1981

Shape optimization of trusses subject to strength, displacement, and frequency constraints.

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https://hdl.handle.net/10945/20666

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

SHAPE OPTIMIZATION OF TRUSSES SUBJECT TO STRENGTH, DISPLACEMENT, AND FREQUENCY CONSTRAINTS

by

Jorge E. Felix

December 1981

Thesis Advisor: Garret N. Vanderplaats

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REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle) Shape Optimization of Trusses Subject to Strength, Displacement, and Frequency Constraints	5. TYPE OF REPORT & PERIOD COVERED Master's Thesis; December 1981
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Jorge E. Felix	8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940	12. REPORT DATE December 1981
	13. NUMBER OF PAGES 72 pages
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report) Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Structural optimization, finite elements, structural configuration, trusses, frequency constraints.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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The finite element displacement method of analysis is used and eigenvalues are calculated using the subspace iteration technique. All gradient information is calculated analytically.

The design problem is cast as a multi-level numerical optimization problem. The joint coordinates are treated as system variables. For each proposed configuration, the member sizes are updated as a sub-optimization problem. This sub-problem is efficiently solved using approximation concepts in the reciprocal variable space. The multi-level approach is shown to be an effective technique which conveniently takes advantage of the most efficient methods available for the member sizing problem.

Examples are presented to demonstrate the method. The optimum configuration is shown to be strongly dependent on the constraints which are imposed on the design.

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Shape Optimization of Trusses Subject to Strength, Displacement, and Frequency Constraints

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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
December 1981

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LIST OF SYMBOLS

A_i	Cross-sectional area of member i
A_{min}, A_{max}	Minimum and maximum allowable area variables, respectively
a_1, b_1	Constants defined by Eq. 43
c	Factor defined by Eq. 31
$\underset{\approx}{D}$	Matrix of direction cosines
E_i	Young's Modulus for member i
$\underset{\sim}{F}_{\ell}$	Vector of applied loads
$G_j(\bar{X})$	Constraint function of coordinate variable
$G_j(\bar{X}_m)$	Constraint function of area variable
$G_k(\bar{X}_m)$	Active constraints of area variable
$\underset{\approx}{I}$	Identity matrix
$\underset{\approx}{K}$	Global stiffness matrix
$\underset{\approx}{K}_i$	Element stiffness matrix
L_i	Length of a bar i
$\underset{\sim}{M}$	Global mass matrix
$\underset{\approx}{M}_G$	Global generalized mass matrix
$\underset{\approx}{M}_L$	Lumped mass matrix
NAC	Number of active constraints
NIC	Number of initial constraints
$\bar{S}$	Direction vector
$\bar{S}_g$	Direction vector on the coordinate design space
$\bar{S}_m$	Direction vector on the area design space

$\underset{\sim}{u}$	Nodal displacement vector
$\bar{\underset{\sim}{u}}_i$	ith nodal displacement vector
u_k^-, u_k^+	Minimum and maximum allowable displacement value
$\bar{X}$	Vector of design variables
$\bar{X}_g$	Vector of coordinate design variables
$\bar{X}_m$	Vector of area design variables
X_ℓ	lth independent design vector
X_k^-, X_k^+	Minimum and maximum allowable coordinate variable
α, μ, ν	Direction cosines
α^*, α_1, α_2	Scalar parameter defining distance of move in the area or coordinate design space.
β	Objective function of direction function problem
δ	Displacement constraint value
δ^-, δ^+	Lower and upper limits on displacement constraints
$\delta_{i\ell}$	Kronecker delta
θ_j	Push-off factor
$\underset{\sim}{\sigma}_i$	Stress in member i
σ_{ci}	Allowable compressive stress in member i
σ_{ti}	Allowable tensile stress in member i
σ_{bi}	Allowable Euler buckling stress in member i
ρ_i	Material density
ϕ_i	Eigenvector corresponding to ω_i
ω_i	ith eigenvalue
ω^-, ω^+	Minimum and maximum allowable fundamental frequency
$\underset{\approx}{\Phi}$	Nodal matrix
$\underset{\approx}{\Omega}^2$	Expectral matrix
λ	Frequency limit

ACKNOWLEDGEMENT

The author wishes to express his deep appreciation to his faculty advisor, Professor Garret N. Vanderplaats, for his continuous guidance, assistance and encouragement as well as his great enthusiasm during the course of this project; to Professor David Salinas who read this project, and gave of his time and engineering expertise; to the distinguished members of the faculty and the staff of the Department of Mechanical Engineering for their support in very way, my sincere appreciation.

This investigation would not have been possible without the facilities and support of the W. R. Church Computer Center.

Infinite thanks and grateful appreciation go to my wife, Yolanda, my son, Paulo, and daughter, Maria Gabriela, whose understanding and encouragement made the difficult time bearable.

I. INTRODUCTION

The process of optimization of structures has undergone important changes since its development in the early 1960s. Minimum weight of elastic truss structures subject to multiple loadings has been an active area of research. Attention has been focused on the problem of least weight, when the overall layout was known in advance, and when the cross-sectional area was the design variable. Some attention has been directed toward the optimum configuration of the structure. Design improvements in this area often exceed those in fixed-geometry and so shape optimization is of major interest.

Pioneer work in shape optimization was conducted in 1964 [Ref. 1] by Dorn, Gomory and Greenberg. The optimal connectivity of nodes for truss members, subject to a single load condition, was found and minimum weight designs were achieved. In their work only planar trusses were tested, and the process was presented as a plastic design problem using linear programming.

Their work was followed by Dobbs and Felton [Ref. 2] who in 1969 investigated the effect of multiple load conditions on the optimum configuration of trusses through the use of non-linear programming methods. Again, only planar

trusses were considered, subject to failure by stress and elastic buckling.

Later, Pedersen [Ref. 3] considered the positions of the joints as continuous design variables in addition to the areas of the bars. Stress, displacement, and buckling constraints were considered. Pedersen's work is significant because the optimization process is carried out by considering two separate design spaces. The optimization is achieved by successive iterations using a gradient method with move limits.

The optimization process was advanced by Vanderplaats and Moses [Ref. 4] who divided the design space into two subspaces, separating the area variables and the joint position variables. Multiple load conditions and constraints on stresses and Euler buckling were considered. The optimization was carried out alternatively between the two spaces until convergence was achieved. Three-dimensional indeterminate trusses were designed subject to multiple loading conditions. This work was extended by Vanderplaats [Ref. 5] to include displacement constraints.

In other research, Spillers [Ref. 6] considered statically indeterminate trusses. The optimization followed an iterative design where the member sizes and node locations were the design variables.

Recently, Imai [Ref. 7] treated the sizing and configuration variables simultaneously for either determinate or indeterminate trusses. The optimization was achieved using the Augmented Lagrange multiplier method.

The design problem considered in this study is the optimum configuration of three-dimensional indeterminate trusses, for multiple prescribed static load conditions. The objective is to minimize the weight of the structure, design variables are the node coordinates and the member sizes. Constraints include stress, Euler buckling, displacement, and the natural frequency of the system.

Some approximation concepts are introduced in order to reduce computational effort and to reduce the nonlinear characteristic of some constraints. Among these, a first order Taylor Series expansion is applied to approximate the constraints.

The optimization proceeds iteratively in two design spaces: the member sizing space, where the structure is optimized for a fix layout, and the coordinate space, where the geometry is allowed to vary. In both optimization processes the minimum weight of the structure was maintained, subject to the requirements that the constraints remain satisfied.

The mathematical formulation is presented in Chapter II. The objective function and constraints are defined in terms

of the design variables. The analytic gradients are also formulated.

The optimization technique is discussed in Chapter III. Several examples are presented and explained in Chapter IV. Conclusions and recommendations for future work are presented.

II. MATHEMATICAL FORMULATION

A. INTRODUCTION

Several features are desirable when finite element methods of structural analysis are used in optimization. First, the number of analyses for the structure should be kept as low as possible. Second, the amount of gradient information required during the design process should be reduced as much as possible.

B. ANALYSIS

1. Static Analysis

The initial layout of the truss, the member sizing and material properties (which may be different for each member), a set of external loads, and support conditions are initially specified.

The analysis for the stresses and deflections must be carried out satisfying the conditions of equilibrium of forces at the nodes and compatibility of deformation. If the material of the structure behaves in a linear manner, Hooke's law will establish the force-deflection relationship.

For a truss, it is also necessary to establish the following assumptions before selecting the method of computing the internal forces. The discrete element is treated as pin-connected, and loads and reactions are supported at

the joints. In this study, the weight of the members is not included as loads.

The Displacement (Stiffness) method [Ref. 8] considers the joint displacement components as the unknowns, and is written in the most general form using matrix notation,

$$\underset{\approx}{K}\underset{\sim}{u} = \underset{\sim}{F} \qquad \text{(Eq. 1)}$$

where K is the global stiffness matrix, F is the vector or vectors of applied loads, and u is the vector or vectors of displacements. Equation 1 is the set of equilibrium equations, and is formulated such that the compatibility is automatically satisfied.

The generality of the method is important if either statically determinate or indeterminate trusses are analyzed. The global stiffness matrix is symmetric and sparce. These features are used to write the code for a computer solution, and the matrix K is stored in compact form for efficient numerical solution.

Once the displacements at every node are known, the internal forces or stresses are calculated by applying force-deflection relations. This is defined as:

$$\underset{\sim}{g}_{ij} = \frac{E_i \underset{\approx}{D}^T}{L_i} \begin{Bmatrix} u_{\ell j} - u_{kj} \\ v_{\ell j} - v_{kj} \\ w_{\ell j} - w_{kj} \end{Bmatrix} \qquad \text{(Eq. 2)}$$

where i and j are element and load condition numbers respectively, and k and ℓ are node numbers associated with the

element i. σ_{ij} is the stress, E_i is the Young's modulus, and L_i is the length. Matrix D = f(α,μ,ν) contains the direction cosines. For brevity, hereafter the second subscript is omitted and it is assumed that the stress or displacement corresponds to the appropriate loading condition.

2. Dynamic Analysis

When system natural frequency constraints are imposed in the design process, the corresponding dynamic analysis of the structure has to be carried out. This requires the solution of an eigen-problem to determine the natural frequencies and normal modes. For linear elastic structures, the finite element approach leads to the well-known equation of motion, considering free vibration conditions,

$$\underset{\approx}{M}\ddot{\underset{\sim}{u}} + \underset{\approx}{K}\underset{\sim}{u} = 0 \qquad \text{(Eq. 3)}$$

where M is the global mass matrix, and u is the linear acceleration. Assuming a solution of the form

$$\underset{\approx}{u} = \underset{\sim}{\phi} e^{i\omega t} \qquad \text{(Eq. 4)}$$

where ω is the angular natural frequency of vibration of the structure, and ϕ is the corresponding eigenvector. After substitution into Eq. 3, the generalized eigenvalue problem becomes,

$$\underset{\approx}{K}\underset{\sim}{\phi} = \omega^2 \underset{\approx}{M}\phi \qquad \text{(Eq. 5)}$$

Written in matrix form for several eigenvalues, Eq. 5 becomes,

$$\underset{\approx}{K}\underset{\approx}{\Phi} = \Omega^2 \underset{\approx}{M}\underset{\approx}{\Phi} \qquad \text{(Eq. 6)}$$

where Φ is the modal matrix, and $\Omega^2 = \text{diag}\ (\omega^2)$ is the spectral matrix.

From the static analysis, the global stiffness matrix is already calculated, then only the global mass matrix evaluation is needed. Both generalized and lumped mass options are coded. These are defined respectively as:

$$\underset{\approx}{M}_G = \sum_{i=1}^{NE} \frac{\rho_i A_i L_i}{6} \begin{bmatrix} 2I_3 & I_3 \\ I_3 & 2I_3 \end{bmatrix} \qquad \text{(Eq. 7)}$$

and

$$\underset{\approx}{M}_L = \sum_{i=1}^{NE} \frac{\rho_i A_i L_i}{6} \begin{bmatrix} I \end{bmatrix}_{6x6} \qquad \text{(Eq. 8)}$$

Where I is the identity matrix, the sub-space iteration method of Bathe and Wilson [Ref. 9] is used to solve for a specified number of lowest eigenvalues and the associated eigenvectors. The method is economically efficient for large problems. The mass matrix may be diagonal or banded. The method is well suited for re-analysis when small changes are made in the design.

