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One-dimensional analysis of steady-flow air-water mixtures in pipes

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ONE-DIMENSIONAL ANALYSIS
OF STEADY-FLOW AIR-WATER
MIXTURES IN PIPES

MERSON BOOTH

1953

ONE-DIMENSIONAL ANALYSIS OF STEADY-FLOW AIR-WATER MIXTURES IN PIPES

by

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B.S., U. S. NAVAL ACADEMY

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May, 1953

ONE-DIMENSIONAL ANALYSIS OF STEADY-FLOW AIR-WATER
MIXTURES IN PIPES

By

Merson Booth, Lieutenant, U. S. Navy

Submitted to the Department of Naval Architecture and Marine Engineering On May 25, 1953, in partial fulfillment of the requirements for the degree of Naval Engineer.

ABSTRACT

The object of this thesis is to investigate a number of simple flow examples of air-water mixtures and to present the results in a simple quick reference form which can be valuable for engineering use in design and understanding the phenomena of flow of two-component mixtures. The problems analyzed are:

1. The one-dimensional trajectory of droplets accelerated in a gas stream

2. The variation of stream properties during the droplet acceleration process

3. The variation of stream properties due to wall friction with water present in the stream.

The procedure used was entirely analytical. The results of the first problem were calculated, tabulated, and plotted in dimensionless form for various water and air velocities, droplet diameters, distance moved down the duct, and water and air properties. The plots should be valuable for engineering use in determining the one-dimensional trajectory of spherical particles in a gas stream. An iteration procedure using the plots can be used where air velocity, droplet diameter, or stream properties vary during the acceleration.

The results of the second and third problem were calculated, tabulated, and plotted in dimensionless form for various water to air velocity ratios, water to air mass rates, air Mach numbers, and stream property ratios. The results should be valuable for engineering use in all regions except near choking conditions in the stream. Near this condition the air velocity increases too rapidly for the droplets to accelerate with the air. Since this was one of the assumptions in the analysis the plots are not correct in this region.

The analysis of the third problem incorporates in it a pseudo-frictional term accounting for the effect of momentum exchange of droplets with the duct wall. The exact nature of this effect is not known, but some correlation with experimental data is given.

Thesis Supervisor: A. H. Shapiro
Title: Professor of Mechanical Engineering

Cambridge, Massachusetts
May 25, 1953

Professor Earl B. Millard
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge, Massachusetts

Dear Sir:

In accordance with the requirements of the degree of Naval Engineer, I herewith submit a thesis entitled, "One-Dimensional Analysis of Steady-Flow Air-Water Mixtures in Pipes."

Respectfully,

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I. INTRODUCTION

The object of this thesis is to investigate a number of simple flow examples of air-water mixtures. The problems to be analyzed are:

(1) the one-dimensional trajectory of droplets accelerated in a gas stream,

(2) the variation of stream properties during the acceleration process, and

(3) the variations of stream properties due to wall friction with water present in the stream.

This investigation is undertaken as part of the Aerothermopressor Project at Massachusetts Institute of Technology. The Aerothermopressor is a device to increase the stagnation pressure of a gas stream by reducing the stagnation temperature through evaporation of water injected into the flow.

In the work on the Aerothermopressor, there has been no quick reference guide to aid in design or in understanding the phenomena of flow of air-water mixtures. The purpose of this thesis is to analyze these separate effects and to present the results in a simple quick reference form which can be valuable for engineering use in problems concerning flow of air-water mixtures.

II. DROPLET TRAJECTORY

Procedure

The example studied is the injection of one droplet at a given initial velocity into a moving gas stream. The problem is to determine the subsequent droplet velocities as a function of distance moved downstream and the properties of the gas and droplet.

The assumptions made are:

(1) the gas velocity is constant,

(2) the droplet is spherical and of constant diameter during the acceleration, and

(3) the table of drag coefficient vs. Reynolds number as given in reference (1) and repeated in Table I is correct.

The details of this analysis are given in the appendix. It was necessary to perform a graphical integration of a function of Reynolds number. The results of one of these integrations is contained in reference (1). The other integration was done using the trapozoidal rule. The results of both these integrations are given in Table II.

Results

Figures I and II are non-dimensional plots of droplet velocity vs. distance moved downstream as a function of gas velocity, droplet diameter, and the properties of the gas and droplet. This same information is given in Table III. Figure I is for droplet velocity less than gas velocity; Figure II is for droplet velocity greater than gas velocity. The plots in Figures I and II are normalized to specific initial droplet velocities. For droplet

velocity less than gas velocity, the initial droplet velocity is assumed zero. For droplet velocity greater than gas velocity, the initial droplet velocity is assumed twice the air velocity. For an initial droplet velocity other than the normalized conditions, it is necessary to subtract the distance read at the initial condition from that of the final condition. This is shown in more detail in the appendix.

Figures I and II are used in the following manner: From given initial droplet velocity, gas velocity, droplet diameter, and gas and droplet properties, calculate $\frac{V_w}{V_a}$ and $\frac{V_a d_w}{\nu_a}$. Enter the plot with these quantities to determine the initial point at the intersection of ordinate $\frac{V_w}{V_a}$ and the curve corresponding to $\frac{V_a d_w}{\nu_a}$. Read on the absicca the initial value $\frac{X}{d_w}\ \frac{\rho_a}{\rho_w}$. Now follow along the same $\frac{V_a d_w}{\nu_a}$ curve to the intersection of that curve and the desired $\frac{V_w}{V_a}$ or $\frac{X}{d_w}\ \frac{\rho_a}{\rho_w}$ distance from the initial point. Read off the value of $\frac{X}{d_w}\frac{\rho_a}{\rho_w}$ which gives the distance from the initial point with the required $\frac{V_w}{V_a}$ or the value of $\frac{V_w}{V_a}$ at that distance respectively.

Figures III and IV are plots of distances required to reach specific values of $\frac{V_w}{V_a}$ = .5,.1 as a function of droplet diameter, air Mach number, and given air properties assuming initial droplet velocity is zero. Figure III is for air stagnation temperature of 70°F and stagnation pressure of 1 atmosphere. Figure IV is for air stagnation temperature of 1500°R and stagnation pressure of 1 atmosphere.

If the gas velocity, droplet diameter, or the stream and droplet properties are not constant, the solution can be approximated by assuming the variation to occur in a stepwise fashion and apply the plots successively over each step. This can be done in the case of evaporation in which a curve of droplet diameter vs. droplet velocity or distance may be assumed, or in the

case of a converging or diverging section in which, because of known area changes, a curve of gas velocity vs. distance can be assumed.

Conclusion

The results obtained are given in dimensionless form in as simple a manner as possible. The plots should be valuable for engineering use in determining one-dimensional trajectory of spherical particles in a gas stream.

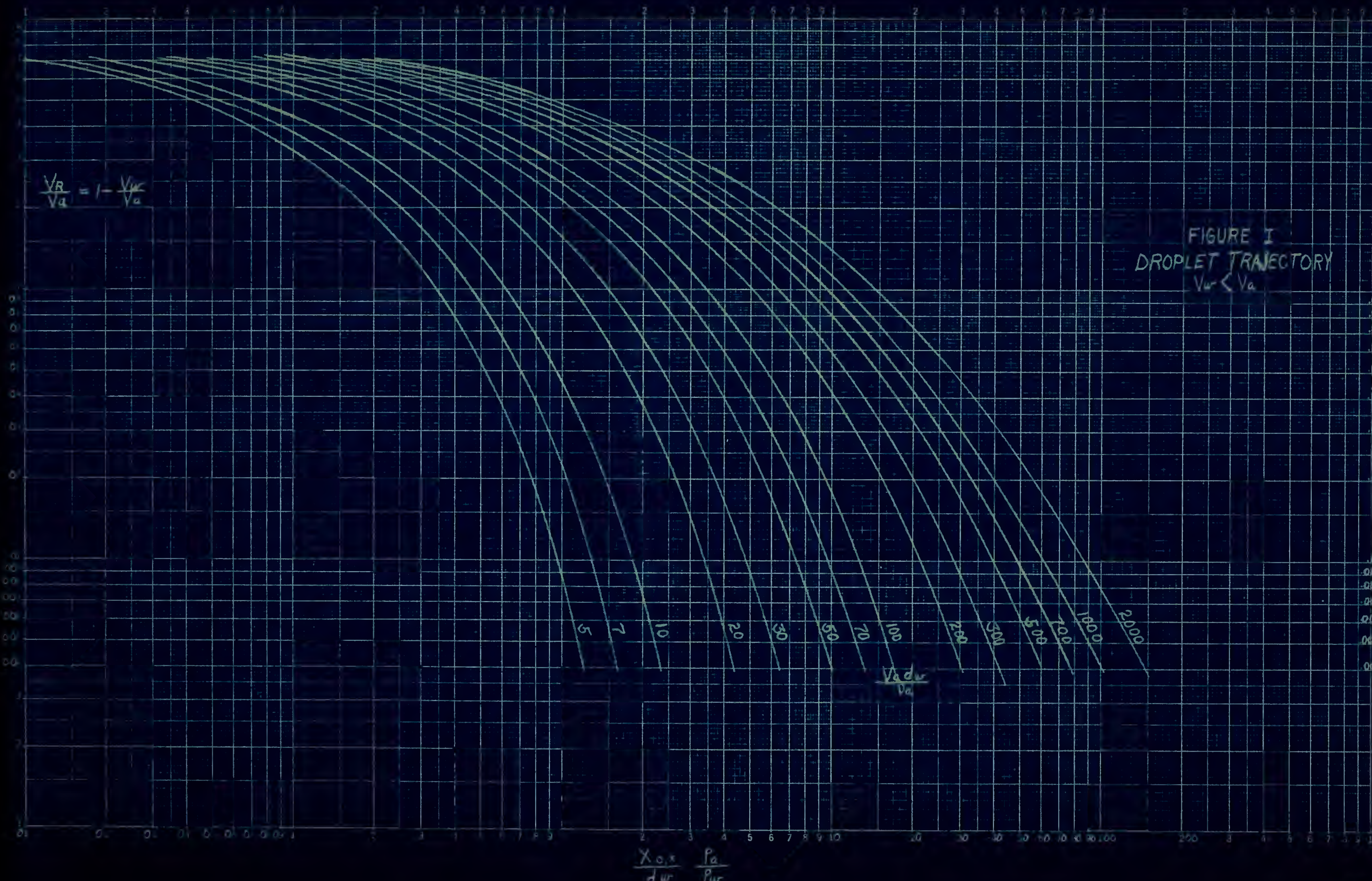
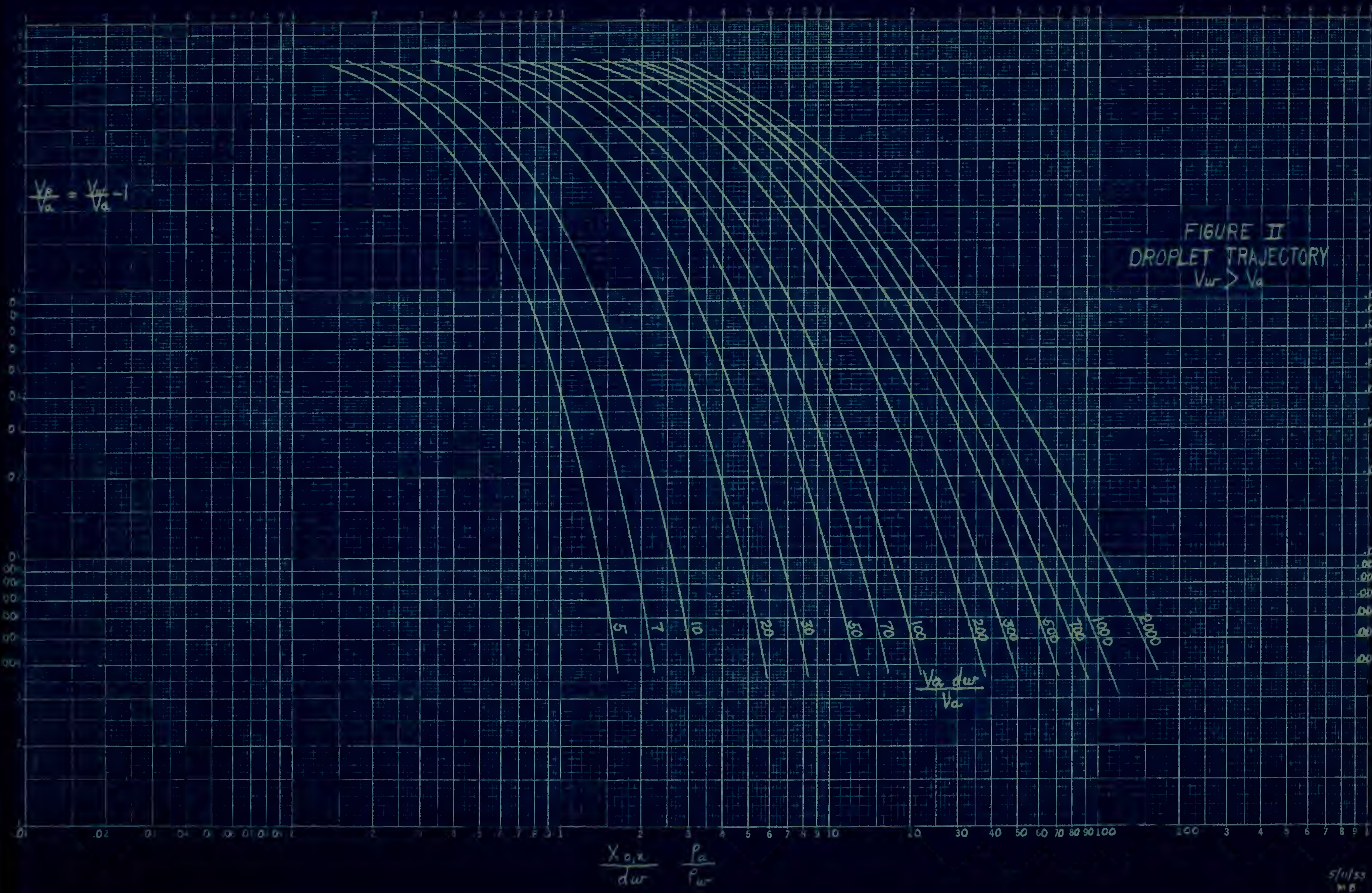


FIGURE I
DROPLET TRAJECTORY
$V_w < V_a$
$\frac{V_R}{V_a} = 1 - \frac{V_w}{V_a}$
5
7
10
20
30
50
70
100
200
300
500
700
1000
2000
$\frac{V_a d_w}{\nu_a}$
$\frac{X_{0,x}}{d_w} \frac{\rho_a}{\rho_w}$



$\frac{V_R}{V_a} = \frac{V_w}{V_a} - 1$
FIGURE II
DROPLET TRAJECTORY
$V_w > V_a$
5
7
10
20
30
50
70
100
200
300
500
700
1000
2000
$\frac{V_a\, d_w}{V_a}$
$\frac{X_{o,x}}{d_w} \quad \frac{\rho_a}{\rho_w}$
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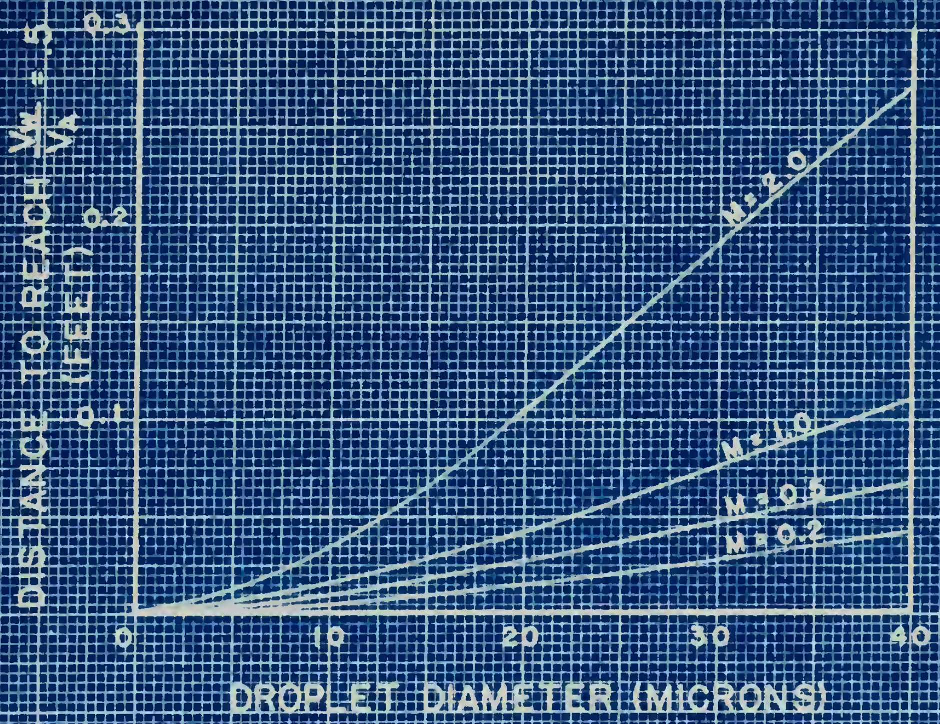
FIGURE III
DROPLET TRAJECTORY
$T_o = 70^oF$
$P_o = 1$ ATMOS.



