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operational test (OT) parameters from
developmental test (DT) parameters via meta-analysis

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Operational Test (OT) Parameters from
Developmental Test (DT) Parameters via
Meta-Analysis**

by

Donald P. Gaver
Patricia A. Jacobs
Arthur Fries

August 1997

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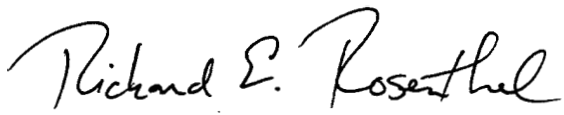
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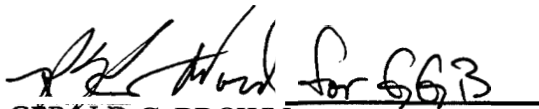

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PREDICTION OF CHANGEOVER PERFORMANCE: OPERATIONAL TEST (OT) PARAMETERS FROM DEVELOPMENTAL TEST (DT) PARAMETERS VIA META-ANALYSIS

Donald P. Gaver

Patricia A. Jacobs

Arthur Fries

1. Introduction

This paper sketches and examines some analytical statistical concepts and methodologies that should usefully inform and sharpen the process of military test and evaluation decisionmaking. The concepts generally fall into the broad category of *combining information (CI)* or *meta-analysis*. See Gaver *et al.* (1992) for examples and references. The term CI does not mean blindly simplistic and uncritical data pooling across either time or the different systems under evaluation. CI in test and evaluation would encourage and systematize quantitative descriptions and comparisons between systems' capabilities and limitations, over time and across comparable systems. It refers to explicit processes whereby judgments, experience and expertise, and data from previous and current military acquisitions, are systematically, transparently, and critically brought to bear on data-taking and analysis for either a particular current system acquisition, or on families of current and future projects. It quantifies aspects of corporate memory.

The formalized CI process illustrated by the examples we provide has not yet proceeded far in practice in any organized way because useful historical data has not been identified, but requirements for more efficient T&E decisionmaking encourage the future development of **such** approaches.

2. Combining Developmental and Operational Testing (DT and OT)

Reliability Data

Developmental Testing (DT) refers to the testing of a new or upgraded system in the course of its technical engineering development. In general, system DT is conducted by technical experts attentive to demonstration of its engineering performance requirements. Operational Testing (OT) is conducted later, and by operational military personnel; the objective is to discover how the system is likely to behave in field operation and to uncover faults and obstacles to such operation. Because of the rigors of field operation there is the expectation that OT failure rates are likely to be higher than those prevailing in DT. We sketch analytical models that can represent such behavior in a quite economical or parsimonious way.

Model I: A Fixed Changeover Effect Multiplier

Suppose that δ_i is the prior-to-changeover failure (or event) rate for system i , and ω_i is the corresponding post-changeover rate; $i = I, 2, \dots, I$. Assume that before changeover system i fails in accordance with a Poisson process, so, over operating (exposure) time x_i , system i fails d_i ($d_i = 0, 1, 2, \dots$) times with probability $e^{-\delta_i x_i} \frac{(\delta_i x_i)^{d_i}}{d_i!}$; after changeover that same system fails w_i ($w_i = 0, 1, 2, \dots$) times in operating time y_i with probability $e^{-\omega_i y_i} \frac{(\omega_i y_i)^{w_i}}{w_i!}$; this independently for $i = I, 2, \dots, I$; this is equivalent to assuming

exponentially distributed times to/between system failures. It is assumed that the data initially available are $(d_i, x_i, w_i, y_i, i = 1, 2, \dots)$. Our objective is to use these data to estimate any consistent change in rates (δ_i, ω_i) from prior- to post-changeover, and to use the estimated relationship to anticipate, and strengthen estimates of, the post-changeover rate of a new system. **The** different analysis approaches used here depend on different ways of characterizing an adjustment factor, κ ; κ is first taken to be a constant in Model I, applicable to all system changeovers. A subsequent setup, Model II, allows the data to indicate the constancy of the relationship.

Suppose there are I different systems for which both DT and OT data are available. Let D_i be the number of failures experienced by system i during developmental testing (DT) during an exposure time x_i . Let W_i be the number of failures experienced by that system during operational testing (OT) during exposure time y_i . Model I assumes that $\{D_i\}$ are independent Poisson random variables with $E[D_i] = \delta_i x_i$ and $\{W_i\}$ are independent Poisson random variables with $E[W_i] = \kappa \delta_i y_i$; that is, κ is an unknown *constant* in this model. The log likelihood is, up to multiplication by irrelevant constants,

$$\ln L \equiv \ell(\delta, \kappa; data) = \sum_{i=1}^I \{(-\delta_i x_i) + d_i \ln \delta_i - (\kappa \delta_i y_i) + w_i [\ln \kappa + \ln \delta_i]\}. \quad (2.1)$$

Setting $\frac{\partial}{\partial \delta_i} = 0$, results in

$$\hat{\delta}_i = \frac{d_i + w_i}{x_i + \kappa y_i}. \quad (2.2)$$

Setting $\frac{\partial}{\partial \kappa} = 0$,

$$\hat{\kappa} = \frac{\sum_i w_i}{\sum_i \hat{\delta}_i y_i}. \quad (2.3)$$

A recursive procedure to find the maximum likelihood estimates works; start with $\hat{\delta}_i(0) = d_i/x_i$. The approximate variance of $\hat{\kappa}$ can be obtained from Fisher information or by bootstrapping; details are omitted here, but see Gaver, Jacobs and Fries (1997).

An important use of the estimate, $\hat{\kappa}$, is to project DT data for a new system into the post-changeover OT phase. Suppose, for instance, that we compute the isolated DT rate estimate for a new (the $I + 1^{\text{st}}$) system, $\hat{\delta}_{I+1} = \frac{d_{I+1}}{x_{I+1}}$. Then a natural point estimate for the failure rate during OT could be

$$\hat{\omega}_{I+1} = \hat{\kappa} \hat{\delta}_{I+1}.$$

Using the obvious independence and asymptotic likelihood approximations (Fisher information) the estimated standard error (se) of $\hat{\omega}_{I+1}$ can be computed:

$$\begin{aligned} SE[\hat{\omega}_{I+1}] &= \sqrt{\hat{Var}[\hat{\kappa}] \hat{Var}[\hat{\delta}_{I+1}] + \hat{Var}[\hat{\kappa}] \left(\hat{E}[\hat{\delta}_{I+1}] \right)^2 + \hat{Var}[\hat{\delta}_{I+1}] \left(\hat{E}[\hat{\kappa}] \right)^2} \\ &= \sqrt{\hat{Var}[\hat{\kappa}] \frac{1}{x_{I+1}} \left(\frac{d_{I+1}}{x_{I+1}} \right) + \hat{Var}[\hat{\kappa}] \left(\frac{d_{I+1}}{x_{I+1}} \right)^2 + \left(\frac{1}{x_{I+1}} \right) \left(\frac{d_{I+1}}{x_{I+1}} \right) (\hat{\kappa})^2}. \end{aligned} \quad (2.4)$$

This can in turn be used to assign approximate standard errors to predicted future OT performance, such as the probability that the future system will exhibit no/zero failures during a test or mission time $x_{I+1}(m)$:

$$\hat{P}\{W_{I+1} = 0 | w_{I+1}, x_{I+1}(m)\} = e^{-\hat{\omega}_{I+1} x_{I+1}(m)}. \quad (2.5)$$

Numerical Examples

Simulation was used to study the coverage properties of various confidence intervals for estimates of κ in Model I. In each replication 20 Poisson random numbers are generated having means $\delta_1 x_1, \dots, \delta_{10} x_{10}, \delta_1 \kappa y_1, \dots, \delta_{10} \kappa y_{10}$, where $\kappa = 4$ and δ_i, x_i, y_i appear in Table 1. This is a simulated version of raw observational data.

