A note on typing variables and references

Volpano, Dennis M.; Smith, Geoffrey.

Monterey, California. Naval Postgraduate School

NPS-CS-96-003

NAVAL POSTGRADUATE SCHOOL
Monterey, California

A Note on Typing Variables and References

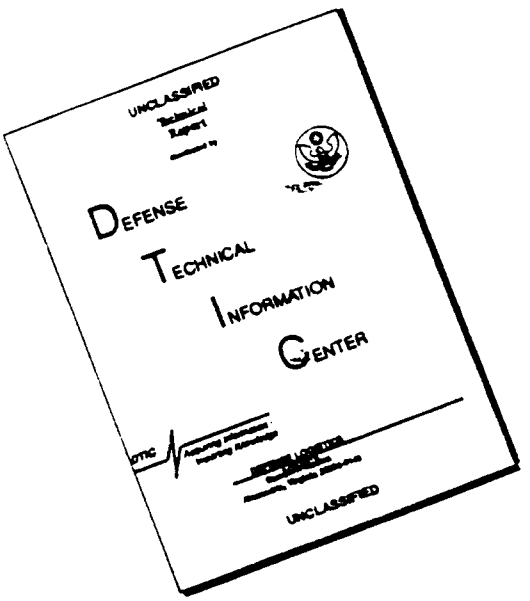
by

Dennis Volpano
Geoffrey Smith

February 1996

Prepared for: Naval Postgraduate School
Monterey, CA 93943

DISCLAIMER NOTICE



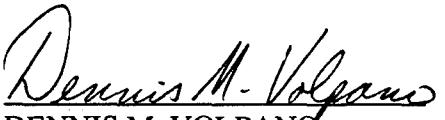
THIS DOCUMENT IS BEST QUALITY AVAILABLE. THE COPY FURNISHED TO DTIC CONTAINED A SIGNIFICANT NUMBER OF PAGES WHICH DO NOT REPRODUCE LEGIBLY.

NAVAL POSTGRADUATE SCHOOL
Monterey, California

Rear Admiral M. J. Evans
Superintendent

Richard Elster
Provost

This report was prepared as part of the Advanced Type Systems Project at the Naval Postgraduate School, which is currently funded by the National Science Foundation under Agreement No. CCR-9400592 and CCR-9414421. Any opinions, findings , and conclusions or recommendations expressed in this report are those of the author and do not necessarily reflect the views of the National Science Foundation.



DENNIS M. VOLPANO
Assistant Professor
Department of Computer Science

Reviewed by:

YUTAKA KANAYAMA
Professor
Department of Computer Science

Released by:

TED LEWIS
Chairman
Department of Computer Science

GORDON SCHACHER
Acting Dean of Research

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION: UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT: Approved for public release; distribution is unlimited
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S): NPSCS-96 -003	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION: Computer Science Dept. Naval Postgraduate School; 6b. OFFICE SYMBOL (if applicable): CS	7a. NAME OF MONITORING ORGANIZATION
6c. ADDRESS (City, State, and ZIP Code): Monterey, CA 93943	7b. ADDRESS (City, State, and ZIP Code)
8a. NAME OF FUNDING/SPONSORING ORGANIZATION: National Science Foundation; 8b. OFFICE SYMBOL (if applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c. ADDRESS (City, State, and ZIP Code): 4201 Wilson Blvd, Arlington, VA 22230	10. SOURCE OF FUNDING NUMBERS: PROGRAM ELEMENT NO. / PROJECT NO. / TASK NO. / WORK UNIT ACCESSION NO.