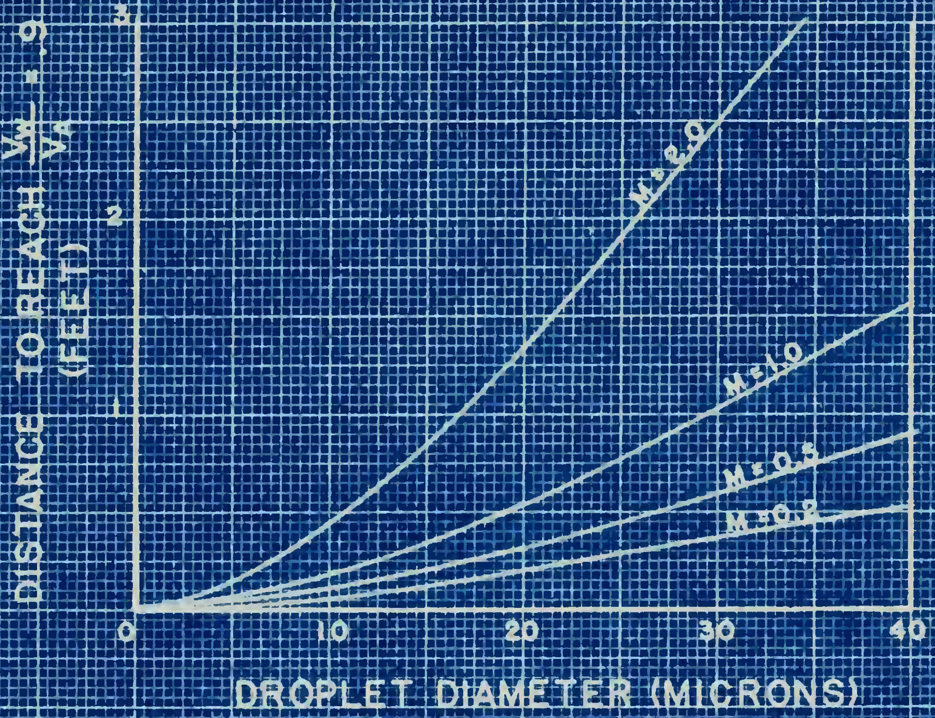


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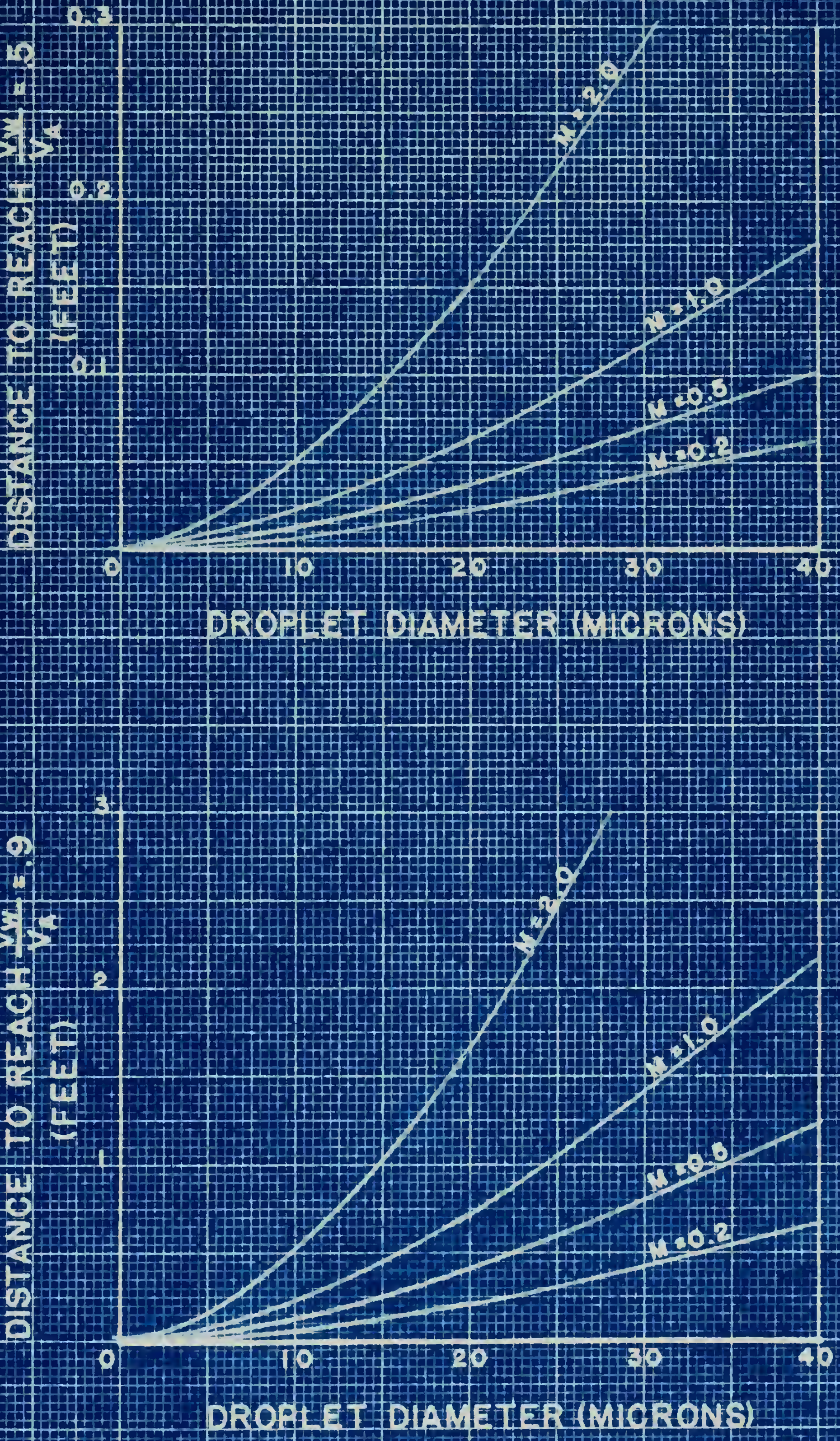
FIGURE IV
DROPLET TRAJECTORY
$T_0 = 1500^{\circ}R$
$P_0 = 1$ ATMOS.



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III. DISCONTINUITY ANALYSIS

Procedure

The example studied is the flow of an air-water mixture past two sections 1 and 2 in the flow. The problem is to determine the stream properties at section 2 as a function of the water velocity at 2, the water rate, and the stream properties at section 1.

The assumptions made are:

(1) Constant area flow

(2) Adiabatic—no change in stagnation temperature

(3) No evaporation of water droplets

(4) No change in temperature of droplets

(5) Perfect gas relation holds for air.

The details of this analysis are given in the appendix. The general method of performing the analysis is given in reference (2).

Results

Values of $\frac{T_o}{T_o^*}$, $\frac{T}{T^*}$, $\frac{P}{P^*}$, and $\frac{P_o}{P_o^*}$ were calculated for $\frac{V_w}{V_a} = 0, 1$ for various water rates and Mach numbers. The star condition is a normalized condition corresponding to $\frac{V_w}{V_a} = 0$ and $M = 1$ and does not depend on the water rate. These values are tabulated in Tables IV through VIII. The guiding principle in using the tables is that $\frac{T_o}{T_o^*}$ is constant in going between tables.

Figures V through X show the effect of droplet acceleration on stream properties assuming that at section 1, $\frac{V_w}{V_a} = 0$, and at section 2, $\frac{V_w}{V_a} = 1$ for various initial Mach numbers and water rates. The numbers on each curve indicate the water rate to air rate ratio, $\frac{w_w}{w_a}$, and the letter S on some branches of the curves indicate that a normal shock in the flow is necessary to reach these states.

Figures V through X are used in the following manner: Knowing the stream properties at section 1, the point of injection of the water, enter the required plot with the M_1 and $\frac{w_w}{w_a}$. Read on the ordinate the value of the ratio of the desired property at section 2 to that at section 1.

Conclusion

The plots given are easy to use and are correct within the assumptions. However, near choking conditions the air velocity increases very rapidly. It is erroneous to assume that the droplets will be able to accelerate with the stream in the absence of very large drag coefficients. Therefore, near choking conditions the plots will not give a correct answer to the variation of stream properties due to droplet acceleration. However, the value of M_1 where choking of the flow at section 2 is indicated is approximately correct.