TABLE 1

System Number	δ_i DT Failure Rate	x_i DT Test Time (hours)	y_i OT Test Time (hours)
1	0.0002	20,000	5000
2	0.0004	10,000	2500
3	0.0006	6,666.67	1666.67
4	0.0008	5000	1250
5	0.001	4000	1000
6	0.002	2000	500
7	0.004	1000	250
8	0.006	666.67	166.67
9	0.008	500	125
10	0.01	400	100

For each replication 7 types of confidence intervals for κ are calculated. The first uses the MLE estimate of κ and the asymptotic normal confidence limits with observed Fisher information. The next three procedures use 2000 bootstrap replications where the bootstrap resampling is from Poissons with means d_i, w_i . One bootstrap confidence interval procedure uses the percentiles of the bootstrap distribution. Another is a percentile- t procedure with the observed Fisher information of the bootstrap sample being used to estimate the standard error; (cf. DiCiccio and Efron (1996)). The third

procedure uses the normal confidence interval procedure with the bootstrap standard error. The last three confidence intervals are also obtained by bootstrapping. However, in this case the 2000 bootstrap re-samples are drawn as follows.

1. Obtain re-samples of pre-changeover (DT) data as random numbers from the Poisson distribution with mean $\hat{\delta}_i x_i$.
2. Obtain re-samples from post-changeover (OT) data as Poisson samples with mean $\hat{\kappa} \hat{\delta}_i y_i$

where $\hat{\delta}_i$ and $\hat{\kappa}$ are the original parameter estimates.

The three confidence interval methods are the percentile, the percentile- t , **and** the normal confidence interval procedure with bootstrap standard error.

Table 2 displays the results of the simulations. Displayed are the number of intervals that cover the true $\kappa = 4$, and the mean and standard deviation of the width of the intervals. The results of the simulation do not differ by much for the different confidence interval procedures. Thus, for practical purposes the convenient asymptotic confidence interval seems adequate. Perhaps by luck the asymptotic interval not only covers as well as any, but is also shorter and less variable.

Simulation is also used to study confidence intervals for the failure rate of the post-changeover OT failure rate that is projected from DT data for a new system, using $\hat{\kappa}$. In each replication, data is simulated using the model with parameters in Table 1 with $\kappa = 4$. In addition, the number of DT failures for **an** 11th (the new) system is simulated by generating d_{11} from a Poisson distribution with mean $0.004 \times 1,000$ conditioned to be positive; that is, $\delta_{11} = 0.004$ and $x_{11} = 1000$. The estimated DT failure rate of the 11th system $\hat{\delta}_{11} =$

d_{11}/x_{11} where d_{11} is the random number and $\hat{\omega}_{11} = \hat{\kappa}\hat{\delta}_{11}$ where $\hat{\kappa}$ is the estimate obtained from the data generated for the 10 systems.

Level	80%			90%			95%		
Interval Proc.	Coverage	Width		Coverage	Width		Coverage	Width	
		Mean	S.D.		Mean	S.D.		Mean	S.D.
Asymptotic Normal	85	2.42	0.59	92	3.11	0.76	95	3.70	0.90
Bootstrap I (2000)									
Normal B. SE	84	2.59	0.67	93	3.32	0.86	95	3.96	1.02
Percentile	80	2.49	0.63	91	3.25	0.81	92	3.94	1.01
t-Percentile	82	2.45	0.60	91	3.17	0.75	92	3.80	0.90
Bootstrap II (2000)									
Normal B. SE	86	2.59	0.68	92	3.32	0.87	95	3.96	1.08
Percentile	80	2.48	0.63	91	3.24	0.83	93	3.94	1.03
t-Percentile	82	2.45	0.60	91	3.17	0.77	93	3.80	0.91

Bootstrap I $D_i(b) \sim \text{Poisson}(d_i)$ $W_i(b) \sim \text{Poisson}(w_i)$

Bootstrap II: $D_i(b) \sim \text{Poisson}(\hat{\delta}_i x_i)$ $W_i(b) \sim \text{Poisson}(\hat{\delta}_i \hat{\kappa} y_i)$

For each replication 5 types of confidence intervals for the new system's OT failure rate, ω_{11} , are calculated. The first uses the MLE estimate of κ , $\hat{\delta}_{11}$, and the asymptotic estimate of standard error and normal percentiles; this confidence interval is called the asymptotic normal interval. The next two confidence intervals use 2000 bootstrap replications, where the bootstrap resampling includes the additional random draw of $d_{11}(b)$ from a Poisson distribution with mean d_{11} , the number of DT failures for the 11th system. The b^{th} bootstrap estimate of ω_{11} is $\hat{\omega}_{11}(b) = \hat{\kappa}(b)d_{11}(b)/x_{11}$. One bootstrap confidence interval is constructed from the percentiles of the bootstrap distribution of $\hat{\omega}_{11}$. The second procedure uses the normal confidence

interval procedure with the mean and standard deviation of the bootstrap estimates. The fourth and fifth confidence intervals are also bootstrap confidence intervals but with the bootstrap samples for the first 10 systems being drawn as follows: $d_i(b)$ = Poisson mean $\hat{\delta}_i x_i$ and $w_i(b)$ = Poisson mean $\hat{\kappa} \hat{\delta}_i y_i$. The fourth confidence interval uses the percentiles of the bootstrap distribution of ω_{11} and the fifth uses the mean and standard deviation of the bootstrap distribution and standard normal percentiles. The results appear in Table 3. Displayed are the number of intervals (out of the 100) that cover the true value of $\omega_{11} = 0.016$ and the mean and standard deviation of the width of the intervals. There is not much practical difference between the 5 confidence intervals procedures.

Level	80%			90%			95%		
Interval Proc.	Coverage	Width		Coverage	Width		Coverage	Width	
		Mean	S.D.		Mean	S.D.		Mean	S.D.
Asymptotic Normal	82	0.023	0.009	87	0.030	0.011	91	0.035	0.013
Normal B. SE	84	0.024	0.009	89	0.031	0.012	91	0.037	0.014
Percentile	82	0.023	0.009	89	0.030	0.012	93	0.036	0.014
Normal B. SE	84	0.024	0.009	89	0.031	0.012	91	0.037	0.014
Percentile	81	0.023	0.009	90	0.030	0.012	93	0.036	0.014

Bootstrap I : $D_i(b)$ = Poisson mean d_i ; $W_i(b)$ = Poisson mean w_i , $i = 1, \dots, 10$

Bootstrap II : $D_i(b)$ = Poisson mean $\hat{\delta}_i x_i$; $W_i(b)$ = Poisson mean $\hat{\delta}_i \hat{\kappa} y_i$ $i = 1, \dots, 10$

Model 11: A Variable/Adaptive Changeover Multiplier

This next model generalizes the previous setup by allowing for possible variability in the DT-OT multiplier, κ : we permit the training data sets, $i = 1, 2, \dots, I$, a chance to *reveal their appropriate κ -variability*: each training data set is thought of as having its own κ -value, each a sample from a population with mean and variance to be estimated. If the variability of this population is sizable then the predictability (and usefulness) of the relation is questionable. We again assume $\{D_i\}$ are independent Poisson random variables with mean $\delta_i x_i$, and we let $\{W_i\}$, number of failures during OT, to be independent random variables. The conditional distribution of W_i , given Z_i , is Poisson with mean $\omega_i y_i = \delta_i Z_i y_i$ where Z_i is conveniently taken to be a gamma-distributed random variable having *mean* $\mu = E[Z_i] = \frac{\beta}{\alpha}$ and shape parameter β (its scale is α), so *each system has its own individual DT-OT multiplier*; these may be close to a mean value μ but not necessarily tightly clustered around that mean. Note that this is *not* the same as a partially/Bayesian analysis of Model I with κ an unknown *constant* described by a gamma prior. In the present model $\{W_i\}$ **turn** out to be independent negative-binomial random variables that depend on the values of α and β , which will be estimated from data. The value of $\sqrt{\widehat{Var}[Z_i]/\widehat{E}[Z_i]}$ allows an analyst to get a rough idea of the cohesiveness of his data sets.