11. TITLE (Include Security Classification): A Note on Typing Variables and References

12. PERSONAL AUTHOR(S): Dennis Volpano and Geoffrey Smith

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Final	FROM _____ TO _____	February 8, 1996	7

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	polymorphic types, references, variables, and weak types

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

We consider the polymorphic typing of variables and references with C's address-of operator '&' in the context of nonweak types. A natural semantics and type system are given for a polymorphically-typed imperative language with first class functions. The type system is proved sound with respect to the natural semantics.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT: [X] UNCLASSIFIED/UNLIMITED [ ] SAME AS RPT. [ ] DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION: UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL: Dennis Volpano, Assistant Professor	22b. TELEPHONE (Include Area Code): (408) 656-3091; 22c. OFFICE SYMBOL: CS/ Vo

A Note on Typing Variables and References

Dennis Volpano
Department of Computer Science
Naval Postgraduate School
Monterey, CA 93943
volpano@cs.nps.navy.mil

Geoffrey Smith
School of Computer Science
Florida International University
Miami, FL 33199
smithg@fiu.edu

February 8, 1996

Polymorphic typing of variables and references is considered in [1]. However, a treatment of the address-of operator '&' in the context of nonweak types is not given. The operator is treated in [2] but only in the context of weak types, since every type in that system is weak. In this note, the semantics and subject reduction theorem of [1] are reformulated in order to accomodate '&' in the presence of nonweak types.

The syntax of the language in [1] is extended as follows:

(*Expressions*) $e ::= \&e \mid l.1$

(*Values*) $v ::= l.0$

Meta-variable l ranges over locations. We say $l.1$ is a *variable* and $l.0$ is a *reference*. Unlike references, variables are not values. Variables and references replace variable locations and reference locations respectively in the syntax of [1].

Typing rules (REFLOC) and (VARLOC) of [1] are changed and a typing rule for '&' is added—see Figure 1. The domain of a location typing is no longer partitioned into variable and reference locations.

Some changes are needed in the evaluation rules. These changes are reflected in the new rules given in Figure 2.

We now turn to subject reduction. First, we introduce some lemmas:

Lemma 1 (Superfluousness) *If* $\lambda;\gamma \vdash e : \tau$ *and* $l \notin dom(\lambda)$, *then* $\lambda[l : \tau'];\gamma \vdash e : \tau$.

Lemma 2 (Substitution) *If* $\lambda;\gamma \vdash v : \sigma$ *and* $\lambda;\gamma[x : \sigma] \vdash e : \tau$, *then* $\lambda;\gamma \vdash [v/x]e : \tau$. *Also, if* $\lambda;\gamma \vdash l.1 : \tau\ var$ *and* $\lambda;\gamma[x : \tau\ var] \vdash e : \tau'$, *then* $\lambda;\gamma \vdash [l.1/x]e : \tau'$.

$$\text{(REFLOC)} \quad \lambda;\gamma \vdash l.0 : \tau\ ref \quad \lambda(l) = \tau$$

$$\text{(VARLOC)} \quad \lambda;\gamma \vdash l.1 : \tau\ var \quad \lambda(l) = \tau$$

$$\text{(ADDRESS)} \quad \frac{\lambda;\gamma \vdash e : \tau\ var, \quad \tau \text{ is weak}}{\lambda;\gamma \vdash \&e : \tau\ ref}$$

Figure 1: New Rules of the Type System

(CONTENTS) $\mu \vdash l.1 \Rightarrow \mu(l), \mu$

(BINDVAR)
$$\frac{\mu \vdash e_1 \Rightarrow v_1, \mu_1 \quad l \notin dom(\mu_1) \quad \mu_1[l := v_1] \vdash [l.1/x]e_2 \Rightarrow v_2, \mu_2}{\mu \vdash \textbf{letvar } x := e_1 \textbf{ in } e_2 \Rightarrow v_2, \mu_2}$$

(UPDATE)
$$\frac{\mu \vdash e \Rightarrow v, \mu'}{\mu \vdash l.1 := e \Rightarrow \textbf{unit}, \mu'[l := v]}$$

$$\frac{\mu \vdash e_1 \Rightarrow l.0, \mu_1 \quad \mu_1 \vdash e_2 \Rightarrow v, \mu_2}{\mu \vdash *e_1 := e_2 \Rightarrow \textbf{unit}, \mu_2[l := v]}$$