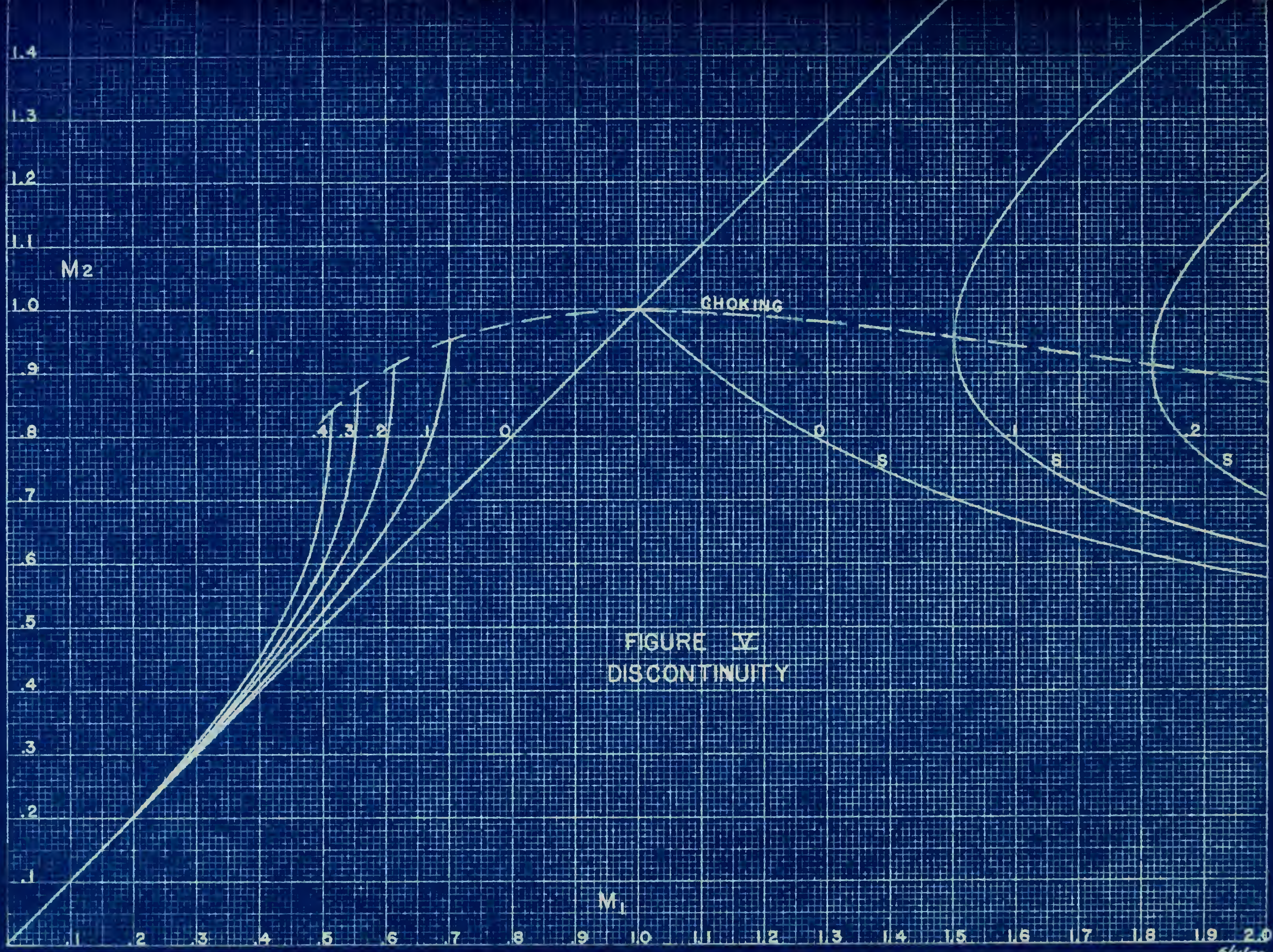


FIGURE V
DISCONTINUITY

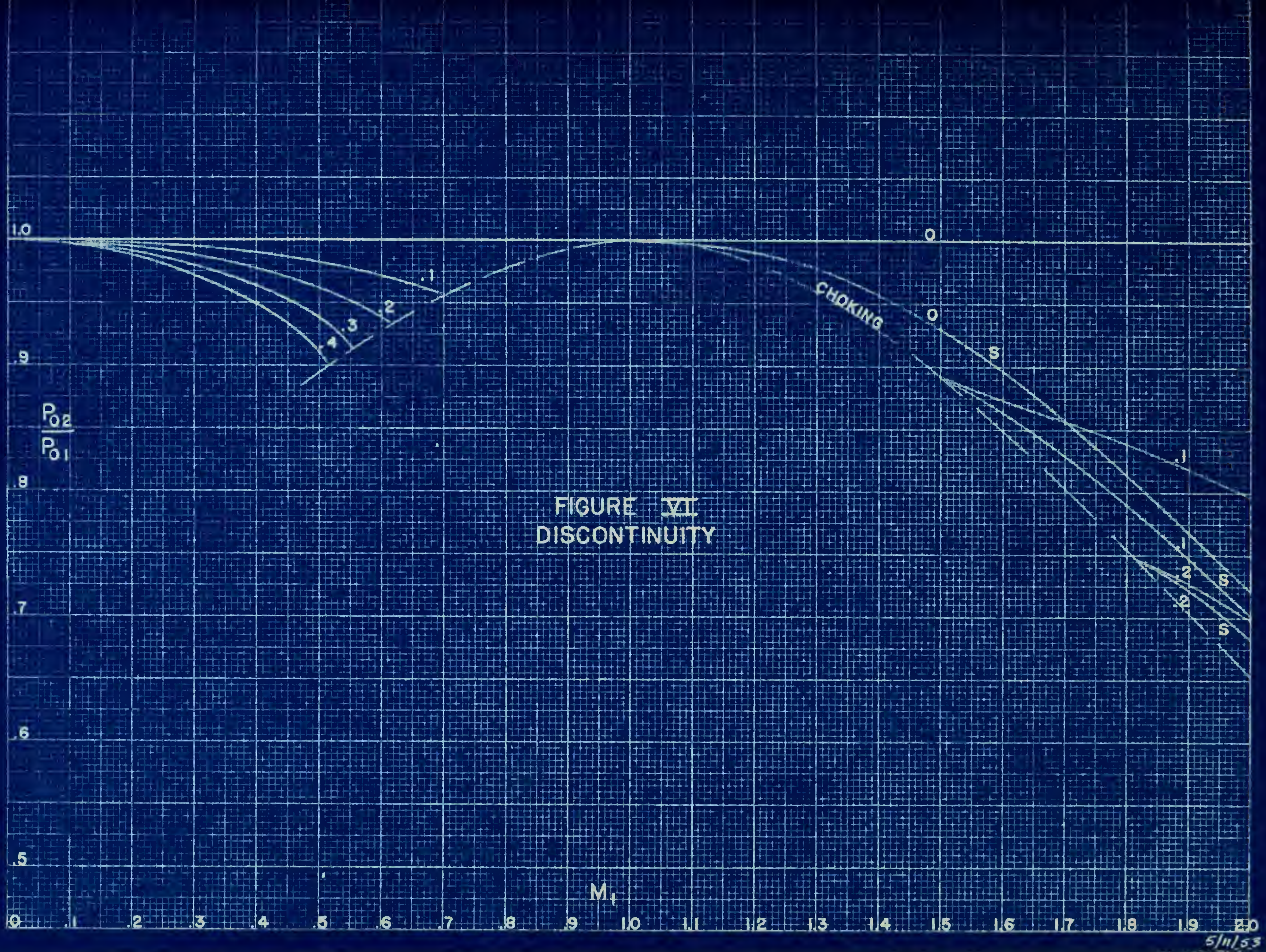


FIGURE VI
DISCONTINUITY

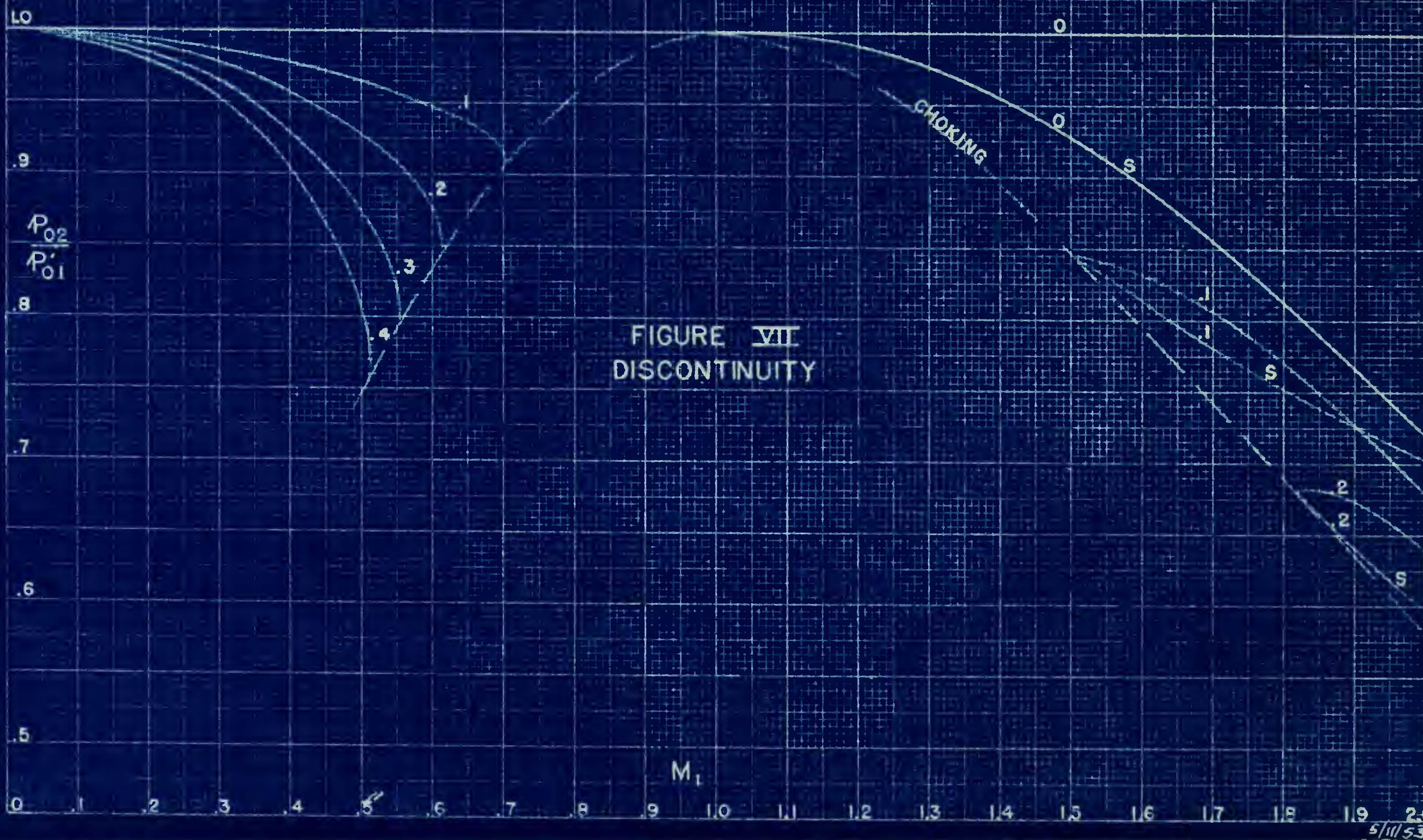


FIGURE VII
DISCONTINUITY

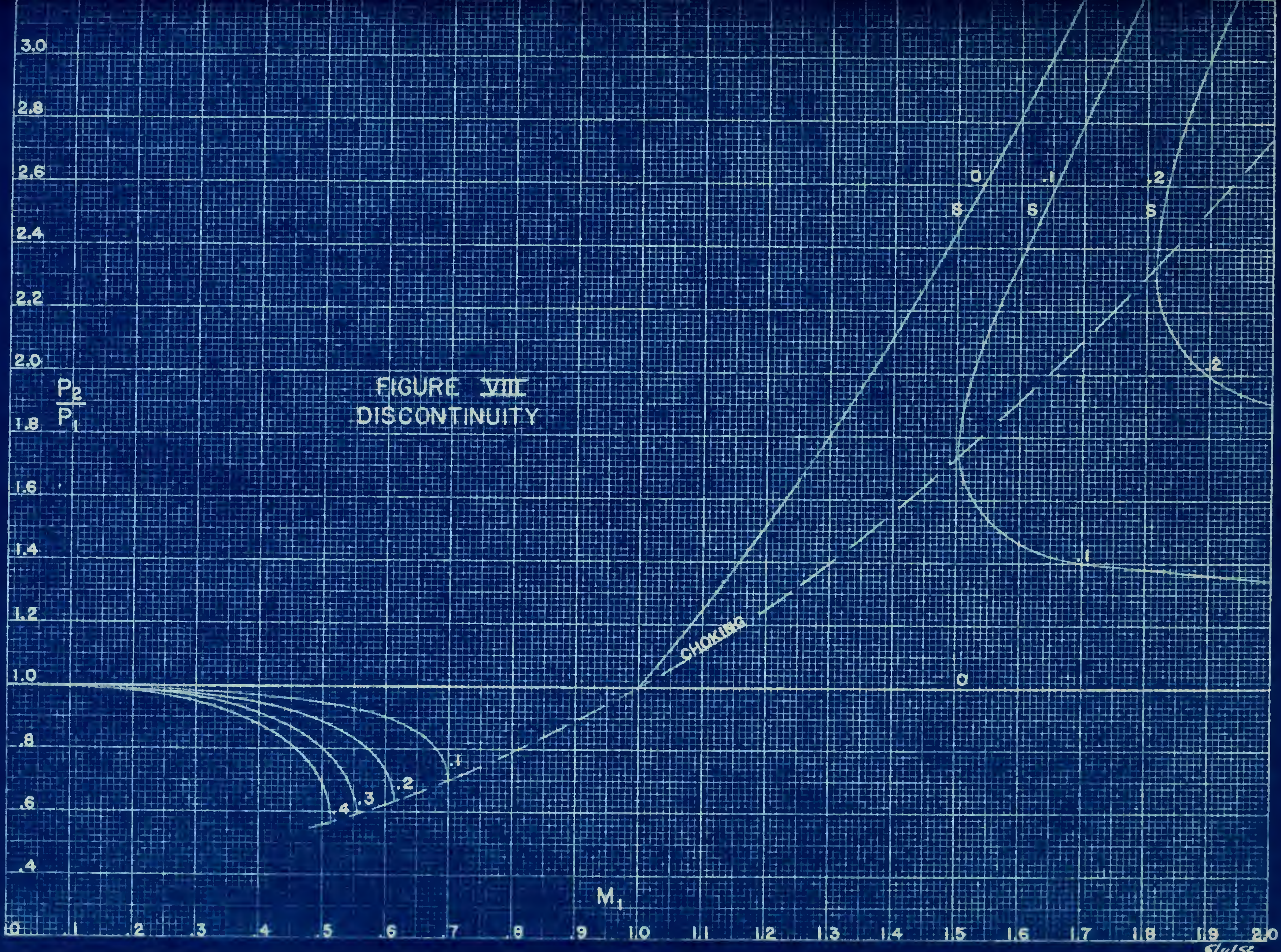


FIGURE VIII
DISCONTINUITY
$\frac{P_2}{P_1}$
M_1
CHOKING
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0

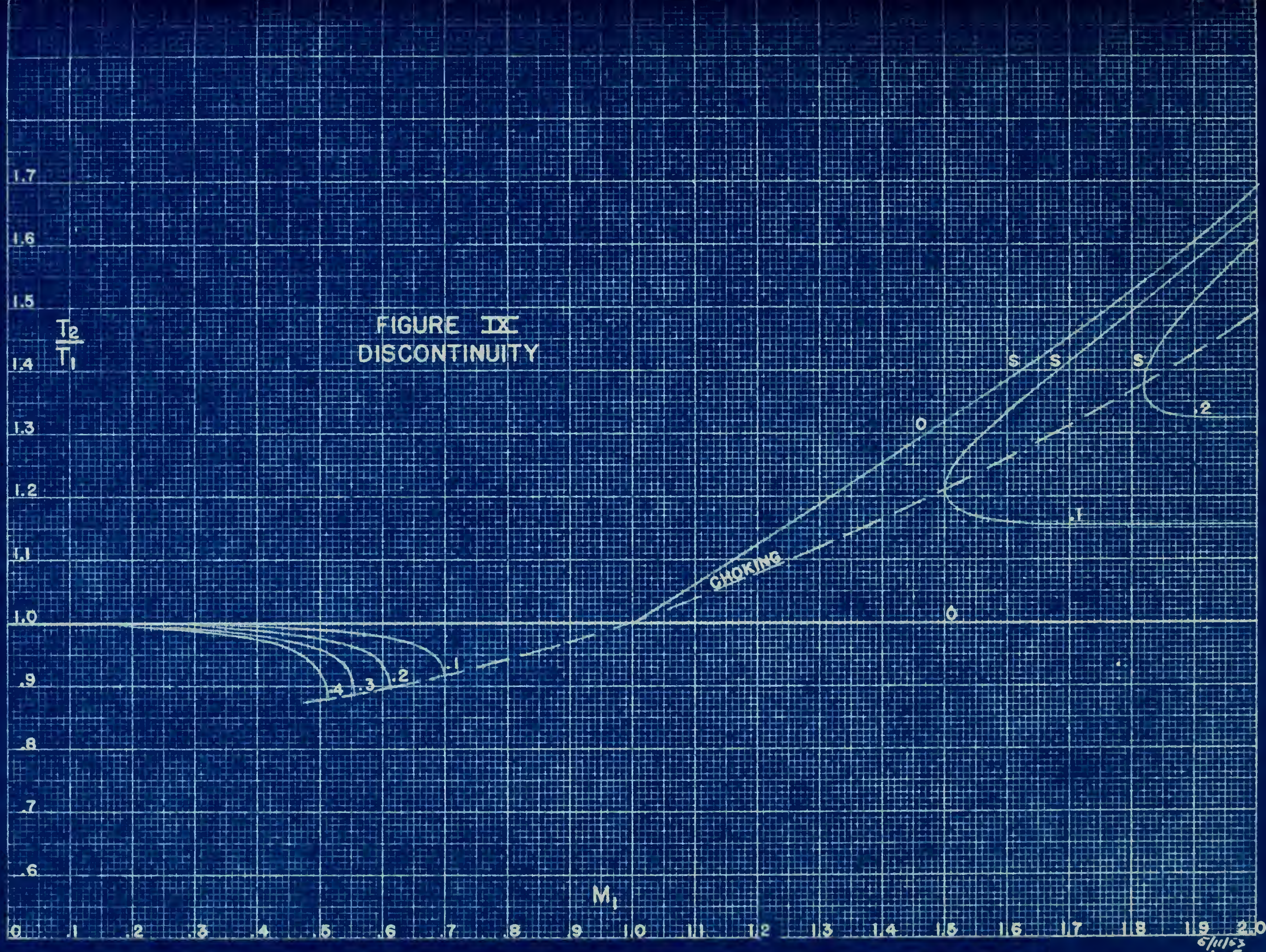


FIGURE IX
DISCONTINUITY
T2/T1
M1
CHOKING
S
S
S
O
O
.1
.2
.3
.4
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
.6
.7
.8
.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7

