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Model of Anoxic-Aerobic Wastewater Treatment

at Phoenix 91st Avenue Plant

Master of Science in Civil Engineering

University of Washington

1993

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(Chairperson of Supervisory Committee)

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Chapter 1: Introduction and Project Objective

Project Objective

The objective of this project was to develop a model that can be used to evaluate the design and performance of a multiple staged anoxic-aerobic wastewater treatment system. This type of system has general application, and was first used at the 91st Ave. wastewater treatment plant in Phoenix, Arizona. The model simulates the concentrations of ammonia, nitrate, and nitrite in each of the stages of the ten-stage system, using a Microsoft Excel spreadsheet. The oxygen consumption rate is also computed in each stage. The staged treatment system performance was evaluated as a function of various design and operating parameters which include the effect of Solids Retention Time (SRT), the internal recycle flow rate and distribution, and aeration tank DO concentrations. Two models were developed for comparison. The first model was based on Michaelis-Menton kinetics. The second model was a modified version of the first model that takes into account soluble substrate uptake and storage in the initial stages.

Background Information

Nitrification and Denitrification Processes

Anoxic/aerobic treatment systems are used in wastewater treatment for the control and removal of nitrogen. Nitrogen concentrations in treatment plant effluents must be controlled in order to avoid adverse effects in receiving waters. High effluent ammonia-nitrogen concentrations may result in depletion of dissolved oxygen in receiving waters, thereby having a negative impact on aquatic life. Significant nitrogen concentrations in effluent can also accelerate the

eutrophication of lakes and allow for the growth of rooted aquatic plants and algae in streams. Therefore, nitrogen control strategies are an important part of wastewater treatment (Metcalf and Eddy, Inc., 1991)

The biological removal of nitrogen from wastewater requires a two step process. The influent ammonia is first converted to nitrate through a process known as biological nitrification. The transformation of ammonia-nitrogen to nitrate-nitrogen does not result in nitrogen removal, but it does eliminate its oxygen demand (Metcalf and Eddy, Inc., 1991). The nitrate can then be used as an electron acceptor under anoxic conditions (without oxygen), and can subsequently be reduced to nitrogen gas through a process known as denitrification (Stensel, 1992(a)). When describing the denitrification process, the term "anoxic" is used rather than "anaerobic". This is because the principal metabolic pathways involved in denitrification are modified aerobic pathways using nitrate as the final electron acceptor, rather than anaerobic reactions. (Metcalf and Eddy, Inc., 1991).

Nitrification is a two step process involving two different genera of nitrifying bacteria. In the first step, *Nitrosomonas* converts ammonia to nitrite. In the second step, nitrite is converted to nitrate by *Nitrobacter* (Metcalf and Eddy, Inc., 1991). This two step oxidation process is generally written as follows:

$2NH_4^+ + 3O_2 \text{ =======> } 2NO_2^- + 4H^+ + 2H_2O$ via *Nitrosomonas*

$2NO_2^- + O_2 \text{ =======> } 2NO_3^-$ via *Nitrobacter*

Nitrification is an autotrophic process. Ammonia oxidation provides the energy for growth of the nitrifying microorganism and carbon dioxide is used for cell

synthesis. This can be contrasted with heterotrophic organisms, which use organic carbon for cell synthesis (Metcalf and Eddy, Inc., 1991). Dissolved oxygen concentrations above 1 mg/l are required to sustain reasonable nitrification rates.

The anoxic process of denitrification can be accomplished by several different genera of heterotrophic bacteria, such as *Pseudomonas, Bacillus, Achromobacter, Flavobacterium*, and *Spirillum.* These heterotrophs are capable of dissimilatory nitrate reduction in the form of the following reaction sequence:

$$NO_3^- \Longrightarrow NO_2^- \Longrightarrow NO \Longrightarrow . N_2O \Longrightarrow N_2$$

This reaction sequence shows that the first step of denitrification involves the conversion of nitrate to nitrite. This is followed by the production of nitric oxide, nitrous oxide, and nitrogen gas. These three gaseous products can be released to the atmosphere, thereby completing the nitrogen removal process (Metcalf and Eddy, Inc., 1991). The presence of dissolved oxygen will suppress the enzyme system needed for this dissimilatory nitrate reduction process to occur.

Phoenix 91st Avenue Anoxic-Aerobic Wastewater Treatment Plant

The City of Phoenix 91st Avenue Wastewater Treatment Plant provides service for seven cities in the Phoenix, AZ area, receiving wastewater that is primarily of domestic origin. This plant uses conventional primary and secondary treatment with a design capacity of approximately 150 MGD. In September 1992, two existing aeration tanks were modified to accomplish nitrogen removal by converting to an anoxic-aerobic system with internal recycle. The

purpose of the modification was to achieve biological nitrification and denitrification without the added expense of constructing new aeration basins. The nitrogen removal goal was an effluent total nitrogen concentration of 10.0 mg/l or less. Test results over a six month period indicated that this goal was being achieved (Stensel, 1993(b)).

Figure 1 shows the layout of one of the two modified aeration tanks. The tank volume is 3.72 million gallons (315 ft x 100 ft x 15.5 ft deep), and it is sub-divided into 4-pass aeration channels with a 25 ft. width. The total flow length of the 4-pass channel system is 1260 ft. The primary effluent feed rate into stage 1 (Ax-1) of the basin is approximately 20 MGD. The internal recycle flow rate into stage 1 (Ax-1) and stage 3 (Ax-3) is 20 MGD and 40 MGD respectively. The return recycle sludge flow rate is approximately 9 MGD, and the operating solids retention time (SRT) is around 5 days (Stensel, 1993(b)).

The basin was sub-divided into several small initial anoxic stages, instead of using only one single anoxic stage of greater volume. This design feature was included in an effort to reduce sludge volume index (SVI) values, and to achieve shorter total anoxic detention times. These initial anoxic stages (Ax-1 and Ax-2), with short detention times and high F/M ratios, serve as biological selector zones that maximize soluble substrate uptake by non-filamentous bacteria (Stensel, 1993(b)). This helps to control sludge settling characteristics. The first two anoxic stages in the basin (Ax-1 and Ax-2) are mixed with course bubble diffusers. The remainder of the anoxic and aerobic stages are mixed with fine bubble diffusers. The operating dissolved oxygen concentration in the first four anoxic zones is maintained at or near zero. The fifth stage (Ax-5) is a very short zone with limited DO concentration. The last five aerobic zones (Ox-2 through Ox-6)

are operated with target DO concentrations of 2 to 3 mg/l. Significant biological storage of SCOD occurs in the first stage of treatment (Stensel, 1993(b)). Approximately 70 to 80 percent of the SCOD removed in the first anoxic zone is taken into cell storage by the microorganism population in the wastewater.

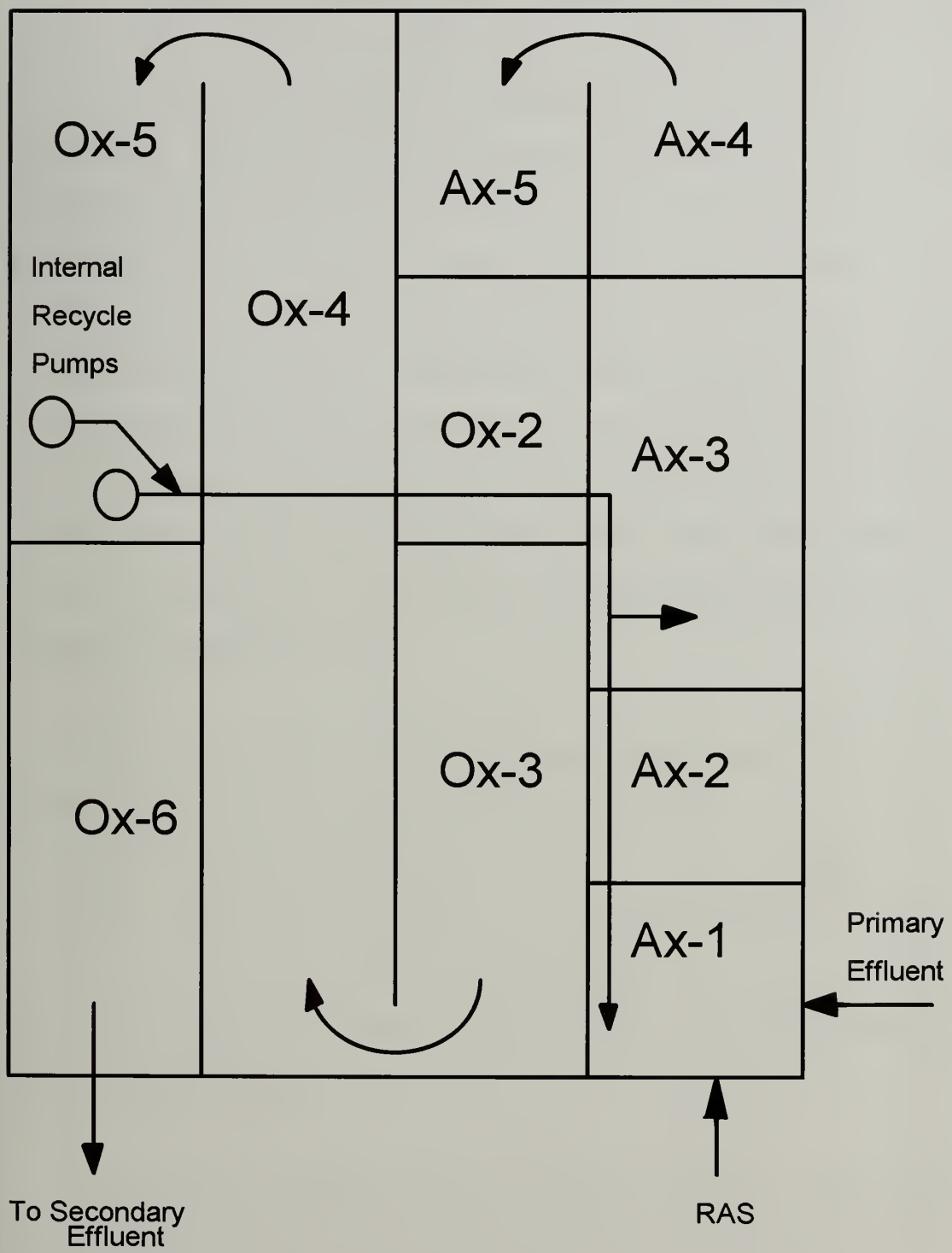

FIGURE 1: Schematic of Phoenix 91st Ave.
Anoxic-Aerobic Treatment System

Chapter 2: Model Based on Michaelis-Menton Kinetics

A Microsoft Excel (version 4.0) spreadsheet has been used to solve the anoxic-aerobic wastewater treatment system model. Ammonia, nitrate, and nitrite concentrations are calculated and plotted through each stage of the ten-stage system. Other model parameters include soluble chemical oxygen demand (SCOD), particulate chemical oxygen demand (PCOD), and the oxygen consumption rate in each stage. Mass balance equations were written for ammonia, nitrate, nitrite, SCOD and PCOD for each of the ten aeration basin stages. The oxygen consumption was based on the amount of nitrogen and COD oxidized in each stage, and appropriate stoichiometry relationships between oxygen consumed and unit substrate removed. Steady-state conditions were assumed for the model solution.

Internal recycle flow is withdrawn from stage 9 of the aeration basin. Therefore, the total flow into the first anoxic stage includes:

($Q_{ir(1)}$) Internal recycle flow from stage 9

(Q_{ras}) Return flow from stage 10 (via secondary clarifier)

(Q_o) Influent flow (from primary effluent)

The total flow into stage 3 includes the above listed flows and the internal recycle flow from stage 9 ($Q_{ir(3)}$).

The following is a listing of typical mass balance equations in stages 1 and 6 for each of the wastewater components being modeled. Descriptions of kinetic terms within the equations are also included (see **Table 1** for coefficient and

Table 1: Coefficient and Parameter Definitions

Symbol	Definition	Typical Value	Cell # on Spread-sheet
$\mu_{m(NS)}$	maximum specific growth rate for nitrosomonas nitrifiers	0.50 d^{-1}	T2
$\mu_{m(NB)}$	maximum specific growth rate for nitrobacter nitrifiers	0.70 d^{-1}	V5
$\mu_{m(H)}$	maximum specific growth rate for heterotrophs	3.2 d^{-1}	P10
Y_{ns}	yield of nitrosomonas nitrifiers	0.10 g TSS/ g NH4	T3
Y_{nb}	yield of nitrobacter nitrifiers	0.05 g TSS/ g NH4	V6
Y_H	yield of heterotrophs	0.35 g TSS/ g COD	N10
K	heterotrohic maximum specific substrate utilization rate	8 g COD/ g TSS-d	N24
k_s	half-saturation constant for soluble substrate (for heterotrophs)	20 mg/l	N13
K_{os}	DO half-saturation constant for heterotrophs	0.1 mg/l	N14
Kp	coefficient for particulate substrate utilization	0.0009 l/mg-d	R2
k_{oh}	DO nitrate reduction inhibition constant	0.1 mg/l	P13
k_{on}	DO half-saturation constant for nitrifiers	0.5 mg/l	P12
$k_{N(ns)}$	Nitrogen half-saturation constant for nitrosomonas	0.5 mg/l	P5
$K_{N(nb)}$	Nitrogen half-saturation constant for nitrobacter	0.60 mg/l	P11
$K_{(sto)}$	coefficient for stored substrate utilization	0.937	P16
k_{ss}	half-saturation constant for stored substrate utilization	0.002 g/g	P17
$S_{S(10)}/X_H$	ratio of SCOD in cell storage to heterotrophic biomass (stage 10)	0.000115 g/g	P19

$S_{s(9)}/X_H$	ratio of SCOD in cell storage to heterotrophic biomass (stage 9)	0.000388 g/g	P20
$k_{d(h)}$	endogenous decay coef. for heterotrophs	0.06 g/g-d	N23
$k_{d(n)}$	endogenous decay coefficient for nitrifiers	0.01 g/g-d	T6
NO_s	nitrate-nitrogen half-saturation constant	0.75 mg/l	N17
F_{DN}	fraction of heterotrophs using nitrate for electron acceptor	0.5	N11
F_n	fraction of g NH_4-N / g TSS	0.1	T5
O	dissolved oxygen concentration	0 / 3 mg/l	N15 and N26
S_o	influent soluble substrate concentration (primary effluent)	228 mg/l	N3
S_{10}	effluent soluble substrate concentration for stage 10	0 mg/l	N3
S_9	soluble substrate concentration in stage 9	0.1 mg/l	N7
S_R	residual (non-biodegradable) SCOD	30 mg/l	P18
NO_o	influent nitrate concentration (primary effluent)	0 mg/l	N19
NO_{10}	nitrate nitrogen concentration in stage 10	10.0 mg/l	N20
NO_9	nitrate concentration in stage 9	6.7 mg/l	N21
$NO2_o$	influent nitrite concentration (primary effluent)	0.1 mg/l	V2
$NO2_{10}$	nitrite concentration in stage 10	0.7 mg/l	V8
$NO2_9$	nitrite concentration in stage 9	0.9 mg/l	V7
P_o	influent particulate COD concentration (primary effluent)	228 mg/l	R3
P_{10}	particulate COD concentration in stage 10	392mg/l	R4
P_9	particulate COD concentration in stage 9	397mg/l	R5
N_o	influent ammonia nitrogen concentration (primary effluent)	45 mg/l	T7
N_{10}	ammonia concentration in stage 10	2.9mg/l	T8
N_9	ammonia concentration in stage 9	6.3 mg/l	T9
V_1	volume of stage 1	0.156 MG	N18
V_2	volume of stage 2	0.145 MG	N27
V_3	volume of stage 3	0.5047 MG	N28
V_4	volume of stage 4	0.124 MG	N29
V_5	volume of stage 5	0.124 MG	N30
V_6	volume of stage 6	0.3394 MG	N31
V_7	volume of stage 7	0.4648 MG	N32

V_8	volume of stage 8	0.9296 MG	N33
V_9	volume of stage 9	0.4648 MG	N34
V_{10}	volume of stage 10	0.4648 MG	N35
V_T	total volume of anoxic-aerobic basin	3.72 MG	P14
θ_c	SRT (solids retention time)	5 days	L2
Q_o	influent flow (primary effluent)	20 MGD	N2
Q_{RAS}	return flow from secondary clarifier	9.35 MGD	N4
$Q_{ir(1)}$	internal recycle flow to stage 1	20 MGD	N6
$Q_{ir(3)}$	internal recycle flow to stage 3	40 MGD	N25
X_H	heterotroph concentration	2977 mg/l	N12
X_{NB}	nitrobacter concentration	31 mg/l	P4
X_{NS}	nitrosomonas concentration	62 mg/l	T4
X_{io}	influent inert solids concentration	20 mg/l	R6
X_i	inert solids concentration	537 mg/l	R7
X_T	total MLSS	3910 mg/l	P21
X_P	particulate solids concentration	300 mg/l	N/A

** The value for $\mu_{m(ns)}$ ($0.50 d^{-1}$) listed in the table above was obtained from Phoenix Plant data (Stensel, H.D.,1993(c)). The value for $\mu_{m(nb)}$ was derived by using a ratio of terms ($\mu_{m(nb)}/\mu_{m(ns)}$ =1.4) obtained from an EPA manual for wastewater treatment. This ratio was multiplied by the value for $\mu_{m(ns)}$ to obtain $\mu_{m(nb)}$ = $0.70 d^{-1}$. The remainder of the coefficients listed in the table above were obtained from a variety of sources, including: (Abbott, 1992), (Metcalf and Eddy, Inc., 1991), and class notes from a course entitled "Biological Treatment Systems", H.D. Stensel, 1993.

parameter definitions, and typical values assigned). For a complete listing of mass balance equations, see Appendix C.

Description of Mass Balance Equations

Ammonia (NH_4-N) Mass Balances

Stage 1 (Ax-1)

$$V\frac{dN_1}{dt} = 0 = (Q_0)(N_0) + (Q_{RAS})(N_{10}) + (Q_{ir(1)})(N_9) - (Q_0 + Q_{RAS} + Q_{ir(1)})(N_1)$$

[Term (a)]

$$-\left(\frac{\mu_{m(NS)}}{Y_{NS}}\right)\frac{(X_{NS})(N_1)}{(K_N + N_1)}\left(\frac{O_1}{K_{0N} + O_1}\right)(V_1)$$

[Term (b)]

$$-Y_H\left[(Q_0 S_0) + (Q_{RAS} S_{10}) + (Q_{ir(1)} S_9) - (Q_0 + Q_{RAS} + Q_{ir(1)})(S_1)\right](F_N)$$

[Term (c)]

$$-Y_H\left[(Q_0 P_0) + (Q_0 + Q_{RAS})(P_{10}) - \left(\frac{V_1 P_{10}}{\theta_c}\right) + (Q_{ir(1)})(P_9)\right](F_N)$$
$$-Y_H\left[-(Q_0 + Q_{RAS} + Q_{ir(1)})(P_1)\right](F_N)$$

[Term (d)]

$$+(K_d)(X_H)(V_1)(F_N)(F_{DN})\left(\frac{NO_1}{NO_s + NO_1}\right)$$

[Term (e)]

Term (a) represents the flow of ammonia into and out of the first anoxic zone in the basin. Term (b) represents the depletion of ammonia by *Nitrosomonas* (nitrifiers). The dissolved oxygen value (O_1) is included to take into account the

inhibiting effect of low DO on nitrification rates. In the anoxic stages, the dissolved oxygen concentration (O_1) is assumed to equal zero. Therefore, in the first five stages, term (b) is equal to zero. Terms (c) and (d) represent the depletion of ammonia by the heterotrophic uptake of soluble COD (S_1) and particulate COD (P_1). Term (e) represents the addition of ammonia to the wastewater due to the decay and death of heterotrophic bacteria. The value for F_{DN} is included in term (e) because only a fraction of the heterotrophic organisms are able to use nitrate as an electron acceptor.

The ammonia mass balance for the first aerobic zone (Ox-2) is the same as the equation for Stage 1 above, except that term (e) is replaced with the following:

$$+(k_d)(X_H)(V_6)(F_N)\left(\frac{O}{k_{os}+O}\right)$$

This ammonia production term (for endogenous decay) takes into consideration the limiting effects of low dissolved oxygen concentration. The entire ammonia mass balance for stage 6 (Ox-2) is written as follows:

$$V\frac{dN_6}{dt} = 0 = (Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(N_5)-(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(N_6)$$

[Term (a)]

$$-\left(\frac{\mu_{m(NS)}}{Y_{NS}}\right)\frac{(X_{NS})(N_6)}{(K_N+N_6)}\left(\frac{O_6}{K_{0N}+O_6}\right)(V_6)$$

[Term (b)]

$$-Y_H\left[(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(S_5)-(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(S_6)\right](F_N)$$

[Term (c)]

$$-Y_H\left[(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(P_5)-(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(P_6)\right](F_N)$$

[Term (d)]

$$+(K_{d(h)})(X_H)(V_6)(F_N)\left(\frac{O_6}{k_{os}+O_6}\right)$$

[Term (e)]

Nitrate (NO_3-N) Mass Balances

Stage 1 (Ax-1)

$$V\frac{dNO_1}{dt} = 0 = (Q_0)(NO_0) + (Q_{RAS})(0.1)(NO_{10}) + (Q_{ir(1)})(NO_9) - (Q_0 + Q_{RAS} + Q_{ir(1)})(NO_1)$$

term (a)

$$-\left(\frac{1-1.42Y_H}{2.86}\right)\left(\frac{\mu_{m(H)}}{Y_H}\right)F_{DN}\left(\frac{S_1X_H}{K_s+S_1}\right)\left(\frac{k_{oh}}{k_{oh}+O_1}\right)\left(\frac{NO_1}{NO_s+NO_1}\right)V_1$$

term (b)

$$-\frac{1.42}{2.86}(k_d)(X_H)(F_{DN})\left(\frac{k_{oh}}{k_{oh}+O_1}\right)\left(\frac{NO_1}{NO_s+NO_1}\right)V_1$$

term (c)

$$-\left(\frac{1-1.42Y_H}{2.86}\right)F_{DN}(K_P)X_H\left(\frac{k_{oh}}{k_{oh}+O_1}\right)\left(\frac{NO_1}{NO_s+NO_1}\right)V_1$$

term (d)

Term (a) represents the flow of nitrate into and out of the first anoxic zone in the basin. Term (b) represents the depletion of nitrate due to heterotrophic uptake of soluble substrate. Term (c) represents the use of nitrate during endogenous decay, and term (d) represents nitrate used due to particulate COD degradation. Term (d) has been multiplied by fractions that account for the inhibiting effect of

high DO or low nitrate concentrations on the rate of nitrate depletion by heterotrophic organisms using NO_3^- as an electron acceptor.

Stage 6 (Ox-2)

The mass balance for nitrate in the first aeration zone (Ox-2) is as follows:

$$V\frac{dNO_6}{dt} = 0 = (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(NO_5) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(NO_6)$$

term (a)

$$+\left(\frac{\mu_{m(NB)}}{Y_{NB}}\right)\left(\frac{(X_{NB})(NO2_6)}{K_N + NO2_6}\right)\left(\frac{O_6}{k_{on} + O_6}\right)V_6$$

term (b)

$$-\left(\frac{1-1.42Y_H}{2.86}\right)\left(\frac{\mu_{m(H)}}{Y_H}\right)F_{DN}\left(\frac{S_6 X_H}{K_s + S_6}\right)\left(\frac{k_{oh}}{k_{oh} + O_6}\right)\left(\frac{NO_6}{NO_s + NO_6}\right)V_6$$

term (c)

$$-\frac{1.42}{2.86}(k_d)(X_H)(F_{DN})\left(\frac{k_{oh}}{k_{oh} + O_6}\right)\left(\frac{NO_6}{NO_s + NO_6}\right)V_6$$

term (d)

$$-\left(\frac{1-1.42Y_H}{2.86}\right)F_{DN}(K_P)X_H\left(\frac{k_{oh}}{k_{oh} + O_6}\right)\left(\frac{NO_6}{NO_s + NO_6}\right)V_6$$

term (e)

Term (b) is added to this mass balance to represent the production of nitrate from nitrite by Nitrobacter.

Nitrite (NO_2^-) Mass Balances

Stage 1 (Ax-1)

$$V\frac{dNO2_1}{dt} = 0 = (Q_0)(NO2_0) + (Q_{RAS})(NO2_{10}) + (Q_{ir(1)})(NO2_9) - (Q_0 + Q_{RAS} + Q_{ir(1)})(NO2_1)$$

This equation describes the flow of nitrite into and out of stage 1. This mass balance assumes that there is no production or depletion of nitrite in the anoxic zones, and that nitrite is not used for denitrification in the oxic zones.

Stage 6 (Ox-2)

$$V\frac{dNO2_6}{dt} = 0 = (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(NO2_5) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(NO2_6)$$

term (a)

$$+\left(\frac{\mu_{m(NS)}}{Y_{NS}}\right)\left(\frac{X_{NS}N_6}{K_N + N_6}\right)\left(\frac{O_6}{K_{on} + O_6}\right)(V_6)$$

term (b)

$$-\left(\frac{\mu_{m(NB)}}{Y_{NB}}\right)\left(\frac{X_{NB}NO2_6}{k_{N(NB)} + NO2_6}\right)\left(\frac{O_6}{k_{on} + O_6}\right)(V_6)$$

term (c)

Term (b) represents the production of nitrite by Nitrosomonas. Term (c) represents the depletion of nitrite by Nitrobacter.

SCOD Mass Balances

Stage 1 (Ax-1)

$$V\frac{dS_1}{dt} = 0 = Q_0 S_0 + Q_{RAS} S_{10} + Q_{ir(1)} S_9 - (Q_0 + Q_{RAS} + Q_{ir(1)})(S_1) - (R_{su})(V_1)$$

where: $$R_{su} = K(F_{DN})\left(\frac{S_1 X_H}{k_s + S_1}\right)\left(\frac{k_{oh}}{k_{oh} + O_1}\right)\left(\frac{NO_1}{NO_s + NO_1}\right)$$

R_{su} represents the rate of depletion of the soluble COD (SCOD).

Stage 6 (Ox-2)

$$V\frac{dS_6}{dt} = 0 = (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(S_5) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(S_6)$$

Term (a)

$$-\left(\frac{KS_6 X_H}{k_s + S_6}\right)\left(\frac{O_6}{O_6 + k_{os}}\right)V_6$$

Term (b)

$$-\left(\frac{KS_6 X_H}{k_s + S_6}\right)\left(\frac{k_{oh}}{k_{oh} + O_6}\right)\left(\frac{NO_6}{NO_s + NO_6}\right)V_6(F_{DN})$$

Term (c)

Term (a) represents the flow of SCOD into and out of the first oxic zone (stage 6). Term (b) describes the depletion of SCOD by heterotrophic organisms using oxygen as an electron acceptor. A low DO concentration inhibits the SCOD utilization rate. Term (c) represents the depletion rate of SCOD by heterotrophic organisms using nitrate as an electron acceptor. High DO or low nitrate concentrations inhibit the rate of SCOD depletion.

PCOD Mass Balances

Stage 1 (Ax-1)

$$V\frac{dP_1}{dt} = 0 = (Q_0)(P_0) + (Q_0 + Q_{RAS})(P_{10}) - \frac{(V_T)(P_{10})}{\theta_c} + (Q_{ir(1)})(P_9) - (Q_0 + Q_{RAS} + Q_{ir(1)})(P_1) - r_p(V_1)$$

where: $$r_p = F_{DN}(K_p P_1 X_H)\left(\frac{k_{oh}}{k_{oh} + O_1}\right)\left(\frac{NO_1}{NO_s + NO_1}\right)$$

The term labeled r_p describes the rate of depletion of PCOD by heterotrophic organisms under anoxic conditions.

Stage 6 (Ox-2)

$$V\frac{dP_6}{dt} = 0 = (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(P_5) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(P_6)$$

Term (a)

$$-(K_p P_6 X_H V_6)\left(\frac{k_{oh}}{k_{oh} + O_6}\right)\left(\frac{NO_6}{NO_s + NO_6}\right)(F_{DN})$$

Term (b)

$$-(K_p P_6 X_H V_6)\left(\frac{O_6}{k_{os} + O_6}\right)$$

Term (c)

Term (b) describes the rate of depletion of particulate COD at low or zero DO concentrations using nitrate. Term (c) represents PCOD degradation at higher DO concentrations, where oxygen is used as the electron acceptor.

Oxygen Consumption

Equations for the dissolved oxygen consumption rate have been included in the five aerobic stages of this model (stages 6 through 10) in units of kg O_2 / hr. A present limitation of this model is that it does not account for the oxygen used in the anoxic stages due to the use of course and fine bubble aeration for mixing. The equation for stage 6 (Ox-2) is shown below as an example:

Stage 6 (Ox-2)

Dissolved Oxygen Consumption Rate (Kg O_2 / hr) per stage

$$= \underbrace{\left[(A)(r_{su}) + 1.42(K_d)\left(\frac{O_6}{K_{os}+O_6}\right)(X_H)\right](V_6)}_{\textbf{term (a)}} + \underbrace{(3.22)(r_{NH_4(ox)})(V_6)}_{\textbf{term (b)}} + \underbrace{(1.11)(r_{NO_2(ox)})(V_6)}_{\textbf{term (c)}}$$

where:

$$r_{su} = \left(\frac{KS_6X_H}{k_s+S_6}\right)\left(\frac{O_6}{O_6+k_{os}}\right) + K_p(P_6)(X_H)\left(\frac{O_6}{O_6+K_{os}}\right)$$

$$A = 1 - 1.42(Y_H)$$

$$r_{NH_4(ox)} = \left(\frac{\mu_{m(NS)}}{Y_{NB}}\right)\left(\frac{X_{NS}(N_6)}{K_N+N_6}\right)\left(\frac{O_6}{O_6+k_{on}}\right)$$

$$r_{NO_2(ox)} = \left(\frac{\mu_{m(NB)}}{Y_{NB}}\right)\left(\frac{(X_{NB})(NO2_6)}{K_N+NO2_6}\right)\left(\frac{O_6}{O_6+k_{on}}\right)$$

Term (a) represents the oxygen consumed for heterotrophic degradation of soluble and particulate substrate, and for endogenous decay. This term also describes the oxygen consumed during the endogenous decay of the heterotrophs. Term (b) represents the oxygen consumed as a result of depletion of ammonia by Nitrosomonas. Term (c) represents the oxygen consumed due to depletion of nitrite by nitrobacter under aerobic conditions.

Approximation of Biomass, Inert Solids and $PCOD_{(10)}$ Concentrations

An important and reasonable simplifying assumption used to solve the model equations was that the biomass concentrations were constant from stage to stage. This required performing a steady state mass balance for solids for the overall system. A similar approach was used for the particulate COD to aid in the model solution. Mass balance equations were done for the entire ten-stage system (as a whole) to approximate the following components:

X_H (Heterotrophic biomass concentration)

X_{NS} (Nitrosomonas Nitrifier concentration)

X_{NB} (Nitrobacter Nitrifier concentration)

X_I (Inert solids concentration)

P_{10} (effluent PCOD concentration)

These mass balance equations have been rearranged, and are listed below:

$$X_H = \left(\frac{Y_H(\Delta S)(SRT)}{[1+(k_{d(H)})(SRT)](\theta)} \right)$$

where: $\Delta S = S_0 - S_{10} + P_0 - \left(\frac{V_T P_{10}}{(SRT) Q_0} \right)$

$$X_{NS} = \frac{Y_{NS}(N_0 - N_{10})(SRT)}{(1 + k_{d(n)}(SRT))\theta}$$

$$X_{NB} = \frac{Y_{NB}(N_0 - N_{10})(SRT)}{(1 + k_{d(n)}(SRT))\theta}$$

$$X_I = \frac{Q_0(X_{I(0)})(SRT)}{V_T}$$

$$X_t = X_H + X_{ns} + X_{nb} + X_I + X_P$$

$$P_{10} = (1.3)X_P$$

[This approximation assumes the relationship between P_{10} and X_p can be defined with the ratio: COD/VSS = 1.3. This ratio is used to convert P_{10} to X_p]

Assume $P_{10} = P_{avg}$

$$P_{avg} = \frac{Q_0 P_0}{\left(\frac{V_T}{(SRT)} + K_p(X_H)(V_T) \right)} = P(10) \quad \textit{(solve for } K_p\textit{)}$$

The value for X_t (total MLSS) is obtained from Phoenix Plant data for the testing date that is used in the model. The last equation can then be rearranged to solve for K_p, with the assumption that $P_{10} = P_{avg}$.

Incorporation of Model into an Excel Spreadsheet

Appendix A shows the layout of the model on the Excel spreadsheet. Column (B) lists the calculated concentrations for each of the components being modeled. These components are calculated using the Excel "solver" function, and are computed for each of the ten stages of the anoxic-aerobic basin. The six components included on the spreadsheet are as follows:

SCOD --- Soluble COD concentration (mg/l)
NO --- Nitrate-nitrogen concentration (mg/l)
PCOD --- Particulate COD concentration (mg/l)
N --- Ammonia-nitrogen concentration (mg/l)
NO2 --- Nitrite-nitrogen concentration (mg/l)
Ox consu --- Oxygen consumed (kg O_2 / hr)

Columns (D) through (H) of the spreadsheet contain the output values produced by mass balance equations for each component in each of the stages. These mass balances have been written in terms of cell references. All kinetic constants and initial input values for this model are located in columns (L) through (X) of the spreadsheet. The lower portion of the spreadsheet contains output graphs and tables, describing the changing concentrations through the ten stage system for each component being modeled. Typical concentration curves based on plant data are also provided for comparison to the curves computed by mass balances.

Model Solution

Any of the input values in Table A.1 of Appendix A [columns (N) through (X) on the Excel spreadsheet] may be altered for the purpose of analyzing the effect on the anoxic-aerobic treatment system. When any of these input values are changed, the "solver function" in Excel must be used first to solve the two simultaneous equations for X_H and $P_{(10)}$. These equations are located in rows 41-42 and columns (N) through (O) on the spreadsheet. This solver command can be activated by clicking the Macro button in cell O-37.

The "solver command" must then be used ten more times for each of the ten stages in the treatment system. Using the first anoxic stage (Ax-1) as an example, the "solver function" would be selected from the Format Menu of Excel. Cell D6 would be used as the target cell in stage 1. Cells D6 through D10 would be set equal to zero as constraints in the "solver" dialog box. Cells B6 through B10 would be designated as the "changing cells". The table below lists the input requirements for the dialog box of the "solver function" in each of the ten stages. A macro button has been installed for each stage to speed up the solving process. Clicking this button with the mouse will automatically activate the "solver function" for the corresponding anoxic or aerobic stage. The button location for each stage is also listed on the following table:

Table 2: Input Requirements for Microsoft Excel "Solver" Function

Stage	Target Cell	Changing Cells	Constraints	Macro Button Location
1	D6 = 0	B6:B10	D6:D10 = 0	A5
2	D14=0	B14:B17	D14:D17=0	A13
3	D22=0	B22:B26	D22:D26=0	A21
4	D30=0	B30:B33	D30:D33=0	A29
5	D38=0	B38:B41	D38:D41=0	A37
6	D46=0	B46:B50	D46:D50=0	A45
7	D54=0	B54:B58	D54:D58=0	A53
8	D62=0	B62:B66	D62:D66=0	A61
9	D70=0	B70:B74	D70:D74=0	A69
10	D78=0	B78:B82	D78:D82=0	A77

There will normally be at least one stage within each run in which the "solver function" cannot find a feasible solution for the set of simultaneous equations. When this occurs, the component that did not achieve a successful solution must be identified, and "re-solved" individually using the "solver function". This component can be identified by observing the values listed in column (D). If the "solver" has found a successful solution, then all of the values in column (D) should be very close to zero. If any of the values in column (D) are not close to zero, then the corresponding component must re-solved. This provides a seed value that can be used to re-solve all of the mass balances for a particular stage simultaneously. The new seed value [re-entered into the appropriate row of column (B)] should allow for a successful solution using the "solver function" . If not, then repeat the procedure for unsolved mass balances until appropriate seed values are obtained **(see Microsoft Excel "User's Guide" for description of "Solver" command).**

Chapter 3: Substrate Storage Model

As previously stated, significant substrate uptake and storage of SCOD occurs in the first anoxic zone in the aeration basin. An additional model has been developed that accounts for substrate storage on an Excel Spreadsheet, and is referred to in this paper as the "storage model". This model accounts for the initial accumulation and subsequent utilization of stored substrate within the cell. Steady-state conditions have again been assumed for this model. The concentrations of the following components are computed for each of the ten anoxic-aerobic stages:

ratio of soluble stored substrate to heterotrophic biomass (S_S/X_H)
ammonia-nitrogen (N)
nitrate-nitrogen (NO)
nitrite-nitrogen (NO2)
soluble substrate as chemical oxygen demand (S)
particulate substrate as chemical oxygen demand (P)

In order to account for cell storage, the value for S_S is divided by X_H throughout this model to create a ratio (S_S/X_H). This normalizes the stored substrate concentration by relating it to the heterotrophic biomass concentration. The following mass balances are based on the assumption that as the amount of stored substrate per unit biomass increases, there will be a higher degradation rate.

Description of Mass Balance Equations

The following is a listing of typical mass balance equations for the "substrate storage model". See Table 1 for coefficient/parameter definitions and typical values. For a complete listing of mass balance equations, see Appendix C.

S_S/X_H Mass Balances

Stage 1 (Ax-1)

$$V\frac{d\left(S_{S(1)}/X_H\right)}{dt} = (Q_{ir(1)})(S_{S(9)}/X_H) + (Q_{RAS})(S_{S(10)}/X_H) - (Q_0 + Q_{ir(1)} + Q_{RAS})(S_{S(1)}/X_H)$$

Term (a)

$$+\left(\frac{1}{X_H}\right)(Q_0S_0 - Q_0S_R)(0.7) - (R_{S_SU})(V_1)$$

Term (b) **Term (c)**

where:

$$R_{S_SU} = k_{sto}\left(\frac{S_{S(1)}/X_H}{k_{ss} + (S_{S(1)}/X_H)}\right)\left(\frac{k_{oh}}{k_{oh} + O_1}\right)\left(\frac{NO_1}{NO_s + NO_1}\right)(F_{DN})$$

Term (a) represents the flow of stored substrate (S_S) into and out of the first anoxic zone. Term (b) describes the uptake of soluble COD into cell storage. Based on observations at the Phoenix Plant, this model assumes that 70 % of all influent SCOD is taken up into storage in the first anoxic stage (Ax-1). In the mass balance equations for stages 2 and 3, this uptake percentage is assumed to be

10 % and 100% respectively. Residual non-biodegradable SCOD is subtracted from the influent total SCOD concentration. Term (c) represents the rate of depletion of stored SCOD.

The stored substrate mass balance equations for the aerobic stages (zones 6 through 10) are similar to the above equation, with the exception that the Rs_Su equation is substituted with the following:

$$R_{S_SU} = k_{sto}\left(\frac{S_{S(6)}/X_H}{k_{ss}+(S_{S(6)}/X_H)}\right)\left(\frac{O_6}{k_{os}+O_6}\right)$$

One potential weakness in this model is the lack of information available for estimating the following coefficients:

K_{sto} (coefficient for stored substrate utilization)
k_{ss} (half-saturation constant for stored substrate utilization)

For the run of the spreadsheet model shown in Appendix B, values of 0.937 1/day and 0.002 g/g were used for K_{sto} and k_{ss} respectively. However, the accuracy of these values is not known. The value for K_{sto} was estimated using an iterative process with the S_S/X_H mass balance equations in stages 5 through 10. A K_{sto} value was chosen that would satisfy the condition that S_S in stage 10 is less than or equal to one one-hundredth (1/100) of S_S in stage 5. In equation form:

$$S_{S(10)} < \text{or} = (0.01)(S_{S(5)})$$

This was based on the assumption that the rate of stored substrate utilization would rapidly start to decrease in the region of the curve where $S_{S(10)}$ is less than $(0.01)(S_{S(5)})$. Oxygen consumption data is expected to be available from the

Phoenix Plant in 1994, and this will hopefully allow for a better approximation of the K_{sto} value.