The log-likelihood is, up to irrelevant constants,

$$\ell \equiv \ln L =$$

$$\sum_{i=1}^I \left\{ -\delta_i x_i + d_i \ln \delta_i + \ln \left[\frac{\Gamma(\beta + w_i)}{\Gamma(\beta)} \right] + \beta \ln \alpha + w_i \ln \delta_i - (\beta + w_i) \ln [\alpha + \delta_i y_i] \right\}. \quad (2.6)$$

Analysis of simulated data shows that procedures to maximize the full likelihood rather frequently misbehave: while a somewhat reasonable estimate of the gamma mean, $\mu = \frac{\beta}{\alpha}$, is usually obtained, the tendency is for the estimate of shape, β , to fly towards $+\infty$, so the variance of the κ -population tends to be badly underestimated. Such misbehavior of likelihoods has been previously noted when attempts are made to estimate one or more basic (“interest”) parameters in the presence of many other (“nuisance”) parameters (this may be nature’s way of telling an analyst to slow down); see Cox and Reid (1993), (1987). A partially Bayesian way of addressing the situation is to treat the unknown δ s as random and integrate them out (marginalize). Simulations are used to demonstrate that this method can tend to be reasonably reliable—much more so than is the full likelihood approach. A similar maneuver has been employed to estimate the common mean of a large number of different-variance normal populations, cf. Barndorff-Nielsen and Cox (1994).

Assume $\{\delta_i\}$ are iid with a Jeffery’s prior. Then to marginalize on $\underline{\delta}$ carry out

$$\begin{aligned}
 P\{D_i = d_i, W_i = w_i | Z_i = \kappa\} &= \int_0^{\infty} e^{-\delta_i x_i} \frac{(\delta_i x_i)^{d_i}}{d_i!} e^{-\kappa_i \delta_i y_i} \frac{[\delta_i y_i \kappa]^{w_i}}{w_i!} \frac{1}{\delta_i} d\delta_i \\
 &\propto \kappa^{w_i} \left[\frac{1}{x_i + \kappa y_i} \right]^{d_i + w_i}.
 \end{aligned} \tag{2.7}$$

Put the mean $\mu = \beta/\alpha$. The integrated likelihood is proportional to

$$\begin{aligned}
 L(\mu, \mathbf{p}; \text{data}) &= \\
 \prod_{i=1}^I \frac{\left[\frac{\beta}{\mu}\right]^{\beta}}{\Gamma(\beta)} \frac{x_i^{\beta-d_i}}{y_i^{w_i+\beta}} \int_0^{\infty} \exp\left\{-\left(\frac{\beta}{\mu}\right) \frac{x_i}{y_i} u + (w_i + \beta - 1) \ln u\right\} \frac{1}{[1+u]^{d_i+w_i}} du.
 \end{aligned} \tag{2.8}$$

After several algebraic steps, the log-likelihood is, up to irrelevant constants,

$$\begin{aligned} \ell(\mu, \beta; data) = & \sum_{i=1}^I \left\{ \beta \ln \left[\frac{\beta}{\mu} \right] - \ln \Gamma(\beta) + \beta \ln x_i - \beta \ln y_i \right. \\ & \left. + \ln \int_0^{\infty} \exp \left\{ - \left(\frac{\beta}{\mu} \right)^{\frac{x_i}{y_i}} u + (w_i + \beta - 1) \ln u \right\} \frac{1}{[1+u]^{d_i+w_i}} du \right\}. \end{aligned} \quad (2.9)$$

Direct numerical integration has been used in what follows.

Numerical Examples

Simulations were used to evaluate the above procedures. In each simulation replication the DT rates, $\delta_i, i = 1, 2, \dots, I$ were generated independently from a uniform distribution over $[0, 10]$. The times $x_i = y_i \equiv 1$ for all simulations. In the first two Cases, (A) and (B), the $\{Z_i\}$ values were generated from a gamma distribution having mean $\mu = 4$ and shape parameter $\beta = 1$. In Cases (C) and (D), the $\{Z_i\}$ were generated from a gamma distribution having mean $\mu = 4$ and shape parameter $\beta = 4$.

For each data set generated, the mean and the shape parameters of the gamma distribution are estimated using the integrated likelihood (2.9), and alternatively the method of moments. The numerical integration uses Simpson's rule with **up** to 10th order difference correction for a step size $k = 0.01$ (cf. Hamming (1973)) over $[0, 20]$ as implemented in *A Graphical Statistical System, AGSS*.

In the present method of moments, the DT failure rate of the i^{th} system is estimated by $\hat{\delta}_i = d_i/x_i$. The multiple of the OT failure rate for the i^{th} system is estimated by $\hat{\kappa}_i = w_i/\hat{\delta}_i y_i$. The moment estimate of the mean of the gamma $\hat{\mu}_M = \frac{1}{I} \sum_{i=1}^I \hat{\kappa}_i$ and the moment estimate of the shape parameter of the gamma is

$$\hat{\beta}_M = \frac{(\hat{\mu}_M)^2}{\frac{1}{I-1} \sum_{i=1}^I (\hat{\kappa}_i - \hat{\mu}_M)^2} \quad (2.10)$$

No attempt has been made to adjust for the (Poisson) variability of d_i or w_i in the above. The integrated log likelihood is searched until parameters change by less than 0.01. The search is started at the method of moments estimates.

Table 4 displays the results of a simulation experiment. Each simulation has 10 replications. Each replication consists of I systems. The mean and shape parameters of the gamma distribution are estimated using moments and integrated likelihood. The table reports the mean (and standard deviation) of the 10 replications of the estimates of the mean and shape parameter of the gamma for the moment estimators and the integrated likelihood estimators for each case.

The results displayed in Table 4 suggest the following. Both procedures estimate the mean of the gamma relatively well; they are both biased high, with the bias smaller for integrated likelihood. The results for $\hat{\beta}$ suggest that the method-of-moments estimate of the shape parameter can be biased on the low side for larger β . The variance of the estimates suggests that the smaller the variance of the gamma distribution (larger β , or shape), the more difficult it is for either of the procedures to accurately estimate the shape parameter β .

The simulation results are not presented as at all exhaustive or definitive, but as suggestive of procedures that might well work in practice (integrated likelihood, but also simple moments), and others to be avoided (full likelihood).

TABLE 4
Estimates of Mean and Shape Parameter of the Gamma Distribution
10 replications

	Cases\Methods	Mean (Std Dev) of Estimates Integrated Likelihood	Mean (Std Dev) of Estimates Moments
(A)	$\mu = 4, \beta = 1, I = 10$	$\bar{\mu}_{ML} = 4.80 (1.75)$ $\bar{\beta}_{ML} = 1.52 (0.50)$	$\bar{\mu}_M = 4.70 (2.45)$ $\bar{\beta}_M = 1.08 (0.39)$
(B)	$\mu = 4, \beta = 1, I = 20$	$\bar{\mu}_{ML} = 4.33 (0.89)$ $\bar{\beta}_{ML} = 1.66 (0.71)$	$\bar{\mu}_M = 4.37 (1.32)$ $\bar{\beta}_M = 1.22 (0.67)$
(C)	$\mu = 4, \beta = 4, I = 10$	$\bar{\mu}_{ML} = 4.55 (1.47)$ $\bar{\beta}_{ML} = 5.38 (3.10)$	$\bar{\mu}_M = 4.93 (1.82)$ $\bar{\beta}_M = 1.81 (0.90)$
(D)	$\mu = 4, \beta = 4, I = 20$	$\bar{\mu}_{ML} = 4.34 (0.90)$ $\bar{\beta}_{ML} = 5.23 (4.28)$	$\bar{\mu}_M = 4.81 (1.22)$ $\bar{\beta}_M = 1.38 (0.80)$

3. Likelihood-Based Pooling of Observations from Sensors with a Particular Range-Dependent Precision

The next example describes the form of an estimate of a target item's range from observations by several co-located sensors with range-dependent precision. Suppose there are s ($s \geq 1$) sensors capable of detection and range determination of targets at various ranges. Here is a model: let

$$R_i = \text{range estimate of } i^{\text{th}} \text{ sensor, } i = 1, 2, \dots, s$$

if r is the true range of target, then suppose that all R_i are independent and normal/Gaussian, with

$$\begin{aligned} E[R_i] &= r \\ \text{Var}[R_i] &= \sigma_i^2 r^2. \end{aligned} \tag{3.1}$$

The objective is to *estimate* r , using *all* information available in the above, which means use the fact that the variance also depends on the mean.