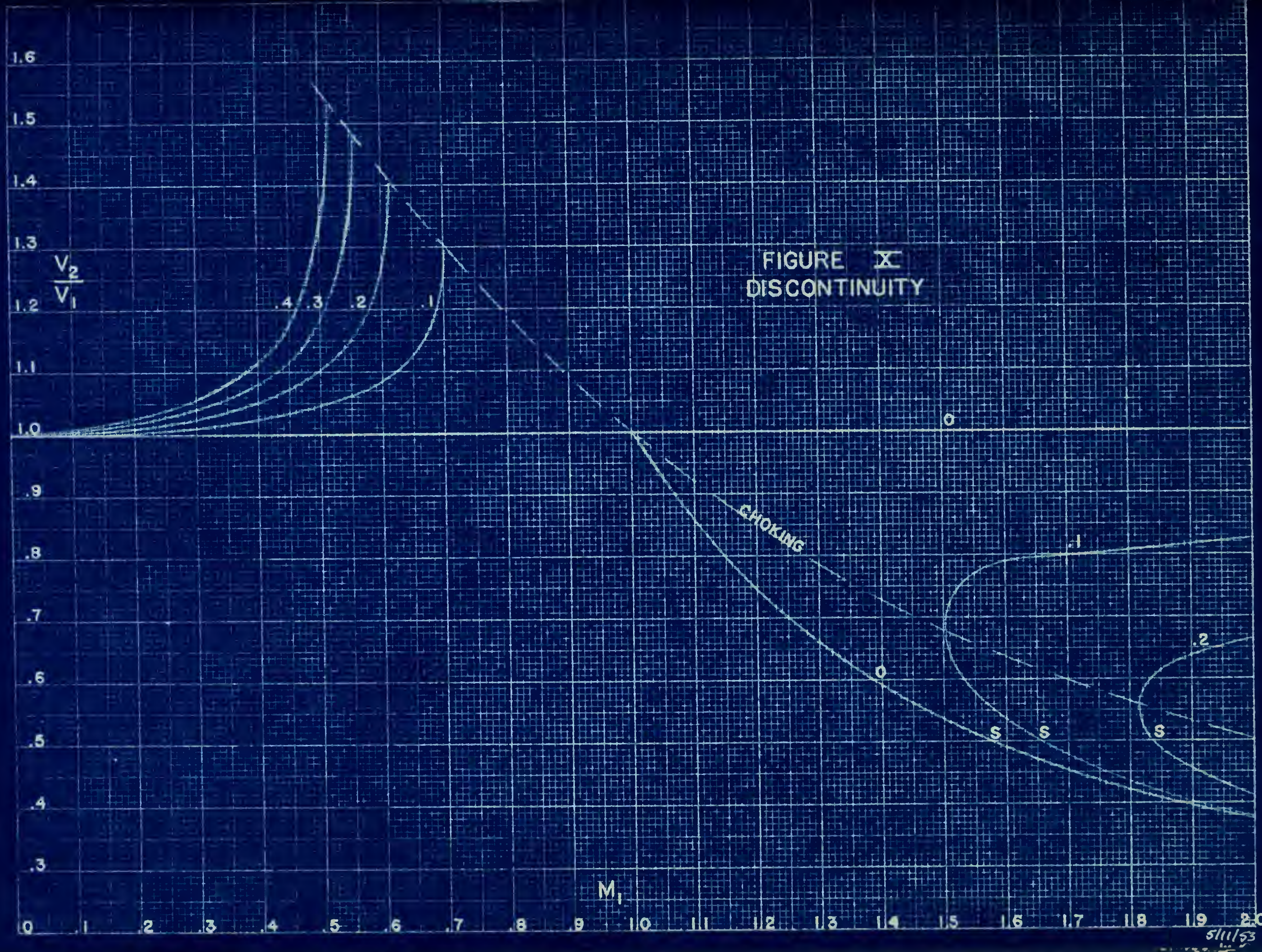


FIGURE X
DISCONTINUITY
V2/V1
M1
CHOKING
.1
.2
.3
.4
0
S
5/11/53

IV. MODIFIED FANNO LINE ANALYSIS

Procedure

The example studied is the flow of an air-water mixture in a pipe with wall friction and droplets striking the wall. The problem is, knowing the initial stream properties and friction factor, to determine the stream properties at any section downstream.

The assumptions made are:

(1) Constant area flow

(2) Adiabatic—no change in stagnation temperature

(3) No evaporation of water droplets

(4) No change in temperature of droplets

(5) Perfect gas relation holds for air

(6) For droplet entrained in air stream; $\frac{V_W}{V_a} = 1$

(7) For droplet on the wall; $\frac{V_W}{V_a} = 0$

(8) In each differential section an amount of water leaves the stream and hits the wall, and an equal amount of water leaves the wall and is picked up by the stream

(9) That this amount of water is proportional to the length of section

The details of this analysis are given in the appendix. The general method of performing the analysis is given in reference (2).

Results

Values of $\left[4 \frac{f}{D} + K\right] L_{max}$, $\frac{P_o}{P_o{*}}$, $\frac{\rho_o}{\rho_o{*}}$, $\frac{P}{P*}$, $\frac{T}{T*}$, $\frac{V}{V*}$, and $\frac{F}{F*}$ were calculated for various water rates and Mach numbers. The star condition is a normalized condition corresponding to choking of the flow. These values,

to slide rule accuracy, are tabulated in Tables XII through XV and plotted in Figures XI through XVII. These figures show the effect of friction and droplet momentum exchange with the walls on the stream properties. The numbers on each curve indicate the water rate to air rate ratio, $\frac{w_w}{w_a}$.

The symbol K in the term $\left[4\frac{f}{D} + K\right] L_{max}$ incorporates as a pseudo-frictional term the effect of momentum exchange of droplets with the wall. This term is a function of the water rate to air rate ratio, $\frac{w_w}{w_a}$, the duct diameter, and the droplet diameter and perhaps other stream properties. The nature of this function is not known. However, using the data of reference (3) for sand together with the low Mach number analysis described in the appendix, it appears that $K = .4 \frac{w_w}{w_a}$ for 1" diameter pipe and sand diameter of 200 μ and 450 μ. It appears that the constant of proportionality is slightly smaller for smaller sand diameters. The effect of duct diameter is no known.

Figures XI through XVII are used in the following manner: Knowing the initial stream properties enter Figure XI with the initial Mach number, M_1, and $\frac{w_w}{w_a}$ and read off the value $\left[4\frac{f}{D} + K\right] L_{max}$. Then knowing $\left[4\frac{f}{D} + K\right]$ and the length of duct from the initial section 1 to any other section 2, calculate $\left[4\frac{f}{D} + K\right] L_{1,2}$. Then calculate $\left[4\frac{f}{D} + K\right] L_{max_2} = \left[4\frac{f}{D} + K\right] L_{max_1} - \left[4\frac{f}{D} + K\right] L_{1,2}$. Enter Figure XI with $\left[4\frac{f}{D} + K\right] L_{max_2}$ and $\frac{w_w}{w_a}$ and read off M_2. Then entering the desired plot of properties with M_1, M_2, and $\frac{w_w}{w_a}$, read off the value of the normalized property at sections 1 and 2. From this form the ratio of the property at 2 to that at 1. Knowing the value of the property at 1, the value at 2 can be calculated.

Figure XVII is used to determine the Mach number after a normal shock knowing the Mach number and $\frac{w_w}{w_a}$ just before the shock and assuming $\frac{V_w}{V_a} = 1$ on both sides of the shock. Enter the plot with initial M and $\frac{w_w}{w_a}$ to find initial

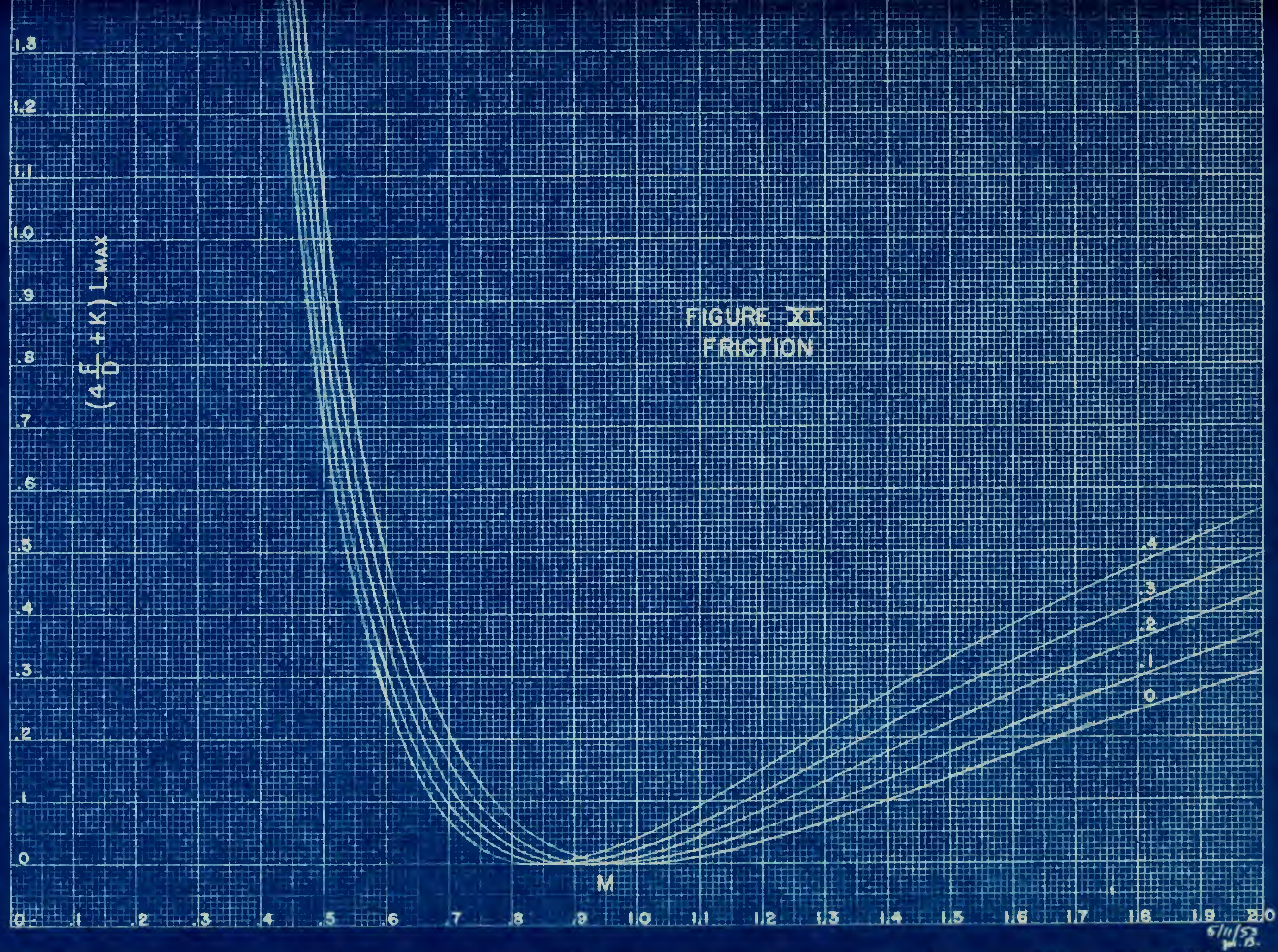
point. Follow across horizontally with constant $\frac{F}{pa}$ until reaching the same $\frac{w_w}{w_a}$ curve. Read the value of M after the shock.

Conclusion

Provided more information can be obtained concerning the pseudo-frictional term, K, the plots given can be valuable for engineering use. This could well form the basis of a thesis to determine whether this simple model of the flow is correct and to correlate data in the field to two-component flows.

In addition, very near choking conditions this model of the flow is not correct since the air velocity increases very rapidly in a very short length. It is erroneous to assume that the droplets can accelerate with the stream in the absence of very large drag coefficients. Therefore, near choking conditions the plots will not give a correct answer to the variation of stream properties due to friction.

