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Analysis of chemical warfare using a transient semi-Markov formulation.

Kierzewski, Michael O.

NAVAL POSTGRADUATE SCHOOL
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THESIS

ANALYSIS OF CHEMICAL WARFARE USING A TRANSIENT SEMI-MARKOV FORMULATION

by

Michael O. Kierzewski

September 1988

Thesis Co-Advisors: Samuel H. Parry
Arthur L. Schoenstadt

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Analysis of Chemical Warfare Using a Transient Semi-Markov Formulation

by

Michael O. Kierzewski
Captain, United States Army
B.S., Virginia Tech University, 1982

Submitted in partial fulfillment of the
requirements for the degree of

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ABSTRACT

This thesis proposes an analytical model to test various assumptions about conventional/chemical warfare. A unit's status in conventional/chemical combat is modeled as states in a semi-Markov chain with transient and absorbing states. The effects of differing chemical threat levels, availability of decontamination assets and assumed personnel degradation rates on expected unit life and capabilities are tested. The model results indicate a possible optimal mix of conventional and chemical weapons. Also the availability of decontamination assets affects expected unit life more than decisions as to when to decontaminate a unit.

THESIS DISCLAIMER

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

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I. INTRODUCTION

Purported Iraqi use of chemical agents on the city of Halabja on 4 April 1988 resulted in as many as 5,000 deaths. The agents used were reported to be hydrogen cyanide, mustard gas and possibly Sarin, a deadly nerve gas [Ref. 1].[1] This incident highlights the continuing presence of chemical weapons in the arsenals of many of the world's armies. Other than the Iran-Iraq war and possible use in the Afghanistan war, modern armies have not seen chemical warfare on a large scale since the end of WW I. With the development of new chemical agents in WW II and new weapons to deliver those agents the historical data from WW I do not reflect the probable outcome of chemical warfare today.

Information from the Iran-Iraq war serves to dramatize the possible effects of large scale chemical warfare. However, these attacks against unprotected civilians and troops do not directly translate to chemical warfare between two modern, fully equipped armies such as the United States and Warsaw Pact forces. Limited information from small

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[1] Hydrogen cyanide(HC) is the gas used in U.S. gas chambers to kill convicted criminals. Mustard gas is a vesicant which kills tissues on contact, causing severe blistering of exposed skin and destruction of mucous membranes. Sarin is an organophosphate compound which kills by disrupting the transmission of nerve impulses. Both HC and Sarin can be fatal in seconds to minutes.

conflicts or scattered use of chemical agents do not enable planners to predict the effects of large scale use of chemical agents or the tactics that would be needed to minimize the effects of such use.

In planning for the eventuality of chemical warfare the tactician's primary tool is simulation and modeling. To be useful, these models need to reflect the full extent of our chemical warfare knowledge. Such models also need to be integrated into current combat models instead of being add-on external models. Integration reflects the U.S. Army view that any future conflict will be one of conventional and nonconventional (nuclear, chemical, biological) weapons.

The purpose of this thesis is to demonstrate the use of an analytical model as an important tool for gauging the effects of chemical warfare. The particular model considered is a semi-Markov chain with transient and absorbing states. The model will be used to determine the effect of the following parameters on the survivability and capability of a unit in combat: frequency of enemy use of chemical weapons, availability/priority of decontamination assets and assumed degradation effects on personnel in a chemical environment.

II. STATEMENT OF THE PROBLEM

In some current high resolution and aggregated Army combat models the effects of chemical agents on the battlefield are portrayed, but not necessarily accurately. The parameters of interest, such as how the degradation of a unit's effectiveness due to being placed into Mission Oriented Protective Posture (MOPP) affects the outcome and pace of battle, are only now being addressed. For most of these models, testing the sensitivity of the model to the assumed chemical parameters is, at best, very difficult and time consuming. Production runs from one of these aggregated combat models can take on the order of days to weeks to run and the postprocessing necessary to determine the effect of model changes on the Measures of Effectiveness(MOEs) can take weeks to months. Less cumbersome methods are needed to test the sensitivity of battle outcome and the pace of battle to the use and effects of chemical agents.

Modeling a unit in combat in a conventional/chemical environment as a semi-Markov chain is one way to accomplish the needed sensitivity analysis. Using widely accepted high resolution input data for conventional conflict, the user can then input assumed chemical warfare effects and find a range of values that the model is extremely sensitive to or

extremely robust against. This thesis proposes such a model and demonstrates the sensitivity and MOE evaluation process using well known techniques involving Markov chains and their properties.

As an analytical model, this semi-Markov model has several advantages. Most importantly there is no need for replications to reduce variance in the results. Once the transition probabilities, transition times and starting state are specified, the results of the model (expected unit lifetime, probability of absorption, etc.) are uniquely determined. Also, the solution techniques for this class of model are widely known and the computations are relatively fast and simple. However, the output from this model must be interpreted with the following caveats. First, the model is very scenario specific and may require simulation model input for the transition probabilities and times. Second, the model is not intended as a replacement for time step simulations but rather as a tool for sensitivity analysis for parameters that may be later used in a simulation model.

This thesis will use a semi-Markov model to investigate how the frequency of enemy chemical usage, availability/priority of decontamination assets and assumed personnel degradation rates effect the survivability and capability of a unit in a conventional/chemical battle.

III. METHODOLOGY

The following definition of a Markov process is extracted from Introduction to Probability Models by Sheldon M. Ross.

> ...consider a stochastic process $\{X_n, n = 0,1,2,...\}$ that takes on a finite or countable number of possible values. Unless otherwise mentioned, this set of possible values of the process will be denoted by the set of nonnegative integers $\{0,1,2,...\}$. If $X_n = i$, then the process is said to be in state i at time n. We suppose that whenever the process is in state i, there is a fixed probability p_{ij} that it will next be in state j. That is we suppose that
>
> $$P\{X_{n+1}=j \mid X_n= i,\ X_{n-1}=i_{n-1},...,X_1=i_1,\ X_0=i_0\} = p_{ij} \quad (1.1)$$
>
> for all states $i_0,i_1,...,i_{n-1},i,j$ and all $n \geq 0$. Such a stochastic process is known as a *Markov chain*. Equation (1.1) may be interpreted as stating that, for a Markov chain, the conditional distribution of any future state X_{n+1} given the past states $X_0, X_1,...,X_{n-1}$ and the present state X_n, is independent of the past states and depends only on the present state. [Ref. 2:p. 132]

Note that for a Markov chain the transition times are all one time step. The definition above specifically refers to a Markov chain with stationary probabilities (i.e., the probability of transitioning from state i to state j is not a function of time).

For a process described by a Markov chain with stationary probabilities, we can answer three questions. First, if there are absorbing states (states which the process can transition into but can never leave) what is the probability that the

process is ultimately absorbed in state j given the process started in state i? Second, what is the expected time until the process first enters an absorbing state? Third, what is the expected number of visits the process makes to each state prior to absorption?

The following method for calculating the quantities of interest comes from An Introduction to Stochastic Modeling by Taylor and Karlin. This method of calculating the absorption probabilities uses matrix algebra and gives an intermediate matrix **W** called the fundamental matrix that can be used to determine the mean number of times a state is visited before absorption occurs and the mean time spent in each state prior to absorption. $p_{ij}^{(n)}$, referred to as the *nth* step probability, denotes the probability of being in state j after n transitions given that the process started in state i.

> Consider a Markov chain whose states are labeled 0, 1,...,N. States 0,1,...,r-1 are transient in that $p_{ij}^{(n)} \to 0$ as $n \to \infty$ for $0 \leq i, j < r$, while states r,...,N are absorbing, or trap, and here $p_{ii} = 1$ for $r \leq i \leq N$. The transition matrix has the form
>
> $$\mathbf{P} = \begin{bmatrix} \mathbf{Q} & \mathbf{R} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \qquad (1)$$
>
> where **0** is an (N - r + 1) matrix all of whose components are zero, **I** is an (N - r + 1) X (N - r + 1) identity matrix and $q_{ij} = p_{ij}$ for $0 \leq i, j < r$. [Ref. 3:p. 116]

The **R** matrix in (1) has a row for every transient state and a column for every absorbing state in the chain. The individual entries, r_{ij}, are the probabilities of transitioning from the transient state i into the jth absorbing state in one time step and are given by $r_{ij} = p_{ij}$ for $0 \leq i, j \geq r$ (r being the state label of the first absorbing state as defined in the previous quote from Taylor).

Once the transition matrix is expressed in this format the fundamental matrix is defined by:

$$\mathbf{W} = (\mathbf{I} - \mathbf{Q})^{-1} \tag{2}$$

where **I** is an (r x r) identity matrix the same size as **Q**.

Taylor shows that the values w_{ij} are the expected number of visits to state j before absorption given that the Markov chain initially started in state i [Ref. 3:pp. 117-118]. If the starting state of the chain was a distribution of states, then the expected number of visits for any state j can be calculated by:

$$\mathbf{V} = \mathbf{W}\,\mathbf{P}_{initial} \tag{3}$$

where $\mathbf{P}_{initial}$ is a column vector of the probabilities that the initial state was state i and **V** is a column vector of the

number of visits for each state given the initial distribution $\mathbf{P}_{initial}$.

Now using the matrix **W** the absorption probabilities are calculated by:

$$\mathbf{U} = \mathbf{W}\,\mathbf{R} \tag{4}$$

using the **R** matrix defined in the **P** matrix in (1).

The resulting **U** matrix has a row for every nonabsorbing state in the chain and a column for every absorbing state. The entries u_{ij} are the probabilities of absorption in state j given that the chain started in state i. [Ref. 3:p. 119]

All of the above calculations are derived from the pure Markov chain but can be used for a more general class of stochastic models called a semi-Markov chain. The distinguishing feature of a semi-Markov chain is that the process can sojourn in the state i for a random time with a mean of μ_i before transitioning to some state j [Ref. 2:p. 292]. Just as with the transition probabilities, the mean sojourn time, μ_i, is independent of how the process reached the state i. For the particular model considered in this thesis, the sojourn time is a degenerate random variable that takes on a constant value with probability one. Transition times are assumed to be a function of both the state the process is leaving and the state it is transitioning to, being

constant for given i and j. Thus the μ_i for a state i is given by:

$$\mu_i = \sum_{j=0}^{N} p_{ij} t_{ij} \tag{5}$$

where the t_{ij} is the constant transition time from state i to state j. The matrix **T** will denote the transition time matrix whose entries are the individual transition times t_{ij}.

Note that the absorption probabilities (4) and expected number of visits per state prior to absorption (1) are functions only of the **P** matrix for the chain. Thus a Markov and a semi-Markov process that have the same **P** matrix, will have identical **W** and **U** matrices. However, the expected time until absorption and the time spent in a state prior to absorption will not be equal for the two processes.

For a Markov process all transition times are one time step; therefore, the time spent in any state is exactly equal to the number of visits to that state. For an initial state i the expected time until absorption is then the sum across the ith row of the **W** matrix (6).

$$(t_{abs})_i = \sum_{j=0}^{N} w_{ij} \tag{6}$$

For a semi-Markov chain the expected times until absorption are given by:

$$\mathbf{T}_{abs} = \mathbf{W}\,\mu \tag{7}$$

where μ is a column vector of the expected sojourn times for each state, $\mathbf{W}$ is the fundamental matrix and $\mathbf{T}_{abs}$ is a column vector of the expected times to absorption given the process starting state. For a semi-Markov chain the expected time prior to absorption spent in a particular state j given the process started in state i is given by $w_{ij}\mu_j$.

The following numerical example demonstrates the above calculations. We assume a semi-Markov process with five states {0,1,2,3,4} where the state space diagram is given by Figure 1. The numbered circles represent the states in which the process can exist. An arrow from state i to state j denotes a positive probability of transitioning from state i into state j. The numbers above the arrows are the transition probabilities and the numbers below the arrows are the transition times.

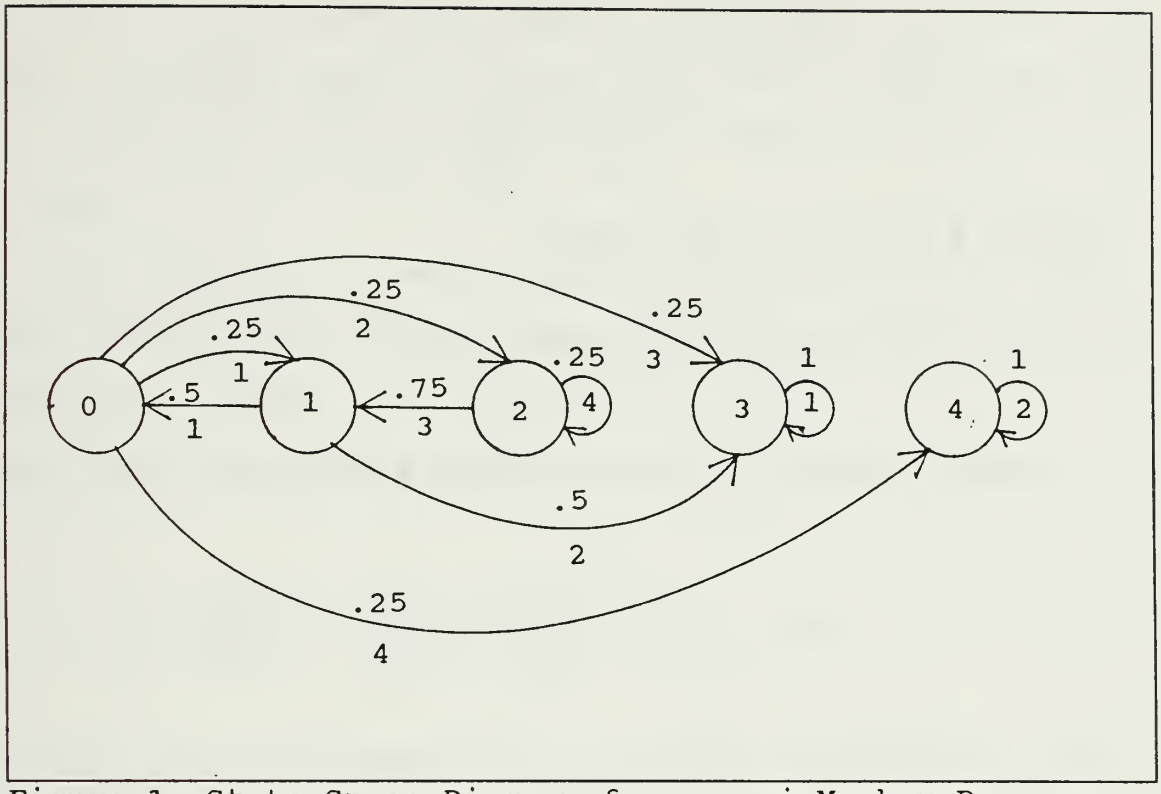

Figure 1 State Space Diagram for a semi-Markov Process.

The one-step transition probability matrix for this process is given below.

$$P = \begin{bmatrix} 0 & .25 & .25 & .25 & .25 \\ 0.5 & 0 & 0 & .5 & 0 \\ 0 & .75 & .25 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

This semi-Markov process also has a transition time matrix T shown below.

$$T = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 0 & 0 & 2 & 0 \\ 0 & 3 & 4 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

For this example the **Q** matrix becomes

$$\mathbf{Q} = \begin{bmatrix} 0 & .25 & .25 \\ .5 & 0 & 0 \\ 0 & .75 & .25 \end{bmatrix}$$

and the **R** matrix is shown below.

$$\mathbf{R} = \begin{bmatrix} .25 & .25 \\ .5 & 0 \\ 0 & 0 \end{bmatrix}$$

Using equation (2) the resulting **W** matrix is shown below.

$$\mathbf{W} = \begin{bmatrix} 1.333 & 0.667 & 0.444 \\ 0.667 & 1.333 & 0.222 \\ 0.667 & 1.333 & 1.556 \end{bmatrix}$$

This "fundamental" matrix is the number of visits prior to absorption. The absorption probabilities and the expected time to absorption for each initial state are listed in Table I below.

Table I. SAMPLE ABSORPTION TIMES AND PROBABILITIES.

Initial State	Probability of absorption in State 3	Probability of absorption in State 4	Expected time to absorption
0	0.667	0.333	5.778
1	0.833	0.167	4.389
2	0.833	0.167	8.722
3	1.000	0	0
4	0	1.000	0

Note that the $\mathbf{U}$ matrix defined by (4) is the first three rows of columns 2 and 3 in Table I and the $\mathbf{T}_{abs}$ vector defined by equation (7) is the first three entries in the last column of Table I.

The solutions to the example problem and to the full model were obtained using the programming language APL on an AST personal computer.

IV. MODEL FORMULATION

A. BASIC CONCEPT

In the "real world" a unit in combat is assigned missions and succeeds or fails on the basis of the factors given by the acronym METT-T. METT-T is an abbreviation for the following:

1. Mission: What is the unit's current mission and what do we want the unit to do in the future?.

2. Equipment: What is the quantity and condition of the unit's equipment? Is the on hand equipment sufficient for some missions and not for others?

3. Troops: Analogous to the question for equipment, what is the quantity and condition of the unit's personnel? Is the unit at full strength for personnel or at twenty-five percent strength for personnel? Included in this category are morale, discipline and leadership, which are very hard to quantify.

4. Terrain: What is the terrain in the unit's area of operations and how does this terrain affect the unit in the performance of its mission? Also if a unit must move to another area of operations, how is the new or intervening terrain expected to affect the unit's operations? Terrain in this instance is defined to include the weather and its effects on the battle.

5. Time: How much time is available for the planning and execution of the mission?

All of these questions are considered when a real commander is either deciding on a mission for his unit or is assessing his unit's probability of mission success given a specific mission. A commander also considers these factors for the enemy he is facing. For a combat scenario, if the enemy threat, terrain and overall unit mission remain relatively constant, the commander is making decisions based on his unit's "state" assuming adequate time. This unit state is determined by the unit's equipment and personnel strengths and the unit's current activity. For example, a unit that is at 20% strength for both personnel and equipment will not be sent back into battle if at all possible but will more likely go to the rear for restorative processes such as resupply and refit. If, however, the tactical situation forces the commander to recommit this unit without restoration, his unit has a high probability of mission failure.

Note that in "real world" combat a unit's state expressed in terms of personnel and equipment strengths does not present the entire picture. A unit that has suffered sustained attrition and is at 50% personnel is likely to be more effective than a unit that is at 50% personnel due to a catastrophic loss. In the former case the unit's chain of command has had time to adapt to the loss of personnel while in the later case the sudden shock has probably disrupted the unit's chain of command and lowered morale more than in the

former case. However these effects are beyond the scope of most models to credibly capture.

B. SEMI-MARKOV CHAIN APPROACH

One approach to modeling a unit's transitions from state to state is to use a semi-Markov chain. In the "real world" units start at various strength levels and cycle through processes that lower strength (combat) and processes that raise strength (decontamination, resupply/refit). Ultimately the battle is over or the units leave the battle due to being ineffective or being rotated to another battle area. A semi-Markov chain can capture this change in status and subsequent absorption. Such an approach assumes the following:

1. The unit can be in specified states and can transition between these states.

2. These unit states can be described in sufficient detail so that the next possible states a unit can transition into can be determined solely from the unit's current state description. This implies that the process is memoryless (i.e., how a unit arrived at a state is not important).

3. It is desirable to have these transition probabilities remain stationary (i.e., invariant over time) so that every time a unit enters a given state its probability of entering any other state is the same.

Note that the memoryless property fails to consider the shock effect, discussed in paragraph A. above. This is a shortcoming of the current model but could be overcome by expanding the state space to include an indicator that tells how a unit arrived at the current state. Units with a catastrophic loss indicator would have different transition probabilities than units with a gradual loss indicator.

C. DEFINITION OF TERMS

To be useful the state description must be detailed enough to allow all of a unit's next transitions to be determined based solely on the current state and broad enough so that the size of the state space does not become unmanageable. For this model, the state description covers three basic areas: the current unit activity, the current unit strength and the current unit environment. The terms used to describe each of these areas are detailed in the following sections.

1. Current Unit Activity

This model considers four unit activities: ENGAGED, DISENGAGED, chemical decontamination (DECON) and resupply/refit (REFIT). ENGAGED describes a unit involved in a generic direct/indirect fire battle. DISENGAGED describes a unit that is in an assembly area or in transit. A unit in DECON is being cleaned of chemical contamination. A unit that is being resupplied or having maintenance performed on its equipment is in REFIT.

2. Current Unit Strength

This model considers two factors that determine a unit's ability to fight and survive. The first factor is the amount of personnel, equipment and supplies the unit has on hand. This factor is a measure of the unit's MTOE (Modified Table of Organization and Equipment) strength, and is called a unit's MTOE factor. It takes on values of HIGH (minimal attrition losses to personnel and equipment), MEDIUM (moderate attrition losses to personnel and equipment) and LOW (severe attrition losses to personnel and equipment). The second factor further describes the condition of the unit's personnel and is called the PERSEFFECT (personnel effectiveness) factor. PERSEFFECT refers to the ability of the unit's personnel to perform their assigned tasks and is a function of the unit's exposure to chemical attacks. This factor has the values of HIGH (no degradation, personnel are totally effective) and LOW (personnel have been seriously degraded by exposure to chemical attacks). Determination of these categorical values is discussed in detail in section E.

3. Current Unit Environment

This refers specifically to the presence or absence of chemical weapons and protective gear. The three values for this indicator are CONVENTIONAL, CHEMICAL-MOPP (Mission Oriented Protective Posture) and CHEMICAL-NO MOPP. CONVENTIONAL refers to a state in which the unit is not

currently exposed to chemical hazards. A unit in CHEMICAL-MOPP is exposed to chemical hazards but has the required chemical protective gear to forestall loss of life strictly due to chemicals. CHEMICAL-NO MOPP refers to a unit exposed to chemical hazards that either does not have chemical protective gear or whose personnel have not yet donned their protective gear.

D. MODEL STATE SPACE

1. Number and Type of States

Using the terms defined in section C. the model consists of 48 states numbered (0-47) with 46 transient states and two absorbing states. In actual combat units are rarely absorbed unless captured or destroyed by catastrophic losses. Replacements and reconstitution of weak units result in units surviving as entities even though all or most of the original personnel and equipment are gone. For the purposes of this model absorption means that the unit must go to the next higher level of restorative processes and is lost to the current level for a long enough time to be considered absorbed. The two absorbing states are COMBAT INEFFECTIVE-ATTRITION (unit has lost enough MTOE equipment/personnel to be totally combat ineffective) and COMBAT INEFFECTIVE-DEGRADATION (personnel are so degraded as to be totally ineffective). Appendix A contains the complete listing of the model state space.