Treating r as an unknown parameter one can write down its likelihood, given observations

$$R_1 = x_1, R_2 = x_2, \dots, R_s = x_s.$$

It is

$$L(r; \underbrace{x_1, x_2, \dots, x_s}_{\text{Data}}) = \prod_{i=1}^s \frac{e^{-\frac{1}{2}(x_i-r)^2/\sigma_i^2 r^2}}{\sqrt{2\pi} \sigma_i r} \quad (3.2)$$

The log-likelihood is

$$\begin{aligned} \ell(r; \text{data}) &= -\frac{1}{2} \sum_{i=1}^s \frac{(x_i - r)^2}{\sigma_i^2 r^2} - s \ln r \\ &= -\frac{1}{2} \sum_{i=1}^s \frac{(x_i/r - 1)^2}{\sigma_i^2} + s \ln(1/r). \end{aligned} \quad (3.3)$$

or, if $\theta = 1/r$

$$\ell(\theta; \text{data}) = -\frac{1}{2} \sum_{i=1}^s \frac{(x_i \theta - 1)^2}{\sigma_i^2} + s \ln \theta. \quad (3.4)$$

Now

$$\begin{aligned} \frac{\partial \ell}{\partial \theta} &= -\frac{1}{2} \sum_{i=1}^s \frac{2(x_i \theta - 1)x_i}{\sigma_i^2} + \frac{s}{\theta} \\ &= -\theta \sum_{i=1}^s \frac{x_i^2}{\sigma_i^2} + \sum_{i=1}^s \frac{x_i}{\sigma_i^2} + \frac{s}{\theta} \end{aligned} \quad (3.5)$$

so setting this equal to zero yields a quadratic equation for θ :

$$\theta^2 \left(\sum \frac{x_i^2}{\sigma_i^2} \right) - \theta \left(\sum \frac{x_i}{\sigma_i^2} \right) - s = 0.$$

The acceptable solution is

$$\hat{\theta} = \frac{\sum_{i=1}^s \frac{x_i}{\sigma_i^2} + \sqrt{\left(\sum_{i=1}^s \frac{x_i}{\sigma_i^2}\right)^2 + 4s \left(\sum_{i=1}^s \frac{x_i^2}{\sigma_i^2}\right)}}{2 \sum_{i=1}^s \frac{x_i^2}{\sigma_i^2}}, \quad (3.6)$$

consequently the point estimate that uses all the information

$$\hat{r} = \frac{1}{\hat{\theta}} = \frac{2 \sum_{i=1}^s \frac{x_i^2}{\sigma_i^2}}{\left(\sum_{i=1}^s \frac{x_i}{\sigma_i^2} + \sqrt{\left(\sum_{i=1}^s \frac{x_i}{\sigma_i^2}\right)^2 + 4s \sum_{i=1}^s \frac{x_i^2}{\sigma_i^2}}\right)}. \quad (3.7)$$

Under certain circumstances (e.g. $\sigma_i = O(O.1)$) the second term inside the radical will be much smaller than the first; neglecting it gives the weighted estimate

$$\hat{r} \cong \frac{\sum_{i=1}^s (x_i^2/\sigma_i^2)}{\sum_{i=1}^s (x_i/\sigma_i^2)} = \frac{\sum_{i=1}^s x_i^4 / (\sigma_i^2 x_i^2)}{\sum_{i=1}^s x_i^3 / (\sigma_i^2 x_i^2)}. \quad (3.8)$$

It is surprising that, while the rightmost formula weights as the inverse of the estimated/observed-range-calibrated variance, i.e. $\Psi(\sigma_i^2 x_i^2)$, a natural surrogate for $\Psi(\sigma_i^2 r^2)$, then, instead of weighting the raw observations x_i , it weights their *fourth power* with the normalizing factor necessarily involving weighted third powers. The accuracy of these estimates should be compared to the simple linearly-weighted estimate that recognizes true range dependence:

$$\hat{r}_L = \frac{\sum_{i=1}^s x_i (1/\sigma_i^2 r^2)}{\sum_{i=1}^s (1/\sigma_i^2 r^2)} = \frac{\sum_{i=1}^s x_i / \sigma_i^2}{\sum_{i=1}^s 1/\sigma_i^2} \quad (3.9)$$

where the unknown true range dependence that influences the variance cancels in this (normal) dispersion model! This is independent of the form of the range dependence, $g(r)$ (here $g(r) = r^2$) provided it is the same **for all** sensors. Numerical investigation shows that the linear estimate (3.9) performs nearly optimally, and is certainly highly convenient in applications.

Numerical Illustration

We have simulated the results of observing a target by five different sensors and then combining the results. The situation and results appear in Table 5, and the figure provides further insights.

As expected the mle approach (3.7) generates estimates that closely concentrate around the true mean ($r = 1000$); the standard error of these estimates is smaller than those of the other two. The approximation (3.8) is a bit high on the average (dropping a positive term in the denominator is the reason). The properties of the linear calculation (3.9) are gratifyingly similar to those of the mle; ease of calculation is welcome.

TABLE 5
Estimates of Range Determination by Combination
500 replications

$$r = 1000; \sigma_1^2 = \sigma_2^2 = 0.09; \sigma_3^2 = \sigma_4^2 = \sigma_5^2 = 0.05$$

Estimate:	MLE	Approximation	Linear
Eq. Number	(3.7)	(3.8)	(3.9)
Mean	1006.4	1068.8	991.2
Std. Dev. of Estimates	134.6	143.1	141.1
Mean Sq. Error	18118.6	25169.2	19939.9

4. Alternatives to the Circular Normal Dispersion Model: "Robust CEP"

Experience shows that in projectile (e.g. missile, gunfire) testing it is often the case that some individual shots deviate from aim point more wildly than described by the customary circular normal model. *One* way of providing a

model for this to guide data analyses and perform CI across the DT to OT changeover is to stochastically **mix**: letting $\boldsymbol{\rho}=1/\sigma^2$ be a precision parameter, think of it as being chosen randomly, perhaps (not necessarily) from shot to shot, and then presume that the random \boldsymbol{p} has a distribution and use it to remove the condition on precision.

If we start with standard circular normal dispersion, conditional on \boldsymbol{p} ,

$$P\{R > r | \boldsymbol{\rho} = (1/\sigma^2) \equiv \rho\} = e^{-\frac{1}{2}r^2\rho} \quad (4.1)$$

then when the condition is removed

$$P\{R > r\} = E\left[e^{-\frac{1}{2}r^2\rho}\right] = \int_0^{\infty} e^{-\left(\frac{1}{2}r^2\right)x} dF_{\rho}(x).$$

Gamma Variation of the Precision Parameter

Let the variability in the precision, ρ , be described by the gamma ($\boldsymbol{a} \boldsymbol{\beta}$) density

$$dF_{\rho}(x) = f_{\rho}(x; \alpha, \beta) dx = e^{-\alpha x} \frac{(\alpha x)^{\beta}}{\Gamma(\beta)} \alpha dx; \quad (4.2)$$

for which $E[\rho] = \beta/\alpha = \bar{\rho}$, $Var[\rho] = \beta/\alpha^2$.

Then

$$P\{R > r | \alpha, \beta\} = \left(\frac{\alpha}{\alpha + r^2/2}\right)^{\beta} = \left(\frac{1}{1 + (r^2/2)(\beta/\alpha) \cdot 1/\beta}\right)^{\beta} = \left(1 + \frac{r^2}{2} \bar{\rho} \frac{1}{\beta}\right)^{-\beta}. \quad (4.3)$$

The CEP satisfies for this distribution (which has a long or fat "Pareto tail"),

$$\left(1 + \frac{r_{\text{CEP}}^2}{2} \bar{\rho} \frac{1}{\beta}\right)^{-\beta} = \frac{1}{2}. \quad (4.4)$$

This gives

$$r_{\text{CEP}} = \sqrt{2\beta(2^{1/\beta} - 1) \frac{1}{\bar{\rho}}} \equiv \sqrt{2\beta(2^{1/\beta} - 1)\sigma} \quad (4.5)$$

The CEP approaches $\sqrt{2\ln 2} / \bar{\rho} = 1.177$ as $\beta \rightarrow \infty$ if $\bar{\rho} = 1$, in which case there is no shot to shot variation in precision. If we maintain the mean of the dispersion distribution at $\bar{\rho} = 1$ and reduce β (increase variance) $\beta = 1$ (and $\bar{\rho} = 1$) we find $r_{\text{CEP}} = \sqrt{2} = 1.414$, a 20% increase; as β decreases further the CEP increases indefinitely, induced by the great shot-to-shot variability.