C. ANALYTIC GRADIENTS OF THE CONSTRAINTS

The necessity to compute gradients of the relevent functions in a design optimization process arise from the fact that efficient mathematical programming algorithms require information on derivatives. Furthermore, approximate methods based on a Taylor Series expansion of functions,

requires determination of derivatives. Based on the static and dynamic analyses, the gradients of forces, displacements, and frequencies with respect to the reciprocal of the cross-sectional areas and the coordinate variables, are formulated.

1. <u>Gradient of Member Stresses with Respect to the Reciprocal of Area Variables</u>

Stress in a member is defined as:

$$\underset{\sim}{\sigma}_i = \frac{F_i}{A_i} \qquad \text{(Eq. 9)}$$

also

$$\underset{\sim}{\sigma}_i = \frac{\underset{\approx}{K}_i \underset{\sim}{u}_i}{A_i} \qquad \text{(Eq. 10)}$$

The partial derivative is then:

$$\frac{\partial \underset{\sim}{\sigma}_i}{\partial \underset{\sim}{X}_e} = \frac{\partial}{\partial \underset{\sim}{X}_e} \left[\frac{1}{A_i} \underset{\approx}{K}_i \right] \underset{\sim}{u}_i + \frac{\underset{\sim}{K}_i}{A_i} \frac{\partial}{\partial \underset{\sim}{X}_e} u_i \qquad \text{(Eq. 11)}$$

The first term of the right hand side is zero, then

$$\frac{\partial \underset{\sim}{\sigma}_i}{\partial \underset{\sim}{X}_\ell} = \frac{\underset{\approx}{K}_i}{A_i} \frac{\partial}{\partial \underset{\sim}{X}_\ell} (\underset{\sim}{u}_i) \qquad \text{(Eq. 12)}$$

This can be written in explicit form as

$$\frac{\partial \underset{\sim}{\sigma}_i}{\partial \underset{\approx}{X}_\ell} = \frac{E_i \underset{\approx}{D}_i^T}{L_i^2} \frac{\partial}{\partial \underset{\approx}{X}_\ell} (\underset{\sim}{u}_i) \qquad \text{(Eq. 13)}$$

2. <u>Gradient of Nodal Displacements with Respect to the Reciprocal of Area Variable</u>

Consider the equation

$$\underset{\approx}{K} \underset{\sim}{u} = \underset{\sim}{F} \qquad \text{(Eq. 14)}$$

The derivative of $[K\,u]$ with respect to some variable X_ℓ is

$$\frac{\partial}{\partial X_\ell}\,[K u] = \frac{\partial}{\partial X_\ell}(F) \qquad \text{(Eq. 15)}$$

In this case $X_\ell = 1/A_\ell$ and the loads are constant and independent of the areas (weight of the truss elements are ignored), then

$$\frac{\partial}{\partial X_\ell}\,(K)(u) + K\,\frac{\partial}{\partial X_\ell}(u) = 0 \qquad \text{(Eq. 16)}$$

or

$$K\,\frac{\partial}{\partial X_\ell}(u) = -\frac{\partial}{\partial X_\ell}(K)u \qquad \text{(Eq. 17)}$$

Finally, then

$$\frac{\partial}{\partial X_\ell}(u) = -K^{-1}\left[\frac{\partial}{\partial X_\ell}(K)\right]u \qquad \text{(Eq. 18)}$$

where K^{-1} is the inverse of K. It is necessary to compute the partial derivative of the global stiffness matrix K defined as:

$$K = \sum_{i=1}^{NE} \frac{E_i A_i}{L_i}\,[D_i] \qquad \text{(Eq. 19)}$$

where D_i is the matrix of direction cosines defined earlier. Therefore, the partial derivative with respect to the reciprocal of the area variables is defined as:

$$\frac{\partial}{\partial X_\ell}(K) = \sum_{i=1}^{NE} \delta_{i\ell}\,\frac{E_i A_i^2}{L_i}\,[D_i] \qquad \text{(Eq. 20)}$$

where $\delta_{i\ell}$ is the kronecker delta defined by $\delta_{i\ell} = 1$ if $i = \ell$ and $\delta_{i\ell} = 0$ if $i \neq \ell$. In practice the full matrix, $\frac{\partial}{\partial X_\ell}(\underset{\approx}{K})$ is not actually stored. Instead the product, $\frac{\partial}{\partial \underset{\approx}{X}_\ell}(K)u$ is created directly. Details of efficient gradient computations are given in [Ref. 10].

3. Gradient of Frequencies with Respect to the Reciprocal of Area Variable

Consider the eigen-problem defined by,

$$[\underset{\approx}{K} - \omega_i^2 \underset{\approx}{M}]\ \underset{\sim}{\phi}_i = 0 \qquad \text{(Eq. 21)}$$

Taking the derivative with respect to the variable X [Ref. 11] gives

$$\frac{\partial}{\partial \underset{\sim}{X}_\ell}\{[\underset{\approx}{K} - \omega_i^2 \underset{\approx}{M}]\ \underset{\sim}{\phi}_i\} = 0 \qquad \text{(Eq. 22)}$$

then

$$\frac{\partial}{\partial \underset{\sim}{X}_\ell}\{[\underset{\approx}{K} - \omega_i^2 \underset{\approx}{M}]\}\ \underset{\sim}{\phi}_i + [\underset{\approx}{K} - \omega_i^2 \underset{\approx}{M}]\ \frac{\partial}{\partial \underset{\sim}{X}_\ell}\ \phi_i = 0 \qquad \text{(Eq. 23)}$$

Pre-multiplying through by ϕ_i^T and applying the condition of symmetry of the matrix $\left[K - \omega_i M\right]$ gives

$$\underset{\sim}{\phi}_i^T\ [\underset{\approx}{K} - \omega_i^2 \underset{\approx}{M}]\ \ \frac{\partial}{\partial \underset{\approx}{X}}(\underset{\sim}{\phi}_i) = -\phi_i^T\ [\frac{\partial \underset{\approx}{K}}{\partial \underset{\sim}{X}_\ell} - \omega_i^2 \frac{\partial \underset{\approx}{M}}{\partial \underset{\sim}{X}_\ell}\ \frac{\partial \omega_i}{\partial \underset{\sim}{X}_\ell}\ \underset{\approx}{M}]\ \underset{\sim}{\phi}_i \qquad \text{(Eq. 24)}$$

The left hand side of the equation is zero because of Eq. 21, and thus

$$\frac{\partial \omega_i^2}{\partial \underset{\sim}{X}_\ell} = \frac{\underset{\sim}{\phi}_i^T \left[\frac{\partial \underset{\approx}{K}}{\partial \underset{\sim}{X}_\ell} - \omega_i^2\ \frac{\partial \underset{\approx}{M}}{\partial \underset{\sim}{X}_\ell}\right] \underset{\sim}{\phi}_i}{\underset{\sim}{\phi}_i^T\ \underset{\approx}{M}\ \underset{\sim}{\phi}_i} \qquad \text{(Eq. 25)}$$

The partial derivative of the generalized mass matrix with respect to the reciprocal of the area variable is defined as:

$$\frac{\partial \underset{\approx}{M}_G}{\partial \underset{\sim}{X}_\ell} = - \sum_{i=1}^{NE} \delta_{i\ell} \frac{\rho_i A_i^2 L_i}{6} \begin{bmatrix} 2I_3 & I_3 \\ I_3 & 2I_3 \end{bmatrix} \qquad \text{(Eq. 26)}$$

The partial derivative of the lumped mass matrix with respect to the reciprocal of the area variable is formulated following the same procedure as the generalized mass matrix.

4. Gradient of Stresses with Respect to the Joint Coordinate Variables

Since stress in a member is

$$\underset{\sim}{\sigma}_i = \frac{\underset{\approx}{K}_i \underset{\sim}{u}_i}{A_i} \qquad \text{(Eq. 27)}$$

the gradient is calculated as

$$\frac{\partial \underset{\sim}{\sigma}_i}{\partial \underset{\sim}{X}_\ell} = \frac{\partial}{\partial \underset{\sim}{X}_\ell} \left(\frac{1}{A_i} \underset{\approx}{K}_i\right) \underset{\sim}{u}_i + \frac{1}{A_i} \underset{\approx}{K}_i \frac{\partial \underset{\sim}{u}_i}{\partial \underset{\sim}{X}_\ell} \qquad \text{(Eq. 28)}$$

In this case, the stiffness matrix is a function of the coordinate variables so the first term on the right hand side of Eq. 28 is not zero.

5. Gradient of Displacements with Respect to the Joint Coordinate Variables

Consider the following equation:

$$\underset{\approx}{K} \underset{\sim}{u} = \underset{\sim}{F} \qquad \text{(Eq. 29)}$$

The gradient of u with respect to X_ℓ is:

$$\frac{\partial}{\partial \underset{\sim}{X}_\ell}(\underset{\sim}{u}) = -\underset{\approx}{K}^{-1} \frac{\partial}{\partial \underset{\sim}{X}_\ell}(\underset{\approx}{K}) \underset{\sim}{u} \qquad \text{(Eq. 30)}$$

The partial derivative of the global stiffness matrix K now is defined as:

$$\frac{\partial}{\partial \underset{\sim}{X}_\ell} (\underset{\approx}{K}) = \sum_{i=1}^{NE} \left\{ \frac{A_i E_i}{L_i^2} \frac{\partial \underset{\approx}{K}_i}{X} - \frac{3A_i E_i}{L_i^2} (-1)^j (dX_i) [\underset{\approx}{K}_i] \right\} \qquad \text{(Eq. 31)}$$

where $(-1)^j$ and j = 1 or 2 defines the sign of the gradient at that particular node. Note that the terms in the summation are only evaluated for members connected to the joint defined by the variable X_ℓ.

6. Gradient of the Natural Frequencies with Respect to the Joint Coordinate Variables

The value of the derivative of the eigenvalue ω^2 found from

$$[\underset{\approx}{K} - \omega_i^2 \underset{\approx}{M}] \underset{\sim}{\phi}_i = 0 \qquad \text{(Eq. 32)}$$

is

$$\frac{\partial \omega_i^2}{\partial \underset{\sim}{X}_\ell} = - \frac{\underset{\sim}{\phi}_i^T \left[\frac{\partial \underset{\approx}{K}}{\partial X_\ell} - \omega^2 \frac{\partial \underset{\approx}{M}}{\partial X_\ell} \right] \underset{\sim}{\phi}_i}{\underset{\sim}{\phi}_i^T \underset{\sim}{M} \underset{\sim}{\phi}_i} \qquad \text{(Eq. 33)}$$

The partial derivative of the generalized mass matrix with respect to the coordinate variables are defined as:

$$\frac{\partial \underset{\approx}{M}_G}{\partial \underset{\sim}{X}_\ell} = - \sum_{i=1}^{NE} (-1)^i \frac{\rho_i A_i dX_i}{6L_i} \begin{bmatrix} 2I_3 & I_3 \\ I_3 & 2I_3 \end{bmatrix} \qquad \text{(Eq. 34)}$$

where $(-1)^j$ and j = 1 or 2 defines the sign of the gradient at the particular node. The partial derivative of the lumped mass matrix follows the same procedure.

D. APPROXIMATION CONCEPTS

At this stage the analysis tools necessary for the design process under static and dynamic conditions may be regarded as available. It has been pointed out that the application of mathematical programming methods for structural design is the most widely used because of its great generality and its simple formulation. However, it is required that a large number of structural analyses and sensitivity analyses be performed. This has motivated the idea of formulating simple and explicit approximations for the most relevant response quantities.

These approximations can only be expected to be of acceptable quality in some finite region of the design space surrounding the point about which the approximations were constructed. The total number of analyses required to find an optimum design using approximation concepts is significantly less than the number previously required.