2. Basic Model Assumptions

Appendix B contains the model connectivity matrix that shows all the allowable one-step transitions. The basic assumptions used to determine what are allowable transitions are listed below with explanations.

a. No Catastrophic Losses

Units do not go from HIGH levels for either MTOE or PERSEFFECT directly to absorbing states. Units will transition through a series of states where these strength indicators are lowered one level at a time. Transitions directly to absorbing states are allowed only from LOW levels of either strength indicator. A model improvement would be to use actual random sojourn times and allow for a positive probability of very short or instantaneous transition times. Such a model would allow for units to transition quickly (possibly instantaneously) from HIGH levels to INEFFECTIVE levels (absorption) thus capturing the effects of catastrophic losses.

b. Decontamination Before Refit

If a unit has contacted chemical hazards it must first be decontaminated before it is allowed to go to refit and get supplies and replacements for attrition losses. This is in keeping with current doctrine so that the supply and maintenance facilities will remain uncontaminated and efficient.

c. MTOE Losses

This model does not consider the personnel and equipment losses that occur through disease, accident and wearout while a unit is not actively engaged with the enemy. However, the MTOE factor of a unit can be lowered by any of the CHEMICAL-NO MOPP states because of the possibility of chemical casualties to personnel not in chemical protective gear.

d. Degradation While Disengaged

In contrast to MTOE losses, this model does consider the loss of personnel effectiveness while units are not in active combat. Personnel will continue to suffer degradation as long as they are exposed to chemical hazards. The rate at which personnel become degraded is faster for ENGAGED units because of the greater exertion required in battle. Also, once they have been decontaminated, personnel will continue to be degraded unless they are given time to rest and recover. This rest and recuperation is handled in the model by having longer DECON sojourn times for units that go from a LOW PERSEFFECT to a HIGH PERSEFFECT factor than for units that are cleaned up in DECON but maintain a LOW PERSEFFECT indicator.

e. MOPP is 100% Protective

Personnel exposed to chemical hazards while wearing MOPP will not be assessed casualties but the PERSEFFECT indicator may be lowered on the next transition.

f. Disengage Before/After Restoration

Units must be in a DISENGAGED state before they can transition into the restorative processes, DECON or REFIT. Also when units leave DECON or REFIT they will go to DISENGAGED states.

3. Interpretation of Transition Probabilities and Times

The individual transition probabilities, p_{ij}'s, can be considered a mix of decisions and externally imposed events. For example, if given a choice, a unit would have a higher probability of going to a disengaged state from an engaged state with both LOW strength levels, than from an engaged state at both HIGH strength levels. Thus the transition probabilities in this case reflect a conscious choice. (If the tactical scenario will allow a choice). On the other hand, the probability that a unit will receive a chemical attack as opposed to a strictly conventional weapon attack is dictated by the scenario and can be considered an external event imposed on the unit.

The individual transition times, t_{ij}'s, also have dual interpretations. The first is that the transition time is the time it takes for the **event** that causes the unit to transition from state i to state j. The transition times can also be varied by the experimenter to represent a capability or efficiency. For example, allowing a unit to remain engaged longer and still be at HIGH MTOE strength is an increased

capability while taking longer for a REFIT or DECON operation denotes a less efficient REFIT or DECON process.

For the purposes of this thesis the transition probabilities and times were determined by the author based on military judgement and the basic model assumptions discussed in section D. 2.

E. CATEGORICAL ASSESSMENTS

One of the key issues in this model is the assigning of HIGH, MEDIUM and LOW strengths for the MTOE and PERSEFFECT factors of a unit. What exactly are these levels and how do they affect the probability of a unit transitioning into other states?

A thesis by Cpt. Paul Crawford, Dynamic Study of Factors Impacting on Combat Power, addresses this issue. In his thesis Crawford considers a unit whose state is described by the percentage of personnel(PER), ammunition(AMMO), combat vehicles(VEH) and POL(Petroleum, Oil and Lubricants) on hand as opposed to the authorized unit levels for these variables. His problem was to map this state vector into a measure called the "unit effectiveness". The unit effectiveness has five levels: Totally Effective, Effective, Marginally Effective, Ineffective and Totally Ineffective. Personnel and combat vehicles were evaluated at three levels (100%, 75%, 50%) and ammunition and POL were evaluated at four levels (100%, 75%,

50%, 25%). Throughout the experiment the unit mission and threat were held constant. [Ref. 4:p. 19]

The experiment consisted of a questionnaire given to 45 Army officers who then made a categorical judgement as to the unit's effectiveness level for a given state vector. The categorical responses were then transformed to an interval scale and the different state vectors were mapped into the interval scale. Crawford determined that the final function that mapped a state vector into the interval scale and thus into a category of effectiveness was elliptical of the form given by (7):

$$Y = a^2 + b^2(PER - X)^2 + c^2(AMMO - X)^2 + d^2(VEH - X)^2 + e^2(POL - X)^2. \quad (7)$$

The best fitting model as determined by Crawford was:

$$Y = 88.978 - .0056xX1 - .0055xX2 - .0054xX3 - .0005xX4, \quad (8)$$

where:

Y = The value of unit effectiveness [interval scale],

X1 = $(PER - 100)^2$,

X2 = $(AMMO - 100)^2$,

X3 = $(VEH - 100)^2$ and

X4 = $(POL - 100)^2$. [Ref. 4:pp. 11-23]

In the semi-Markov model considered in this thesis the state variable becomes the effectiveness (categorical) for the levels of HIGH, MEDIUM, LOW and INEFFECTIVE. The MTOE effectiveness in this model is exactly analogous to the unit effectiveness considered by Crawford. The PERSEFFECT effectiveness could also be calculated with Crawford's method but the state vector used would be different. The DEGRADATION state vector would include, type of chemical agent used, temperature, exertion level of troops, protective equipment used and time spent in the protective gear.

This section has presented a method and reference for the user to determine what constitutes a HIGH, MEDIUM, LOW and INEFFECTIVE level for a unit. Even though categorical assessments are subjective, Crawford showed in his thesis that a group of decision makers can agree on a range of state vectors that map into a specified level of unit effectiveness. Also, by being able to fit a model to the results of the decision makers, Crawford showed a common systematic approach that was consciously or unconsciously used by them in their assessment. Thus if the method used for the categorical assessments, HIGH, MEDIUM, LOW and INEFFECTIVE, are explicitly stated then different users can use this model and assign subjective transition probabilities and times.

F. DEMONSTRATION OF MODEL

In this section the working of the model and the output available from the model is demonstrated. A concern throughout the model development is, whether the model behaves reasonably and gives results that are believable. Assume that a unit can start in one of the following initial states:

1. State 0: ENGAGED-CONVENTIONAL-HIGH-HIGH
2. State 9: DISENGAGED-CONVENTIONAL-MEDIUM-LOW
3. State 12: ENGAGED-CHEMICAL-MOPP-HIGH-HIGH
4. State 19: ENGAGED-CHEMICAL-NO MOPP-HIGH-LOW
5. State 35: DISENGAGED-CHEMICAL-NO MOPP-LOW-LOW
6. State 44: REFIT-LOW-HIGH

Figure 2 shows the expected time until absorption for the unit given the various initial states and transition probabilities and times given in Appendices C and D.

The unit lifetimes vary from a high of 38.8 Hrs to a low of 13.8 Hrs. The lifetimes also follow a relatively intuitive pattern with the longest expected lifetime for the unit in REFIT and the shortest expected lifetime for the unit that is in chemical combat without protective gear and already at a LOW PERSEFFECT level.

Reporting the expected lifetime is only part of the story. Also important is what was the unit capable of doing while "alive"? To gauge how a unit spent its useful life, the following classes of states were considered.

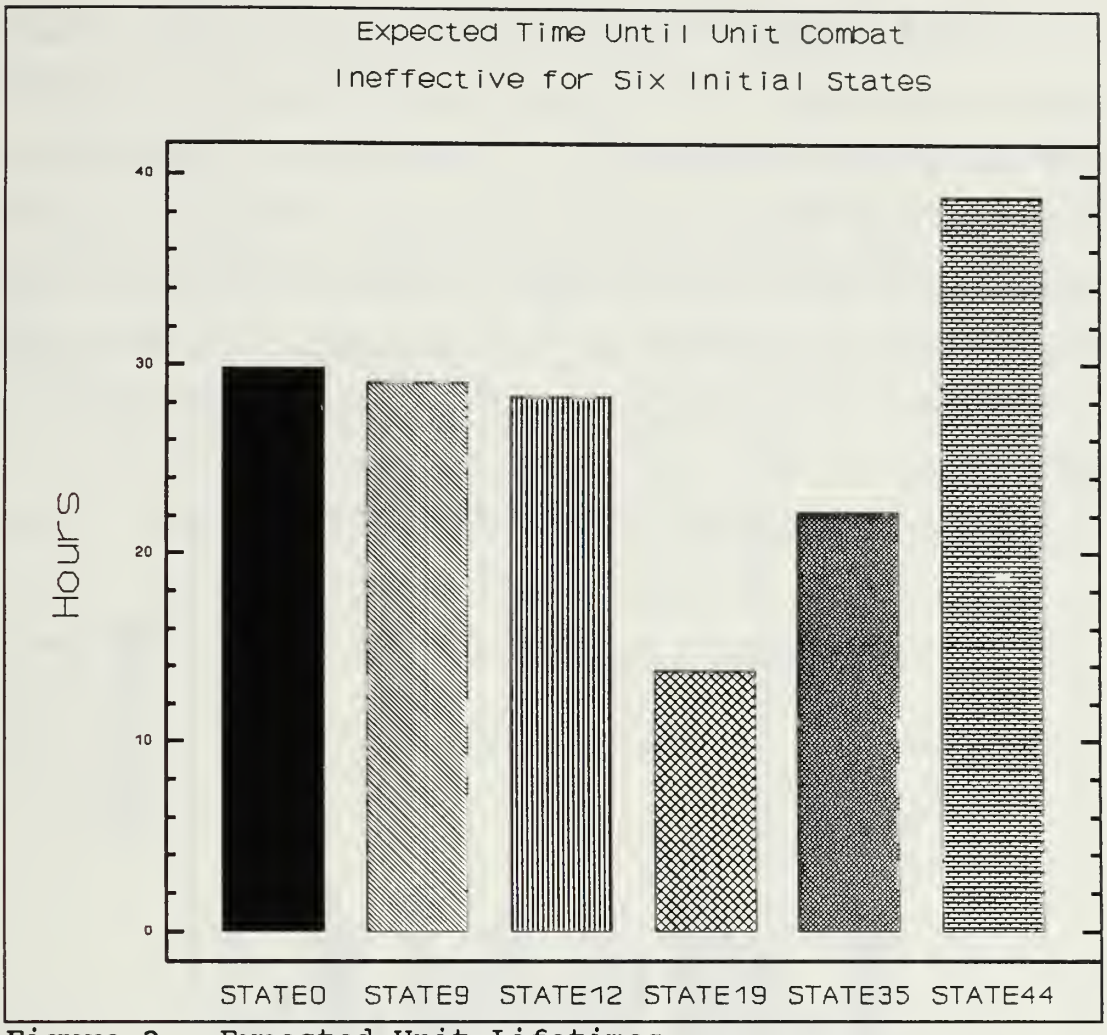

Figure 2. Expected Unit Lifetimes

1. COMCLEAN: All ENGAGED-CONVENTIONAL states, numbers 0-5.

2. COMDIRTY: All ENGAGED-CHEMICAL states, numbers 12-23.

3. DISCLEAN: All DISENGAGED-CONVENTIONAL states, numbers 6-11.

4. DISDIRTY: All DISENGAGED-CHEMICAL states, numbers 24-35.

5. DECON: All the DECON (chemical decontamination) states, numbers 36-41.

6. REFIT: All REFIT states, numbers 42-45.

For the purposes of this thesis these are the classes of states considered. Now that the expected unit lifetimes have been determined the question is, what did the units actually do before they became combat ineffective? Figure 3 shows the percentage of time the unit spent in each of the above classes of states for each of the six initial states in this example.

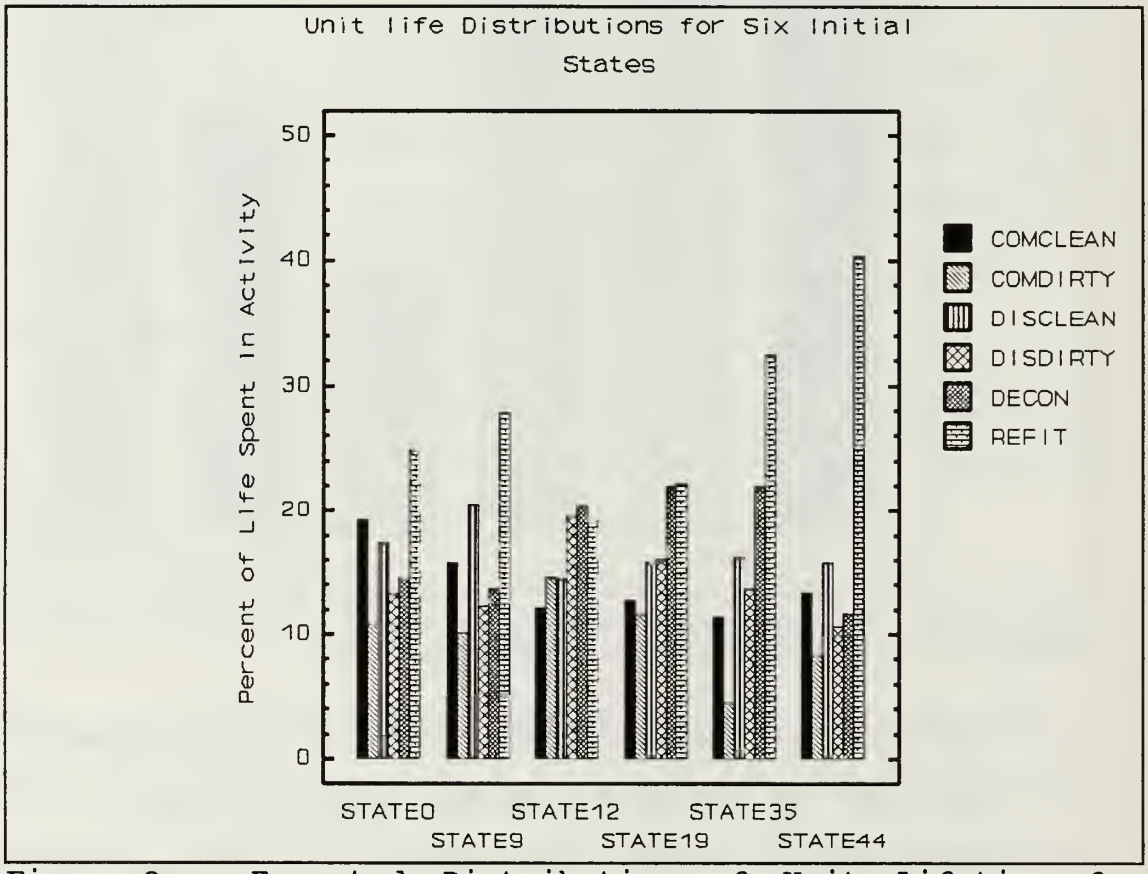

Figure 3. Expected Distribution of Unit Lifetime for Various Initial States.

Note that starting in state 44 gives the longest expected unit lifetime but that 40% of its life is spent in REFIT. This highlights the necessity to report expected unit lifetimes in conjunction with a breakout showing were the unit spent its useful life. Actually the times spent in COMCLEAN for the

initial states of 44, 0 and 6 are approximately equal, being between 5.7 to 5.2 hours for all three cases.

The last area to address is the cause of the unit becoming combat ineffective. Did the unit become COMBAT INEFFECTIVE-ATTRITION (ATTRIT) or COMBAT INEFFECTIVE-DEGRADATION (DEGRAD)? Using this model one cannot say what caused a specific unit to become combat ineffective but we can give a probability that a unit was absorbed in one or the other of the absorbing states. Figure 4 shows the probabilities of absorption for the six initial states in this example.

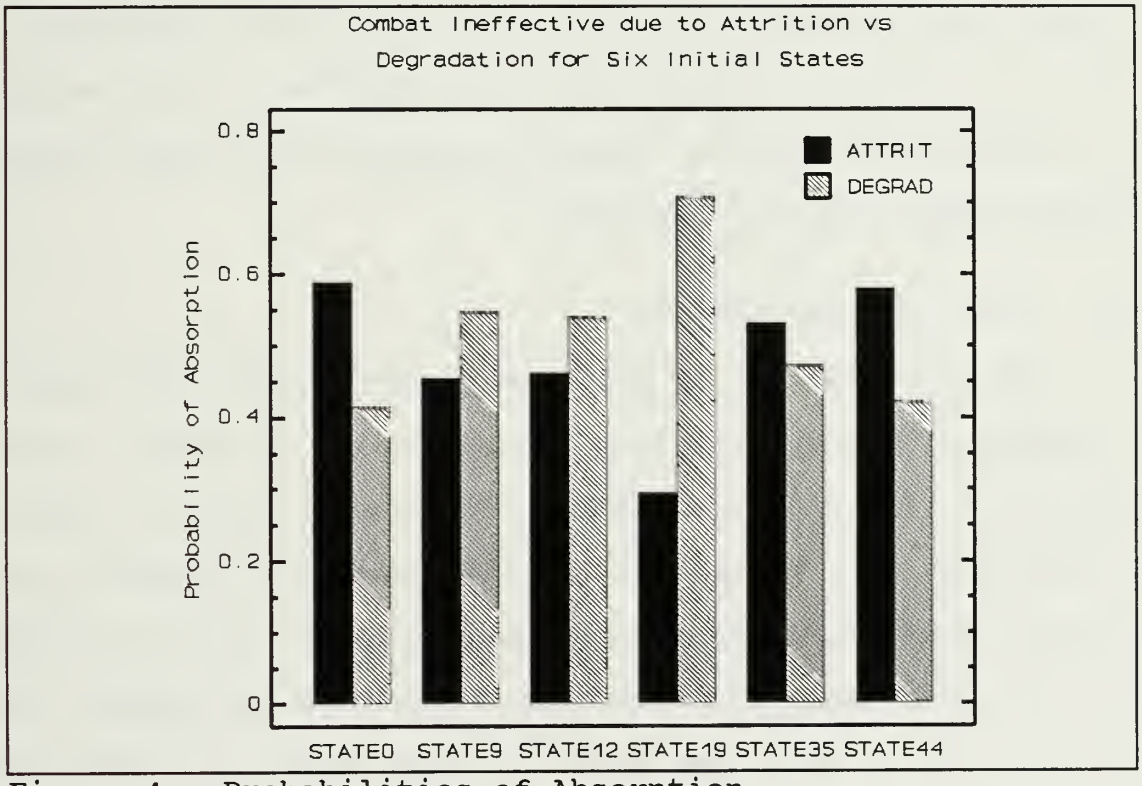

Figure 4. Probabilities of Absorption

As one would expect, the initial state of ENGAGED-CHEMICAL-NO MOPP-HIGH-LOW (State 19) has a very high probability of absorption in COMBAT INEFFECTIVE-DEGRADATION as opposed to COMBAT INEFFECTIVE-ATTRITION.

An example of the output used to develop these results is contained in Appendix E. The summary report gives the information found in Figure 2 and Figure 3. A more detailed report is also provided by the computer program. The detailed report shown in Appendix E. gives the number of visits, expected sojourn time per visit and the total time spent in each state for which the unit spends at least one percent of its total life. The solution time required to run one case (a fixed **P** and **T** matrix) is approximately 3-5 minutes using an 80286 based personal computer.

G. MODEL USE AND CAVEATS

Because this is an analytical model, the user needs to understand the abstractions and limits of the model. First, unlike a time step simulation, this model does not track an individual unit throughout its lifetime. For example a unit that "lives" nine hours could not possibly visit 23 states when travel times alone between activities are greater than nine hours. What the model is providing is a nine hour expected lifetime for all units that start in the given initial state. Also, if a large number of units start in the same initial state, we expect they will spend their lifetimes

distributed among the 23 states in the given proportions. Second, the expected time until absorption must always be considered in conjunction with the distribution of states in which the unit spent its useful life. Also, because the user makes the categorical judgments as to what constitutes a HIGH, MEDIUM and LOW strength unit any use of the model must consider how the judgments were made. Finally, because the user sets the P and T matrices the model is very scenario specific as are most simulations with user input parameters.

V. ANALYSIS USING MODEL

With the model in place, sensitivity analysis and testing of various assumptions can now be done. This analysis will concentrate on the three questions presented in the introduction : estimated threat, availability and priority of decontamination, and assumed degradation rates for personnel in a chemical environment.

The following five step structure will be used to examine each of the questions. First, pose the question as a tactical decision maker would. Second, identify the MOEs (Measures of Effectiveness) that answer the question. Third, describe what parameters in the model will be varied for the test. Fourth, present and analyze the raw model output. Fifth, frame the answer to the question in terms that are useful to a tactical decision maker.

A. ESTIMATED THREAT

1. Question to be Answered

The first question considered is, how does the frequency of chemical agent employment by the enemy affect the survivability of our force? The frequency of enemy chemical usage could be varied independently for each of two types of attacks: chemical weapons only and combined chemical/conventional weapons attacks. This analysis will

consider the relative frequency of all attacks that involve chemicals whether or not conventional weapons are involved.

2. Supporting MOEs

Several Measures of Effectiveness (MOE) address the somewhat vague term of survivability. The first MOE will be the time until a unit becomes combat ineffective given an initial starting state. The second MOE is the percentage of its useful life the unit spends in each of the six classes of states outlined in Chapter IV. The third MOE is the probability of absorption in each of the two absorbing states (i.e. the probability a unit becomes combat ineffective due to ATTRITION as opposed to DEGRADATION).

For purposes of this analysis all units will start in state 6, DISENGAGED-CONVENTIONAL-HIGH-HIGH (i.e., not fighting, in an environment free from chemical hazards, at full MTOE strength and with individual personnel at full effectiveness (not degraded)). Using the first MOE, time to absorption (unit lifetime), by itself can give misleading results. What is important is not only how long a unit survives but also what a unit was capable of doing while it was alive. A unit that survives 6 days but spends all its time in refit and decontamination is much less useful than a unit that survives 3 days but spends all its time in combat inflicting losses on the enemy. A unit's time to absorption must be reported in conjunction with the time and percentage

of time a unit spent in the six classes of states from Chapter IV.

Once a unit has lived out its useful life, what finally causes the unit to become combat ineffective; degradation or attrition losses? To the next higher level unit that needs to restore the combat ineffective unit this information is vital. Knowing what proportion of times units will become combat ineffective for each cause, the supporting unit(s) can determine the proper mix of restorative processes needed to support subordinate units.

These three MOEs will be used to evaluate the survivability and combat effectiveness of the unit throughout the analysis presented here. The primary MOEs are time to absorption and the proportion of time the unit spends in each of the classes outlined above. The secondary MOE is the probability of absorption in either of the two absorbing states in this process.

3. Experimental Design

To simulate various levels of chemical usage by the enemy the following procedure is used. All transition probabilities for transitions that lead from CONVENTIONAL states to CHEMICAL states are varied by a common factor from the base case. Using a common factor for all the transition probabilities keeps the relative frequency of each type of chemical attack constant while increasing or decreasing the

overall enemy usage of chemicals. The five variations tested and their relation to the base case are listed in Table II.