SCOD Mass Balances

The SCOD mass balance for stage 1 in the "Storage Model" is shown below:

Stage 1

$$V\left(\frac{dS_1}{dt}\right) = 0 = Q_0 S_0 + (Q_{ir(1)})(S_9) + (Q_{RAS})(S_{10}) - (Q_0 + Q_{ir(1)} + Q_{RAS})(S_1)$$

Term (a)

$$-(Q_0 S_0 - Q_0 S_R)(0.70)$$

Term (b)

Term (b) assumes 70 % uptake into cell storage of soluble degradable COD. For the mass balance equations in stages 2 and 3, 10% and 100% uptake is assumed respectively. Therefore, after the third anoxic stage, all degradable SCOD will theoretically have been depleted. These SCOD mass balances assume that none of the soluble substrate is metabolized by the microorganisms. All SCOD that is removed from the wastewater in the first three stages is taken up into cell storage. After the third anoxic zone (Ax-3), all that remains is the residual non-biodegradable SCOD (S_R), which remains at a constant concentration through the remaining seven stages in the basin.

The remainder of the storage model mass balances are identical to the equations listed for the model in Chapter 2 (for NH_4, NO_3, NO_2^-, PCOD, and oxygen consumption), with the exception of the following three modifications:

Modification #1

For NH_4 mass balances, term (c) is replaced with the following (using stage 1 and 6 as an example):

$$-Y_H[X_H(R_{S_sU})(V_1)](F_N)$$

where:

$$R_{S_sU} = k_{sto}\left(\frac{S_{S(1)}/X_H}{k_{ss}+(S_{s(1)}/X_H)}\right)\left(\frac{k_{oh}}{k_{oh}+O_1}\right)\left(\frac{NO_1}{NO_s+NO_1}\right)(F_{DN}) \qquad \{in\ anoxic\ zones\}$$

and

$$R_{S_sU} = k_{sto}\left(\frac{S_{S(6)}/X_H}{k_{ss}+(S_{s(6)}/X_H)}\right)\left(\frac{O_6}{k_{os}+O_6}\right) \qquad \{in\ aerobic\ zones\}$$

This portion of the ammonia mass balance represents the depletion of NH_4-N used for cell synthesis during heterotrophic utilization of stored soluble substrate.

Modification #2

For NO_3 mass balances, term (b) is replaced with the following (using stage 1 as an example):

$$-\left(\frac{1-1.42(Y_H)}{2.86}\right)[(X_H)(R_{S_sU})(V_1)]$$

where:

$$R_{S_sU} = k_{sto}\left(\frac{S_{S(1)}/X_H}{k_{ss}+(S_{S(1)}/X_H)}\right)\left(\frac{k_{oh}}{k_{oh}+O_1}\right)\left(\frac{NO_1}{NO_s+NO_1}\right)(F_{DN})$$

The basic form of this term can be used in the NO_3 mass balance equations for both the anoxic and aerobic zones (all ten stages). This portion of the ammonia mass balance represents the depletion of NO_3-N by heterotrophic utilization of stored soluble substrate.

Modification #3

For dissolved oxygen consumption, the heterotrophic substrate utilization rate (r_{su}) is replaced with the following equation for stored substrate utilization rate:

$$R_{S_SU} = k_{sto}\left(\frac{S_{S(6)}/X_H}{k_{ss}+(S_{s(6)}/X_H)}\right)\left(\frac{O_6}{k_{os}+O_6}\right)+k_P(P_6)(X_H)\left(\frac{O_6}{k_{os}+O_6}\right)$$

Incorporation of Model into Excel Spreadsheet

Appendix B shows the layout of the storage model on the Excel spreadsheet. An additional component (S_S/X_H) has been added under column (B) of each stage, along with the associated output value produced by mass balance equations in columns (D) through (F). A chart displaying the fluctuation of the S_S/X_H value in each of the ten stages has also been added on the lower portion of the spreadsheet. The remainder of the spreadsheet layout is identical to the model previously described in Chapter 2.

Model Solution

The procedures for operation of this storage model are similar to those described in Chapter 2 of this paper. The only difference is that one additional mass balance equation (for S_S/X_H) must be incorporated into the solution process for each stage. Table 3 lists the input requirements for the dialog box of the "solver function" in each of the ten stages (for manual use of the "solver" command). Macro buttons have again been installed for each stage to speed up the solution process.

Table 3: Input Requirements for Microsoft Excel "Solver" Function (for Storage Model)

Stage	Target Cell	Changing Cells	Constraints	Macro Button Location
1	D6=0	B6:B10;B12	D6:D10;D12=0	A5
2	D14=0	B14:B17;B20	D14:D17;D20=0	A13
3	D22=0	B22:B26;B28	D22:D26;D28=0	A21
4	D31=0	B31:B33;B36	D31:D33;D36=0	A29
5	D39=0	B39:B41;B44	D39:D41;D44=0	A37
6	D47=0	B47:B50;B52	D47:D50;D52=0	A45
7	D55=0	B55:B58;B60	D55:D58;D60=0	A53
8	D63=0	B63:B66;B68	D63:D66;D68=0	A61
9	D71=0	B71:B74;B76	D71:D74;D76=0	A69
10	D79=0	B79:B82;B84	D79:D82;D84=0	A77

Chapter 4: Results and Discussion

Introduction

The storage model was run using the typical kinetic coefficients shown in Table 1. The spreadsheet results are provided in Appendix B. Profile data from the Phoenix Treatment Plant was available for eight different testing dates between October 1992 to April 1993. A data set from 17 November 1992 was selected for analysis in this paper because the concentrations of the various components were in the average range when compared to the other data sets.

Some coefficient and parameter values from the Phoenix Plant profile data were input directly into the model. These include values for:

Q_o (influent flow) = 20 MGD
Q_{ras} (return flow) = 9.34 MGD
X_T (total MLSS) = 3910 mg/l
X_{ns} (nitrosomonas concentration) = 62 mg/l
X_{nb} (nitrobacter concentration) = 31 mg/l

The values for X_{ns} and X_{nb} were computed in a summary report on the Phoenix Plant operations(Stensel, 1993(c)). These calculations were based on the average influent TKN, SRT, and COD removed over a two week period. The original intention was to use the following equations for calculation of X_{ns} and X_{nb}:

$$X_{NS} = \frac{Y_{NS}(N_0 - N_{10} - N_{syn})(SRT)}{[1 + k_{d(n)}(SRT)]\theta}$$

$$X_{NB} = \frac{Y_{NB}(N_0 - N_{10} - N_{syn})(SRT)}{[1+k_{d(n)}(SRT)]\theta}$$

where: $$N_{syn} = \frac{X_T(V_T)}{(SRT)(Q_0)}(0.09)$$

*** N_{syn} = amount of influent nitrogen used for cell growth

However, use of these equations yielded values that were 15 percent higher than the average values obtained directly from available plant data. Therefore, concentrations of nitrosomonas and nitrobacter (X_{ns} and X_{nb}) were fixed for the purpose of this analysis (for the SRT analysis, the X_{ns} and X_{nb} values were prorated to account for changing SRT).

Table 1 shows that there are many different coefficients that could conceivably be adjusted to achieve a "better fitting curve". However, for the purposes of this analysis, the only coefficients that were considered for adjustment in curve fitting are as follows:

$K_{N(nb)}$ (nitrogen half saturation constant for nitrobacter) = 0.6 mg/l

$\mu_{m(nb)}$ (maximum specific growth rate for nitrobacter nitrifiers) = 0.7 d^{-1}

X_{IO} (influent inert solids concentration) = 20 mg/l

The values for $K_{N(ns)}$ and $\mu_{m(ns)}$ were fixed at 0.5 mg/l and 0.5 d^{-1} based on testing data obtained from Phoenix Plant (Stensel(b), 1993).

In order to run the spreadsheet model, several initial assumptions were made for influent concentrations, and concentrations in stages 9 and 10. The internal recycle withdrawal is located in stage 9, and therefore required an assumed concentration for each component. Assumed concentrations for stage 10 were needed due to the return flow that is cycled through a secondary clarifier. Assumed concentrations for the following components are taken directly from the Phoenix Plant Data (for the particular testing date being analyzed):

SCOD (influent)
SCOD (stage 9)
SCOD (stage 10)
Nitrate (stage 10)
Nitrate (stage 9)
Ammonia (influent)
Ammonia (stage 10)
Ammonia (stage 9)
Nitrite (influent)
Nitrite (stage 10)
Nitrite (stage 9)

These values were used to initiate the computations in the spreadsheet. The calculated values (computed in the model) were normally very close to the concentrations that were initially assumed. Influent nitrate was always assumed to equal zero in this model, and the influent PCOD concentration was assumed to equal the influent SCOD concentration (as a rough approximation).

Figures 2 through 8 show the results for the storage model using coefficient and parameter values as listed in Appendix B. The calculated NH_4, NO_3, and NO_2^- concentrations (produced by the model) correspond fairly well to the

Phoenix Plant Profile Data in each stage of the ten stage system. The calculated concentration for NH_4 decreases steadily in the aerobic zones (stages 6 through 10) as NH_4 is transformed into NO_2^- and NO 3 through the nitrification process. As shown in Figure 2 for ammonia, the calculated influent concentration is 25 % greater than the influent Plant Data concentration. This difference is due to the assumption that the influent flow for the Storage Model profile for ammonia includes TKN (Total Kjeldahl Nitrogen). However, the Phoenix Plant Data only includes the inorganic NH_4-N, as evidenced by historical Plant Data. The influent TKN value was assumed to be approximately 25% greater than the inorganic NH_4-N influent value. All of the organic nitrogen is assumed to be hydrolyzed in the first anoxic zone (stage 1). Consequently, the calculated (model) curve and the Plant Data curve are directly comparable in later stages (Figure 2), due to the fact that most of the inorganic NH_4-N should be released as the number of stages increases.

The calculated curve for Nitrite (Figure 4) does not correspond to the Plant Data curve as well as for NH_4 and NO_3. The Plant Data profile shows nitrite depletion in stages 1 and 2. The mass balance equations for the Storage Model do not include depletion terms for NO_2^- in the anoxic stages. Therefore, the model curve rises well above the Plant Data curve in stages 1 and 2. The remainder of the stages correspond fairly well.

The model and plant data curves for SCOD (Figure 5) fit very well, with the assumption that 70% of the degradable SCOD is taken up into cell storage in the first anoxic zone. The figure shows that most of the biodegradable SCOD is depleted after reaching the third anoxic zone, leaving only the non-biodegradable (residual SCOD) in the remaining seven stages.

There is no plant data available for comparison when evaluating the profile curves for PCOD, oxygen consumption rate, and S_S/X_H (ratio of stored SCOD to heterotrohic biomass concentration). The PCOD curve is shown in Figure 6. The PCOD concentration accumulates in stage 1 (due mostly to the internal recycle flow from stage 9), and is then gradually degraded in stages 2 through 10. Figure 7 is a bar chart showing the oxygen consumption rates computed by the model in stages 6 through 10. The rates are computed in units of kg/hr, and converted to mg/l-hr for plotting on the bar chart. Figure 7 shows that the O_2 consumption rate is fairly constant in stages 6 through 9, and drops slightly in stage 10 due to declining NH_4 and NO_2^- concentrations. No oxygen consumption is assumed to occur in the anoxic stages (zones 1 through 5).

The stored substrate curve (S_S/X_H) is shown in Figure 8. Although there is no plant data available for comparison, the curve behaves as would be expected. The peak amount of SCOD in cell storage occurs in the first anoxic zone, with gradual depletion to near zero in stage ten.

Analysis of Parameter Adjustments (Using Storage Model)

The Storage Model as shown in Appendix B was analyzed to determine the effect of adjusting various operating parameters. The following adjustments were made (individually) using the coefficients and parameters listed in Appendix B as the default values:

1) Vary SRT (Solids Retention Time)
2) Vary DO (dissolved oxygen concentration) in aerobic stages

(6 through 10)

3) Change the internal recycle withdrawal location from stage 9 to stage 10
4) Vary the percent distribution of internal recycle flow from stage 9 to stages 1and 3.
5) Increase the internal recycle flow from stage 9

Effect of SRT

The effect of SRT was varied from 3, 5, 7 and 9 days. The effect of raising the SRT on NH_4, NO_3 and NO_2^- concentration can be seen in Figures 9, 10, and 11. As SRT increases, the effluent NH_4 concentration decreases and the effluent NO_3 concentration increases (see Table 4). This can be attributed to the fact that the nitrifier biomass increases as SRT increases, thereby resulting in a greater nitrification rate, and more rapid depletion of NH_4. Figure 11 shows that the effluent NO_2^- concentration drops sharply as SRT increases, due to decreased amounts of NH_4 available for transformation to NO_2^- by *nitrosomonas* in stage 10.. The SCOD concentration remains fairly constant as SRT is increased, while peak stored substrate (S_S/X_H) ratios decline (see Figure 12). The latter effect is mainly due to the increases in heterotrophic biomass concentration that occurs as SRT increases.

Effect of Dissolved Oxygen Concentration

The dissolved oxygen concentration was varied in the aerobic zones (stages 6 through 10). DO concentrations of 0.5, 1.0, 2.0, 3.0 mg/l were used.. The effect

of raising DO levels on NH_4 and NO_3 concentration profiles can be seen in Figures 13 and 14. As DO is decreased, effluent NH_4 concentrations increase and effluent NO_3 concentrations decrease (see Table 4). This change is to be expected due to the inhibitory effects of low dissolved oxygen concentrations on the nitrification rate. As the DO level falls below 2 mg/l, the rate in which NH_4 is converted into NO_2^- and NO_3 is reduced, resulting in higher effluent NH_4 concentrations. There is little change in the profile for nitrite, with only a small decrease in effluent NO_2^- concentration in stage 10 as DO increases.

Effect of Internal Recycle Withdrawal Location

With the current aeration basin design, the internal recycle flow is withdrawn from stage 9. One-third of this flow is directed to stage 1 (20 MGD), and the remainder (40 MGD) feeds into stage 3. As part of this analysis, the Storage Model was modified so that the internal recycle flow was withdrawn from stage 10 instead of stage 9. The effect on the NH_4, NO_3 and NO_2^- concentration profiles can be seen in Figures 15 through 17. Changing to stage 10 withdrawal lowers the NH_4 and NO_2^- curves slightly. This is due to the lower concentrations of ammonia and nitrite that are being recycled from stage 10 (in comparison to stage 9). The change in withdrawal location raises the NO_3 profile (Figure 16) due to the higher nitrate concentration that exists in stage 10. However, the effluent concentrations of each component remain approximately the same (see Table 4). This would suggest that little benefit would be realized as a result of relocating the withdrawal location. There is little change observed in the SCOD, PCOD and stored substrate curves as a result of changing the withdrawal location.

Effect of Internal Recycle Flow Distribution

The internal recycle flow distribution to stages 1 and 3 (from stage 9) was varied with five different scenarios as follows:

	$Q_{ir(1)}$	$Q_{ir(3)}$
60 MGD =	20 MGD	40 MGD
	30 MGD	30 MGD
	40 MGD	20 MGD
	50 MGD	10 MGD
	60 MGD	0 MGD

As can be seen in Figures 18 and 19, the re-distribution of internal recycle flow does not have a significant effect on the NH_4 and NO_3 profiles, except for in stages 1 and 2. As the percentage of internal recycle flow into stage 1 is increased, the concentration of NH_4 in stages 1 and 2 is decreased, and the concentrations of NO_3 and NO_2^- in stages 1 and 2 are increased. The effluent concentrations for all five distribution scenarios remain fairly constant (see Table 4). SCOD concentrations drop slightly in stages 1 and 2 as increased flow is fed into stage 1 (SCOD declines by approximately 5% as flow into stage 1 is increased from 20 MGD to 60 MGD). Similarly, PCOD concentration also fell slightly in the first two anoxic stages, with the effluent levels remaining essentially the same. Figure 20 shows the effect of varying internal recycle distribution on stored SCOD concentration, with declining S_S/X_H values as flow to stage 1 is increased.

Effect of Increased Internal Recycle Flow

The existing internal recycle flow from stage 9 is approximately 60 MGD. The Storage Model was used to analyze the effects of varying this withdrawal flow as follows:

	$Q_{ir(1)}$	$Q_{ir(3)}$
40 MGD =	13.33 MGD	26.67 MGD
60 MGD =	20 MGD	40 MGD
75 MGD =	25 MGD	50 MGD

Each of these three scenarios assumes that one-third of the internal recycle flow is directed into stage 1, with the remainder being fed into stage 3. Figures 21 and 22 show the effect of increased flow on the ammonia and nitrate concentrations. As internal recycle flow increases, the NH_4 concentration decreases in stages 1 through 7, while NO_3 concentrations increase in stages 1 through 7, and decrease through the remainder of the aerobic zones. Table 4 shows that NH_4 effluent concentrations change very little with increased flow. Little change occurs in the NO_2^- profile, with only small concentration increases in the anoxic zones as internal recycle flow increases. The stored SCOD curve lowers only slightly as flow increases.

Analysis of Alternate Data Set (Phoenix Plant Data from 4 November 1992)

The spreadsheet run shown in Appendix B was based on Phoenix plant data dated 17 November 1992. In order to determine how well the Storage Model would "fit" to another set of plant data, an additional run was performed using the 4 November 1992 data set. The results are shown in Figures 23 through 29. As can be seen, the ammonia and nitrate profiles do not fit as well as was observed with the 17 November Plant Data. However, there still seems to be an adequate correlation with the Plant Data curve. The SCOD model curve (Figure 26) falls below the Plant Data curve, suggesting that the percentage of SCOD uptake into cell storage is significantly less than the assumed 70% (in stage 1). Figure 30 is provided to show the wide variability in NH_4 Plant Profile Data for different testing dates.

Comparison of Michaelis-Menton Model to Storage Model

The Michaelis-Menton Model (described in Chapter 2) has been run using the same coefficients and parameters used for the Storage Model (Appendix B). The results of the Michaelis-Menton Model are provided in Appendix A, including concentration profile charts for each component being modeled. The curves for NH_4 and NO_2^- are similar to the Storage Model in terms of correspondence with the Plant profile data. The PCOD concentration profiles for the two models are also very similar. The NO_3 profile in the Michaelis-Menton model, however, is considerably lower than for the Storage Model (see Figure 31). The calculated SCOD curve in the Michaelis-Menton Model (Figure 32) does not take into account non-biodegradable SCOD. This explains the poor correspondence between the model and Plant Data curves for SCOD. This can be compared with

the SCOD profile for the Storage Model (Figure 5), which does take the residual (non-biodegradable) SCOD into account, and provides a much closer approximation of the actual concentrations in each stage. The SDNR (Specific Denitrification Rate) curve shown for the Michaelis-Menton Model in Appendix B fits much better to the Plant Data than the SDNR curve for the Storage Model (shown in Appendix A). This suggests that the Storage Model is not sufficiently calibrated due to lack of information for kinetic coefficients.

Chapter 5: Conclusions

Two models have been developed to simulate various components of an anoxic-aerobic treatment system in Phoenix, Arizona. An initial observation suggests that both models appear to be equally sufficient in their ability to approximate NH_4, NO_3^-, and NO_2^- concentrations in each of the ten stages. When comparing modeled concentrations to Phoenix Plant Data, neither model produces a precise estimate of actual concentrations within the aeration basin. Only a rough approximation is obtained. The Storage Model should provide a more reasonable correlation, due to the fact that it takes into consideration the uptake of soluble degradable COD into cell storage. The Michaelis-Menton Model does not take this into account. However, in view of the results, no conclusion can be reached concerning the superiority of one model over the other.

An analysis was performed using the Storage Model to determine the effects of adjusting various operating parameters. The model responded in a logical manner, predicting decreased NH_4 concentrations and increased NO_3 concentrations as the SRT and DO levels were increased. The internal recycle withdrawal location was changed from stage 9 to stage 10 as part of the analysis. The model showed that this modification would have little effect on the effluent NH_4, NO_3, and NO_2^- concentrations. However, the model could not be calibrated due to lack of certainty in the kinetic coefficients. Appropriate values for the K_{sto} and k_{ss} storage terms are not known, and had to be roughly approximated for the purposes of this model. Therefore, the model cannot be used to predict profile concentrations of the various components. Nevertheless, the Storage Model could be used to predict trends in profile concentrations as a result of changing various operating parameters.

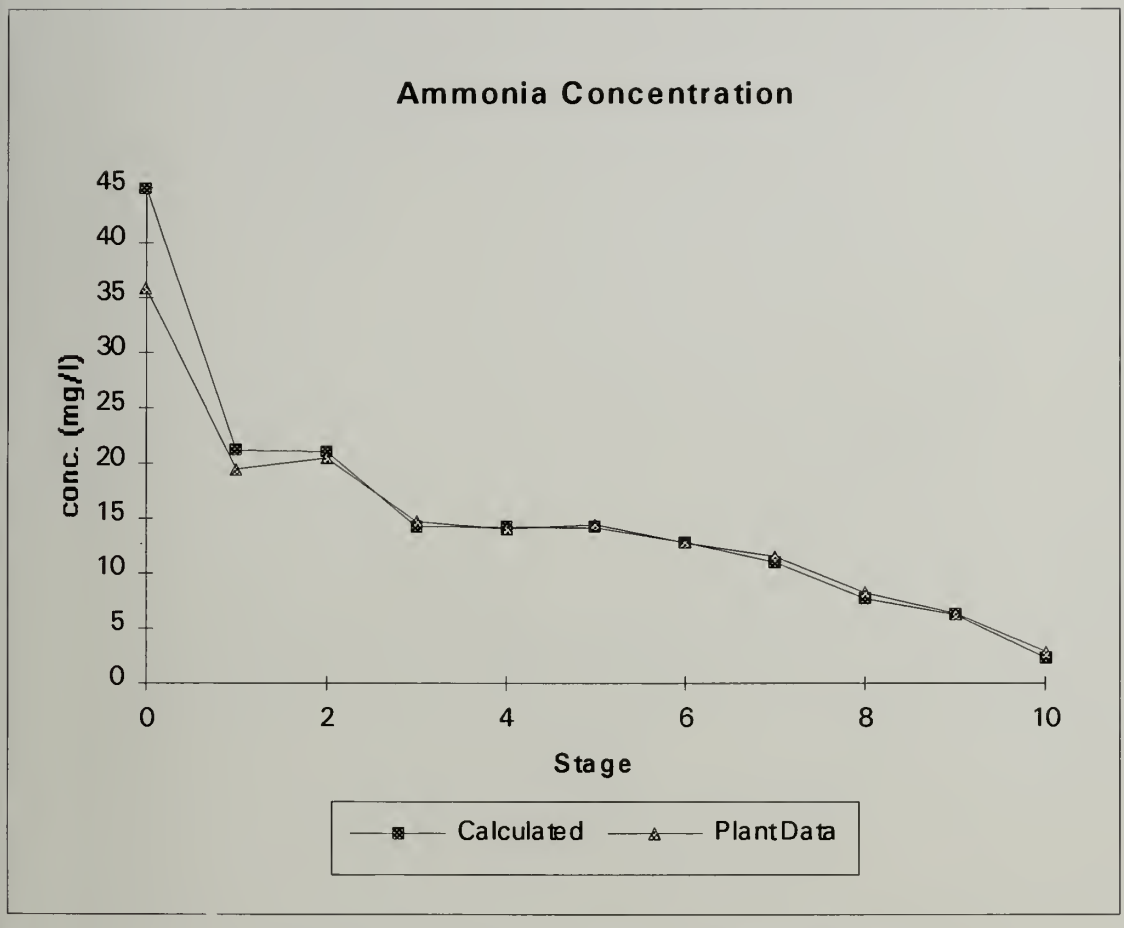

FIGURE 2: Storage Model; Ammonia Profile

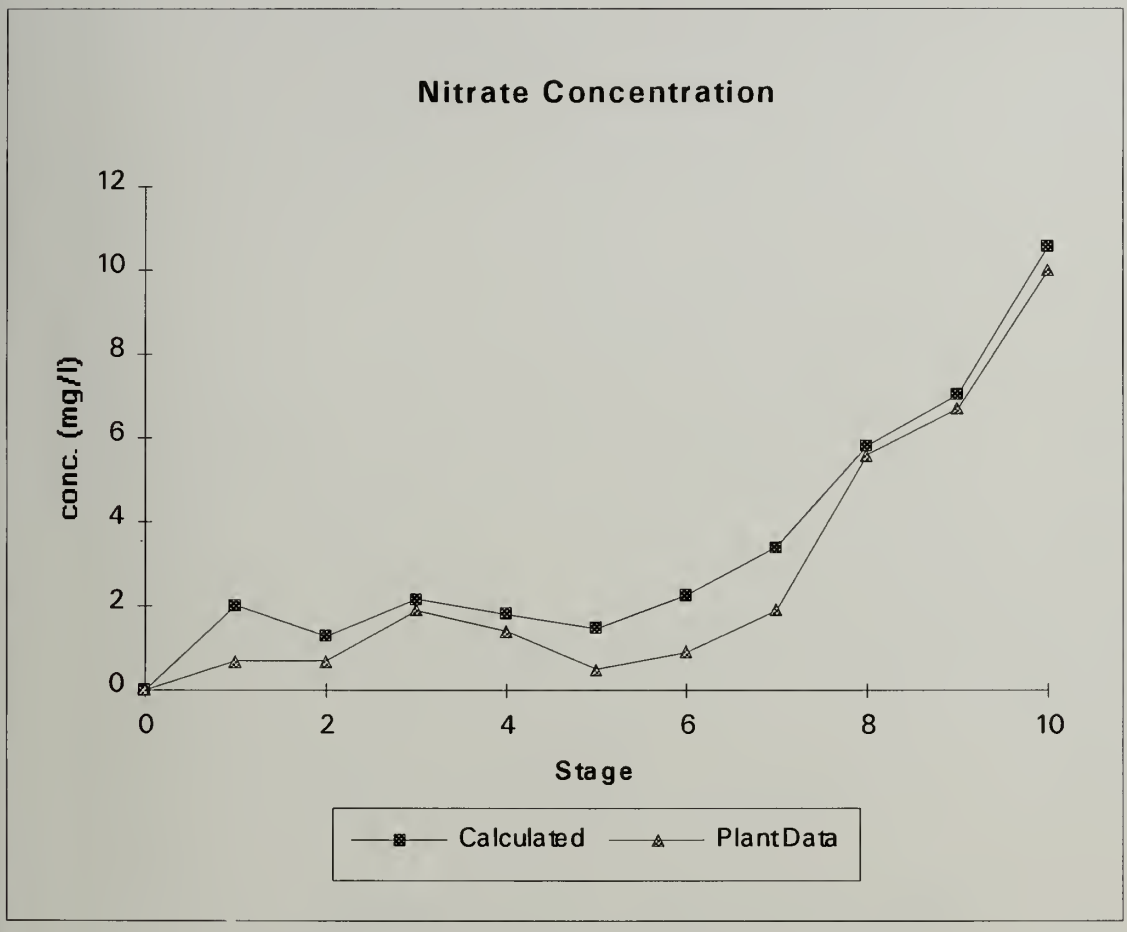

FIGURE 3: Storage Model ; Nitrate Profile

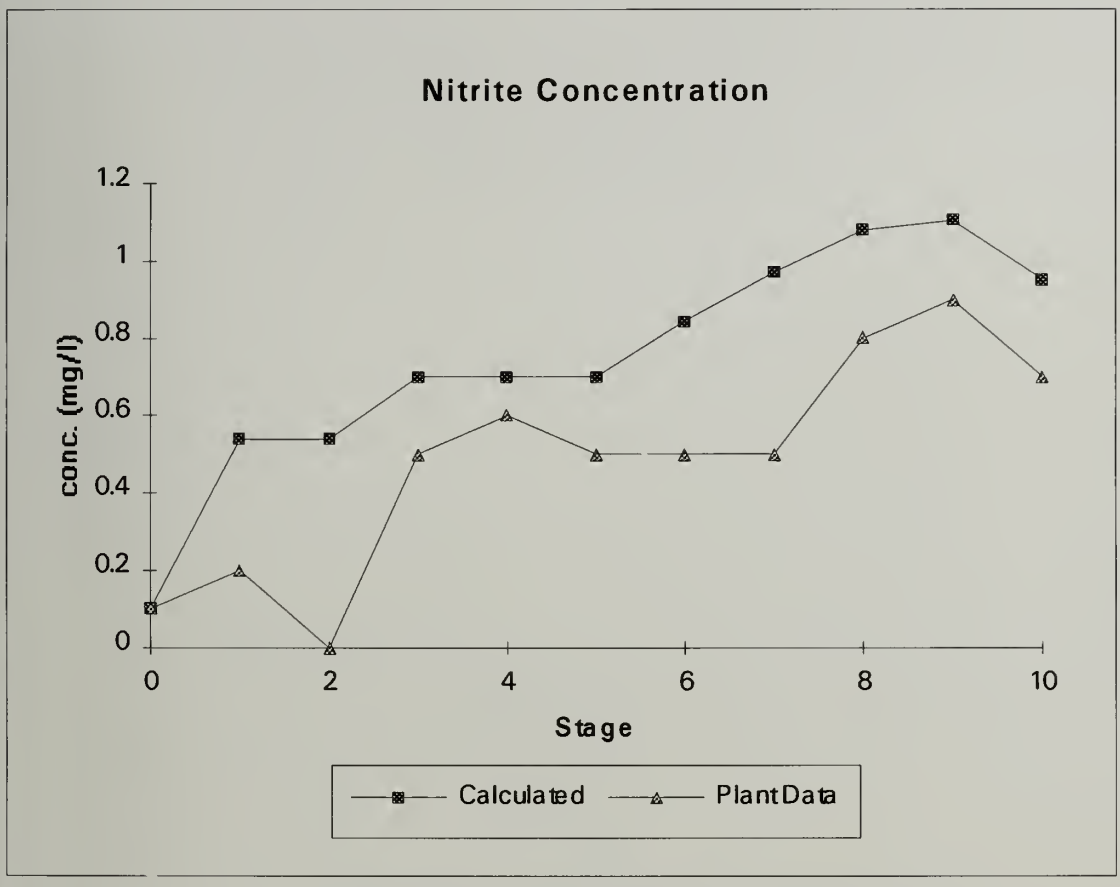

FIGURE 4: Storage Model; Nitrite Profile

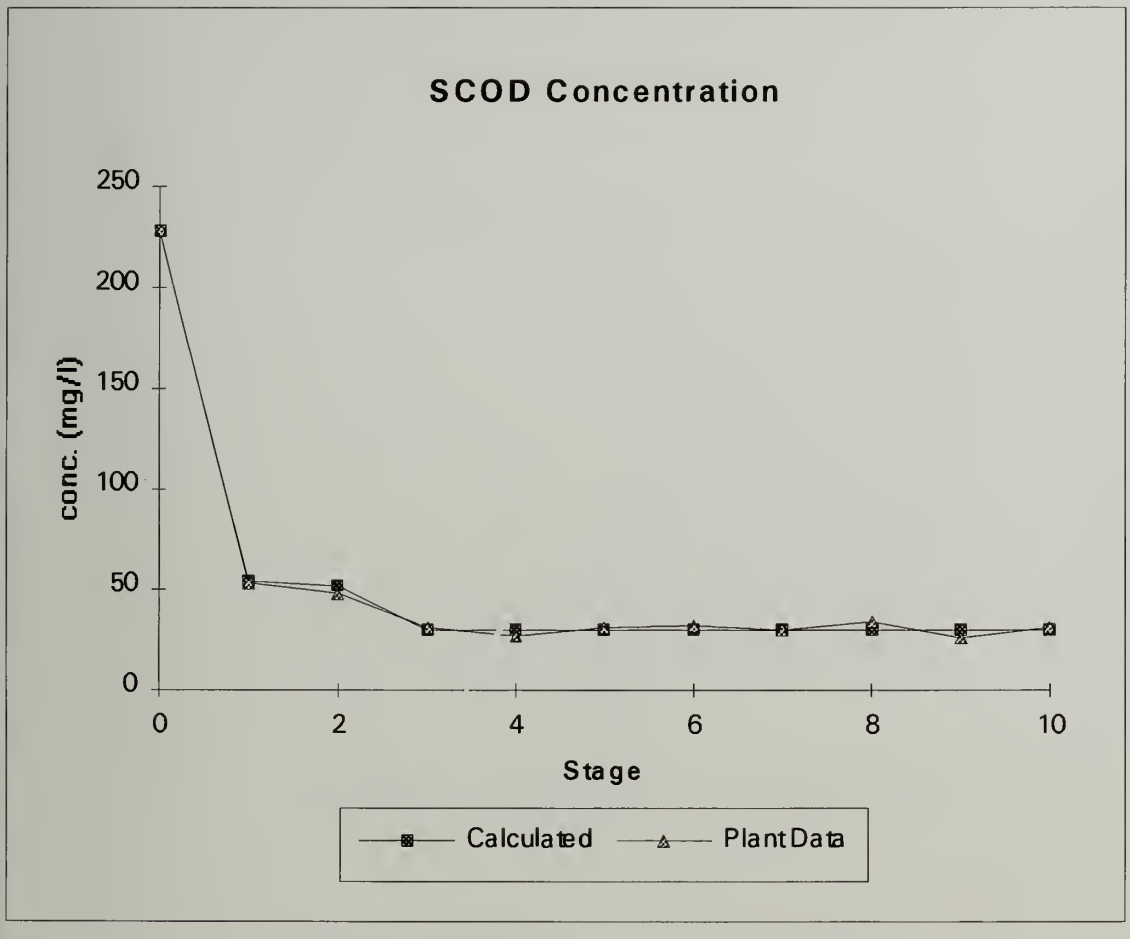

FIGURE 5: Storage Model; SCOD Profile

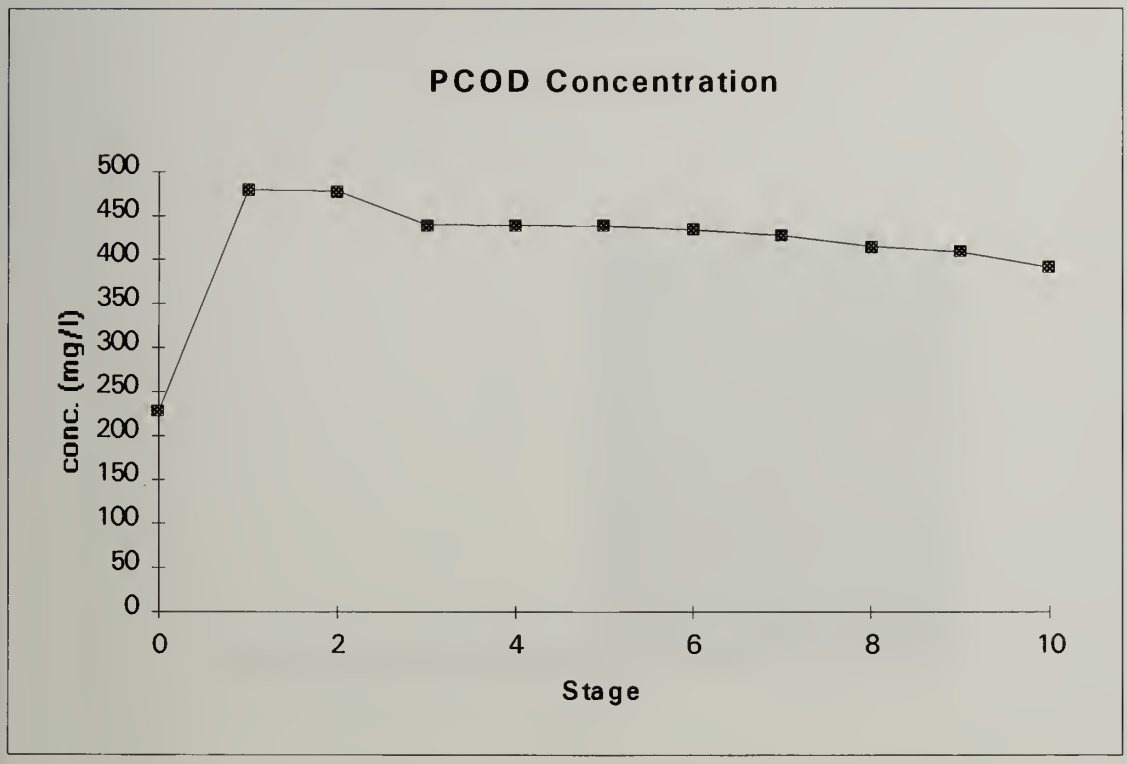

FIGURE 6: Storage Model; PCOD Profile

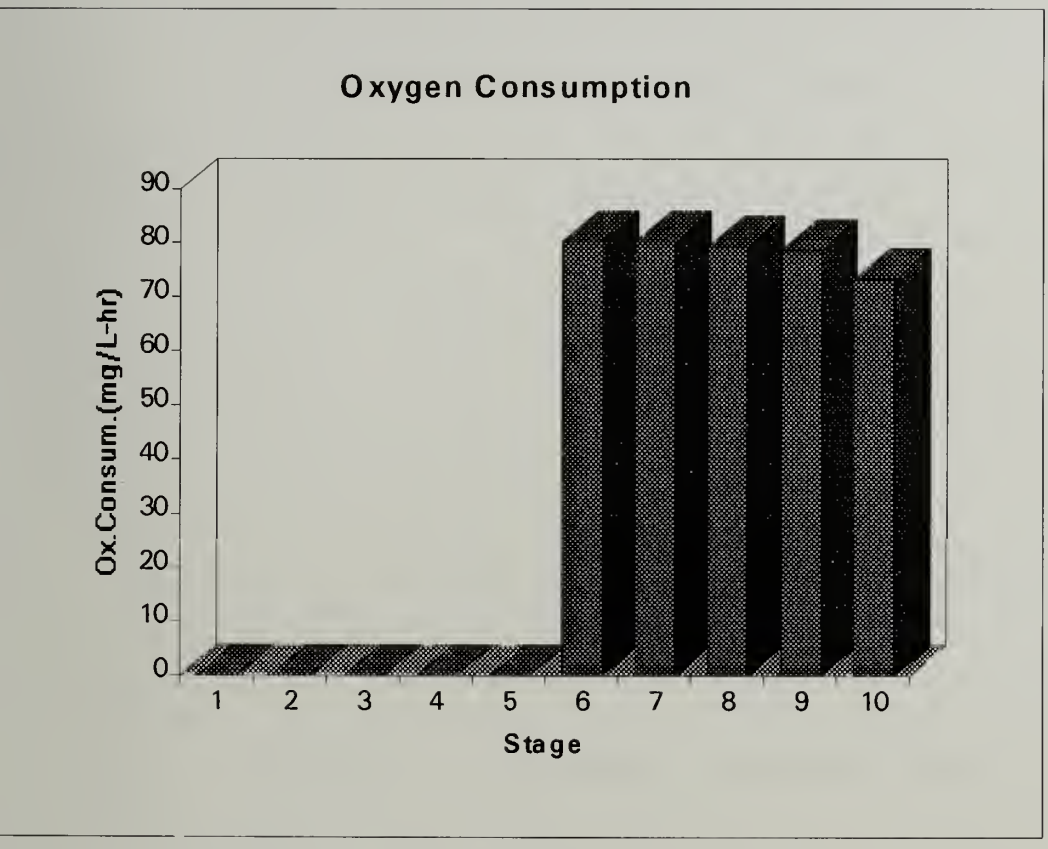

FIGURE 7: Storage Model; Oxygen Consumption Rate

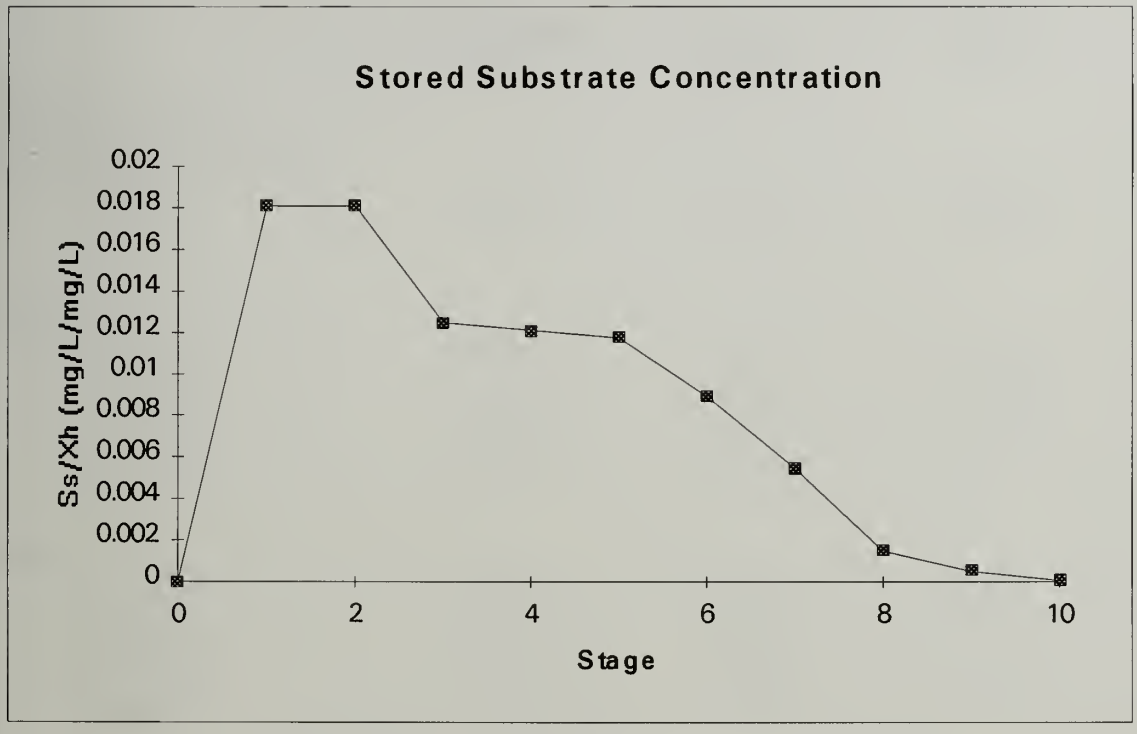

FIGURE 8: Storage Model; Stored SCOD Profile

TABLE 4: Effluent Concentrations of NH_4, NO_3 and NO_2^-

EFFLUENT CONCENTRATIONS
(Storage Model)