Some sources of simplification can be considered. First, the dimension of the design space can be reduced if a proper subspace can be identified. Second, linking of design variables which is imposed because of symmetry or practical considerations also reduces the dimension of the design space.

The objective function for the design of trusses is a relatively simple explicit function of the design variables.

On the other hand nonlinear constraint functions are very complicated. From a computational standpoint, a small portion of the constraints play an active role in the optimization process; therefore, deletion of non-critical constraints avoids effort of evaluation of irrelevant constraints.

A method to deal with complicated constraint which is very effective in reducing computational effort is to deduce simple and explicit expressions for the constraints. Linearization is directly and efficiently accomplished by Taylor series expansions. The order of the expansion selected is decided based on the degree of nonlinearity and the approximation required; and a trade-off must be made between the computational effort required for the highest order derivatives versus improvement of approximation.

Application of a first order Taylor series expansion on stress constraints has been found to be sufficient. This was not the case for the natural frequency constraints, which may still be numerically unstable during the optimization. This suggests further expansion up to a second order. However, the computational effort needed to do this usually exceeds the benefits. Therefore, a first order expansion with move limits is used.

First order Taylor series expansion of a function, W, of the variable, X, about a point, Xo, is written as

$$W(p) \approx W(x) = W(Xo) + (X-Xo)\,\nabla W(Xo)$$

The philosophy underlying the use of linear approximations is not to transform the problem into a sequence of linear programs, but to replace the constraints by simple and explicit approximation functions. In this study the Taylor series expansion was used for all constraints with respect to the reciprocal of the member sizing variables, A_i. The objective function, in reciprocal space, is now non-linear but still explicit and easily evaluated.

E. OBJECTIVE FUNCTION

The function whose least value is sought in the optimization procedure is defined as the total weight of the truss which is given by

$$W = \sum_{i=1}^{NE} \rho_i A_i L_i \qquad \text{(Eq. 35)}$$

Where NE is the total number of members, and ρ_i is the material density, both are prescribed constants. A_i and L_i are the area and length of the ith member respectively.

F. CONSTRAINTS

The restrictions to be satisfied in order for the design to be acceptable are formulated explicitly; behavioral and side constraints are defined accordingly.

1. <u>Stress</u>

$$\sigma_{ci} \leq \sigma_i \leq \sigma_{ti} \qquad \text{(Eq. 36)}$$

where σ_{ci} and σ_{ti} are the allowable compressive and tensile stresses respectively for member i.

2. Euler Buckling

The Euler buckling stress is formulated as

$$\bar{\sigma}_{bi} = - \frac{cA_i E_i}{L_i^2} \qquad \text{(Eq. 37)}$$

where c is a prescribed constant.

3. Frequency

$$\omega_K^- \leq \omega_K \leq \omega_K^+ \qquad \text{(Eq. 38)}$$

where ω_K^- and ω_K^+ are the lower and upper bounds respectively on the first natural frequency of the system.

4. Limits on Areas

$$A_{min_i} \leq A_i \leq A_{max_i} \qquad \text{(Eq. 39)}$$

where, A_{min_i}, and A_{max_i} are the minimum and maximum allowable cross-sectional areas of the ith member, and are taken to be the same for all members. When symmetry of design is to be preserved, linking of the variables is required. This is defined as

$$A_K = A_i \qquad \text{(Eq. 40)}$$

where K and i are symmetric members.

5. Displacement

$$u_K^- \leq u_K \leq u_K^+ \qquad \text{(Eq. 41)}$$

where u_K^- and u_K^+ are the lower and upper bounds respectively on joint displacements u_K.

6. Limits on Coordinate Variables

$$X_K^- \le X_K \le X_K^+ \qquad \text{(Eq. 42)}$$

where again X_K^- and X_K^+ are the lower and upper bounds respectively on the Kth coordinate variable, and if symmetry must be preserved, linking of the variables is required. This is obtained by

$$X_\ell = a_\ell + b_\ell X_K \qquad \text{(Eq. 43)}$$

where a_ℓ and b_ℓ are constants and X_K is the coordinate variable.

G. GENERAL FORMULATION

The inequality constraints are of the form,

$$G_j(\bar{X}) \le 0 \qquad j = 1....NIC \qquad \text{(Eq. 44)}$$

The constraints are normalized as follows:

Stresses

$$\frac{\sigma}{\bar{\sigma}_t} - 1 \le 0$$

$$\frac{\sigma}{\bar{\sigma}_c} - 1 \le 0 \qquad \text{(Eq. 45)}$$

Euler Buckling

$$\frac{\sigma}{\bar{\sigma}_b} - 1 \le 0 \qquad \text{(Eq. 46)}$$

Note that because σ and $\bar{\sigma}_b$ depend on the member area A_i, this constraint is treated as a nonlinear function. The Taylor series expansion is performed on σ and the value of $\bar{\sigma}_b$ is continually updated.

Displacements $\frac{\delta}{\bar{\delta}-} - 1 \leq 0$

$$\frac{\delta}{\bar{\delta}+} - 1 \leq 0 \qquad \text{(Eq. 47)}$$

Frequency $\frac{\omega}{\bar{\omega}} - 1 \leq 0$

The side constraints are

$$A_\ell - A_i \leq 0 \qquad \text{(Eq. 49)}$$

$$X_K^- - X_K \leq 0$$

$$X_K^+ - X_K \leq 0 \qquad \text{(Eq. 50)}$$

III. OPTIMIZATION

A. INTRODUCTION

The main goal of structural engineering is to design structural systems that efficiently perform specified purposes. Selection of a specific algorithm is required and this algorithm must minimize the number of times the structure has to be analyzed and the amount of specific gradient information required. Finally, the algorithm should provide reasonable assurance that it will attain an optimum or near-optimum design.

The next two sections are a brief explanation of the algorithm used for this work.

B. GENERAL FORMULATION

The general constrained minimization design problem is defined as

minimize

$$W(\bar{X}) \qquad \text{(Eq. 1)}$$

subject to

$$G_j(\bar{X}) \leq 0 \qquad j = 1,\ldots.m \qquad \text{(Eq. 2)}$$

where, $W(\bar{X})$ is the objective function. Functions $G_j(\bar{X})$ are the set of inequality constraints. The vector of design variables $\bar{X}$ includes member sizing variables $\bar{X}_{mg}$ and geometric design variables $\bar{X}_g$.

C. OPTIMUM GEOMETRY DESIGN

The procedure used here was to treat the geometric design parameters as independent design variables. The member sizing parameters are handled as dependent variables which are determined as a sub-problem.

Beginning with an initial geometric design vector $\bar{X}_o$, the design proceeds iteratively using the following relationship:

$$\bar{X}_g^{q+1} = \bar{X}_g^q + \alpha_g^* \bar{S}_g^q \qquad \text{(Eq. 3)}$$

where q is the iteration number and $\bar{S}_g$ is the search direction to be determined. α_g^* is the scalar parameter determining the distance of travel in the design space.

For each proposed geometric vector, $\bar{X}_g$, the structure was optimized with respect to the member sizing variables, $\bar{X}_m$, by the sub-optimization problem defined in Section D.

Assume that for the initial geometry the structure has been optimized with respect to the cross-sectional areas, and that, from this subproblem there are ℓ active constraints of the form:

$$G_j(\bar{X}) = 0 \qquad k = 1,\ldots\ell \qquad \text{(Eq. 4)}$$

where, $G_j(\bar{X})$ is defined as active if its value is close to zero.

Now, it is necessary to find the search direction, $\bar{S}_g$, so that by moving in this direction in the coordinate design space, the objective function is minimized. This direction is found by solving the following subproblem.

Minimize

$$\nabla F(\bar{X}).\bar{S} \qquad \text{(Eq. 5)}$$

Subject to:

$$G_K(X)\ .\ \bar{S} \leq 0 \qquad k = 1,\ldots.\ell \qquad \text{(Eq. 6)}$$

$$\bar{S}\ .\ \bar{S} \leq 1 \qquad \text{(Eq. 7)}$$

At this point, $\bar{S}$, provides a search direction in the combined $[\bar{X}_g + \bar{X}_m]$ space which is projected onto the active constraints. Only the $\bar{S}_g$ part of $\bar{S}$ is used.

Once, $\bar{S}_g$, is known it is substituted back into Eq. 3 and a one-dimensional search on α_g^* is performed to update the vector, $\bar{X}_g$. The Optimum-Geometry design problem can be summarized in the following algorithm.

1. Given an initial coordinate design vector, $\bar{X}_o$, and area design vector, A_o. Specify, D_{max} and P_{max}. (D_{max}, maximum change in the coordinate at each iteration, and P_{max}, total number of iterations.)

2. Solve the fixed geometry problem and calculate the minimum weight (W*) for the current geometry.

3. Determine the set of active constraints.

4. Determine the search direction vector, $\bar{S}$.

5. Find the move parameter α_1^* for a D_{max} change of some coordinate variable, or such that some coordinate constraints becomes active.

6. Solve the fixed geometry problem for coordinates. $\bar{X}_g = \bar{X}_g + \alpha_1^*\bar{S}_g$ and calculate the optimum weight, $W(\bar{X})$.

7. Find the parameter α_2^*, which minimizes the weight using 2-point quadratic interpolation. If $\alpha_2^* > \alpha_i^*$ do to step 9.

8. Solve the fixed geometry design problem for coordinates $\bar{X}_g^{q+1} = \bar{X}_g^q + \alpha_2^* \bar{S}_g^q$ and calculate the optimal weight.

9. Check convergence; if satisfied, terminate; otherwise, set q = q+1, update $\bar{X}$ and return to step 3.

D. FIXED GEOMETRY DESIGN

As stated earlier, a sub-optimization problem has to be solved for a proposed geometry design vector, $\bar{X}$. The structure is now optimized based on the cross-sectional area subspace. This is defined in general as:

Minimize,

$$W(\bar{X}) \qquad \text{(Eq. 8)}$$

Subject to,

$$G_j(\bar{X}) < 0 \qquad j = 1, \ldots. \ell \qquad \text{(Eq. 9)}$$

The design proceeds iteratively. Given an initial vector of design variables $\bar{X}_o$, find the $\bar{X}$ vector at the (q+1) ith iteration defined as:

$$\bar{X}_m^{q+1} = X_m^q + X_m^* \bar{S}_m^q \qquad \text{(Eq. 10)}$$

where, α_m^*, is a scaler multiplier, and, $\bar{S}_m$, is a vector move direction in the design space.

Now the problem becomes one of finding the direction, S, and the move parameter, α*. Zoutendijk [Ref. 12] shows

that the direction may be found by solving the following problem.

Maximize

$$\beta \qquad \text{(Eq. 11)}$$

Subject to

$$\nabla F(\bar{X}) \cdot \bar{S} + \beta \leq 0 \qquad \text{(Eq. 12)}$$

$$G_j(\bar{X}_m) \cdot \bar{S} + \theta_j \beta \leq 0 \qquad j = 1,\ldots NAC. \qquad \text{(Eq. 13)}$$

$$\bar{S} \cdot \bar{S} \leq 1 \qquad \text{(Eq. 14)}$$

where the scalars θ_j are named as "push-off" factors.

If some of the constraints are violated, this algorithm is modified in order to find a feasible design [Ref. 13].

For the fixed geometry sub-problem, the reciprocals of the member sizes are used as design variables and approximation techniques are employed [Ref. 5]. The optimization program CONMIN [Ref. 14] is used to solve this sub-problem.

IV. NUMERICAL EXAMPLES

A. INTRODUCTION

Design of planar trusses and space towers are presented here and the corresponding numerical results are summarized to demonstrate the purpose of this study.

The examples begin with an 18-bar truss. In this case, it is shown how the optimum geometry is dependent on the constraints imposed. A single load condition was considered and the cross-sectional areas are linked.

Next, a 47-bar planar tower is designed to support a set of load conditions given in Table VI. The design is subject to constraints on the member stresses, Euler buckling, displacement and first natural frequency. Linking was imposed for symmetry in both cross-sectional area and coordinate variables.