FIGURE XI
FRICTION



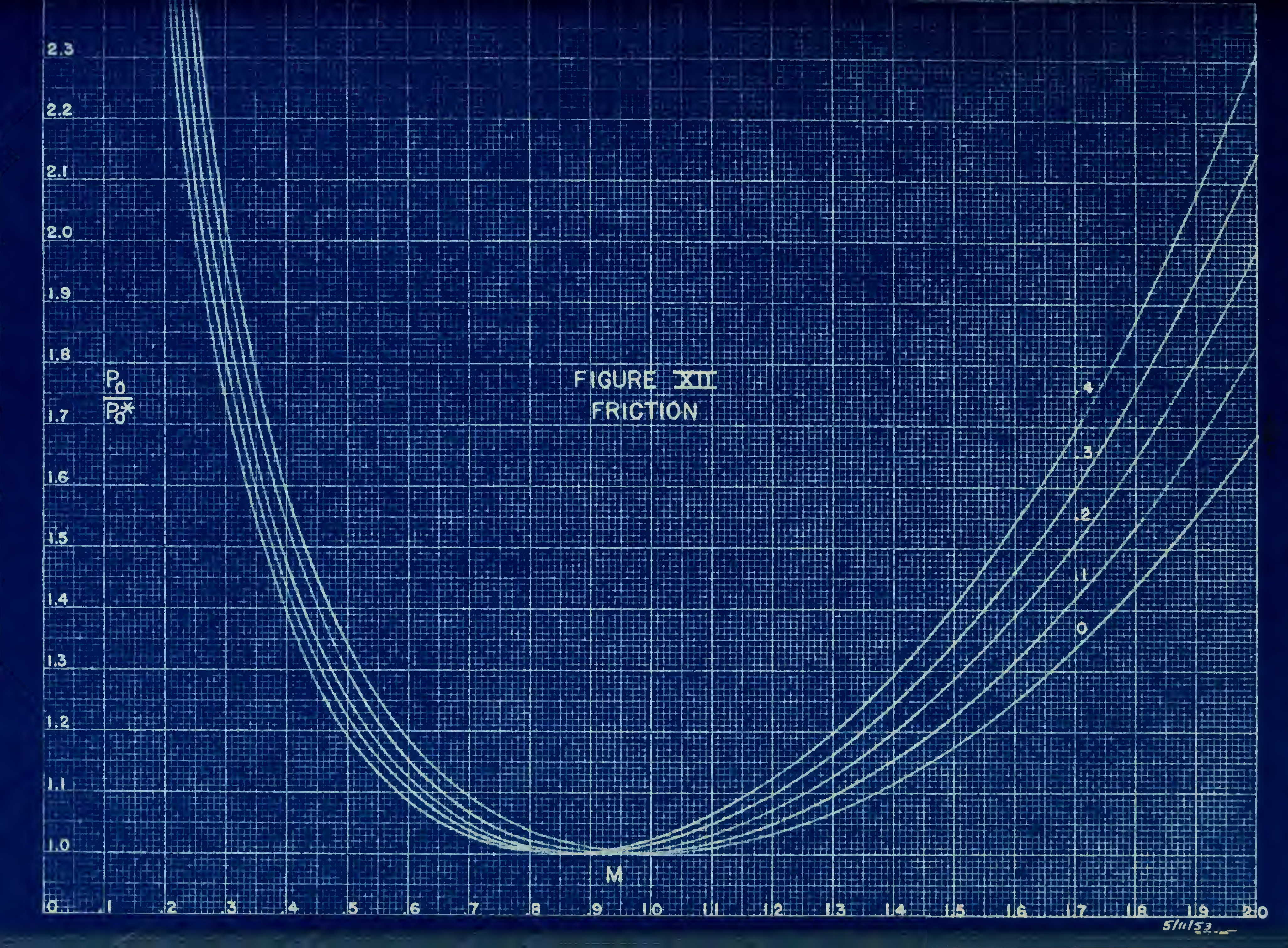


FIGURE XII
FRICTION
$\frac{P_0}{P_0^*}$
M
.4
.3
.2
.1
0
2.3
2.2
2.1
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
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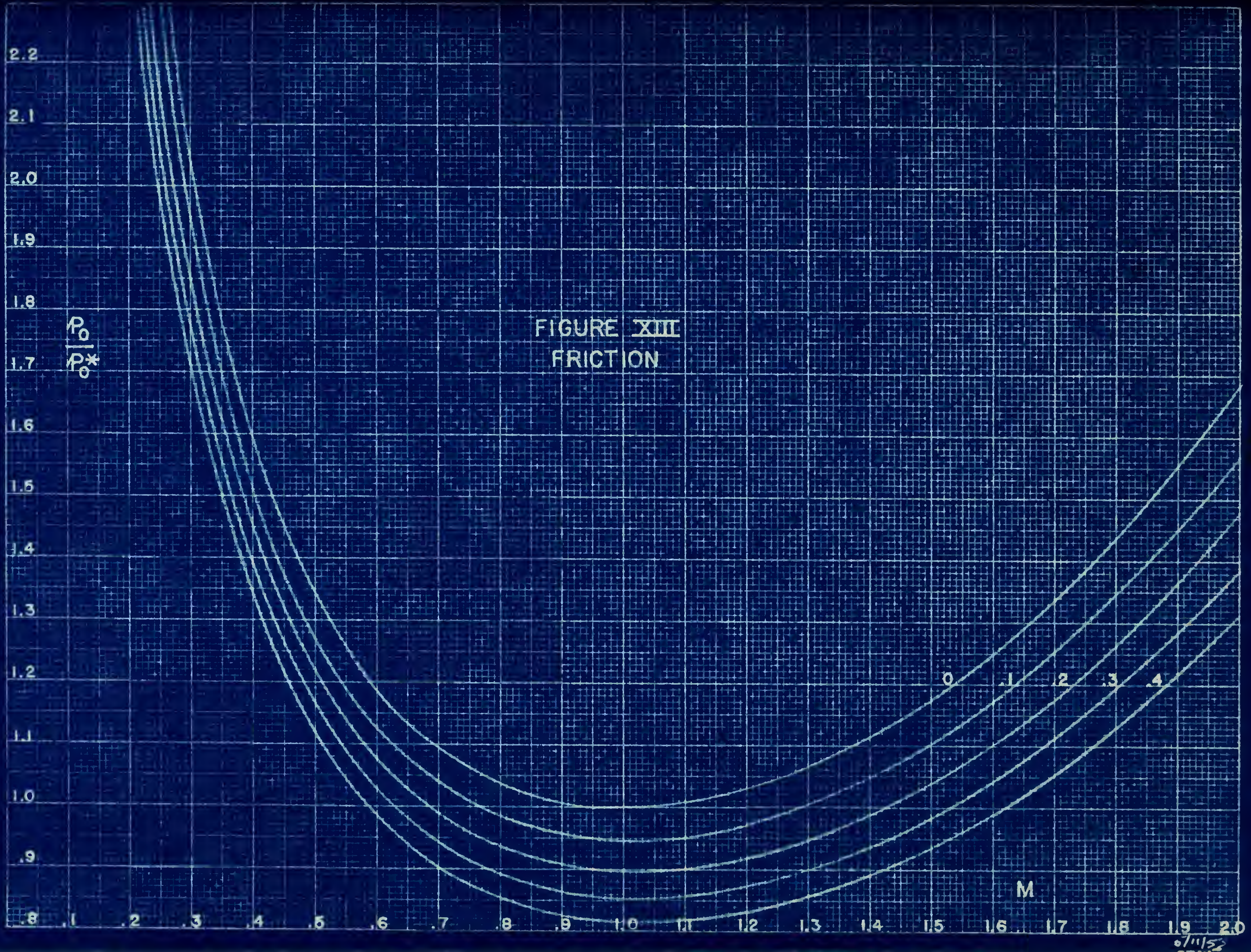


FIGURE XIII
FRICTION
P_o / P_o^*
M
2.2
2.1
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
.9
.8
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
0
.1
.2
.3
.4

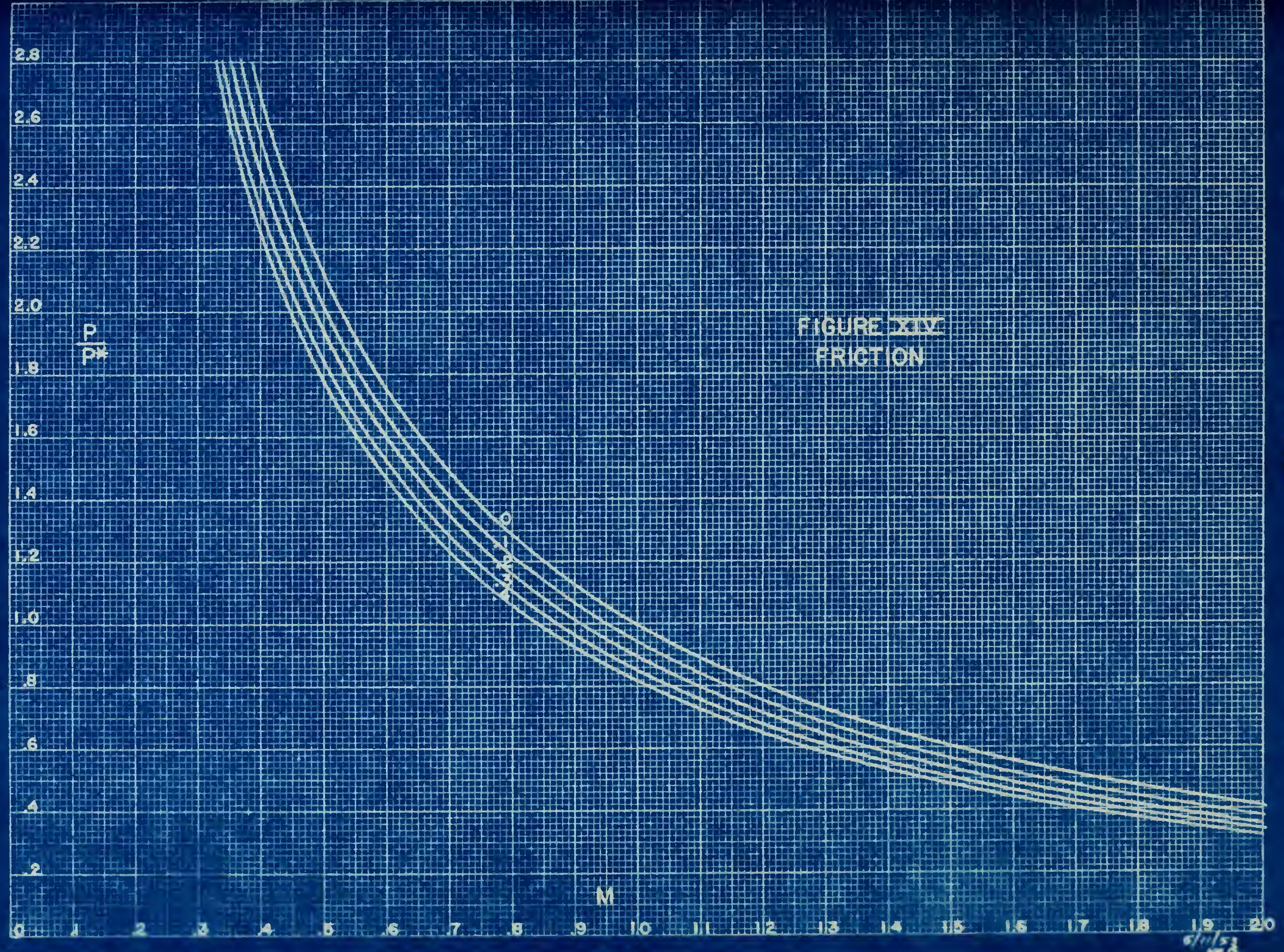


FIGURE XIV
FRICTION

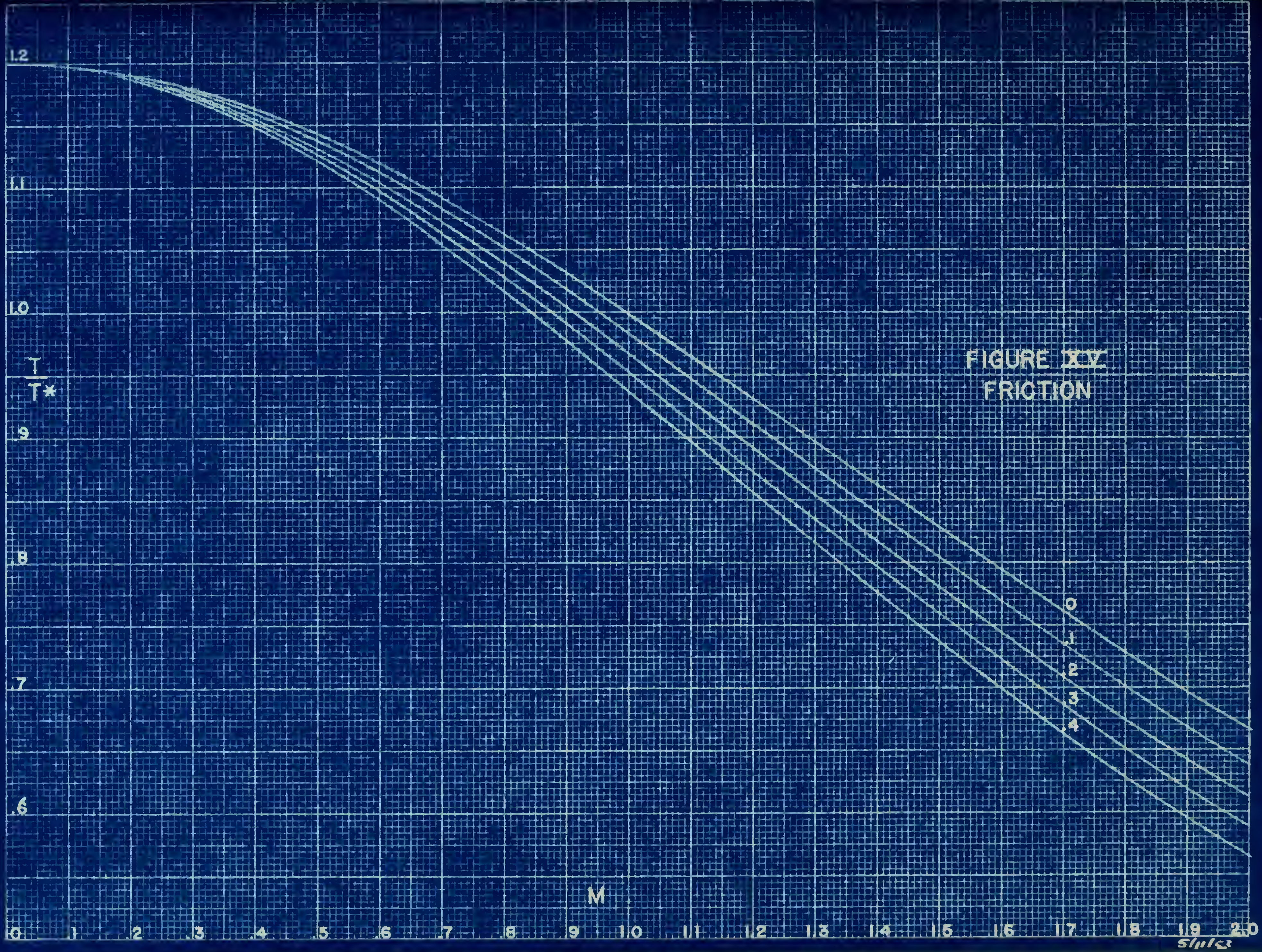


FIGURE XV
FRICTION
T/T*
1.2
1.1
1.0
.9
.8
.7
.6
M
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
0
1
2
3
4