	Ammonia (mg/l)	Nitrate (mg/l)	Nitrite (mg/l)
Internal Recycle Withdrawal Location			
Stage 9	2.32638	10.5886	0.95286
Stage 10	2.7225	9.78687	0.98328
SRT			
3 days	5.98667	7.70259	1.10224
5 days	2.32638	10.5886	0.95286
7 days	0.56086	12.6163	0.48776
9 days	0.16648	13.6194	0.199
DO			
0.5 mg/l	6.08655	5.66895	1.0997
1.0 mg/l	4.21585	8.12662	1.07492
2.0 mg/l	2.85036	9.89541	1.00315
3.0 mg/l	2.32638	10.5886	0.95286
Distribution of Internal Recycle Flow			
Qir(1) Qir(3)			
20 MGD 40 MGD	2.32638	10.5886	0.95286
30 MGD 30 MGD	2.32554	10.5291	0.95274
40 MGD 20 MGD	2.32546	10.507	0.95273
50 MGD 10 MGD	2.32557	10.4987	0.95274
60 MGD 0 MGD	2.32573	10.4966	0.95276
Increased Internal Recycle Flow			
Qir(1) Qir(3)			
40 MGD = 13.33 + 26.67	2.27488	11.3958	0.95736
60 MGD = 20 + 40	2.32638	10.5886	0.95286
75 MGD = 25 + 50	2.35185	10.3736	0.94915

TABLE 4 (continued): Effluent Concentrations

	Ammonia	**Nitrate**	**Nitrite**
Compare Storage Model and Michaelis-Menton Model			
Storage Model	2.32638	10.5886	0.95286
Michaelis-Menton Model	2.39589	9.23093	0.95974
Compare 4 NOV and 17 NOV Plant Data Using Storage Model			
17 NOV	2.32638	10.5886	0.95286
4 NOV	2.72203	11.8047	0.97504

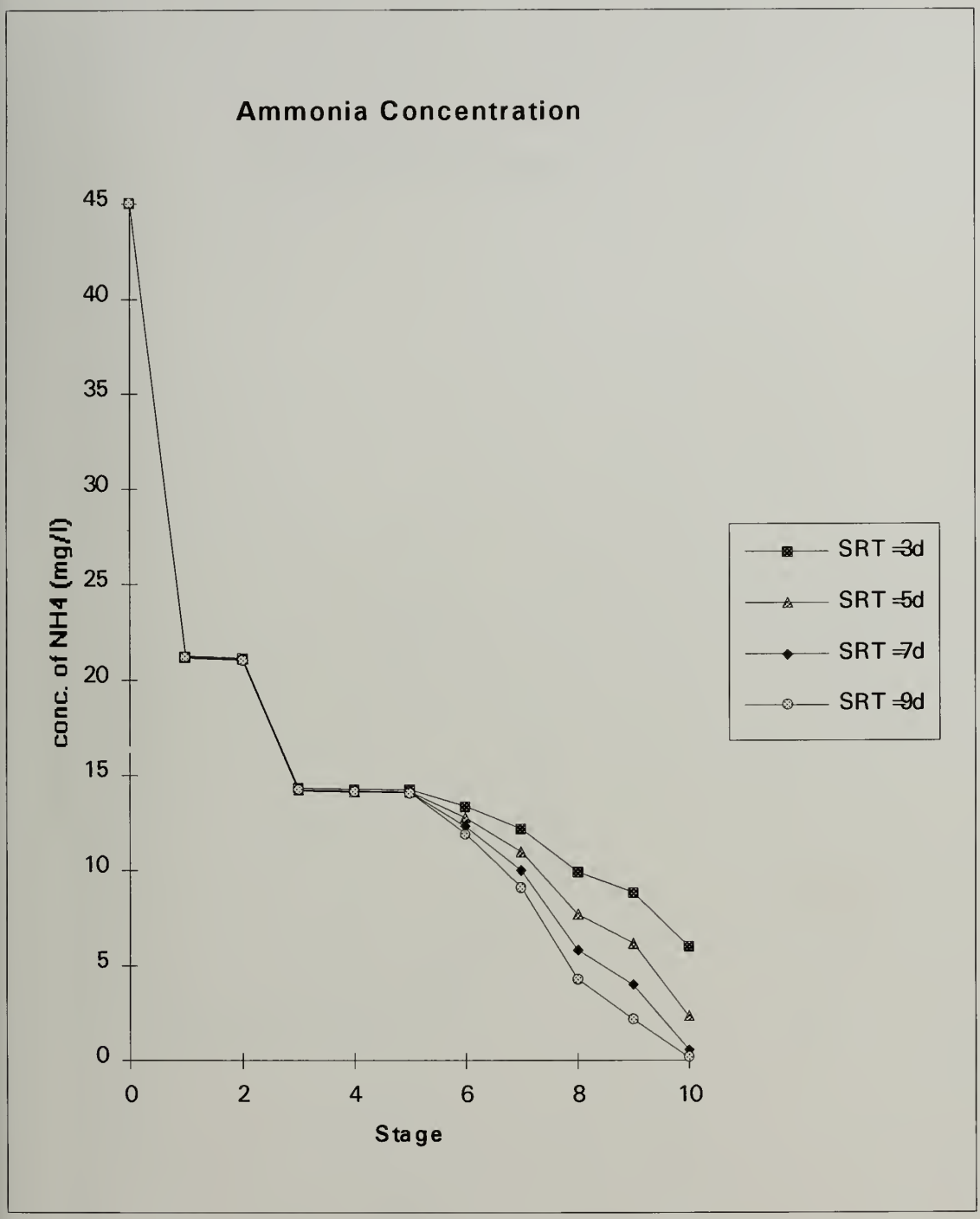

FIGURE 9: Effect of SRT on Ammonia Concentrations

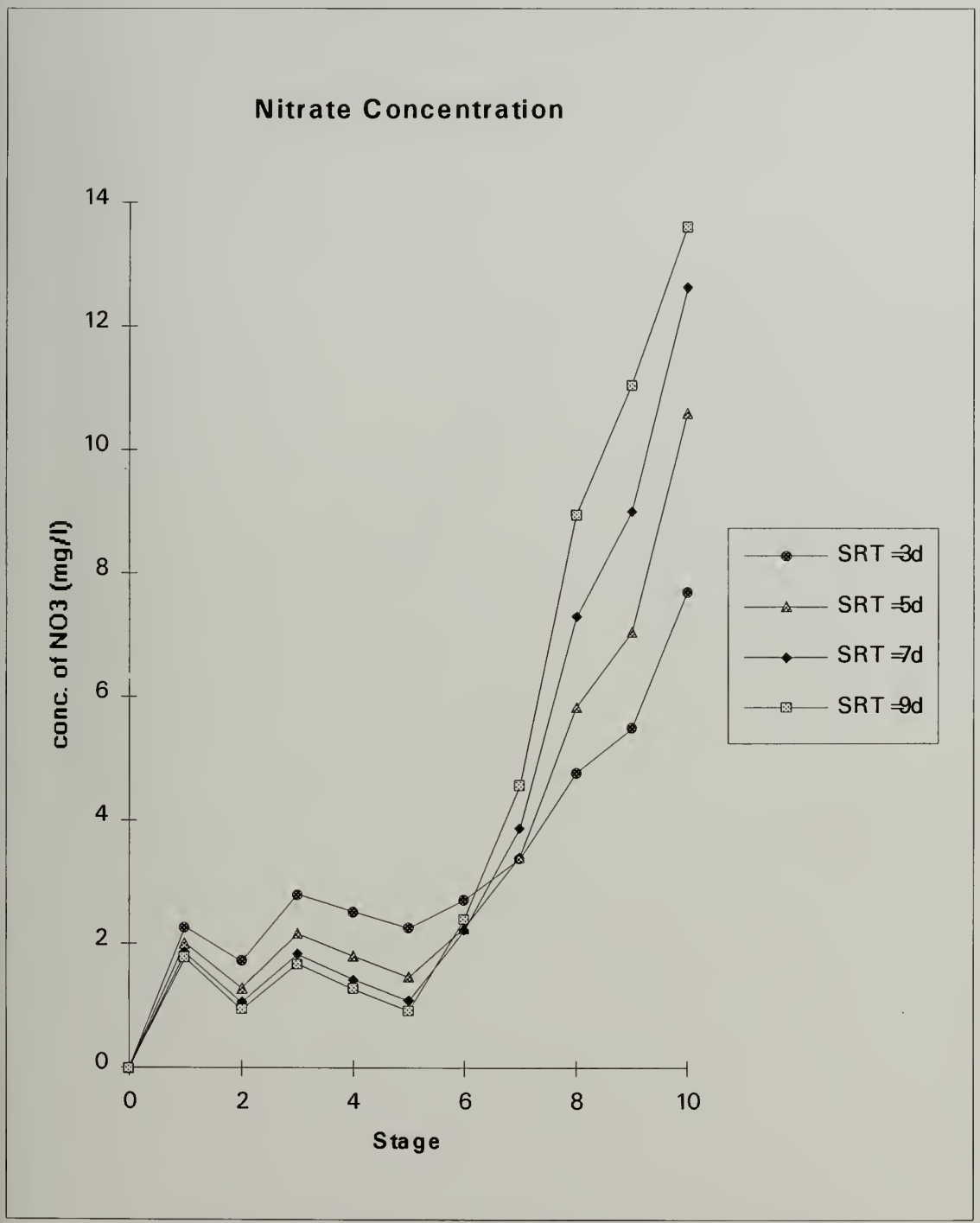

FIGURE 10: Effect of SRT on Nitrate Concentrations

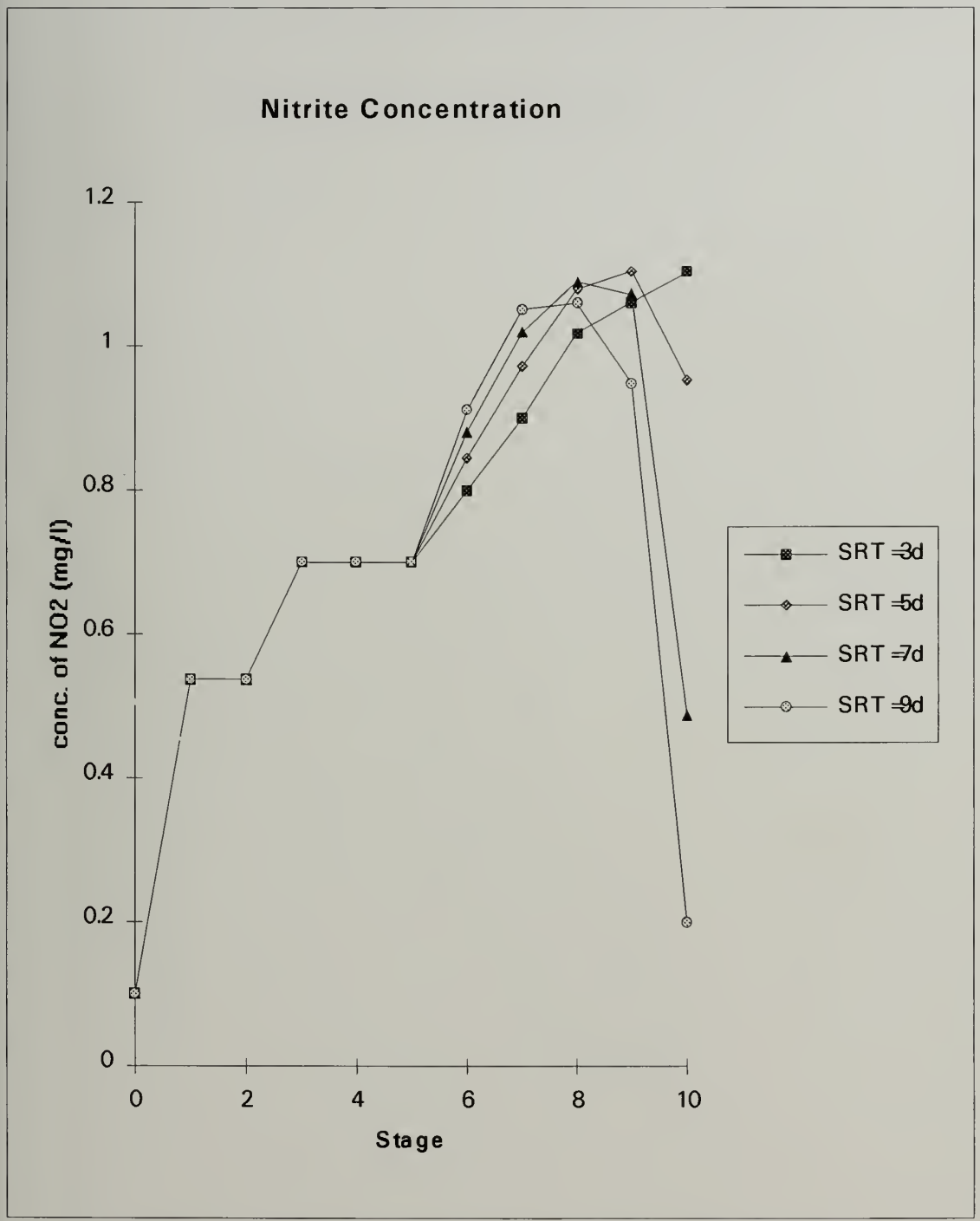

FIGURE 11: Effect of SRT on Nitrite Concentrations

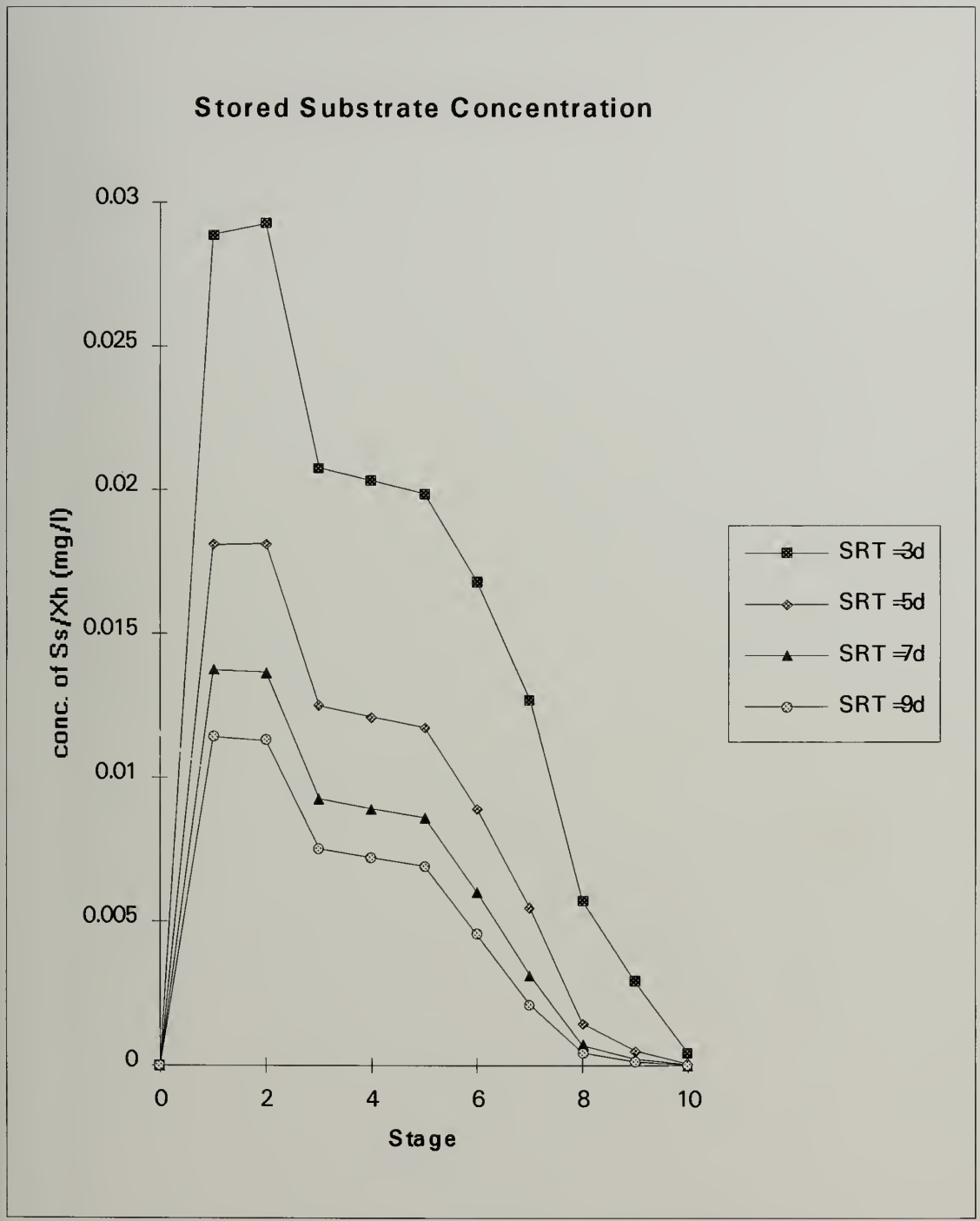

FIGURE 12: Effect of SRT on Stored SCOD / X_H Ratio

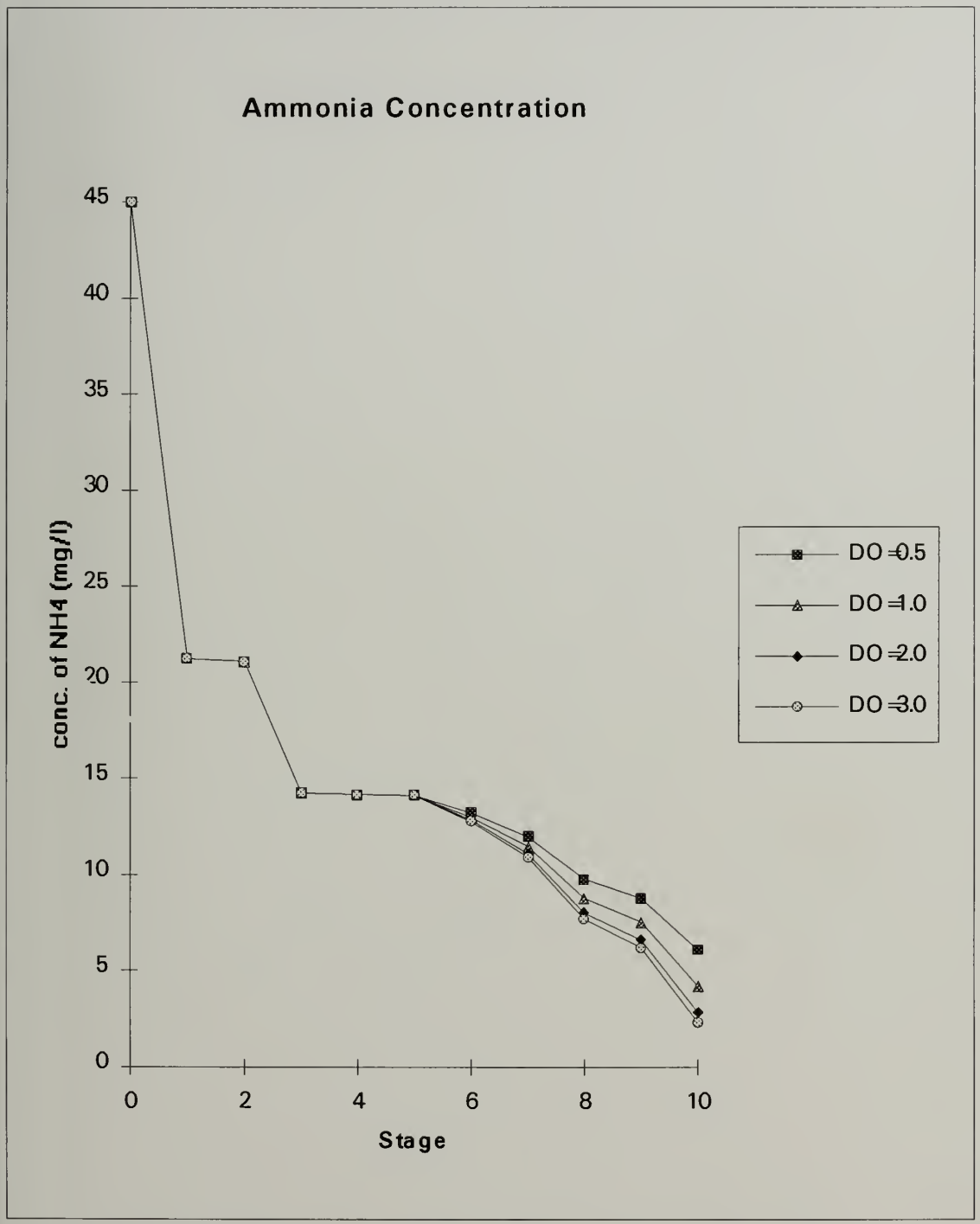

FIGURE 13: Effect of DO on Ammonia Concentration

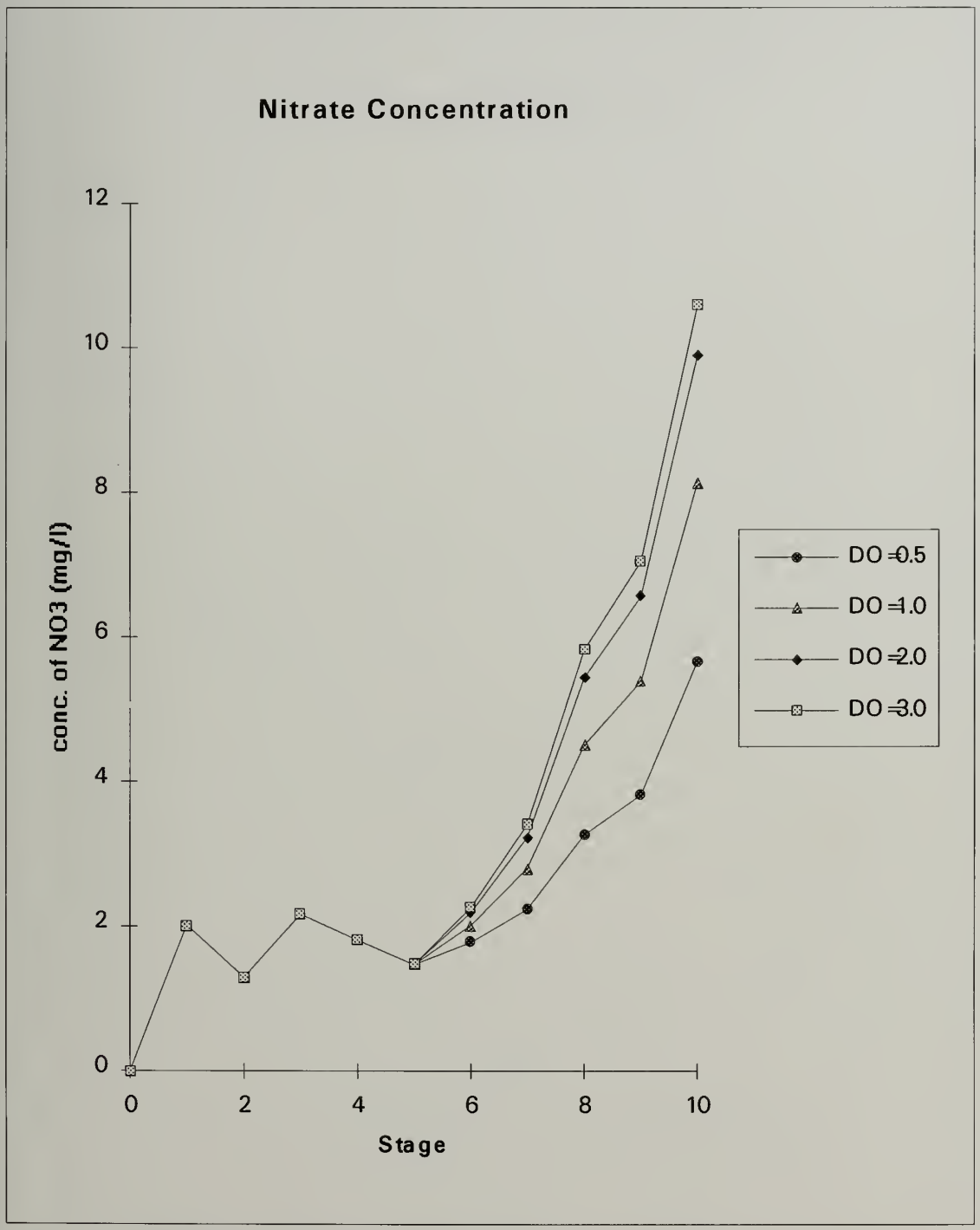

FIGURE 14: Effect of DO on Nitrate Concentration

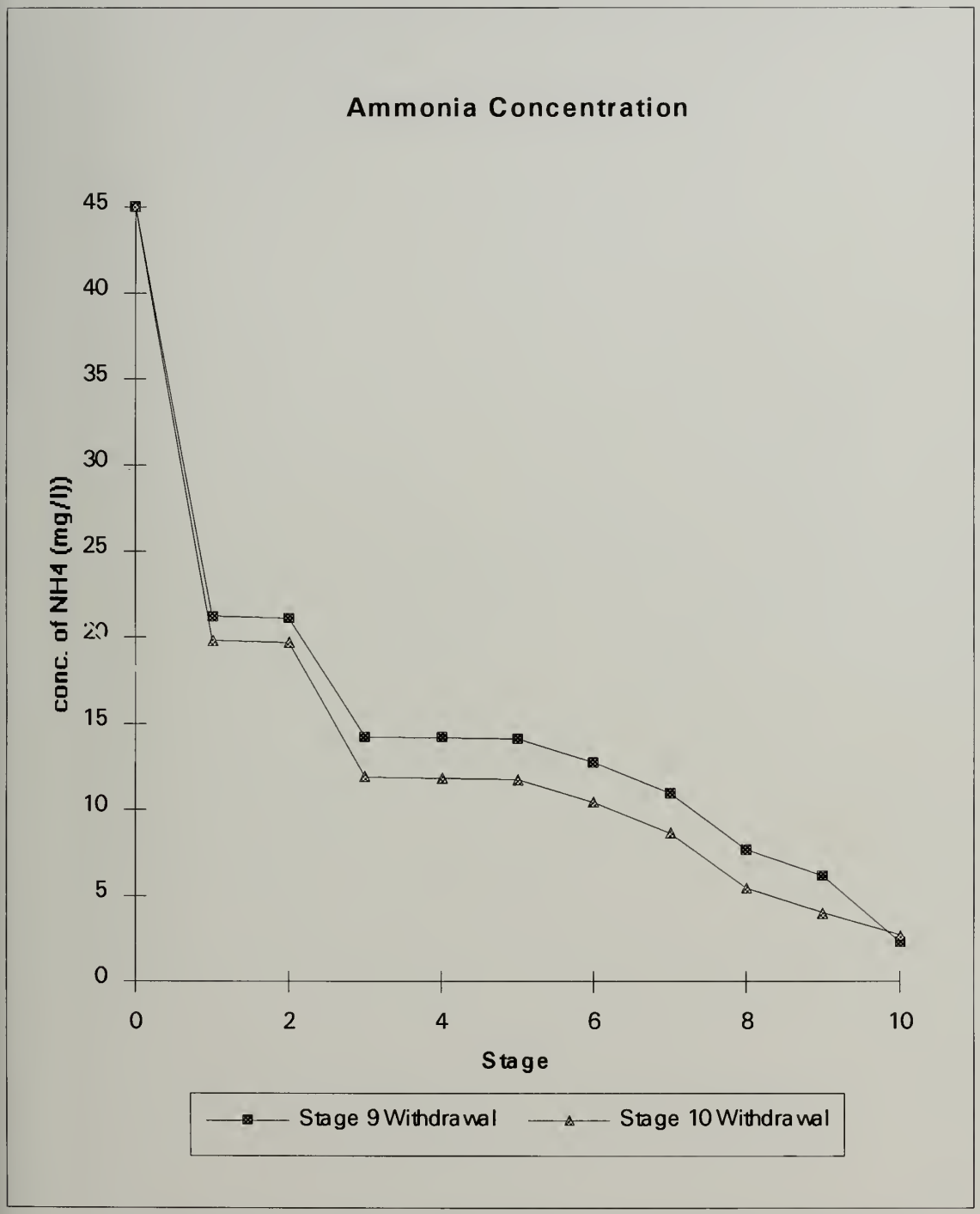

FIGURE 15: Effect of Internal Recycle Withdrawal Location on Ammonia Concentration

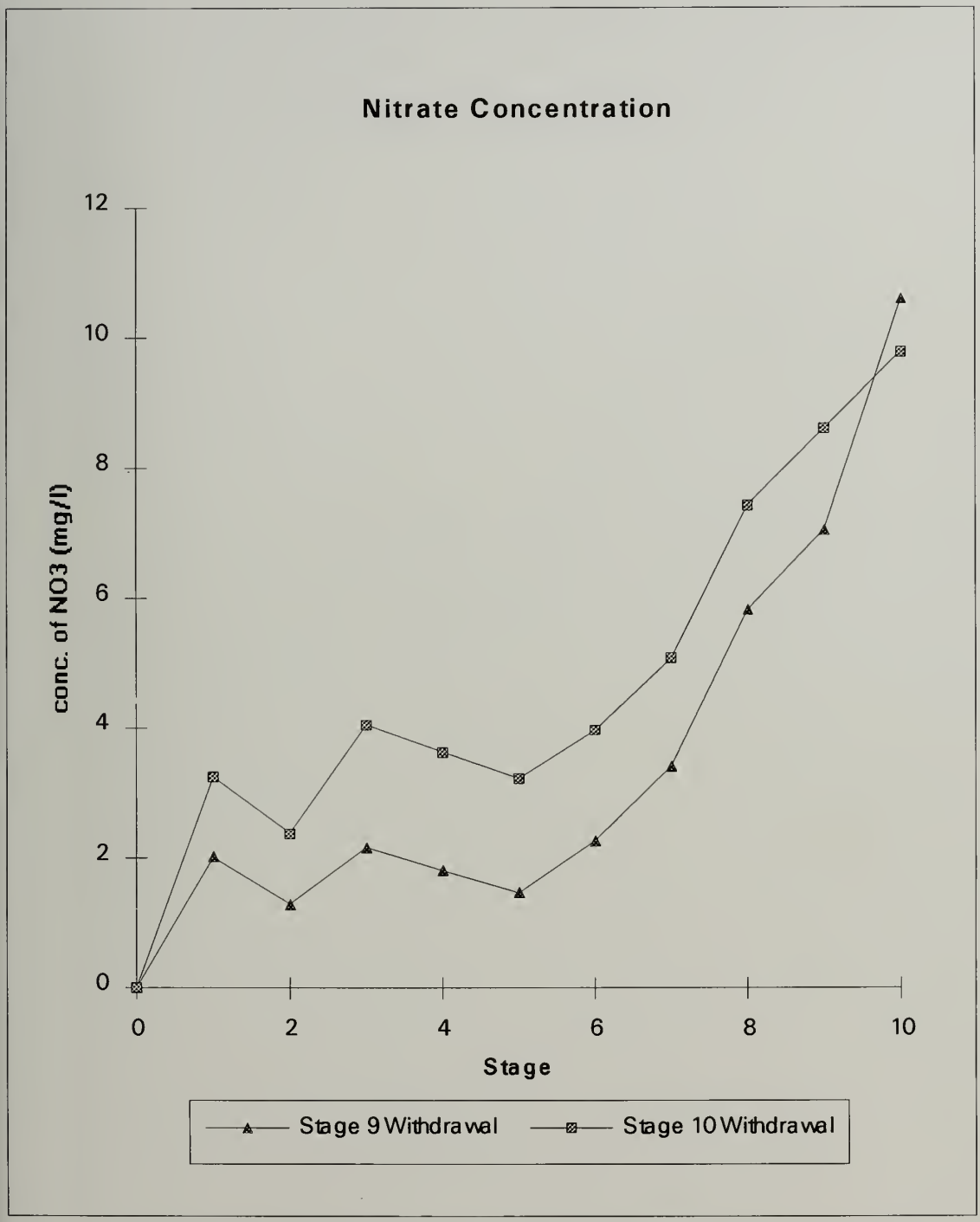

FIGURE 16: Effect of Internal Recycle Withdrawal Location on Nitrate Concentration

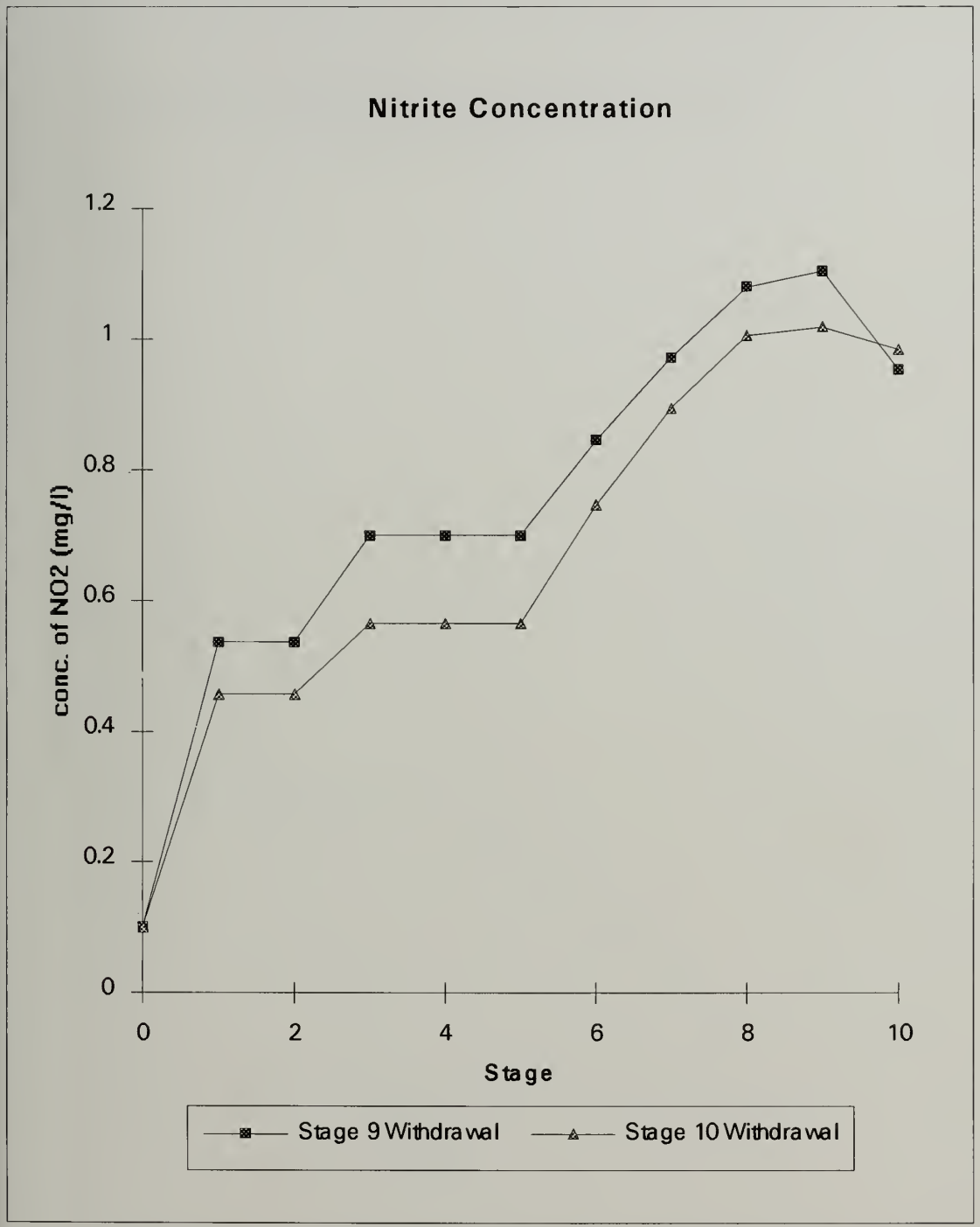

FIGURE 17: Effect of Internal Recycle Withdrawal Location on Nitrite Concentration

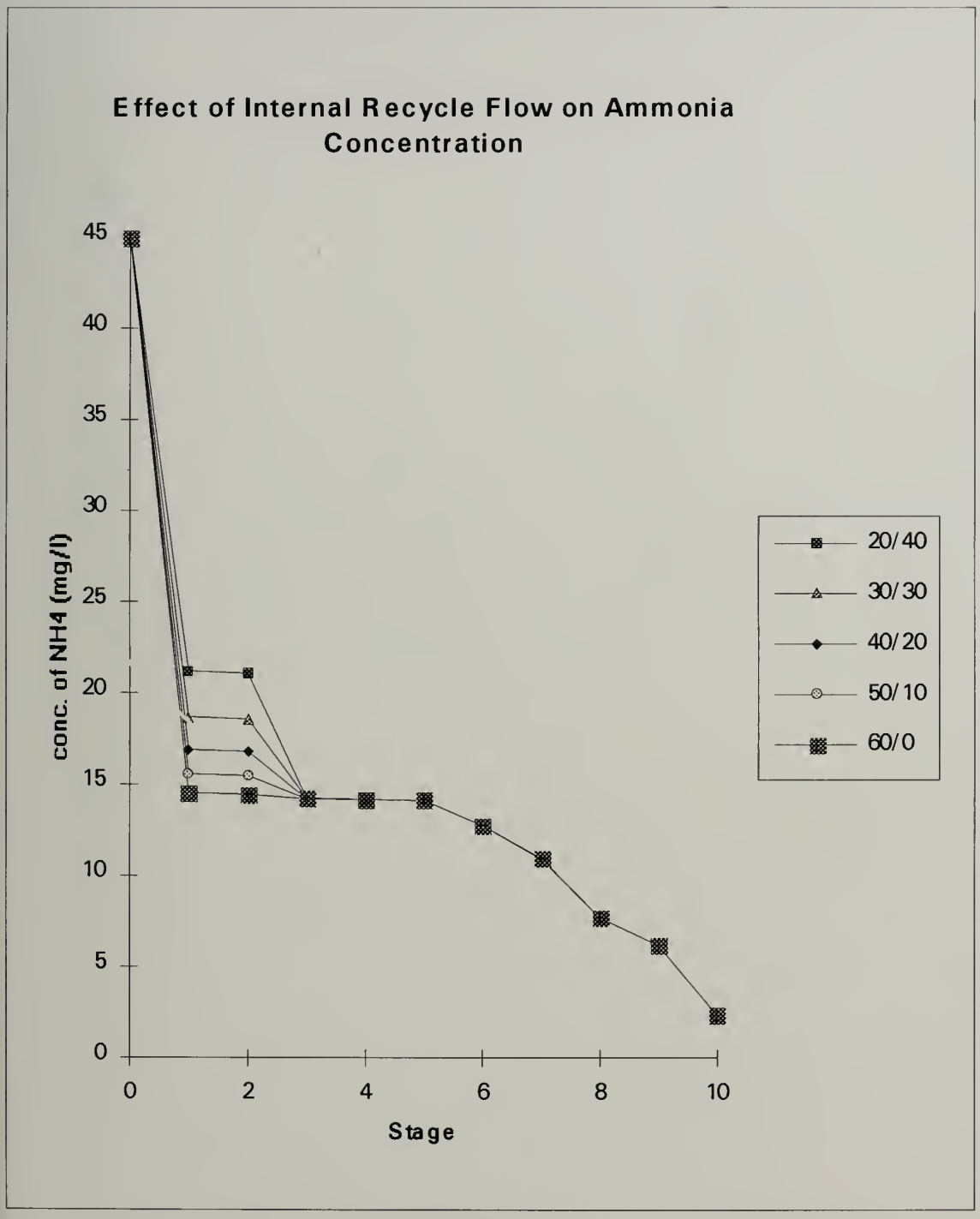

FIGURE 18: Effect of Internal Recycle Flow Distribution on Ammonia Concentration $[Q_{ir(1)} / Q_{ir(3)}]$