Finally, a 25-bar space tower was designed. Constraints on stresses, Euler buckling, displacement, and frequencies were imposed. The truss was required to support two different load conditions. Symmetry for the member areas and coordinates was established.

B. CASE 1: 18-BAR PLANAR TRUSS

A cantilever truss, as shown in Fig. A.1, has been used previously as an example for the design of trusses of a

specified geometry [Ref. 7]. This structure was designed for optimum geometry subject to a single set of load conditions given in Table I. The allowable stresses are specified as

$$-20{,}000 \leq \sigma_i \leq 20{,}000 \text{ psi}$$

Young's modulus is taken as 10^7 psi and the material density ρ = 0.1 lb./cu. in. The allowable stress at which Euler buckling occurs is

$$\sigma_b = -\frac{cAE}{L^2}$$

The independent coordinate variables were taken as X3, Y3, X5, Y5, X7, U7, X9, Y9. The member areas were linked in the following groups: A1=A4=A8=A12=A16; A2=A6=A10=A14=A18; A3=A7=A11=A15; A5=A9=A13=A17. There are a total of eight independent coordinate variables and four independent area variables.

1. Case 1a

The structure was designed subject to stress constraints only. The resulting geometry is shown in Fig. 1a, and the design information is given in Table II. Weight versus iteration history is plotted in Fig. A.2. The number of analyses for this design is 59, and the execution time 2.53 seconds on an IBM 3033 computer.

2. Case 1b

Stress and Euler buckling constraints were imposed for this case. Design information is given in Table III. Figure 1b shows the final geometry layout and the weight versus iteration history is shown in Fig. A.3. The number of analyses for this design is 78 and the execution time was 3.94 seconds.

3. Case 1c

This design is based on stress, Euler buckling and displacement constraints. The latter was applied at node number 1 in the Y-direction. The results are shown in Table IV. Figure 1c presents the final layout and the weight versus iteration is plotted in Fig. A.4. The number of analyses is 91 and the execution time was 3.94 seconds.

4. Case 1d

This final case includes all the constraints mentioned before plus a constraint that the first natural frequency be greater than or equal to 3 Hz. A non-structural mass of fixed value W = 1,000 lbs was placed at node 1. Steady convergence is achieved and results are summarized in Table V. The final geometry and weight iteration history are shown in Figures 1d and A.5, respectively. The number of analyses is 96 and the execution time was 4.73 seconds.

When the mass is removed from the structure, the design fails to converge even when move limits are imposed. First, it is known by definition that the frequency is

proportional to the stiffness and is inversely proportional to the mass. Second, both the K and M matrices are functions of area and coordinate variables. Therefore, as K increases, M also increases, and the frequency is kept closed to the initial range.

C. CASE 2: 47-BAR PLANAR TOWER

The initial layout of the tower is shown in Fig. A.6. Stress constraints were imposed as well as constraints on Euler buckling, displacement, and first natural frequency:

$$-15{,}000 \leq \sigma_i \leq 20{,}000 \text{ psi} \qquad i=1,47$$

and

$$\sigma_i \geq \sigma_{bi} = -\frac{10.1\pi E A_i}{8L_i^2} \text{ psi}$$

$$\lambda \geq 12 \text{ hz.}$$

respectively. The members are assumed tubular with D/t=10. Young's modulus of 3.x 10^7 psi, and material density ℓ = .3 lbs/cu. in. Minimum allowable area of 10^{-6} in. was imposed. Symmetry about the y-axis was desired during optimization so linking of variables is necessary. Joints 15, 16, 17, 22 are kept fixed, and joints 1 and 2 allowed to move in the x-axis direction (Y=0). This gave a total of 27 area design variables and 17 coordinate variables. Nonstructural masses were attached at nodes 19 and 20, of W=500 lbs each. The results are tabulated in Tables VII through XII. The final geometry for the case where all

constraints were imposed is shown in Fig. A.7 and the iteration history is shown in Figs. A.8 through A.10. When all constraints are imposed, the design required 220 analysis and the execution time was 160.7 seconds.

D. CASE 3: 25-BAR SPACE TOWER

The 25-bar space tower shown in Fig. A.11 was designed to support two independent load conditions given in Table XIII. The allowable stresses were specified as

$$-40{,}000 \le \sigma_i \le 40{,}000 \text{ psi} \qquad i=1,25$$

Young's modulus was selected as 10^7 psi and the material density ρ = .1 lb./cu. in. The members are assumed to be turbular with a nominal diameter to thickness ratio of D/t = 100., so that the stress at which Euler buckling occurs is

$$\sigma_{bi} = - \frac{10.1\ \pi\ EA_i}{8L_1^2} \text{ psi.}$$

Symmetry with respect to both x-z plane and the y-z plane was imposed, so linking of variables were made as follows: the member areas were grouped in the following sequence: A1, A2=A3=A4=A5, A6=A7=A8=A9, A10=A11, A12=A13, A14=A15=A16=A17, A18=A19=A20=A21, and A22=A23=A24=A25. For the coordinates X4, Y4, Z4, X8, Y8 were considered as variables. The joints 1 and 2 were held fixed and joints 7 through 10 were required to lie in the x-z plane. Nonstructural masses of W=500 lbs. were attached at nodes 1 and 2, respectively. The first

natural frequency was limited to a value $\lambda \geq 16$ Hz. Results of this example are shown in Table XIV and the iteration history is shown in Fig. A.12.

V. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

The final layout dependency on constraints has been presented for the elastic design of trusses for optimal geometry. The truss may be planar or three-dimensional and may be indeterminate. Multiple load conditions were considered. The design procedure was separated as: analysis, design for fixed geometry, design for optimum geometry.

The displacement method for static analysis and the subspace iteration method for dynamic analysis were applied.

The sequential optimization based on two design subspaces present substantial advantages in the reduced number of analyses and allow the designer to keep control of the optimization process.

Several examples were considered. In every case, significant weight reduction was efficiently achieved. Also, the geometries obtained appear quite acceptable from an aesthetic as well as structural point of view.

The graphs of Weight vs. Iteration number show that an acceptable design can be achieved in few iterations.

B. RECOMMENDATIONS

The following recommendations may be of theoretical and practical value.

1. Weight due to the structure itself and other design dependent external forces can easily be taken into account and should be considered in future studies.

2. Application should be made to reasonable sized structures such as offshore towers and long span roof trusses.

3. The principles and the procedure described herein can also be used for optimal design of frames as well as other structures.

APPENDIX A

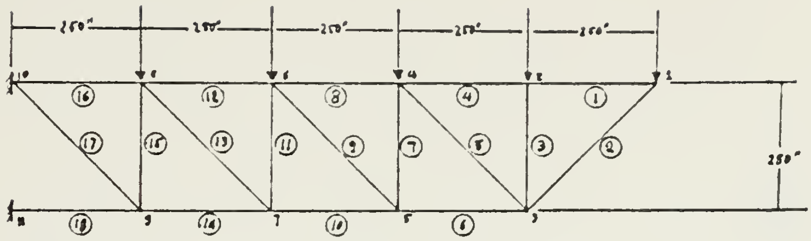
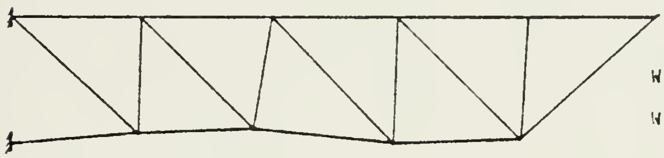


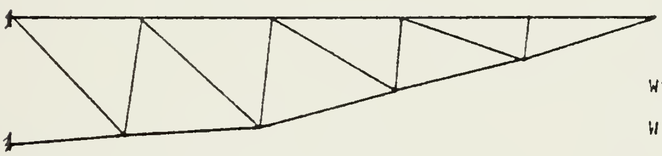
Figure 1. 18-Bar Planar Truss. Initial Geometry.



Wi= 4,780. (lbs)

Wf= 4,524. (lbs)

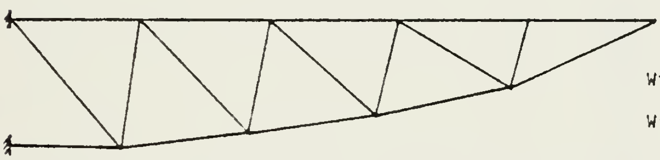
Figure 1a. Stress.



Wi= 6,430. (lbs)

Wf= 5,724. (lbs)

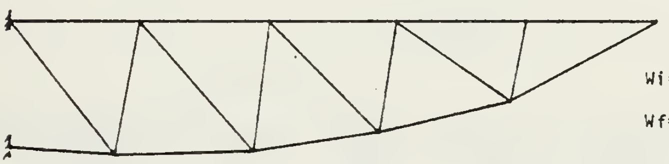
Figure 1b. Stress, Euler Buckling.



Wi= 7,413. (lbs)

Wf= 6,465. (lbs)

Figure 1c. Stress, Euler Buckling, Displacement.



Wi= 13,686. (lbs)

Wf= 11,528. (lbs)

Figure 1d. Stress, Euler Buckling, Displacement, and Frequency.

Figure A.1. 18-Bar Planar Truss. Stress, Euler Buckling, Displacement, and Frequency Constraint.

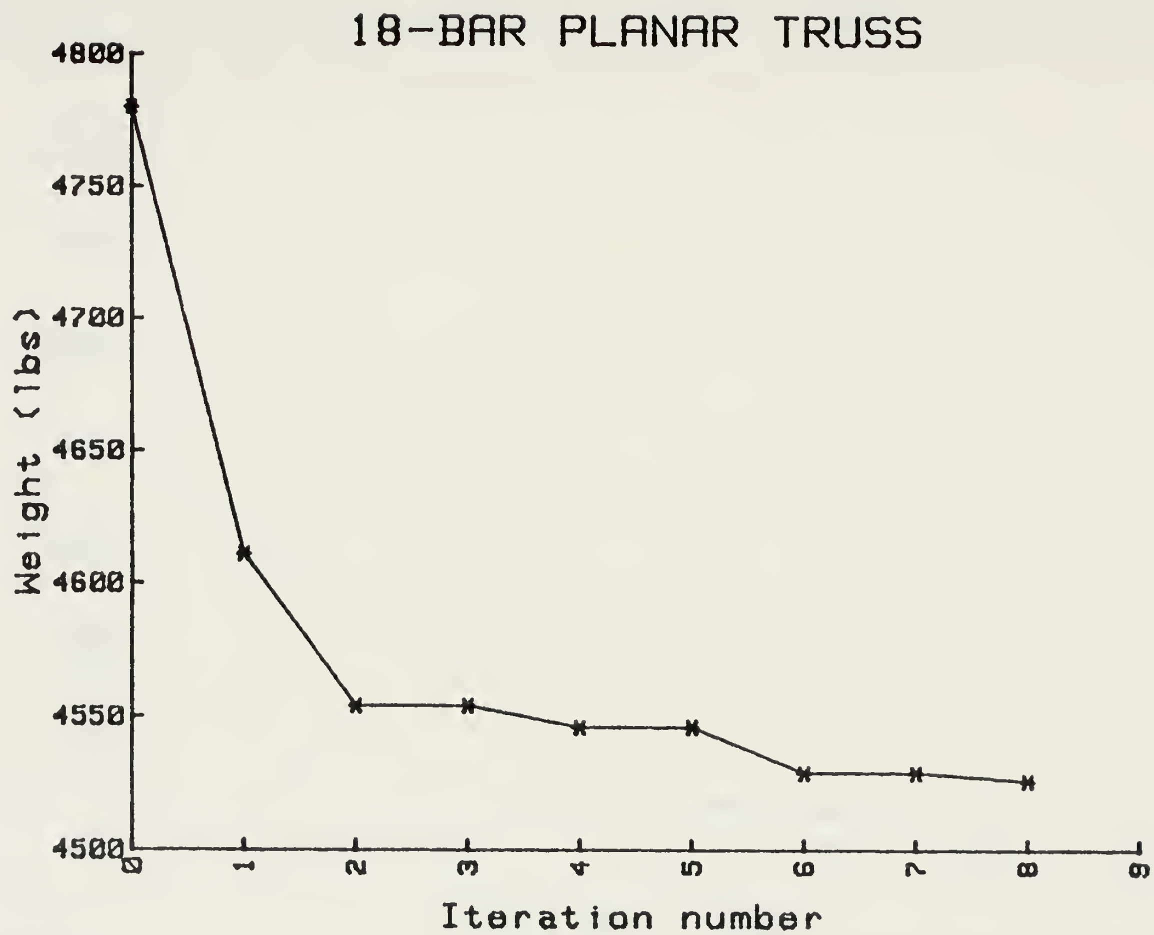


Figure A.2. 18-Bar Planar Truss. Weight vs. Iteration Number. Stress.