For the present base case chemical weapons are used 20% of the time if the unit is at MEDIUM-HIGH or better and 10% of the time if the unit is at MEDIUM-LOW or worse. The worst case tested assumes a chemical usage rate of 40% of the time if the attacked unit is at MEDIUM-HIGH or better and 20% of the time if the attacked unit is at MEDIUM-LOW or worse.

Table II TEST CASES FOR THE CHEMICAL THREAT ASSESSMENT

Case Name	Variation from Base Case
Base	None (see base case in Appendix C.)
Nochem	All transition probabilities from clean to dirty states are set to zero.
Chemonly	All transition probabilities from disengaged to engaged states are set to zero.
Halfchem	All transition probabilities from clean to dirty states are set to one half the base case value
Twicchem	All transition probabilities from clean to dirty states are set to twice the base case value.

4. Analysis of Model Output

The effect of various assumed chemical threats on the time until a unit becomes combat ineffective is shown in

Figure 5. Under a conventional weapons-only scenario the expected life of a unit is approximately 69 hours. In a chemical weapons-only scenario the expected life of a unit is reduced to approximately 43 hours. The minimum life expectancy of a unit, 26 hours, occurs at a chemical weapons usage which is twice the current base case. The worst case expected lifetimes are 38% of the conventional weapons-only and 59% of the chemical weapons-only lifetimes.

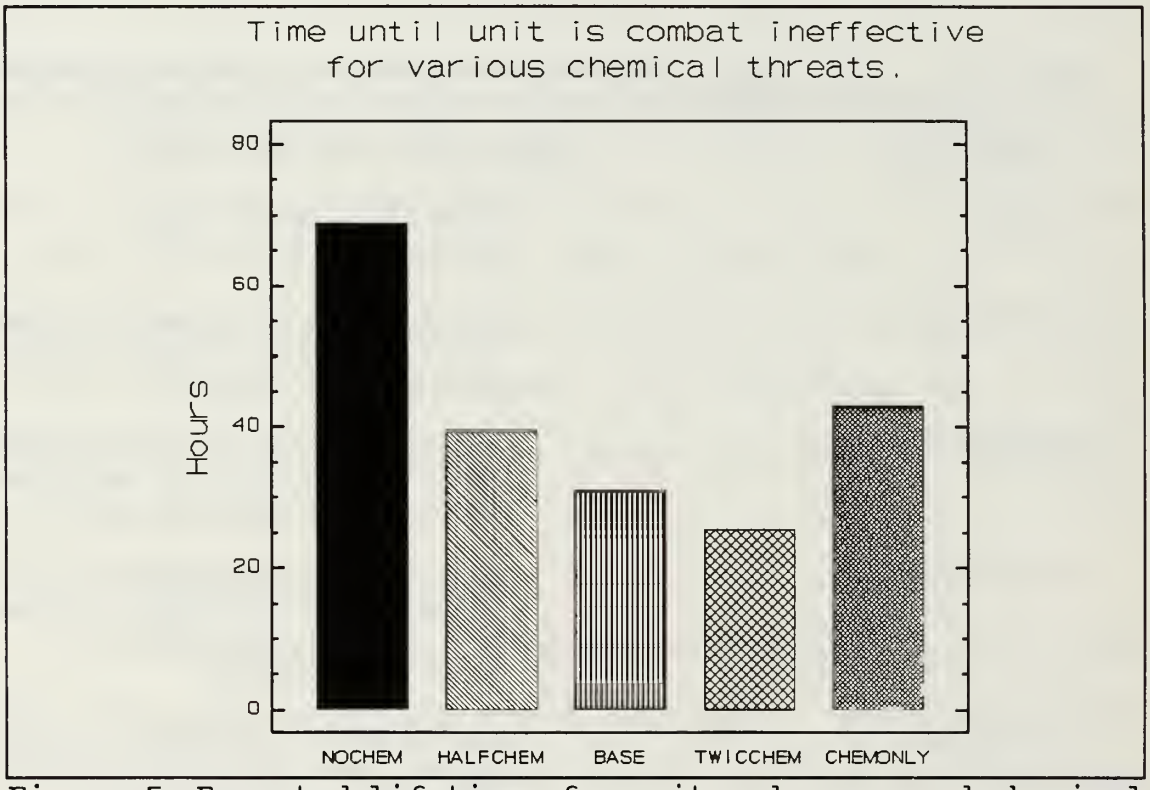

Figure 5 Expected lifetime of a unit under assumed chemical usage rates.

The worst case scenario assumes a chemical usage rate of 40% if the attacked unit is at MEDIUM-HIGH or better and 20% if the attacked unit is at MEDIUM-LOW or worse. The major consideration seems to be the introduction of chemical weapons

into the battle. Once they are being used, a variation of 400% in their frequency of use results in only a 38% variation in the expected lifetime of a unit. Figure 6 shows the distribution of its lifetime a unit spends in the classes of states mentioned in Chapter IV.

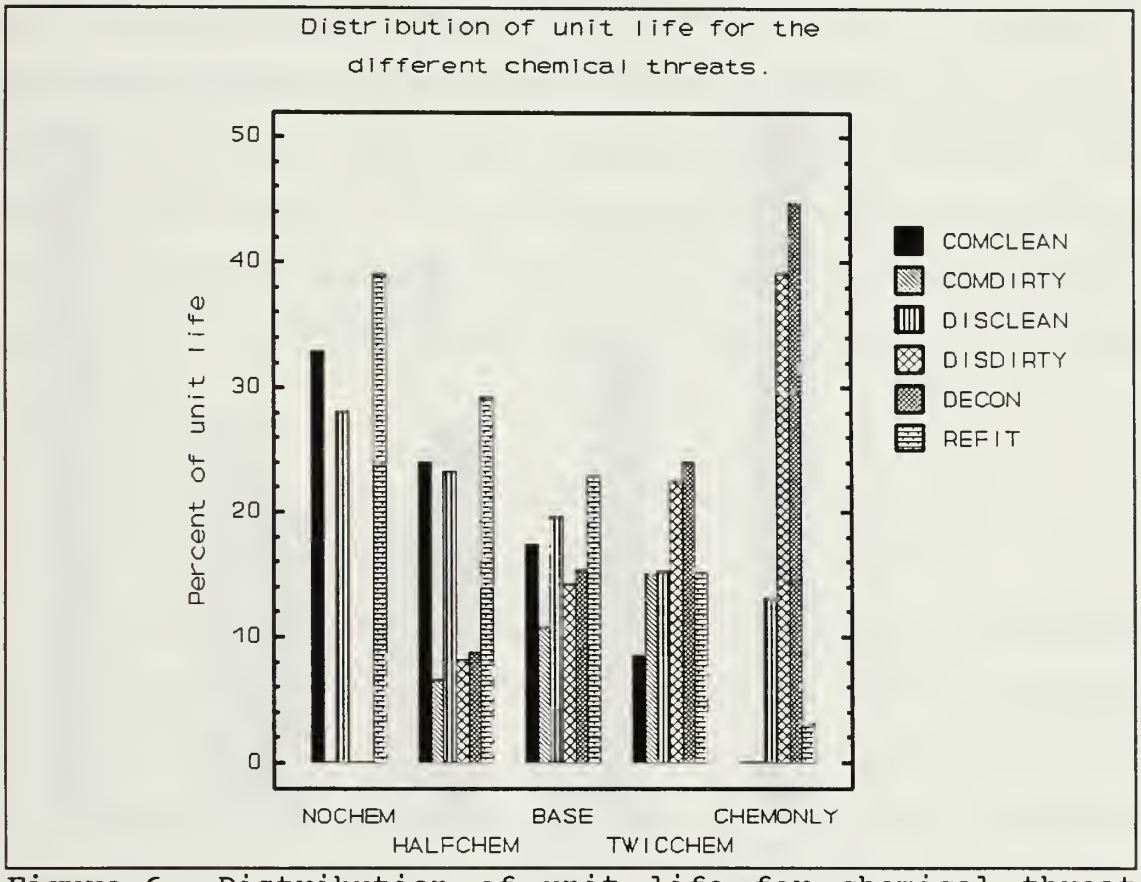

Figure 6 Distribution of unit life for chemical threat cases.

Finally, what causes the unit to become combat ineffective, attrition losses or degradation due to the use of chemicals and chemical protective gear? Figure 7 is a comparison of the probabilities of absorption due to attrition and degradation. Note there is a positive probability that

units will become combat ineffective due to attrition losses in the chemical weapons-only case. This is the result of chemical attacks on unwarned, unprotected units that result in both attrition loses and degradation.

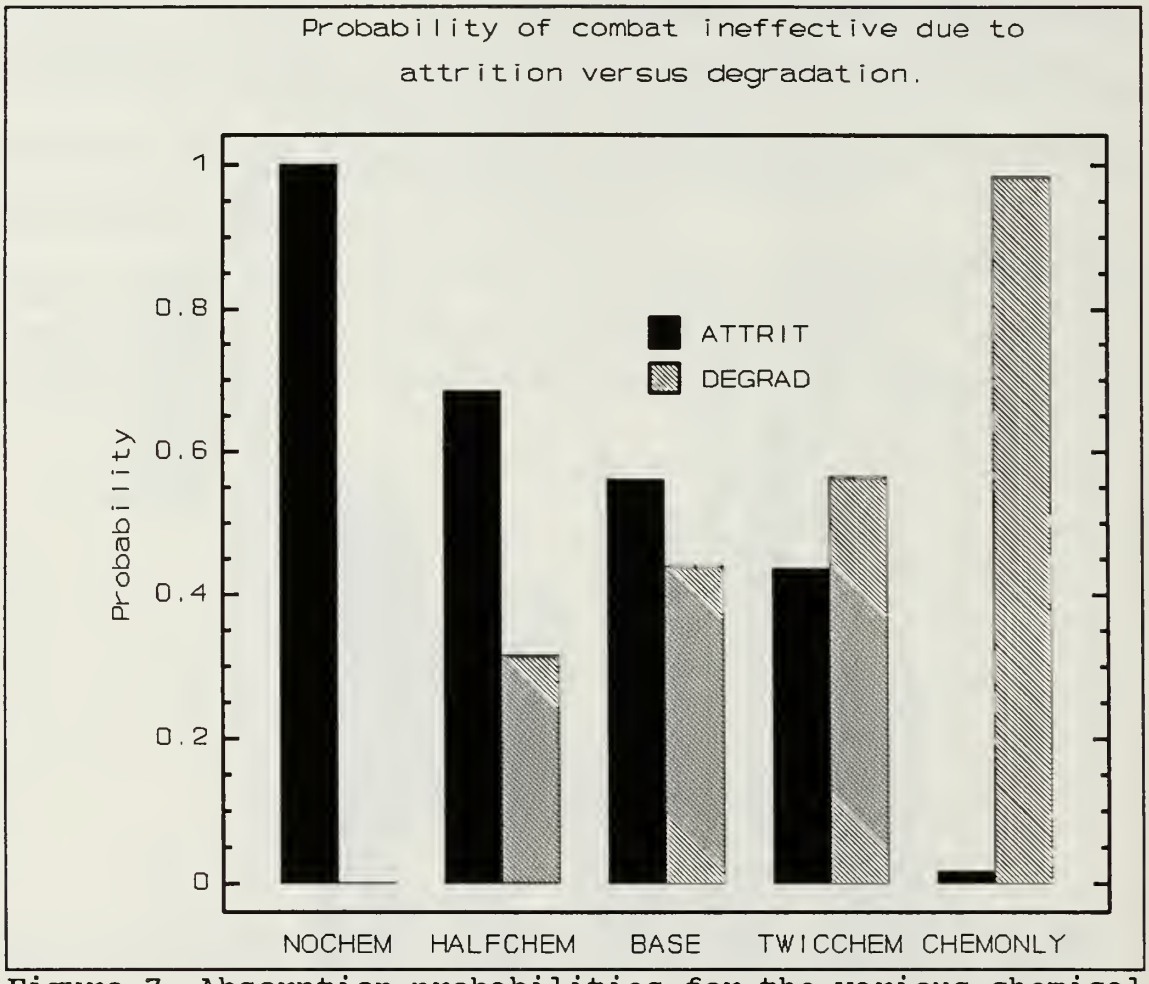

Figure 7 Absorption probabilities for the various chemical threat cases.

5. Discussion

To achieve the minimum expected lifetimes for friendly units, an enemy's best tactic is to use a combination of chemical and conventional weapons. Using conventional or chemical weapons exclusively does not result in the shortest

expected lives for friendly units. This result is not unexpected because in U.S. and Warsaw Pact doctrine, chemical weapons are seen as combat multipliers, weapons that may have a synergistic effect in conjunction with other weapons. This is accounted for in the model by having the fastest transitions to worse states for units involved in combined chemical/conventional battles. What is interesting is the appearance of an apparent optimal mix of chemical and conventional weapons. To this author's knowledge, no studies have been done using large simulation models to determine the existence or actual value of such a chemical to conventional weapons mix.

B. PRIORITY AND AVAILABILITY OF DECONTAMINATION

1. Question to be Answered

This is a two part question with both parts affecting the survivability of the force in this model. First, how does the decision to let a unit fight dirty or go immediately to decontamination affect the life time of the unit and where the unit spends its useful life? Second, given a base case of decontamination decisions, how does the lack of decontamination assets affect the life of a unit?

2. Supporting MOEs

The MOEs are the same ones used to answer the previous question about the frequency of enemy chemical agent use: time to absorption, portion of time the unit spends in each class

of states and probability of becoming combat ineffective due to attrition losses as opposed to chemical degradation.

3. Experimental Design

The second part of this question is the more straightforward one to answer. Starting with the base case probabilities of entering decon, decrease all these decon probabilities by a common factor until no decon assets are available. The various scenarios for this analysis are listed in Table III.

Table III SCENARIOS FOR THE AVAILABILITY OF DECON ASSETS

Case	Variation from Base Case
Base	None.(see Appendix C.)
D75%	All probabilities decreased to 75% of base case.
D50%	All probabilities decreased to 50% of base case.
D25%	All probabilities decreased to 25% of base case.
Dnone	All probabilities decreased to 0. No decon available.

The first portion of this question concerns when the decision to decon a unit is made based on the unit's strength and degradation level. A unit must be disengaged in order to

proceed to decon and in actual combat may be restrained from going to decon due to the tactical situation. For the purposes of this analysis the probability of going to decon from various disengaged states is considered as a decision variable. The various decon decision scenarios are given in Table IV and Table V.

Table IV DECON DECISION SCENARIOS

Case Name and Description	States to,from	Transition Probability old	new
BASE (see Appendix C.)			
DLOW (only decon units that are LOW degradation)	24,36	.4	0
	26,38	.4	0
	28,40	.5	0
	30,36	.3	0
	32,38	.2	0
	34,40	.3	0
	(all other probabilities remain the same as BASE case)		
DNMP (shift priority of decon to units w/o chem protective gear).	24,36	.4	.2
	25,37	.4	.2
	26,38	.4	.2
	27,39	.6	.3
	28,40	.5	.25
	29,41	.7	.35
	30,36	.3	.6
	31,37	.3	.6
	32,38	.2	.4
	33,39	.3	.6
	34,40	.3	.6
	35,41	.3	.6

4. Analysis of Model Output

The supporting MOEs are the same as for the threat analysis and the model results for the various decontamination availability cases are given in Figures 8, 9 and 10. As decon

Table V FURTHER DECISION SCENARIOS

Case Name	States to,from	Transition Probabilities old	new
DHIGH	24,36	.4	.6
(conserve	25,37	.4	.6
fighting ability	26,38	.4	.4
by shifting	27,39	.6	.6
decon priority	28,40	.5	.25
to higher	29,41	.7	.35
strength units).	30,36	.3	.45
	31,37	.3	.45
	32,38	.2	.2
	33,39	.3	.3
	34,40	.3	.15
	35,41	.3	.15

becomes less available the units experience shorter expected lives. Also, the decrease in unit lifetimes is nearly linear with a loss of approximately five hours for every 25% decrease in the availability of decontamination from the base case. The distribution of time a unit spends in classes of states is shown in Figure 9. When decon is totally unavailable at this level the probability that a unit goes ineffective due to degradation approaches 0.62 (Figure 10).

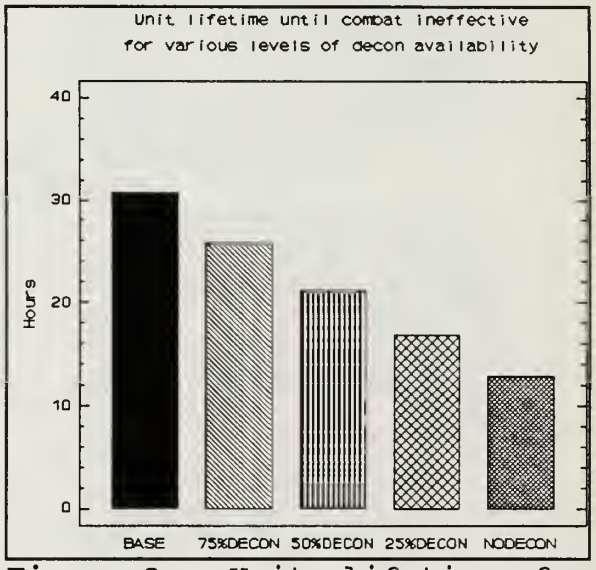

Figure 8 Unit lifetime for various decontamination availability cases.

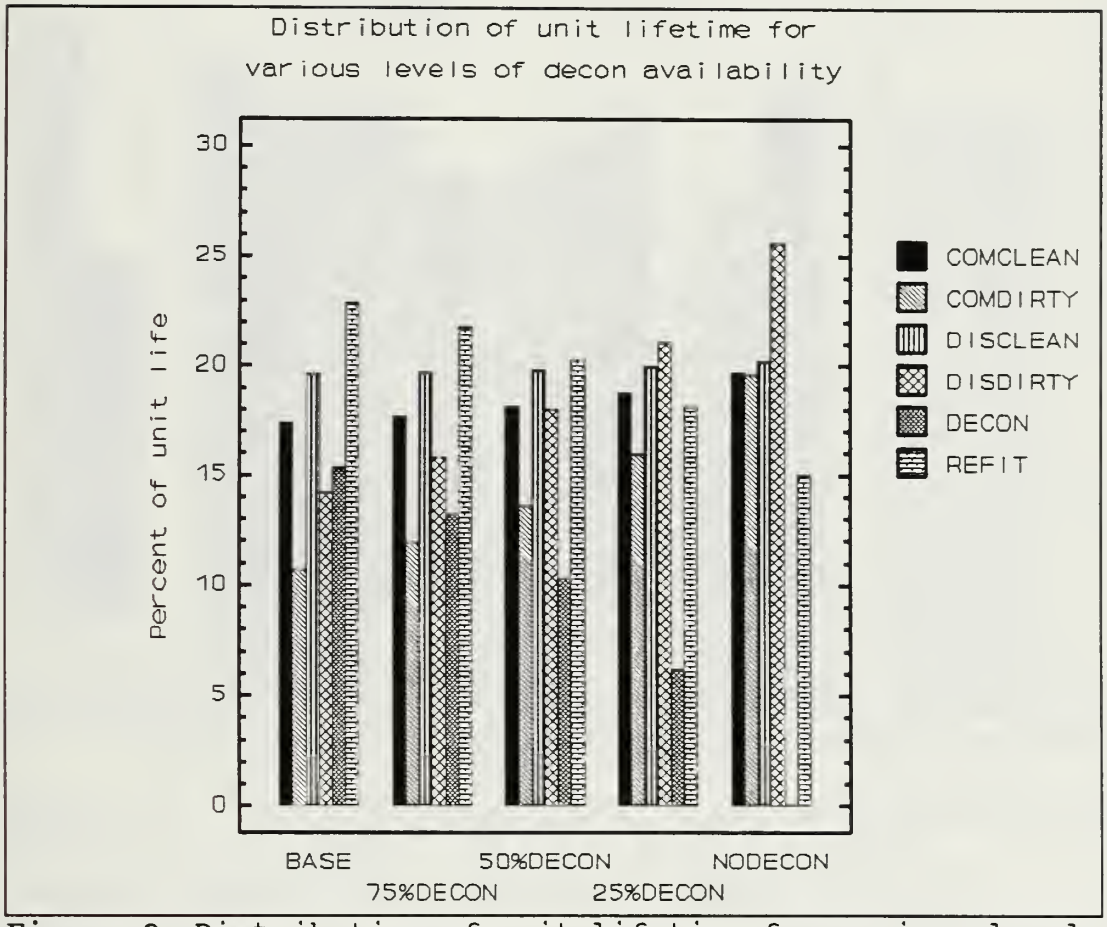

Figure 9 Distribution of unit lifetime for various levels of decontamination availability.

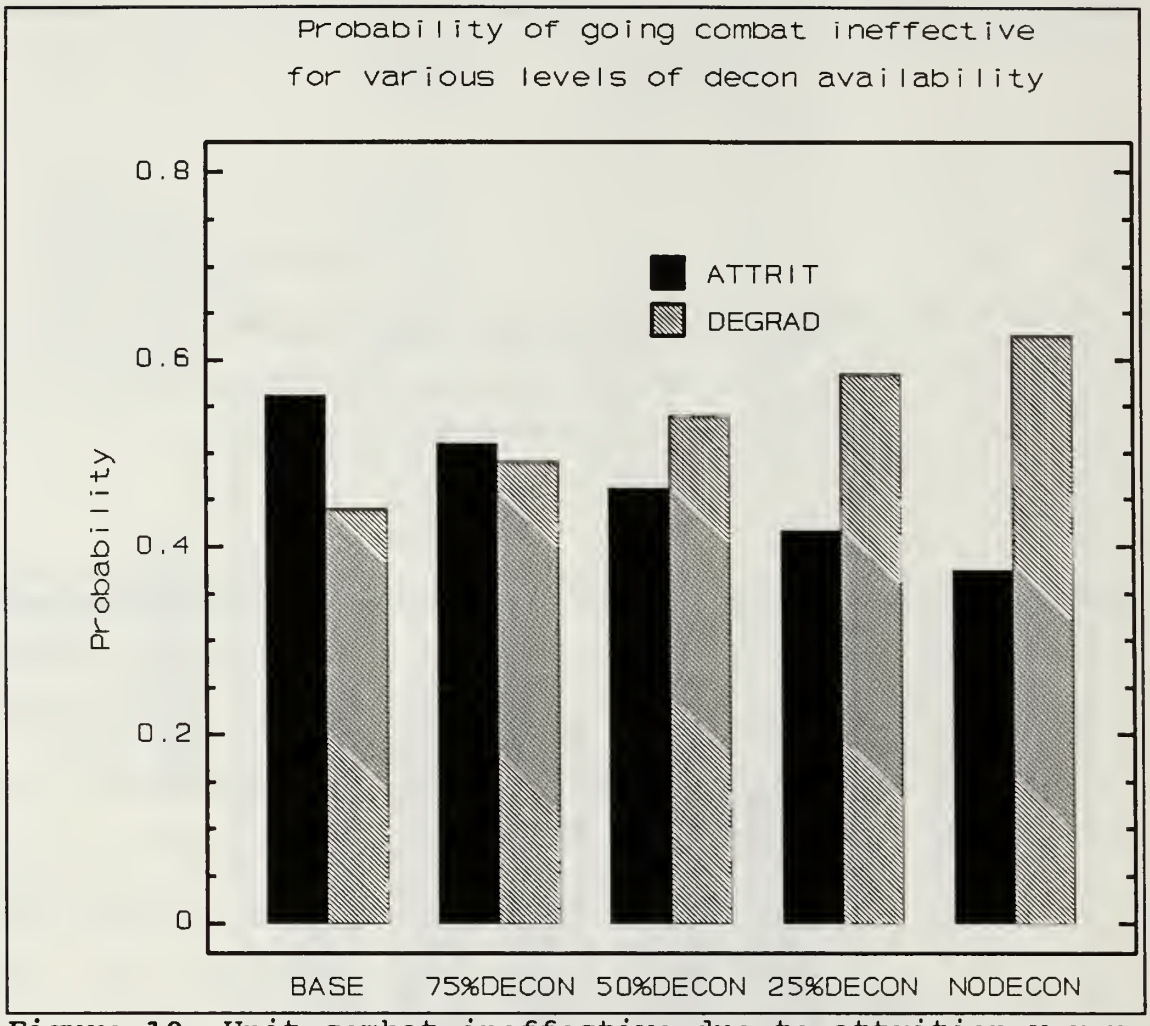

Figure 10 Unit combat ineffective due to attrition versus degradation for various decon decision scenarios.