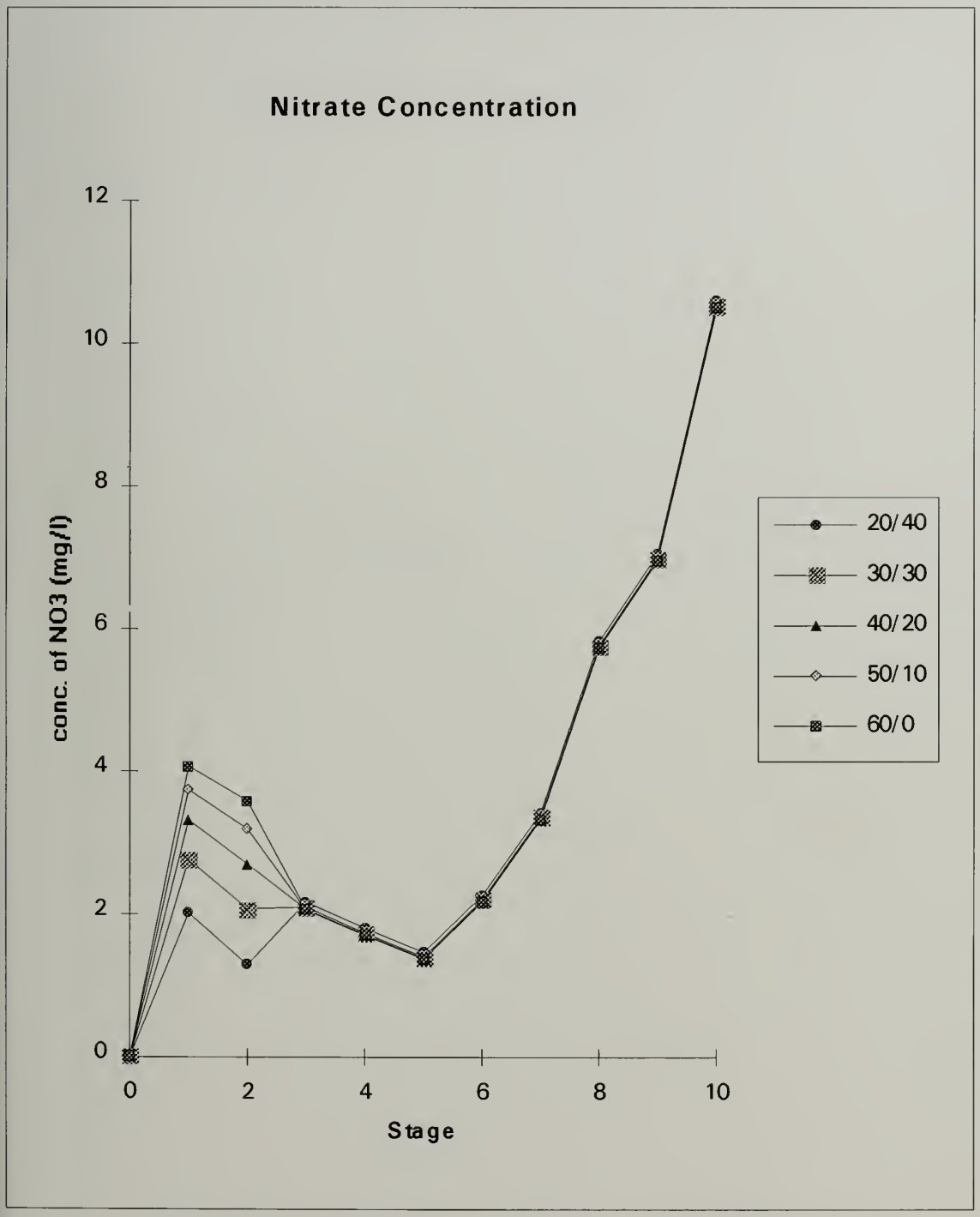

FIGURE 19: Effect of Internal Recycle Flow Distribution on Nitrate Concentration [$Q_{ir(1)}$ / $Q_{ir(3)}$]

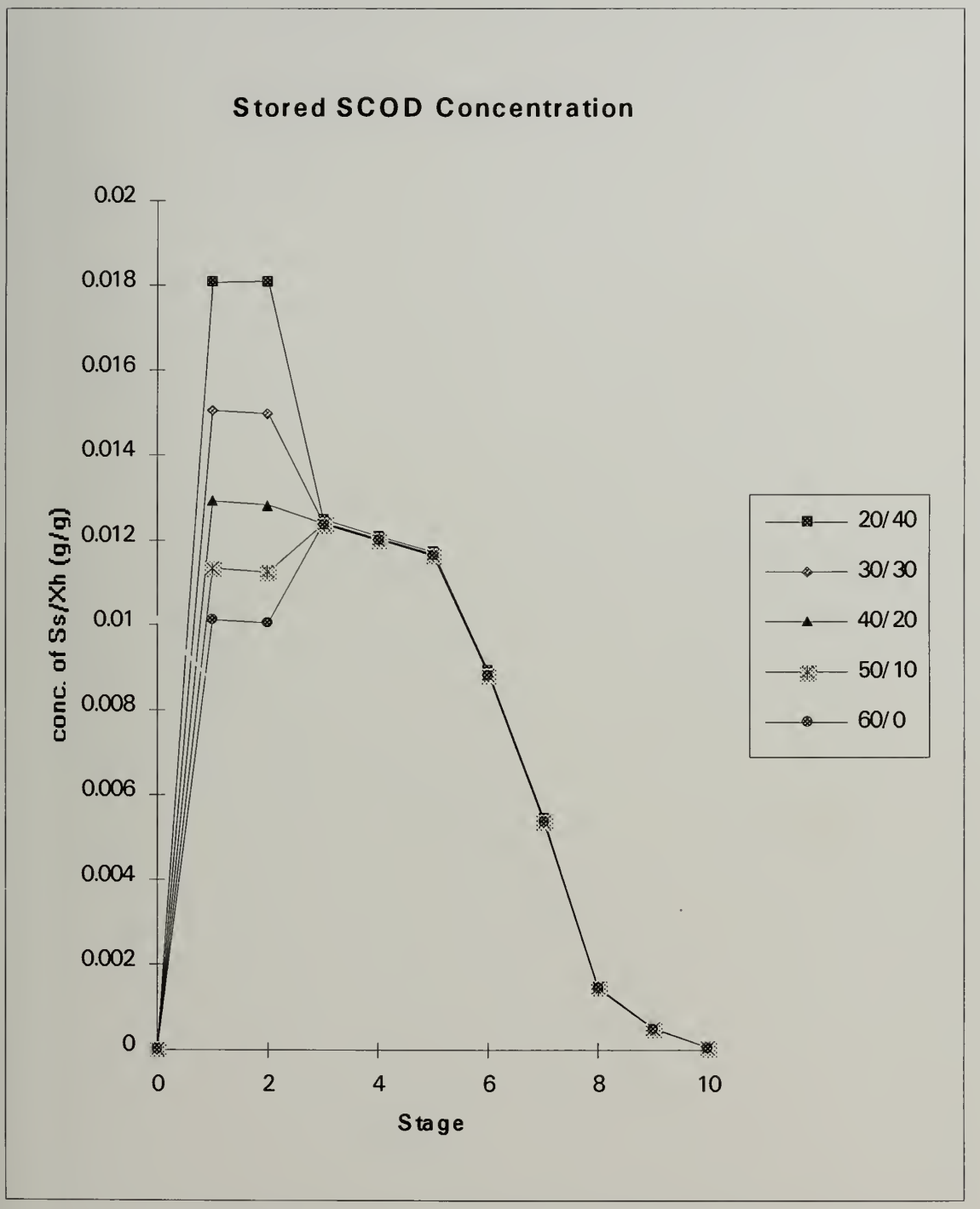

FIGURE 20: Effect of Internal Recycle Flow Distribution on Stored SCOD Profile [$Q_{ir(1)}$ / $Q_{ir(3)}$]

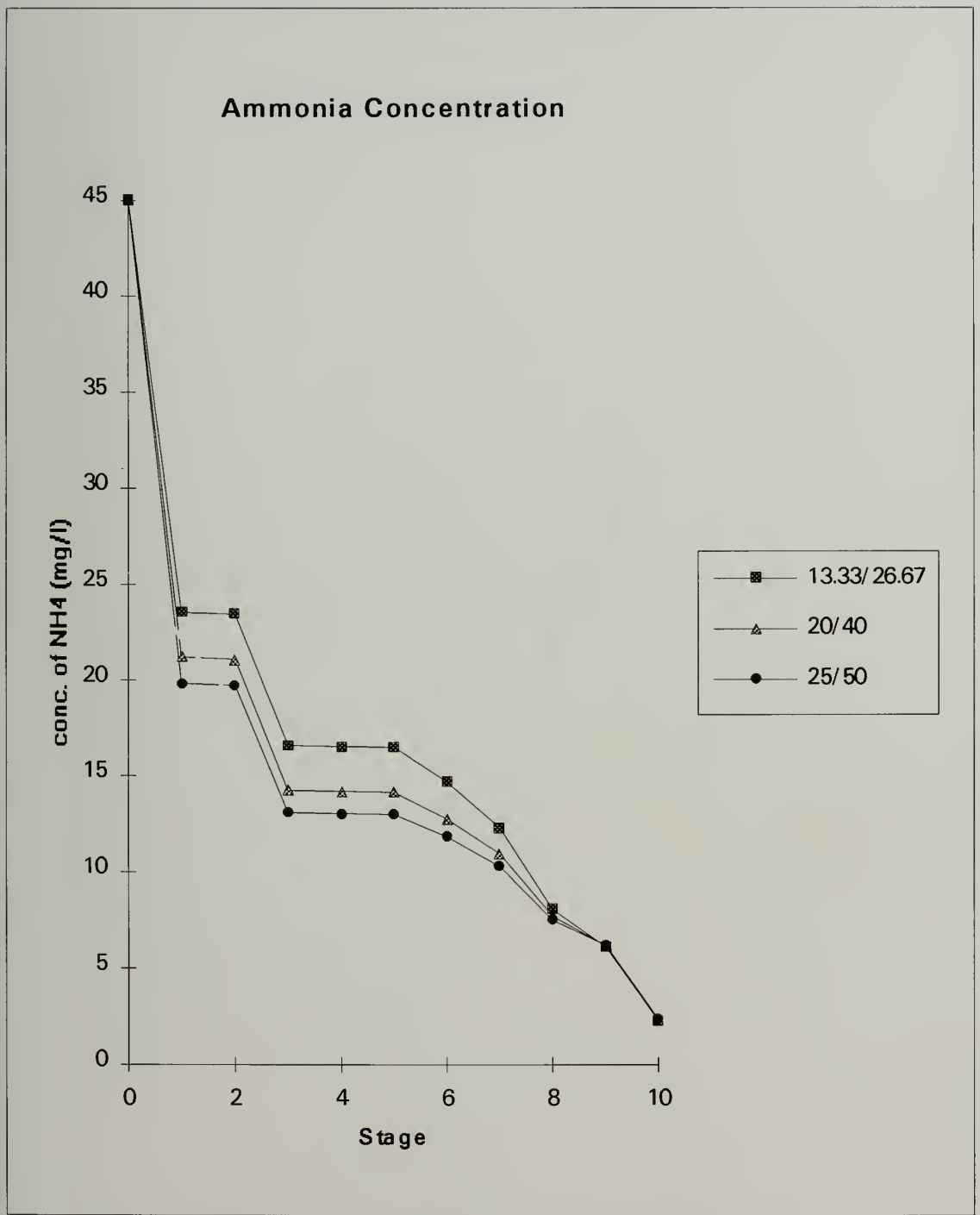

FIGURE 21: Effect of Internal Recycle Flow Amount on Ammonia Concentration

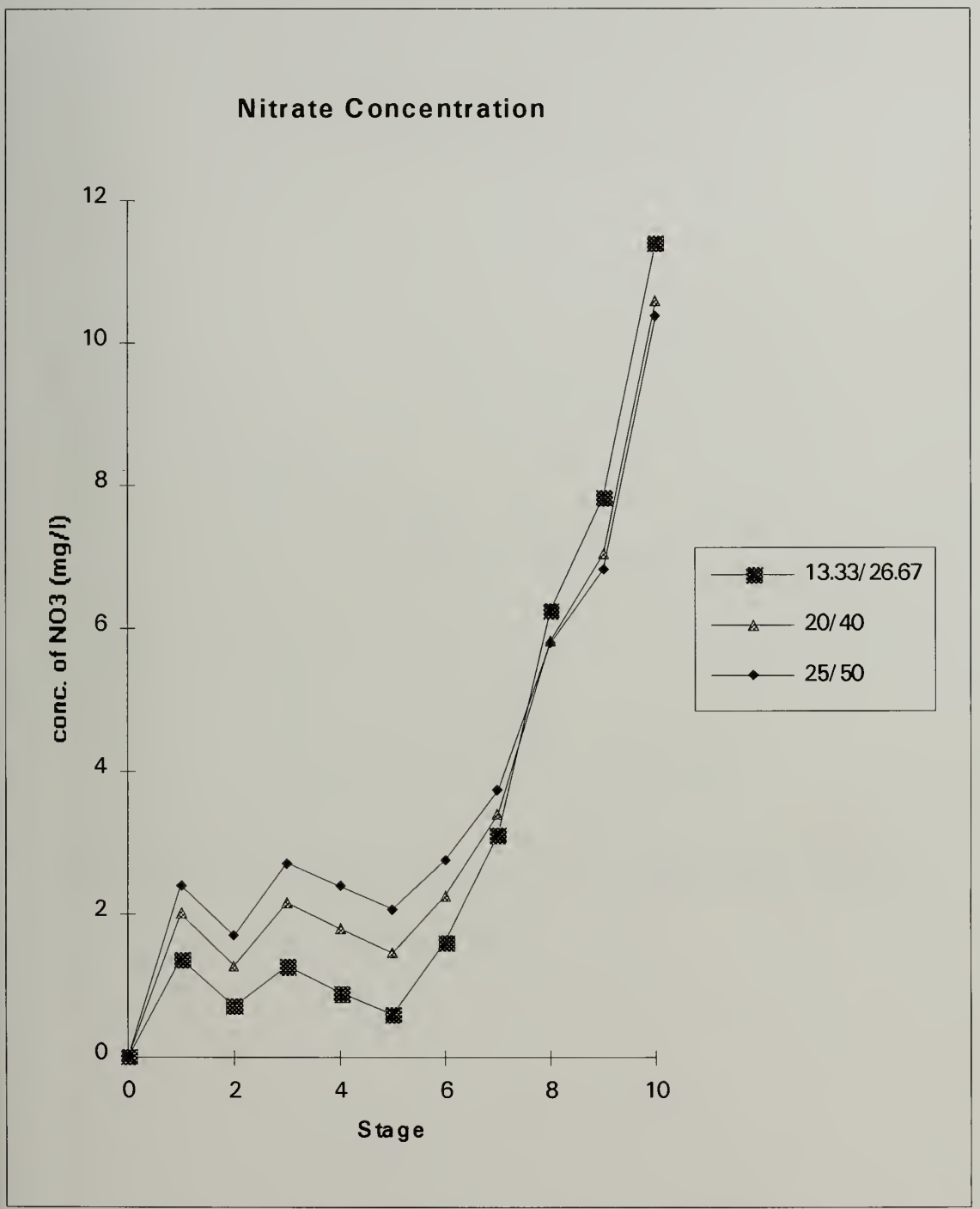

FIGURE 22: Effect of Internal Recycle Flow Amount on Nitrate Concentration

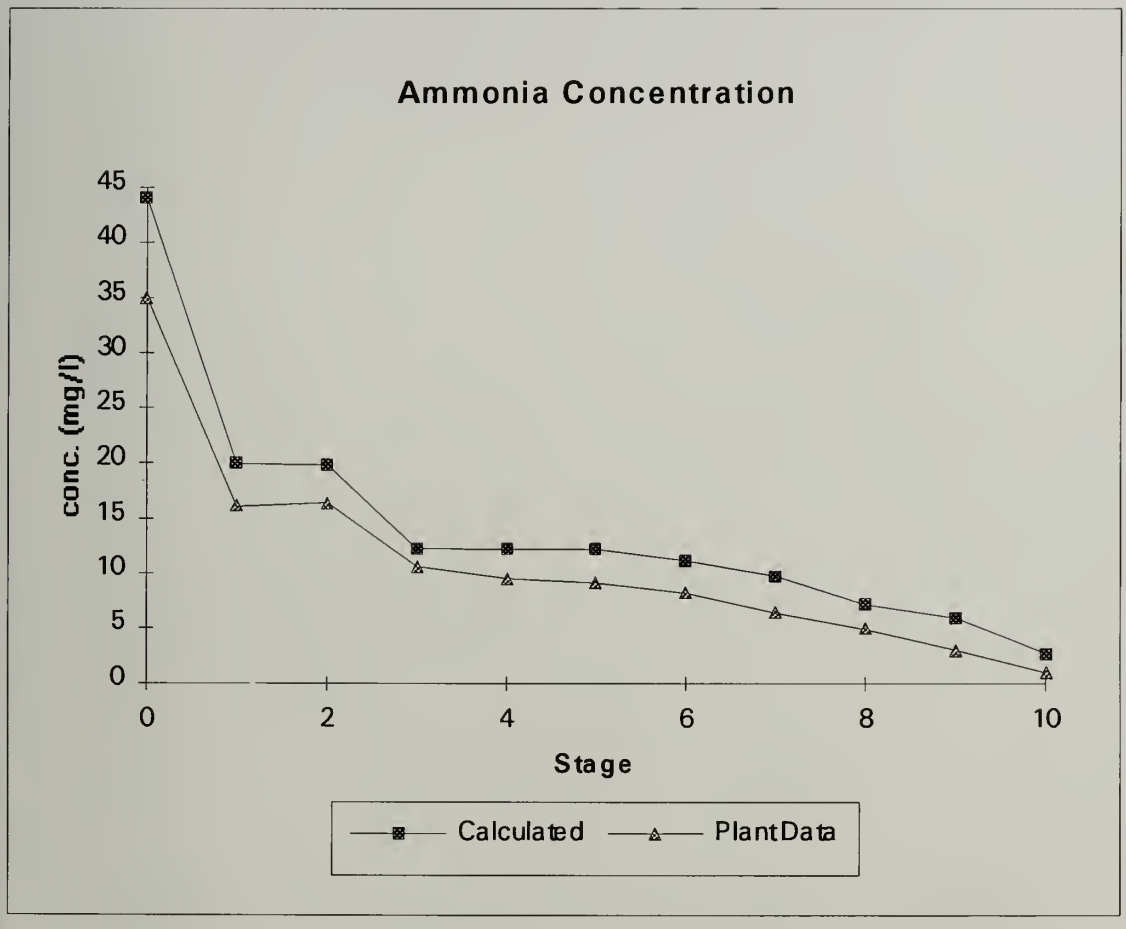

FIGURE 23: 4 November 1992 Data Set; Ammonia Concentration; Storage Model

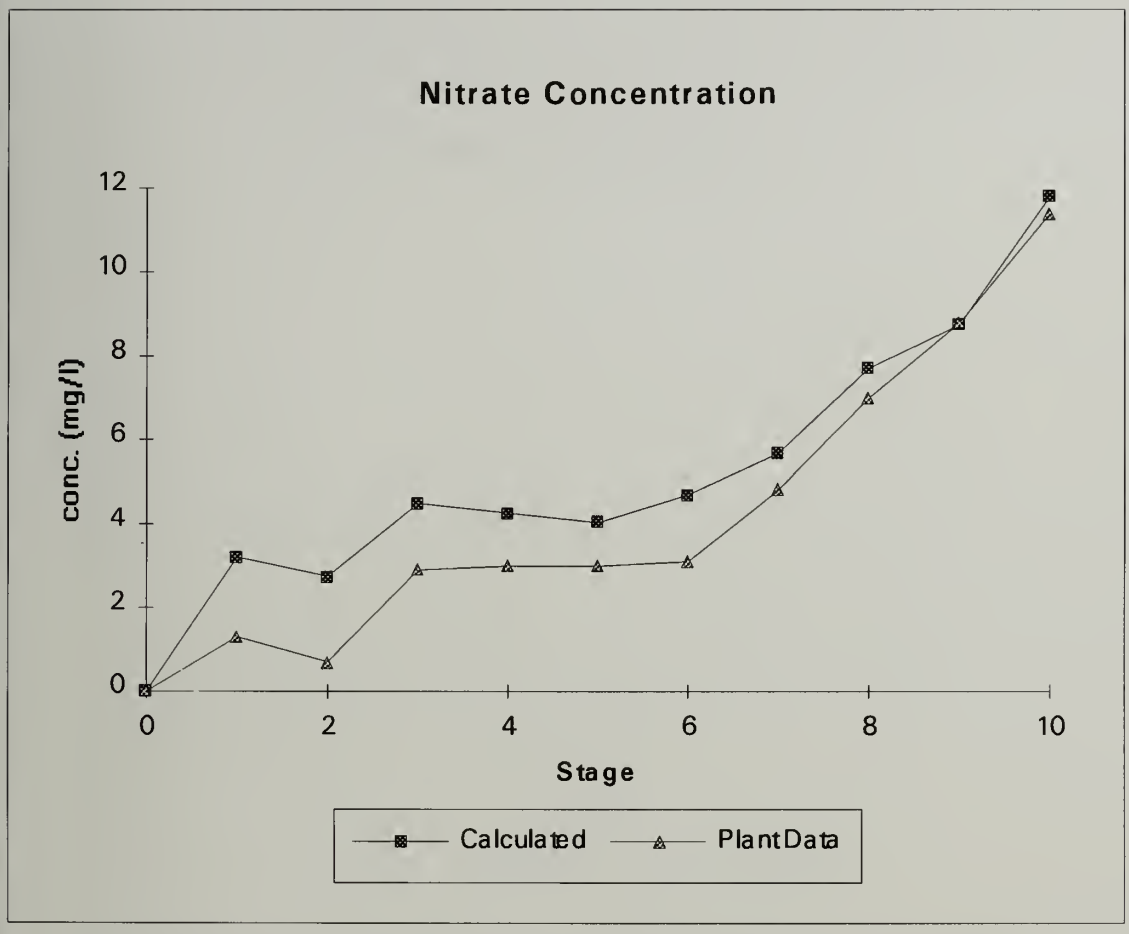

FIGURE 24: 4 November 1992 Data Set; Nitrate Concentration; Storage Model

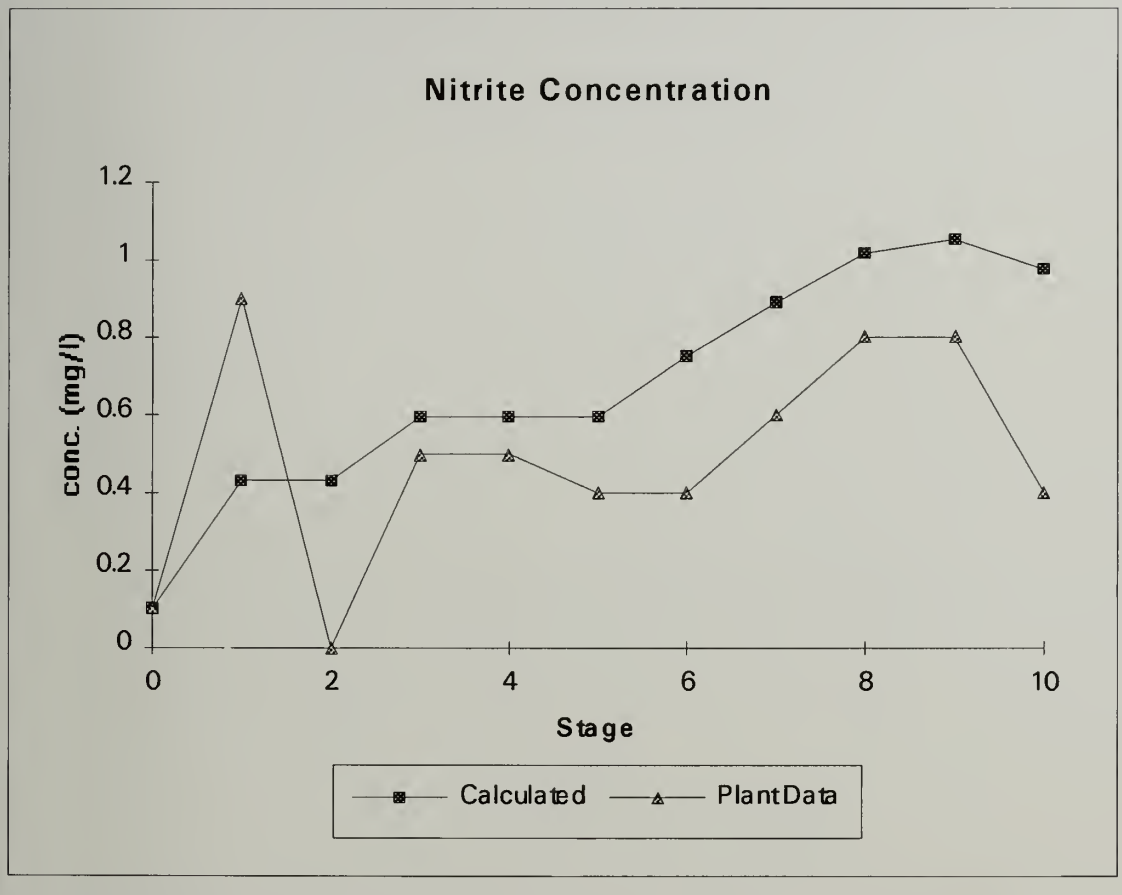

FIGURE 25: 4 November 1992 Data Set; Nitrite Concentration; Storage Model

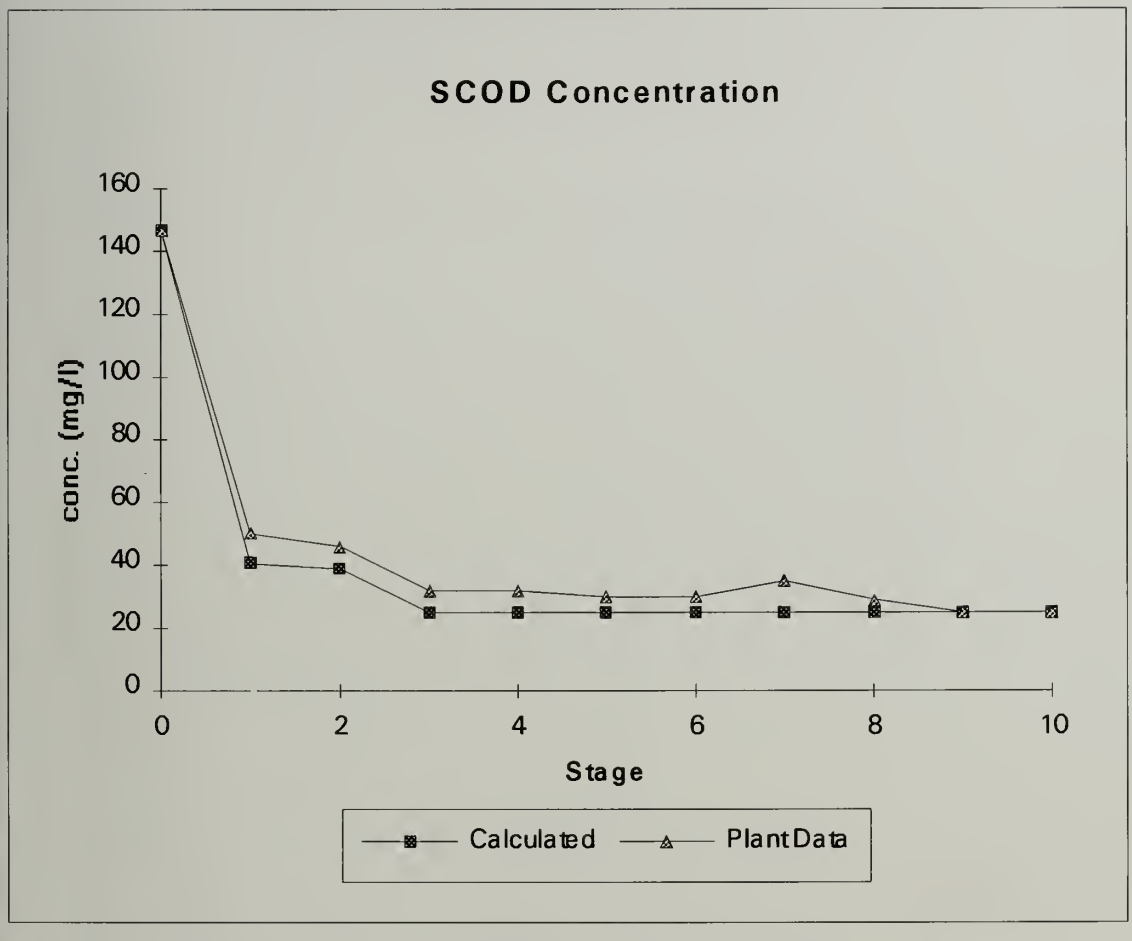

FIGURE 26: 4 November 1992 Data Set; SCOD Concentration; Storage Model

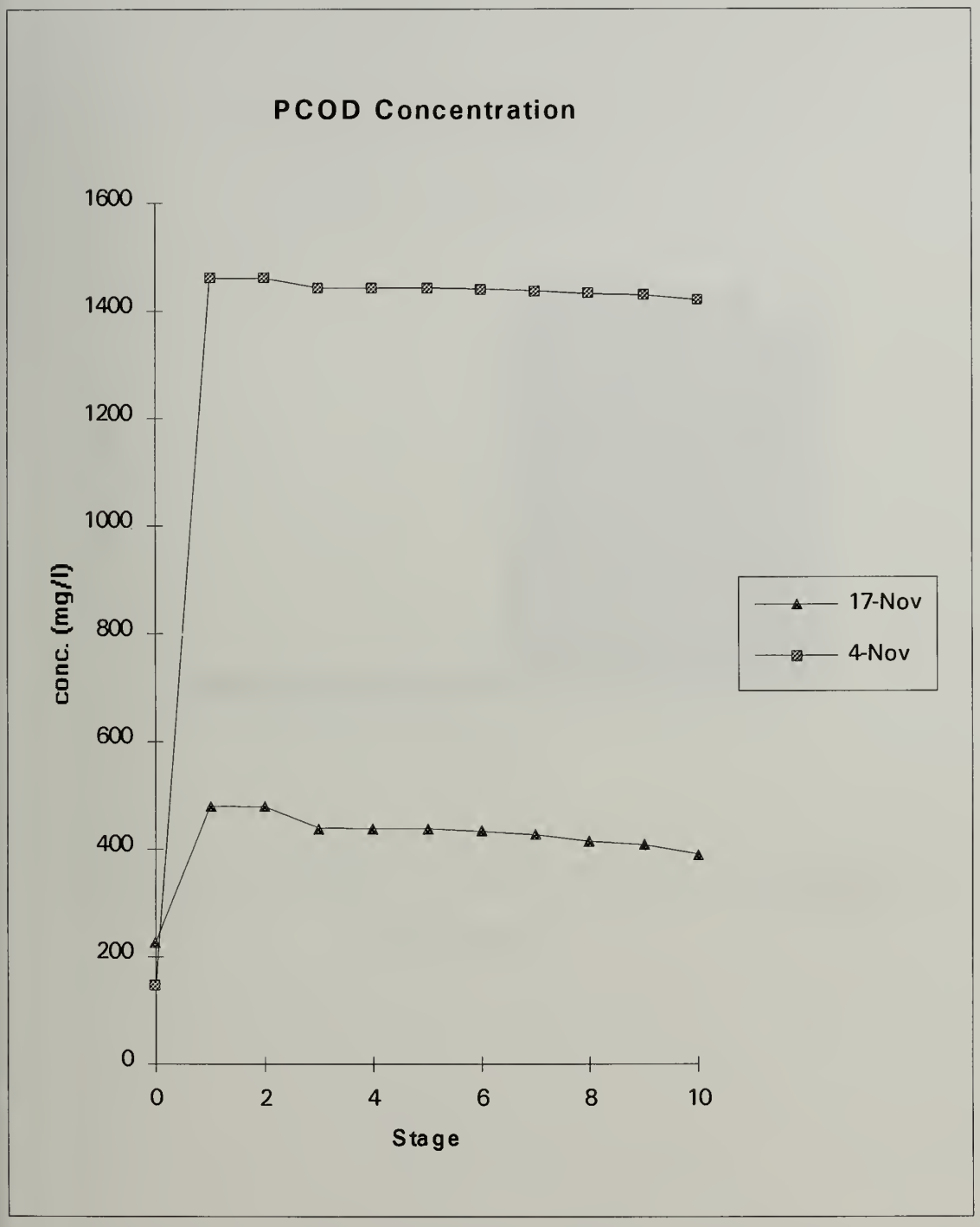

FIGURE 27: Compare 4 November and 17 November 1992 Data Sets; PCOD Concentration; Storage Model

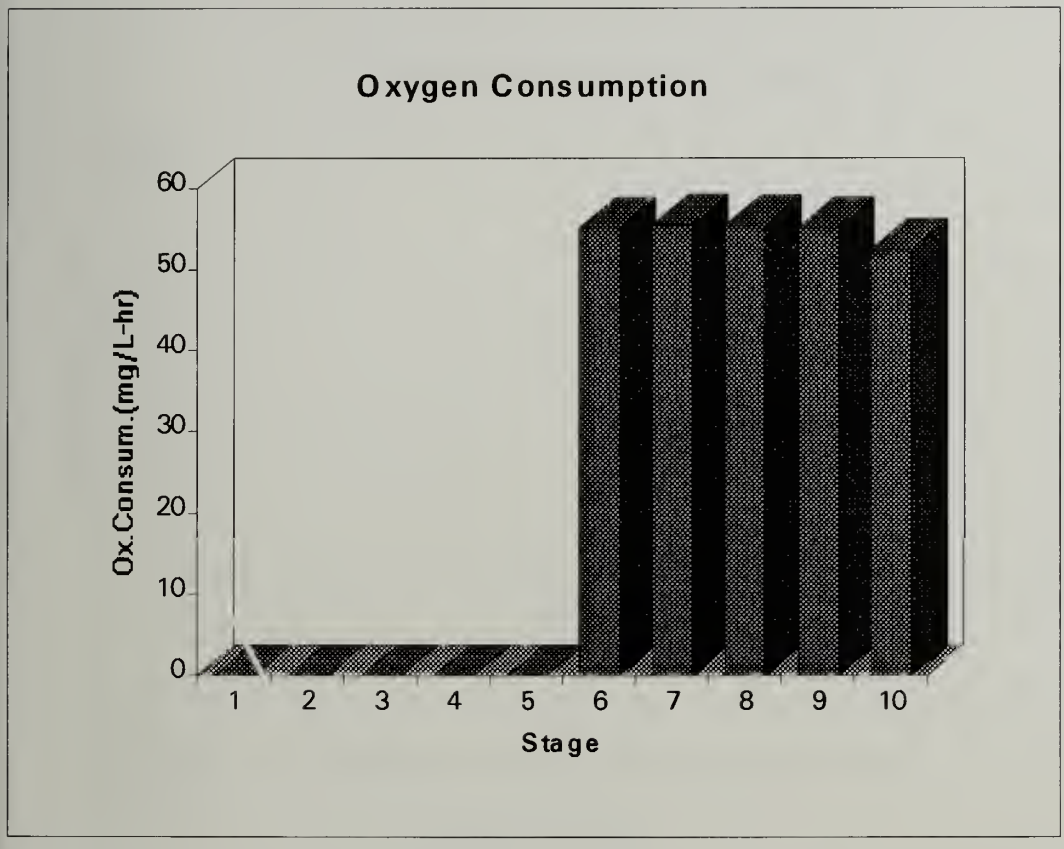

FIGURE 28: 4 November 1992 Data Set; Oxygen Consumption Rate; Storage Model

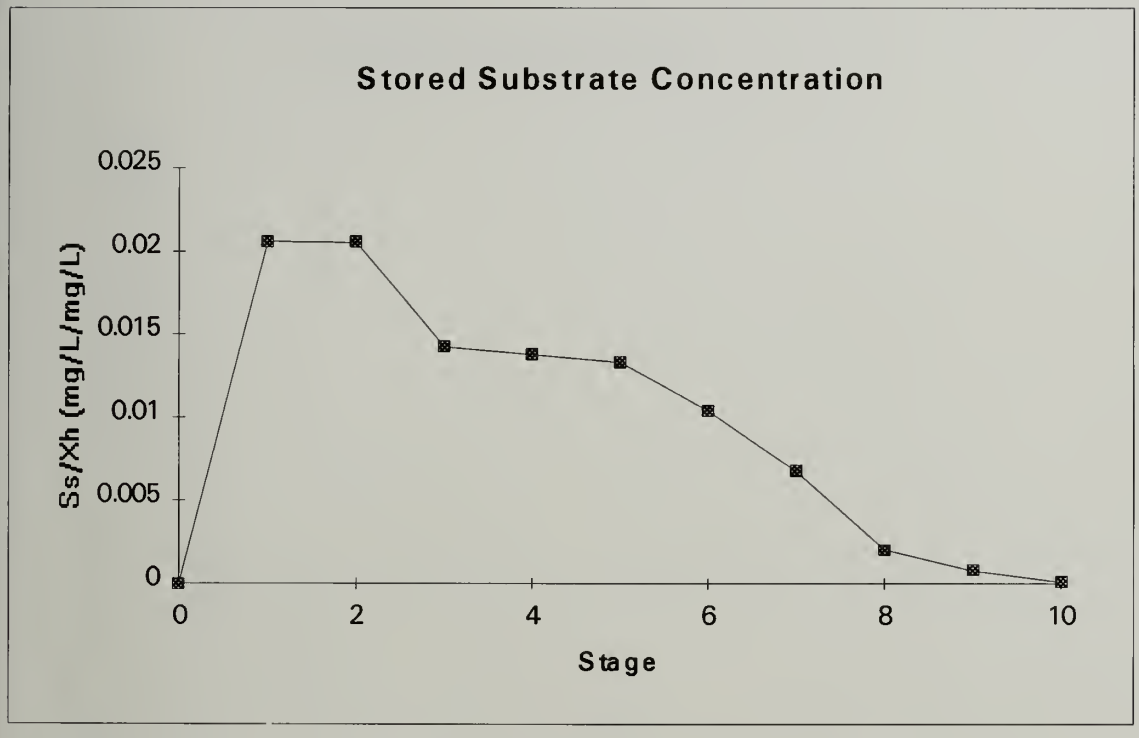

FIGURE 29: 4 November 1992 Data Set; Stored SCOD ; Storage Model

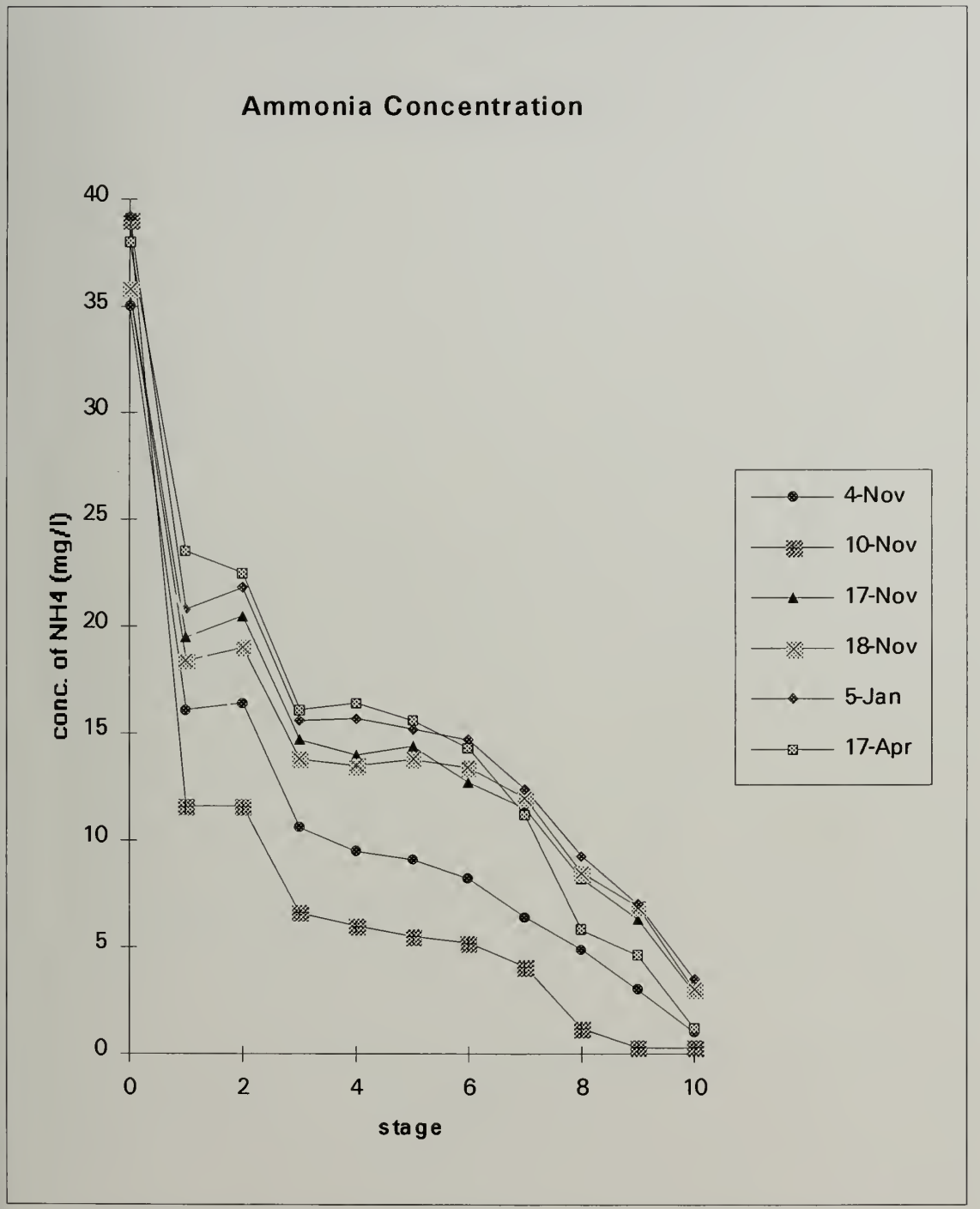

FIGURE 30: Variation of Ammonia Concentration Profiles from Different Phoenix Plant Data Sets