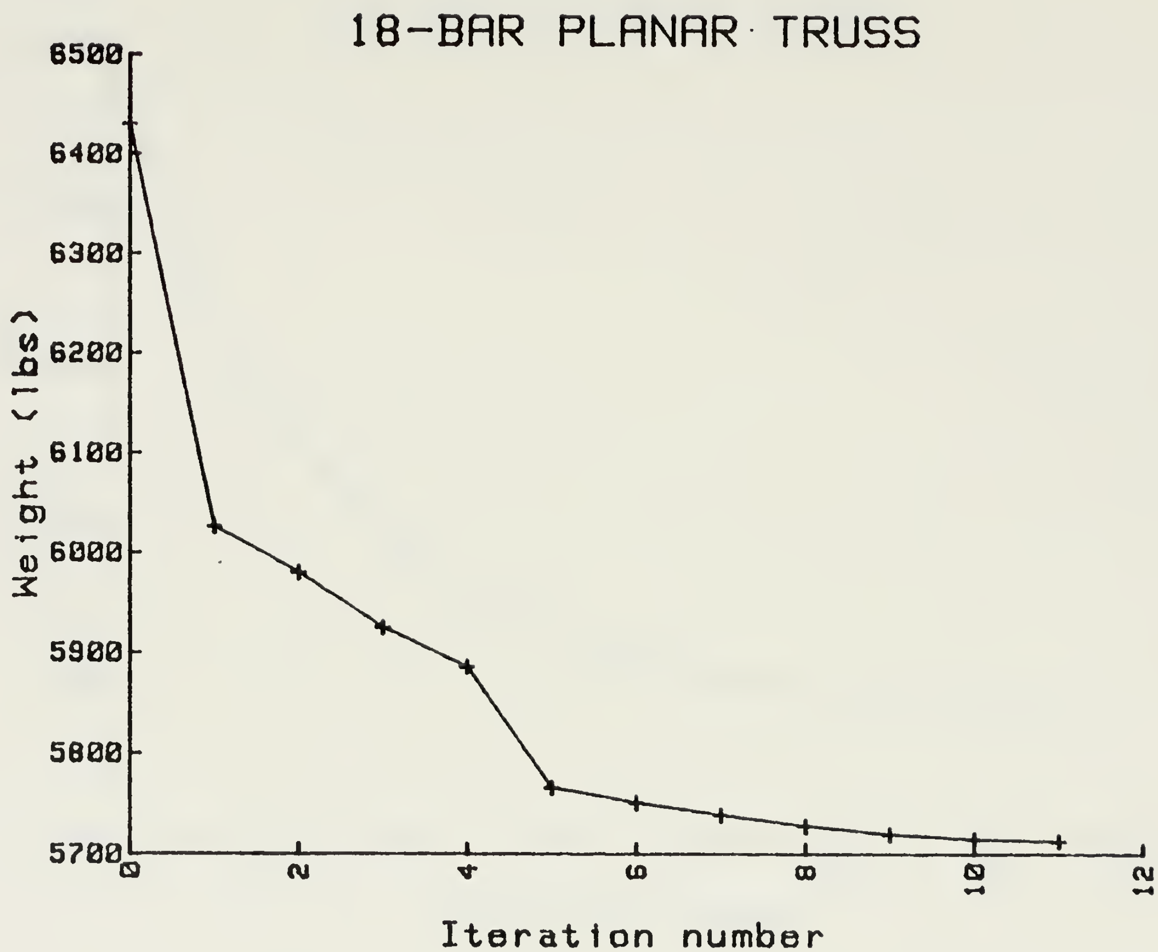


Figure A.3. 18-Bar Planar Truss. Weight vs. Iteration Number. Stress, Euler Buckling.

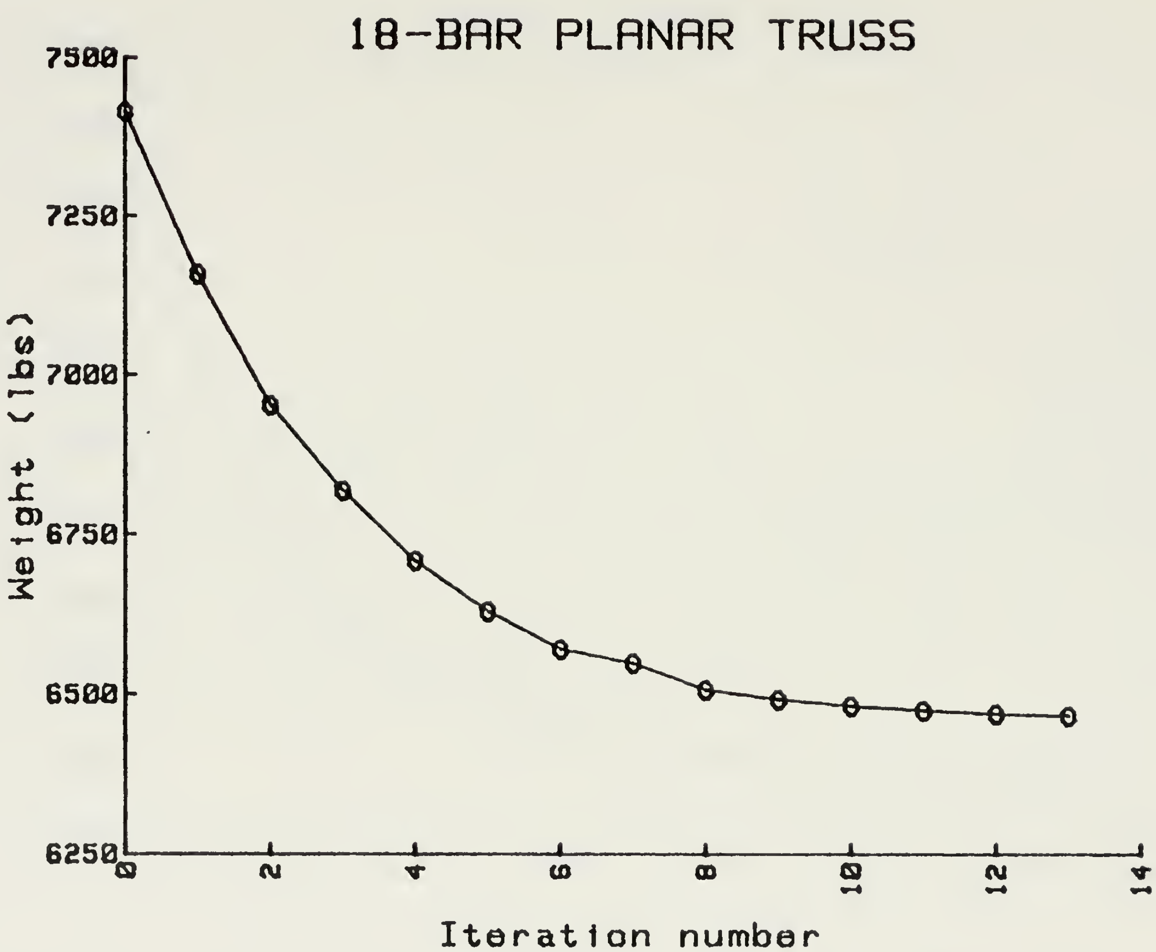


Figure A.4. 18-Bar Planar Truss. Weight vs. Iteration Number. Stress, Euler Buckling, Displacement.

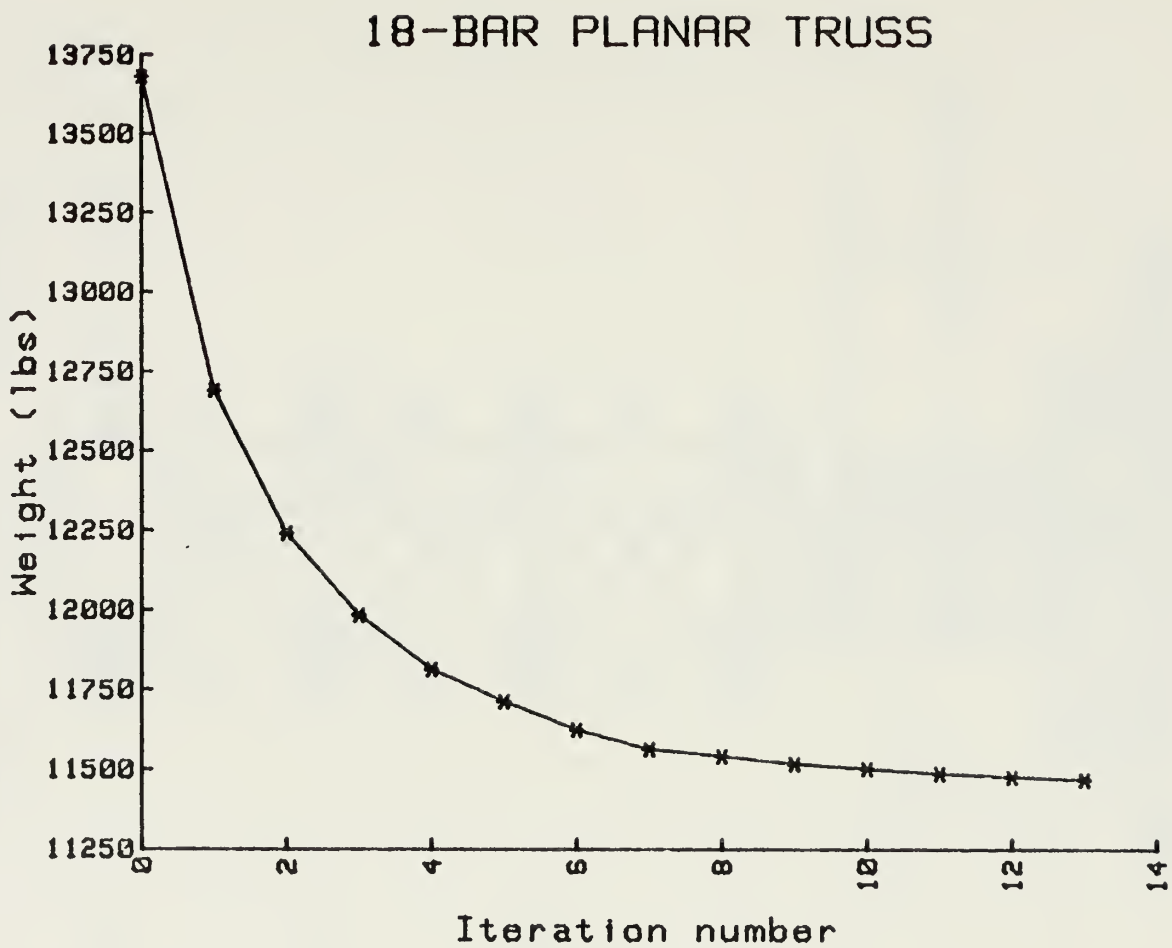


Figure A.5. 18-Bar Planar Truss. Weight vs. Iteration Number. Stress, Euler Buckling, Displacement, and Frequency.