(ADDROF) $\mu \vdash \& l.1 \Rightarrow l.0, \mu$

$$\frac{\mu \vdash e \Rightarrow l.0, \mu'}{\mu \vdash \& * e \Rightarrow l.0, \mu'}$$

(ALLOC)
$$\frac{\mu \vdash e \Rightarrow v, \mu' \quad l \notin dom(\mu')}{\mu \vdash \textbf{ref } e \Rightarrow l.0, \mu'[l := v]}$$

(DEREF)
$$\frac{\mu \vdash e \Rightarrow l.0, \mu'}{\mu \vdash *e \Rightarrow \mu'(l), \mu'}$$

Figure 2: The New Evaluation Rules

Lemma 3 ($\forall$-intro) *If $\lambda; \gamma \vdash e : \sigma$ and α does not occur free in λ or in γ, then $\lambda; \gamma \vdash e : \forall \alpha \,.\, \sigma$.*

Lemma 4 *If $\lambda[l : \tau]; \gamma \vdash e : \tau'$ and l does not occur in e, then $\lambda; \gamma \vdash e : \tau'$.*

The preceding lemmas are straightforward variants of those in [1].

The subject reduction theorem now becomes:

Theorem 5 *Suppose that $\mu \vdash e \Rightarrow v, \mu'$, $\lambda \vdash e : \tau$, $\mu : \lambda$, and $\lambda(l)$ is weak if $l.1$ occurs in the range of μ or in a λ-abstraction in e, or $l.0$ occurs in the range of μ or in e. Then there exists a λ' such that $\lambda \subseteq \lambda'$, $\mu' : \lambda'$, $\lambda' \vdash v : \tau$, and $\lambda'(l)$ is weak if $l.1$ or $l.0$ occurs in the range of μ' or in v.*

Proof. The proof is by induction on the structure of the derivation of $\mu \vdash e \Rightarrow v, \mu'$.

For brevity, we present only the interesting cases: (BIND), when e_1 is not a value, and the evaluation rules of Figure 2.

(BIND). Suppose e_1 is not a value. Then the evaluation must end with

$$\frac{\mu \vdash e_1 \Rightarrow v_1, \mu_1 \qquad \mu_1 \vdash [v_1/x]e_2 \Rightarrow v_2, \mu_2}{\mu \vdash \textbf{let } x = e_1 \textbf{ in } e_2 \Rightarrow v_2, \mu_2}$$

while the typing must end with

$$\frac{\lambda \vdash e_1 : \tau_1 \qquad \lambda; [x : \mathit{AppClose}_\lambda(\tau_1)] \vdash e_2 : \tau_2}{\lambda \vdash \textbf{let } x = e_1 \textbf{ in } e_2 : \tau_2}$$

Also, $\mu : \lambda$ and $\lambda(l)$ is weak if either $l.1$ occurs in the range of μ or in a λ-abstraction in e_1 or e_2, or $l.0$ occurs in the range of μ or in e_1 or e_2.

By induction, there exists a λ_1 such that $\lambda \subseteq \lambda_1$, $\mu_1 : \lambda_1$, $\lambda_1 \vdash v_1 : \tau_1$, and $\lambda_1(l)$ is weak if $l.1$ or $l.0$ occurs in the range of μ_1 or in v_1.

Now to apply induction again we want to show that

$$\lambda_1 \vdash [v_1/x]e_2 : \tau_2.$$

By Lemma 1 we have

$$\lambda_1; [x : \mathit{AppClose}_\lambda(\tau_1)] \vdash e_2 : \tau_2,$$

so we can apply Lemma 2 to get what we want provided that we can show

$$\lambda_1 \vdash v_1 : \mathit{AppClose}_\lambda(\tau_1).$$

Now, applying Lemma 3 to $\lambda_1 \vdash v_1 : \tau_1$ we can get $\lambda_1 \vdash v_1 : \mathit{AppClose}_{\lambda_1}(\tau_1)$, but this is not good enough, because λ_1 may contain free strong type variables that are not free in λ. To proceed, we exploit our knowledge about what locations can occur in v_1.