For the decision cases the model output is shown in Figures 11, 12 and 13. The model shows the unit lifetimes and distributions to be very robust against changes in the decon priority, with approximately a 27% difference between the high and low expected unit lifetimes. The optimum scenario (longest expected unit lifetime) is the BASE (original decontamination decision) case.

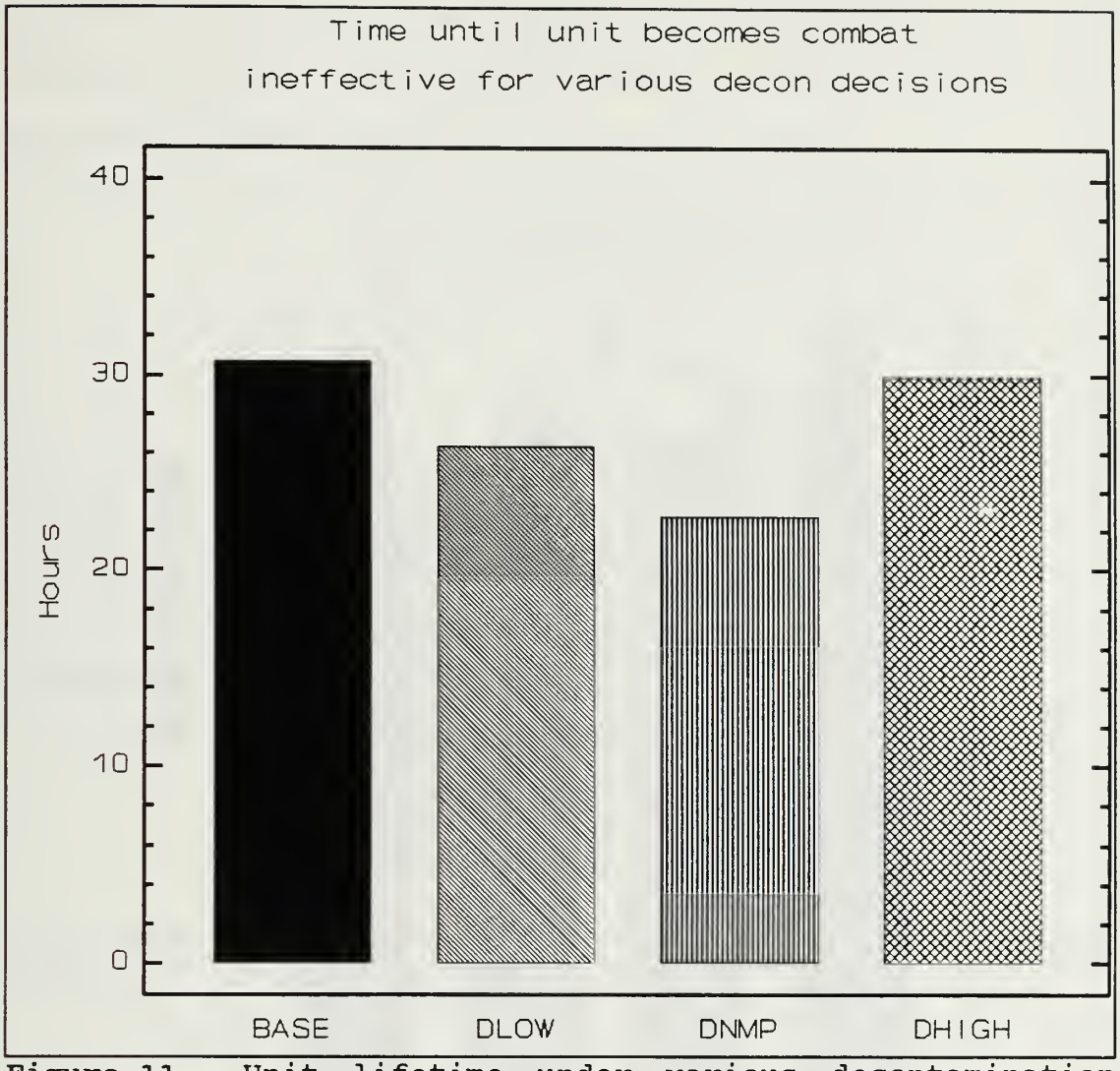

Figure 11 Unit lifetime under various decontamination decision scenarios.

5. Discussion

For a commander facing the proposed BASE chemical threat, not having decontamination assets results in unit survival times that are approximately 1/3 the BASE case with decon assets.

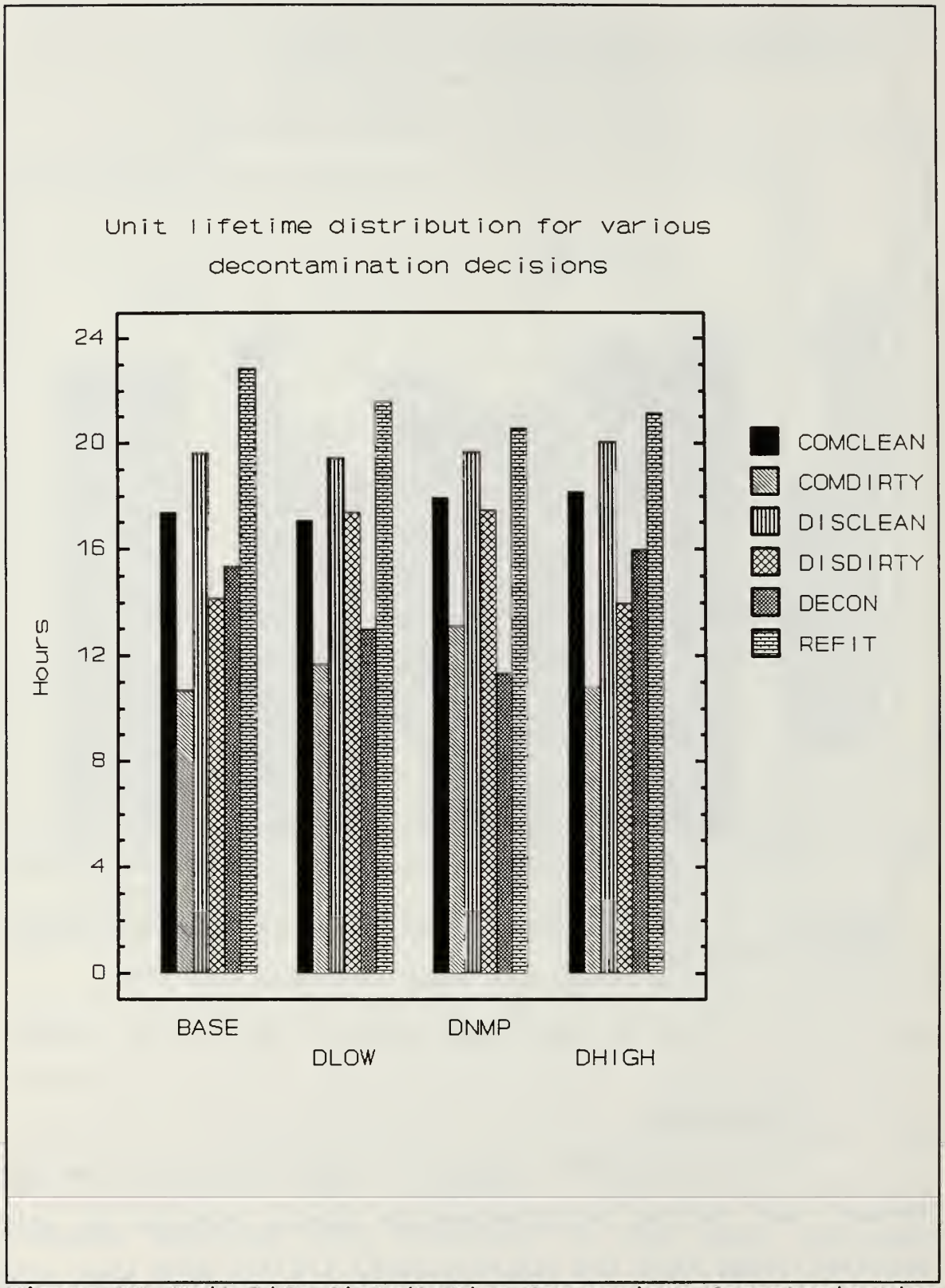

Figure 12 Unit life distribution for various decontamination decision scenarios.

In the current model the units are getting a double penalty because of the lack of decontamination assets. An underlying assumption is that units must be clean to go to refit and get supplies and replacements for attrition losses.

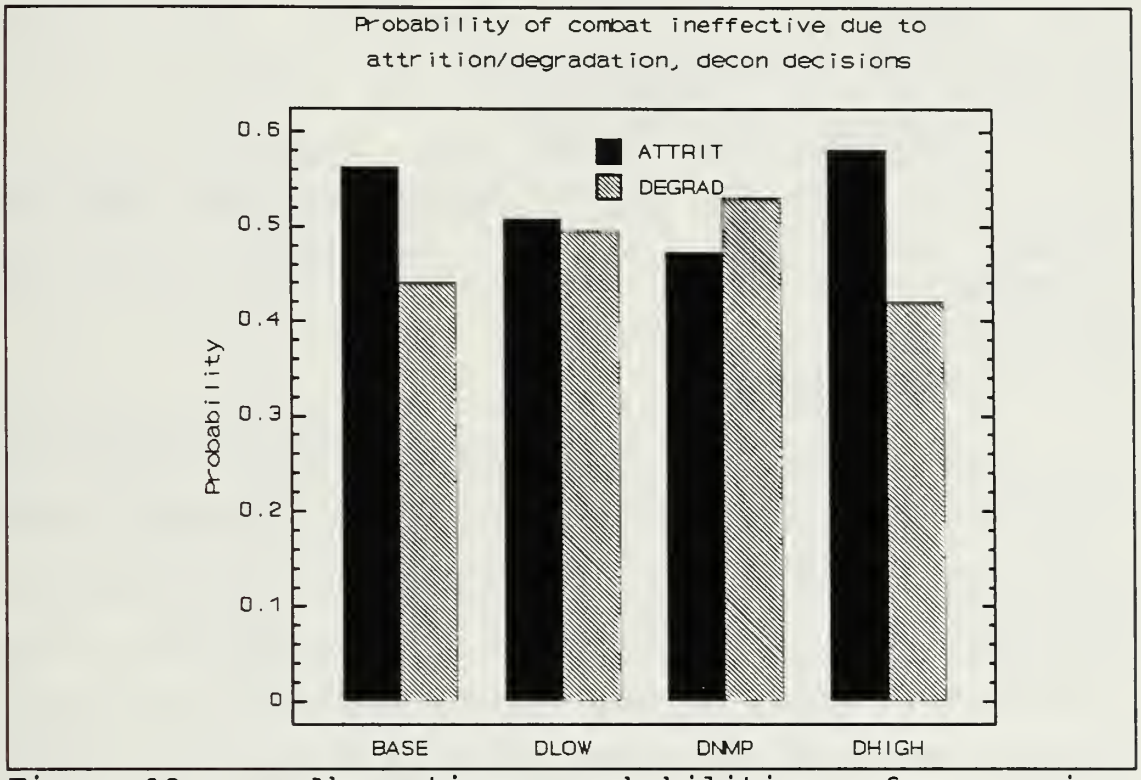

Figure 13 Absorption probabilities for various decontamination decision scenarios.

This is in keeping with current doctrine so that the supply and maintenance facilities will remain clean and efficient. A commander faced with no refit at all or doing it "dirty" will undoubtedly designate a lucky unit to perform the dirty refits and accept the contamination dirty units bring in. This can be reflected in the model by the creation of new dirty refit states that have a longer turnaround time. Also

units undergoing dirty refit would have a small probability of becoming ineffective due to degradation. The restoring unit would also have a positive probability of going ineffective due to degradation.

C. MOPP GEAR DEGRADATION EFFECTS

1. Question to be Answered

How does the assumed degradation that MOPP gear imposes on the personnel in a unit affect the unit's performance?

2. Supporting MOEs

The time until a unit becomes combat ineffective and the distribution of states in which the unit spends its useful life are the MOEs for this analysis. Probability of absorption will not be considered for this question. In the experimental design the transition probability matrix is not changed, therefore the probabilities of absorption will remain constant for all cases considered.

3. Experimental Design

The degradation effects of a chemical environment are assumed to cause a loss in the unit's ability to sustain attrition and degradation strength levels as compared to a non-chemical environment. Both the degradation caused by the

chemicals themselves and the degradation due to the wearing of MOPP gear are considered.[1]

In this model these decreased capabilities are represented by decreased transition times to the next lower strength level. For example a unit that is HIGH-HIGH and in conventional combat can sustain HIGH-HIGH for one hour before going to MEDIUM-HIGH. Also this unit can disengage at 75% of it's HIGH to MEDIUM transition time and maintain it's HIGH MTOE strength rating. In contrast, a unit that is engaged and in MOPP will transition to MEDIUM in one hour but must disengage at 60% of this transition time to maintain it's HIGH MTOE rating. Also, if the PERSEFFECT indicator is already at LOW the unit's transition time to MEDIUM MTOE is .6 hours. For units that are engaged in chemicals and are not wearing MOPP the transition times are one half the MOPP transition times.

This test used the BASE case transition time matrix (the TBASE case) and two other boundary cases, TBAD and TGOOD. In the base case units in CONVENTIONAL states can sustain strengths longer than units in CHEMICAL-MOPP, which in turn can sustain strength levels longer than units in CHEMICAL-NO MOPP.

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[1] Degradation effects of chemical are those effects that hamper personnel but do not require medical attention to preserve the life of the soldier. Such effects include pinpointing of the pupils (miosis, nerve agents) and minor blistering from mustard agents. MOPP gear degrades troops by sensory deprivation and thermal stress (these suits get HOT).

In the TBAD case the transition times for CHEMICAL-MOPP units are 1.1 times the transition times for CHEMICAL-NO MOPP units. This represents the case were MOPP gear either does not protect adequately or is so stressful for troops to wear that it is only slightly better than no chemical protective gear.

In the TGOOD case the transition times for CHEMICAL-MOPP units are 90% of the transition times for CONVENTIONAL units. This represents the case were MOPP gear protects adequately and only slightly restricts troops in their mission performance. Also for TBAD and TGOOD, the transitions while DISENGAGED that have no counterpart in the CONVENTIONAL states are decreased and increased appropriately.

4. Analysis of Model Output

The assumed degradation rates were tested against the different chemical threat levels described in Table II and the results are shown in Figures 14, 15 and 16. Figure 14 shows that as the assumed degradation rate increases (transition times decrease) the expected unit lifetime decreases. Though the lifetimes are affected more in the CHEMONLY case, the model does not exhibit any clear breakpoints, a case where the change in assumed degradation rates makes a dramatic or unexpected change in the expected unit lifetime. The effects of the assumed degradation rates on the unit lifetime distribution are shown in Figures 15 and 16.

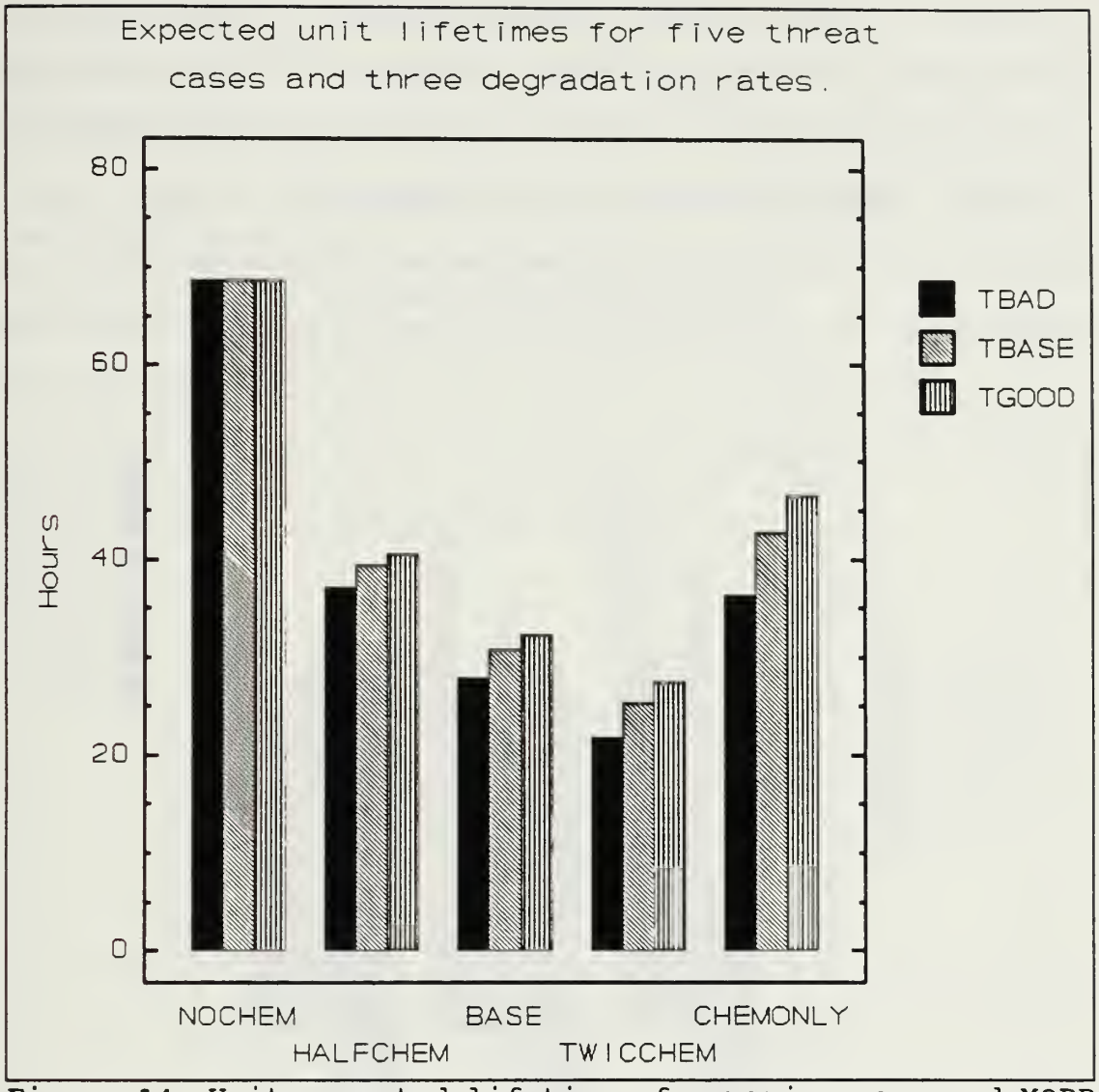


Figure 14 Unit expected lifetimes for various assumed MOPP degradation rates.

The degradation rates affect the CHEMONLY case much more than they affect the BASE threat case. Note that under TBAD, highest degradation effects, a unit spends 53% of its time in decontamination (see Figure 16). For the CHEMONLY case and TGOOD time matrix the unit spends only 41% of its time in DECON and the time spent in DISDIRTY states jumps from

28% to 45%. Note that for the BASE case the distribution of time spent in the different classes of states is almost constant with the only change in ranking occurring between the DISDIRTY and DECON states (see Figure 15).

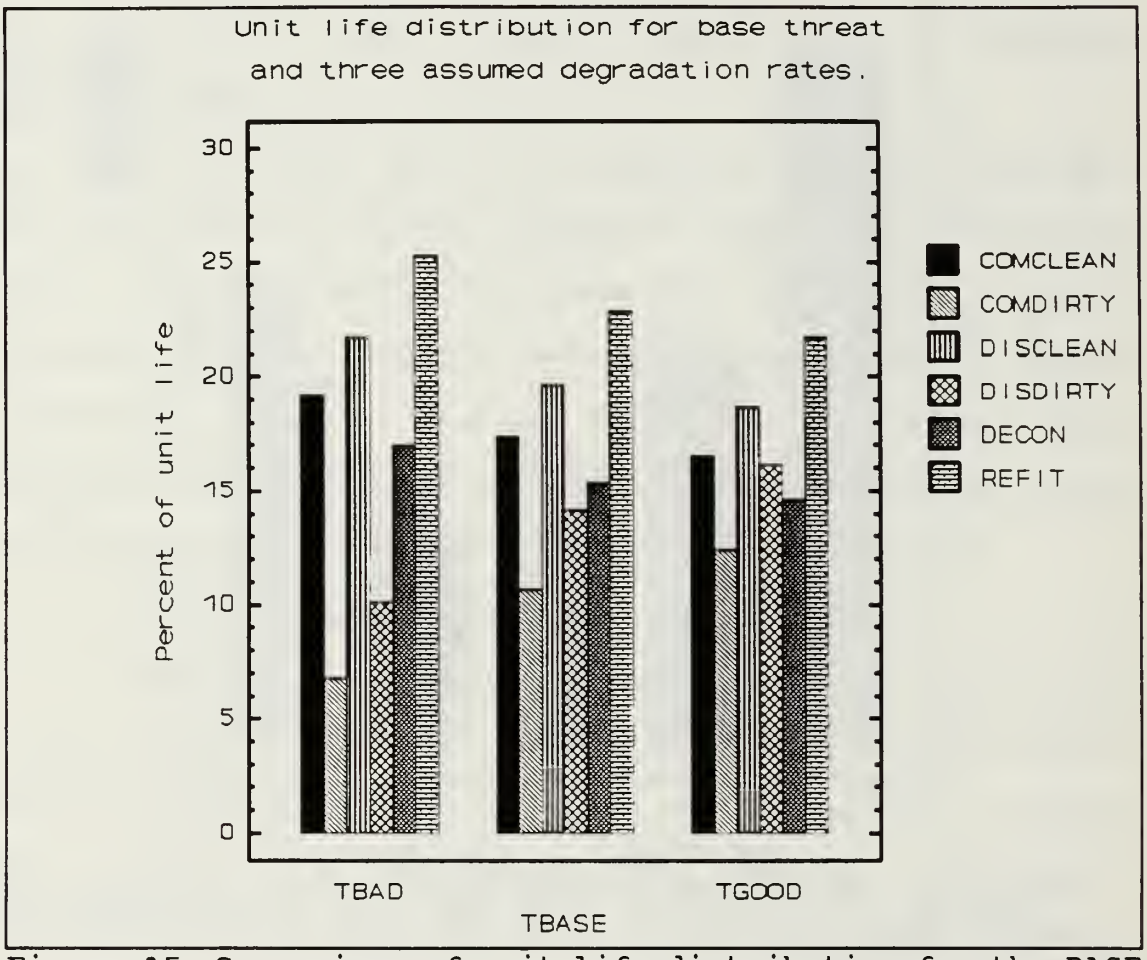


Figure 15 Comparison of unit life distribution for the BASE threat case and three different degradation rates.