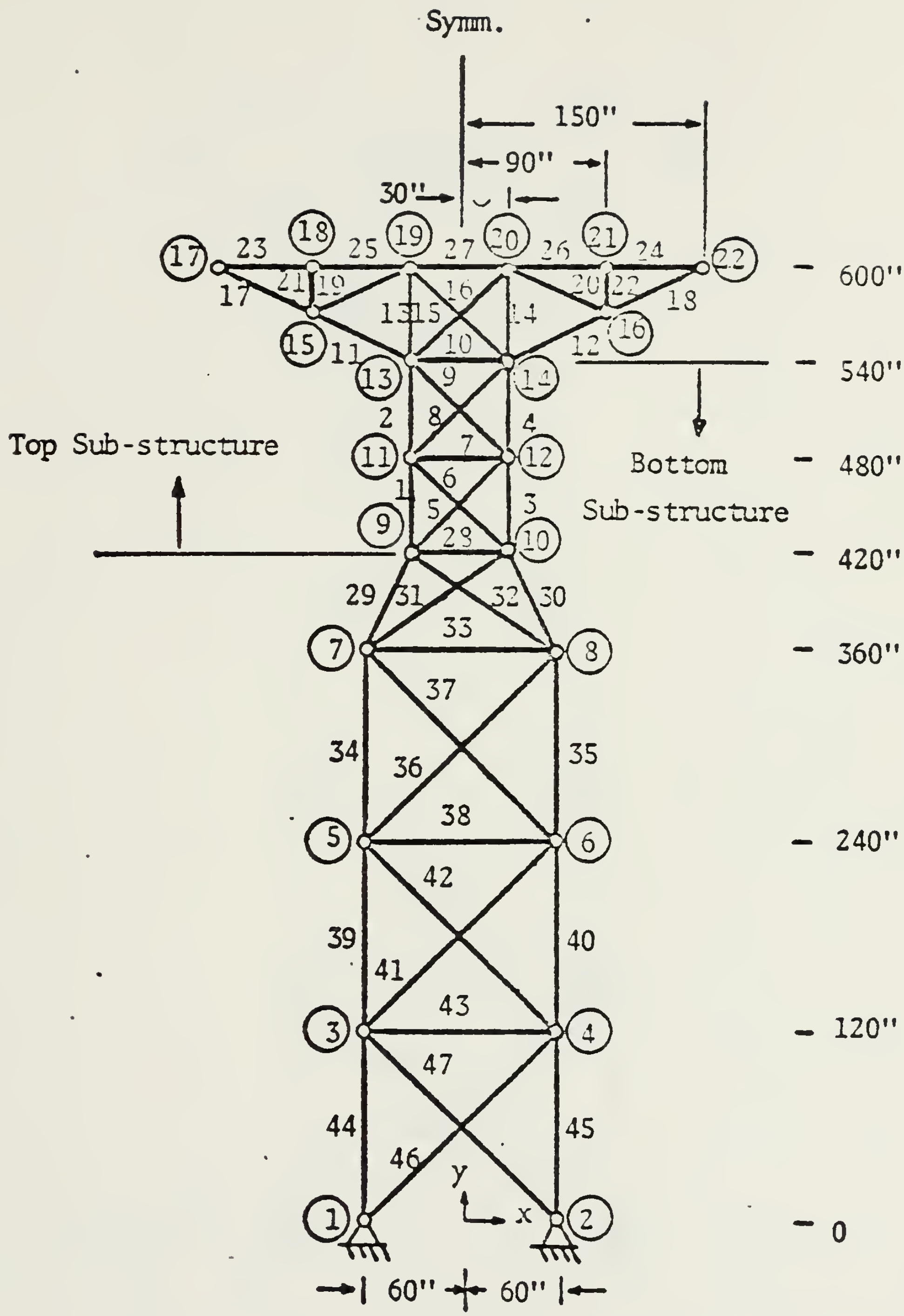


Figure A.6. 47-Bar Planar Tower.

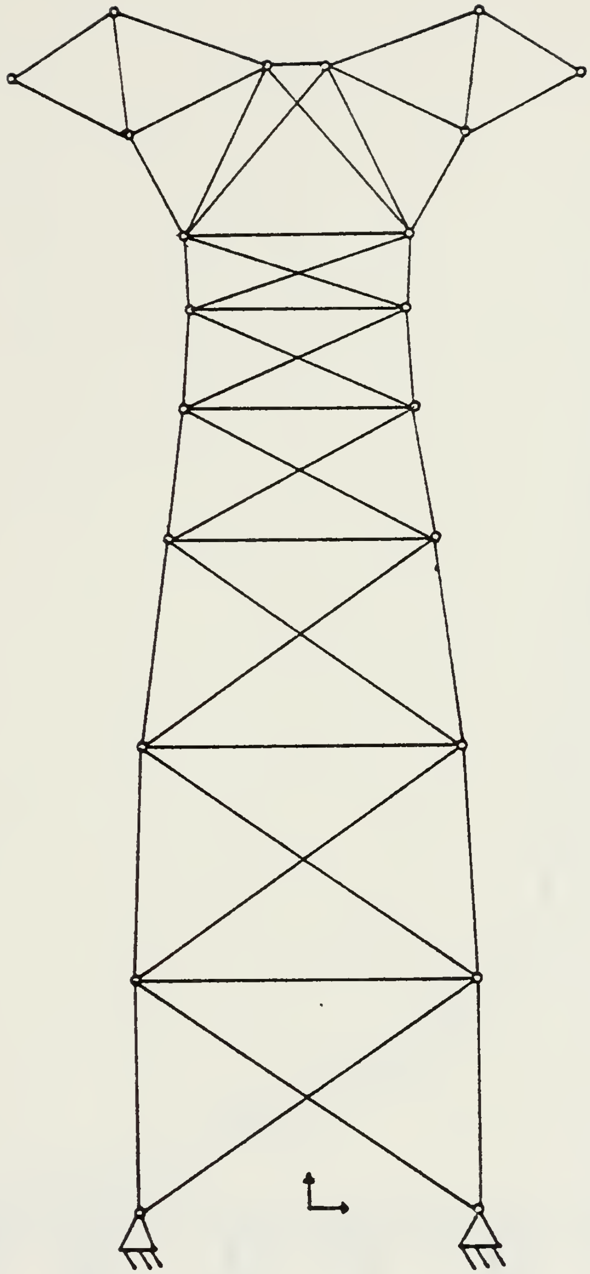


Figure A.7. 47-Bar Planar Tower. Stress, Euler Buckling, Displacement, Frequency.

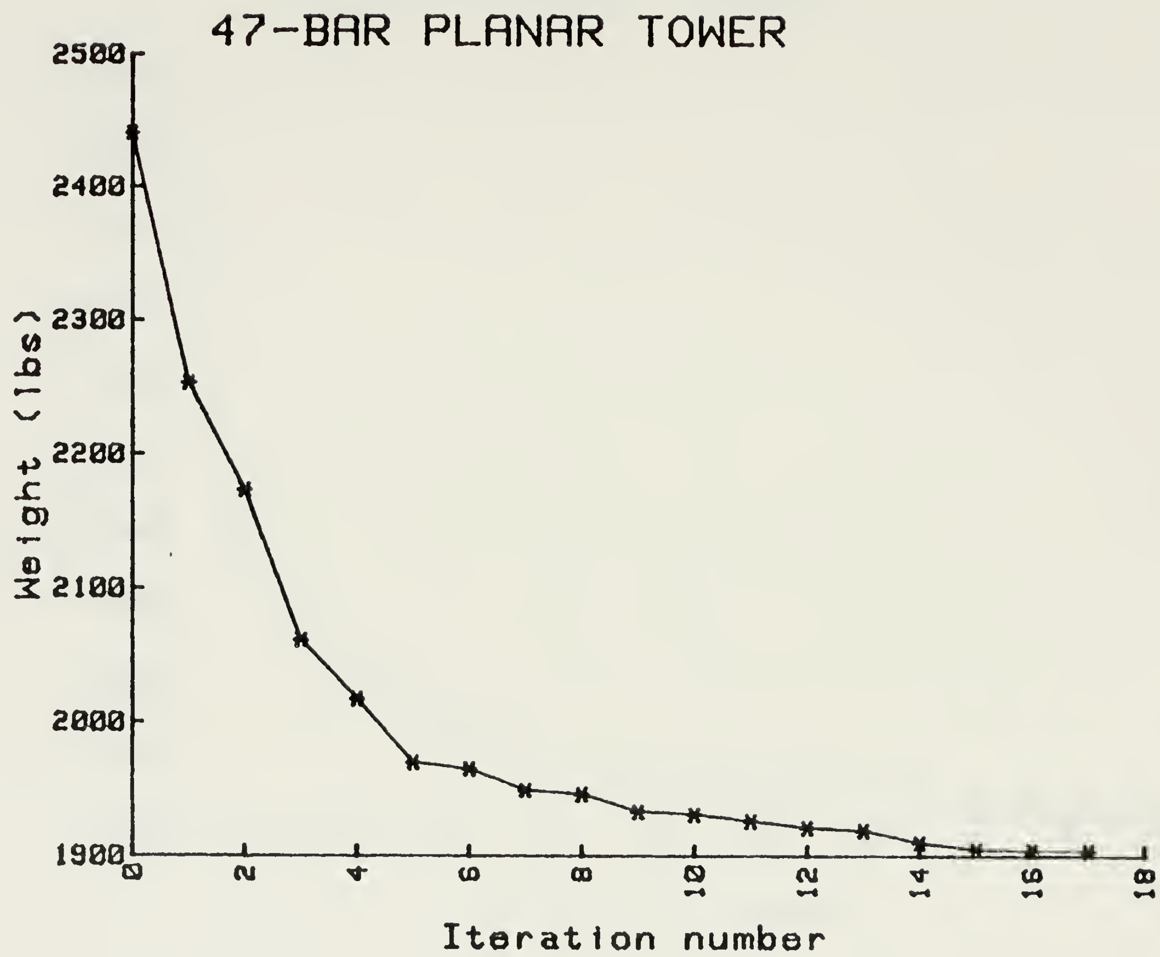


Figure A.8. 47-Bar Planar Tower. Weight vs. Iteration Number. Stress, Euler Buckling.

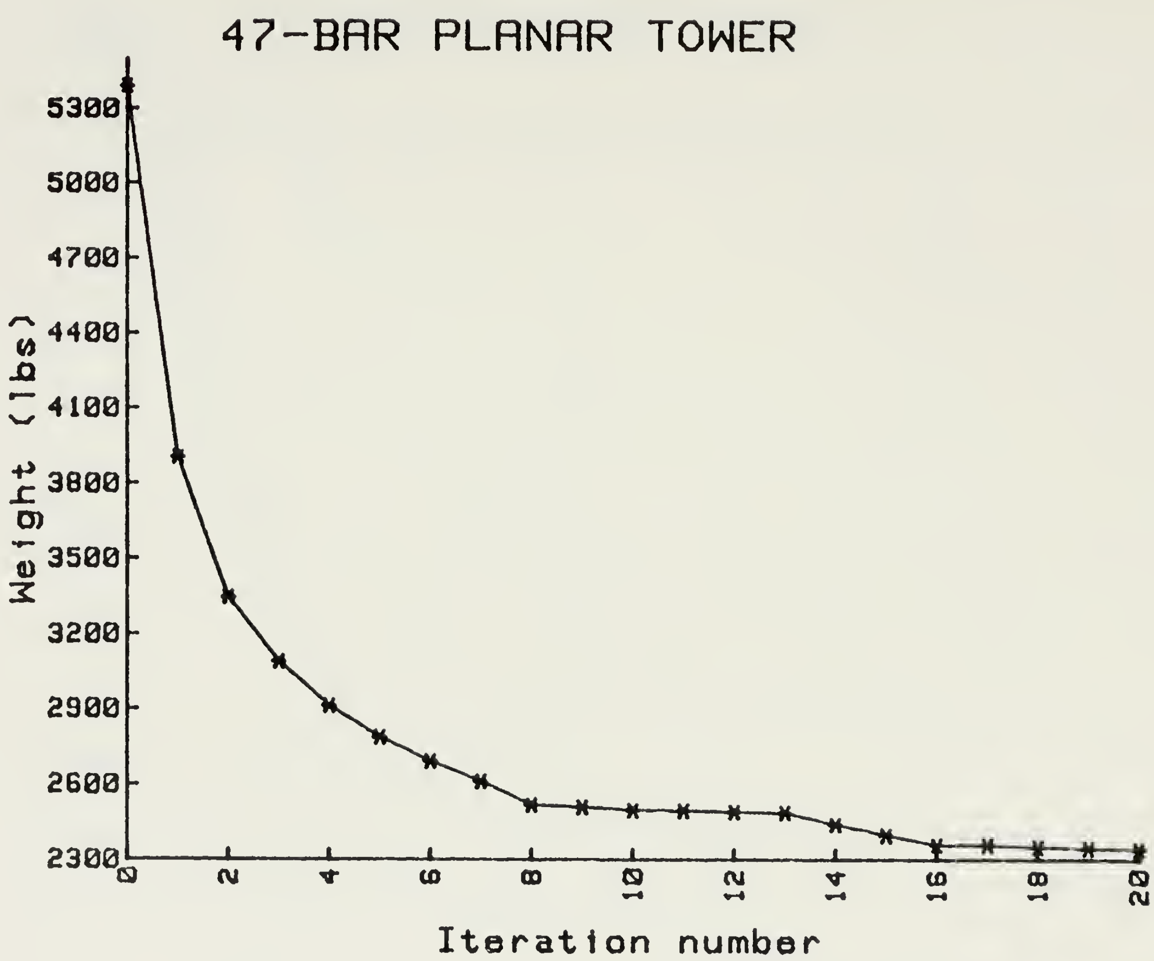


Figure A.9. 47-Bar Planar Tower. Weight vs. Iteration Number. Stress, Euler Buckling, Displacement