Let λ_1^- be formed by removing from λ_1 any typings $l : \tau$ such that τ is not weak. By the above use of induction, this process does not remove any typings of locations that occur in v_1, as all such locations have weak types. So by Lemma 4, $\lambda_1^- \vdash v_1 : \tau_1$. Hence, by Lemma 3, $\lambda_1^- \vdash v_1 : \mathit{AppClose}_\lambda(\tau_1)$, since λ_1^- contains no strong type

variables. Lemma 1 then gives $\lambda_1 \vdash v_1 : AppClose_\lambda(\tau_1)$, and finally by Lemma 2 we get $\lambda_1 \vdash [v_1/x]e_2 : \tau_2$.

By the use of induction above, $\lambda_1(l)$ is weak if $l.1$ or $l.0$ occurs in the range of μ_1. If a variable $l.1$ occurs in a λ-abstraction in $[v_1/x]e_2$, then either it occurs in v_1 or in a λ-abstraction in e_2. In the first case, $\lambda_1(l)$ is weak by the above use of induction; in the second case, $\lambda(l)$ is weak by the hypothesis, and so $\lambda_1(l)$ is weak since $\lambda \subseteq \lambda_1$. Furthermore, if a reference $l.0$ occurs in $[v_1/x]e_2$, then either it occurs in v_1 or e_2. In the former case, $\lambda_1(l)$ is weak by the above use of induction, and in the latter, $\lambda(l)$ is weak by the hypothesis, and so $\lambda_1(l)$ is weak.

Hence we can use induction a second time to show that there exists a λ' such that $\lambda_1 \subseteq \lambda'$, $\mu_2 : \lambda'$, $\lambda' \vdash v_2 : \tau_2$, and $\lambda'(l)$ is weak if $l.1$ or $l.0$ occurs in the range of μ_2 or in v_2. Since $\lambda \subseteq \lambda_1 \subseteq \lambda'$, we are done.

(BINDVAR). The evaluation must end with

$$\frac{\begin{array}{l}\mu \vdash e_1 \Rightarrow v_1, \mu_1 \\ l \notin dom(\mu_1) \\ \mu_1[l := v_1] \vdash [l.1/x]e_2 \Rightarrow v_2, \mu_2\end{array}}{\mu \vdash \textbf{letvar } x := e_1 \textbf{ in } e_2 \Rightarrow v_2, \mu_2}$$

while the typing must end with

$$\frac{\begin{array}{l}\lambda \vdash e_1 : \tau_1 \\ \lambda; [x : \tau_1\ var] \vdash e_2 : \tau_2 \\ \text{If } x \text{ occurs in a } \lambda\text{-abstraction in } e_2 \text{ then } \tau_1 \text{ is weak.}\end{array}}{\lambda \vdash \textbf{letvar } x := e_1 \textbf{ in } e_2 : \tau_2}$$

Also, $\mu : \lambda$ and $\lambda(l')$ is weak if either $l'.1$ occurs in the range of μ or in a λ-abstraction in e_1 or e_2, or $l'.0$ occurs in the range of μ or in e_1 or e_2.

By induction, there exists a λ_1 such that $\lambda \subseteq \lambda_1$, $\mu_1 : \lambda_1$, $\lambda_1 \vdash v_1 : \tau_1$, and $\lambda_1(l')$ is weak if $l'.1$ or $l'.0$ occurs in the range of μ_1 or in v_1.

Since $l \notin dom(\lambda_1)$, $\lambda_1 \subseteq \lambda_1[l : \tau_1]$.

Since $\lambda_1[l : \tau_1] \vdash l.1 : \tau_1\ var$ and (by Lemma 1) $\lambda_1[l : \tau_1]; [x : \tau_1\ var] \vdash e_2 : \tau_2$, we can apply Lemma 2 to get

$$\lambda_1[l : \tau_1] \vdash [l.1/x]e_2 : \tau_2$$

Also, $\mu_1[l := v_1] : \lambda_1[l : \tau_1]$ by Lemma 1.