5. Discussion

In this case the model provides consistent results. If personnel are degraded worse the unit can expect to become combat ineffective quicker and this effect is more pronounced

for higher rates of enemy chemical weapons usage. Also, if MOPP gear is more efficient troops can fight in it longer and spend less percentage of their time in DECON. The model also is very robust to changes in the transition times. Some of the transition times are varied by a factor of five yet the expected unit lifetime and life distributions do not exhibit jumps of that magnitude (see Figures 14, 15 and 16).

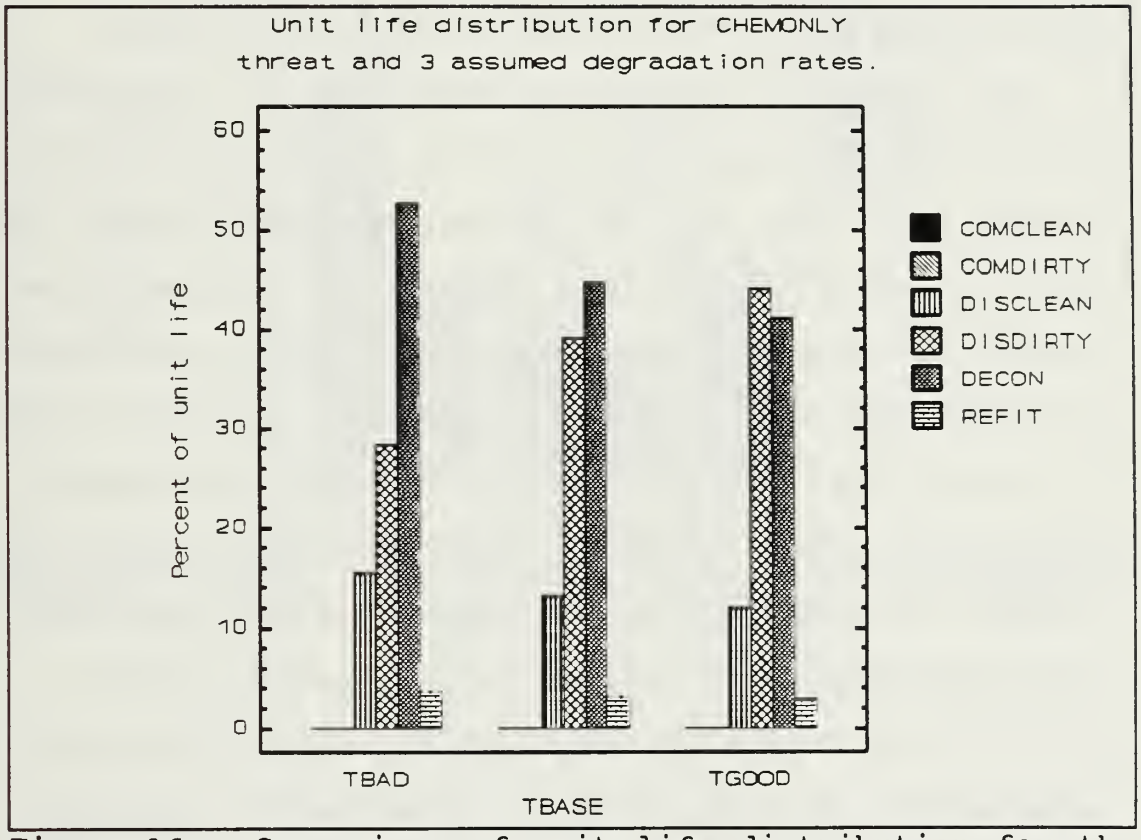


Figure 16. Comparison of unit life distribution for the CHEMONLY threat case and three different degradation rates.

D. SENSITIVITY ANALYSIS

Sensitivity analysis is concerned with how general are the results from an experiment? Are the results robust,

applicable over a range of input parameters, or are the results so sensitive that one minor change in the inputs will cause entirely different conclusions? For this particular model the inputs are the **P** and **T** matrices. Because the experiment with the assumed degradation rates showed the expected unit lifetime to be less sensitive to changes in the **T** matrix than in the **P** matrix, a sensitivity analysis was performed on the **P** matrix (transition probabilities).

The original threat-cases experiment was performed by systematically varying part of the **P** matrix for the model and holding the rest of the transition probabilities fixed relative to one another (see Appendix F). How sensitive are results of the previous analysis to changes in the portion of the model that are not being varied in a systematic manner? To test this sensitivity the following experiment was conducted. The five chemical threat levels were tested against the base **P** matrix and against five matrices that had been perturbed using a Normal(0,.01) random variable. The N(0,.01) random variable was added to a matrix of all ones the same size as the model **P** matrix. The **P** matrix, with the test entries removed, was the multiplied by this randomized matrix. The resulting probabilities were then normalized so that the rows still summed to one when the removed probabilities were added back in. The results for the total unit lifetimes are shown in Figure 17. The randomized matrices caused no change

in the relative ranking of the threats but did affect the absolute lifetime of the unit particularly at the extreme cases of no chemicals and all chemicals. P10T1 is the original transition probability matrix given in Appendix C.

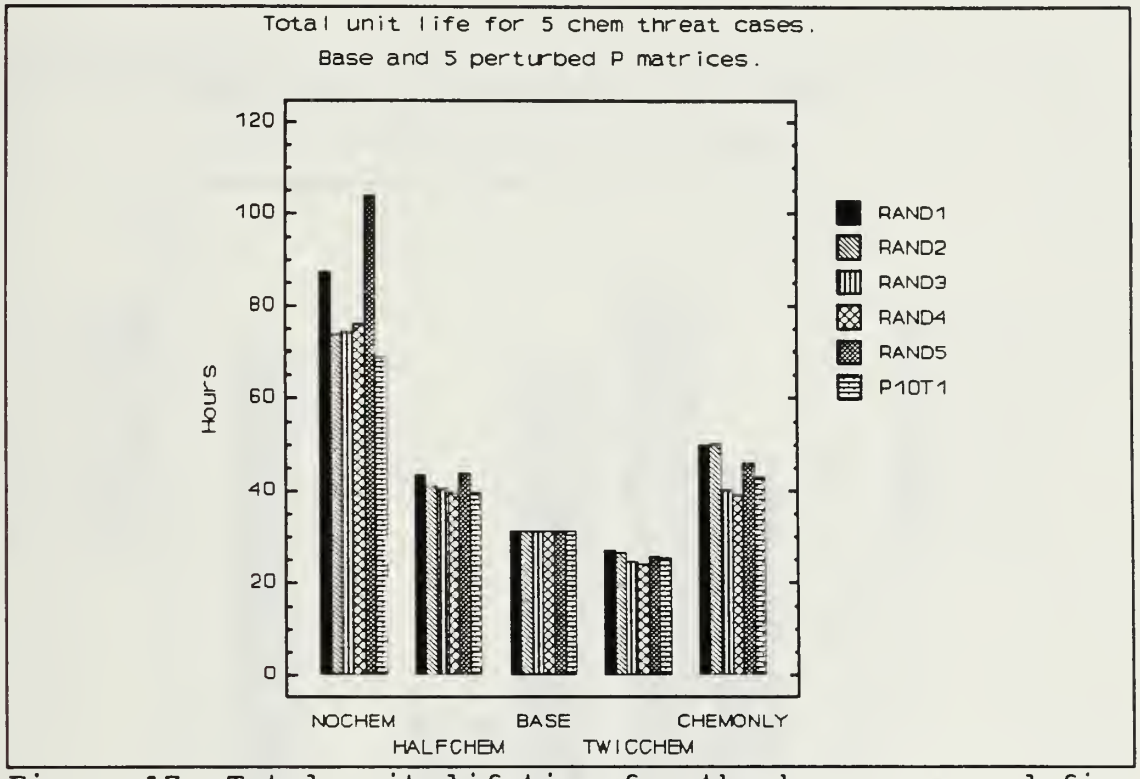


Figure 17 Total unit lifetime for the base case and five randomized P matrices.

Figure 18 and Figure 19 show the unit lifetime distribution for the NOCHEM and CHEMONLY threat cases. These were the cases whose expected unit lifetimes showed the most variance. The distribution of unit lifetime appears to be more robust than the expected unit life.

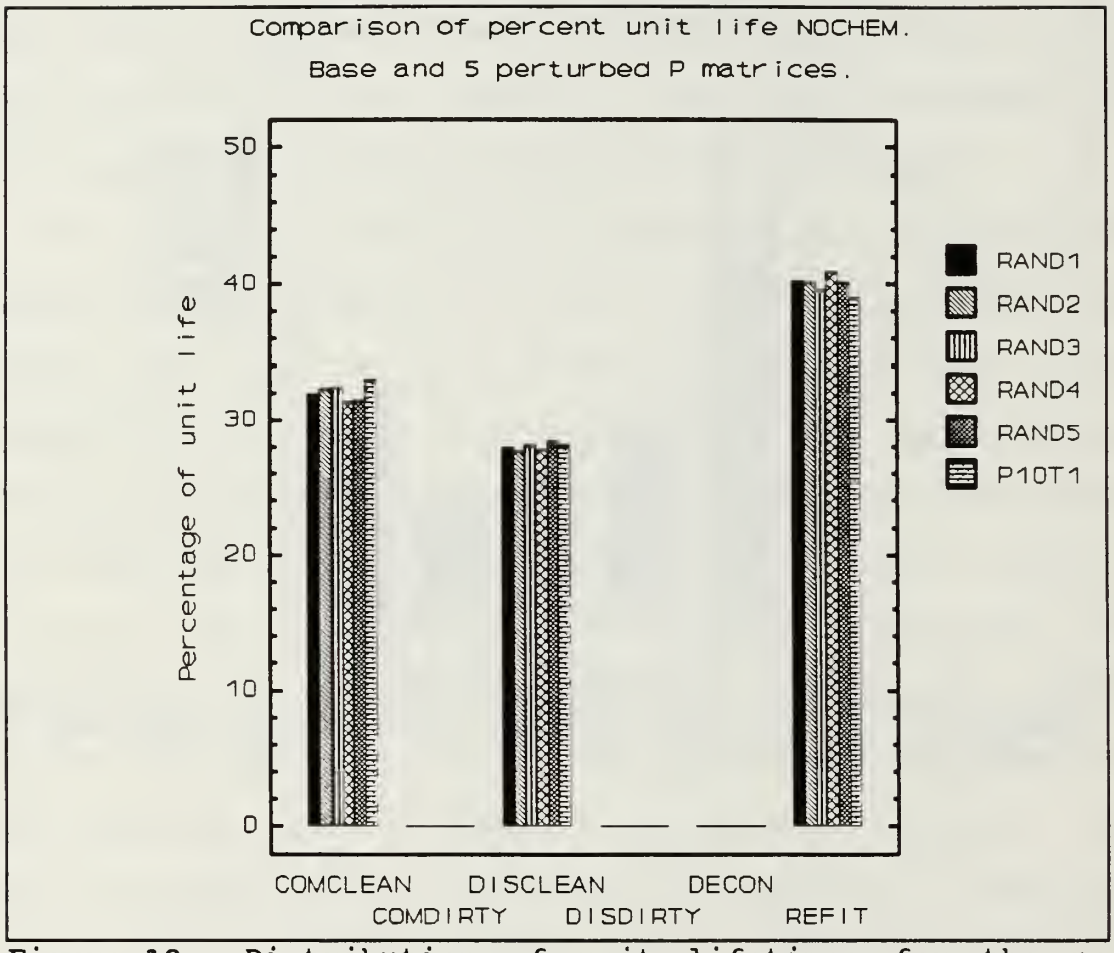


Figure 18 Distribution of unit lifetimes for the no chemicals case.

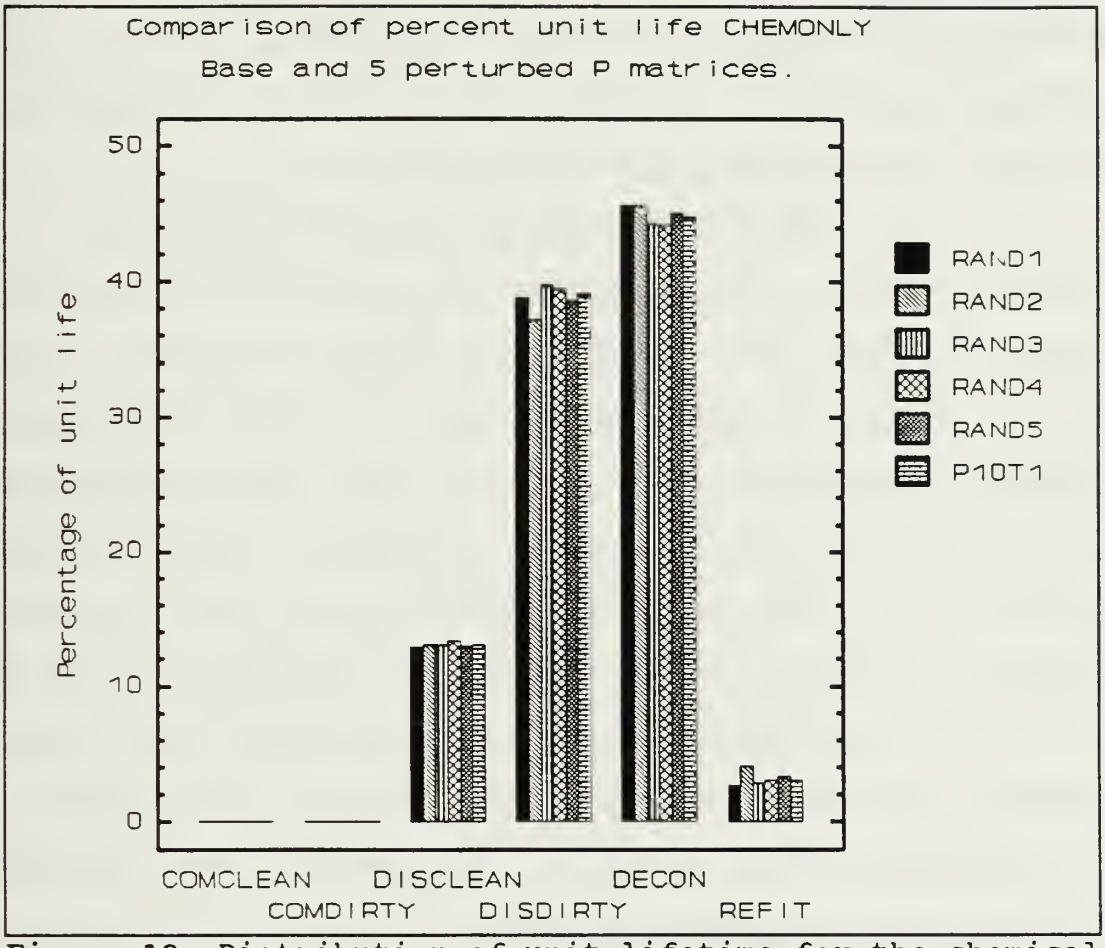


Figure 19 Distribution of unit lifetime for the chemical only case.

VI. CONCLUSION AND RECOMMENDATIONS

The semi-Markov model presented in this thesis has shown itself to be a useful tool for sensitivity analysis and the ranking of alternative threats and courses of action. The model provides the following insights into the three questions posed in the Introduction to the thesis.

The major factor that determines the expected lifetime and capabilities of a unit is whether or not chemical weapons have been introduced into the battle. Units can expect to live about 70 hours in a strictly conventional battle whereas their expected lifetime drops to 40 hours with the introduction of chemical weapons at the lowest frequency of usage tested in this model. Once chemical weapons have been introduced, however, increasing their use does not continue to lower unit lifetimes as dramatically. A 400% increase in the use of chemical weapons results in approximately a 38% decrease in the expected life of a unit. As chemical usage increases, however, more of a unit's lifetime is spent in restorative processes and not in productive combat. Also, the model points to the existence of an optimal conventional/chemical weapons mix.

As regards the availability of decontamination assets and their use, the model shows that the availability of assets

affects the survivability of units more than the decision of when to decontaminate a unit. The decontamination priority scheme chosen for the base case of the model gave the best results for expected unit lifetime by a small margin over the decision to decontaminate units at HIGH MTOE strengths. As might be expected, the more that chemical gear degrades individual soldiers, the shorter the expected unit life will be. The results of the degradation experiment show how robust the model is to changes in an individual transition time or even a group of transition times. Varying individual transition times by a factor of five did not vary unit lifetimes by even a factor of one-half. This robustness also points out that varying the transition times alone may not be an adequate experiment. Further experiments should consider varying both the transition times and transition probabilities to test the effects of assumed degradation rates.

The model is not intended as a replacement for time step, high-resolution simulations. However, using such simulations as input for the P and T matrices the model is a quick way to answer " What if...?" questions concerning changes in the threat and changes in tactics.

This class of analytical models is limited only by the user's patience in designing the state space and experiments.

APPENDIX A. STATE SPACE LIST

The state space used for this model is given below. The terms are defined in Chapter IV Model Formulation. The bold numbers are the state number and the written portion is the state description.

0 ENGAGED-CONVENTIONAL-HIGH-HIGH
1 ENGAGED-CONVENTIONAL-HIGH-LOW
2 ENGAGED-CONVENTIONAL-MEDIUM-HIGH
3 ENGAGED-CONVENTIONAL-MEDIUM-LOW
4 ENGAGED-CONVENTIONAL-LOW-HIGH
5 ENGAGED-CONVENTIONAL-LOW-LOW
6 DISENGAGED-CONVENTIONAL-HIGH-HIGH
7 DISENGAGED-CONVENTIONAL-HIGH-LOW
8 DISENGAGED-CONVENTIONAL-MEDIUM-HIGH
9 DISENGAGED-CONVENTIONAL-MEDIUM-LOW
10 DISENGAGED-CONVENTIONAL-LOW-HIGH
11 DISENGAGED-CONVENTIONAL-LOW-LOW
12 ENGAGED-CHEMICAL-MOPP-HIGH-HIGH
13 ENGAGED-CHEMICAL-MOPP-HIGH-LOW
14 ENGAGED-CHEMICAL-MOPP-MEDIUM-HIGH
15 ENGAGED-CHEMICAL-MOPP-MEDIUM-LOW
16 ENGAGED-CHEMICAL-MOPP-LOW-HIGH
17 ENGAGED-CHEMICAL-MOPP-LOW-LOW
18 ENGAGED-CHEMICAL-NO MOPP-HIGH-HIGH
19 ENGAGED-CHEMICAL-NO MOPP-HIGH-LOW
20 ENGAGED-CHEMICAL-NO MOPP-MEDIUM-HIGH
21 ENGAGED-CHEMICAL-NO MOPP-MEDIUM-LOW
22 ENGAGED-CHEMICAL-NO MOPP-LOW-HIGH
23 ENGAGED-CHEMICAL-NO MOPP-LOW-LOW
24 DISENGAGED-CHEMICAL-MOPP-HIGH-HIGH
25 DISENGAGED-CHEMICAL-MOPP-HIGH-LOW
26 DISENGAGED-CHEMICAL-MOPP-MEDIUM-HIGH
27 DISENGAGED-CHEMICAL-MOPP-MEDIUM-LOW
28 DISENGAGED-CHEMICAL-MOPP-LOW-HIGH
29 DISENGAGED-CHEMICAL-MOPP-LOW-LOW
30 DISENGAGED-CHEMICAL-NO MOPP-HIGH-HIGH
31 DISENGAGED-CHEMICAL-NO MOPP-HIGH-LOW
32 DISENGAGED-CHEMICAL-NO MOPP-MEDIUM-HIGH
33 DISENGAGED-CHEMICAL-NO MOPP-MEDIUM-LOW
34 DISENGAGED-CHEMICAL-NO MOPP-LOW-HIGH
35 DISENGAGED-CHEMICAL-NO MOPP-LOW-LOW
36 DECONTAMINATION-HIGH-HIGH

```
37  DECONTAMINATION-HIGH-LOW
38  DECONTAMINATION-MEDIUM-HIGH
39  DECONTAMINATION-MEDIUM-LOW
40  DECONTAMINATION-LOW-HIGH
41  DECONTAMINATION-LOW-LOW
42  REFIT-MEDIUM-HIGH
43  REFIT-MEDIUM-LOW
44  REFIT-LOW-HIGH
45  REFIT-LOW-LOW
46  COMBAT INEFFECTIVE-ATTRITION
47  COMBAT INEFFECTIVE-DEGRADATION
```

APPENDIX B. CONNECTIVITY MATRIX

The connectivity matrix for the model is given on the next page. The bold numbers are the state numbers and the entries in the cells denote whether or not a transition from state i to state j is allowed in one transition. An "X" denotes a positive probability of transitioning from state i, the row number, to state j, the column number. A blank denotes that the transition is not allowed or at least not considered in the present model.

APPENDIX C. ONE-STEP TRANSITION PROBABILITY MATRIX

This appendix contains the the numerical values for the transition probabilities that correspond to the "X"s on the connectivity matrix in Appendix B. The probabilities are broken down by rows and each section is labeled with the current state, row, number. The notation p(N,M) is the probability of going from state N to state M in one transition. All probabilities not expressly assigned in this appendix are equal to zero for this model.