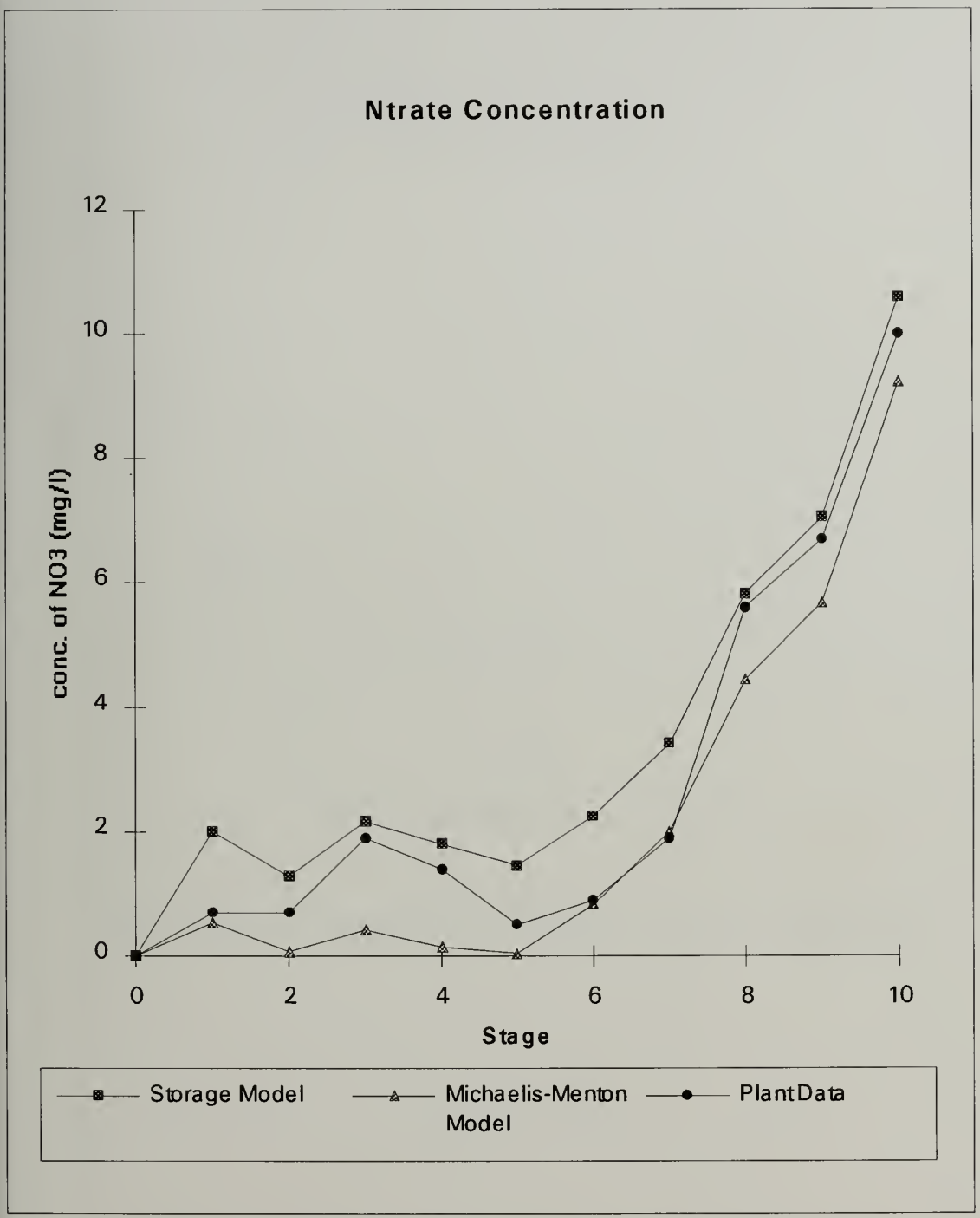

FIGURE 31: Michaelis-Menton Model; Nitrate Concentration

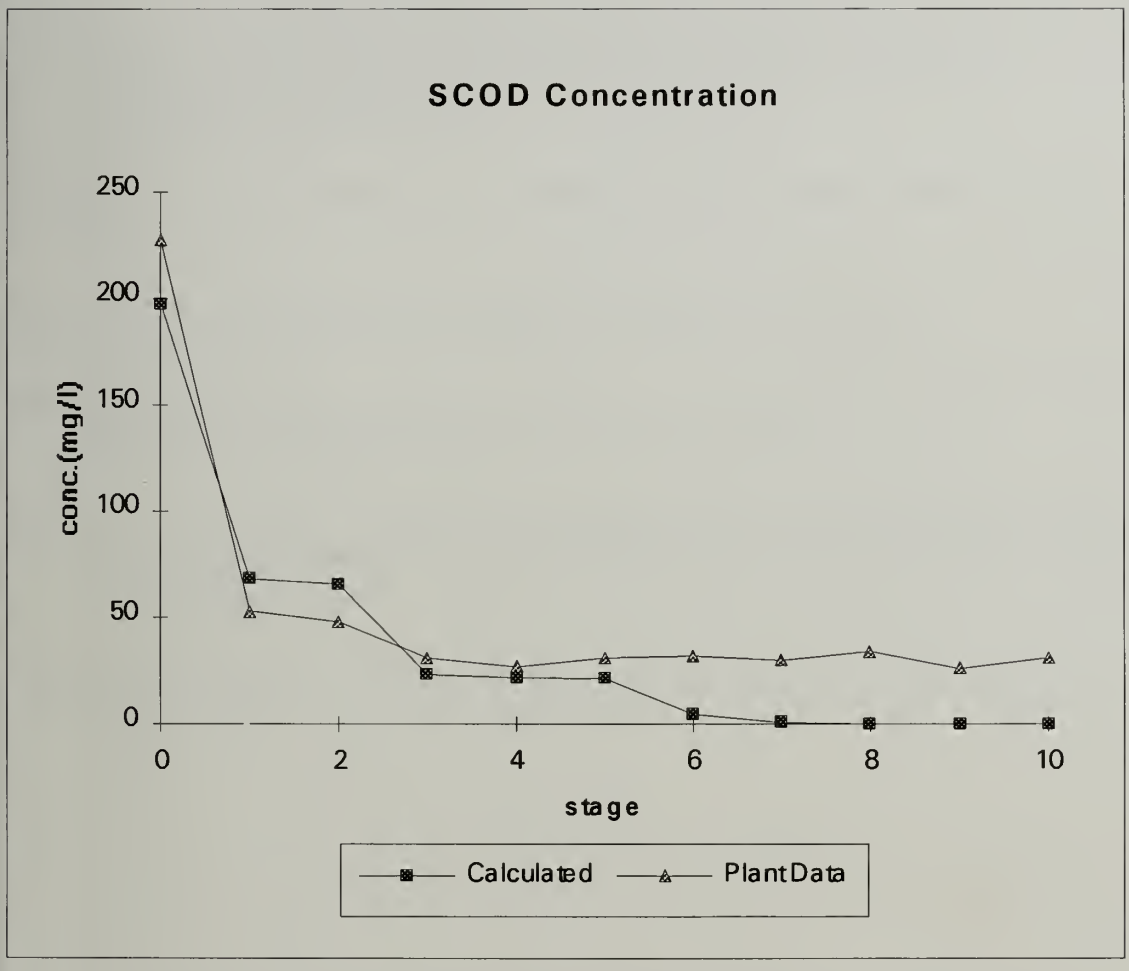

FIGURE 32: Michaelis-Menton Model; SCOD Concentration

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Appendix A: Model Based on Michaelis-Menton Kinetics

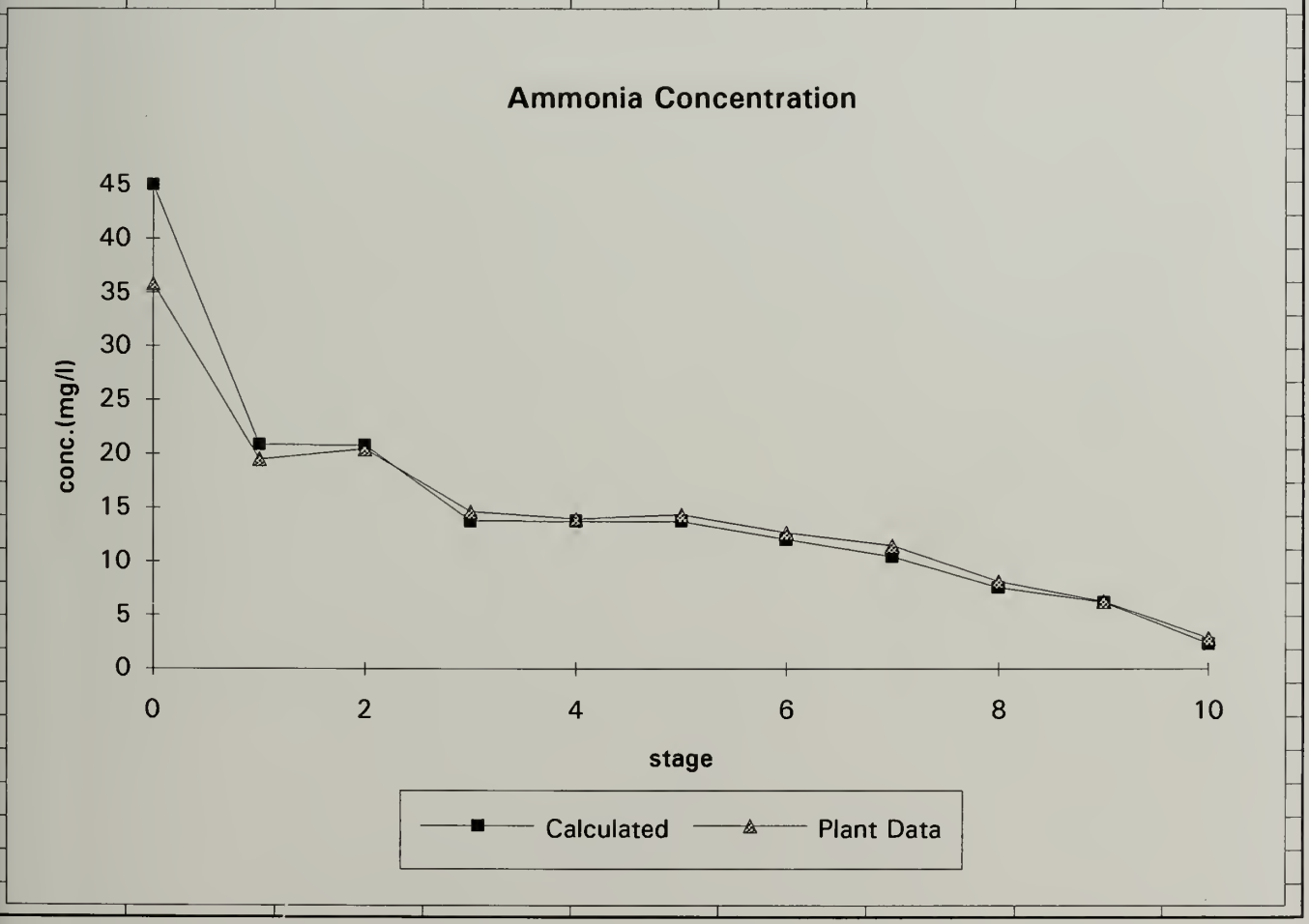
K	L	M	N	O	P	Q	R
		TABLE A.2 (INPUT VALUES FOR M-M MODEL)					
SRT =	5	Qo =	20			K(part.) =	0.000982
		So =	198			Po =	228
		Qras =	9.35	Xnb =	31	P(10) =	392.4781
		S(10) =	0	Kn(ns) =	0.5	P(9) =	397.4781
		Qir(ax-1)	20	A =	0.175874	X(i)(o) =	20
		S9 =	0			X(i) =	537.6344
		S1 =		Fh =	0.1		
				Kno =	0.75		
		Y(h) =	0.35	Um(H) =	2.8		
		Fdn =	0.5	Kn(nb) =	0.6		
		Xh =	2977.459	Kon(nitrifi)	0.5		
		Ks =	20	Koh(inhib)	0.1		
		Kos(het) =	0.1	V(total) =	3.72		The Macro
		O1 =	0				
		NO1 =		K (sto) =	0.937		
		NOs =	0.75	Kss (sto) =	0.002		
		V1 =	0.156				
		NOo =	0				
		NO10 =	10				
		NO9 =	6.7	X(total) =	3910		
		A =	0.175874				
		Kd(het) =	0.06				
		K(hetero)	8				
		Qir(ax-3) =	40				
		O6 =	3				
		V2 =	0.145				
		V3 =	0.5047				
		V4 =	0.124				
		V5 =	0.124				
		V6 =	0.3394				
		V7 =	0.4648				
		V8 =	0.9296				
		V9 =	0.4648				
		V10 =	0.4648				
	Solve for Xh and P(10)						
	X(h) and the initially assumed						
	P(10) value are estimated by						
	solving two equations simultaneously						
	X(h) =	2977.459	4.55E-13	411.3998			
	P(10) =	392.4781	0				

S	T	U	V	W	X
Um(ns) =	0.5	NO2o =	0.1	A =	0.503
Y(ns) =	0.1				
Xns =	62				
Fn =	0.1	Umb =	0.7		
Kd (nitrif) =	0.01	Ynb =	0.05		
No =	45	NO2(9) =	0.9		
N10 =	2.9	NO2(10) =	0.7		
N9 =	6.3				
ssociated with this file is [SOLV_MM.XLM]					

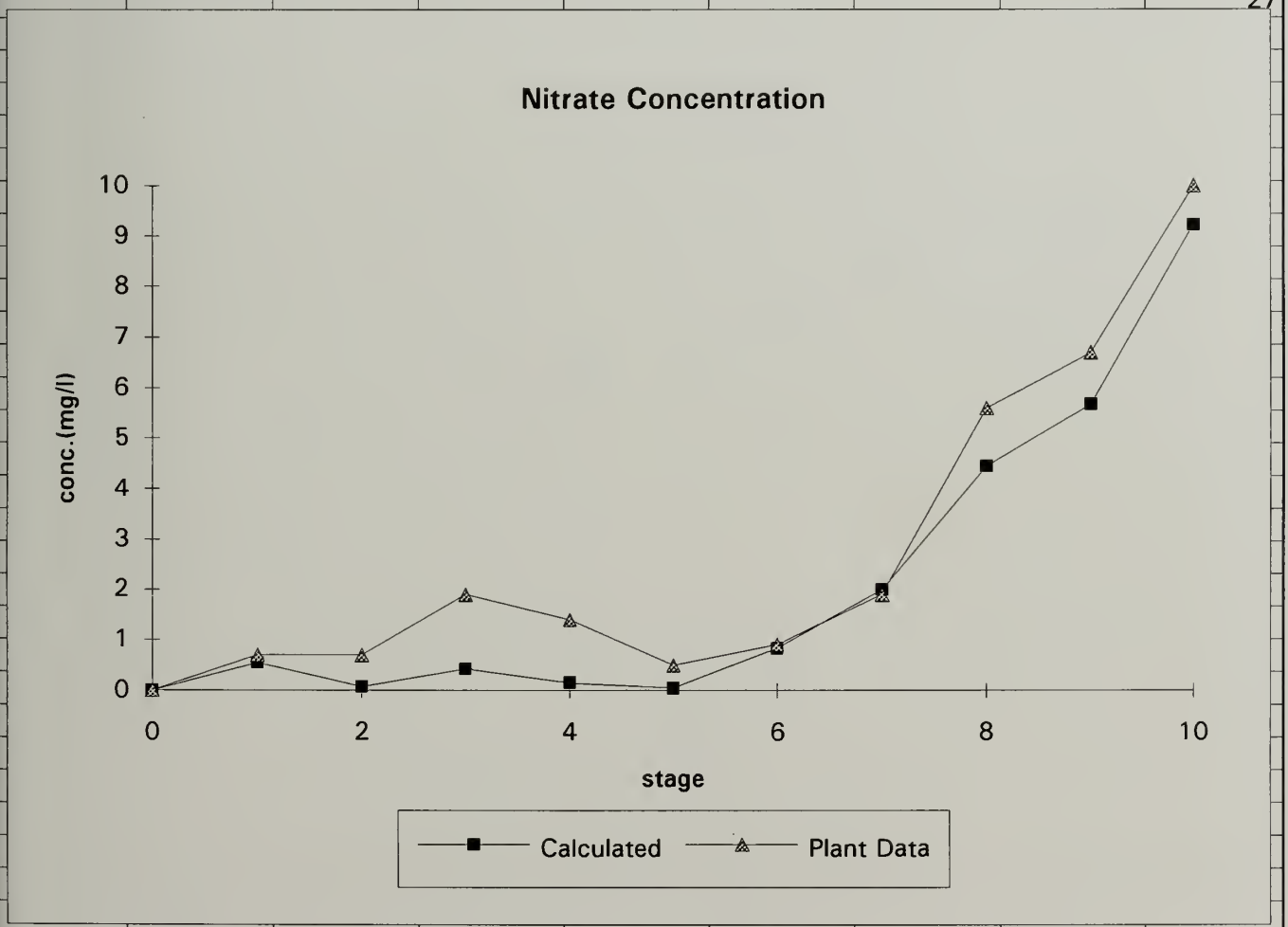
A	B	C	D	E	F	G	H	I
		PHOENIX PROJECT			(MICHAELIS - MENTON KINETICS MOD			
Final Analysis Model (based on PHOENX27.XLS).						Amended 23 NOV 93		
Comp.	Stage		Mass Balance Equation					
	AX-1							
SCOD =	68.06068		-1.3E-07	3853.882				
NO =	0.540159		-2.4E-08	105.7365	2.896626	8.06002		
PCOD =	480.06		-5.3E-09	293.7714				
N =	20.89265		-1E-06	0	21.0422	1.603992	0.583405	
NO2 =	0.537893		3.55E-14					
Ox consu	0							
	AX-2							
SCOD =	65.65949		2.13E-07					
NO =	0.073741		3.79E-08	20.84084	0.575672	1.601222		
PCOD =	479.8755		2.33E-09	62.78873				
N =	20.8045		-1.7E-12	0	4.147451	0.318653	0.115945	
NO2 =	0.537893							
Ox consu	0							
	AX-3							
SCOD =	23.21862		-2.1E-08					
NO =	0.423677		-4E-09	205.0185	8.079959	20.68507		
PCOD =	441.6717		-1.1E-09	233.0353				
N =	13.82667		-1E-06	0	40.79991	4.116452	1.627372	
NO2 =	0.7		-1E-06					
Ox Consu	0							
	AX-4							
SCOD =	21.82914		-7.1E-08	1001.212				
NO =	0.144015		-6.4E-08	21.83482	0.885877	2.267146		
PCOD =	441.5274		-2.9E-07	103.9575				
N =	13.77499		-1E-06	0	4.345259	0.451175	0.178423	
NO2 =	0.7							
Ox Consu	0							
	AX-5							
SCOD =	21.3406		-1.7E-08	352.0229				
NO =	0.045552		-3.7E-08	7.677053	0.314881	0.805753		
PCOD =	441.4762		-1.9E-07	36.94691				
N =	13.7568		-1E-06	0	1.527779	0.16035	0.06342	
NO2 =	0.7							
Ox Consu	0							

A	B	C	D	E	F	G	H	I
	Ox-2 /6							
SCOD =	4.657131		-7.9E-11	1477.689	12.97904			
NO =	0.835624		-3.4E-09	73.77928	2.282677	0.255888	0.647773	
PCOD =	436.7418		2.07E-09	3.683165	419.3357			
N =	12.10356		-8.4E-10	86.60573	52.17339	14.80566	5.867708	
NO2 =	0.843553		4.56E-09	86.60573	73.77928			
Ox Consu	220.5524		1037.525	278.8705	81.895	5589.349		
	Ox-3 /7							
SCOD =	0.677953		-1.7E-10	351.2799	4.259734			
NO =	2.003096		-9.6E-08	106.7535	0.749177	0.483809	1.206775	
PCOD =	430.3321		8.06E-09	6.861583	565.8419			
N =	10.51041		-2.4E-08	117.8955	12.44389	20.04462	8.035683	
NO2 =	0.968254		1.21E-07	117.8955	106.7535			
Ox Consu	169.3293		575.4189	379.6235	118.4963	1973.154		
	Ox-4 /8							
SCOD =	0.05163		-5.4E-14	55.17513	0.786792			
NO =	4.443684		-2.4E-09	222.0987	0.138376	1.137867	2.755932	
PCOD =	417.8581		1.21E-11	15.66991	1098.88			
N =	7.637138		-8.8E-10	231.8302	1.958667	39.00924	16.07137	
NO2 =	1.077168		3.27E-09	231.8302	222.0987			
Ox Consu	284.1862		808.703	746.4932	246.5295	1241.453		
	Ox-5 /9							
SCOD =	0.007291		-2.5E-13	3.904244	0.057475			
NO =	5.674778		-4.5E-07	111.9972	0.010108	0.587333	1.401601	
PCOD =	411.7101		1.96E-09	7.969341	541.3559			
N =	6.230772		-3.4E-07	114.3294	0.13866	19.22638	8.035683	
NO2 =	1.10327		7.88E-07	114.3294	111.9972			
Ox Consu	138.9334		388.3726	368.1407	124.3169	1173.107		
	Ox-6 /10							
SCOD =	0.000373		3.21E-15	0.199943	0.003082			
NO =	9.230935		-9.8E-08	106.3925	0.000542	0.61499	1.403743	
PCOD =	393.7959		-1.6E-11	7.981523	517.8006			
N =	2.39589		-1.1E-07	102.18	0.007106	18.40237	8.035683	
NO2 =	0.959743		2.12E-07	102.18	106.3925			
Ox Consu	129.6188		374.661	329.0196	118.0957	1114.459		

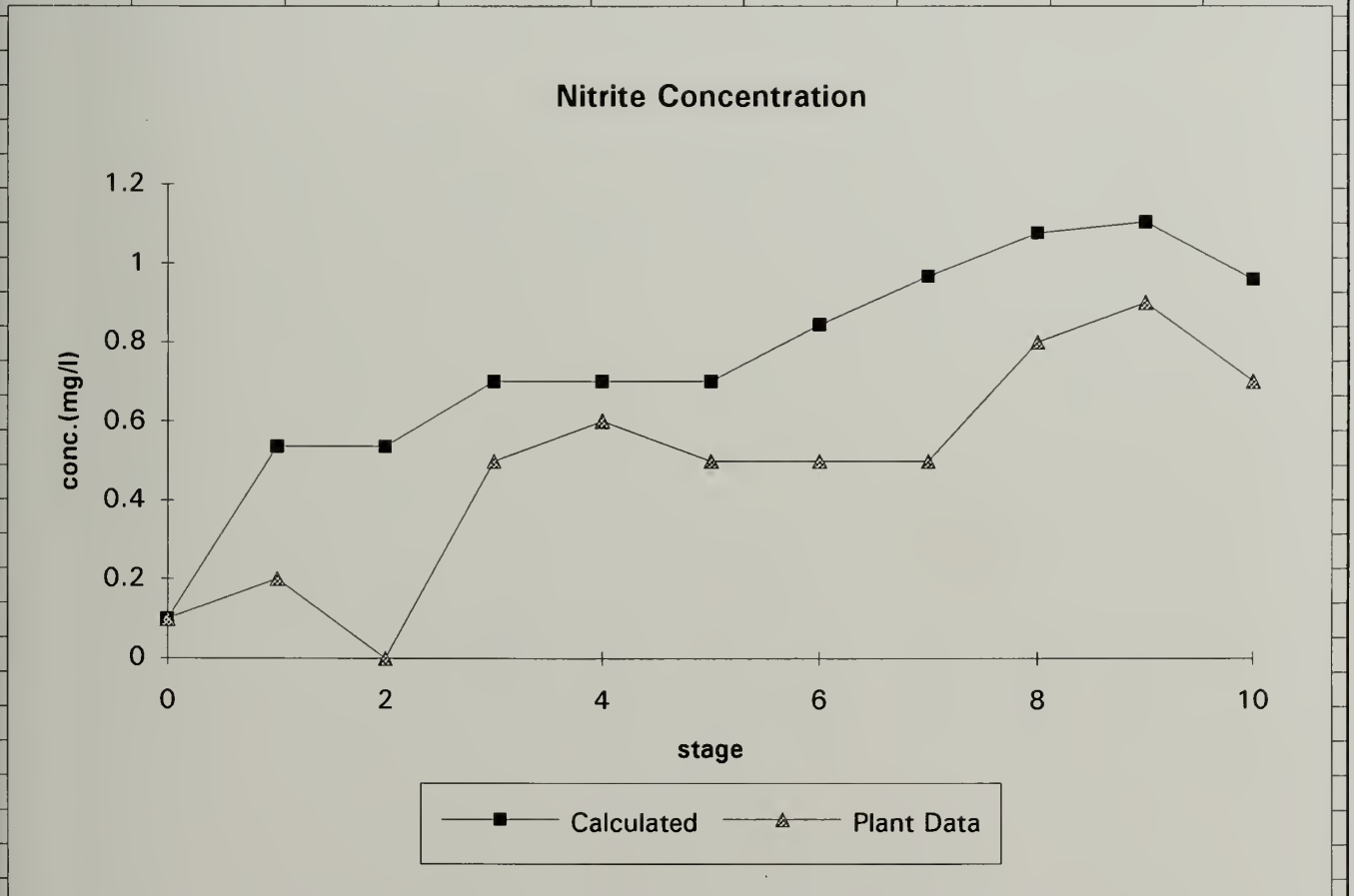
A	B	C	D	E	F	G	H	I
AMMONIA								
	(concentration)		(concentratio)					
stage	Calculated		Plant Data					
0	45		35.8			17-Nov		
1	20.89265		19.5					
2	20.8045		20.5					
3	13.82667		14.7					
4	13.77499		14					
5	13.7568		14.4					
6	12.10356		12.7					
7	10.51041		11.5					
8	7.637138		8.2					
9	6.230772		6.3					
10	2.39589		2.9					27



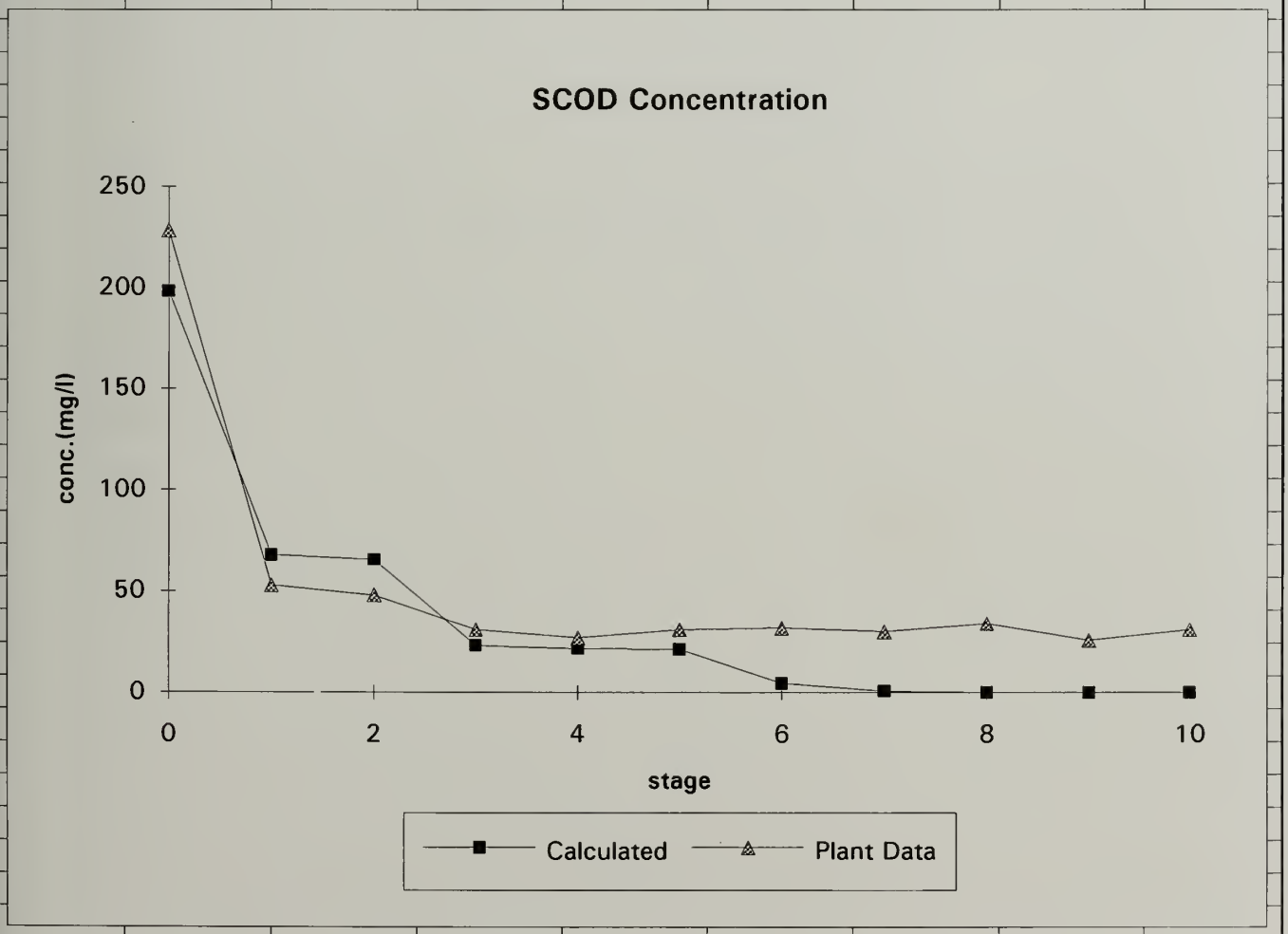
A	B	C	D	E	F	G	H	I
NITRATE								
	(concentration)		(concentratio)					
stage	Calculated		Plant Data					
0	0		0			17-Nov		
1	0.540159		0.7					
2	0.073741		0.7					
3	0.423677		1.9					
4	0.144015		1.4					
5	0.045552		0.5					
6	0.835624		0.9					
7	2.003096		1.9					
8	4.443684		5.6					
9	5.674778		6.7					
10	9.230935		10					
								27



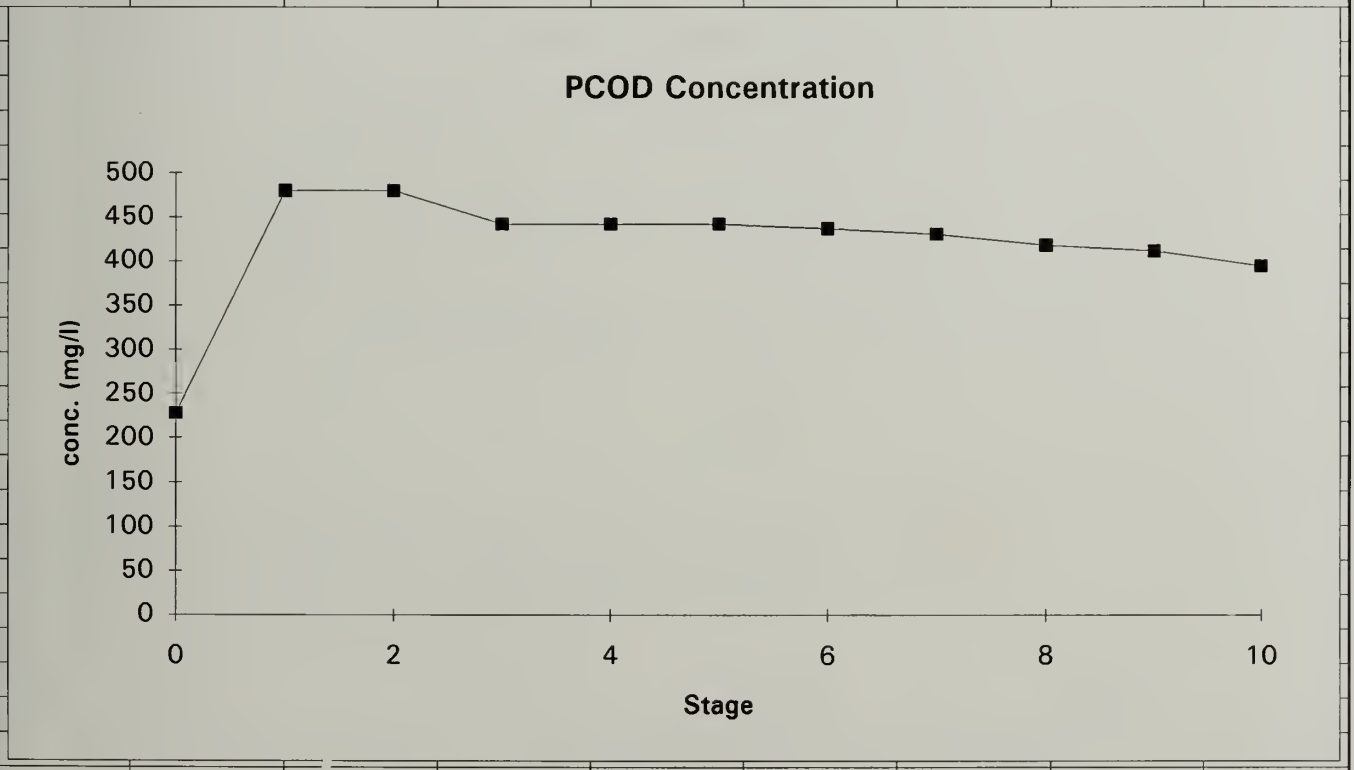
A	B	C	D	E	F	G	H	I
NITRITE								
	(concentration)		(concentratio)					
Stage	Calculated		Plant Data					
0	0.1		0.1			17-Nov		
1	0.537893		0.2					
2	0.537893		0					
3	0.7		0.5					
4	0.7		0.6					
5	0.7		0.5					
6	0.843553		0.5					
7	0.968254		0.5					
8	1.077168		0.8					
9	1.10327		0.9					
10	0.959743		0.7					22



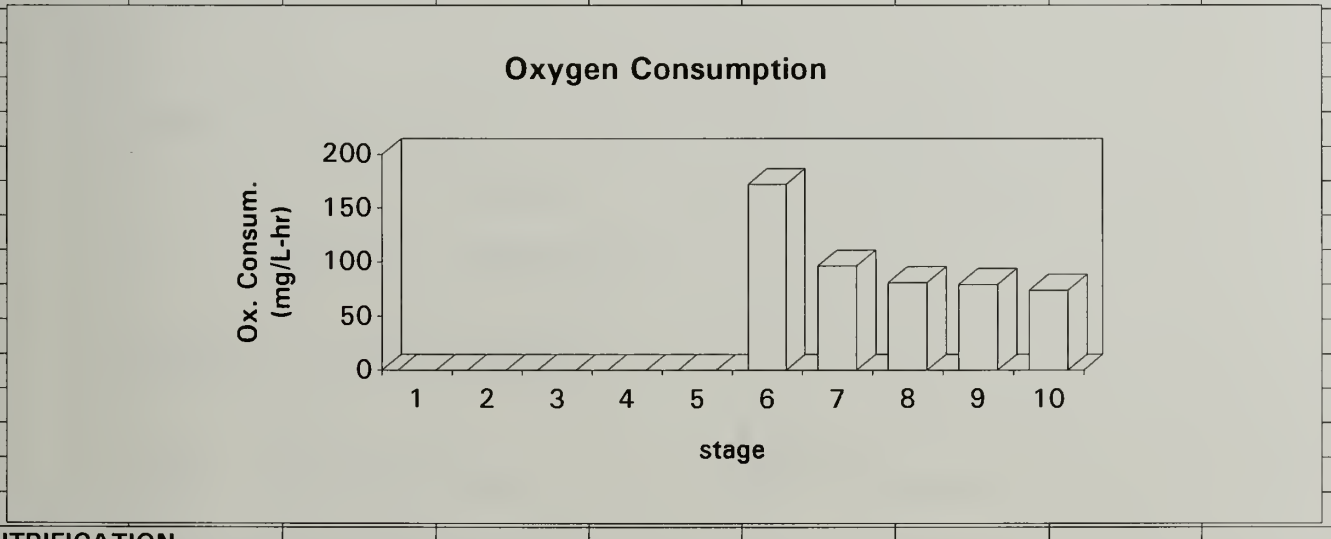
A	B	C	D	E	F	G	H	I
SOLUBLE COD								
	(concentration)		(concentratio)					
Stage	Calculated		Plant Data					
0	198		228			17-Nov		
1	68.06068		53					
2	65.65949		48					
3	23.21862		31					
4	21.82914		27					
5	21.3406		31					
6	4.657131		32					
7	0.677953		30					
8	0.05163		34					
9	0.007291		26					
10	0.000373		31					



A	B	C	D	E	F	G	H	I
PARTICULATE COD								
	(concentration)		Concentration					
Stage	Calculated		from Plant Data					
0	228							
1	480.06		XXXXXXX					
2	479.8755		XXXXXXX					
3	441.6717		XXXXXXX					
4	441.5274		XXXXXXX					
5	441.4762		XXXXXXX					
6	436.7418		XXXXXXX					
7	430.3321		XXXXXXX					
8	417.8581		XXXXXXX					
9	411.7101		XXXXXXX					
10	393.7959		XXXXXXX					

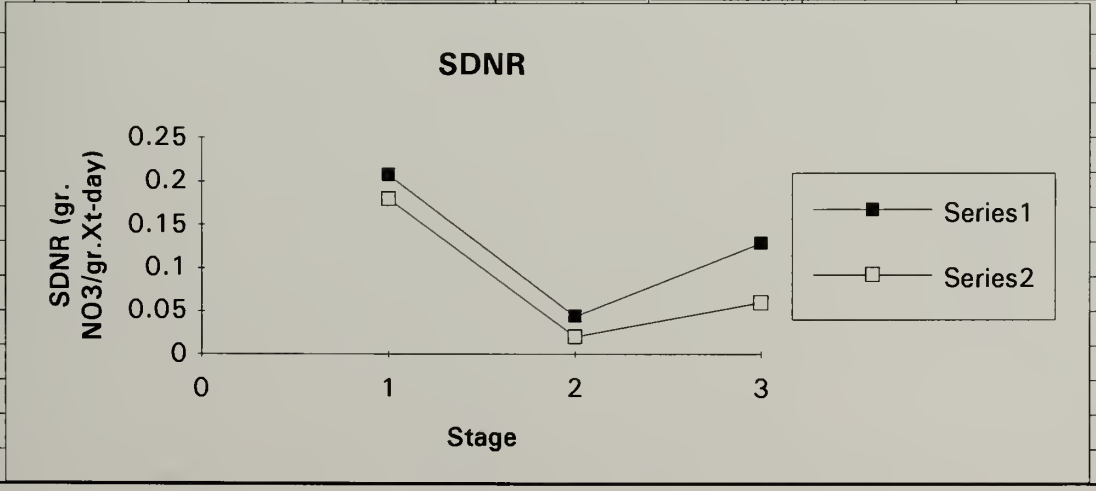


A	B	C	D	E	F	G	H	I
OXYGEN CONSUMPTION								
	Calculated Oxygen			Calculated Oxygen				
Stage	Consumption (Kg/hr)			Consumption (mg/L-hr)				
1	0			0				
2	0			0				
3	0			0				
4	0			0				
5	0			0				
6	220.5524			171.6656				
7	169.3293			96.23862				
8	284.1862			80.7589				
9	138.9334			78.96305				
10	129.6188			73.66908				