Next, by the use of induction above, $\lambda_1(l')$ is weak if $l'.1$ or $l'.0$ occurs in the range of $\mu_1[l := v_1]$. Thus, $\lambda_1[l : \tau_1](l')$ is weak since $\lambda_1 \subseteq \lambda_1[l : \tau_1]$. Now suppose that a variable $l'.1$ occurs in a λ-abstraction in $[l.1/x]e_2$. Then either $l'.1$ occurs in a λ-abstraction in e_2, or else $l' = l$ and x occurs in a λ-abstraction in e_2. In the first case, by the hypothesis, $\lambda(l')$ is weak and so $\lambda_1[l : \tau_1](l')$ is weak. In the second case, by the restriction on the (LETVAR) rule, τ_1 is weak, and so $\lambda_1[l : \tau_1](l')$ is weak. Finally, if $l'.0$ occurs in $[l.1/x]e_2$ then it occurs in e_2. Thus, by the hypothesis, $\lambda(l')$ is weak and so $\lambda_1[l : \tau_1](l')$ is weak.

So by a second use of induction, there exists a λ' such that $\lambda_1[l : \tau_1] \subseteq \lambda'$, $\mu_2 : \lambda'$, $\lambda' \vdash v_2 : \tau_2$, and $\lambda'(l')$ is weak if $l'.1$ or $l'.0$ occurs in the range of μ_2 or in v_2. Since $\lambda \subseteq \lambda_1 \subseteq \lambda_1[l : \tau_1] \subseteq \lambda'$, we are done.

(ADDROF). Suppose the evaluation ends with

$$\mu \vdash \& l.1 \Rightarrow l.0, \mu$$

while the typing ends with

$$\frac{\lambda \vdash l.1 : \tau\ var,\ \ \tau \text{ is weak}}{\lambda \vdash \&\, l.1 : \tau\ ref}$$

Also, $\mu : \lambda$ and $\lambda(l')$ is weak if either $l'.1$ or $l'.0$ occurs in the range of μ. Since $\lambda \vdash l.1 : \tau\ var$, we have $\lambda(l) = \tau$ by rule (VARLOC). Thus, $\lambda \vdash l.0 : \tau\ ref$ by (REFLOC). Furthermore, by the restriction on rule (ADDRESS), τ, or $\lambda(l)$, is weak.

Now suppose the evaluation ends with

$$\frac{\mu \vdash e \Rightarrow l.0, \mu'}{\mu \vdash \& * e \Rightarrow l.0, \mu'}$$

while the typing ends with

$$\frac{\dfrac{\lambda \vdash e : \tau\ ref}{\lambda \vdash *e : \tau\ var,\ \ \tau \text{ is weak}}}{\lambda \vdash \& * e : \tau\ ref}$$

Also, $\mu : \lambda$ and $\lambda(l')$ is weak if $l'.1$ occurs in the range of μ or in a λ-abstraction in e, or $l'.0$ occurs in the range of μ or in e.

By induction, there is a λ' such that $\lambda \subseteq \lambda'$, $\mu' : \lambda'$, $\lambda' \vdash l.0 : \tau\ ref$, $\lambda'(l)$ is weak, and $\lambda'(l')$ is weak if $l'.1$ or $l'.0$ occurs in the range of μ'. And, we're done.

(CONTENTS). The evaluation must end with

$$\mu \vdash l.1 \Rightarrow \mu(l), \mu$$

while the typing must end with

$$\frac{\lambda \vdash l.1 : \tau\ var}{\lambda \vdash l.1 : \tau}$$

Also, $\mu : \lambda$ and $\lambda(l')$ is weak if either $l'.1$ or $l'.0$ occurs in the range of μ. From $\mu : \lambda$, we have $\lambda \vdash \mu(l) : \lambda(l)$. Since $\lambda \vdash l.1 : \tau\ var$, we have $\lambda(l) = \tau$, so $\lambda \vdash \mu(l) : \tau$.