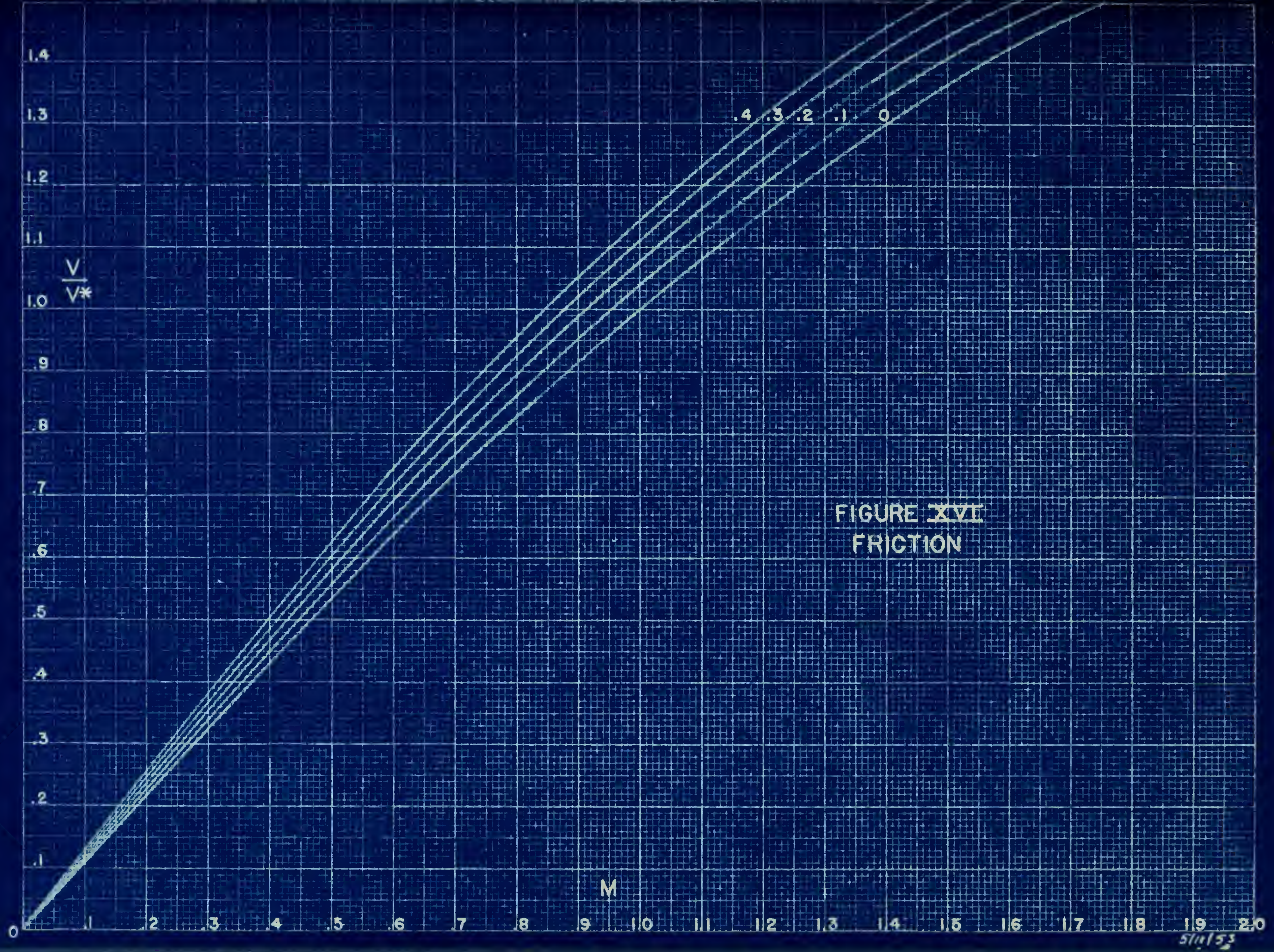


FIGURE XVI
FRICTION

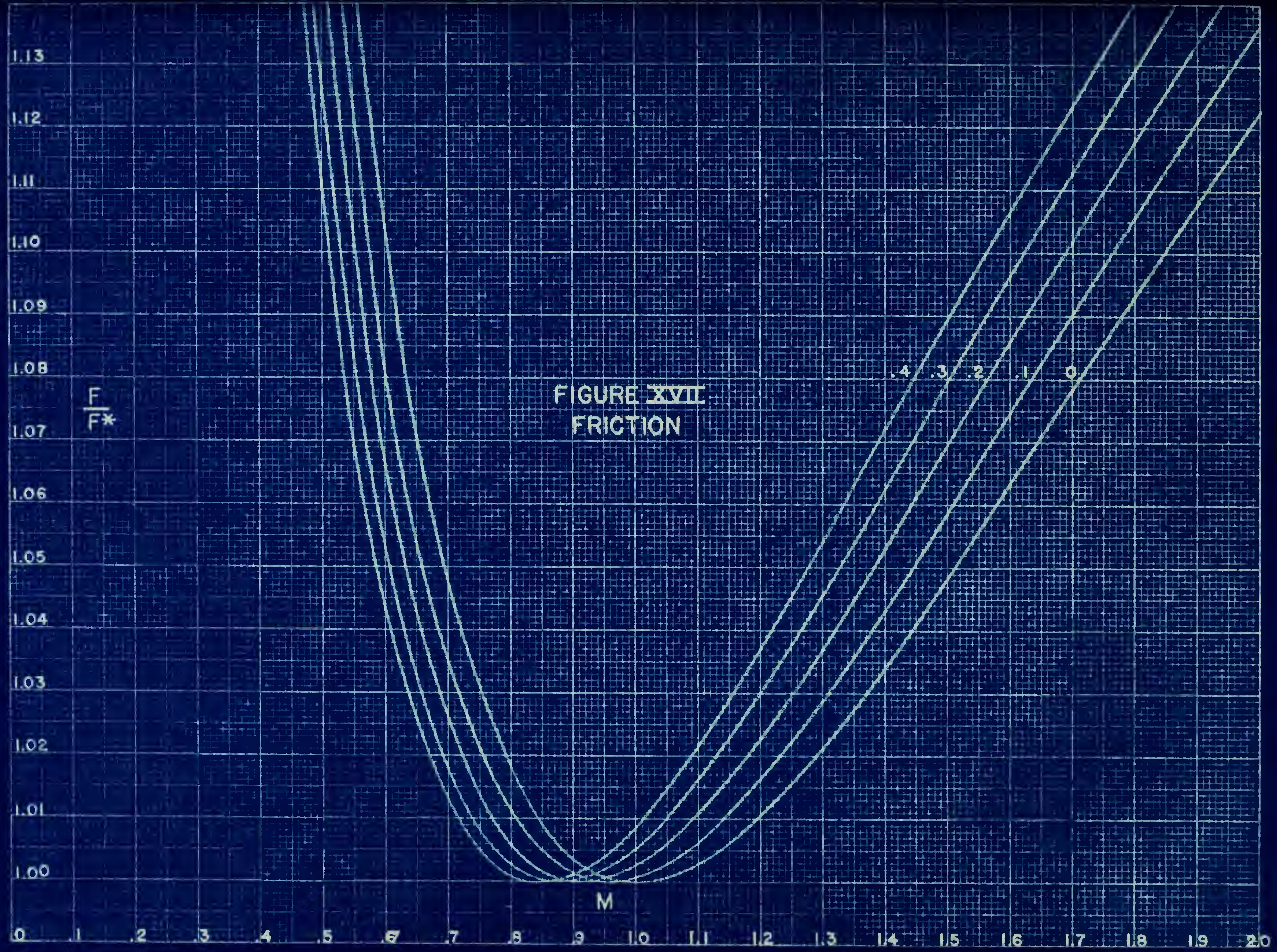


FIGURE XVII
FRICTION
F/F*
M
.4 .3 .2 .1 0
1.13
1.12
1.11
1.10
1.09
1.08
1.07
1.06
1.05
1.04
1.03
1.02
1.01
1.00
0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0

APPENDIX

SYMBOLS

A = CROSS SECTIONAL AREA
C = SPEED OF SOUND
C_D = DRAG COEFFICIENT
C_P = SPECIFIC HEAT AT CONSTANT PRESSURE
D = HYDRAULIC DIAMETER OF DUCT
d_w = DROPLET DIAMETER
f = FRICTION COEFFICIENT OF DUCT
F = IMPULSE FUNCTION $(PA + \rho A V^2)$
h = SPECIFIC ENTHALPY
K = RATIO OF SPECIFIC HEATS (C_P/C_V)
L_{MAX} = MAXIMUM LENGTH OF DUCT FOR CHOKING FLOW
M = MACH NUMBER
m = MASS OF DROPLET
P = STATIC PRESSURE
p_0, P_0 = ISENTROPIC STAGNATION PRESSURE
R = REYNOLDS NUMBER
T = TEMPERATURE (ABSOLUTE)
T_0 = ISENTROPIC STAGNATION TEMPERATURE (ABSOLUTE)
t = TIME
V = VELOCITY
w = MASS RATE OF FLOW
X = DISTANCE ALONG DUCT
ρ = MASS DENSITY
ν = KINEMATIC VISCOSITY
$(\ )^*$ = REFERS TO A NORMALIZED CHOKING CONDITION
$(\ )_a$ = REFERS TO AIR
$(\ )_w$ = REFERS TO WATER
$(\ )_0$ = REFERS TO ISENTROPIC STAGNATION CONDITION
$(\ )_1$ = REFERS TO SECTION 1
$(\ )_2$ = REFERS TO SECTION 2
$(\ )_R$ = REFERS TO CONDITION RELATIVE TO AIR

DROPLET TRAJECTORY - DETAILS OF ANALYSIS

INDIVIDUAL DROPLETS INJECTED INTO MOVING AIR STREAM

LET: V_a = CONSTANT

DEFINE: $V_R \equiv V_a - V_w$ (FOR $V_a > V_w$)

$V_R \equiv V_w - V_a$ (FOR $V_a < V_w$)

$$R_R \equiv \frac{\rho_a \, d_w \, V_R}{\mu_a}$$

FORCE EQUATION ON DROPLET

$$m \frac{dV_R}{dt} = -C_D \frac{\rho_a}{2} A V_R^2$$

$$\frac{dR_R}{C_D R_R^2} = -\frac{\mu_a A}{2 d_w m} \, dt$$

$$A = \frac{1}{4} \pi d_w^2$$

$$m = \frac{1}{6} \rho_w \pi d_w^3$$

$$\therefore \frac{dR_R}{C_D R_R^2} = -\frac{3}{4} \frac{\mu_a}{\rho_w d_w^2} \, dt$$

$$t_{1,2} = \frac{4}{3} \frac{\rho_w d_w^2}{\mu_a} \int_{R_{R2}}^{R_{R1}} \frac{dR_R}{C_D R_R^2} \qquad (1)$$

$$\frac{dV_R}{dt} = V_R \frac{dV_R}{dx_R}$$

$$\therefore \frac{dR_R}{C_D R_R} = -\frac{\rho_a A}{2m} \, dx_R$$

DROPLET TRAJECTORY - CON'T

$$\frac{d\,R_R}{C_D R_R} = -\frac{3}{4}\,\frac{\rho_a}{\rho_w\, d_w}\, d\,X_R$$

$$X_{R\,1,2} = \frac{4}{3}\,\frac{\rho_w\, d_w}{\rho_a}\int_{R_{R2}}^{R_{R1}} \frac{d\,R_R}{C_D\, R_R} \qquad (2)$$

IN TERMS OF REAL DISTANCES

$$X_{1,2} = V_a\, t_{1,2} + X_{R\,1,2} \qquad (3)$$

NOTE: IF $V_a > V_w$, X_R IS IN NEGATIVE DIRECTION

IN EQUATION (4) SIGN OF LAST TERM IS:

$+$ IF $V_a < V_w$
$-$ IF $V_a > V_w$

$$\frac{X_{1,2}}{d_w}\,\frac{\rho_a}{\rho_w} = \frac{4}{3}\left[\frac{\rho_a V_a d_w}{\mu_a}\int_{R_{R2}}^{R_{R1}} \frac{d\,R_R}{C_D\, R_R^2} \pm \int_{R_{R2}}^{R_{R1}} \frac{d\,R_R}{C_D\, R_R}\right] \qquad (4)$$

NORMALIZING EQUATION (4) SO THAT INITIALLY, AT POINT 1, $V_R = V_a$; SO THAT:

$V_w = 0$ FOR $V_a > V_w$
$V_w = 2V_a$ FOR $V_a < V_w$

LET THIS NORMALIZED POSITION BE POINT 0, AND ANY OTHER POINT DOWNSTREAM BE POINT X.

DROPLET TRAJECTORY - CON'T

$$\frac{X_{o,x}}{d_w}\,\frac{\rho_a}{\rho_w} = \frac{4}{3}\left[\frac{V_a d_w}{\nu_a}\int_{\frac{V_{Rx} d_w}{\nu_a}}^{\frac{V_a d_w}{\nu_a}} \frac{dR_R}{C_D R_R^2} \pm \int_{\frac{V_{Rx} d_w}{\nu_a}}^{\frac{V_a d_w}{\nu_a}} \frac{dR_R}{C_D R_R}\right] \qquad (5)$$

FOR ANY INJECTED V_w OTHER THAN THE NORMALIZED CONDITION:

$$\frac{X_{x_1,x_2}}{d_w}\,\frac{\rho_a}{\rho_w} = \frac{X_{o,x_2}}{d_w}\,\frac{\rho_a}{\rho_w} - \frac{X_{o,x_1}}{d_w}\,\frac{\rho_a}{\rho_w} \qquad (6)$$

$\frac{X_{o,x}}{d_w}\frac{\rho_a}{\rho_w}$ HAS BEEN CALCULATED (TABLE III)

AND PLOTTED (FIGURES I AND II) FOR VALUES OF

$\frac{V_a d_w}{\nu_a}$ FROM 5 TO 2000 AND $\frac{V_{RX}}{V_a}$ FROM .01 TO 1.

Table I

Values of $C_D = f(R)$ for spherical particles

R	C_D	$C_D R$	$C_D R^2$
0.1	240	24.0	2.4
0.2	120	24.0	4.8
0.3	80	24.0	7.2
0.5	49.5	24.8	12.4
0.7	36.5	25.6	17.9
1.0	26.5	26.5	26.5
2	14.4	28.8	57.6
3	10.4	31.2	93.7
5	6.9	34.5	173
7	5.4	37.8	265
10	4.1	41.0	410
20	2.55	51.0	1.02×10^3
30	2.00	60.0	1.80
50	1.50	75.0	3.75
70	1.27	89.0	6.23
100	1.07	107	10.7
200	0.77	154	3.08×10^4
300	.65	195	5.85
500	.55	275	13.75
700	.50	350	24.5
1000	.46	460	46.0
2000	.42	840	1.68×10^6
3000	.40	1200	3.60
5000	.385	1920	9.60
7000	.390	2730	19.1
10000	.405	4050	40.5
2×10^4	.45	9000	$.180 \times 10^9$
3	.47	14,200	.426
5	.49	24,500	1.23
7	.50	35,000	2.45
10	.48	48,000	4.8
2×10^5	.42	84,000	16.8×10^9

Table II

Values of $X = f(R)$, $Y = f(R)$

$$X = \int_{R}^{2 \times 10^5} \frac{dR}{C_D R^2}$$

$$Y = \int_{0}^{R} \frac{dR}{C_D R}$$

R	X	Y
2×10^5	0	17.047
1	1.09×10^{-5}	16.311
7×10^4	2.04×10^{-5}	15.570
5	3.18	14.876
3	5.62	13.764
2	9.42	12.856
1	21.52	11.066
7×10^3	3.23×10^{-4}	10.146
5	4.71	9.259
3	8.14	7.905
2	12.22	6.894
1	23.9	5.2119
700	3.27×10^{-3}	4.4573
500	4.40	3.8080
300	6.62	2.9316
200	8.88	2.3505
100	14.38	1.5585
70	1.309×10^{-2}	12.495×10^{-1}
50	2.214	10.038
30	2.979	7.036
20	3.704	5.2242
10	5.244	3.0242
7	6.161×10^{-2}	22.615×10^{-2}
5	7.077	17.070
3	8.67	10.966
2	10.00	7.628
1	12.50	4.0134

Table XI - Cont.

R	X	Y
.7	.1367	28.055×10^{-3}
.5	.1517	20.697
.3	.1727	12.500
.2	.1900	8.333
.1	.2185	4.166
.05	.2472	20.83×10^{-4}
.01	.3142	4.166
.005	.3432	20.83×10^{-5}
10^{-3}	.4102	4.166
10^{-4}	.5062	4.166×10^{-6}
10^{-5}	.6022	4.166×10^{-7}
0	∞	0

Table III

Values of $Z = f\left(\frac{V_r}{V_a}\right)$ for various value of R_1

$$R_1 = \frac{\rho_a V_a d_w}{\mu_a}$$

$$Z = \frac{x}{d_w} \cdot \frac{\rho_a}{\rho_w}$$

$\left[\begin{array}{l} Z_1 \text{ for } V_w < V_a \\ Z_2 \text{ for } V_w > V_a \end{array}\right]$

$R_1 = 2000$			$R_1 = 1000$		
$\frac{V_r}{V_a}$	Z_1	Z_2	$\frac{V_r}{V_a}$	Z_1	Z_2
.50	.273	5.358	.70	.1672	2.1789
.35	2.210	8.709	.50	.8079	4.0507
.25	4.550	12.568	.30	2.3090	8.6702
.15	9.115	19.678	.20	4.0369	12.4604
.10	14.566	28.477	.10	11.2689	20.0327
.05	27.073	42.196	.07	15.6462	24.2113
.035	37.452	52.499	.05	20.7174	31.9361
.025	47.921	63.624	.03	30.8149	42.5335
.015	67.016	84.419	.02	39.9373	52.4396
.010	87.002	103.990	.01	60.1723	73.2610

Table III - Cont.

R_1 = 700

$\frac{V_r}{V_a}$	Z_1	Z_2
.714	.1899	1.9199
.428	1.0921	5.1596
.286	2.4263	8.0431
.143	6.5026	14.2308
.100	9.9525	18.1045
.0714	13.0041	22.2111
.0428	19.7424	29.7492
.0286	26.2856	36.7560
.0143	40.3420	51.4190
.0100	46.7970	60.0771

R_1 = 500

$\frac{V_r}{V_a}$	Z_1	Z_2
.60	.3114	2.6479
.40	1.0431	4.9288
.20	3.6531	9.6503
.14	5.7139	12.5349
.10	8.0857	15.5617
.06	12.7845	21.0603
.04	17.3749	26.1343
.02	27.3457	36.6916
.014	33.3559	42.9000
.01	39.3871	49.0841

R_1 = 300

$\frac{V_r}{V_a}$	Z_1	Z_2
.667	.1292	1.6784
.333	1.2729	4.9036
.233	2.3446	6.8291
.167	3.6367	8.7762
.100	6.2960	12.2358
.0667	8.9535	15.3764
.0333	14.3187	21.0201
.0233	18.3042	25.5968
.0167	21.9733	29.3339
.01	28.2704	35.7936

R_1 = 200

$\frac{V_r}{V_a}$	Z_1	Z_2
.50	.4106	2.5220
.35	.9378	3.9230
.25	1.7400	5.3303
.15	3.3796	7.7697
.10	5.0706	9.9443
.05	8.0830	14.3432
.035	11.2261	16.8890
.025	13.5942	19.4065
.015	17.7545	23.7265
.01	21.2596	27.3166

Table III - Cont.

n_i = 100

$\frac{V_r}{V_a}$	z_1	z_2
.7	.0826	.9060
.5	.2950	1.7738
.3	.9146	3.1935
.2	1.6395	4.4017
.1	3.3990	6.7478
.07	4.5198	8.0717
.05	5.6668	9.3667
.03	7.7063	11.5638
.02	9.4347	13.3663
.01	12.7206	16.7654

n_i = 70

$\frac{V_r}{V_a}$	z_1	z_2
.714	.0504	.7064
.428	.3643	1.6191
.286	.7990	2.7374
.143	1.9427	4.4677
.100	2.6960	5.4249
.0714	3.4775	6.3538
.0428	4.8217	7.9204
.0286	6.0782	9.2060
.0143	8.3627	11.5870
.01	9.3203	12.8861

n_i = 50

$\frac{V_r}{V_a}$	z_1	z_2
.6	.1100	.9098
.4	.3514	1.6348
.2	1.0045	2.9545
.14	1.5941	3.6672
.10	2.1307	4.3017
.06	3.1138	5.4974
.04	3.9557	6.4284
.02	5.5710	8.1402
.014	6.4686	9.0667
.01	7.3247	9.9456

n_i = 30

$\frac{V_r}{V_a}$	z_1	z_2
.667	.0481	.8317
.333	.3707	1.4408
.233	.6358	1.9091
.167	.9282	2.3494
.100	1.4835	3.0674
.0667	1.9708	3.6435
.0333	2.9227	4.6922
.0233	3.4553	5.2554
.0167	3.9646	5.7856
.01	4.7935	6.6365

Table III - Cont.