State 0: p(0,2)= 0.6, p(0,6)= 0.2, p(0,12)= 0.15, p(0,18)= 0.05

State 1: p(1,3)= 0.5, p(1,7)= 0.3, p(1,13)= 0.15, p(1,19)= 0.05

State 2: p(2,4)= 0.3, p(2,8)= 0.5, p(2,14)= 0.15, p(2,20)= 0.05

State 3: p(3,5)= 0.2, p(3,9)= 0.7, p(3,15)= 0.075, p(3,21)= 0.025

State 4: p(4,10)= 0.7, p(4,16)= 0.075, p(4,22)= 0.025, p(4,46)= 0.2

State 5: p(5,11)= 0.6, p(5,17)= 0.075, p(5,23)= 0.025, p(5,46)= 0.3

State 6: p(6,0)= 0.8, p(6,12)= 0.075, p(6,18)= 0.025, p(6,24)= 0.075, p(6,30)= 0.025

State 7: p(7,1)= 0.8, p(7,13)= 0.075, p(7,19)= 0.025, p(7,25)= 0.075, p(7,31)= 0.025

State 8: p(8,2)= 0.6, p(8,14)= 0.075, p(8,20)= 0.025, p(8,26)= 0.075, p(8,32)= 0.025

State 9: p(9,3)= 0.6, p(9,15)= 0.0375, p(9,21)= 0.0125,
p(9,27)= 0.0375), p(9,33)= 0.0125, p(9,43)= 0.3

State 10: p(10,4)= 0.3, p(10,16)= 0.0375, p(10,22)= 0.0125,
p(10,28)= 0.0375, p(10,34)= 0.0125, p(10,44)= 0.6

State 11: p(11,5)= 0.3, p(11,17)= 0.0375, p(11,23)= 0.0125,
p(11,29)= 0.0375, p(11,35)= 0.0125, p(11,45)= 0.6

State 12: p(12,13)= 0.1, p(12,14)= 0.2, p(12,15)= 0.1,
p(12,24)= 0.6

State 13: p(13,15)= 0.3, p(13,25)= 0.6, p(13,47)= 0.1

State 14: p(14,15)= 0.2, p(14,16)= 0.3, p(14,17)= 0.2
p(14,26)= 0.3

State 15: p(15,17)= 0.3, p(15,27)= 0.6, p(15,47)= 0.1

State 16: p(16,17)= 0.2, p(16,28)= 0.4, p(16,46)= 0.4

State 17: p(17,29)= 0.3, p(17,46)= 0.5, p(17,47)= 0.2

State 18: p(18,12)= 0.2, p(18,19)= 0.1, p(18,20)= 0.2
p(18,21)= 0.1, p(18,30)= 0.4

State 19: p(19,13)= 0.2, p(19,21)= 0.4, p(19,31)= 0.2
p(19,47)= 0.2

State 20: p(20,14)= 0.2, p(20,21)= 0.2, p(20,22)= 0.3
p(20,23)= 0.2, p(20,32)= 0.1

State 21: p(21,15)= 0.2, p(21,23)= 0.4, p(21,33)= 0.2
p(21,47)= 0.2

State 22: p(22,16)= 0.2, p(22,23)= 0.2, p(22,34)= 0.2
p(22,46)= 0.4

State 23: p(23,17)= 0.2, p(23, 35)= 0.2, p(23,46)= 0.4
p(23,47)= 0.2

State 24: p(24,12)= 0.3, p(24,25)= 0.3, p(24,36)= 0.4

State 25: p(25,13)= 0.4, p(25,37)= 0.4, p(25,47)= 0.2

State 26: p(26,14)= 0.3, p(26,27)= 0.3, p(26,38)= 0.4

State 27: p(27,15)= 0.2, p(27,39)= 0.6, p(27,47)= 0.2

State 28: p(28,16)= 0.2, p(28,30)= 0.3, p(28,40)= 0.5

State 29: p(29,17)= 0.1, p(29,41)= 0.7, p(29,47)= 0.2

State 30: p(30,18)= 0.1, p(30.24)= 0.3, p(30,31)= 0.1
p(30,33)= 0.2, p(30,36)= 0.3

State 31: p(31,19)= 0.1, p(31,25)= 0.3, p(31,33)= 0.2
p(31,37)= 0.3, p(31,47)= 0.1

State 32: p(32,20)= 0.1, p(32,26)= 0.3, p(32,33)= 0.2,
p(32,35)= 0.2, p(32,38)= 0.2

State 33: p(33,21)= 0.1, p(33,27)= 0.3, p(33,35)= 0.2,
p(33,39)= 0.3, p(33,47)= 0.1

State 34: p(34,22)= 0.1, p(34,28)= 0.3, p(34,35)= 0.2,
p(34,40)= 0.3, p(34,46)= 0.1

State 35: p(35,23)= 0.1, p(35,29)= 0.3, p(35,41)= 0.3,
p(35,46)= 0.2, p(35,47)= 0.1

State 36: p(36,6)= 1.0

State 37: p(37,6)= 0.4, p(37,7)= 0.6

State 38: p(38,8)= 0.5, p(38,42)= 0.5

State 39: p(39,8)= 0.2, p(39,9)= 0.3, p(39,43)= 0.5

State 40: p(40,10)= 0.4. p(40,44)= 0.6

State 41: p(41,10)= 0.2, p(41,11)= 0.3, p(41,45)= 0.5

State 42: p(42,6)= 1.0

State 43: p(43,7)= 1.0

State 44: p(44,6)= 0.6, p(44,8)= 0.4

State 45: p(45,7)= 0.7, p(45,9)= 0.3

State 46: p(46,46)= 1.0

State 47: p(47,47)= 1.0

APPENDIX D. TRANSITION TIME MATRIX

This appendix contains the the numerical values for the transition times that correspond to the "X"s on the connectivity matrix in Appendix B. The times are broken down by rows and each section is labeled with the current state (row) number. The notation t(N,M) is the transition time for going from state N to state M in one transition. Note that times are only given for transitions with a positive transition probability from Appendix C. Times are expressed in hours.

State 0: t(0,2)= 1.0, t(0,6)= 0.75, t(0,12)= 0.1, t(0,18)= 0.1

State 1: t(1,3)= 0.9, t(1,7)= 0.68, t(1,13)= 0.1, t(1,19)= 0.1

State 2: t(2,4)= 2.0, t(2,8)= 1.5, t(2,14)= 0.1, t(2,20)= 0.1

State 3: t(3,5)= 1.8, t(3,9)= 1.4, t(3,15)= 0.1, t(3,21)= 0.1

State 4: t(4,10)= 0.53, t(4,16)= 0.1, t(4,22)= 0.1, t(4,46)= 0.7

State 5: t(5,11)= 0.38, t(5,17)= 0.1, t(5,23)= 0.1, t(5,46)= 0.5

State 6: t(6,0)= 1.0, t(6,12)= 1.0, t(6,18)= 1.0, t(6,24)= 1.0, t(6,30)= 1.0

State 7: t(7,1)= 1.0, t(7,13)= 1.0, t(7,19)= 1.0, t(7,25)= 1.0, t(7,31)= 1.0

State 8: t(8,2)= 1.0, t(8,14)= 1.0, t(8,20)= 1.0, t(8,26)= 1.0, t(8,32)= 2.0

State 9: t(9,3)= 1.0, t(9,15)= 1.0, t(9,21)= 1.0,
t(9,27)= 1.0, t(9,33)= 1.0, t(9,43)= 2.0

State 10: t(10,4)= 1.0, t(10,16)= 1.0, t(10,22)= 1.0,
t(10,28)= 1.0, t(10,34)= 1.0, t(10,44)= 2.0

State 11: t(11,5)= 1.0, t(11,17)= 1.0, t(11,23)= 1.0,
t(11,29)= 1.0, t(11,35)= 1.0, t(11,45)= 2.0

State 12: t(12,13)= 3.0, t(12,14)= 1.0, t(12,15)= 1.5,
t(12,24)= 0.6

State 13: t(13,15)= 0.6, t(13,25)= 0.36, t(13,47)= 0.5

State 14: t(14,15)= 3.0, t(14,16)= 2.0, t(14,17)= 1.7,
t(14,26)= 1.2

State 15: t(15,17)= 1.2, t(15,27)= 0.72, t(15,47)= 0.5

State 16: t(16,17)= 3.0, t(16,28)= 1.8, t(16,46)= 0.7

State 17: t(17,29)= 0.3, t(17,46)= 0.53, t(17,47)= 0.5

State 18: t(18,12)= 0.2, t(18,19)= 1.5, t(18,20)= 0.5,
t(18,21)= 0.75, t(18,30)= 0.25

State 19: t(19,13)= 0.2, t(19,21)= 0.3, t(19,31)= 0.15,
t(19,47)= 0.5

State 20: t(20,14)= 0.2, t(20,21)= 1.5, t(20,22)= 1.0,
t(20,23)= 0.85, t(20,32)= 0.5

State 21: t(21,15)= 0.2, t(21,23)= 0.6, t(21,33)= 0.3,
t(21,47)= 0.5

State 22: t(22,16)= 0.2, t(22,23)= 1.5, t(22,34)= 0.75,
t(22,46)= 0.3

State 23: t(23,17)= 0.2, t(23, 35)= 0.1, t(23,46)= 0.2,
t(23,47)= 0.5

State 24: t(24,12)= 0.9, t(24,25)= 4.0, t(24,36)= 2.0

State 25: t(25,13)= 0.9, t(25,37)= 2.0, t(25,47)= 2.0

State 26: t(26,14)= 0.9, t(26,27)= 4.0, t(26,38)= 2.0

State 27: t(27,15)= 0.9, t(27,39)= 2.0, t(27,47)= 2.0

State 28: t(28,16)= 0.9, t(28,30)= 4.0, t(28,40)= 2.0

State 29: t(29,17)= 0.9, t(29,41)= 2.0, t(29,47)= 2.0

State 30: t(30,18)= 0.5, t(30.24)= 0.1, t(30,31)= 0.3,
t(30,33)= 0.5, t(30,36)= 2.0

State 31: t(31,19)= 0.5, t(31,25)= 0.1, t(31,33)= 0.5,
t(31,37)= 0.5, t(31,47)= 2.0

State 32: t(32,20)= 0.5, t(32,26)= 0.1, t(32,33)= 0.5,
t(32,35)= 1.0, t(32,38)= 2.0

State 33: t(33,21)= 0.5, t(33,27)= 0.1, t(33,35)= 0.6,
t(33,39)= 2.0, t(33,47)= 0.7

State 34: t(34,22)= 0.5, t(34,28)= 0.1, t(34,35)= 0.4,
t(34,40)= 2.0, t(34,46)= 0.5

State 35: t(35,23)= 0.5, t(35,29)= 0.1, t(35,41)= 2.0,
t(35,46)= 0.5, t(35,47)= 0.7

State 36: t(36,6)= 4.0

State 37: t(37,6)= 6.0, t(37,7)= 4.0

State 38: t(38,8)= 4.0, t(38,42)= 4.0

State 39: t(39,8)= 6.0, t(39,9)= 4.0, t(39,43)= 4.0

State 40: t(40,10)= 4.0, t(40,44)= 4.0

State 41: t(41,10)= 7.0, t(41,11)= 5.0, t(41,45)= 5.0

State 42: t(42,6)= 5.0

State 43: t(43,7)= 5.0

State 44: t(44,6)= 8.0, t(44,8)= 8.0

State 45: t(45,7)= 8.0, t(45,9)= 8.0

State 46: t(46,46)= 1.0

State 47: t(47,47)= 1.0

APPENDIX E. SAMPLE MODEL OUTPUT

This appendix contains the model outputs that were used for the sample runs in Chapter IV. The first series of reports are the Summary reports that are given for each initial state. The report shows the total time until absorption, the actual time (hours) spent in each class of states and the percent of the unit's total life that was spent in each class of states. Below the Summary report are three numbers; the initial state number, the probability of absorption in COMBAT INEFFECTIVE-ATTRITION and the probability of absorption in COMBAT INEFFECTIVE-DEGRADATION. The first set of reports are created by the function AUTO and are displayed by the function REPORT (see Appendix G. APL FUNCTIONS).

```
      (⎕FREAD 1 4)[0;;]
   FOR THE BASE P10T1 T10T1 CASE
   AND INITIAL  STATE OF ENGAGED-CONVENTIONAL-HIGH-HIGH

   THE TOTAL UNIT LIFE UNTIL COMBAT INEFFECTIVE WAS  29.840 HRS

   OF ITS USEFUL LIFE THE UNIT SPENT IT IN THE FOLLOWING AREAS

                    HOURS          PERCENT OF LIFE

COMBAT CLEAN        5.740                19.24
COMBAT DIRTY        3.239                10.85
DISENGAGED CLEAN    5.189                17.39
DISENGAGED DIRTY    3.949                13.23
DECONTAMINATION     4.339                14.54
REFIT               7.384                24.75

      0             0.5866583374  0.4133416626


      ^
   (⎕FREAD 1 4)[9;;]
   FOR THE BASE P10T1 T10T1 CASE
   AND INITIAL  STATE OF DISENGAGED-CONVENTIONAL-MEDIUM-LOW

   THE TOTAL UNIT LIFE UNTIL COMBAT INEFFECTIVE WAS  29.092 HRS

   OF ITS USEFUL LIFE THE UNIT SPENT IT IN THE FOLLOWING AREAS

                    HOURS          PERCENT OF LIFE

COMBAT CLEAN        5.339                18.35
COMBAT DIRTY        1.523                 5.23
DISENGAGED CLEAN    7.198                24.74
DISENGAGED DIRTY    2.682                 9.22
DECONTAMINATION     3.647                12.54
REFIT               8.704                29.92

      9             0.4534389037  0.5465610963


   FOR THE BASE P10T1 T10T1 CASE
   AND INITIAL  STATE OF ENGAGED-CHEMICAL-MOPP-HIGH-HIGH

   THE TOTAL UNIT LIFE UNTIL COMBAT INEFFECTIVE WAS  28.304 HRS

   OF ITS USEFUL LIFE THE UNIT SPENT IT IN THE FOLLOWING AREAS

                    HOURS          PERCENT OF LIFE

COMBAT CLEAN        3.429                12.12
COMBAT DIRTY        4.109                14.52
DISENGAGED CLEAN    4.084                14.43
DISENGAGED DIRTY    5.510                19.47
DECONTAMINATION     5.755                20.33
REFIT               5.416                19.14

     12             0.460368813   0.539631187
```

```
(□FREAD 1 4)[19;;]
   FOR THE BASE P10T1 T10T1 CASE
   AND INITIAL  STATE OF ENGAGED-CHEMICAL-NO MOPP-HIGH-LOW

   THE TOTAL UNIT LIFE UNTIL COMBAT INEFFECTIVE WAS  13.768 HRS

   OF ITS USEFUL LIFE THE UNIT SPENT IT IN THE FOLLOWING AREAS

                      HOURS            PERCENT OF LIFE

COMBAT CLEAN          1.746                 12.68
COMBAT DIRTY          1.589                 11.54
DISENGAGED CLEAN      2.174                 15.79
DISENGAGED DIRTY      2.200                 15.98
DECONTAMINATION       3.010                 21.86
REFIT                 3.050                 22.15

      19              0.2929669279  0.7070330721


(□FREAD 1 4)[35;;]
   FOR THE BASE P10T1 T10T1 CASE
   AND INITIAL  STATE OF DISENGAGED-CHEMICAL-NO MOPP-LOW-LOW

   THE TOTAL UNIT LIFE UNTIL COMBAT INEFFECTIVE WAS  22.178 HRS

   OF ITS USEFUL LIFE THE UNIT SPENT IT IN THE FOLLOWING AREAS

                      HOURS            PERCENT OF LIFE

COMBAT CLEAN          2.510                 11.32
COMBAT DIRTY           .993                  4.48
DISENGAGED CLEAN      3.594                 16.21
DISENGAGED DIRTY      3.022                 13.62
DECONTAMINATION       4.855                 21.89
REFIT                 7.204                 32.48

      35              0.5294819827  0.4705180173


(□FREAD 1 4)[44;;]
   FOR THE BASE P10T1 T10T1 CASE
   AND INITIAL  STATE OF REFIT-LOW-HIGH

   THE TOTAL UNIT LIFE UNTIL COMBAT INEFFECTIVE WAS  38.834 HRS

   OF ITS USEFUL LIFE THE UNIT SPENT IT IN THE FOLLOWING AREAS

                      HOURS            PERCENT OF LIFE

COMBAT CLEAN          5.159                 13.28
COMBAT DIRTY          3.218                  8.29
DISENGAGED CLEAN      6.152                 15.84
DISENGAGED DIRTY      4.135                 10.65
DECONTAMINATION       4.515                 11.63
REFIT                15.655                 40.31

      44              0.5787679768  0.4212320232
```

This next report is a sample of the Detailed report also created by the function AUTO. This report shows the states visited, the number of visits, the sojourn time for each visit and the total time spent in each state for all states in which the unit spent at least one percent of it's total life. The heading information is the same as that for the Summary report.

```
DFREAD 1 5
   FOR THE BASE P10T1 T10T1 CASE
         AND INITIAL STATE OF ENGAGED-CONVENTIONAL-HIGH-HIGH

   THE TOTAL UNIT LIFE UNTIL COMBAT INEFFECTIVE WAS  29.840 HRS

   OF ITS USEFUL LIFE THE UNIT SPENT IT IN THE FOLLOWING STATES
```

STATES	VISITS	TIME EACH VISIT	TOTAL TIME IN STATE
ENGAGED-CONVENTIONAL-HIGH-HIGH	1.979	.770	1.524
ENGAGED-CONVENTIONAL-HIGH-LOW	.516	.674	.348
ENGAGED-CONVENTIONAL-MEDIUM-HIGH	1.932	1.370	2.646
ENGAGED-CONVENTIONAL-MEDIUM-LOW	.569	1.350	.768
ENGAGED-CONVENTIONAL-LOW-HIGH	.757	.521	.394
DISENGAGED-CONVENTIONAL-HIGH-HIGH	1.223	1.000	1.223
DISENGAGED-CONVENTIONAL-HIGH-LOW	.645	1.000	.645
DISENGAGED-CONVENTIONAL-MEDIUM-HIGH	1.241	1.200	1.489
DISENGAGED-CONVENTIONAL-MEDIUM-LOW	.519	1.300	.674
DISENGAGED-CONVENTIONAL-LOW-HIGH	.590	1.600	.944
ENGAGED-CHEMICAL-MOPP-HIGH-HIGH	.551	1.010	.556
ENGAGED-CHEMICAL-MOPP-MEDIUM-HIGH	.612	1.900	1.163
ENGAGED-CHEMICAL-MOPP-MEDIUM-LOW	.444	.842	.374
ENGAGED-CHEMICAL-MOPP-LOW-HIGH	.308	1.600	.493
DISENGAGED-CHEMICAL-MOPP-HIGH-HIGH	.448	2.270	1.017
DISENGAGED-CHEMICAL-MOPP-HIGH-LOW	.407	1.560	.635
DISENGAGED-CHEMICAL-MOPP-MEDIUM-HIGH	.291	2.270	.660
DISENGAGED-CHEMICAL-MOPP-MEDIUM-LOW	.391	1.780	.696
DISENGAGED-CHEMICAL-MOPP-LOW-HIGH	.152	2.380	.362
DISENGAGED-CHEMICAL-MOPP-LOW-LOW	.176	1.890	.332
DECONTAMINATION-HIGH-HIGH	.205	4.000	.820
DECONTAMINATION-HIGH-LOW	.174	4.800	.834
DECONTAMINATION-MEDIUM-HIGH	.126	4.000	.503
DECONTAMINATION-MEDIUM-LOW	.252	4.400	1.110
DECONTAMINATION-LOW-HIGH	.083	4.000	.332
DECONTAMINATION-LOW-LOW	.137	5.400	.740
REFIT-MEDIUM-HIGH	.311	5.000	1.555
REFIT-MEDIUM-LOW	.282	5.000	1.409
REFIT-LOW-HIGH	.404	8.000	3.231
REFIT-LOW-LOW	.149	8.000	1.189

```
      FF
```

APPENDIX F. EXPERIMENTAL IDENTIFIER MATRICES AND DATA

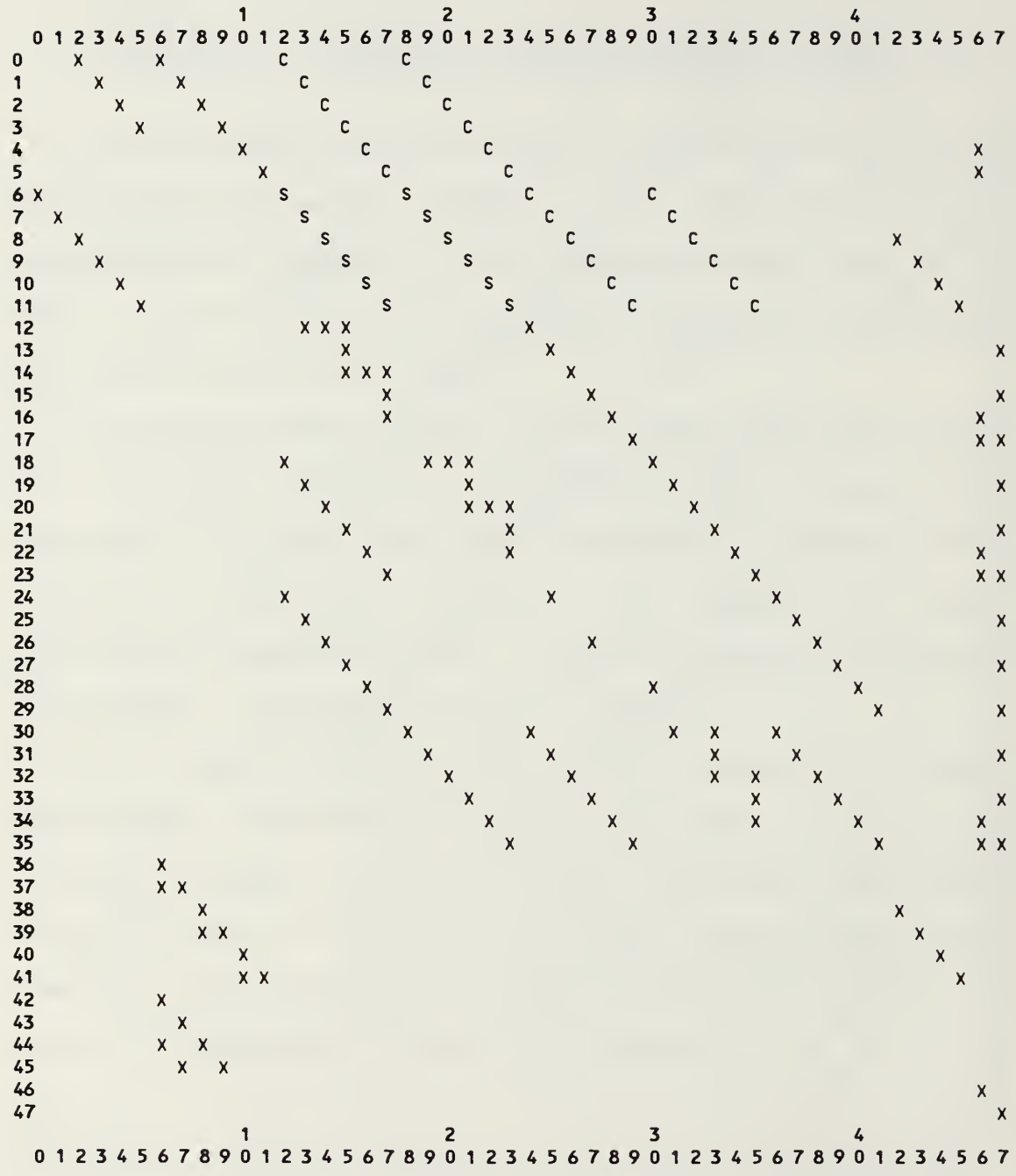
This appendix contains connectivity matrices that identify which probabilities and transition times were changed for the experiments in Chapter V. The notation is the same as that used in Appendix B with the exception of special letters which are explained when the appear.