Parameter Estimation by Likelihood

Maximum likelihood estimation of the parameters of the miss-distance distribution (4.3) can be easily done when all miss distances of a series of n shots are recorded: r_1, r_2, \dots, r_n . That is, none have been (prematurely) deleted as outliers. The density function of a miss distance is

$$f_R(r; \alpha, \beta) = \left(1 + \frac{r^2}{2} \bar{\rho} \cdot \frac{1}{\beta}\right)^{-(\beta+1)} r \bar{\rho} \quad ; \quad (4.6)$$

the log-likelihood function is

$$\ell(\alpha, \beta; r) = -(\beta+1) \sum_{i=1}^n \left(\ln \left(1 + r_i^2 \frac{\bar{\rho}}{2\beta} \right) \right) + n \ln \bar{\rho}. \quad (4.7)$$

To maximize, differentiate and set the derivatives equal to zero; solve. The equations appear as

$$\frac{\partial \ell}{\partial \bar{\rho}} = -\frac{(\beta+1)}{2\beta} \sum_{i=1}^n \left(\frac{r_i^2}{1 + r_i^2 \bar{\rho}/2\beta} \right) + \frac{n}{\bar{\rho}} = 0 \quad (4.8,a)$$

or

$$\frac{1}{\bar{\rho}} = \frac{\beta+1}{2\beta} \cdot \frac{1}{n} \sum_{i=1}^n \frac{r_i^2}{1 + r_i^2 \bar{\rho}/2\beta} \quad (4.8,b)$$

and

$$\frac{\partial \ell}{\partial \beta} = -\sum_{i=1}^n \left[\ln \left(1 + r_i^2 \frac{\bar{\rho}}{2\beta} \right) \right] + \frac{(\beta+1)}{2\beta^2} \sum_{i=1}^n \left[\frac{r_i^2}{1 + r_i^2 \frac{\bar{\rho}}{2\beta}} \right] = 0. \quad (4.9,a)$$

Because of (4.8,b) the second term allows

$$\frac{1}{n} \sum_{i=1}^n \ln \left(1 + r_i^2 \frac{\bar{\rho}}{2\beta} \right) = \frac{1}{\beta}. \quad (4.9,b)$$

An iterative solution of (4.8,b) and (4.9,b) is promising: start from $\hat{\beta}(0) = 1$, $\bar{\rho}(0) = 2\hat{\beta}(0) \left(2^{1/\hat{\beta}(0)} - 1 \right) / (r_{MED})^2$ where r_{MED} = median of the ordered miss distances. Use of Fisher information or bootstrapping will furnish standard errors of estimates.

Modeling DT-OT Miss-Distance Data Combination

A convenient approach to represent the difference between DT and OT miss distances in the current context is to utilize a proportional hazard or Lehmann alternative device. If $R_D(R_W)$ are respectively location errors or shot shot deviations for DT and OT we put in (4.3) for the DT dispersion

$$\begin{aligned} P\{R_W > r | \alpha, \beta\} &= \left(P\{R_D > r | \alpha, \beta\} \right)^\kappa \\ &= \left(1 - \left(r^2/2 \right) \bar{\rho} / \beta \right)^{\beta\kappa}. \end{aligned} \quad (4.10)$$

The corresponding CEP turns out to be

$$r_{CEP} = \sqrt{2\beta \left(2^{1/\beta\kappa} - 1 \right) \frac{1}{\bar{\rho}}} \quad (4.11)$$

so here κ turns out to be small if there is pronounced degradation of precision in OT over that in DT.

The density of OT miss distances is seen to be

$$f_W(r; \alpha, \beta, \kappa) = \left(1 + \frac{2}{\beta} \frac{\bar{\rho}}{\rho} \frac{1}{\beta}\right)^{-(\beta\kappa+1)} \frac{r \bar{\rho}^\kappa}{r \rho^\kappa} \quad (4.12)$$

DT-OT Data, and Parameter Estimation

Suppose we have DT miss-distance data from system i , $i = 1, 2, \dots, I$, denoted by $r_{ij}^2(\text{DT}) = u_{ij}$, $j = 1, 2, \dots, n_i$, and corresponding OT miss distances, $r_{ik}^2(\text{OT}) = v_{ik}$, $j = 1, 2, \dots, m_i$. Under Model I assumptions, i.e. a fixed (unknown) κ , the likelihood is

$$L(\underline{\bar{\rho}}, \underline{\beta}, \kappa; \text{data}) = \prod_{i=1}^I \prod_{j=1}^{n_i} \left(1 + \frac{u_{ij}}{2} \frac{\bar{\rho}_i}{\beta_i}\right)^{-(\beta_i+1)} \frac{1}{\bar{\rho}_i} \cdot \prod_{k=1}^{m_i} \left(1 + \frac{v_{ik}}{2} \frac{\bar{\rho}_i}{\beta_i}\right)^{-(\beta_i\kappa+1)} \frac{1}{\bar{\rho}_i^\kappa} \quad (4.13)$$

dropping factors independent of parameters. Hence the log-likelihood is

$$\begin{aligned} \ell(\underline{\bar{\rho}}, \underline{\beta}, \kappa; \text{data}) = \sum_{i=1}^I \left[-(\beta_i + 1) \sum_{j=1}^{n_i} \ln \left(1 + \frac{u_{ij}}{2} \frac{\bar{\rho}_i}{\beta_i}\right) - (\beta_i \kappa + 1) \sum_{k=1}^{m_i} \ln \left(1 + \frac{v_{ik}}{2} \frac{\bar{\rho}_i}{\beta_i}\right) \right. \\ \left. + \sum_{j=1}^{n_i} \ln \bar{\rho}_i + \sum_{k=1}^{m_i} (\ln \bar{\rho}_i + \ln \kappa) \right]. \end{aligned} \quad (4.14)$$

From this,

$$\frac{\partial \ell}{\partial \bar{\rho}_i} = -\frac{\beta_i + 1}{2\beta_i} \sum_{j=1}^{n_i} \frac{u_{ij}}{1 + \frac{u_{ij}}{2} \frac{\bar{\rho}_i}{\beta_i}} - \frac{(\beta_i \kappa + 1)}{2\beta_i} \sum_{k=1}^{m_i} \frac{v_{ik}}{1 + \frac{v_{ik}}{2} \frac{\bar{\rho}_i}{\beta_i}} + \frac{n_i + m_i}{\bar{\rho}_i} = 0 \quad (4.15,a)$$

or

$$\frac{1}{\bar{\rho}_i} = \frac{1}{n_i + m_i} \left[\frac{\beta_i + 1}{2\beta_i} \sum_{j=1}^{n_i} \frac{u_{ij}}{1 + \frac{u_{ij}}{2} \frac{\bar{\rho}_i}{\beta_i}} + \frac{(\beta_i \kappa + 1)}{2\beta_i} \sum_{k=1}^{m_i} \frac{v_{ik}}{1 + \frac{v_{ik}}{2} \frac{\bar{\rho}_i}{\beta_i}} \right] \quad (4.15,b)$$

$$\frac{\partial \ell}{\partial \beta_i} = -\sum_{j=1}^{n_i} \ln\left(1 + \frac{u_{ij}}{2} \bar{\rho}_i \frac{1}{\beta_i}\right) + \frac{(\beta_i + 1)}{2\beta_i^2} \sum_{j=1}^{m_i} \frac{u_{ij}}{1 + \frac{u_{ij}}{2} \bar{\rho}_i \frac{1}{\beta_i}} \cdot \bar{\rho}_i \quad (4.16,a)$$

$$-\kappa \sum_{k=1}^{m_i} \ln\left(1 + \frac{v_{ik}}{2} \bar{\rho}_i \frac{1}{\beta_i}\right) + \frac{(\beta_i \kappa + 1)}{2\beta_i^2} \sum_{k=1}^{m_i} \frac{v_{ik}}{1 + \frac{v_{ik}}{2} \bar{\rho}_i \frac{1}{\beta_i}} \bar{\rho}_i = 0$$