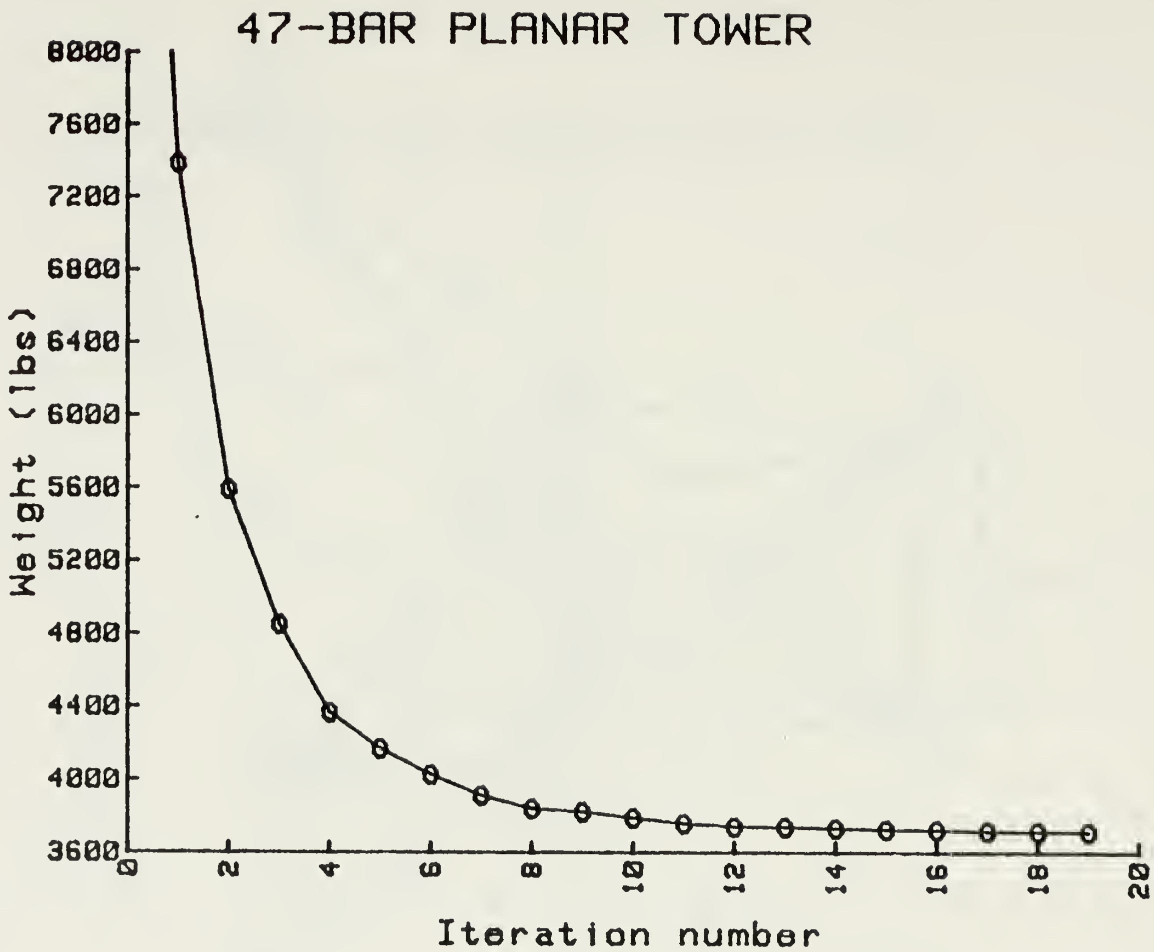


Figure A.10. 47-Bar Planar Tower. Weight vs. Iteration Number. Stress, Euler Buckling, Displacement, Frequency.

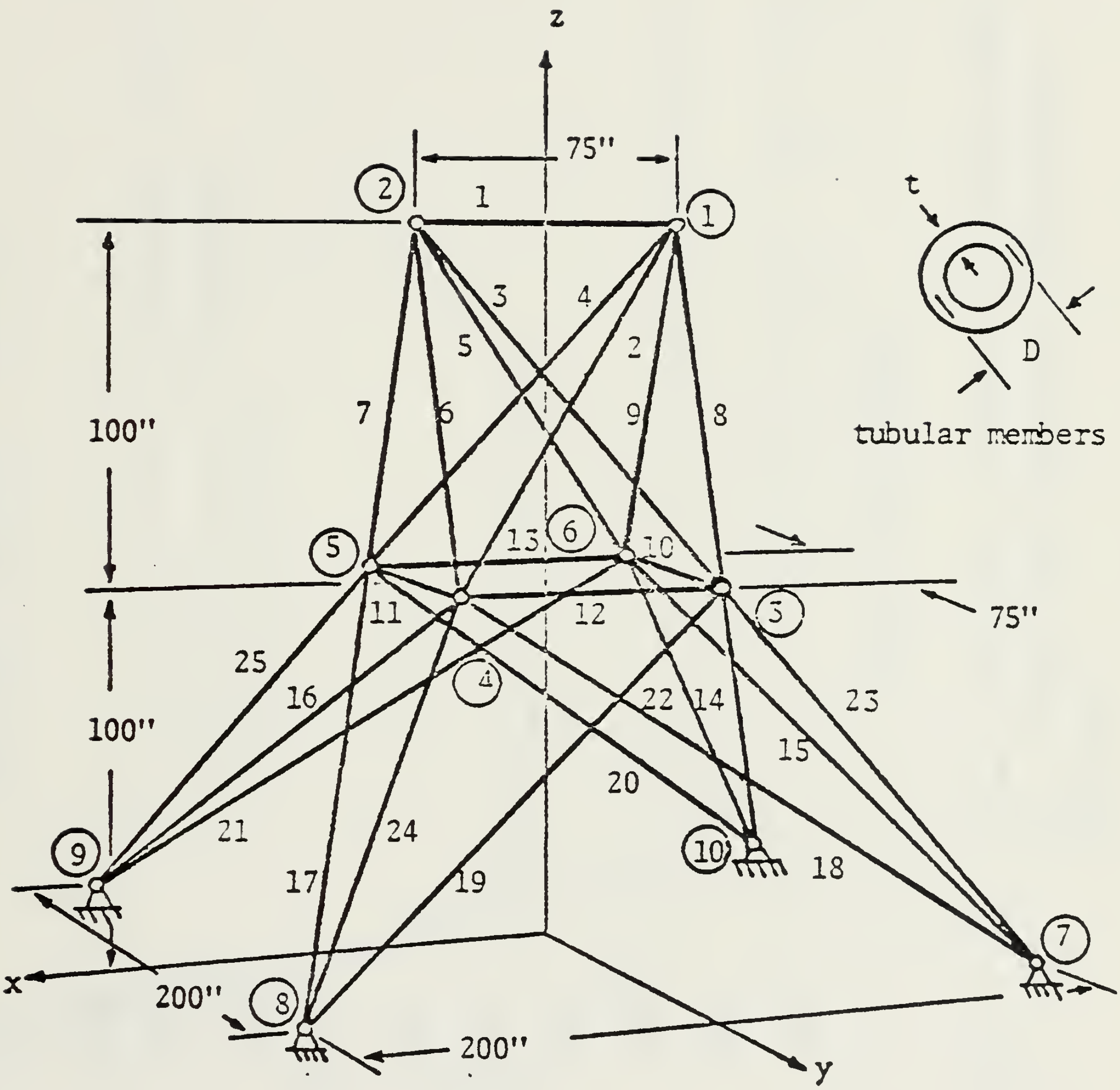


Figure A.11. 25-Bar Space Tower.

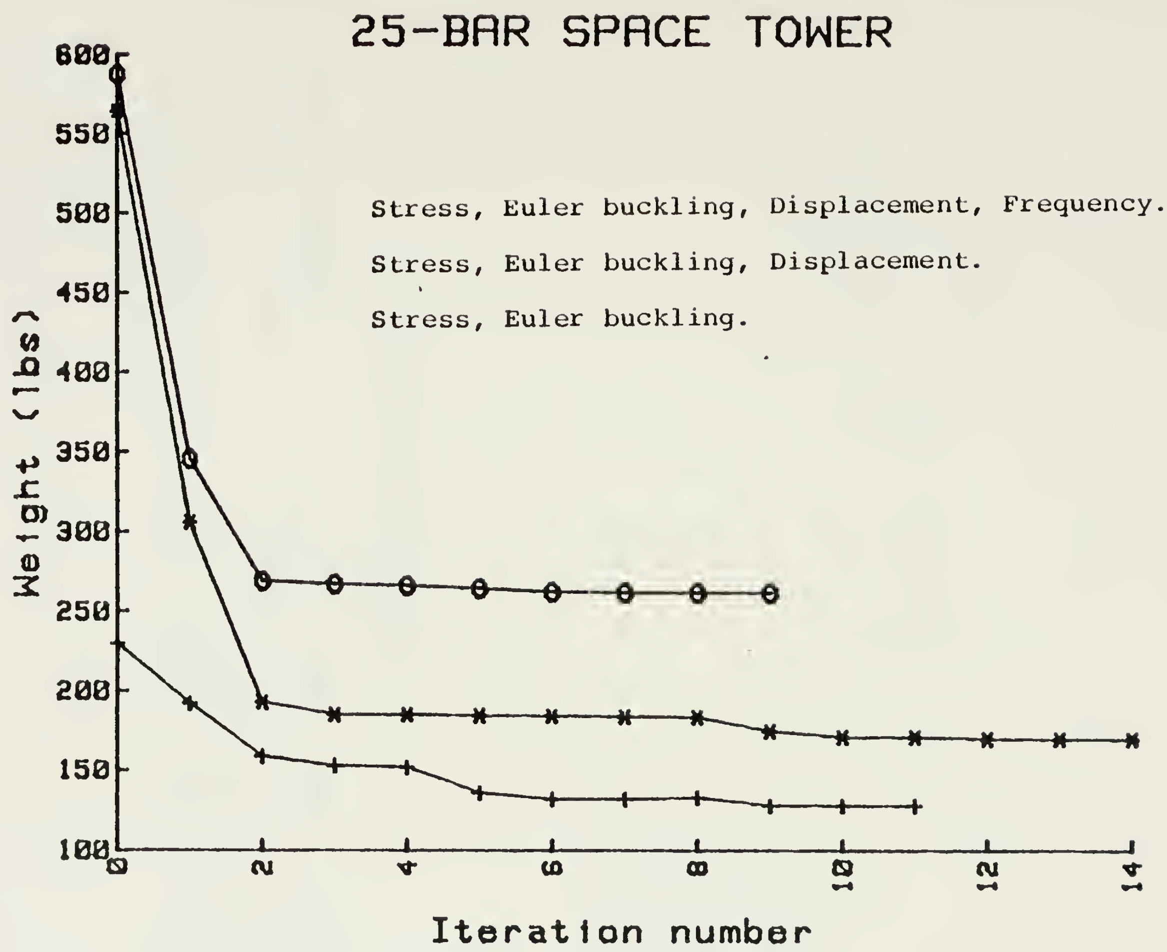


Figure A.12. 25-Bar Space Tower. Weight vs. Iteration Number. Stress, Euler Buckling, Displacement, Frequency.