The first matrix on the following page shows the entries which were set to zero for the CHEMONLY case. These entries correspond to transitions from DISENGAGED to ENGAGED states (conventional weapons attacks). The letter "E" denotes conventional weapons attacks only and "S" denotes simultaneous conventional and chemical weapons attacks.

The second matrix shows the transitions between CONVENTIONAL to CHEMICAL states which were varied for the HALFCHEM, TWICCHEM, and NOCHEM cases. The letter "C'" denotes chemical weapons only attacks and "S" denotes simultaneous chemical and conventional weapons attacks. How they are varied is described in Table II., Chapter V.

Below is the conventional weapons attacks identifier matrix.

Below is the chemical weapons attacks identifier matrix.

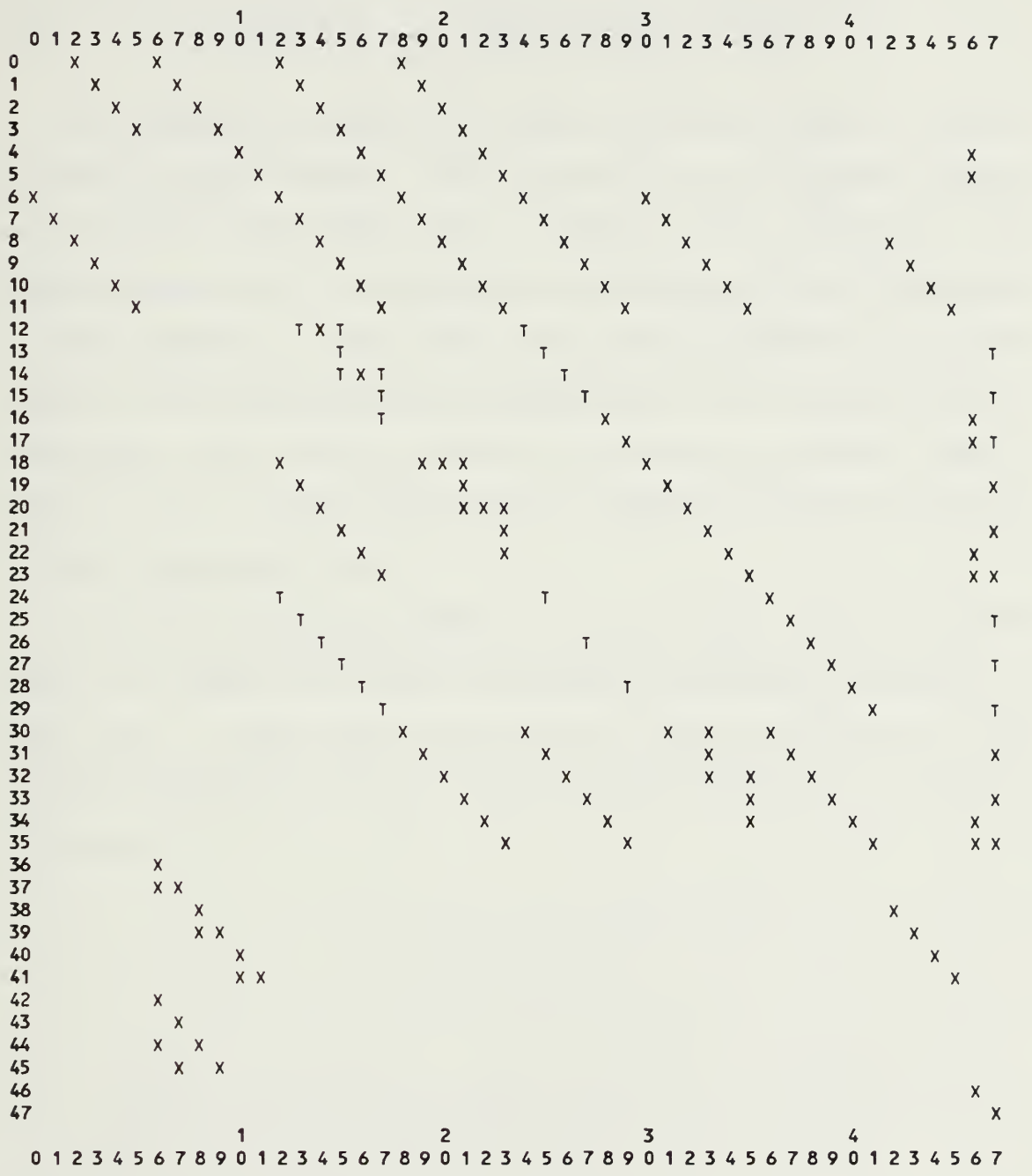


The matrix below shows which transition times were varied for the TBAD case given in Chapter V. (shown by the letter "T").The actual values for the TBAD case are given below the matrix.

t(12,13)= 1.65, t(12,14)= 0.55, t(12,15)= 0.825, t(12,24)= 0.275
t(13,15)= 0.33, t(13.25)= 0.165
t(14,15)= 1.65, t(14,16)= 1.1, t(14,17)= 0.935, t(14,26)= 0.55
t(15,17)= 0.66, t(15,27)= 0.33
t(16,17)= 1.65, t(16,28)= 0.825, t(16,46)= 0.33
t(17,29)= 0.11, t(17,46)= 0.22
t(24,12)= 0.55, t(24,25)= 0.33
t(25,13)= 0.55, t(25,47)= 0.77
t(26,14)= 0.55, t(26,27)= 0.55

t(27,15)= 0.55, t(27,47)= 0.77
t(28,16)= 0.55, t(28,29)= 0.44
t(29,17)= 0.55, t(29,47)= 0.77

The matrix below shows which transition times were varied for the TGOOD case given in Chapter V. (shown by the letter "T").The actual values for the TGOOD case are given below the matrix.



t(12,13)= 4.0, t(12,15)= 2.0, t(12,24)= 0.75
t(13,15)= 0.81, t(13.25)= 0.608, T(13,47)= 0.7
t(14,15)= 4.0, t(14,17)= 2.0, t(14,26)= 1.5
t(15,17)= 1.62, t(15,27)= 1.22, T(15,47)= 0.7
t(16,17)= 4.0
t(17,47)= 0.7
t(24,12)= 1.0, t(24,25)= 6.0
t(25,13)= 1.0, t(25,47)= 3.0
t(26,14)= 1.0, t(26,27)= 6.0

```
t(27,15)= 1.0, t(27,47)= 3.0
t(28,16)= 1.0, t(28,29)= 6.0
t(29,17)= 1.0, t(29,47)= 3.0
```

APPENDIX G. APL FUNCTIONS USED IN THESIS

This appendix contains the APL functions used for model analysis in this thesis. The APL used is the APL*PLUS System for the PC from STSC, Inc. Some of the functions are documented and an introduction is included for each function in this appendix. Some of the functions are general in nature but the majority of them are "hardwired" to the specific model considered here and the author's particular hardware setup. Lines of code without bracketed line numbers are continuations of the lines above them.

The first function, AUTO, is the overall function which solves the model and writes the output to the micro-computer's hard disk. AUTO uses the function FUND to solve for the fundamental matrix used in the solution technique from Taylor and Karlin.

```
      ⎕VR 'AUTO'
    ∇ T AUTO P;A;Q;R;CASE;SIZE;TABS;TSP;COMCLEAN;COMDIRTY;DI
      SCLEAN;DISDIRTY;DECON;REFIT;LINE;LINE3;LINE5;LINE7;W;M
      USOJ;LINED;OUTPUT;NUM;ALLTABS;LT;OUTDET;TSPOUT;TSPMAT;
      C1;C2;C3;C4;U;TABHEAD;TABNUM;TABOUT;DESCRIBE;FILE;HEAD
      ER;N;ST;AT;TABLE;I
[1]
[2]   ⍝  THIS FUNCTION PROVIDES THE COMPLETE BREAKOUT OF WHER
      E A UNIT
[3]   ⍝  SPENT ITS USEFUL LIFE.  THE SOJOURN TIMES ARE COMBIN
      ED INTO THE
[4]   ⍝  FOLLOWING AREAS; COMBAT CLEAN, COMBAT DIRTY, DISENGA
      GED CLEAN,
[5]   ⍝  DISENGAGED DIRTY, DECONTAMINATION AND REFIT.  THE AC
      TUAL TIME AND
[6]   ⍝  THE PERCENTAGE OF THE TOTAL UNIT LIFE ARE GIVEN.
[7]
[8]    '  INPUT THE NUMBER OF ABSORBING STATES '
[9]    A←⎕
[10]
[11]   '  INPUT THE CASE NAME FOR THIS RUN '
[12]   CASE←⍞
[13]
[14]  ⍝  CREATE A FILE TO HOLD THE FUNCTION OUTPUT
[15]
[16]   '  INPUT THE NAME OF THE FILE FOR YOUR OUTPUT'
[17]   '  THE NAMES CURRENTLY IN USE ARE SHOWN BELOW'
[18]   ⍞←⎕FNAMES
[19]   FILE←⍞
[20]   '  INPUT THE FILE TIE NUMBER FOR YOUR FILE'
[21]   '  THE TIE NUMBERS CURRENTLY IN USE ARE SHOWN BELOW'
[22]   ⍞←⎕FNUMS
[23]   NUM←⎕
[24]
[25]   FILE ⎕FCREATE NUM
[26]
[27]  ⍝  THE FIRST COMPONENT OF THE FILE IS THIS DESCRIBE VAR
      IABLE
[28]  ⍝  WHICH DESCRIBES THE CONTENTS OF THE FILE
[29]
[30]   DESCRIBE← 12 80 ⍴' '
[31]   DESCRIBE[0;]←80↑'THIS FILE CONTAINS THE ',CASE,' RUN.'
[32]   DESCRIBE[1;]←80↑'THE COMPONENTS IN THIS FILE ARE AS FO
      LLOWS:'
[33]   DESCRIBE[2;]←80↑'COMP 1: THIS DESCRIPTION OF FILE CONT
      ENTS.'
[34]   DESCRIBE[3;]←80↑'COMP 2: THE ABSORPTION PROBABILITIES
      FOR THIS CASE.'
[35]   DESCRIBE[4;]←80↑'COMP 3: THE TIMES TO ABSORPTION ALL S
      TATES'
[36]   DESCRIBE[5;]←80↑'COMP 4: SUMMARIZED LIFE HISTORY OF UN
```

```
      IT FOR EACH INITIAL STATE.'
[37]  DESCRIBE[6;]←80↑'       THIS COMPONENT IS IN THE FORM
      OF A 46×15×75 CHARACTER'
[38]  DESCRIBE[7;]←80↑'       MATRIX WHERE EACH PAGE CORRESP
      ONDS TO AN INITIAL STATE.'
[39]  DESCRIBE[8;]←80↑'COMPS 5→50:  DETAILED LIFE HISTORY OF
       UNIT FOR EACH INITIAL STATE.'
[40]  DESCRIBE[9;]←80↑'             EACH COMPONENT IS FOR AN
       INITIAL STARTING STATE.'
[41]  DESCRIBE[10;]←80↑'            TO GET THE REPORT FOR IN
      ITIAL STATE N REFER TO COMPONENT'
[42]  DESCRIBE[11;]←80↑'            N+5.'
[43]
[44]  ⎕TCFF
[45]
[46]  DESCRIBE ⎕FAPPEND NUM
[47]
[48] ⍝  FIND THE FUNDAMENTAL MATRIX
[49]
[50]  W←A FUND P
[51]
[52] ⍝  PULL THE P MATRIX INTO ITS COMPONENT PARTS
[53]
[54]  SIZE←⍴P
[55]  Q←(SIZE-A)↑P
[56]  R←((SIZE[1]-A),0-A)↑P
[57]
[58] ⍝  ASSIGN THE PROBABILITIES OF ABSORPTION
[59]
[60]  U←W+.×R
[61]
[62] ⍝  FIND THE MEAN SOJOURN TIME FOR EACH STATE
[63]
[64]  MUSOJ←(0-A)↓+/P×T
[65]
[66] ⍝  THE TIME TO ABSORPTION FROM EACH INITIAL STATE
[67]
[68]  ALLTABS←,W+.×((⍴MUSOJ),1)⍴MUSOJ
[69]  ((46 1 ⍴(⍳46)),U)⎕FAPPEND NUM
[70]
[71]  TABHEAD← 6 70 ⍴' '
[72]  TABHEAD[0;]←70↑'THE TOTAL TIME TO ABSORPTION FROM EACH
       OF THE INITIAL'
[73]  TABHEAD[1;]←70↑'STATES FOR THE ',CASE,' CASE '
[74]  TABHEAD[2;]←70↑'IS SHOWN BELOW'
[75]  TABHEAD[3;]←70⍴' '
[76]  TABHEAD[4;]←70↑'   STATE        TABS        STATE
         TABS '
[77]  TABHEAD[5;]←70⍴' '
[78]  ST←⍉ 2 23 ⍴⍳46
[79]  AT←⍉ 2 23 ⍴ALLTABS
```

```
[80]  TABNUM←'I6,F15.4,I10,F15.4' ⎕FMT(ST[;0];AT[;0];ST[;1];
      AT[;1])
[81]  TABOUT←TABHEAD,[0] 23 70 ↑TABNUM
[82]
[83]  TABOUT ⎕FAPPEND NUM
[84]
[85]
[86]
[87] ⍝  CREATE AN EMPTY BASKET TO HOLD THE GROUPED OUTPUT
[88]
[89]  OUTPUT← 46 15 70 ⍴' '
[90]
[91]  INIT←0
[92]
[93]  TSPOUT←(⍴MUSOJ)⍴0
[94] ⍝  START A LOOP
[95]
[96] LOOP1:
[97]
[98] ⍝  FIND THE TIME SPENT IN EACH STATE
[99]
[100]  TSP←MUSOJ×W[INIT;]
[101]
[102] ⍝  FIND THE TIMES FOR EACH CATEGORY
[103]
[104]  COMCLEAN←+/((6⍴1),40⍴0)/TSP
[105]  COMDIRTY←+/((12⍴0),(12⍴1),22⍴0)/TSP
[106]  DISCLEAN←+/((6⍴0),(6⍴1),34⍴0)/TSP
[107]  DISDIRTY←+/((24⍴0),(12⍴1),10⍴0)/TSP
[108]  DECON←+/((36⍴0),(6⍴1),4⍴0)/TSP
[109]  REFIT←+/((42⍴0),4⍴1)/TSP
[110]
[111] ⍝  OUTPUT SECTION
[112]
[113]  OUTPUT[INIT;0;]←'          FOR THE ',CASE,' CASE ',((7
      0-23)-⍴CASE)⍴' '
[114]  OUTPUT[INIT;1;]←'           AND INITIAL STATE OF ',STA
      TNAME[INIT;]
[115]  OUTPUT[INIT;2;]←70⍴' '
[116]
[117]  LINE3←'          THE TOTAL UNIT LIFE UNTIL COMBAT INEF
      FECTIVE WAS ',(7 3 ⍕ALLTABS[INIT]),' HRS'
[118]  OUTPUT[INIT;3;]←LINE3,(70-⍴LINE3)⍴' '
[119]
[120]  OUTPUT[INIT;4;]←70⍴' '
[121]
[122]  LINE5←'          OF ITS USEFUL LIFE THE UNIT SPENT IT
      IN THE FOLLOWING AREAS  '
[123]  OUTPUT[INIT;5;]←LINE5,(70-⍴LINE5)⍴' '
[124]
[125]  OUTPUT[INIT;6;]←70⍴' '
```

```
[126]
[127]  LINE7←'                              HOURS              PERCE
      NT OF LIFE                '
[128]  OUTPUT[INIT;7;]←LINE7,(70-ρLINE7)ρ' '
[129]
[130]  OUTPUT[INIT;8;]←70ρ' '
[131]
[132]  LINE←'  COMBAT CLEAN          ',(6 3 ⍕COMCLEAN),'
                 ', 6 2 ⍕(100×COMCLEAN÷ALLTABS[INIT])
[133]  OUTPUT[INIT;9;]←LINE,(70-ρLINE)ρ' '
[134]
[135]  LINE←'  COMBAT DIRTY          ',(6 3 ⍕COMDIRTY),'
                 ', 6 2 ⍕(100×COMDIRTY÷ALLTABS[INIT])
[136]  OUTPUT[INIT;10;]←LINE,(70-ρLINE)ρ' '
[137]
[138]  LINE←'  DISENGAGED CLEAN      ',(6 3 ⍕DISCLEAN),'
                 ', 6 2 ⍕(100×DISCLEAN÷ALLTABS[INIT])
[139]  OUTPUT[INIT;11;]←LINE,(70-ρLINE)ρ' '
[140]
[141]  LINE←'  DISENGAGED DIRTY      ',(6 3 ⍕DISDIRTY),'
                 ', 6 2 ⍕(100×DISDIRTY÷ALLTABS[INIT])
[142]  OUTPUT[INIT;12;]←LINE,(70-ρLINE)ρ' '
[143]
[144]  LINE←'  DECONTAMINATION       ',(6 3 ⍕DECON),'
               ', 6 2 ⍕(100×DECON÷ALLTABS[INIT])
[145]  OUTPUT[INIT;13;]←LINE,(70-ρLINE)ρ' '
[146]
[147]  LINE←'  REFIT                 ',(6 3 ⍕REFIT),'
               ', 6 2 ⍕(100×REFIT÷ALLTABS[INIT])
[148]  OUTPUT[INIT;14;]←LINE,(70-ρLINE)ρ' '
[149]
[150]  TSPOUT←TSPOUT,TSP
[151]
[152]  →(INIT=45)ρENDOFLOOP1
[153]  INIT←INIT+1
[154]  →LOOP1
[155]
[156] ENDOFLOOP1:OUTPUT □FAPPEND NUM
[157]
[158] ⍝  BECAUSE THE DETAILED OUTPUT IS TOO LARGE TO BE COMP
      UTED AS A MATRIX
[159] ⍝  THIS SECTION COMPUTES AND FEEDS EACH PAGE OF OUTDET
       TO THE FILE
[160] ⍝  AS A SEPARATE COMPONENT
[161]
[162]  TSPMAT←((ρMUSOJ),ρMUSOJ)ρ(ρMUSOJ)↓TSPOUT
[163]
[164]  INIT←0
[165]
[166] ⍝  THIS SECTION PROVIDES THE MOST COMPLETE BREAKOUT OF
       WHERE A
```

```
[167] ⍝  UNIT SPENT ITS USEFUL LIFE.  EACH STATE IN WHICH TH
      E UNIT STAYED
[168] ⍝  FOR AT LEAST ONE PERCENT OF ITS LIFE IS LISTED ALON
      G WITH ITS
[169] ⍝  PROBABILITY OF ABSORPTION IN EACH FINAL STATE.
[170]
[171]  HEADER← 10 75 ⍴' '
[172]
[173] LOOP2:
[174]
[175] ⍝  THIS SECTION MAKES THE HEADERS FOR EACH PLANE OF TH
      E VARIABLE OUTDET
[176]
[177]  HEADER[0;]←'          FOR THE ',CASE,' CASE ',((75-23)
      -⍴CASE)⍴' '
[178]  HEADER[1;]←'                AND INITIAL STATE OF ',STA
      TNAME[INIT;]
[179]  HEADER[2;]←75⍴' '
[180]
[181]  LINED←'          THE TOTAL UNIT LIFE UNTIL COMBAT INEF
      FECTIVE WAS ',(7 3 ⍕ALLTABS[INIT]),' HRS'
[182]  HEADER[3;]←LINED,(75-⍴LINED)⍴' '
[183]
[184]  HEADER[4;]←75⍴' '
[185]
[186]  LINED←'          OF ITS USEFUL LIFE THE UNIT SPENT IT
      IN THE FOLLOWING STATES '
[187]  HEADER[5;]←LINED,(75-⍴LINED)⍴' '
[188]
[189]  HEADER[6;]←75⍴' '
[190]
[191]  LINED←'           STATES                              VIS
      ITS   TIME EACH   TOTAL TIME'
[192]  HEADER[7;]←LINED,(75-⍴LINED)⍴' '
[193]
[194]  LINED←'
             VISIT       IN STATE  '
[195]  HEADER[8;]←LINED,(75-⍴LINED)⍴' '
[196]  HEADER[9;]←75⍴' '
[197]
[198]  LT←,ALLTABS
[199]  I←((TSPMAT[INIT;]÷LT[INIT])≥0.01)
[200]  N←+/I
[201]  C1←I⌿(¯2 0)↓STATNAME
[202]  C2←(10 3)⍕((N,1)⍴(I/,W[INIT;]))
[203]  C3←(10 3)⍕(N,1)⍴(I/,MUSOJ)
[204]  C4←(10 3)⍕((N,1)⍴(I/,TSPMAT[INIT;]))
[205]  TABLE←(N,75)↑C1,C2,C3,C4
[206]  OUTDET←HEADER,[0]TABLE
[207]
[208]  OUTDET ⎕FAPPEND NUM
```

```
[209]
[210]  →(INIT=45)ρENDOFLOOP2
[211]  INIT←INIT+1
[212]  →LOOP2
[213]
[214] ENDOFLOOP2:'THE COMPLETE RUN FOR THE ',CASE,' CASE IS
      DONE'
[215]  '  FOR A COMPLETE BREAKOUT OF WHAT THE UNIT DID SEE'
[216]  '  THE FILE CALLED ',FILE,'.'
[217]  '  THE TIE NUMBER FOR THIS FILE IS ',(⍕NUM),'.'
[218]  '  TO GET A DESCRIPTION OF THE FILE CONTENTS TYPE ★ □
      FREAD ',(⍕(NUM,1)),' ★.'
[219]
    ∇
```

The following function, FUND, solves for the fundamental matrix W. This function is called by the function AUTO.

```
      □VR 'FUND'
    ∇ W←A FUND P;Q;I;SIZE
[1]   ⍝  THIS FUNCTION FINDS THE FUNDAMENTAL MATRIX GIVEN THE
       ONE STEP
[2]   ⍝  PROBABILITY MATRIX, P, AND THE NUMER OF ABSORBING ST
      ATES, A, FOR
[3]   ⍝  A SEMI-MARKOV PROCESS.
[4]   ⍝
[5]   ⍝  FIND THE SIZE OF THE Q MATRIX AND NEEDED IDENTITY MA
      TRIX
[6]
[7]   SIZE←(ρP)-A
[8]   Q←SIZE↑P
[9]   I←SIZEρ1,(1↑SIZE)ρ0
[10]
[11]  ⍝  SOLVE FOR THE FUNDAMENTAL MATRIX W.
[12]
[13]  W←⌹(I-Q)
    ∇
```

