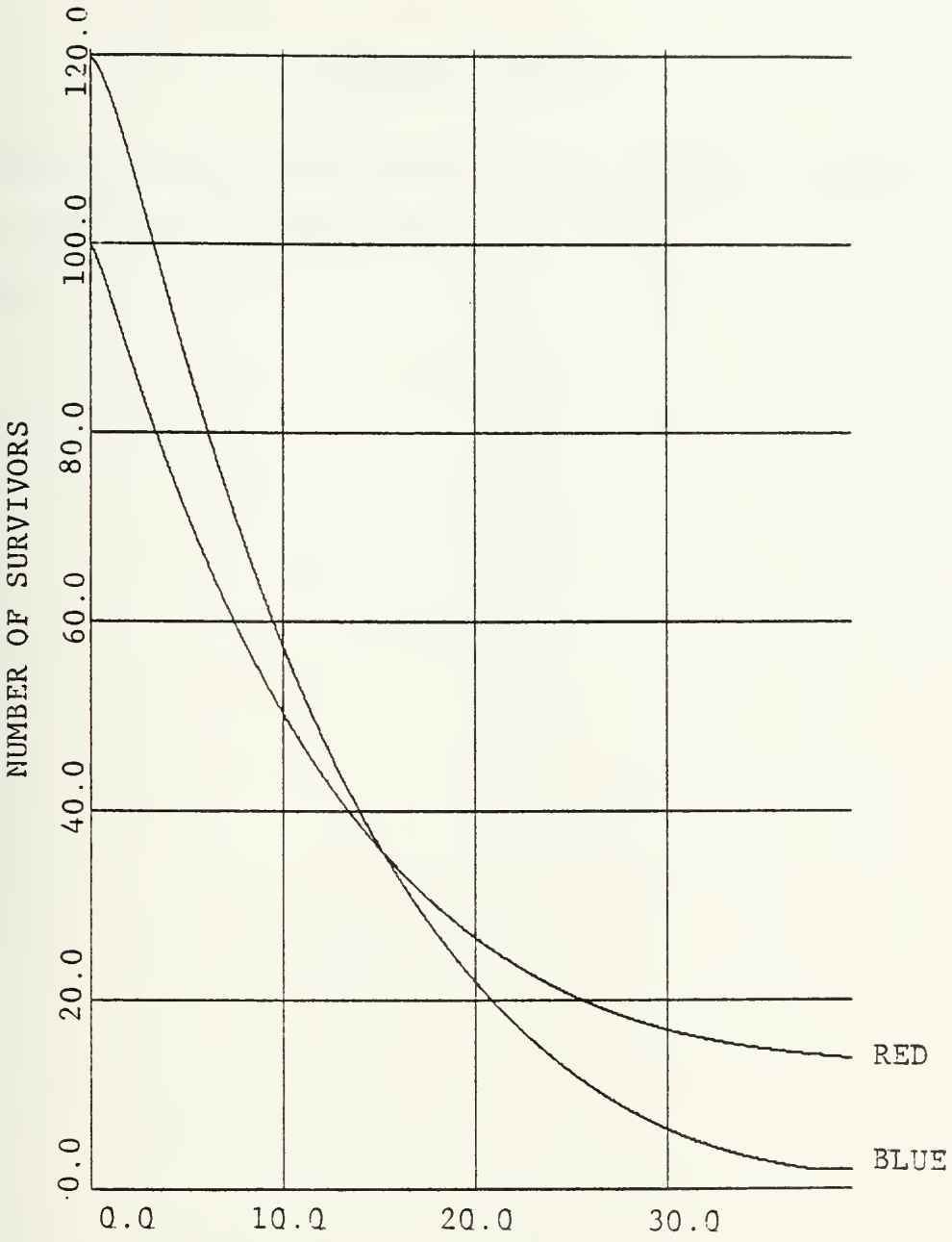


Exhibit 30.

$b_{ua} = 1.5$ $\quad r_{ua} = 0.5$

$\rho_{uu} = \rho_{ua} = 0.015$ $\quad \eta_{uu} = \eta_{ua} = 0.01$

$\rho_{au} = \rho_{aa} = 0.1$ $\quad \eta_{au} = \eta_{aa} = 0.15$

$B(0) = 120$ $\quad R(0) = 100$

COMPUTER PROGRAM

In this section we displayed the computer programs that are used to produce the graphs that we presented before.

COMBAT UNDER CONDITIONS OF MUTUAL ATTRITION WITH
COORDINATED-AIMED FIRE IN THE CASE OF
GRADUAL TRANSITION OF INFORMATION

```
      DIMENSION X(30),XDOT(30),C(15)
      C(10)= 1.
    1 CALL INTEG2(T,X,XDOT,C)
      IF((X(2)+X(1)).LE.1.0) GO TO 2
      IF(X(1).LE.10.0**(-50)) X(1)=0.0
      GO TO 6
    2 IF(X(1).LE.10.0**(-50)) X(1)=0.0
      X(2)=1.0-X(1)
    6 IF ((X(3)+X(4)).LE.1.0) GO TO 3
      IF(X(3).LE.10.0**(-50)) X(3)=0.0
      GO TO 5
    3 IF(X(3).LE.10.0**(-50)) X(3)=0.0
      X(4)=1.0-X(3)
    5 XDOT(1)=-C(4)*X(1)-C(5)*X(1)*X(3)/C(8)-C(6)*X(1)*X(4)/(X(2)+X(1))
      XDOT(2)=C(4)*X(1)-C(5)*X(2)*X(3)/C(8)-C(6)*X(2)*X(4)/(X(2)+X(1))
      XDOT(3)=-C(1)*X(3)-C(2)*X(3)*X(1)/C(7)-C(3)*X(3)*X(2)/(X(3)+X(4))
      XDOT(4)=C(1)*X(3)-C(2)*X(4)*X(1)/C(7)-C(3)*X(4)*X(2)/(X(3)+X(4))
      X(5)=X(1)+X(2)
      X(6)=X(3)+X(4)
      GO TO 1
      END
```