NITRIFICATION								
Term (a)	Depletion of NH4 via heterotrphic uptake of soluble COD							
Term (b)	Depletion of NH4 via heterotrophic uptake of particulate COD							
Term (c)	Adding NH4 by death of heterotrophs							
Term (d)	Depletion of NH4 by Nitrosomonas							
Stage	Term (a)		Term (b)		Term (c)		Term (d)	
	mg/L-d	mg L-hr	mg/L-day	mg/L-hr	mg/L-d	mg/L-hr	mg/L-d	mg/L-hr
1	134.8859	5 620245	10.282	0.428417	3.739775	0.155824	0	0
2	28.60311	1.191796	2.197605	0.091567	0.799622	0.033318	0	0
3	80.83992	3.36833	8.156236	0.339843	3.224434	0.134351	0	0
4	35.04241	1.4601	3.638512	0.151605	1.438896	0.059954	0	0
5	12.3208	0.513367	1.293142	0.053881	0.511449	0.02131	0	0
6	153.7224	6.405101	43.62304	1.817627	17.28847	0.720353	255.173	10.63221
7	26.77256	1.115523	43.12526	1.796886	17.28847	0.720353	253.6478	10.56866
8	2.107	0.087792	41.96347	1.748478	17.28847	0.720353	249.387	10.39113
9	0.298322	0.01243	41.36485	1.723536	17.28847	0.720353	245.9755	10.24898
10	0.015288	0.000637	39.59203	1.649668	17.28847	0.720353	219.8365	9.159852

A	B	C	D	E	F	G	H	I
Total Nitrification								
Stage	mg/L-d	mg/L-hr						
1	148.9076	6.204485						
2	31.60034	1.316681						
3	92.22059	3.842525						
4	40.11982	1.671659						
5	14.12539	0.588558						
6	469.807	19.57529						
7	340.8341	14.20142						
8	310.746	12.94775						
9	304.9272	12.7053						
10	276.7322	11.53051						
	MLSS =	X(total) =	Xh +	Xns +	Xnb +	X(inerts)		
			2977.459	62	31	537.6344		
		X(total) =	3608.094	mg/l				
	SPECIFIC DENITRIFICATION RATE							
		SDNR			SDNR (gr. NO3/gr. Xt-day)			
	Stage	gr. NO3/gr.Xt-day			Plant Data			
	1	0.207321	116.6932		0.18			
	2	0.043996	23.01773		0.02			
	3	0.128382	233.7835		0.06			



Appendix B: Storage Model

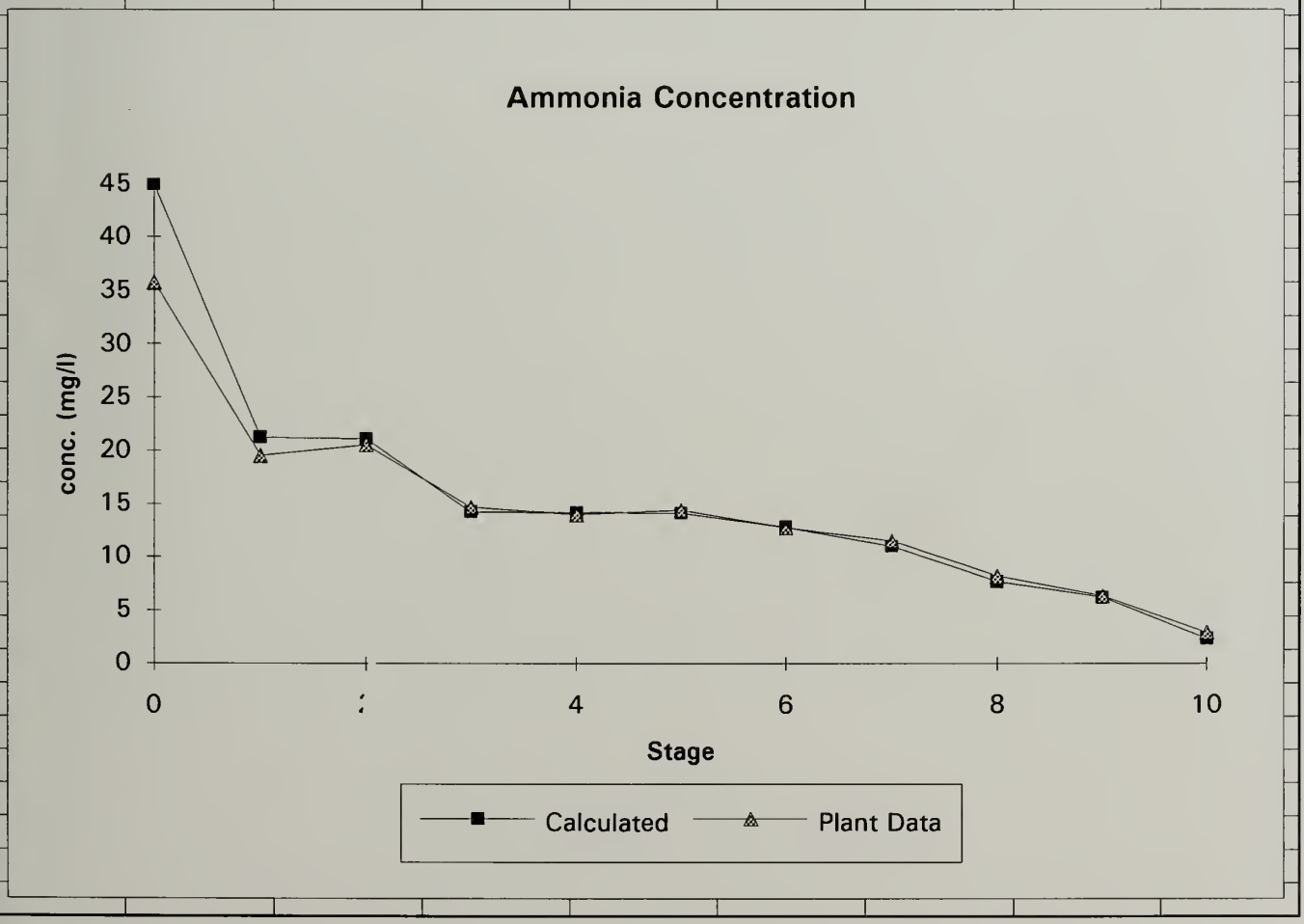
J	K	L	M	N	O	P
	TABLE A.1 (INPUT VALUES FOR STORAGE MODEL)					
	SRT =	5	Qo =	20		
			So =	228		
			Qras =	**9.35**	Xnb =	**31**
			S(10) =	30	Kn(ns) =	**0.5**
			Qir(ax-1)	20	A =	0.175874
			S9 =	30		
			S1 =		Fh =	0.1
					Kno =	0.75
			Y(h) =	0.35	Um(H) =	2.8
			Fdn =	0.5	Kn(nb) =	**0.6**
			Xh =	2977.459	Kon(nitrifi)	0.5
			Ks =	20	Koh(inhib)	0.1
			Kos(het) =	0.1	V(total) =	3.72
			O1 =	0		
			NO1 =		K (sto) =	0.937
			NOs =	0.75	Kss (sto) =	0.002
			V1 =	0.156	Sr =	**30**
			NOo =	0	Ss(10)/Xh	0.000115
			NO10 =	10	Ss(9)/Xh	0.000388
			NO9 =	6.7	X(total) =	**3910**
			A =	0.175874		
			Kd(het) =	0.06		
			K(hetero)	8		
			Qir(ax-3) =	40		
			O6 =	3		
			V2 =	0.145		
			V3 =	0.5047		
			V4 =	0.124		
			V5 =	0.124		
			V6 =	0.3394		
			V7 =	0.4648		
			V8 =	0.9296		
			V9 =	0.4648		
			V10 =	0.4648		
		solve for X(h) and P(10)				
		X(h) and the initially assumed				
		P(10) value are estimated by				
		solving two equations simultaneously				
		X(h) =	**2977.459**	4.55E-13	411.3998	
		P(10) =	**392.4781**	0		
	clear all concentrations					

Q	R	S	T	U	V	W	X
	TABLE A.1 (CONTINUED)						
K(part.) =	0.000982	Um(ns) =	0.5	NO2o =	0.1	A =	0.503
Po =	228	Y(ns) =	0.1				
P(10) =	392.4781	Xns =	62				
P(9) =	397.4781	Fn =	0.1	Umb =	0.7		
K(i)(o) =	20	Kd (nitrif) =	0.01	Ynb =	0.05		
K(i) =	537.6344	No =	45	NO2(9) =	0.9		
		N10 =	2.9	NO2(10) =	0.7		
		N9 =	6.3				
		Macro Name: [SOLV_STO.XLM]					

A	B	C	D	E	F	G	H	I
		PHOENIX PROJECT			(STORAGE MODEL)			
"Fit" Model FOR ANALYSIS (based on STORGE13.XLS)								
Q(o) = 20; Q(ras) = 9.35; Xi(o) = 20			Xns and Xnb are fixed					
Comp.	Stage		Mass Balance Equation					
	AX-1							
SCOD =	54.07295		-1E-06	2772				
NO =	2.010418		2.02E-07	25.09641	5.038781	14.0007	0.307213	
PCOD =	479.3755		8.46E-07	510.2971				
N =	21.20262		7.05E-09	0	4.994335	2.786222	1.014853	0.307213
NO2 =	0.537893		3.55E-14					
Ox consu	0							
Ss/Xh =	0.018073		8.5E-11	0.047925				
	AX-2							
SCOD =	51.66565		-1E-06	118.8				
NO =	1.28935		2.36E-08	20.25265	4.06571	11.26634	0.266726	
PCOD =	478.0775		1E-07	441.7871				
N =	21.09211		7.88E-10	0	4.030398	2.242069	0.818868	0.266726
NO2 =	0.537893							
Ox consu	0							
Ss/Xh =	0.018098		9.48E-12	0.038675				
	AX-3							
SCOD =	30		-1E-06	1069.2				
NO =	2.164613		-1.7E-07	79.2693	16.62348	42.32849	0.299933	
PCOD =	439.3013		-3E-07	476.8673				
N =	14.23665		-1.5E-08	0	15.77506	8.423622	3.34811	0.299933
NO2 =	0.7		-1E-06					
Ox Consu	0							
Ss/Xh =	0.012494		-1.8E-10	0.151376				
	AX-4							
SCOD =	30							
NO =	1.804206		1.31E-07	18.44067	3.884555	9.877117	0.283993	
PCOD =	438.6728		6.35E-07	452.9044				
N =	14.18234		2.56E-09	0	3.669804	1.965605	0.782382	0.283993
NO2 =	0.7							
Ox Consu	0							
Ss/Xh =	0.0121		3.12E-11	0.035215				
	AX-5							
SCOD =	30							
NO =	1.467539		-6.6E-09	17.20042	3.639397	9.241358	0.264892	
PCOD =	438.0847		-3.2E-08	423.7524				
N =	14.13165		-1.3E-10	0	3.422986	1.839085	0.733005	0.264892
NO2 =	0.7							
Ox Consu	0							
Ss/Xh =	0.011733		-1.6E-12	0.032847				
	Ox-2 /6							
SCOD =	30							
NO =	2.261081		-2E-07	73.83104	1.647538	0.364614	0.915883	0.00927
PCOD =	433.3695		-3.5E-07	5.207603	416.0978			

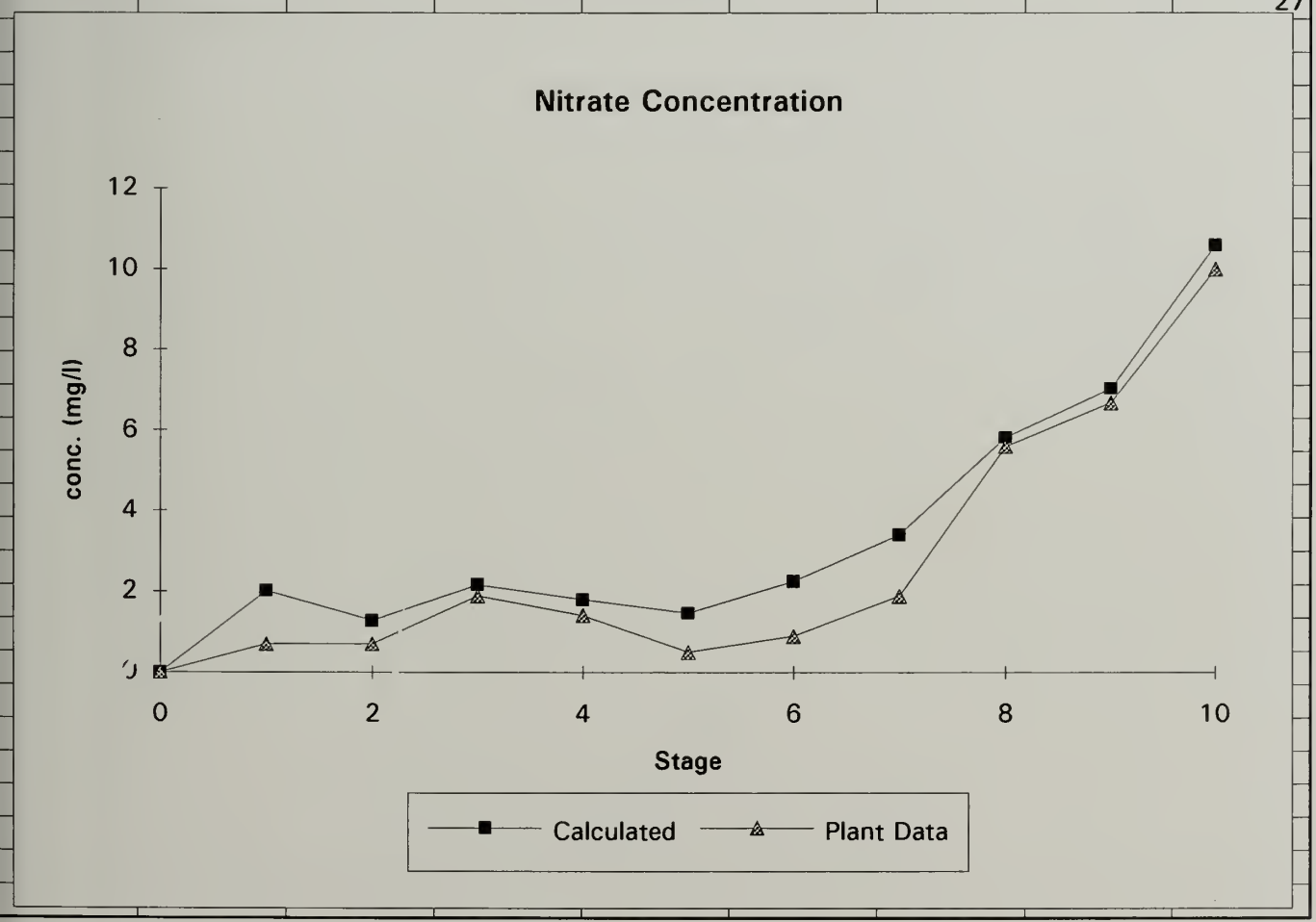
A	B	C	D	E	F	G	H	I
N =	12.7678		2.89E-13	86.78485	26.19746	14.74569	5.867708	0.740685
NO2 =	0.844978		-3.6E-13	86.78485	73.83104			
Ox Consu	***103.1783***		*292.7451*	279.4472	81.95246	1226.721		
Ss/Xh =	0.008919		-6.2E-07	0.251388				
	Ox-3 /7							
SCOD =	30							
NO =	3.411185		-3.3E-08	106.8637	2.207626	0.545107	1.349143	0.00907
PCOD =	426.9998		-6.2E-08	7.671071	561.4603			
N =	10.95301		-6E-14	118.1122	32.15546	19.9196	8.035683	0.663857
NO2 =	0.970871		-1.4E-13	118.1122	106.8637			
Ox Consu	***141.2656***		*396.6764*	380.3214	118.6187	1208.625		
Ss/Xh =	0.005466		-1.1E-07	0.308561				
	Ox-4 /8							
SCOD =	30							
NO =	5.822783		-4.1E-08	222.2464	2.760602	1.178162	2.831391	0.005671
PCOD =	414.6164		-9.7E-08	16.09896	1090.355			
N =	7.687354		1.6E-12	231.9233	37.2083	38.72588	16.07137	0.384087
NO2 =	1.079174		-1.6E-12	231.9233	222.2464			
Ox Consu	***279.2338***		*776.8415*	746.7929	246.6935	1173.313		
Ss/Xh =	0.00147		-6.3E-07	0.357047				
	Ox-5 /9							
SCOD =	30							
NO =	7.046403		-3.6E-09	112.0305	0.676028	0.60099	1.423057	0.002777
PCOD =	408.5141		-1.1E-08	8.091338	537.1535			
N =	6.184886		6.75E-14	114.2664	8.931178	19.08357	8.035683	0.184386
NO2 =	1.104199		-1.8E-13	114.2664	112.0305			
Ox Consu	***138.2708***		*384.338*	367.938	124.3538	1155.85		
Ss/Xh =	0.00051		-6.4E-07	0.085703				
	Ox-6 /10							
SCOD =	30							
NO =	10.5886		-1.2E-07	106.0975	0.10677	0.620973	1.406388	0.000439
PCOD =	390.7364		-4.5E-07	7.996558	513.7777			
N =	2.32638		3.1E-10	101.6556	1.365169	18.2621	8.035683	0.028184
NO2 =	0.952856		-2.7E-10	101.6556	106.0975			
Ox Consu	***128.9668***		*372.5435*	327.3309	117.7682	1105.402		
Ss/Xh =	6.42E-05		-1.6E-07	0.0131				

A	B	C	D	E	F	G	H	I
AMMONIA								
	(concentration)		(concentration)					
stage	Calculated		Plant Data					
0	45		35.8			17-Nov		
1	21.20262		19.5					
2	21.09211		20.5					
3	14.23665		14.7					
4	14.18234		14					
5	14.13165		14.4	Delta				
6	12.7678		12.7	0.067798				
7	10.95301		11.5	-0.54699				
8	7.687354		8.2	-0.51265				27
9	6.184886		6.3	-0.11511				
10	2.32638		2.9	-0.57362				

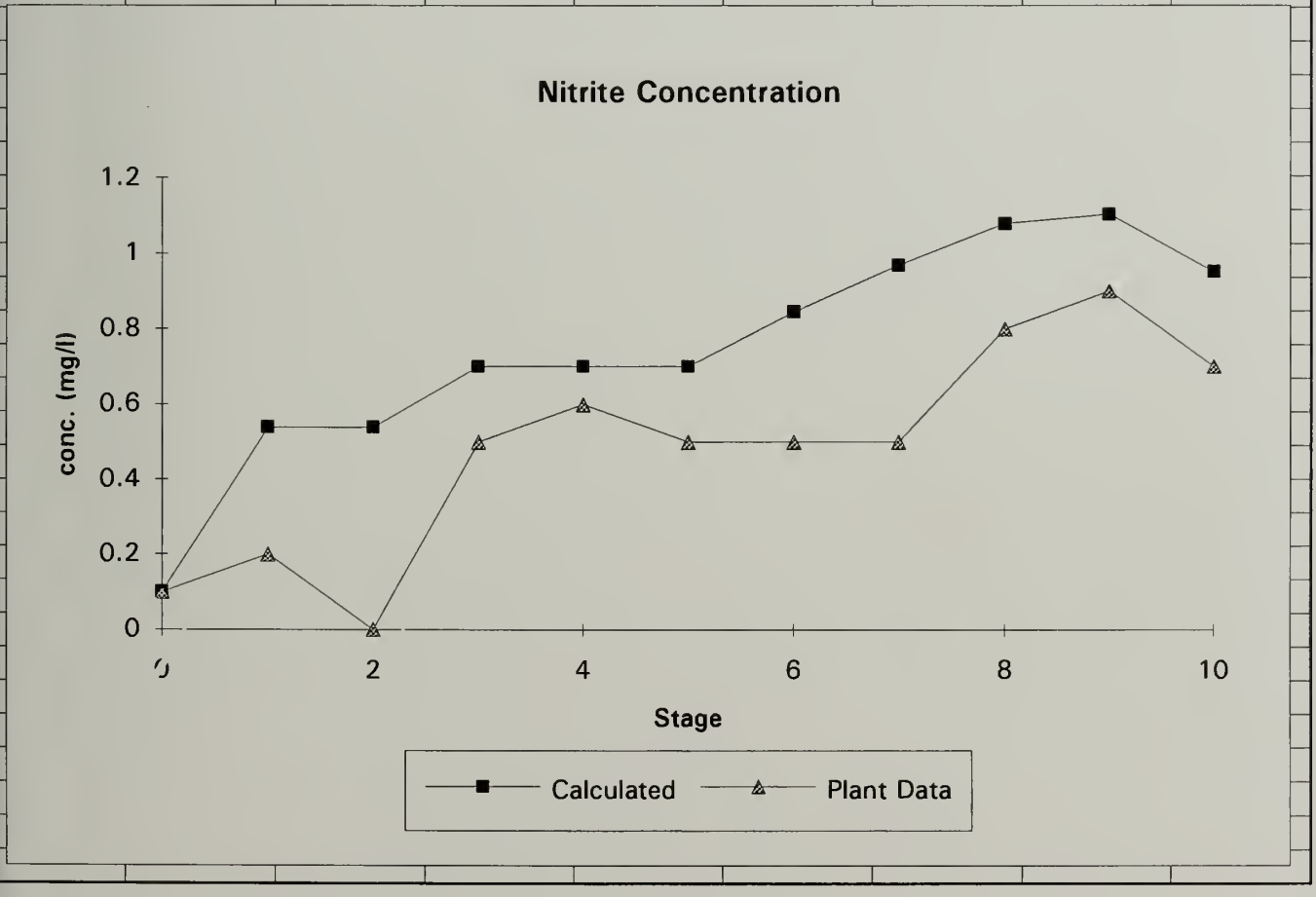


A	B	C	D	E	F	G	H	I
NITRATE								
	(concentration)		(concentration)					
stage	Calculated		Plant Data					
0	0		0			17-Nov		
1	2.010418		0.7					
2	1.28935		0.7					
3	2.164613		1.9					
4	1.804206		1.4					
5	1.467539		0.5	Delta				
6	2.261081		0.9	1.361081				
7	3.411185		1.9	1.511185				
8	5.822783		5.6	0.222783				
9	7.046403		6.7	0.346403				
10	10.5886		10	0.588595				

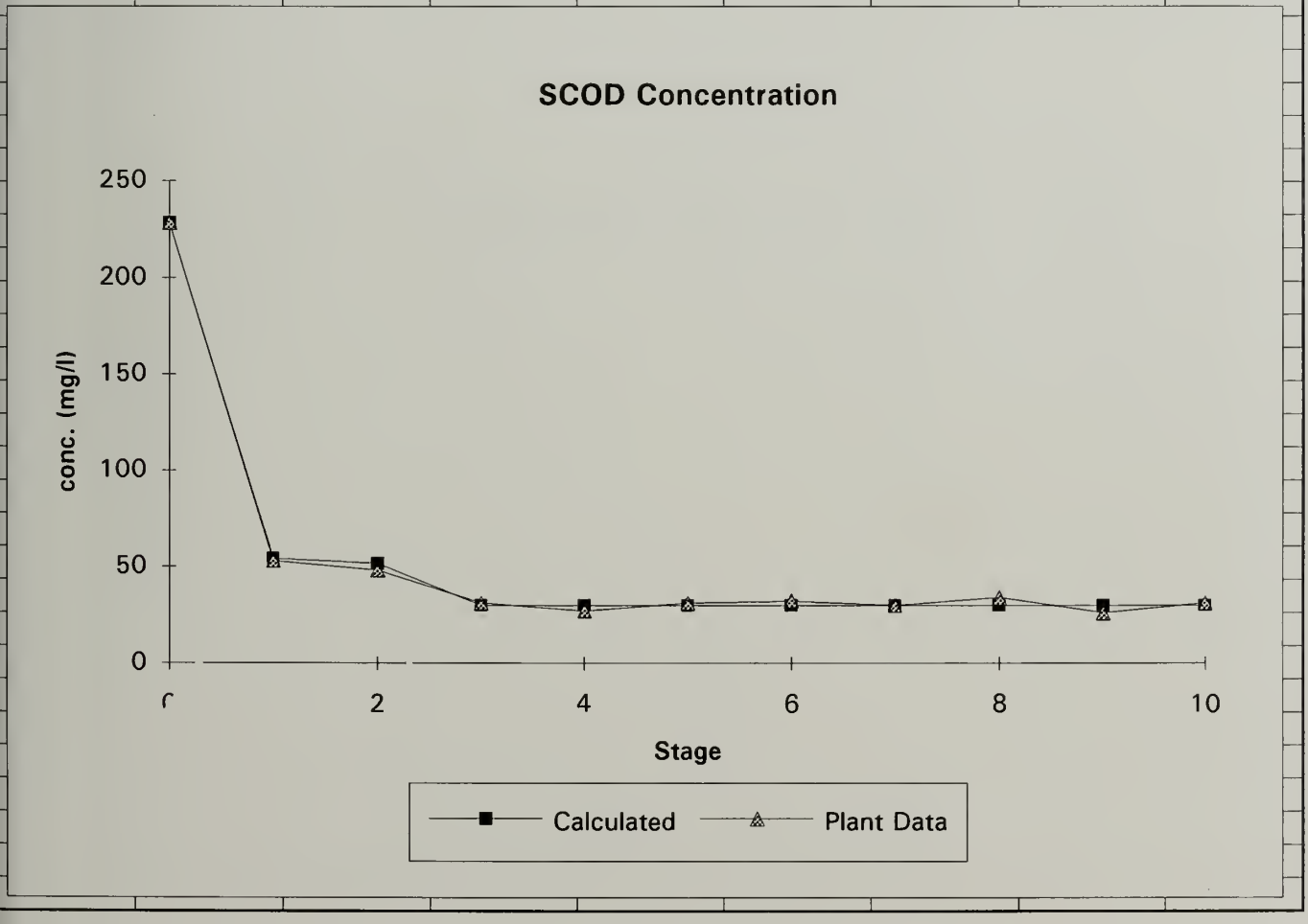
27



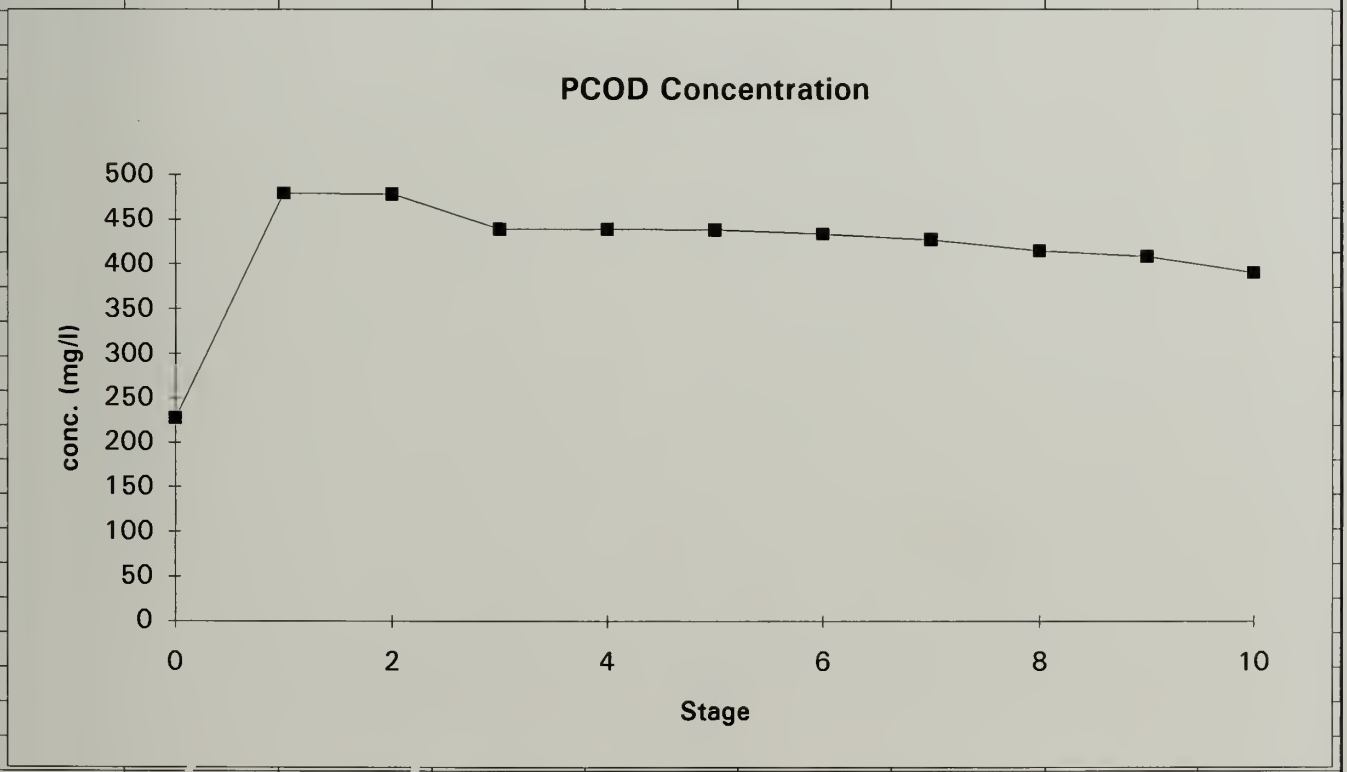
A	B	C	D	E	F	G	H	I
NITRITE								
	(concentration)		(concentration)					
Stage	Calculated		Plant Data					
0	0.1		0.1			17-Nov		
1	0.537893		0.2					
2	0.537893		0					
3	0.7		0.5					
4	0.7		0.6					
5	0.7		0.5	Delta				
6	0.844978		0.5	0.344978				
7	0.970871		0.5	0.470871				
8	1.079174		0.8	0.279174				
9	1.104199		0.9	0.204199				
10	0.952856		0.7	0.252856				
								22



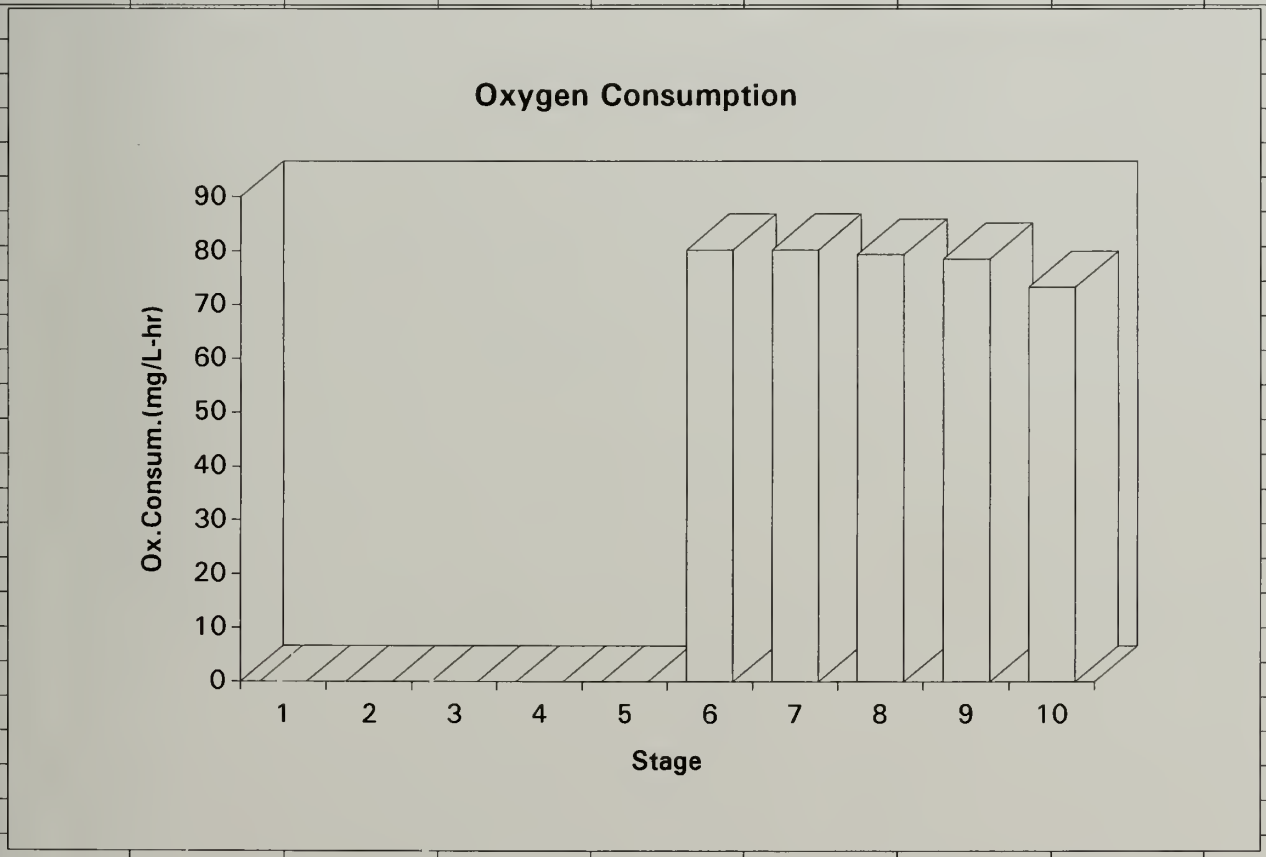
A	B	C	D	E	F	G	H	I
SOLUBLE COD								
	(concentration)		(concentration)					
Stage	Calculated		Plant Data					
0	228		228			17-Nov		
1	54.07295		53					
2	51.66565		48					
3	30		31					
4	30		27					
5	30		31					
6	30		32					
7	30		30					
8	30		34					
9	30		26					
10	30		31					



A	B	C	D	E	F	G	H	I
PARTICULATE COD								
	(concentration)		Concentration					
Stage	Calculated		from Plant Data					
0	228							
1	479.3755		XXXXXXX					
2	478.0775		XXXXXXX					
3	439.3013		XXXXXXX					
4	438.6728		XXXXXXX					
5	438.0847		XXXXXXX					
6	433.3695		XXXXXXX					
7	426.9998		XXXXXXX					
8	414.6164		XXXXXXX					
9	408.5141		XXXXXXX					
10	390.7364		XXXXXXX					

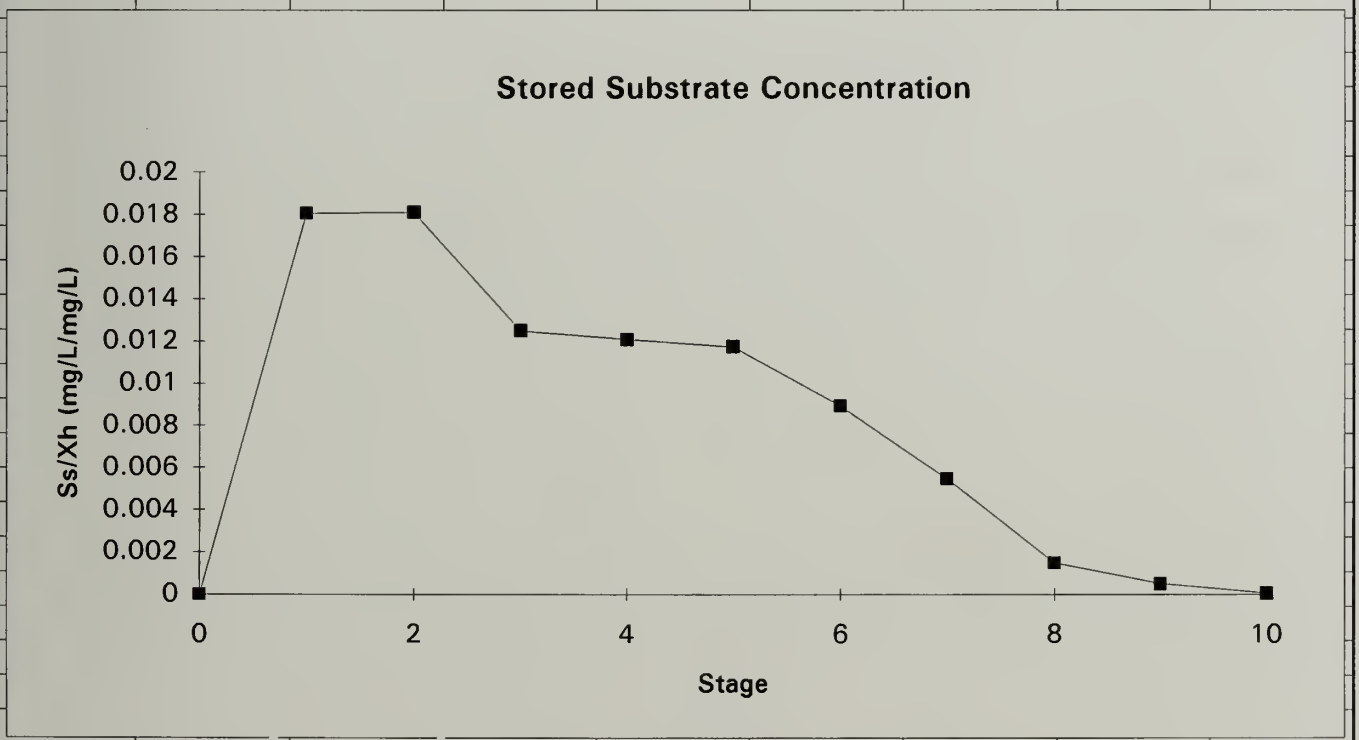


A	B	C	D	E	F	G	H	I
OXYGEN CONSUMPTION								
	Calculated Oxygen			Calculated Oxygen				
Stage	Consumption (Kg/hr)			Consumption (mg/L-hr)				
1	0			0				
2	0			0				
3	0			0				
4	0			0				
5	0			0				
6	103.1783			80.30819				
7	141.2656			80.28857				
8	279.2338			79.35155				
9	138.2708			78.58649				
10	128.9668			73.29851				

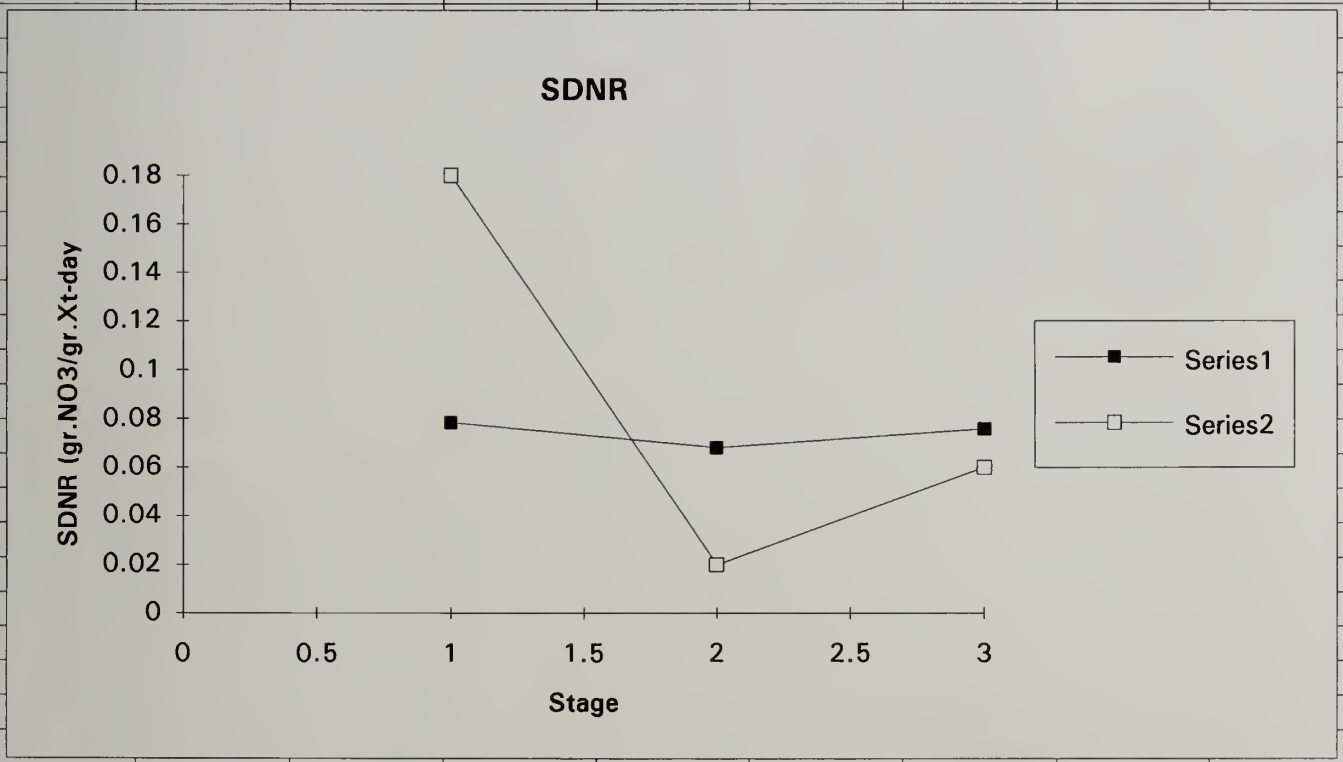


MLSS =	X(total) =	Xh +	Xns +	Xnb +	X(inerts) +	X(part.)
		2977.459	62	31	537.6344	301.9062
	X(total) =	3910	mg/l			