The function CHEMPROB (following pages) is used to vary the probability of chemical agent attacks from various states.

```
      ⎕VR 'CHEMPROB'
    ∇ CHEMPROB P;FACTOR;B;SCHEM;DELTA;COUNT
[1]
[2]  ⍝  THIS FUNCTION ALLOWS THE USER TO VARY THE PROBABILIT
      Y OF
[3]  ⍝  COMING UNDER CHEMICAL ATTACK FOR ALL STATES THAT LEA
      D TO
[4]  ⍝  POSSIBLE CHEM ATTACKS.
[5]
[6]  ⍝  INPUT SECTION
[7]
[8]   '  INPUT THE FACTOR TO CHANGE THE PROBABILITY OF CHEMI
      CAL ATTACK'
[9]   '  FACTORS GREATER THAN ONE WILL INCREASE AND FACTORS
      LESS THAN '
[10]  '  ONE WILL DECREASE THE OVERALL CHANCE OF CHEMICAL AT
      TACKS.  '
[11]  '  INPUT FACTORS CAN BE BETWEEN 0 TO 3 AND DO NOT HAVE
       TO BE INTEGER.'
[12]
[13]  FACTOR←⎕
[14]
[15]  '  TO VARY ONLY THE CHANCE OF CHEMICAL ATTACKS WHILE E
      NGAGED '
[16]  '  INPUT ***E***.  TO VARY THE CHANCE OF CHEMICAL ONLY
       ATTACKS'
[17]  '  WHILE NOT ENGAGED INPUT ***C***.  TO VARY THE CHANC
      E OF '
[18]  '  CHEMICAL ATTACKS FOR ALL POSSIBLE STATES INPUT ***A
      ***'
[19]
[20]  B←⎕
[21]
[22] ⍝  GUTS OF THE PROGRAM.   THIS SECTION IS HARDWIRED FOR
       A PARTICULAR
[23] ⍝  STATE SPACE.
[24]
[25]  COUNT←0
[26]  PNEW←P
[27]
[28]
[29] ⍝  JUMP TO THE CHEM ONLY SECTION IF REQUESTED
[30]
[31]  →(B='C')⍴CHEMRESET
[32]
[33] ⍝  LOOP TO DO THE ENGAGED CHEM ATTACKS
[34]
[35] ATTKLOOP:SCHEM←P[COUNT;(12+COUNT)]+P[COUNT;(18+COUNT)]
[36]  PNEW[COUNT;(12+COUNT)]←FACTOR×P[COUNT;(12+COUNT)]
[37]  PNEW[COUNT;(18+COUNT)]←FACTOR×P[COUNT;(18+COUNT)]
[38]  DELTA←PNEW[COUNT;(12+COUNT)]+PNEW[COUNT;(18+COUNT)]-SC
```

```
      HEM
[39]  →(COUNT≥4)ρBLOTTOCASE
[40]
[41]  PNEW[COUNT;(2+COUNT)]←P[COUNT;(2+COUNT)]-P[COUNT;(2+CO
      UNT)]×DELTA÷(1-SCHEM)
[42]  PNEW[COUNT;(6+COUNT)]←P[COUNT;(6+COUNT)]-P[COUNT;(6+CO
      UNT)]×DELTA÷(1-SCHEM)
[43]  COUNT←COUNT+1
[44]  →ATTKLOOP
[45] BLOTTOCASE:
[46]  PNEW[COUNT;46]←P[COUNT;46]-P[COUNT;46]×DELTA÷(1-SCHEM)
[47]  PNEW[COUNT;(6+COUNT)]←P[COUNT;(6+COUNT)]-P[COUNT;(6+CO
      UNT)]×DELTA÷(1-SCHEM)
[48]  COUNT←COUNT+1
[49]  →(COUNT≤5)ρATTKLOOP
[50]
[51] A  JUMP TO CONTINUE ***ALL*** CALCULATIONS IF REQUESTED
[52]  →(B='A')ρALL
[53]
[54] SECONDCASE:SCHEM←P[COUNT;(6+COUNT)]+P[COUNT;(12+COUNT)]
[55]  PNEW[COUNT;(6+COUNT)]←FACTOR×P[COUNT;(6+COUNT)]
[56]  PNEW[COUNT;(12+COUNT)]←FACTOR×P[COUNT;(12+COUNT)]
[57]  DELTA←PNEW[COUNT;(6+COUNT)]+PNEW[COUNT;(12+COUNT)]-SCH
      EM
[58]  PNEW[COUNT;(COUNT-6)]←P[COUNT;(COUNT-6)]-P[COUNT;(COUN
      T-6)]×DELTA÷(1-SCHEM)
[59]  PNEW[COUNT;(18+COUNT)]←P[COUNT;(18+COUNT)]-P[COUNT;(18
      +COUNT)]×DELTA÷(1-SCHEM)
[60]  PNEW[COUNT;(24+COUNT)]←P[COUNT;(24+COUNT)]-P[COUNT;(24
      +COUNT)]×DELTA÷(1-SCHEM)
[61]
[62]  COUNT←COUNT+1
[63]  →(COUNT≤7)ρSECONDCASE
[64]
[65] THIRDCASE:SCHEM←P[COUNT;(6+COUNT)]+P[COUNT;(12+COUNT)]
[66]  PNEW[COUNT;(6+COUNT)]←FACTOR×P[COUNT;(6+COUNT)]
[67]  PNEW[COUNT;(12+COUNT)]←FACTOR×P[COUNT;(12+COUNT)]
[68]  DELTA←PNEW[COUNT;(6+COUNT)]+PNEW[COUNT;(12+COUNT)]-SCH
      EM
[69]  PNEW[COUNT;(COUNT-6)]←P[COUNT;(COUNT-6)]-P[COUNT;(COUN
      T-6)]×DELTA÷(1-SCHEM)
[70]  PNEW[COUNT;(18+COUNT)]←P[COUNT;(18+COUNT)]-P[COUNT;(18
      +COUNT)]×DELTA÷(1-SCHEM)
[71]  PNEW[COUNT;(24+COUNT)]←P[COUNT;(24+COUNT)]-P[COUNT;(24
      +COUNT)]×DELTA÷(1-SCHEM)
[72]  PNEW[COUNT;(34+COUNT)]←P[COUNT;(34+COUNT)]-P[COUNT;(34
      +COUNT)]×DELTA÷(1-SCHEM)
[73]
[74]  COUNT←COUNT+1
[75]  →(COUNT≤11)ρTHIRDCASE
[76]  →END
```

```
[77]
[78]
[79] ⍝  ENTER INTO THE CHEM ONLY LOOP
[80] CHEMRESET:COUNT←6
[81]
[82] CHEM:SCHEM←P[COUNT;(18+COUNT)]+P[COUNT;(24+COUNT)]
[83]  PNEW[COUNT;(18+COUNT)]←FACTOR×P[COUNT;(18+COUNT)]
[84]  PNEW[COUNT;(24+COUNT)]←FACTOR×P[COUNT;(24+COUNT)]
[85]  DELTA←PNEW[COUNT;(18+COUNT)]+PNEW[COUNT;(24+COUNT)]-SC
      HEM
[86]  PNEW[COUNT;(COUNT-6)]←P[COUNT;(COUNT-6)]-P[COUNT;(COUN
      T-6)]×DELTA÷(1-SCHEM)
[87]  PNEW[COUNT;(6+COUNT)]←P[COUNT;(6+COUNT)]-P[COUNT;(6+CO
      UNT)]×DELTA÷(1-SCHEM)
[88]  PNEW[COUNT;(12+COUNT)]←P[COUNT;(12+COUNT)]-P[COUNT;(12
      +COUNT)]×DELTA÷(1-SCHEM)
[89]  COUNT←COUNT+1
[90]  →(COUNT≤7)⍴CHEM
[91]
[92] CHEMREFIT:SCHEM←P[COUNT;(18+COUNT)]+P[COUNT;(24+COUNT)]
[93]  PNEW[COUNT;(18+COUNT)]←FACTOR×P[COUNT;(18+COUNT)]
[94]  PNEW[COUNT;(24+COUNT)]←FACTOR×P[COUNT;(24+COUNT)]
[95]  DELTA←PNEW[COUNT;(18+COUNT)]+PNEW[COUNT;(24+COUNT)]-SC
      HEM
[96]  PNEW[COUNT;(COUNT-6)]←P[COUNT;(COUNT-6)]-P[COUNT;(COUN
      T-6)]×DELTA÷(1-SCHEM)
[97]  PNEW[COUNT;(6+COUNT)]←P[COUNT;(6+COUNT)]-P[COUNT;(6+CO
      UNT)]×DELTA÷(1-SCHEM)
[98]  PNEW[COUNT;(12+COUNT)]←P[COUNT;(12+COUNT)]-P[COUNT;(12
      +COUNT)]×DELTA÷(1-SCHEM)
[99]  PNEW[COUNT;(34+COUNT)]←P[COUNT;(34+COUNT)]-P[COUNT;(34
      +COUNT)]×DELTA÷(1-SCHEM)
[100]
[101]  COUNT←COUNT+1
[102]  →(COUNT≤11)⍴CHEMREFIT
[103]  →END
[104]
[105] ⍝  FINAL SECTION OF CALCULATIONS FOR THE ***ALL*** CAS
      E.
[106]
[107] ALL:SCHEM←P[COUNT;(6+COUNT)]+P[COUNT;(12+COUNT)]+P[COU
      NT;(18+COUNT)]+P[COUNT;(24+COUNT)]
[108]  PNEW[COUNT;(6+COUNT)]←FACTOR×P[COUNT;(6+COUNT)]
[109]  PNEW[COUNT;(12+COUNT)]←FACTOR×P[COUNT;(12+COUNT)]
[110]  PNEW[COUNT;(18+COUNT)]←FACTOR×P[COUNT;(18+COUNT)]
[111]  PNEW[COUNT;(24+COUNT)]←FACTOR×P[COUNT;(24+COUNT)]
[112]  DELTA←PNEW[COUNT;(6+COUNT)]+PNEW[COUNT;(12+COUNT)]+PN
      EW[COUNT;(18+COUNT)]+PNEW[COUNT;(24+COUNT)]-SCHEM
[113]  PNEW[COUNT;(COUNT-6)]←P[COUNT;(COUNT-6)]-P[COUNT;(COU
      NT-6)]×DELTA÷(1-SCHEM)
[114]
```

```
[115]  COUNT←COUNT+1
[116]  →(COUNT≤7)ρALL
[117]
[118] ALLSEC:SCHEM←P[COUNT;(6+COUNT)]+P[COUNT;(12+COUNT)]+P[
      COUNT;(18+COUNT)]+P[COUNT;(24+COUNT)]
[119]  PNEW[COUNT;(6+COUNT)]←FACTOR×P[COUNT;(6+COUNT)]
[120]  PNEW[COUNT;(12+COUNT)]←FACTOR×P[COUNT;(12+COUNT)]
[121]  PNEW[COUNT;(18+COUNT)]←FACTOR×P[COUNT;(18+COUNT)]
[122]  PNEW[COUNT;(24+COUNT)]←FACTOR×P[COUNT;(24+COUNT)]
[123]  DELTA←PNEW[COUNT;(6+COUNT)]+PNEW[COUNT;(12+COUNT)]+PN
      EW[COUNT;(18+COUNT)]+PNEW[COUNT;(24+COUNT)]-SCHEM
[124]  PNEW[COUNT;(COUNT-6)]←P[COUNT;(COUNT-6)]-P[COUNT;(COU
      NT-6)]×DELTA÷(1-SCHEM)
[125]  PNEW[COUNT;(34+COUNT)]←P[COUNT;(34+COUNT)]-P[COUNT;(3
      4+COUNT)]×DELTA÷(1-SCHEM)
[126]
[127]  COUNT←COUNT+1
[128]  →(COUNT≤11)ρALLSEC
[129]
[130] END:'  THE PROGRAM IS DONE.  YOUR NEW TRANSITION PROBA
      BILITIES ARE IN'
[131]  '  THE MATRIX PNEW'
[132]
    ∇
```

The function CONPROB (following pages) is used in the same manner as CHEMPROB but is used to vary the probabilities for conventional weapons attacks. Both these functions are interactive.

```
      ⎕VR 'CONPROB'
    ∇ CONPROB P;FACTOR;B;SUM;DELTA;C;ROW;NEWROW;NEWPROB;DPRO
      B;ELSE
[1]
[2]   ⍝  THIS FUNCTION ALLOWS THE USER TO VARY THE PROBABILIT
      Y OF
[3]   ⍝  COMING UNDER CONVENTIONAL ATTACK FOR ALL STATES THAT
       LEAD TO
[4]   ⍝  POSSIBLE CONVENTIONAL WEAPONS ATTACKS.
[5]
[6]   ⍝  INPUT SECTION
[7]
[8]    '  INPUT THE FACTOR TO CHANGE THE PROBABILITY OF CONVE
      NTIONAL ATTACK'
[9]    '  FACTORS GREATER THAN ONE WILL INCREASE AND FACTORS
      LESS THAN '
[10]   '  ONE WILL DECREASE THE OVERALL CHANCE OF CONVENTIONA
      NL ATTACKS.  '
[11]   '  INPUT FACTORS CAN BE BETWEEN 0 TO 3 AND DO NOT HAVE
       TO BE INTEGER.'
[12]
[13]   FACTOR←⎕
[14]
[15]   '  TO VARY THE CHANCE OF CONVENTIONAL ONLY ATTACKS INP
      UT ***C***.'
[16]   ' '
[17]   '  TO VARY THE CHANCE OF SIMULTANEOUS CONVENTIONAL'
[18]   '  AND CHEMICAL ATTACKS INPUT ***S***.'
[19]   ' '
[20]   '  TO VARY THE CHANCE OF CONVENTIONAL'
[21]   '  ATTACKS BOTH WITH AND WITHOUT CHEMICALS INPUT ***A*
      **.'
[22]
[23]   B←⎕
[24]
[25]  ⍝  GUTS OF THE PROGRAM.
[26]  ⍝  THIS SECTION USES THE METHOD FROM FUNCTION
[27]  ⍝  DP BECAUSE THERE IS ONLY ONE PROBABILITY PER ROW THA
      T NEEDS CHANGING.
[28]
[29]   C←0
[30]   PNEW←P
[31]
[32]   →(B='C')⍴CONVENONLY
[33]   →(B='S')⍴SIMUL
[34]   →ALL
[35]
[36]  CONVENONLY:
[37]  ⍝  PULL OUT THE ROW IN QUESTION
[38]
[39]  CLEAN:ROW←P[(C+6);]
```

```
[40]  DPROB←P[(C+6);C]
[41]  NEWPROB←DPROB×FACTOR
[42]
[43]  DELTA←NEWPROB-DPROB
[44]  ELSE←1-DPROB
[45]  NEWROW←ROW-ROW×DELTA÷ELSE
[46]  NEWROW[C]←NEWPROB
[47]  PNEW[(C+6);]←NEWROW
[48]
[49]  C←C+1
[50]  →(C≤5)ρCLEAN
[51]
[52] DIRTY:ROW←P[(C+18);]
[53]  DPROB←P[(C+18);C+6]
[54]  NEWPROB←DPROB×FACTOR
[55]
[56]  DELTA←NEWPROB-DPROB
[57]  ELSE←1-DPROB
[58]  NEWROW←ROW-ROW×DELTA÷ELSE
[59]  NEWROW[C+6]←NEWPROB
[60]  PNEW[(C+18);]←NEWROW
[61]
[62]  C←C+1
[63]  →(C≤17)ρDIRTY
[64]  →END
[65]
[66] ⍝  JUMP TO THE SIMULTANEOUS CHEM AND CONVENTIONAL ATTAC
      KS IF REQUESTED
[67]
[68]
[69] SIMUL:SUM←P[(C+6);C+12]+P[(C+6);C+18]
[70]  PNEW[(C+6);C+12]←FACTOR×P[(C+6);C+12]
[71]  PNEW[(C+6);C+18]←FACTOR×P[(C+6);C+18]
[72]  DELTA←PNEW[(C+6);C+12]+PNEW[(C+6);C+18]-SUM
[73]
[74]  PNEW[(C+6);C]←P[(C+6);C]-P[(C+6);C]×DELTA÷(1-SUM)
[75]  PNEW[(C+6);C+24]←P[(C+6);C+24]-P[(C+6);C+24]×DELTA÷(1-
      SUM)
[76]  PNEW[(C+6);C+30]←P[(C+6);C+30]-P[(C+6);C+30]×DELTA÷(1-
      SUM)
[77]  PNEW[(C+6);C+40]←P[(C+6);C+40]-P[(C+6);C+40]×DELTA÷(1-
      SUM)
[78]  C←C+1
[79]  →(C≤5)ρSIMUL
[80]  →END
[81]
[82] ⍝  DO THE CASE OF ALL CONVENTIONAL ATTACKS WHETHER COMB
      INED WITH
[83] ⍝  CHEMICALS OR NOT.
[84]
[85] ALL:SUM←P[(C+6);C]+P[(C+6);C+12]+P[(C+6);C+18]
```

```
[86]   PNEW[(C+6);C]←FACTOR×P[(C+6);C]
[87]   PNEW[(C+6);C+12]←FACTOR×P[(C+6);C+12]
[88]   PNEW[(C+6);C+18]←FACTOR×P[(C+6);C+18]
[89]   DELTA←PNEW[(C+6);C]+PNEW[(C+6);C+12]+PNEW[(C+6);C+18]-
       SUM
[90]
[91]   PNEW[(C+6);C+24]←P[(C+6);C+24]-P[(C+6);C+24]×DELTA÷(1-
       SUM)
[92]   PNEW[(C+6);C+30]←P[(C+6);C+30]-P[(C+6);C+30]×DELTA÷(1-
       SUM)
[93]   PNEW[(C+6);C+40]←P[(C+6);C+40]-P[(C+6);C+40]×DELTA÷(1-
       SUM)
[94]   C←C+1
[95]   →(C≤5)ρALL
[96]
[97]  ALLTWO:ROW←P[(C+18);]
[98]   DPROB←P[(C+18);C+6]
[99]   NEWPROB←DPROB×FACTOR
[100]
[101]   DELTA←NEWPROB-DPROB
[102]   ELSE←1-DPROB
[103]   NEWROW←ROW-ROW×DELTA÷ELSE
[104]   NEWROW[C+6]←NEWPROB
[105]   PNEW[(C+18);]←NEWROW
[106]
[107]   C←C+1
[108]   →(C≤17)ρALLTWO
[109]   →END
[110]
[111]  END:'  THE PROGRAM IS DONE.  YOUR NEW TRANSITION PROBA
       BILITIES ARE IN'
[112]   '  THE MATRIX PNEW'
     ∇
```

The function DP (following page) allows the user to change one probability and still have the row of the P matrix sum to one. The ratio between individual probabilities not targeted by the user remains constant.

```
      ⎕VR 'DP'
    ∇ DPOUT←DP P;ROW;DPROB;NEWPROB;DELTA;ELSE;INDEX;NEWROW
[1]
[2]   ⍝  THIS IS AN INTERACTVIE FUNCTION THAT ALLOWS YOU TO C
      HANGE
[3]   ⍝  THE PROBABILITY OF ENTERING DECON AT A LEVEL AND AUT
      OMATICALLY
[4]   ⍝  UPDATE THE OTHER PROBABILITIES SO THAT THE ROW SUM R
      EMAINS ONE.
[5]   ⍝  ACTUALLY THIS FUNCTION WILL WORK FOR ANY PROBABILITY
       YOU WANT
[6]   ⍝  TO CHANGE AS LONG AS YOU ONLY WANT TO CHANGE ONE PER
       ROW
[7]   ⍝  INDEPENDENT OF THE OTHER PROBABILITIES.
[8]
[9]    DPOUT←P
[10]
[11] START:'  INPUT THE ROW AND COLUMN OF THE PROBABILITY YO
      U WANT TO CHANGE'
[12]   INDEX←⎕
[13]
[14] ⍝  PULL OUT THE ROW IN QUESTION
[15]
[16]   ROW←P[INDEX[0];]
[17]   DPROB←P[INDEX[0];INDEX[1]]
[18]
[19]   '  THE OLD PROBABILITY IS ',⍕DPROB
[20]   '  ENTER THE NEW PROBABILITY'
[21]   NEWPROB←⎕
[22]   DELTA←NEWPROB-DPROB
[23]   ELSE←1-DPROB
[24]   NEWROW←ROW-ROW×DELTA÷ELSE
[25]   NEWROW[INDEX[1]]←NEWPROB
[26]   DPOUT[INDEX[0];]←NEWROW
[27]
[28]   '  DO YOU WANT TO CHANGE ANOTHER PROBABILITY Y OR N'
[29]   →(⎕='Y')⍴START
[30]
[31]
    ∇
```

The functions CHEMRAND and CONRAND were used during the sensitivity analysis. These functions "protect" the probabilities of interest from the randomizing matrix used to perturb the rest of the **P** matrix.

```
      ⎕VR 'CHEMRAND'
    ∇ PRAND←R CHEMRAND P;BETA;PTILDA;GAMMA;RATIO;RATIOMAT;PT
      ILDAFINAL
[1]   BETA←+/P×DROPCHEM
[2]   PTILDA←R×P×DROPCHEM
[3]   GAMMA←+/PTILDA
[4]   RATIO←BETA÷GAMMA
[5]   RATIOMAT←⍉ 48 48 ⍴RATIO
[6]   PTILDAFINAL←PTILDA×RATIOMAT
[7]   PRAND←PTILDAFINAL+P×KEEPCHEM
    ∇
```

```
      ⎕VR 'CONRAND'
    ∇ PRAND←R CONRAND P;BETA;PTILDA;GAMMA;RATIO;RATIOMAT;PTI
      LDAFINAL
[1]   BETA←+/P×DROPCON
[2]   PTILDA←R×P×DROPCON
[3]   GAMMA←+/PTILDA
[4]   RATIO←BETA÷GAMMA
[5]   RATIOMAT←⍉ 48 48 ⍴RATIO
[6]   PTILDAFINAL←PTILDA×RATIOMAT
[7]   PRAND←PTILDAFINAL+P×KEEPCON
    ∇
```

The function REPORT is used to call up the Summary Report for the initial State 6.

```
      ⎕VR 'REPORT'
    ∇ REPORT
[1]   ' '
[2]   (⎕FREAD 1 4)[6;;]
[3]   ' '
[4]   ' '
[5]   (⎕FREAD 1 2)[6;]
[6]   ' '
[7]   ⎕FUNTIE 1
    ∇
```

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12. Dr. Donald P. Gaver, Code 55Gv — 1
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Naval Postgraduate School
Monterey, CA 93943-5000

13. CPT Michael O. Kierzewski — 3
c/o H.J. Kierzewski
HCR 71, Box 19
Mine Run, VA 22568