(UPDATE). Suppose the evaluation ends with

$$\frac{\mu \vdash e \Rightarrow v, \mu'}{\mu \vdash l.1 := e \Rightarrow \mathbf{unit}, \mu'[l := v]}$$

while the typing ends with

$$\frac{\begin{array}{l}\lambda \vdash l.1 : \tau\ var \\ \lambda \vdash e : \tau\end{array}}{\lambda \vdash l.1 := e : unit}$$

Also, $\mu : \lambda$ and $\lambda(l')$ is weak if $l'.1$ occurs in the range of μ or in a λ-abstraction in e, or $l'.0$ occurs in the range of μ or in e.

By induction, there exists a λ' such that $\lambda \subseteq \lambda'$, $\mu' : \lambda'$, $\lambda' \vdash v : \tau$, and $\lambda'(l')$ is weak if $l'.1$ or $l'.0$ occurs in the range of μ' or in v.

By rule (LIT), $\lambda' \vdash \mathbf{unit} : unit$. Since $\lambda \vdash l.1 : \tau\ var$, $\lambda(l) = \tau$ by (VARLOC). So $l \in dom(\lambda')$ since $\lambda \subseteq \lambda'$, and thus $dom(\mu'[l := v]) = dom(\lambda')$. If l' is a location such

that $l' \neq l$, then $\lambda' \vdash \mu'(l') : \lambda'(l')$ since $\mu' : \lambda'$. If $l' = l$ then $\mu'[l := v](l') = v$. So $\lambda' \vdash \mu'[l := v](l') : \tau$ since $\lambda' \vdash v : \tau$. Thus, $\mu'[l := v] : \lambda'$. Finally, by the above use of induction, $\lambda'(l')$ is weak if $l'.1$ or $l'.0$ occurs in the range of $\mu'[l := v]$.

Now suppose the evaluation ends with

$$\frac{\mu \vdash e_1 \Rightarrow l.0, \mu_1 \quad \mu_1 \vdash e_2 \Rightarrow v, \mu_2}{\mu \vdash *e_1 := e_2 \Rightarrow \mathbf{unit}, \mu_2[l := v]}$$

while the typing ends with

$$\frac{\lambda \vdash *e_1 : \tau\ var \quad \lambda \vdash e_2 : \tau}{\lambda \vdash *e_1 := e_2 : unit}$$

Also, $\mu : \lambda$ and $\lambda(l')$ is weak if $l'.1$ occurs in the range of μ or in a λ-abstraction in e_1 or e_2, or $l'.0$ occurs in the range of μ or in e_1 or e_2.

By rule (L-VAL), $\lambda \vdash e_1 : \tau\ ref$. By induction, there exists a λ_1 such that $\lambda \subseteq \lambda_1$, $\mu_1 : \lambda_1$, $\lambda_1 \vdash l.0 : \tau\ ref$, $\lambda_1(l)$ is weak, and $\lambda_1(l')$ is weak if $l'.1$ or $l'.0$ occurs in the range of μ_1. By Lemma 1, $\lambda_1 \vdash e_2 : \tau$. Suppose that a variable $l'.1$ occurs in a λ-abstraction in e_2. Then by the hypothesis, $\lambda(l')$ is weak and so is $\lambda_1(l')$ since $\lambda \subseteq \lambda_1$. Likewise, if $l'.0$ occurs in e_2, then $\lambda(l')$ is weak and thus so is $\lambda_1(l')$.

So by a second use of induction, there is a λ' such that $\lambda_1 \subseteq \lambda'$, $\mu_2 : \lambda'$, $\lambda' \vdash v : \tau$, and $\lambda'(l')$ is weak if $l'.1$ or $l'.0$ occurs in the range of μ_2 or in v. The proof is now similar to the first (UPDATE) case above.