A	B	C	D	E	F	G	H	I
STORED SUBSTRATE								
	concentration							
Stage	Calculated							
0	0							
1	0.018073							
2	0.018098							
3	0.012494							
4	0.0121							
5	0.011733							
6	0.008919							
7	0.005466							
8	0.00147							
9	0.00051							
10	6.42E-05							



A	B	C	D	E	F	G	H	I
	SPECIFIC DENITRIFICATION RATE							
		SDNR			SDNR (gr. NO3/gr. Xt-day)			
	Stage	gr. NO3/gr.Xt-day			Plant Data			
	1	0.078413	44.13589		0.18			
	2	0.068017	35.5847		0.02			
	3	0.075904	138.2213		0.06			



A	B	C	D	E	F	G	H	I
NITRIFICATION								
Term (a)	Depletion of NH4 by heterotrophic utilization of soluble degradable COD in cell storage							
Term (b)	Depletion of NH4 via heterotrophic uptake of particulate COD							
Term (c)	Adding NH4 by death of heterotrophs							
Term (d)	Depletion of NH4 by Nitrosomonas							
Stage	Term (a)		Term (b)		Term (c)		Term (d)	
	mg/L-d	mg/L-hr	mg/L-day	mg/L-hr	mg/L-d	mg/L-hr	mg/L-d	mg/L-hr
1	32.01497	1.333957	17.8604	0.744183	6.505469	0.271061	0	0
2	27.79585	1.15816	15.46255	0.644273	5.647369	0.235307	0	0
3	31.25632	1.302347	16.69036	0.695431	6.633862	0.276411	0	0
4	29.59519	1.233133	15.85165	0.660486	6.309534	0.262897	0	0
5	27.60473	1.150197	14.83133	0.617972	5.911334	0.246306	0	0
6	77.18755	3.216148	43.44634	1.810264	17.28847	0.720353	255.7008	10.6542
7	69.18127	2.882553	42.85627	1.785678	17.28847	0.720353	254.1141	10.58809
8	40.02614	1.667756	41.65865	1.735777	17.28847	0.720353	249.4872	10.3953
9	19.2151	0.800629	41.05759	1.710733	17.28847	0.720353	245.84	10.24333
10	2.937111	0.12238	39.29023	1.637093	17.28847	0.720353	218.7082	9.112841

Total Nitrification		
Stage	mg/L-d	mg/L-hr
1	56.38083	2.349201
2	48.90576	2.03774
3	54.58054	2.274189
4	51.75638	2.156516
5	48.3474	2.014475
6	393.6232	16.40096
7	383.4401	15.97667
8	348.4604	14.51918
9	323.4012	13.47505
10	278.224	11.59267

Appendix C: List of Mass Balance Equations for Storage Model and Michaelis-Menton Model

Mass Balance Equations for Michaelis-Menton Model

Ammonia (NH_4-N) Mass Balances

Stage 1

$$V\frac{dN_1}{dt} = 0 = (Q_0)(N_0) + (Q_{RAS})(N_{10}) + (Q_{ir(1)})(N_9) - (Q_0 + Q_{RAS} + Q_{ir(1)})(N_1)$$

$$-\left(\frac{\mu_{m(NS)}}{Y_{NS}}\right)\frac{(X_{NS})(N_1)}{(K_N + N_1)}\left(\frac{O_1}{K_{0N} + O_1}\right)(V_1)$$

$$-Y_H\left[(Q_0 S_0) + (Q_{RAS} S_{10}) + (Q_{ir(1)} S_9) - (Q_0 + Q_{RAS} + Q_{ir(1)})(S_1)\right](F_N)$$

$$-Y_H\left[(Q_0 P_0) + (Q_0 + Q_{RAS})(P_{10}) - \left(\frac{V_1 P_{10}}{\theta_c}\right) + (Q_{ir(1)})(P_9)\right](F_N)$$

$$-Y_H\left[-(Q_0 + Q_{RAS} + Q_{ir(1)})(P_1)\right](F_N)$$

$$+(K_d)(X_H)(V_1)(F_N)(F_{DN})\left(\frac{NO_1}{NO_s + NO_1}\right)$$

Stage 2

$$V\frac{dN_2}{dt} = 0 = (Q_0 + Q_{RAS} + Q_{ir(1)})(N_1) - (Q_0 + Q_{RAS} + Q_{ir(1)})(N_2)$$

$$-\left(\frac{\mu_{m(NS)}}{Y_{NS}}\right)\frac{(X_{NS})(N_2)}{(K_N + N_2)}\left(\frac{O_2}{K_{0N} + O_2}\right)(V_2)$$

$$-Y_H\left[(Q_0 + Q_{RAS} + Q_{ir(1)})(S_1) - (Q_0 + Q_{RAS} + Q_{ir(1)})(S_2)\right](F_N)$$

$$-Y_H\left[(Q_0+Q_{RAS}+Q_{ir(1)})(P_1)-\left[(Q_0+Q_{RAS}+Q_{ir(1)})(P_2)\right]\right](F_N)$$

$$+(K_d)(X_H)(V_2)(F_N)(F_{DN})\left(\frac{NO_2}{NO_s+NO_2}\right)$$

Stage 3

$$V\frac{dN_3}{dt}=0=(Q_0+Q_{RAS}+Q_{ir(1)})(N_2)+(Q_{ir(3)})(N_9)-(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(N_3)$$

$$-\left(\frac{\mu_{m(NS)}}{Y_{NS}}\right)\frac{(X_{NS})(N_3)}{(K_N+N_3)}\left(\frac{O_3}{K_{0N}+O_3}\right)(V_3)$$

$$-Y_H\left[(Q_0+Q_{RAS}+Q_{ir(1)})(S_2)+(Q_{ir(3)})(S_9)-(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(S_3)\right](F_N)$$

$$-Y_H\left[(Q_0+Q_{RAS}+Q_{ir(1)})(P_2)+(Q_{ir(3)})(P_9)-\left[(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(P_3)\right]\right](F_N)$$

$$+(K_d)(X_H)(V_3)(F_N)(F_{DN})\left(\frac{NO_3}{NO_s+NO_3}\right)$$

Stage 4

$$V\frac{dN_4}{dt}=0=(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(N_3)-(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(N_4)$$

$$-\left(\frac{\mu_{m(NS)}}{Y_{NS}}\right)\frac{(X_{NS})(N_4)}{(K_N+N_4)}\left(\frac{O_4}{K_{0N}+O_4}\right)(V_4)$$

$$-Y_H\left[(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(S_3)-(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(S_4)\right](F_N)$$

$$-Y_H\left[(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(P_3)-\left[(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(P_4)\right]\right](F_N)$$

$$+(K_d)(X_H)(V_4)(F_N)(F_{DN})\left(\frac{NO_4}{NO_s+NO_4}\right)$$

Stage 5

$$V\frac{dN_5}{dt} = 0 = (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(N_4) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(N_5)$$

$$-\left(\frac{\mu_{m(NS)}}{Y_{NS}}\right)\frac{(X_{NS})(N_5)}{(K_N + N_5)}\left(\frac{O_5}{K_{0N} + O_5}\right)(V_5)$$

$$-Y_H\left[(Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(S_4) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(S_5)\right](F_N)$$

$$-Y_H\left[(Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(P_4) - \left[(Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(P_5)\right]\right](F_N)$$

$$+(K_d)(X_H)(V_5)(F_N)(F_{DN})\left(\frac{NO_5}{NO_s + NO_5}\right)$$

Stage 6

$$V\frac{dN_6}{dt} = 0 = (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(N_5) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(N_6)$$

$$-\left(\frac{\mu_{m(NS)}}{Y_{NS}}\right)\frac{(X_{NS})(N_6)}{(K_N + N_6)}\left(\frac{O_6}{K_{0N} + O_6}\right)(V_6)$$

$$-Y_H\left[(Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(S_5) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(S_6)\right](F_N)$$

$$-Y_H\left[(Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(P_5) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(P_6)\right](F_N)$$

$$+(K_{d(h)})(X_H)(V_6)(F_N)\left(\frac{O_6}{k_{os} + O_6}\right)$$

Stage 7

$$V\frac{dN_7}{dt} = 0 = (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(N_6) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(N_7)$$

$$-\left(\frac{\mu_{m(NS)}}{Y_{NS}}\right)\frac{(X_{NS})(N_7)}{(K_N + N_7)}\left(\frac{O_7}{K_{0N} + O_7}\right)(V_7)$$

$$-Y_H\left[(Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(S_6) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(S_7)\right](F_N)$$

$$-Y_H\left[(Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(P_6) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(P_7)\right](F_N)$$

$$+(K_{d(h)})(X_H)(V_7)(F_N)\left(\frac{O_7}{k_{os} + O_7}\right)$$

Stage 8

$$V\frac{dN_8}{dt} = 0 = (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(N_7) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(N_8)$$

$$-\left(\frac{\mu_{m(NS)}}{Y_{NS}}\right)\frac{(X_{NS})(N_8)}{(K_N + N_8)}\left(\frac{O_8}{K_{0N} + O_8}\right)(V_8)$$

$$-Y_H\left[(Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(S_7) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(S_8)\right](F_N)$$

$$-Y_H\left[(Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(P_7) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(P_8)\right](F_N)$$

$$+(K_{d(h)})(X_H)(V_8)(F_N)\left(\frac{O_8}{k_{os} + O_8}\right)$$

Stage 9

$$V\frac{dN_9}{dt} = 0 = (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(N_8) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(N_9)$$

$$-\left(\frac{\mu_{m(NS)}}{Y_{NS}}\right)\frac{(X_{NS})(N_9)}{(K_N + N_9)}\left(\frac{O_9}{K_{0N} + O_9}\right)(V_9)$$

$$-Y_H\left[(Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(S_8) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(S_9)\right](F_N)$$

$$-Y_H\left[(Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(P_8) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(P_9)\right](F_N)$$

$$+(K_{d(h)})(X_H)(V_9)(F_N)\left(\frac{O_9}{k_{os} + O_9}\right)$$

Stage 10

$$V\frac{dN_{10}}{dt} = 0 = (Q_0 + Q_{RAS})(N_9) - (Q_0 + Q_{RAS})(N_{10})$$

$$-\left(\frac{\mu_{m(NS)}}{Y_{NS}}\right)\frac{(X_{NS})(N_{10})}{(K_N + N_{10})}\left(\frac{O_{10}}{K_{0N} + O_{10}}\right)(V_{10})$$

$$-Y_H\left[(Q_0 + Q_{RAS})(S_9) - (Q_0 + Q_{RAS})(S_{10})\right](F_N)$$

$$-Y_H\left[(Q_0 + Q_{RAS})(P_9) - (Q_0 + Q_{RAS})(P_{10})\right](F_N)$$

$$+(K_{d(h)})(X_H)(V_{10})(F_N)\left(\frac{O_{10}}{k_{os} + O_{10}}\right)$$

Nitrate (NO_3-N) Mass Balances

Stage 1

$$V\frac{dNO_1}{dt} = 0 = (Q_0)(NO_0) + (Q_{RAS})(0.1)(NO_{10}) + (Q_{ir(1)})(NO_9) - (Q_0 + Q_{RAS} + Q_{ir(1)})(NO_1)$$

$$-\left(\frac{1-1.42Y_H}{2.86}\right)\left(\frac{\mu_{m(H)}}{Y_H}\right)F_{DN}\left(\frac{S_1X_H}{K_s+S_1}\right)\left(\frac{k_{oh}}{k_{oh}+O_1}\right)\left(\frac{NO_1}{NO_s+NO_1}\right)V_1$$

$$-\frac{1.42}{2.86}(k_d)(X_H)(F_{DN})\left(\frac{k_{oh}}{k_{oh}+O_1}\right)\left(\frac{NO_1}{NO_s+NO_1}\right)V_1$$

$$-\left(\frac{1-1.42Y_H}{2.86}\right)F_{DN}(K_P)(P_1)X_H\left(\frac{k_{oh}}{k_{oh}+O_1}\right)\left(\frac{NO_1}{NO_s+NO_1}\right)V_1$$

Stage 2

$$V\frac{dNO_2}{dt} = 0 = (Q_0 + Q_{ras} + Q_{ir(1)})(NO_1) - (Q_0 + Q_{RAS} + Q_{ir(1)})(NO_2)$$

$$-\left(\frac{1-1.42Y_H}{2.86}\right)\left(\frac{\mu_{m(H)}}{Y_H}\right)F_{DN}\left(\frac{S_2X_H}{K_s+S_2}\right)\left(\frac{k_{oh}}{k_{oh}+O_2}\right)\left(\frac{NO_2}{NO_s+NO_2}\right)V_2$$

$$-\frac{1.42}{2.86}(k_d)(X_H)(F_{DN})\left(\frac{k_{oh}}{k_{oh}+O_2}\right)\left(\frac{NO_2}{NO_s+NO_2}\right)V_2$$

$$-\left(\frac{1-1.42Y_H}{2.86}\right)F_{DN}(K_P)(P_2)X_H\left(\frac{k_{oh}}{k_{oh}+O_2}\right)\left(\frac{NO_2}{NO_s+NO_2}\right)V_2$$

Stage 3

$$V\frac{dNO_3}{dt} = 0 = (Q_0 + Q_{ras} + Q_{ir(1)})(NO_2) + (Q_{ir(3)})(NO_9) - (Q_0 + Q_{RAS} + Q_{ir(1)})(NO_3)$$

$$-\left(\frac{1-1.42Y_H}{2.86}\right)\left(\frac{\mu_{m(H)}}{Y_H}\right)F_{DN}\left(\frac{S_3X_H}{K_s+S_3}\right)\left(\frac{k_{oh}}{k_{oh}+O_3}\right)\left(\frac{NO_3}{NO_s+NO_3}\right)V_3$$

$$-\frac{1.42}{2.86}(k_d)(X_H)(F_{DN})\left(\frac{k_{oh}}{k_{oh}+O_3}\right)\left(\frac{NO_3}{NO_s+NO_3}\right)V_3$$

$$-\left(\frac{1-1.42Y_H}{2.86}\right)F_{DN}(K_P)(P_3)X_H\left(\frac{k_{oh}}{k_{oh}+O_3}\right)\left(\frac{NO_3}{NO_s+NO_3}\right)V_3$$

Stage 4

$$V\frac{dNO_4}{dt} = 0 = (Q_0 + Q_{ras} + Q_{ir(1)} + Q_{ir(3)})(NO_3) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(NO_4)$$

$$-\left(\frac{1-1.42Y_H}{2.86}\right)\left(\frac{\mu_{m(H)}}{Y_H}\right)F_{DN}\left(\frac{S_4X_H}{K_s+S_4}\right)\left(\frac{k_{oh}}{k_{oh}+O_4}\right)\left(\frac{NO_4}{NO_s+NO_4}\right)V_4$$

$$-\frac{1.42}{2.86}(k_d)(X_H)(F_{DN})\left(\frac{k_{oh}}{k_{oh}+O_4}\right)\left(\frac{NO_4}{NO_s+NO_4}\right)V_4$$

$$-\left(\frac{1-1.42Y_H}{2.86}\right)F_{DN}(K_P)(P_4)X_H\left(\frac{k_{oh}}{k_{oh}+O_4}\right)\left(\frac{NO_4}{NO_s+NO_4}\right)V_4$$

Stage 5

$$V\frac{dNO_5}{dt}=0=(Q_0+Q_{ras}+Q_{ir(1)}+Q_{ir(3)})(NO_4)-(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(NO_5)$$

$$-\left(\frac{1-1.42Y_H}{2.86}\right)\left(\frac{\mu_{m(H)}}{Y_H}\right)F_{DN}\left(\frac{S_5X_H}{K_s+S_5}\right)\left(\frac{k_{oh}}{k_{oh}+O_5}\right)\left(\frac{NO_5}{NO_s+NO_5}\right)V_5$$

$$-\frac{1.42}{2.86}(k_d)(X_H)(F_{DN})\left(\frac{k_{oh}}{k_{oh}+O_5}\right)\left(\frac{NO_5}{NO_s+NO_5}\right)V_5$$

$$-\left(\frac{1-1.42Y_H}{2.86}\right)F_{DN}(K_P)(P_5)X_H\left(\frac{k_{oh}}{k_{oh}+O_5}\right)\left(\frac{NO_5}{NO_s+NO_5}\right)V_5$$

Stage 6

$$V\frac{dNO_6}{dt}=0=(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(NO_5)-(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(NO_6)$$

$$+\left(\frac{\mu_{m(NB)}}{Y_{NB}}\right)\left(\frac{(X_{NB})(NO2_6)}{K_N+NO2_6}\right)\left(\frac{O_6}{k_{on}+O_6}\right)V_6$$

$$-\left(\frac{1-1.42Y_H}{2.86}\right)\left(\frac{\mu_{m(H)}}{Y_H}\right)F_{DN}\left(\frac{S_6X_H}{K_s+S_6}\right)\left(\frac{k_{oh}}{k_{oh}+O_6}\right)\left(\frac{NO_6}{NO_s+NO_6}\right)V_6$$

$$-\frac{1.42}{2.86}(k_d)(X_H)(F_{DN})\left(\frac{k_{oh}}{k_{oh}+O_6}\right)\left(\frac{NO_6}{NO_s+NO_6}\right)V_6$$

$$-\left(\frac{1-1.42Y_H}{2.86}\right)F_{DN}(K_P)(P_6)X_H\left(\frac{k_{oh}}{k_{oh}+O_6}\right)\left(\frac{NO_6}{NO_s+NO_6}\right)V_6$$

Stage 7

$$V\frac{dNO_7}{dt} = 0 = (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(NO_6) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(NO_7)$$

$$+\left(\frac{\mu_{m(NB)}}{Y_{NB}}\right)\left(\frac{(X_{NB})(NO2_7)}{K_N + NO2_7}\right)\left(\frac{O_7}{k_{on} + O_7}\right)V_7$$

$$-\left(\frac{1-1.42Y_H}{2.86}\right)\left(\frac{\mu_{m(H)}}{Y_H}\right)F_{DN}\left(\frac{S_7X_H}{K_s + S_7}\right)\left(\frac{k_{oh}}{k_{oh} + O_7}\right)\left(\frac{NO_7}{NO_s + NO_7}\right)V_7$$

$$-\frac{1.42}{2.86}(k_d)(X_H)(F_{DN})\left(\frac{k_{oh}}{k_{oh} + O_7}\right)\left(\frac{NO_7}{NO_s + NO_7}\right)V_7$$

$$-\left(\frac{1-1.42Y_H}{2.86}\right)F_{DN}(K_P)(P_7)X_H\left(\frac{k_{oh}}{k_{oh} + O_7}\right)\left(\frac{NO_7}{NO_s + NO_7}\right)V_7$$

Stage 8

$$V\frac{dNO_8}{dt} = 0 = (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(NO_7) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(NO_8)$$

$$+\left(\frac{\mu_{m(NB)}}{Y_{NB}}\right)\left(\frac{(X_{NB})(NO2_8)}{K_N + NO2_8}\right)\left(\frac{O_8}{k_{on} + O_8}\right)V_8$$

$$-\left(\frac{1-1.42Y_H}{2.86}\right)\left(\frac{\mu_{m(H)}}{Y_H}\right)F_{DN}\left(\frac{S_8X_H}{K_s + S_8}\right)\left(\frac{k_{oh}}{k_{oh} + O_8}\right)\left(\frac{NO_8}{NO_s + NO_8}\right)V_8$$

$$-\frac{1.42}{2.86}(k_d)(X_H)(F_{DN})\left(\frac{k_{oh}}{k_{oh} + O_8}\right)\left(\frac{NO_8}{NO_s + NO_8}\right)V_8$$

$$-\left(\frac{1-1.42Y_H}{2.86}\right)F_{DN}(K_P)(P_8)X_H\left(\frac{k_{oh}}{k_{oh}+O_8}\right)\left(\frac{NO_8}{NO_s+NO_8}\right)V_8$$

Stage 9

$$V\frac{dNO_9}{dt}=0=(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(NO_8)-(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(NO_9)$$

$$+\left(\frac{\mu_{m(NB)}}{Y_{NB}}\right)\left(\frac{(X_{NB})(NO2_9)}{K_N+NO2_9}\right)\left(\frac{O_9}{k_{on}+O_9}\right)V_9$$

$$-\left(\frac{1-1.42Y_H}{2.86}\right)\left(\frac{\mu_{m(H)}}{Y_H}\right)F_{DN}\left(\frac{S_9X_H}{K_s+S_9}\right)\left(\frac{k_{oh}}{k_{oh}+O_9}\right)\left(\frac{NO_9}{NO_s+NO_9}\right)V_9$$

$$-\frac{1.42}{2.86}(k_d)(X_H)(F_{DN})\left(\frac{k_{oh}}{k_{oh}+O_9}\right)\left(\frac{NO_9}{NO_s+NO_9}\right)V_9$$

$$-\left(\frac{1-1.42Y_H}{2.86}\right)F_{DN}(K_P)(P_9)X_H\left(\frac{k_{oh}}{k_{oh}+O_9}\right)\left(\frac{NO_9}{NO_s+NO_9}\right)V_9$$

Stage 10

$$V\frac{dNO_{10}}{dt}=0=(Q_0+Q_{RAS})(NO_9)-(Q_0+Q_{RAS})(NO_{10})$$

$$+\left(\frac{\mu_{m(NB)}}{Y_{NB}}\right)\left(\frac{(X_{NB})(NO2_{10})}{K_N+NO2_{10}}\right)\left(\frac{O_{10}}{k_{on}+O_{10}}\right)V_{10}$$

$$-\left(\frac{1-1.42Y_H}{2.86}\right)\left(\frac{\mu_{m(H)}}{Y_H}\right)F_{DN}\left(\frac{S_{10}X_H}{K_s+S_{10}}\right)\left(\frac{k_{oh}}{k_{oh}+O_{10}}\right)\left(\frac{NO_{10}}{NO_s+NO_{10}}\right)V_{10}$$

$$-\frac{1.42}{2.86}(k_d)(X_H)(F_{DN})\left(\frac{k_{oh}}{k_{oh}+O_{10}}\right)\left(\frac{NO_{10}}{NO_s+NO_{10}}\right)V_{10}$$

$$-\left(\frac{1-1.42Y_H}{2.86}\right)F_{DN}(K_P)(P_{10})X_H\left(\frac{k_{oh}}{k_{oh}+O_{10}}\right)\left(\frac{NO_{10}}{NO_s+NO_{10}}\right)V_{10}$$

Nitrite (NO_2^-) Mass Balances

Stage 1

$$V\frac{dNO2_1}{dt}=0=(Q_0)(NO2_0)+(Q_{RAS})(NO2_{10})+(Q_{ir(1)})(NO2_9)-(Q_0+Q_{RAS}+Q_{ir(1)})(NO2_1)$$

Stage 2

$$V\frac{dNO2_2}{dt}=0=(Q_0+Q_{ras}+Q_{ir(1)})(NO2_1)-(Q_0+Q_{RAS}+Q_{ir(1)})(NO2_2)$$

Stage 3

$$V\frac{dNO2_3}{dt}=0=(Q_0+Q_{ras}+Q_{ir(1)})(NO2_2)+(Q_{ir(3)})(NO2_9)-(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(NO2_3)$$

Stage 4

$$V\frac{dNO2_4}{dt}=0=(Q_0+Q_{ras}+Q_{ir(1)}+Q_{ir(3)})(NO2_3)-(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(NO2_4)$$

Stage 5

$$V\frac{dNO2_5}{dt}=0=(Q_0+Q_{ras}+Q_{ir(1)}+Q_{ir(3)})(NO2_4)-(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(NO2_5)$$

Stage 6

$$V\frac{dNO2_6}{dt} = 0 = (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(NO2_5) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(NO2_6)$$
$$+\left(\frac{\mu_{m(NS)}}{Y_{NS}}\right)\left(\frac{X_{NS}N_6}{K_N + N_6}\right)\left(\frac{O_6}{K_{on} + O_6}\right)(V_6)$$
$$-\left(\frac{\mu_{m(NB)}}{Y_{NB}}\right)\left(\frac{X_{NB}NO2_6}{k_{N(NB)} + NO2_6}\right)\left(\frac{O_6}{k_{on} + O_6}\right)(V_6)$$

Stage 7

$$V\frac{dNO2_7}{dt} = 0 = (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(NO2_6) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(NO2_7)$$
$$+\left(\frac{\mu_{m(NS)}}{Y_{NS}}\right)\left(\frac{X_{NS}N_7}{K_N + N_7}\right)\left(\frac{O_7}{K_{on} + O_7}\right)(V_7)$$
$$-\left(\frac{\mu_{m(NB)}}{Y_{NB}}\right)\left(\frac{X_{NB}NO2_7}{k_{N(NB)} + NO2_7}\right)\left(\frac{O_7}{k_{on} + O_7}\right)(V_7)$$

Stage 8

$$V\frac{dNO2_8}{dt} = 0 = (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(NO2_7) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(NO2_8)$$
$$+\left(\frac{\mu_{m(NS)}}{Y_{NS}}\right)\left(\frac{X_{NS}N_8}{K_N + N_8}\right)\left(\frac{O_8}{K_{on} + O_8}\right)(V_8)$$
$$-\left(\frac{\mu_{m(NB)}}{Y_{NB}}\right)\left(\frac{X_{NB}NO2_8}{k_{N(NB)} + NO2_8}\right)\left(\frac{O_8}{k_{on} + O_8}\right)(V_8)$$

Stage 9

$$V\frac{dNO2_9}{dt} = 0 = (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(NO2_8) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(NO2_9)$$

$$+\left(\frac{\mu_{m(NS)}}{Y_{NS}}\right)\left(\frac{X_{NS}N_9}{K_N + N_9}\right)\left(\frac{O_9}{K_{on} + O_9}\right)(V_9)$$

$$-\left(\frac{\mu_{m(NB)}}{Y_{NB}}\right)\left(\frac{X_{NB}NO2_9}{k_{N(NB)} + NO2_9}\right)\left(\frac{O_9}{k_{on} + O_9}\right)(V_9)$$

Stage 10

$$V\frac{dNO2_{10}}{dt} = 0 = (Q_0 + Q_{RAS})(NO2_9) - (Q_0 + Q_{RAS})(NO2_{10})$$

$$+\left(\frac{\mu_{m(NS)}}{Y_{NS}}\right)\left(\frac{X_{NS}N_{10}}{K_N + N_{10}}\right)\left(\frac{O_{10}}{K_{on} + O_{10}}\right)(V_{10})$$

$$-\left(\frac{\mu_{m(NB)}}{Y_{NB}}\right)\left(\frac{X_{NB}NO2_{10}}{k_{N(NB)} + NO2_{10}}\right)\left(\frac{O_{10}}{k_{on} + O_{10}}\right)(V_{10})$$

SCOD Mass Balances

Stage 1

$$V\frac{dS_1}{dt} = 0 = Q_0S_0 + Q_{RAS}S_{10} + Q_{ir(1)}S_9 - (Q_0 + Q_{RAS} + Q_{ir(1)})(S_1) - (R_{su})(V_1)$$

where: $$R_{su} = K(F_{DN})\left(\frac{S_1X_H}{k_s + S_1}\right)\left(\frac{k_{oh}}{k_{oh} + O_1}\right)\left(\frac{NO_1}{NO_s + NO_1}\right)$$

Stage 2

$$V\frac{dS_2}{dt} = 0 = (Q_0 + Q_{ras} + Q_{ir(1)})(S_1) - (Q_0 + Q_{RAS} + Q_{ir(1)})(S_2) - (R_{su})(V_2)$$

where: $$R_{su} = K(F_{DN})\left(\frac{S_2 X_H}{k_s + S_2}\right)\left(\frac{k_{oh}}{k_{oh} + O_2}\right)\left(\frac{NO_2}{NO_s + NO_2}\right)$$

Stage 3

$$V\frac{dS_3}{dt} = 0 = (Q_0 + Q_{ras} + Q_{ir(1)})(S_2) + (Q_{ir(3)})(S_9) - (Q_0 + Q_{RAS} + Q_{ir(1)})(S_3) - (R_{su})(V_3)$$

where: $$R_{su} = K(F_{DN})\left(\frac{S_3 X_H}{k_s + S_3}\right)\left(\frac{k_{oh}}{k_{oh} + O_3}\right)\left(\frac{NO_3}{NO_s + NO_3}\right)$$

Stage 4

$$V\frac{dS_4}{dt} = 0 = (Q_0 + Q_{ras} + Q_{ir(1)} + Q_{ir(3)})(S_3) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(S_4) - (R_{su})(V_4)$$

where: $$R_{su} = K(F_{DN})\left(\frac{S_4 X_H}{k_s + S_4}\right)\left(\frac{k_{oh}}{k_{oh} + O_4}\right)\left(\frac{NO_4}{NO_s + NO_4}\right)$$

Stage 5

$$V\frac{dS_5}{dt} = 0 = (Q_0 + Q_{ras} + Q_{ir(1)} + Q_{ir(3)})(S_4) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(S_5) - (R_{su})(V_5)$$

where: $$R_{su} = K(F_{DN})\left(\frac{S_5 X_H}{k_s + S_5}\right)\left(\frac{k_{oh}}{k_{oh} + O_5}\right)\left(\frac{NO_5}{NO_s + NO_5}\right)$$

Stage 6

$$V\frac{dS}{dt}=0=(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(S_5)-(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(S_6)$$

$$-\left(\frac{KS_6X_H}{k_s+S_6}\right)\left(\frac{O_6}{O_6+k_{os}}\right)V_6$$

$$-\left(\frac{KS_6X_H}{k_s+S_6}\right)\left(\frac{k_{oh}}{k_{oh}+O_6}\right)\left(\frac{NO_6}{NO_s+NO_6}\right)V_6$$

Stage 7

$$V\frac{dS_7}{dt}=0=(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(S_6)-(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(S_7)$$

$$-\left(\frac{KS_7X_H}{k_s+S_7}\right)\left(\frac{O_7}{O_7+k_{os}}\right)V_7$$

$$-\left(\frac{KS_7X_H}{k_s+S_7}\right)\left(\frac{k_{oh}}{k_{oh}+O_7}\right)\left(\frac{NO_7}{NO_s+NO_7}\right)V_7$$

Stage 8

$$V\frac{dS_8}{dt}=0=(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(S_7)-(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(S_8)$$

$$-\left(\frac{KS_8X_H}{k_s+S_8}\right)\left(\frac{O_8}{O_8+k_{os}}\right)V_8$$

$$-\left(\frac{KS_8X_H}{k_s+S_8}\right)\left(\frac{k_{oh}}{k_{oh}+O_8}\right)\left(\frac{NO_8}{NO_s+NO_8}\right)V_8$$

Stage 9

$$V\frac{dS_9}{dt} = 0 = (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(S_8) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(S_9)$$

$$-\left(\frac{KS_9X_H}{k_s + S_9}\right)\left(\frac{O_9}{O_9 + k_{os}}\right)V_9$$

$$-\left(\frac{KS_9X_H}{k_s + S_9}\right)\left(\frac{k_{oh}}{k_{oh} + O_9}\right)\left(\frac{NO_9}{NO_s + NO_9}\right)V_9$$

Stage 10

$$V\frac{dS_{10}}{dt} = 0 = (Q_0 + Q_{RAS})(S_9) - (Q_0 + Q_{RAS})(S_{10})$$

$$-\left(\frac{KS_{10}X_H}{k_s + S_{10}}\right)\left(\frac{O_{10}}{O_{10} + k_{os}}\right)V_{10}$$

$$-\left(\frac{KS_{10}X_H}{k_s + S_{10}}\right)\left(\frac{k_{oh}}{k_{oh} + O_{10}}\right)\left(\frac{NO_{10}}{NO_s + NO_{10}}\right)V_{10}$$

PCOD Mass Balances

Stage 1

$$V\frac{dP_1}{dt} = 0 = (Q_0)(P_0) + (Q_0 + Q_{RAS})(P_{10}) - \frac{(V_T)(P_{10})}{\theta_c} + (Q_{ir(1)})(P_9) - (Q_0 + Q_{RAS} + Q_{ir(1)})(P_1) - r_p(V_1)$$

where: $r_p = F_{DN}(K_pP_1X_H)\left(\frac{k_{oh}}{k_{oh} + O_1}\right)\left(\frac{NO_1}{NO_s + NO_1}\right)$

Stage 2

$$V\frac{dP_2}{dt} = 0 = (Q_0 + Q_{ras} + Q_{ir(1)})(P_1) - (Q_0 + Q_{RAS} + Q_{ir(1)})(P_2) - r_p(V_2)$$

where: $$r_p = F_{DN}(K_p P_2 X_H)\left(\frac{k_{oh}}{k_{oh} + O_2}\right)\left(\frac{NO_2}{NO_s + NO_2}\right)$$

Stage 3

$$V\frac{dP_3}{dt} = 0 = (Q_0 + Q_{ras} + Q_{ir(1)})(P_2) + (Q_{ir(3)})(P_9) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(P_3) - r_p(V_3)$$

where: $$r_p = F_{DN}(K_p P_3 X_H)\left(\frac{k_{oh}}{k_{oh} + O_3}\right)\left(\frac{NO_3}{NO_s + NO_3}\right)$$

Stage 4

$$V\frac{dP_4}{dt} = 0 = (Q_0 + Q_{ras} + Q_{ir(1)} + Q_{ir(3)})(P_3) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(P_4) - r_p(V_4)$$

where: $$r_p = F_{DN}(K_p P_4 X_H)\left(\frac{k_{oh}}{k_{oh} + O_4}\right)\left(\frac{NO_4}{NO_s + NO_4}\right)$$

Stage 5

$$V\frac{dP_5}{dt} = 0 = (Q_0 + Q_{ras} + Q_{ir(1)} + Q_{ir(3)})(P_4) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(P_5) - r_p(V_5)$$

where: $$r_p = F_{DN}(K_p P_5 X_H)\left(\frac{k_{oh}}{k_{oh} + O_5}\right)\left(\frac{NO_5}{NO_s + NO_5}\right)$$

Stage 6

$$V\frac{dP_6}{dt} = 0 = (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(P_5) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(P_6)$$

$$-(K_p P_6 X_H V_6)\left(\frac{k_{oh}}{k_{oh} + O_6}\right)\left(\frac{NO_6}{NO_s + NO_6}\right)$$

$$-(K_p P_6 X_H V_6)\left(\frac{O_6}{k_{os} + O_6}\right)$$

Stage 7

$$V\frac{dP_7}{dt} = 0 = (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(P_6) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(P_7)$$

$$-(K_p P_7 X_H V_7)\left(\frac{k_{oh}}{k_{oh} + O_7}\right)\left(\frac{NO_7}{NO_s + NO_7}\right)$$

$$-(K_p P_7 X_H V_7)\left(\frac{O_7}{k_{os} + O_7}\right)$$

Stage 8

$$V\frac{dP_8}{dt} = 0 = (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(P_7) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(P_8)$$

$$-(K_p P_8 X_H V_6)\left(\frac{k_{oh}}{k_{oh} + O_8}\right)\left(\frac{NO_8}{NO_s + NO_8}\right)$$

$$-(K_p P_8 X_H V_8)\left(\frac{O_8}{k_{os}+O_8}\right)$$

Stage 9

$$V\frac{dP_9}{dt}=0=(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(P_8)-(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(P_9)$$

$$-(K_p P_9 X_H V_9)\left(\frac{k_{oh}}{k_{oh}+O_9}\right)\left(\frac{NO_9}{NO_s+NO_9}\right)$$

$$-(K_p P_9 X_H V_9)\left(\frac{O_9}{k_{os}+O_9}\right)$$

Stage 10

$$V\frac{dP_{10}}{dt}=0=(Q_0+Q_{RAS})(P_9)-(Q_0+Q_{RAS})(P_{10})$$

$$-(K_p P_{10} X_H V_{10})\left(\frac{k_{oh}}{k_{oh}+O_{10}}\right)\left(\frac{NO_{10}}{NO_s+NO_{10}}\right)$$

$$-(K_p P_{10} X_H V_{10})\left(\frac{O_{10}}{k_{os}+O_{10}}\right)$$

Oxygen Consumption

Stage 6

Dissolved Oxygen Consumption Rate (Kg O_2 / hr) per stage

$$=\left[(A)(r_{su}+1.42(K_d)(X_H)\right](V_6)+(3.22)(r_{NH_4(ox)})(V_6)+(1.11)(r_{NO_2(ox)})(V_6)$$

where:

$$r_{su}=\left(\frac{KS_6X_H}{k_s+S_6}\right)\left(\frac{O_6}{O_6+k_{os}}\right)$$

$$A=1-1.42(Y_H)$$

$$r_{NH_4(ox)}=\left(\frac{\mu_{m(NS)}}{Y_{NB}}\right)\left(\frac{X_{NS}(N_6)}{K_N+N_6}\right)\left(\frac{O_6}{O_6+k_{on}}\right)$$

$$r_{NO_2(ox)}=\left(\frac{\mu_{m(NB)}}{Y_{NB}}\right)\left(\frac{(X_{NB})(NO2_6)}{K_N+NO2_6}\right)\left(\frac{O_6}{O_6+k_{on}}\right)$$

Stage 7

Dissolved Oxygen Consumption Rate (Kg O_2 / hr) per stage

$$=\left[(A)(r_{su}+1.42(K_d)(X_H)\right](V_7)+(3.22)(r_{NH_4(ox)})(V_7)+(1.11)(r_{NO_2(ox)})(V_7)$$

where:

$$r_{su}=\left(\frac{KS_7X_H}{k_s+S_7}\right)\left(\frac{O_7}{O_7+k_{os}}\right)$$

$$A = 1 - 1.42(Y_H)$$

$$r_{NH_4(ox)} = \left(\frac{\mu_{m(NS)}}{Y_{NB}}\right)\left(\frac{X_{NS}(N_7)}{K_N + N_7}\right)\left(\frac{O_7}{O_7 + k_{on}}\right)$$

$$r_{NO_2(ox)} = \left(\frac{\mu_{m(NB)}}{Y_{NB}}\right)\left(\frac{(X_{NB})(NO2_7)}{K_N + NO2_7}\right)\left(\frac{O_7}{O_7 + k_{on}}\right)$$

Stage 8

Dissolved Oxygen Consumption Rate (Kg O_2 / hr) per stage

$$= \left[(A)(r_{su} + 1.42(K_d)(X_H)\right](V_8) + (3.22)(r_{NH_4(ox)})(V_8) + (1.11)(r_{NO_2(ox)})(V_8)$$

where:

$$r_{su} = \left(\frac{KS_8 X_H}{k_s + S_8}\right)\left(\frac{O_8}{O_8 + k_{os}}\right)$$

$$A = 1 - 1.42(Y_H)$$

$$r_{NH_4(ox)} = \left(\frac{\mu_{m(NS)}}{Y_{NB}}\right)\left(\frac{X_{NS}(N_8)}{K_N + N_8}\right)\left(\frac{O_8}{O_8 + k_{on}}\right)$$

$$r_{NO_2(ox)} = \left(\frac{\mu_{m(NB)}}{Y_{NB}}\right)\left(\frac{(X_{NB})(NO2_8)}{K_N + NO2_8}\right)\left(\frac{O_8}{O_8 + k_{on}}\right)$$