$n_1 = 20$

$\frac{V_r}{V_a}$	z_1	z_2
.50	.1173	.7036
.35	.2502	1.0499
.25	.4304	1.3681
.15	.7747	1.8753
.10	1.0849	2.2742
.05	1.7032	2.9890
.035	2.0320	3.3685
.025	2.3881	3.7256
.015	2.9370	4.2954
.01	3.3926	4.7632

$n_1 = 10$

$\frac{V_r}{V_a}$	z_1	z_2
.7	.0264	.2241
.5	.0688	.4199
.3	.2002	.7142
.2	.3331	.9359
.1	.6161	1.3174
.07	.7849	1.6148
.05	.9476	1.6986
.03	1.2166	1.9895
.02	1.4416	2.2257
.01	1.8159	2.8112

$n_1 = 7$

$\frac{V_r}{V_a}$	z_1	z_2
.714	.0115	.1594
.428	.0709	.3095
.286	.1505	.5201
.143	.3436	.6396
.100	.4562	.9027
.0714	.5068	1.1145
.0428	.7518	1.3214
.0286	.9077	1.4804
.0143	1.1600	1.7596

$n_1 = 5$

$\frac{V_r}{V_a}$	z_1	z_2
.6	.0247	.1873
.4	.0688	.3206
.2	.1872	.5353
.14	.2631	.6419
.10	.3392	.7391
.06	.4685	.8902
.04	.5782	1.0111
.02	.7626	1.2066
.01	.9311	1.4007

DISCONTINUITY ANALYSIS

EFFECT OF DROPLET ACCELERATION ON AIR STREAM

ASSUMPTIONS:

1. ADIABATIC - NO CHANGE IN STAGNATION TEMPERATURE
2. NO EVAPORATION OF WATER DROPLETS
3. NO CHANGE IN TEMPERATURE OF WATER DROPLETS
4. PERFECT GAS RELATION HOLDS FOR AIR

CONTINUITY:

$$\frac{w_a}{A} = \rho_a V_a = \sqrt{\frac{K}{R}}\,\frac{P_1}{\sqrt{T_1}}\,M_1 = \sqrt{\frac{K}{R}}\,\frac{P_2}{\sqrt{T_2}}\,M_2$$

$$\therefore \frac{M_2}{M_1} = \frac{P_1}{P_2}\sqrt{\frac{T_2}{T_1}}$$

MOMENTUM:

$$P_1 + \frac{w_a}{A}V_{a1} + \frac{w_w}{A}V_{w1} = P_2 + \frac{w_a}{A}V_{a2} + \frac{w_w}{A}V_{w2}$$

$$P_1 + \left[1 + \frac{w_w}{w_a}\,\frac{V_{w1}}{V_{a1}}\right]\rho_{a1}V_{a1} = P_2 + \left[1 + \frac{w_w}{w_a}\,\frac{V_{w2}}{V_{a2}}\right]\rho_{a2}V_{a2}$$

$$\text{BUT:}\quad \rho V^2 = KPM^2$$

$$\therefore \frac{P_2}{P_1} = \frac{1 + \left(1 + \frac{w_w}{w_a}\,\frac{V_{w1}}{V_{a1}}\right)KM_1^2}{1 + \left(1 + \frac{w_w}{w_a}\,\frac{V_{w2}}{V_{a2}}\right)KM_2^2}$$

ENERGY:

$$h_{a01} + \frac{w_w}{w_a}h_{w01} = h_{a02} + \frac{w_w}{w_a}h_{w02}$$

$$T_1 C_{pa} + \left[1 + \frac{w_w}{w_a}\,\frac{V_{w1}^2}{V_{a1}^2}\right]\frac{V_{a1}^2}{2} = T_2 C_{pa} + \left[1 + \frac{w_w}{w_a}\,\frac{V_{w2}^2}{V_{a2}^2}\right]\frac{V_{a2}^2}{2}$$

$$\text{BUT:}\quad V^2 = M^2C^2 = KRTM^2 \quad ; \quad \frac{KR}{2C_p} = \frac{K-1}{2}$$

DISCONTINUITY ANALYSIS - CON'T

$$\therefore \frac{T_2}{T_1} = \frac{1 + \frac{K-1}{2}\left(1 + \frac{\omega_w}{\omega_a}\frac{V_{w_1}^2}{V_{a_1}^2}\right)M_1^2}{1 + \frac{K-1}{2}\left(1 + \frac{\omega_w}{\omega_a}\frac{V_{w_2}^2}{V_{a_2}^2}\right)M_2^2}$$

DEFINE: $$T_0 \equiv T\left[1 + \frac{K-1}{2}\left(1 + \frac{\omega_w}{\omega_a}\frac{V_w^2}{V_a^2}\right)M^2\right]$$

COMBINING:

$$\frac{T_{02}}{T_{01}} = 1 = \frac{M_2^2}{M_1^2}\frac{\left[1+\left(1+\frac{\omega_w}{\omega_a}\frac{V_{w_1}}{V_{a_1}}\right)KM_1^2\right]^2}{\left[1+\left(1+\frac{\omega_w}{\omega_a}\frac{V_{w2}}{V_{a2}}\right)KM_2^2\right]^2}\;\frac{\left[1+\frac{K-1}{2}\left(1+\frac{\omega_w}{\omega_a}\frac{V_{w2}^2}{V_{a2}^2}\right)M_2^2\right]}{\left[1+\frac{K-1}{2}\left(1+\frac{\omega_w}{\omega_a}\frac{V_{w_1}^2}{V_{a_1}^2}\right)M_1^2\right]}$$

NORMALIZING ALL EQUATIONS TO * CONDITION

WHERE; $\frac{V_w}{V_a} = 0$; $M = 1$

$$\frac{T_0}{T_0^*} = \frac{M^2\left[1+\frac{K-1}{2}\left(1+\frac{\omega_w}{\omega_a}\frac{V_w^2}{V_a^2}\right)M^2\right]}{\left[1+K\left(1+\frac{\omega_w}{\omega_a}\frac{V_w}{V_a}\right)M^2\right]^2}\;2(K+1)$$

$$\frac{T}{T^*} = \frac{(K+1)}{2\left[1+\frac{K-1}{2}\left(1+\frac{\omega_w}{\omega_a}\frac{V_w^2}{V_a^2}\right)M^2\right]}$$

$$\frac{P}{P^*} = \frac{K+1}{\left[1+K\left(1+\frac{\omega_w}{\omega_a}\frac{V_w}{V_a}\right)M^2\right]}$$

ISENTROPIC STAGNATION CONDITIONS:

1. ASSUMING DROPLETS DECELERATE ISENTROPICALLY.

$$P_0 = P\left[1+\frac{K-1}{2}\left(1+\frac{\omega_w}{\omega_a}\frac{V_w^2}{V_a^2}\right)M^2\right]^{\frac{K}{K-1}}$$

$$\therefore \frac{P_0}{P_0^*} = \frac{\left[1+\frac{K-1}{2}\left(1+\frac{\omega_w}{\omega_a}\frac{V_w^2}{V_a^2}\right)M^2\right]^{\frac{K}{K-1}}}{\left[1+K\left(1+\frac{\omega_w}{\omega_a}\frac{V_w}{V_a}\right)M^2\right]}\;\frac{2^{\frac{K}{K-1}}}{[K+1]^{1/K-1}}$$

DISCONTINUITY ANALYSIS - CON'T

2. ASSUMING DROPLETS ARE NOT DECELERATED

$$p_0 = p\left[1+\frac{K-1}{2}M^2\right]^{\frac{K}{K-1}}$$

$$\therefore \frac{p_0}{p_0^*} = \frac{\left[1+\frac{K-1}{2}M^2\right]^{K/K-1}}{\left[1+K\left(1+\frac{w_w}{w_a}\frac{V_w}{V_a}\right)M^2\right]} \quad \frac{2^{K/K-1}}{[K+1]^{1/K-1}}$$

VALUES OF $\frac{T_0}{T_0^*}$, $\frac{P_0}{P_0^*}$, $\frac{P}{P^*}$, $\frac{T}{T^*}$ ARE TABULATED IN TABLES IV THROUGH VIII FOR M FROM 0 TO 2; $\frac{w_w}{w_a} = .1, .2, .3, .4$; $\frac{V_w}{V_a} = 0$ AND 1; $K=1.4$.

VALUES OF M_2, $\frac{P_{02}}{P_{01}}$, $\frac{p_{02}}{p_{01}}$, $\frac{P_2}{P_1}$, $\frac{T_2}{T_1}$, $\frac{V_2}{V_1}$ ARE PLOTTED VS M_1 IN FIGURES V THROUGH X AND M_2 IS TABULATED VS M_1 IN TABLE IX FOR M_1 FROM 0 TO 2; $\frac{w_w}{w_a} = 0, .1, .2, .3, .4$; $K=1.4$; ASSUMING $\frac{V_{w1}}{V_{a1}} = 0$ AND $\frac{V_{w2}}{V_{a2}} = 1$.

Table IV

Discontinuity Analysis

For all values of $\frac{W_W}{W_3}$, $\frac{V_W}{V_3} = 0$, Perfect Gas, K = 1.4

M	$\frac{T_o}{T_o^*}$	$\frac{P_o}{P_o^*}$	$\frac{P}{P^*}$	$\frac{T}{T^*}$
0	0	1.2679	2.4000	1.2000
.10	.04678	1.2591	2.3669	1.1970
.20	.17355	1.2346	2.2727	1.1905
.30	.34686	1.1985	2.1314	1.1788
.35	.43894	1.1779	2.0487	1.1713
.40	.52903	1.1566	1.9608	1.1628
.42	.56376	1.1480	1.9247	1.1591
.44	.59748	1.1394	1.8882	1.1553
.46	.63007	1.1308	1.8515	1.1513
.48	.66139	1.1224	1.8147	1.1471
.50	.69136	1.1140	1.7778	1.1429
.52	.71990	1.1059	1.7410	1.1384
.54	.74695	1.0979	1.7043	1.1339
.56	.77248	1.09010	1.6678	1.1292
.58	.79647	1.08255	1.6316	1.1244
.60	.81892	1.07525	1.5957	1.1194
.62	.83982	1.06821	1.5603	1.1144
.64	.85920	1.06146	1.5253	1.1091
.66	.87709	1.05502	1.4908	1.10383
.68	.89350	1.04890	1.4569	1.09842
.70	.90850	1.04310	1.4235	1.09290
.72	.92212	1.03764	1.3907	1.08727
.74	.93442	1.03253	1.3585	1.08155
.76	.94546	1.02776	1.3270	1.07573
.78	.95528	1.02337	1.2961	1.06982

Table IV - Cont.

M	$\frac{T_o}{T_o^*}$	$\frac{P_o}{P_o^*}$	$\frac{P}{P^*}$	$\frac{T}{T^*}$
.80	.96394	1.01934	1.2658	1.06383
.82	.97152	1.01569	1.2362	1.05775
.84	.97807	1.0124	1.2073	1.05160
.86	.98363	1.00951	1.1791	1.04537
.88	.98828	1.00698	1.1515	1.03907
.90	.99207	1.00485	1.1246	1.03270
.92	.99506	1.00310	1.09842	1.02627
.94	.99729	1.00174	1.07285	1.01978
.96	.99883	1.00077	1.04792	1.01324
.98	.99972	1.00019	1.02364	1.00664
1.00	1.00000	1.00000	1.00000	1.00000
1.05	.99633	1.00121	.94358	.98320
1.10	.99392	1.00486	.89086	.96618
1.15	.98721	1.01092	.84166	.94899
1.20	.97872	1.01941	.79576	.93168
1.25	.96886	1.03032	.75294	.91429
1.30	.95798	1.04365	.71301	.89686
1.40	.93425	1.07765	.64102	.86207
1.50	.90928	1.1215	.57831	.82759
1.60	.88419	1.1756	.52356	.79365
1.70	.85970	1.2402	.47563	.76046
1.80	.83628	1.3159	.43353	.72816
1.90	.81414	1.4033	.39643	.69686
2.00	.79339	1.5031	.36364	.66667

Table V

Discontinuity Analysis

$\frac{M_1}{M_2} = .1, \quad \frac{V_4}{V_3} = 1, \quad$ Perfect Gas, $K = 1.4$

M	$\frac{T_o}{T_o^*}$	$\frac{P_o}{P_o^*}$	$\frac{P}{P^*}$	$\frac{T}{T^*}$
0	0	1.2679	2.400	1.2000
.10	.04566	1.2583	2.3636	1.1974
.20	.17186	1.2315	2.2607	1.1895
.30	.33923	1.1918	2.1079	1.1767
.35	.42738	1.1667	[illegible]	1.1685
.40	.51176	1.1482	1.92[illegible]	1.1592
.42	.54392	1.1391	1.8873	1.1552
.44	.57493	1.1302	1.8488	1.1510
.46	.60467	1.1215	1.8101	1.1466
.48	.63305	1.1127	1.7715	1.1421
.50	.65999	1.10413	1.7329	1.1374
.52	.68543	1.09584	1.6944	1.1326
.54	.70934	1.08786	1.6562	1.1277
.56	.73172	1.07994	1.6184	1.1225
.58	.75254	1.07233	1.5810	1.1173
.60	.77183	1.06604	1.5440	1.1119
.62	.78960	1.05830	1.5076	1.1064
.64	.80590	1.05154	1.4717	1.1008
.66	.82075	1.04524	1.4364	1.0951
.68	.83421	1.03937	1.4016	1.0892
.70	.84634	1.03399	1.3678	1.0832
.72	.85717	1.02872	1.3346	1.0772
.74	.86679	1.02433	1.3020	1.0710
.76	.87523	1.02002	1.2702	1.0647
.78	.88258	1.01586	1.2391	1.0583

Table V - Cont.