(ALLOC). The evaluation must end with

$$\frac{\mu \vdash e \Rightarrow v, \mu' \quad l \notin dom(\mu')}{\mu \vdash \mathbf{ref}\, e \Rightarrow l.0, \mu'[l := v]}$$

while the typing ends with

$$\frac{\lambda \vdash e : \tau, \quad \tau \text{ is weak}}{\lambda \vdash \mathbf{ref}\, e : \tau\ ref}$$

Also, $\mu : \lambda$ and $\lambda(l')$ is weak if $l'.1$ occurs in the range of μ or in a λ-abstraction in e, or $l'.0$ occurs in the range of μ or in e.

By induction, there exists a λ' such that $\lambda \subseteq \lambda'$, $\mu' : \lambda'$, $\lambda' \vdash v : \tau$, and $\lambda'(l')$ is weak if $l'.1$ or $l'.0$ occurs in the range of μ' or in v.

Now $\lambda' \subseteq \lambda'[l : \tau]$ since $l \notin dom(\mu')$.

By Lemma 1 and the above use of induction, $\mu'[l := v] : \lambda'[l : \tau]$. Furthermore, $\lambda'[l : \tau] \vdash l.0 : \tau\ ref$ by rule (REFLOC). Again by the above use of induction, $\lambda'(l')$ is weak if $l'.1$ or $l'.0$ occurs in the range of $\mu'[l := v]$, and hence $\lambda'[l : \tau](l')$ is weak since $\lambda' \subseteq \lambda'[l : \tau]$. Finally, $\lambda'[l : \tau](l) = \tau$ and τ is weak by the restriction on rule (REF).

(DEREF). The evaluation must end with

$$\frac{\mu \vdash e \Rightarrow l.0, \mu'}{\mu \vdash *e \Rightarrow \mu'(l), \mu'}$$

while the typing ends with

$$\frac{\frac{\lambda \vdash e : \tau\ ref}{\lambda \vdash *e : \tau\ var}}{\lambda \vdash *e : \tau}$$

Also, $\mu : \lambda$ and $\lambda(l')$ is weak if $l'.1$ occurs in the range of μ or in a λ-abstraction in e, or $l'.0$ occurs in the range of μ or in e.

By induction, there exists a λ' such that $\lambda \subseteq \lambda'$, $\mu' : \lambda'$, $\lambda' \vdash l.0 : \tau\ ref$, $\lambda'(l)$ is weak, and $\lambda'(l')$ is weak if $l'.1$ or $l'.0$ occurs in the range of μ'.

Since $\lambda' \vdash l.0 : \tau\ ref$, $\lambda'(l) = \tau$ by rule (REFLOC). Now $\lambda' \vdash \mu'(l) : \lambda'(l)$, since $\mu' : \lambda'$, so $\lambda' \vdash \mu'(l) : \tau$. □

References

[1] Smith, G. and Volpano, D., Polymorphic Typing of Variables and References, to appear in *ACM Trans. on Prog. Lang. and Systems*, 1996.

[2] Smith, G. and Volpano, D., Towards an ML-style Polymorphic Type System for C, to appear at *1996 European Symp. on Prog.*, Linköping Sweden, 22-24 April 1996.

[3] Tofte, M., Type Inference for Polymorphic References, *Information and Computation*, 89, pp. 1–34, 1990.

Distribution List

Defense Technical Information Center Cameron Station Alexandria, VA 22314	2 copies
Library, Code 52 Naval Postgraduate School Monterey, CA 93943	2 copies
Director of Research Administration Code 012 Naval Postgraduate School Monterey, CA 93943	1 copy
Prof. Luqi Computer Science Department Code CS/Lq Naval Postgraduate School Monterey, CA 93943	1 copy
Dr. Geoffrey Smith School of Computer Science Florida International University University Park Miami, FL 33199	10 copies
Dr. Dennis Volpano Computer Science Department Code CS/Vo Naval Postgraduate School Monterey, CA 93943	10 copies