Stage 9

Dissolved Oxygen Consumption Rate (Kg O_2 / hr) per stage

$$=\left[(A)(r_{su}+1.42(K_d)(X_H)\right](V_9)+(3.22)(r_{NH_4(ox)})(V_9)+(1.11)(r_{NO_2(ox)})(V_9)$$

where:

$$r_{su}=\left(\frac{KS_9X_H}{k_s+S_9}\right)\left(\frac{O_9}{O_9+k_{os}}\right)$$

$$A=1-1.42(Y_H)$$

$$r_{NH_4(ox)}=\left(\frac{\mu_{m(NS)}}{Y_{NB}}\right)\left(\frac{X_{NS}(N_9)}{K_N+N_9}\right)\left(\frac{O_9}{O_9+k_{on}}\right)$$

$$r_{NO_2(ox)}=\left(\frac{\mu_{m(NB)}}{Y_{NB}}\right)\left(\frac{(X_{NB})(NO2_9)}{K_N+NO2_9}\right)\left(\frac{O_9}{O_9+k_{on}}\right)$$

Stage 10

Dissolved Oxygen Consumption Rate (Kg O_2 / hr) per stage

$$=\left[(A)(r_{su}+1.42(K_d)(X_H)\right](V_{10})+(3.22)(r_{NH_4(ox)})(V_{10})+(1.11)(r_{NO_2(ox)})(V_{10})$$

where:

$$r_{su}=\left(\frac{KS_{10}X_H}{k_s+S_{10}}\right)\left(\frac{O_{10}}{O_{10}+k_{os}}\right)$$

$$A=1-1.42(Y_H)$$

$$r_{NH_4(ox)}=\left(\frac{\mu_{m(NS)}}{Y_{NB}}\right)\left(\frac{X_{NS}(N_{10})}{K_N+N_{10}}\right)\left(\frac{O_{10}}{O_{10}+k_{on}}\right)$$

$$r_{NO_2(ox)}=\left(\frac{\mu_{m(NB)}}{Y_{NB}}\right)\left(\frac{(X_{NB})(NO2_{10})}{K_N+NO2_{10}}\right)\left(\frac{O_{10}}{O_{10}+k_{on}}\right)$$

Storage Model Mass Balance Equations

(Mass balance equations for PCOD and Nitrite in the Storage Model are identical to the mass balance equations listed previously in the Michaelis-Menton Model)

S_S/X_H Mass Balances

Stage 1

$$V\frac{d\left(S_{S(1)}/X_H\right)}{dt} = (Q_{ir(1)})(S_{S(9)}/X_H) + (Q_{RAS})(S_{S(10)}/X_H) - (Q_0 + Q_{ir(1)} + Q_{RAS})(S_{S(1)}/X_H)$$

$$+\left(\frac{1}{X_H}\right)(Q_0S_0 - Q_0S_R)(0.7) - (R_{S_SU})(V_2)$$

where:

$$R_{S_SU} = k_{sto}\left(\frac{S_{S(1)}/X_H}{k_{ss} + S_{s(1)}/X_H}\right)\left(\frac{k_{oh}}{k_{oh} + O_1}\right)\left(\frac{NO_1}{NO_s + NO_1}\right)(F_{DN})$$

Stage 2

$$V\frac{d\left(S_{S(2)}/X_H\right)}{dt} = (Q_0 + Q_{ras} + Q_{ir(1)})(S_{S(1)}/X_H) - (Q_0 + Q_{ir(1)} + Q_{RAS})(S_{S(2)}/X_H)$$

$$+\left(\frac{1}{X_H}\right)\left[(Q_0 + Q_{ras} + Q_{ir(1)})(S_1) - (Q_0 + Q_{ras} + Q_{ir(2)})(S_R)\right](0.10) - (R_{S_SU})(V_2)$$

where:

$$R_{S_sU} = k_{sto}\left(\frac{S_{S(2)}/X_H}{k_{ss}+S_{s(2)}/X_H}\right)\left(\frac{k_{oh}}{k_{oh}+O_2}\right)\left(\frac{NO_2}{NO_s+NO_2}\right)(F_{DN})$$

Stage 3

$$V\frac{d\left(S_{S(3)}/X_H\right)}{dt} = (Q_0+Q_{ras}+Q_{ir(1)})(S_{S(2)}/X_H)+(Q_{ir(3)})(S_{S(9)}/X_H)-(Q_0+Q_{ir(1)}+Q_{RAS}+Q_{ir(3)})(S_{S(3)}/X_H)$$

$$+\left(\frac{1}{X_H}\right)\left[(Q_0+Q_{ras}+Q_{ir(1)})(S_2)-(Q_0+Q_{ras}+Q_{ir(2)})(S_R)\right](1.0)-\left(R_{S_sU}\right)(V_3)$$

where:

$$R_{S_sU} = k_{sto}\left(\frac{S_{S(3)}/X_H}{k_{ss}+S_{s(3)}/X_H}\right)\left(\frac{k_{oh}}{k_{oh}+O_3}\right)\left(\frac{NO_3}{NO_s+NO_3}\right)(F_{DN})$$

Stage 4

$$V\frac{d\left(S_{S(4)}/X_H\right)}{dt} = (Q_0+Q_{ras}+Q_{ir(1)}+Q_{ir(3)})(S_{S(3)}/X_H)-(Q_0+Q_{ir(1)}+Q_{RAS}+Q_{ir(3)})(S_{S(4)}/X_H)$$

$$-\left(R_{S_sU}\right)(V_4)$$

where:

$$R_{S_sU} = k_{sto}\left(\frac{S_{S(4)}/X_H}{k_{ss}+S_{s(4)}/X_H}\right)\left(\frac{k_{oh}}{k_{oh}+O_4}\right)\left(\frac{NO_4}{NO_s+NO_4}\right)(F_{DN})$$

Stage 5

$$V\frac{d\left(S_{S(5)}/X_H\right)}{dt}=\left(Q_0+Q_{ras}+Q_{ir(1)}+Q_{ir(3)}\right)\left(S_{S(4)}/X_H\right)-\left(Q_0+Q_{ir(1)}+Q_{RAS}+Q_{ir(3)}\right)\left(S_{S(5)}/X_H\right)$$

$$-\left(R_{S_SU}\right)(V_5)$$

where:

$$R_{S_SU}=k_{sto}\left(\frac{S_{S(5)}/X_H}{k_{ss}+S_{s(5)}/X_H}\right)\left(\frac{k_{oh}}{k_{oh}+O_5}\right)\left(\frac{NO_5}{NO_s+NO_5}\right)(F_{DN})$$

Stage 6

$$V\frac{d\left(S_{S(6)}/X_H\right)}{dt}=\left(Q_0+Q_{ras}+Q_{ir(1)}+Q_{ir(3)}\right)\left(S_{S(5)}/X_H\right)-\left(Q_0+Q_{ir(1)}+Q_{RAS}+Q_{ir(3)}\right)\left(S_{S(6)}/X_H\right)$$

$$-\left(R_{S_SU}\right)(V_6)$$

where:

$$R_{S_SU}=k\left(\frac{S_{S(6)}/X_H}{k_{ss}+S_{S(6)}/X_H}\right)\left(\frac{O_6}{k_{os}+O_6}\right)$$

Stage 7

$$V\frac{d\left(S_{S(7)}/X_H\right)}{dt}=\left(Q_0+Q_{ras}+Q_{ir(1)}+Q_{ir(3)}\right)\left(S_{S(6)}/X_H\right)-\left(Q_0+Q_{ir(1)}+Q_{RAS}+Q_{ir(3)}\right)\left(S_{S(7)}/X_H\right)$$

$$-\left(R_{S_SU}\right)(V_7)$$

where:

$$R_{S_SU}=k\left(\frac{S_{S(7)}/X_H}{k_{ss}+S_{S(7)}/X_H}\right)\left(\frac{O_7}{k_{os}+O_7}\right)$$

Stage 8

$$V\frac{d\left(S_{S(8)}/X_H\right)}{dt}=\left(Q_0+Q_{ras}+Q_{ir(1)}+Q_{ir(3)}\right)\left(S_{S(7)}/X_H\right)-\left(Q_0+Q_{ir(1)}+Q_{RAS}+Q_{ir(3)}\right)\left(S_{S(8)}/X_H\right)$$

$$-\left(R_{S_sU}\right)\left(V_8\right)$$

where:

$$R_{S_sU}=k\left(\frac{S_{S(8)}/X_H}{k_{ss}+S_{S(8)}/X_H}\right)\left(\frac{O_8}{k_{os}+O_8}\right)$$

Stage 9

$$V\frac{d\left(S_{S(9)}/X_H\right)}{dt}=\left(Q_0+Q_{ras}+Q_{ir(1)}+Q_{ir(3)}\right)\left(S_{S(8)}/X_H\right)-\left(Q_0+Q_{ir(1)}+Q_{RAS}+Q_{ir(3)}\right)\left(S_{S(9)}/X_H\right)$$

$$-\left(R_{S_sU}\right)\left(V_9\right)$$

where:

$$R_{S_sU}=k\left(\frac{S_{S(9)}/X_H}{k_{ss}+S_{S(9)}/X_H}\right)\left(\frac{O_9}{k_{os}+O_9}\right)$$

Stage 10

$$V\frac{d\left(S_{S(10)}/X_H\right)}{dt}=\left(Q_0+Q_{ras}\right)\left(S_{S(9)}/X_H\right)-\left(Q_0+Q_{RAS}\right)\left(S_{S(10)}/X_H\right)$$

$$-\left(R_{S_sU}\right)\left(V_{10}\right)$$

where:

$$R_{S_sU}=k\left(\frac{S_{S(10)}/X_H}{k_{ss}+S_{S(10)}/X_H}\right)\left(\frac{O_{10}}{k_{os}+O_{10}}\right)$$

SCOD Mass Balances (Storage Model)

Stage 1

$$V\left(\frac{dS_1}{dt}\right) = 0 = Q_0 S_0 + \left(Q_{ir(1)}\right)\left(S_9\right) + \left(Q_{RAS}\right)\left(S_{10}\right) - \left(Q_0 + Q_{ir(1)} + Q_{RAS}\right)\left(S_1\right)$$

$$-\left(Q_0 S_0 - Q_0 S_R\right)(0.70)$$

Stage 2

$$V\left(\frac{dS_2}{dt}\right) = 0 = \left(Q_0 + Q_{RAS} + Q_{ir(1)}\right)\left(S_1\right) - \left(Q_0 + Q_{ir(1)} + Q_{RAS}\right)\left(S_2\right)$$

$$-\left[\left(Q_0 + Q_{ras} + Q_{ir(1)}\right)\left(S_1\right) - \left(Q_0 + Q_{ras} + Q_{ir(1)}\right)\left(S_R\right)\right](0.10)$$

Stage 3

$$V\left(\frac{dS_3}{dt}\right) = 0 = \left(Q_0 + Q_{RAS} + Q_{ir(1)}\right)\left(S_2\right) + \left(Q_{ir(3)}\right)\left(S_9\right) - \left(Q_0 + Q_{ir(1)} + Q_{RAS} + Q_{ir(3)}\right)\left(S_3\right)$$

$$-\left[\left(Q_0 + Q_{ras} + Q_{ir(1)}\right)\left(S_2\right) - \left(Q_0 + Q_{ras} + Q_{ir(1)}\right)\left(S_R\right)\right](1.0)$$

$$\mathbf{S_3 = S_4 = S_5 = S_6 = S_7 = S_8 = S_9 = S_{10}}$$

Ammonia (NH_4-N) Mass Balances

Stage 1

$$V\frac{dN_1}{dt} = 0 = (Q_0)(N_0) + (Q_{RAS})(N_{10}) + (Q_{ir(1)})(N_9) - (Q_0 + Q_{RAS} + Q_{ir(1)})(N_1)$$

$$-\left(\frac{\mu_{m(NS)}}{Y_{NS}}\right)\frac{(X_{NS})(N_1)}{(K_N + N_1)}\left(\frac{O_1}{K_{0N} + O_1}\right)(V_1)$$

$$-Y_H[X_H(R_{S_sU})(V_1)](F_N)$$

$$-Y_H\left[(Q_0P_0) + (Q_0 + Q_{RAS})(P_{10}) - \left(\frac{V_1P_{10}}{\theta_c}\right) + (Q_{ir(1)})(P_9)\right](F_N)$$

$$-Y_H\left[-(Q_0 + Q_{RAS} + Q_{ir(1)})(P_1)\right](F_N)$$

$$+(K_d)(X_H)(V_1)(F_N)(F_{DN})\left(\frac{NO_1}{NO_s + NO_1}\right)$$

where:

$$R_{S_sU} = k_{sto}\left(\frac{S_{S(1)}/X_H}{k_{ss} + (S_{s(1)}/X_H)}\right)\left(\frac{k_{oh}}{k_{oh} + O_1}\right)\left(\frac{NO_1}{NO_s + NO_1}\right)(F_{DN})$$

Stage 2

$$V\frac{dN_2}{dt} = 0 = (Q_0 + Q_{RAS} + Q_{ir(1)})(N_1) - (Q_0 + Q_{RAS} + Q_{ir(1)})(N_2)$$

$$-\left(\frac{\mu_{m(NS)}}{Y_{NS}}\right)\frac{(X_{NS})(N_2)}{(K_N + N_2)}\left(\frac{O_2}{K_{0N} + O_2}\right)(V_2)$$

$$-Y_H[X_H(R_{S_sU})(V_2)](F_N)$$

$$-Y_H\left[(Q_0 + Q_{RAS} + Q_{ir(1)})(P_1) - \left[(Q_0 + Q_{RAS} + Q_{ir(1)})(P_2)\right]\right](F_N)$$

$$+(K_d)(X_H)(V_2)(F_N)(F_{DN})\left(\frac{NO_2}{NO_s+NO_2}\right)$$

where:

$$R_{S_sU}=k_{sto}\left(\frac{S_{S(2)}/X_H}{k_{ss}+(S_{s(2)}/X_H)}\right)\left(\frac{k_{oh}}{k_{oh}+O_2}\right)\left(\frac{NO_2}{NO_s+NO_2}\right)(F_{DN})$$

Stage 3

$$V\frac{dN_3}{dt}=0=(Q_0+Q_{RAS}+Q_{ir(1)})(N_2)+(Q_{ir(3)})(N_9)-(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(N_3)$$

$$-\left(\frac{\mu_{m(NS)}}{Y_{NS}}\right)\frac{(X_{NS})(N_3)}{(K_N+N_3)}\left(\frac{O_3}{K_{0N}+O_3}\right)(V_3)$$

$$-Y_H[X_H(R_{S_sU})(V_3)](F_N)$$

$$-Y_H\left[(Q_0+Q_{RAS}+Q_{ir(1)})(P_2)+(Q_{ir(3)})(P_9)-\left[(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(P_3)\right]\right](F_N)$$

$$+(K_d)(X_H)(V_3)(F_N)(F_{DN})\left(\frac{NO_3}{NO_s+NO_3}\right)$$

where:

$$R_{S_sU}=k_{sto}\left(\frac{S_{S(3)}/X_H}{k_{ss}+(S_{s(3)}/X_H)}\right)\left(\frac{k_{oh}}{k_{oh}+O_3}\right)\left(\frac{NO_3}{NO_s+NO_3}\right)(F_{DN})$$

Stage 4

$$V\frac{dN_4}{dt} = 0 = (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(N_3) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(N_4)$$

$$-\left(\frac{\mu_{m(NS)}}{Y_{NS}}\right)\frac{(X_{NS})(N_4)}{(K_N + N_4)}\left(\frac{O_4}{K_{0N} + O_4}\right)(V_4)$$

$$-Y_H[X_H(R_{S_sU})(V_4)](F_N)$$

$$-Y_H\left[(Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(P_3) - \left[(Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(P_4)\right]\right](F_N)$$

$$+(K_d)(X_H)(V_4)(F_N)(F_{DN})\left(\frac{NO_4}{NO_s + NO_4}\right)$$

where:

$$R_{S_sU} = k_{sto}\left(\frac{S_{S(4)}/X_H}{k_{ss} + (S_{s(4)}/X_H)}\right)\left(\frac{k_{oh}}{k_{oh} + O_4}\right)\left(\frac{NO_4}{NO_s + NO_4}\right)(F_{DN})$$

Stage 5

$$V\frac{dN_5}{dt} = 0 = (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(N_4) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(N_5)$$

$$-\left(\frac{\mu_{m(NS)}}{Y_{NS}}\right)\frac{(X_{NS})(N_5)}{(K_N + N_5)}\left(\frac{O_5}{K_{0N} + O_5}\right)(V_5)$$

$$-Y_H[X_H(R_{S_sU})(V_5)](F_N)$$

$$-Y_H\left[(Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(P_4) - \left[(Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(P_5)\right]\right](F_N)$$

$$+(K_d)(X_H)(V_5)(F_N)(F_{DN})\left(\frac{NO_5}{NO_s+NO_5}\right)$$

where:

$$R_{S_sU}=k_{sto}\left(\frac{S_{S(5)}/X_H}{k_{ss}+(S_{s(5)}/X_H)}\right)\left(\frac{k_{oh}}{k_{oh}+O_5}\right)\left(\frac{NO_5}{NO_s+NO_5}\right)(F_{DN})$$

Stage 6

$$V\frac{dN_6}{dt}=0=(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(N_5)-(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(N_6)$$

$$-\left(\frac{\mu_{m(NS)}}{Y_{NS}}\right)\frac{(X_{NS})(N_6)}{(K_N+N_6)}\left(\frac{O_6}{K_{0N}+O_6}\right)(V_6)$$

$$-Y_H[X_H(R_{S_sU})(V_6)](F_N)$$

$$-Y_H\left[(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(P_5)-(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(P_6)\right](F_N)$$

$$+(K_{d(h)})(X_H)(V_6)(F_N)\left(\frac{O_6}{k_{os}+O_6}\right)$$

where:

$$R_{S_sU}=k_{sto}\left(\frac{S_{S(6)}/X_H}{k_{ss}+(S_{s(6)}/X_H)}\right)\left(\frac{O_6}{k_{os}+O_6}\right)$$

Stage 7

$$V\frac{dN_7}{dt} = 0 = (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(N_6) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(N_7)$$

$$-\left(\frac{\mu_{m(NS)}}{Y_{NS}}\right)\frac{(X_{NS})(N_7)}{(K_N + N_7)}\left(\frac{O_7}{K_{0N} + O_7}\right)(V_7)$$

$$-Y_H[X_H(R_{S_sU})(V_7)](F_N)$$

$$-Y_H\left[(Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(P_6) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(P_7)\right](F_N)$$

$$+(K_{d(h)})(X_H)(V_7)(F_N)\left(\frac{O_7}{k_{os} + O_7}\right)$$

where:

$$R_{S_sU} = k_{sto}\left(\frac{S_{S(7)}/X_H}{k_{ss} + (S_{s(7)}/X_H)}\right)\left(\frac{O_7}{k_{os} + O_7}\right)$$

Stage 8

$$V\frac{dN_8}{dt} = 0 = (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(N_7) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(N_8)$$

$$-\left(\frac{\mu_{m(NS)}}{Y_{NS}}\right)\frac{(X_{NS})(N_8)}{(K_N + N_8)}\left(\frac{O_8}{K_{0N} + O_8}\right)(V_8)$$

$$-Y_H[X_H(R_{S_sU})(V_8)](F_N)$$

$$-Y_H\left[(Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(P_7) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(P_8)\right](F_N)$$

$$+(K_{d(h)})(X_H)(V_8)(F_N)\left(\frac{O_8}{k_{os}+O_8}\right)$$

where:

$$R_{S_sU}=k_{sto}\left(\frac{S_{S(8)}/X_H}{k_{ss}+(S_{s(8)}/X_H)}\right)\left(\frac{O_8}{k_{os}+O_8}\right)$$

Stage 9

$$V\frac{dN_9}{dt}=0=(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(N_8)-(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(N_9)$$

$$-\left(\frac{\mu_{m(NS)}}{Y_{NS}}\right)\frac{(X_{NS})(N_9)}{(K_N+N_9)}\left(\frac{O_9}{K_{0N}+O_9}\right)(V_9)$$

$$-Y_H[X_H(R_{S_sU})(V_9)](F_N)$$

$$-Y_H\left[(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(P_8)-(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(P_9)\right](F_N)$$

$$+(K_{d(h)})(X_H)(V_9)(F_N)\left(\frac{O_9}{k_{os}+O_9}\right)$$

where:

$$R_{S_sU}=k_{sto}\left(\frac{S_{S(9)}/X_H}{k_{ss}+(S_{s(9)}/X_H)}\right)\left(\frac{O_9}{k_{os}+O_9}\right)$$

Stage 10

$$V\frac{dN_{10}}{dt} = 0 = (Q_0 + Q_{RAS})(N_9) - (Q_0 + Q_{RAS})(N_{10})$$

$$-\left(\frac{\mu_{m(NS)}}{Y_{NS}}\right)\frac{(X_{NS})(N_{10})}{(K_N + N_{10})}\left(\frac{O_{10}}{K_{0N} + O_{10}}\right)(V_{10})$$

$$-Y_H[X_H(R_{S_sU})(V_{10})](F_N)$$

$$-Y_H\left[(Q_0 + Q_{RAS})(P_9) - (Q_0 + Q_{RAS})(P_{10})\right](F_N)$$

$$+(K_{d(h)})(X_H)(V_{10})(F_N)\left(\frac{O_{10}}{k_{os} + O_{10}}\right)$$

where:

$$R_{S_sU} = k_{sto}\left(\frac{S_{S(10)}/X_H}{k_{ss} + (S_{s(10)}/X_H)}\right)\left(\frac{O_{10}}{k_{os} + O_{10}}\right)$$

Nitrate (NO_3-N) Mass Balances

Stage 1

$$V\frac{dNO_1}{dt} = 0 = (Q_0)(NO_0) + (Q_{RAS})(0.1)(NO_{10}) + (Q_{ir(1)})(NO_9) - (Q_0 + Q_{RAS} + Q_{ir(1)})(NO_1)$$

$$-\left(\frac{1-1.42(Y_H)}{2.86}\right)\left[(X_H)(R_{S_sU})(V_1)\right]$$

$$-\frac{1.42}{2.86}(k_d)(X_H)(F_{DN})\left(\frac{k_{oh}}{k_{oh} + O_1}\right)\left(\frac{NO_1}{NO_s + NO_1}\right)V_1$$

$$-\left(\frac{1-1.42Y_H}{2.86}\right)F_{DN}(K_P)(P_1)X_H\left(\frac{k_{oh}}{k_{oh} + O_1}\right)\left(\frac{NO_1}{NO_s + NO_1}\right)V_1$$

where:

$$R_{S_sU} = k_{sto}\left(\frac{S_{S(1)}/X_H}{k_{ss}+(S_{S(1)}/X_H)}\right)\left(\frac{k_{oh}}{k_{oh}+O_1}\right)\left(\frac{NO_1}{NO_s+NO_1}\right)(F_{DN})$$

Stage 2

$$V\frac{dNO_2}{dt} = 0 = (Q_0+Q_{ras}+Q_{ir(1)})(NO_1)-(Q_0+Q_{RAS}+Q_{ir(1)})(NO_2)$$

$$-\left(\frac{1-1.42(Y_H)}{2.86}\right)\left[(X_H)(R_{S_sU})(V_2)\right]$$

$$-\frac{1.42}{2.86}(k_d)(X_H)(F_{DN})\left(\frac{k_{oh}}{k_{oh}+O_2}\right)\left(\frac{NO_2}{NO_s+NO_2}\right)V_2$$

$$-\left(\frac{1-1.42Y_H}{2.86}\right)F_{DN}(K_P)(P_2)X_H\left(\frac{k_{oh}}{k_{oh}+O_2}\right)\left(\frac{NO_2}{NO_s+NO_2}\right)V_2$$

where:

$$R_{S_sU} = k_{sto}\left(\frac{S_{S(2)}/X_H}{k_{ss}+(S_{S(2)}/X_H)}\right)\left(\frac{k_{oh}}{k_{oh}+O_2}\right)\left(\frac{NO_2}{NO_s+NO_2}\right)(F_{DN})$$

Stage 3

$$V\frac{dNO_3}{dt} = 0 = (Q_0 + Q_{ras} + Q_{ir(1)})(NO_2) + (Q_{ir(3)})(NO_9) - (Q_0 + Q_{RAS} + Q_{ir(1)})(NO_3)$$

$$-\left(\frac{1-1.42(Y_H)}{2.86}\right)\left[(X_H)(R_{S_SU})(V_3)\right]$$

$$-\frac{1.42}{2.86}(k_d)(X_H)(F_{DN})\left(\frac{k_{oh}}{k_{oh}+O_3}\right)\left(\frac{NO_3}{NO_s+NO_3}\right)V_3$$

$$-\left(\frac{1-1.42Y_H}{2.86}\right)F_{DN}(K_P)(P_3)X_H\left(\frac{k_{oh}}{k_{oh}+O_3}\right)\left(\frac{NO_3}{NO_s+NO_3}\right)V_3$$

where:

$$R_{S_SU} = k_{sto}\left(\frac{S_{S(3)}/X_H}{k_{ss}+(S_{S(3)}/X_H)}\right)\left(\frac{k_{oh}}{k_{oh}+O_3}\right)\left(\frac{NO_3}{NO_s+NO_3}\right)(F_{DN})$$

Stage 4

$$V\frac{dNO_4}{dt} = 0 = (Q_0 + Q_{ras} + Q_{ir(1)} + Q_{ir(3)})(NO_3) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(NO_4)$$

$$-\left(\frac{1-1.42(Y_H)}{2.86}\right)\left[(X_H)(R_{S_SU})(V_4)\right]$$

$$-\frac{1.42}{2.86}(k_d)(X_H)(F_{DN})\left(\frac{k_{oh}}{k_{oh}+O_4}\right)\left(\frac{NO_4}{NO_s+NO_4}\right)V_4$$

$$-\left(\frac{1-1.42Y_H}{2.86}\right)F_{DN}(K_P)(P_4)X_H\left(\frac{k_{oh}}{k_{oh}+O_4}\right)\left(\frac{NO_4}{NO_s+NO_4}\right)V_4$$

where:

$$R_{S_SU} = k_{sto}\left(\frac{S_{S(4)}/X_H}{k_{ss}+(S_{S(4)}/X_H)}\right)\left(\frac{k_{oh}}{k_{oh}+O_4}\right)\left(\frac{NO_4}{NO_s+NO_4}\right)(F_{DN})$$

Stage 5

$$V\frac{dNO_5}{dt} = 0 = (Q_0+Q_{ras}+Q_{ir(1)}+Q_{ir(3)})(NO_4)-(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(NO_5)$$

$$-\left(\frac{1-1.42(Y_H)}{2.86}\right)\left[(X_H)(R_{S_SU})(V_5)\right]$$

$$-\frac{1.42}{2.86}(k_d)(X_H)(F_{DN})\left(\frac{k_{oh}}{k_{oh}+O_5}\right)\left(\frac{NO_5}{NO_s+NO_5}\right)V_5$$

$$-\left(\frac{1-1.42Y_H}{2.86}\right)F_{DN}(K_P)(P_5)X_H\left(\frac{k_{oh}}{k_{oh}+O_5}\right)\left(\frac{NO_5}{NO_s+NO_5}\right)V_5$$

where:

$$R_{S_SU} = k_{sto}\left(\frac{S_{S(5)}/X_H}{k_{ss}+(S_{S(5)}/X_H)}\right)\left(\frac{k_{oh}}{k_{oh}+O_5}\right)\left(\frac{NO_5}{NO_s+NO_5}\right)(F_{DN})$$

Stage 6

$$V\frac{dNO_6}{dt} = 0 = (Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(NO_5)-(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(NO_6)$$

$$+\left(\frac{\mu_{m(NB)}}{Y_{NB}}\right)\left(\frac{(X_{NB})(NO2_6)}{K_N+NO2_6}\right)\left(\frac{O_6}{k_{on}+O_6}\right)V_6$$

$$-\left(\frac{1-1.42(Y_H)}{2.86}\right)\left[(X_H)(R_{S_SU})(V_6)\right]$$

$$-\frac{1.42}{2.86}(k_d)(X_H)(F_{DN})\left(\frac{k_{oh}}{k_{oh}+O_6}\right)\left(\frac{NO_6}{NO_s+NO_6}\right)V_6$$

$$-\left(\frac{1-1.42Y_H}{2.86}\right)F_{DN}(K_P)(P_6)X_H\left(\frac{k_{oh}}{k_{oh}+O_6}\right)\left(\frac{NO_6}{NO_s+NO_6}\right)V_6$$

where:

$$R_{S_sU}=k_{sto}\left(\frac{S_{S(6)}/X_H}{k_{ss}+(S_{S(6)}/X_H)}\right)\left(\frac{k_{oh}}{k_{oh}+O_6}\right)\left(\frac{NO_6}{NO_s+NO_6}\right)(F_{DN})$$

Stage 7

$$V\frac{dNO_7}{dt}=0=(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(NO_6)-(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(NO_7)$$

$$+\left(\frac{\mu_{m(NB)}}{Y_{NB}}\right)\left(\frac{(X_{NB})(NO2_7)}{K_N+NO2_7}\right)\left(\frac{O_7}{k_{on}+O_7}\right)V_7$$

$$-\left(\frac{1-1.42(Y_H)}{2.86}\right)\left[(X_H)(R_{S_sU})(V_7)\right]$$

$$-\frac{1.42}{2.86}(k_d)(X_H)(F_{DN})\left(\frac{k_{oh}}{k_{oh}+O_7}\right)\left(\frac{NO_7}{NO_s+NO_7}\right)V_7$$

$$-\left(\frac{1-1.42Y_H}{2.86}\right)F_{DN}(K_P)(P_7)X_H\left(\frac{k_{oh}}{k_{oh}+O_7}\right)\left(\frac{NO_7}{NO_s+NO_7}\right)V_7$$

where:

$$R_{S_SU} = k_{sto}\left(\frac{S_{S(7)}/X_H}{k_{ss}+(S_{S(7)}/X_H)}\right)\left(\frac{k_{oh}}{k_{oh}+O_7}\right)\left(\frac{NO_7}{NO_s+NO_7}\right)(F_{DN})$$

Stage 8

$$V\frac{dNO_8}{dt} = 0 = (Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(NO_7)-(Q_0+Q_{RAS}+Q_{ir(1)}+Q_{ir(3)})(NO_8)$$

$$+\left(\frac{\mu_{m(NB)}}{Y_{NB}}\right)\left(\frac{(X_{NB})(NO2_8)}{K_N+NO2_8}\right)\left(\frac{O_8}{k_{on}+O_8}\right)V_8$$

$$-\left(\frac{1-1.42(Y_H)}{2.86}\right)\left[(X_H)(R_{S_SU})(V_8)\right]$$

$$-\frac{1.42}{2.86}(k_d)(X_H)(F_{DN})\left(\frac{k_{oh}}{k_{oh}+O_8}\right)\left(\frac{NO_8}{NO_s+NO_8}\right)V_8$$

$$-\left(\frac{1-1.42Y_H}{2.86}\right)F_{DN}(K_P)(P_8)X_H\left(\frac{k_{oh}}{k_{oh}+O_8}\right)\left(\frac{NO_8}{NO_s+NO_8}\right)V_8$$

where:

$$R_{S_SU} = k_{sto}\left(\frac{S_{S(8)}/X_H}{k_{ss}+(S_{S(8)}/X_H)}\right)\left(\frac{k_{oh}}{k_{oh}+O_8}\right)\left(\frac{NO_8}{NO_s+NO_8}\right)(F_{DN})$$

Stage 9

$$V\frac{dNO_9}{dt} = 0 = (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(NO_8) - (Q_0 + Q_{RAS} + Q_{ir(1)} + Q_{ir(3)})(NO_9)$$

$$+\left(\frac{\mu_{m(NB)}}{Y_{NB}}\right)\left(\frac{(X_{NB})(NO2_9)}{K_N + NO2_9}\right)\left(\frac{O_9}{k_{on} + O_9}\right)V_9$$

$$-\left(\frac{1-1.42(Y_H)}{2.86}\right)\left[(X_H)(R_{S_sU})(V_9)\right]$$

$$-\frac{1.42}{2.86}(k_d)(X_H)(F_{DN})\left(\frac{k_{oh}}{k_{oh} + O_9}\right)\left(\frac{NO_9}{NO_s + NO_9}\right)V_9$$

$$-\left(\frac{1-1.42Y_H}{2.86}\right)F_{DN}(K_P)(P_9)X_H\left(\frac{k_{oh}}{k_{oh} + O_9}\right)\left(\frac{NO_9}{NO_s + NO_9}\right)V_9$$

where:

$$R_{S_sU} = k_{sto}\left(\frac{S_{S(9)}/X_H}{k_{ss} + (S_{S(9)}/X_H)}\right)\left(\frac{k_{oh}}{k_{oh} + O_9}\right)\left(\frac{NO_9}{NO_s + NO_9}\right)(F_{DN})$$

Stage 10

$$V\frac{dNO_{10}}{dt} = 0 = (Q_0 + Q_{RAS})(NO_9) - (Q_0 + Q_{RAS})(NO_{10})$$

$$+\left(\frac{\mu_{m(NB)}}{Y_{NB}}\right)\left(\frac{(X_{NB})(NO2_{10})}{K_N + NO2_{10}}\right)\left(\frac{O_{10}}{k_{on} + O_{10}}\right)V_{10}$$

$$-\left(\frac{1-1.42(Y_H)}{2.86}\right)\left[(X_H)(R_{S_sU})(V_{10})\right]$$

$$-\frac{1.42}{2.86}(k_d)(X_H)(F_{DN})\left(\frac{k_{oh}}{k_{oh}+O_{10}}\right)\left(\frac{NO_{10}}{NO_s+NO_{10}}\right)V_{10}$$

$$-\left(\frac{1-1.42Y_H}{2.86}\right)F_{DN}(K_P)(P_{10})X_H\left(\frac{k_{oh}}{k_{oh}+O_{10}}\right)\left(\frac{NO_{10}}{NO_s+NO_{10}}\right)V_{10}$$

where:

$$R_{S_SU}=k_{sto}\left(\frac{S_{S(10)}/X_H}{k_{ss}+(S_{S(10)}/X_H)}\right)\left(\frac{k_{oh}}{k_{oh}+O_{10}}\right)\left(\frac{NO_{10}}{NO_s+NO_{10}}\right)(F_{DN})$$

Nitrite (NO_2^-) Mass Balances

(see mass balance equations for nitrite listed previously for Michaelis-Menton Model)

PCOD Mass Balances

(see mass balance equations for PCOD listed previously for Michaelis-Menton Model)

Oxygen Consumption

Stage 6

Dissolved Oxygen Consumption Rate (Kg O_2 / hr) per stage

$$=\left[(A)(r_{su}+1.42(K_d)(X_H)\right](V_6)+(3.22)(r_{NH_4(ox)})(V_6)+(1.11)(r_{NO_2(ox)})(V_6)$$

where:

$$R_{S_sU}=k_{sto}\left(\frac{S_{S(6)}/X_H}{k_{ss}+(S_{s(6)}/X_H)}\right)\left(\frac{O_6}{k_{os}+O_6}\right)+k_P(P_6)(X_H)\left(\frac{O_6}{k_{os}+O_6}\right)$$

$$A=1-1.42(Y_H)$$

$$r_{NH_4(ox)}=\left(\frac{\mu_{m(NS)}}{Y_{NB}}\right)\left(\frac{X_{NS}(N_6)}{K_N+N_6}\right)\left(\frac{O_6}{O_6+k_{on}}\right)$$

$$r_{NO_2(ox)}=\left(\frac{\mu_{m(NB)}}{Y_{NB}}\right)\left(\frac{(X_{NB})(NO2_6)}{K_N+NO2_6}\right)\left(\frac{O_6}{O_6+k_{on}}\right)$$

Stage 7

Dissolved Oxygen Consumption Rate (Kg O_2 / hr) per stage

$$=\left[(A)(r_{su}+1.42(K_d)(X_H)\right](V_7)+(3.22)(r_{NH_4(ox)})(V_7)+(1.11)(r_{NO_2(ox)})(V_7)$$

where:

$$R_{S_sU} = k_{sto}\left(\frac{S_{S(7)}/X_H}{k_{ss}+(S_{s(7)}/X_H)}\right)\left(\frac{O_7}{k_{os}+O_7}\right)+k_P(P_7)(X_H)\left(\frac{O_7}{k_{os}+O_7}\right)$$

$$A = 1-1.42(Y_H)$$

$$r_{NH_4(ox)} = \left(\frac{\mu_{m(NS)}}{Y_{NB}}\right)\left(\frac{X_{NS}(N_7)}{K_N+N_7}\right)\left(\frac{O_7}{O_7+k_{on}}\right)$$

$$r_{NO_2(ox)} = \left(\frac{\mu_{m(NB)}}{Y_{NB}}\right)\left(\frac{(X_{NB})(NO2_7)}{K_N+NO2_7}\right)\left(\frac{O_7}{O_7+k_{on}}\right)$$

Stage 8

Dissolved Oxygen Consumption Rate (Kg O_2 / hr) per stage

$$=\left[(A)(r_{su}+1.42(K_d)(X_H)\right](V_8)+(3.22)(r_{NH_4(ox)})(V_8)+(1.11)(r_{NO_2(ox)})(V_8)$$

where:

$$R_{S_sU} = k_{sto}\left(\frac{S_{S(8)}/X_H}{k_{ss}+(S_{s(8)}/X_H)}\right)\left(\frac{O_8}{k_{os}+O_8}\right)+k_P(P_8)(X_H)\left(\frac{O_8}{k_{os}+O_8}\right)$$

$$A = 1-1.42(Y_H)$$

$$r_{NH_4(ox)} = \left(\frac{\mu_{m(NS)}}{Y_{NB}}\right)\left(\frac{X_{NS}(N_8)}{K_N+N_8}\right)\left(\frac{O_8}{O_8+k_{on}}\right)$$

$$r_{NO_2(ox)} = \left(\frac{\mu_{m(NB)}}{Y_{NB}}\right)\left(\frac{(X_{NB})(NO2_8)}{K_N+NO2_8}\right)\left(\frac{O_8}{O_8+k_{on}}\right)$$

Stage 9

Dissolved Oxygen Consumption Rate (Kg O_2 / hr) per stage

$$=\left[(A)(r_{su}+1.42(K_d)(X_H)\right](V_9)+(3.22)(r_{NH_4(ox)})(V_9)+(1.11)(r_{NO_2(ox)})(V_9)$$

where:

$$R_{S_sU}=k_{sto}\left(\frac{S_{S(9)}/X_H}{k_{ss}+(S_{s(9)}/X_H)}\right)\left(\frac{O_9}{k_{os}+O_9}\right)+k_P(P_9)(X_H)\left(\frac{O_9}{k_{os}+O_9}\right)$$

$$A=1-1.42(Y_H)$$

$$r_{NH_4(ox)}=\left(\frac{\mu_{m(NS)}}{Y_{NB}}\right)\left(\frac{X_{NS}(N_9)}{K_N+N_9}\right)\left(\frac{O_9}{O_9+k_{on}}\right)$$

$$r_{NO_2(ox)}=\left(\frac{\mu_{m(NB)}}{Y_{NB}}\right)\left(\frac{(X_{NB})(NO2_9)}{K_N+NO2_9}\right)\left(\frac{O_9}{O_9+k_{on}}\right)$$

Stage 10

Dissolved Oxygen Consumption Rate (Kg O_2 / hr) per stage

$$=\left[(A)(r_{su}+1.42(K_d)(X_H)\right](V_{10})+(3.22)(r_{NH_4(ox)})(V_{10})+(1.11)(r_{NO_2(ox)})(V_{10})$$

where:

$$R_{S_sU}=k_{sto}\left(\frac{S_{S(10)}/X_H}{k_{ss}+(S_{s(10)}/X_H)}\right)\left(\frac{O_{10}}{k_{os}+O_{10}}\right)+k_P(P_{10})(X_H)\left(\frac{O_{10}}{k_{os}+O_{10}}\right)$$

$$A=1-1.42(Y_H)$$

$$r_{NH_4(ox)} = \left(\frac{\mu_{m(NS)}}{Y_{NB}}\right)\left(\frac{X_{NS}(N_{10})}{K_N + N_{10}}\right)\left(\frac{O_{10}}{O_{10} + k_{on}}\right)$$

$$r_{NO_2(ox)} = \left(\frac{\mu_{m(NB)}}{Y_{NB}}\right)\left(\frac{(X_{NB})(NO2_{10})}{K_N + NO2_{10}}\right)\left(\frac{O_{10}}{O_{10} + k_{on}}\right)$$

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