Because of (4.15,b) the second term allows

$$\frac{1}{\beta_i} = \frac{1}{n_i + m_i} \left[\sum_{j=1}^{n_i} \ln\left(1 + \frac{u_{ij}}{2} \frac{\bar{\rho}_i}{\beta_i}\right) + \kappa \sum_{k=1}^{m_i} \ln\left(1 + \frac{v_{ik}}{2} \frac{\bar{\rho}_i}{\beta_i}\right) \right] \quad (4.16,b)$$

$$\frac{\partial \ell}{\partial \kappa} = \sum_{i=1}^I -\beta_i \sum_{k=1}^{m_i} \ln\left(1 + \frac{v_{ik}}{2} \bar{\rho}_i \frac{1}{\beta_i}\right) + \frac{m_i}{\kappa} = 0. \quad (4.17)$$

Thus,

$$\frac{1}{\kappa} = \frac{\sum_{i=1}^I \beta_i \sum_{k=1}^{m_i} \ln\left(1 + \frac{v_{ik}}{2} \frac{\bar{\rho}_i}{\beta_i}\right)}{\sum_{i=1}^I m_i}. \quad (4.18)$$

An iterative solution of (4.15,b), (4.16,b) and (4.18) is promising: start from $\hat{\beta}_i(0) = 1, \rho_i(0) = 2/\left[\text{median}(r_{i,n_1}, \dots, r_{i,n_i})\right]^2$ and then solve for $\hat{\kappa}(0)$ using (4.18).

Notice that a Model II version of the present setup can be explicitly carried out: if the mixing distribution is gamma, as before, all necessary integrals come out in closed form and the analysis carried forward. This work must be postponed for the present.

Numerical Example

Suppose there is historical data on 5 similar systems. Each system has 20 observations during DT, and 20 observations during OT. Odd-numbered systems have $(\beta, \bar{\rho}) = (0.5, 1)$. Even-numbered systems have $(\beta, \bar{\rho}) = (1, 2)$.

The common $\kappa = 0.8$. Figures 2 - 4 present results from 500 replications of a simulation: "data" were sampled from the above model and the parameters estimated. Figure 2 presents histograms for the estimate of (β, ρ) for the odd numbered systems, and Figure 3 presents histograms for estimates of (β, ρ) for the even numbered systems. All estimates result from the iterative schemes of (4.15,a) - (4.18). Figure 4 presents a histogram of estimates of κ . The estimates appear well-behaved in that their histograms cluster well around the true (here known) values, doing so in an appropriately normal fashion.

5. Discussion

The present paper examines a selection of problem types typical of the testing environment. Emphasis is placed on the issue of borrowing information from the DT period to strengthen decisions concerning OT; this can be useful to inform the decision maker of the advisability of the immediate initiation of OT. Some attention is also given to auxiliary information, i.e. with respect to shot-to-shot variability change with range, and the occurrence of "fat-tailed" outlier-prone shot dispersion distributions. We plan to further address such problems in future.

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500 REPL; 5 OBS; R=1000; SIGI2=SIG22=.09;OTHERS=.05

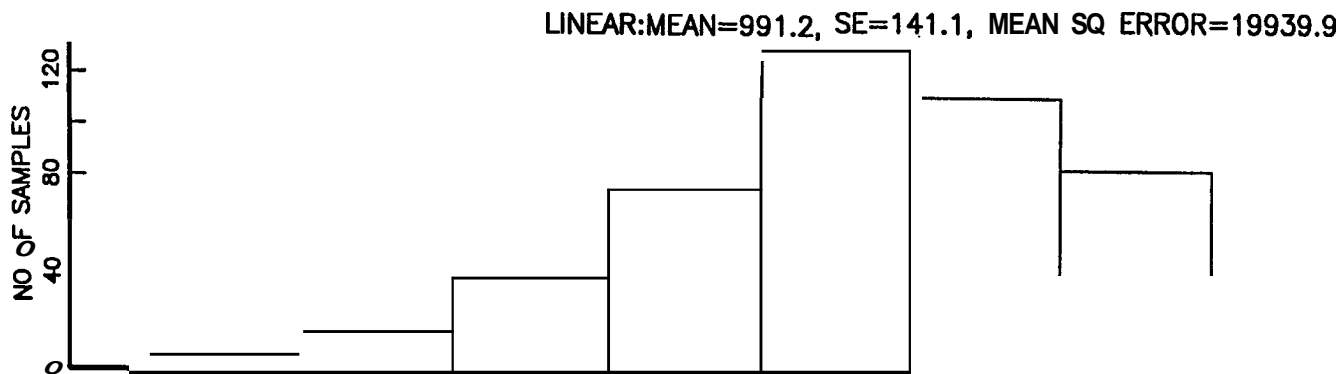
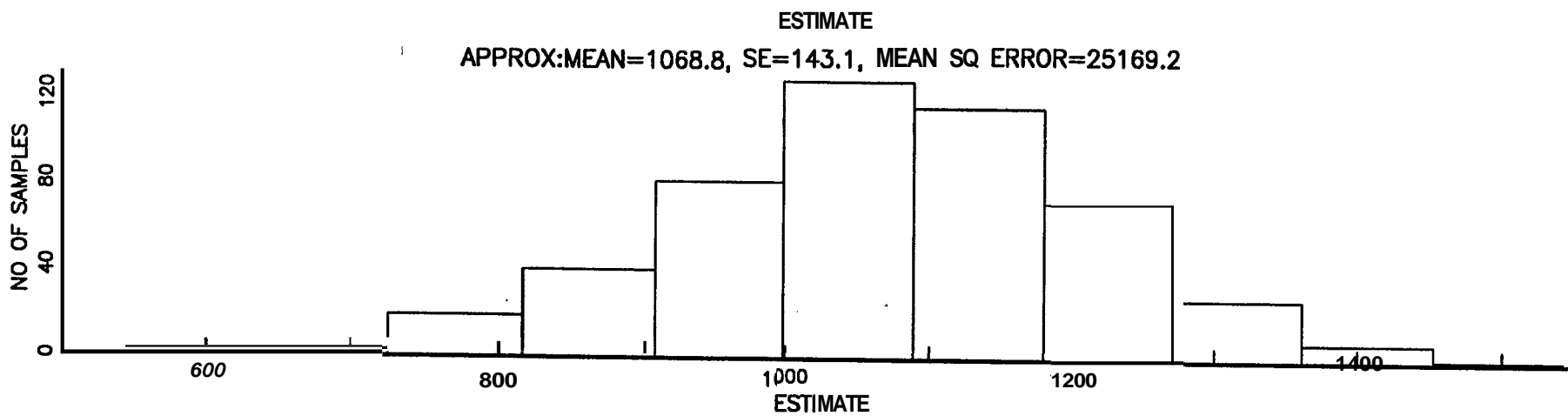
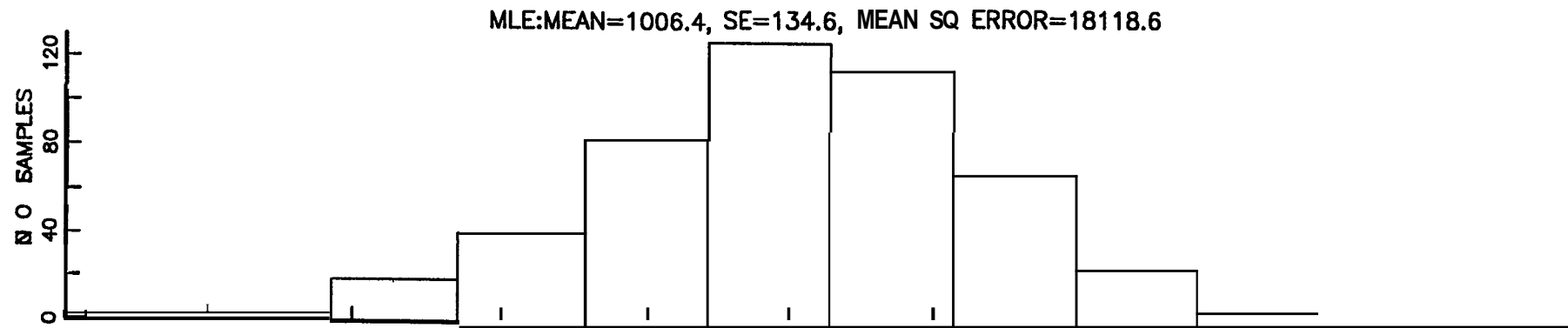
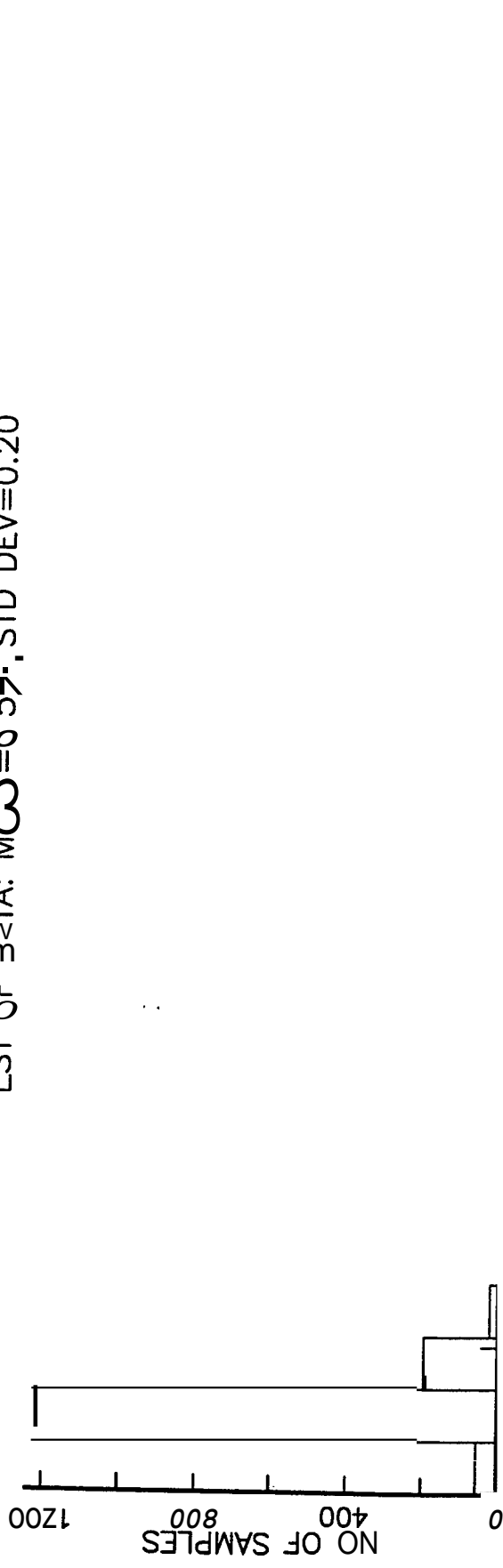