COMBAT UNDER CONDITIONS OF MUTUAL ATTRITION WITH
COORDINATED-AIMED FIRE IN THE CASE OF
INSTANTANEOUS TRANSITION OF INFORMATION

```
      DIMENSION X(30),XDOT(30),C(15)
      C(10)= 1.
    1 CALL INTEG2(T,X,XDOT,C)
      IF(X(1).LT.1.0) X(1)=0.0
      IF(X(2).LT.1.0) X(2)=0.0
      IF (T.GE.(1.0/C(1)))GO TO 20
      IF (T.GE.(1.0/C(4)))GO TO 10
      XDOT(1)=-C(5)*X(1)*X(2)/C(8)
      XDOT(2)=-C(2)*X(1)*X(2)/C(7)
      GO TO 1
   10 XDOT(1)=-C(5)*X(1)*X(2)/C(8)
      XDOT(2)=-C(3)*X(1)
      GO TO 1
   20 XDOT(1)=-C(6)*X(2)
      XDOT(2)=-C(3)*X(1)
      GO TO 1
      END
```

COMBAT UNDER CONDITIONS OF MUTUAL ATTRITION WITH
UNCOORDINATED-AIMED FIRE IN THE CASE OF
GRADUAL TRANSITION OF INFORMATION

```
      DIMENSION X(30),XDOT(30),C(15)
      C(10)= 1.
    1 CALL INTEG2(T,X,XDOT,C)
      IF((X(2)+X(1)).LE.2.0) GO TO 2
      IF(X(1).LE.10.0**(-50)) X(1)=0.0
      GO TO 6
    2 IF(X(1).LE.10.0**(-50)) X(1)=0.0
      X(2)=2.0-X(1)
    6 IF ((X(3)+X(4)).LE.2.0) GO TO 3
      IF(X(3).LE.10.0**(-50)) X(3)=0.0
      GO TO 5
    3 IF(X(3).LE.10.0**(-50)) X(3)=0.0
      X(4)=2.0-X(3)
    5 XDOT(1)=-C(4)*X(1)-C(5)*X(1)*X(3)/C(8)-C(6)*X(1)*(1.0-(1.0-1.0/(X(
     11)+X(2)))**X(4))
      XDOT(2)=C(4)*X(1)-C(5)*X(2)*X(3)/C(8)-C(6)*X(2)*(1.0-(1.0-1.0/(X(1
     1)+X(2)))**X(4))
      XDOT(3)=-C(1)*X(3)-C(2)*X(3)*X(1)/C(7)-C(3)*X(3)*(1.0-(1.0-1.0/(X(
     13)+X(4)))**X(2))
      XDOT(4)=C(1)*X(3)-C(2)*X(4)*X(1)/C(7)-C(3)*X(4)*(1.0-(1.0-1.0/(X(3
     1)+X(4)))**X(2))
      X(5)=X(1)+X(2)
      X(6)=X(3)+X(4)
      GO TO 1
      END
```

COMBAT UNDER CONDITIONS OF MUTUAL ATTRITION
ONE SIDE WITH,THE OTHER WITHOUT,COORDINATION
IN THE CASE OF GRADUAL TRANSITION OF INFORMATION

```
      DIMENSION X(30),XDOT(30),C(15)
      C(10)= 1.
    1 CALL INTEG2(T,X,XDOT,C)
      IF((X(2)+X(1)).LE.2.0) GO TO 2
      IF(X(1).LE.10.0**(-50)) X(1)=0.0
      GO TO 6
    2 IF(X(1).LE.10.0**(-50)) X(1)=0.0
      X(2)=2.0-X(1)
    6 IF ((X(3)+X(4)).LE.2.0) GO TO 3
      IF(X(3).LE.10.0**(-50)) X(3)=0.0
      GO TO 5
    3 IF(X(3).LE.10.0**(-50)) X(3)=0.0
      X(4)=2.0-X(3)
    5 XDOT(1)=-C(4)*X(1)-C(5)*X(1)*X(3)/C(8)-C(6)*X(1)*(1.0-(1.0-1.0/(X(
     11)+X(2)))**X(4))
      XDOT(2)=C(4)*X(1)-C(5)*X(2)*X(3)/C(8)-C(6)*X(2)*(1.0-(1.0-1.0/(X(1
     1)+X(2)))**X(4))
      XDOT(3)=-C(1)*X(3)-C(2)*X(3)*X(1)/C(7)-C(3)*X(3)*X(2)/(X(3)+X(4))
      XDOT(4)=C(1)*X(3)-C(2)*X(4)*X(1)/C(7)-C(3)*X(4)*X(2)/(X(3)+X(4))
      X(5)=X(1)+X(2)
      X(6)=X(3)+X(4)
      GO TO 1
      END
```

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