TABLE I

18-Bar Planar Truss. Loads and Constants.

LOAD CONDITION (lbs.)

Joint	Fx	Fy
1	0.	-20,000.
2	0.	-20,000.
4	0.	-20,000.
6	0.	-20,000.
8	0.	-20,000.

TABLE OF CONSTANTS

Young's Modulus	E= 0.1E08	psi.	(.69E08 KN/m)
Allowable Stress	$\bar{\sigma}$ = 0.2E05	psi.	(.138E06 KN/m)
Density	ρ = 0.1	lbs/cu.in	(.276E04 Kg/m)
Buckling Coefficient	c= .4E08	psi.	(.276E09 KN/m)

TABLE II

18-Bar Planar Truss. Design Information. Stress.

A R E A * (sq.in.)

Member	Initial	Final
A1=A4=A8=A12=A16	10.0	11.05
A2=A6=A10=A14=A18	15.0	15.07
A3=A7=A11=A15	5.0	4.54
A4=A9=A13=A17	7.07	5.33

COORDINATES (in.)

Joint	Initial		Final	
	x	y	x	y
3	1000	0.0	991.17	19.686
5	750	0.0	745.88	15.12
7	500	0.0	494.61	34.538
9	250	0.0	249.54	23.609

Optimum Weight for Initial Geometry = 4,780.5 (lbs.)

Optimum Weight for Final Geometry = 4,524.7 (lbs.)

* Areas are the optimum values for the initial and final geometry.

TABLE III

18-Bar Planar Truss. Design Information. Stress, Euler Buckling.

A R E A * (sq.in.)

Member	Initial	Final
A1=A4=A8=A12=A16	10.00	11.34
A2=A6=A10=A14=A18	21.65	19.28
A3=A7=A11=A15	12.5	10.97
A4=A9=A13=A17	7.071	5.306

COORDINATES (in.)

Joint	Initial		Final	
	x	y	x	y
3	1000	0.0	994.57	162.31
5	750	0.0	747.36	102.92
7	500	0.0	482.90	32.962
9	250	0.0	221.71	17.105

Optimum Weight for Initial Geometry = 6,430.7 (lbs.)
Optimum Weight for Final Geometry = 5,713.0 (lbs.)

* Areas are the optimum values for the initial and final geometry.

TABLE IV

18-Bar Planar Truss. Design Information. Stress, Euler Buckling, Displacement.

A R E A * (sq.in.)

Member	Initial	Final
A1=A4=A8=A12=A16	14.18	16.27
A2=A6=A10=A14=A18	21.66	20.06
A3=A7=A11=A15	12.5	11.18
A4=A9=A13=A17	10.32	7.863

COORDINATES (in.)

Joint	Initial		Final	
	x	y	x	y
3	1000	0.0	962.19	109.93
5	750	0.0	703.95	55.309
7	500	0.0	452.7	27.307
9	250	0.0	208.48	- 2.884

Optimum Weight for Initial Geometry = 7,413.1 (lbs.)
Optimum Weight for Final Geometry = 6,466.9 (lbs.)

* Areas are the optimum values for the initial and final geometry.

TABLE V

18-Bar Planar Truss. Design Information. Stress, Euler Buckling, Displacement, Frequency.

A R E A * (sq.in.)

Member	Initial	Final
A1=A4=A8=A12=A16	31.68	29.66
A2=A6=A10=A14=A18	40.58	35.29
A3=A7=A11=A15	12.5	13.03
A4=A9=A13=A17	21.10	16.09

COORDINATES (in.)

Joint	Initial		Final	
	x	y	x	y
3	1000	0.0	972.52	88.0
5	750	0.0	717.08	27.84
7	500	0.0	468.75	- 3.91
9	250	0.0	200.98	-19.29

Optimun Weight for Initial Geometry = 13,686.0 (lbs.)
Optimum Weight for Final Geometry = 11,528.0 (lbs.)

* Areas are the optimum values for the initial and final geometry.

TABLE VI

47-Bar Planar Tower. Load Conditions.

LOAD CONDITION 1 (lbs.)

JOINT	Fx	Fy
17	6,000.	-14,000.
22	0.	0.

LOAD CONDITION 2 (lbs.)

Joint	Fx	Fy
17	0.	0.
22	6,000.	-14,000.

LOAD CONDITION 3 (lbs.)

Joint	Fx	Fy
17	6,000.	-14,000.
22	6,000.	-14,000.

TABLE VII

47-Bar Planar Tower. Area. Stress, Euler Buckling.

Member	Initial Area * (sq.in.)	Final Area * (sq.in.)
3	3.764	2.727
4	3.315	2.468
5	0.787	0.727
7	0.864	0.213
8	0.856	0.938
10	1.754	1.076
12	2.087	1.691
14	1.188	0.695
15	1.529	1.058
18	2.087	1.412
20	0.648	0.263
22	0.843	0.811
24	1.700	1.060
26	1.700	1.052
27	1.354	0.820
28	0.847	0.302
30	3.609	2.766
31	1.435	0.657
33	0.638	0.207
35	2.842	2.897
36	0.676	0.266
38	1.596	1.408
40	3.686	3.429
41	1.526	0.991
43	0.677	0.170
45	4.486	3.650
46	1.532	1.005

* Optimum areas for initial and final geometry.

TABLE VIII

47-Bar Planar Tower. Coordinates. Stress, Euler Bucling.

Joint	Initial Coordinate (in.)		Final Coordinate (in.)	
	x	y	x	y
2	60.	0.	85.2	0.
4	60.	120.0	73.8	119.5
6	60.	240.0	60.9	237.7
8	60.	370.0	51.5	360.0
10	30.	420.0	45.9	423.0
12	30.	480.0	42.5	480.6
14	30.	540.0	44.7	524.4
16	90.	570.0	90.0	570.0
20	30.	600.0	22.7	602.9
21	90.	600.0	91.0	620.4
22	150.	600.0	150.0	600.0

Optimum Weight for Initial Geometry = 2,440 (lbs.)

Optimal Weight for Final Geometry = 1,904 (lbs.)

TABLE IX

47-Bar Planar Tower. Coordinates. Stress, Euler Buckling, Displacement.

Joint	Initial Coordinate (in.)		Final Coordinate (in.)	
	x	y	x	y
2	60.	0.	90.0	0.
4	60.	120.0	89.78	121.36
6	60.	240.0	78.88	239.66
8	60.	360.0	67.88	357.68
10	30.	420.0	60.0	428.13
12	30.	480.0	59.09	480.59
14	30.	540.0	60.0	519.32
16	90.	570.0	90.0	570.0
20	30.	600.0	19.25	625.84
21	90.	600.0	98.03	673.17
22	150.	600.0	150.0	600.0

Optimum Weight for Initial Geometry = 7,503 (lbs.)

Optimum Weight for Final Geometry = 2,971 (lbs.)

TABLE X

47-Bar Planar Tower. Areas. Stress, Euler Buckling, Displacement.

Member	Initial Area * (sq.in.)	Final Area * (sq.in.)
3	15.28	4.478
4	16.55	3.671
5	3.066	1.037
7	2.691	0.561
8	2.435	1.156
10	3.706	1.170
12	5.263	2.318
14	4.048	0.869
15	5.077	1.385
18	4.780	1.169
20	1.650	0.562
22	2.231	1.295
24	4.665	1.526
26	4.816	1.511
27	3.693	1.036
28	2.923	0.566
30	17.91	5.231
31	7.091	0.949
33	2.566	0.558
35	17.59	6.175
36	6.640	1.217
38	3.385	1.076
40	28.11	7.730
41	6.792	1.570
43	2.701	0.555
45	37.11	9.878
46	7.363	2.153

* Optimum ares for initial and final geometry.

TABLE XI

47-Bar Planar Tower. Areas. Stress, Euler Buckling, Displacement, and Frequency.

Member	Initial Area * (sq.in.)	Final Area * (sq.in.)
3	16.59	4.825
4	15.37	4.605
5	1.321	0.938
7	1.871	0.348
8	1.356	1.081
10	5.152	1.089
12	8.593	3.025
14	5.988	1.088
15	8.468	1.691
18	8.593	1.656
20	1.416	0.635
22	1.836	1.724
24	8.470	2.480
26	8.470	2.186
27	5.999	1.342
28	1.949	0.452
30	14.54	4.833
31	7.096	0.855
33	1.709	0.307
35	9.710	4.579
36	1.414	0.368
38	1.385	1.439
40	10.43	4.396
41	1.484	1.126
43	1.465	0.270
45	11.17	4.365
46	1.482	1.664

* Optimum areas for initial and final geometry.

TABLE XII

47-Bar Planar Tower. Coordinates. Stress, Euler Buckling, Displacement, and Frequency.

Joint	Initial Coordinate (in.)		Final Coordinate (in.)	
	x	y	x	y
2	60.	0.	90.0	0.
4	60.	120.0	90.00	123.49
6	60.	240.0	83.49	244.50
8	60.	360.0	70.54	355.17
10	30.	420.0	60.0	425.02
12	30.	480.0	58.24	478.01
14	30.	540.0	59.66	519.53
16	90.	570.0	90.0	570.0
20	30.	600.0	15.00	607.63
21	90.	600.0	96.98	633.72
22	150.	600.0	150.0	600.0

Optimum Weight for Initial Geometry = 12,325 (lbs.)

Optimum Weight for Final Geometry = 3,714 (lbs.)

TABLE XIII

25-Bar Space Tower. Load Conditions.

LOAD CONDITION 1 (lbs.)

Joint	1	2	3	6
Fx	0.	0.	0.	0.
Fy	20,000	-20,000	0.	0.
Fz	-5,000	-5,000	0.	0.

LOAD CONDITION 2 (lbs.)

Joint	1	2	3	6
Fx	1,000	0.	500	500
Fy	10,000	10,000	0.	0.
Fz	-5,000	-5,000	0.	0.

TABLE XIV

25-Bar Space Tower. Design Information.

COORDINATES (in.)

Joint		Initial	Strs-Bck.	Displacement	Frequency
	x	37.5	21.487	33.512	64.913
4	y	37.5	48.271	602.83	47.271
	z	100.	100.27	114.26	128.77
8	x	100.	22.146	47.83	107.56
	y	100.	96.353	140.53	110.077

AREAS * (sq.in.)

MEMBER	STRESS-BUCKL		DISPLACEMENT		FREQUENCY	
	INITIAL	FINAL	INITIAL	FINAL	INITIAL	FINAL
1	0.9E-3	0.013	0.668	0.029	0.548	0.070
2	0.782	0.414	2.032	0.450	2.533	0.738
6	0.754	0.842	3.100	1.219	2.635	1.209
10	0.1E-3	0.033	0.533	0.015	0.547	0.054
12	0.130	0.101	0.549	0.118	0.496	0.206
14	0.558	0.121	0.668	0.084	0.533	0.350
18	0.982	0.739	1.592	0.751	1.449	0.720
22	0.801	0.554	2.653	0.668	3.363	1.130

	Stress-Buckl.		Displacement		Frequency	
	Initial	Final	Initial	Fianal	Initial	Final
Weight (lbs.)	229.52	128.55	565.2	169.9	587.73	261.52
Iteration		12		44		9
Time (sec.)		5.38		37.14		21.09

* Optimum areas for initial and final geometry.

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