Figure 1
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ODD NUMBER SYSTEMS: $\text{TRU} \approx \beta = 0.5$; $\text{TRU} \approx \rho = 1$

EST OF β : $\mu = 0.57$; STD DEV = 0.20



EST OF ρ : $\text{MEAN} = 1.0$; $\text{STD DEV} = 0.3$

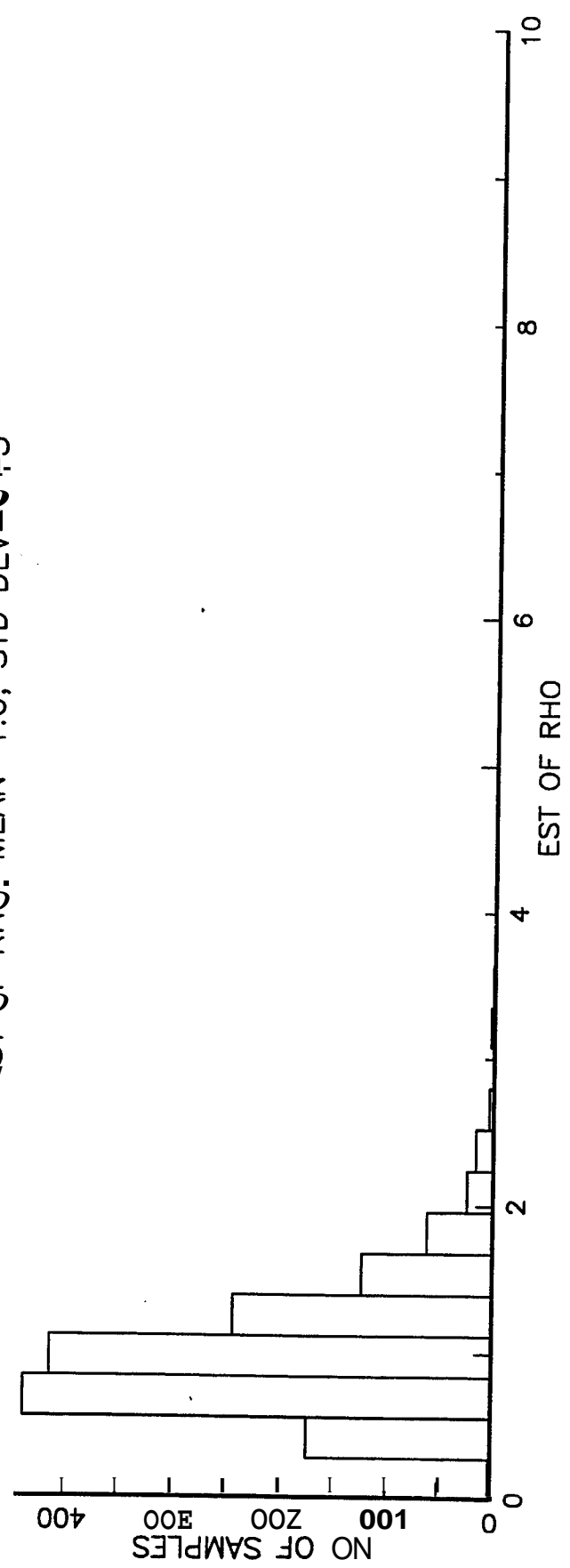


Figure 2

≠ EVEN NUMBER SYSTEMS: TRUE BETA=1.0; TRUE RHO=2

EST OF BETA: MEAN=1.24; STD DEV=0.69

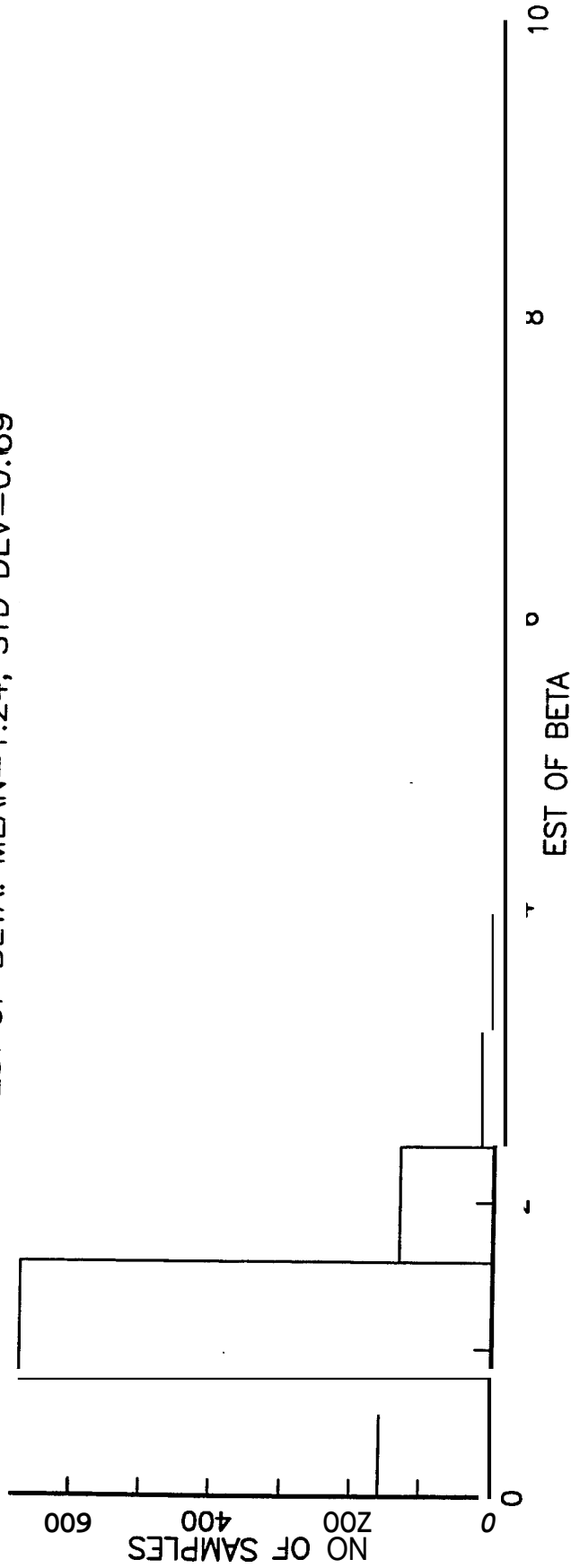
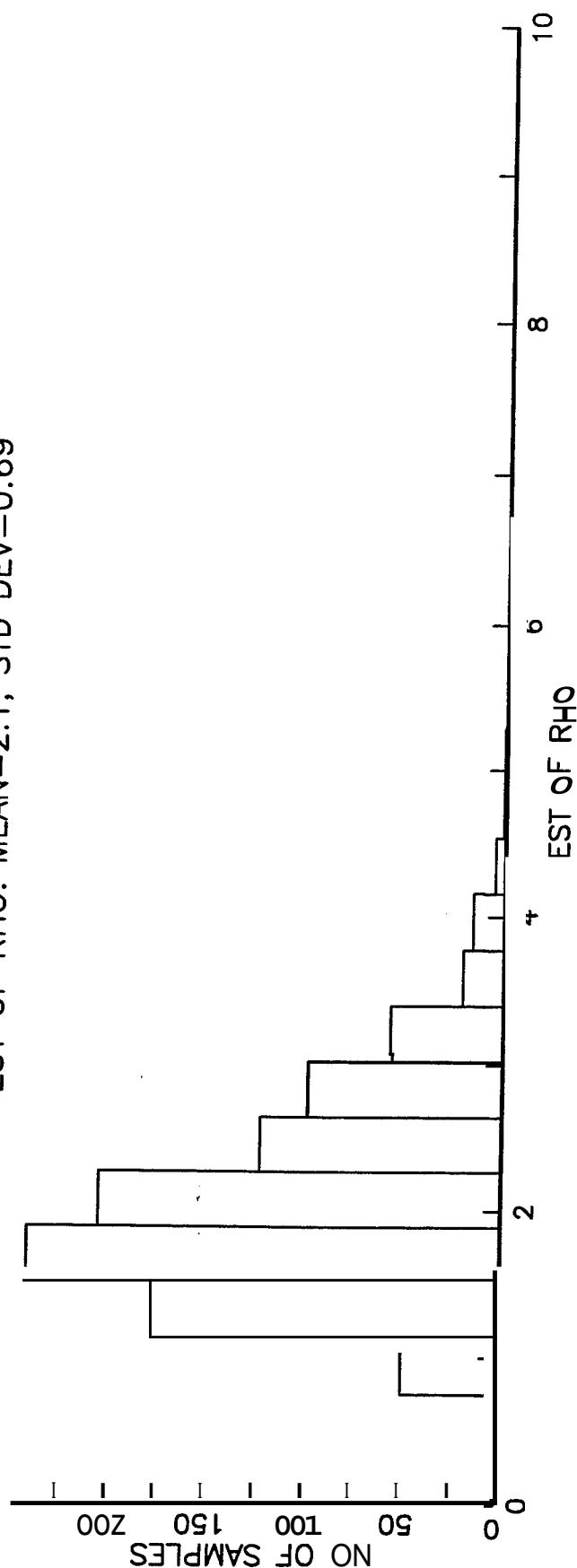


Figure 3

EST OF RHO: MEAN=2.1; STD DEV=0.69



5 SYSTEMS, 500 REPLICATIONS:TRUE KAPPA=0.8

EST OF KAPPA: MEAN=0.80; STD DEV=0.11

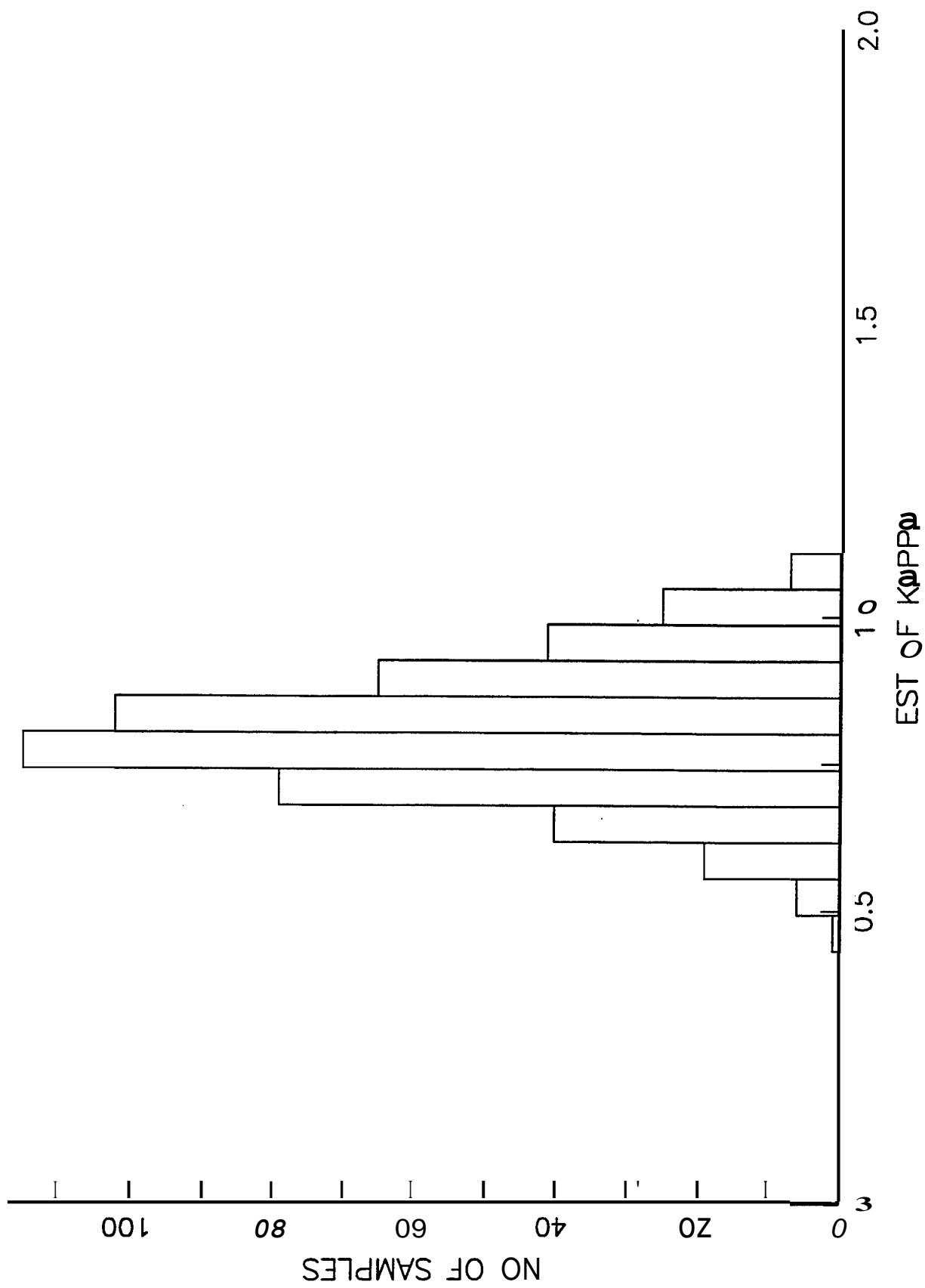


Figure 4

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