M	$\frac{T_o}{T_o^*}$	$\frac{P_o}{P_o^*}$	$\frac{P}{P^*}$	$\frac{T}{T^*}$
.80	.88889	1.01261	1.2087	1.0519
.82	.89422	1.00946	1.1791	1.0454
.84	.89863	1.00679	1.1502	1.0388
.86	.90218	1.00467	1.1220	1.0321
.88	.90494	1.00301	1.09460	1.0253
.90	.90695	1.00156	1.06790	1.0185
.92	.90828	1.00058	1.04191	1.0116
.94	.90896	1.00014	1.01663	1.0047
.96	.90906	1.000016	.99204	.9977
.98	.90862	1.00045	.96813	.9907
1.00	.90768	1.00119	.94488	.9836
1.05	.90344	1.00512	.88960	.9658
1.10	.89694	1.01148	.83816	.9477
1.15	.88870	1.02078	.79034	.9295
1.20	.87914	1.03245	.74590	.9113
1.25	.86861	1.04706	.70459	.8930
1.30	.85741	1.06413	.66619	.8748
1.40	.83386	1.10657	.59725	.8385
1.50	.80988	1.1601	.53751	.8027
1.60	.78636	1.2252	.48559	.7677
1.70	.76380	1.3023	.44032	.7336
1.80	.74250	1.3921	.40069	.7006
1.90	.72259	1.4954	.36589	.6688
2.00	.70410	1.6133	.33520	.6383

Table VI

Discontinuity Analysis

$\frac{w_w}{w_a} = .2, \quad \frac{V_w}{V_a} = 1, \quad$ Perfect Gas, $\quad K = 1.4$

M	$\frac{T_o}{T_o^*}$	$\frac{P_o}{P_o^*}$	$\frac{p}{p^*}$	$\frac{T}{T^*}$
0	0	1.2679	2.4000	1.2000
.10	.04654	1.2574	2.3503	1.1971
.20	.17020	1.2285	2.2469	1.1866
.30	.33201	1.1856	2.0643	1.1746
.35	.41630	1.1637	1.9904	1.1657
.40	.49530	1.1402	1.8916	1.1556
.42	.52317	1.1308	1.8513	1.1513
.44	.55570	1.1216	1.8110	1.1467
.46	.58087	1.11245	1.7706	1.1420
.48	.60600	1.10357	1.7303	1.1371
.50	.63083	1.09486	1.6901	1.1321
.52	.65355	1.08649	1.6503	1.1269
.54	.67468	1.07839	1.6109	1.1215
.56	.69425	1.07056	1.5719	1.1160
.58	.71237	1.06307	1.5334	1.1104
.60	.72894	1.05592	1.4955	1.1046
.62	.74405	1.04919	1.4583	1.0986
.64	.75773	1.04278	1.4217	1.0926
.66	.77004	1.03691	1.3858	1.0864
.68	.78104	1.03133	1.3507	1.0801
.70	.79070	1.02624	1.3164	1.0737
.72	.79933	1.02156	1.2828	1.0672
.74	.80676	1.01731	1.2500	1.0606
.76	.81312	1.01362	1.2163	1.0539
.78	.81848	1.01031	1.1869	1.0471

Table VI - Cont.

M	$\frac{T_o}{T_o^*}$	$\frac{P_o}{P_o^*}$	$\frac{P}{P^*}$	$\frac{T}{T^*}$
.80	.82292	1.00744	1.1565	1.0402
.82	.82648	1.00503	1.1270	1.0333
.84	.82923	1.00311	1.09619	1.0262
.86	.83134	1.00164	1.07022	1.0191
.88	.83255	1.00066	1.04303	1.0119
.90	.83322	1.00012	1.01660	1.0047
.92	.83330	1.00005	.99004	.9974
.94	.83284	1.00046	.96441	.9900
.96	.83190	1.00162	.94181	.9827
.98	.83060	1.00264	.91832	.9752
1.00	.82869	1.00443	.89552	.9677
1.05	.82265	1.01096	.84146	.9489
1.10	.81482	1.02041	.79135	.9299
1.15	.80567	1.03276	.74493	.9109
1.20	.79554	1.04801	.70192	.8918
1.25	.78478	1.06619	.66207	.8727
1.30	.77359	1.08732	.62513	.8537
1.40	.75067	1.1366	.55908	.8161
1.50	.72793	1.2022	.50209	.7792
1.60	.70601	1.2757	.45276	.7433
1.70	.68528	1.3689	.40989	.7085
1.80	.66591	1.4737	.37249	.6751
1.90	.64797	1.5940	.33971	.6429
2.00	.63143	1.7313	.31075	.6122

Table VII

Discontinuity Analysis

$\frac{V_M}{V_a} = .3$, $\frac{V_W}{V_a} = 1$, Perfect Gas, $K = 1.4$

M	$\frac{T_o}{T_o^*}$	$\frac{P_o}{P_o^*}$	$\frac{p}{p^*}$	$\frac{T}{T^*}$
0	0	1.2679	2.4000	1.2000
.10	.04642	1.2566	2.3571	1.1969
.20	.16856	1.2255	2.2371	1.1876
.30	.32842	1.1813	2.0622	1.1726
.35	.40887	1.1570	2.9625	1.1630
.40	.47982	1.1325	1.8567	1.1521
.42	.50743	1.1227	1.8107	1.1474
.44	.53370	1.1133	1.7747	1.1425
.46	.55853	1.1041	1.7327	1.1374
.48	.58187	1.09502	1.6909	1.1322
.50	.60368	1.08630	1.6495	1.1268
.52	.62394	1.07779	1.6094	1.1212
.54	.64288	1.06975	1.5670	1.1154
.56	.65902	1.06202	1.5279	1.1095
.58	.67554	1.05461	1.4896	1.1035
.60	.68977	1.04767	1.4500	1.0973
.62	.70268	1.04119	1.4121	1.0910
.64	.71404	1.03511	1.3750	1.0845
.66	.72421	1.02950	1.3387	1.0779
.68	.73314	1.02440	1.3032	1.0712
.70	.74091	1.01972	1.2686	1.0644
.72	.74755	1.01550	1.2349	1.0575
.74	.75321	1.01184	1.2020	1.0504
.76	.75783	1.00864	1.1700	1.0433
.78	.76166	1.00600	1.1389	1.0361

Table VII - Cont.

M	$\frac{T_o}{T_o^*}$	$\frac{P_o}{P_o^*}$	$\frac{p}{p^*}$	$\frac{T}{T^*}$
.80	.76460	1.00375	1.10965	1.0298
.82	.76677	1.00205	1.07925	1.0214
.84	.76822	1.00091	1.05070	1.0140
.86	.76902	1.00018	1.02299	1.0065
.88	.76922	1.00010	.99610	.9989
.90	.76888	1.00035	.95610	.9912
.92	.76803	1.00116	.94472	.9836
.94	.76673	1.00255	.92019	.9758
.96	.76502	1.00433	.89642	.9680
.98	.76293	1.00673	.87339	.9602
1.00	.76053	1.00955	.85106	.9524
1.05	.75326	1.01890	.79826	.9327
1.10	.74460	1.03136	.74940	.9128
1.15	.73490	1.04701	.70444	.8930
1.20	.72462	1.06582	.66234	.8731
1.25	.71306	1.08751	.62439	.8533
1.30	.70235	1.1130	.58884	.8337
1.40	.68086	1.1734	.52549	.7949
1.50	.65942	1.2475	.47100	.7571
1.60	.63906	1.3361	.42405	.7205
1.70	.62002	1.4400	.38340	.6852
1.80	.60239	1.5606	.34799	.6513
1.90	.58617	1.6990	.31703	.6190
2.00	.57131	1.8563	.28986	.5882

Table VIII

Discontinuity Analysis

$\frac{W_W}{U_a} = .4, \quad \frac{U_W}{V_a} = 1, \quad$ Perfect Gas, $\gamma = 1.4$

M	$\frac{T_o}{T_o^*}$	$\frac{P_o}{P_o^*}$	$\frac{P}{P^*}$	$\frac{T}{T^*}$
0	0	1.2679	2.4000	1.2000
.10	.04630	1.2557	2.3539	1.1966
.20	.16744	1.2341	2.2285	1.1867
.30	.32002	1.1759	2.0401	1.1705
.35	.39047	1.1505	1.9398	1.1602
.40	.45202	1.1282	1.8270	1.1489
.42	.49363	1.1183	1.7824	1.1435
.44	.51482	1.10562	1.7296	1.1383
.46	.53793	1.09524	1.6904	1.1320
.48	.55671	1.08711	1.6534	1.1273
.50	.57535	1.07831	1.6107	1.1215
.52	.59544	1.06985	1.5686	1.1155
.54	.61301	1.06152	1.5271	1.1094
.56	.62807	1.05427	1.4886	1.1031
.58	.64166	1.04704	1.4484	1.0967
.60	.65388	1.04035	1.4071	1.0901
.62	.66473	1.03418	1.3686	1.0834
.64	.67430	1.02842	1.3318	1.0765
.66	.68265	1.02320	1.2947	1.0695
.68	.68934	1.01848	1.2596	1.0624
.70	.69596	1.01431	1.2242	1.0552
.72	.70107	1.01064	1.1904	1.0479
.74	.70536	1.00751	1.1576	1.0405
.76	.70833	1.00494	1.1257	1.0329
.78	.71102	1.00289	1.09466	1.0253

Table VIII - Cont.

M	$\frac{T_o}{T_o^*}$	$\frac{P_o}{P_o^*}$	$\frac{P}{P^*}$	$\frac{T}{T^*}$
.80	.71277	1.00143	1.06458	1.0176
.82	.71383	1.00047	1.03542	1.0099
.84	.71427	1.00002	1.00714	1.0020
.86	.71414	1.00019	.97970	.9941
.88	.71349	1.00084	.95320	.9862
.90	.71247	1.00203	.92750	.9782
.92	.71082	1.00385	.90261	.9701
.94	.70891	1.00616	.87852	.9539
.96	.70668	1.00900	.85521	.9539
.98	.70408	1.01238	.83264	.9457
1.00	.70124	1.01633	.81081	.9376
1.05	.69317	1.02851	.75928	.9169
1.10	.68402	1.04415	.71133	.8963
1.15	.67415	1.06314	.66813	.8757
1.20	.66382	1.08560	.62783	.8552
1.25	.65325	1.1115	.59077	.8348
1.30	.64262	1.1410	.55653	.8146
1.40	.62161	1.2108	.49570	.7740
1.50	.60147	1.2958	.44362	.7362
1.60	.58288	1.3970	.39883	.6990
1.70	.56507	1.5154	.36012	.6633
1.80	.54899	1.6526	.32651	.6292
1.90	.53428	1.8101	.29707	.5968
2.00	.52087	1.9898	.27149	.5660

Table IX

Discontinuity Analysis

Values of $M_2 = f(M_1, \frac{u_w}{u_s})$, Perfect Gas, $k = 1.4$

M_2 for $\frac{u_w}{u_s}$ =

M_1	.1	.2	.3	.4
.2	.201	.203	.204	.209
.25	.252	.254	.257	.260
.30	.304	.308	.312	.317
.31	.315	.319	.324	.329
.32	.325	.331	.336	.342
.33	.335	.342	.348	.354
.34	.345	.353	.360	.367
.35	.356	.364	.372	.380
.36	.367	.376	.384	.393
.37	.378	.387	.397	.407
.38	.388	.398	.408	.421
.39	.399	.410	.422	.437
.40	.411	.422	.436	.452
.41	.422	.433	.449	.467
.42	.433	.447	.464	.484
.43	.444	.458	.478	.502
.44	.456	.472	.493	.520
.45	.467	.486	.510	.542
.46	.478	.498	.525	.562
.47	.489	.512	.542	.586
.48	.500	.526	.560	.612
.49	.513	.541	.581	.643
.50	.525	.557	.603	.687

Table III - Cont.

M_2 for $\frac{EM}{W_n}$ =

M_1	.1	.2	.3	.4
.51	.537	.573	.625	.745
.516				.845
.52	.549	.589	.651	
.53	.563	.607	.683	
.54	.575	.625	.718	
.55	.588	.644	.770	
.557			.877	
.56	.601	.665		
.57	.614	.688		
.58	.629	.716		
.59	.642	.744		
.60	.657	.782		
.61	.673	.847		
.614		.913		
.62	.688			
.63	.707			
.64	.723			
.65	.743			
.66	.763			
.67	.789			
.68	.827			
.69	.865			
.70	.925			
.702	.953			

Table II - Cont.

M_2 for $\frac{W_d}{W_a} = .1$

M_1	Shock	Shockless
1.50	.902	.953
1.52	.870	1.050
1.54	.842	1.085
1.56	.817	1.123
1.58	.793	1.158
1.60	.782	1.177
1.62	.769	1.200
1.64	.756	1.227
1.66	.745	1.250
1.68	.734	1.270
1.70	.723	1.294
1.72	.716	1.310
1.74	.707	1.330
1.76	.700	1.340
1.78	.692	1.360
1.80	.685	1.375

M_2 for $\frac{W_d}{W_a} =$

	.1		.2	
M_1	Shock	Shockless	Shock	Shockless
1.815			.913	.913
1.82	.677	1.406	.877	.946
1.84	.669	1.425	.825	1.010
1.86	.663	1.444	.800	1.045
1.88	.656	1.463	.779	1.077
1.90	.649	1.482	.761	1.106
1.92	.643	1.500	.748	1.131
1.94	.638	1.520	.736	1.153
1.96	.633	1.537	.725	1.175
1.98	.628	1.553	.715	1.193
2.00	.624	1.571	.706	1.210

MODIFIED FANNO LINE ANALYSIS

FRICTIONAL, ADIABATIC, CONSTANT-AREA.

ASSUMPTIONS:

1. ADIABATIC - NO CHANGE IN STAGNATION TEMPERATURE
2. NO EVAPORATION OF WATER DROPLETS
3. NO CHANGE IN TEMPERATURE OF WATER DROPLETS
4. PERFECT GAS RELATION HOLDS FOR AIR
5. FOR DROPLET ENTRAINED IN AIR STREAM; $\frac{V_w}{V_a} = 1$
6. FOR DROPLET ON THE WALL; $\frac{V_w}{V_a} = 0$
7. IN EACH DIFFERENTIAL SECTION, LENGTH dx, AN AMOUNT OF WATER, dw_1, LEAVES THE STREAM AND HITS THE WALL. IN ADDITION AN AMOUNT OF WATER, dw_2, LEAVES THE WALL AND IS PICKED UP BY THE STREAM.

CONTINUITY:

$$\rho V = \text{CONSTANT}$$

$$\frac{d\rho}{\rho} + \frac{dV}{V} = 0 \qquad (1)$$

EQUATION OF STATE:

$$P = \rho R T$$

$$\frac{dP}{P} = \frac{d\rho}{\rho} + \frac{dT}{T} \qquad (2)$$

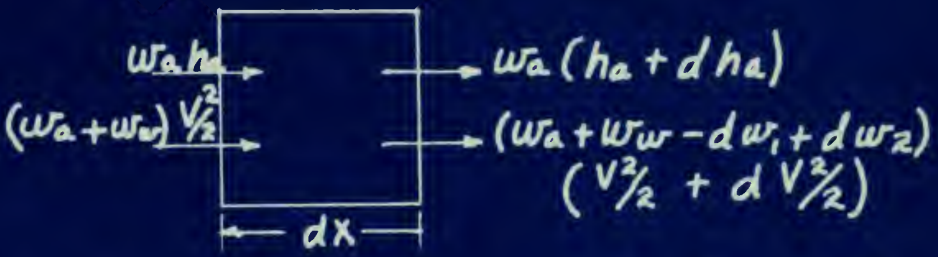
DEFINITION OF M:

$$M^2 = \frac{V^2}{C^2} = \frac{V^2}{KRT}$$

$$\frac{dM^2}{M^2} = \frac{dV^2}{V^2} - \frac{dT}{T} \qquad (3)$$

MODIFIED FANNO LINE ANALYSIS - CON'T

ENERGY:



$$0 = w_a\,dh_a + (w_a + w_w)\,d\frac{V^2}{2} - (dw_1 - dw_2)\left(\frac{V^2}{2} + \frac{dV^2}{2}\right)$$

DROPPING 2ND ORDER DIFFERENTIAL TERM AND REARRANGING; NOTING THAT:

$$dh_a = C_{pa}\,dT\ ,$$

$$\frac{V^2}{2} = M^2\,\frac{KRT}{2}\ ,$$

$$\frac{KR}{2C_p} = \frac{K-1}{2}\ ,$$

$$dw_1 - dw_2 = dw_w$$

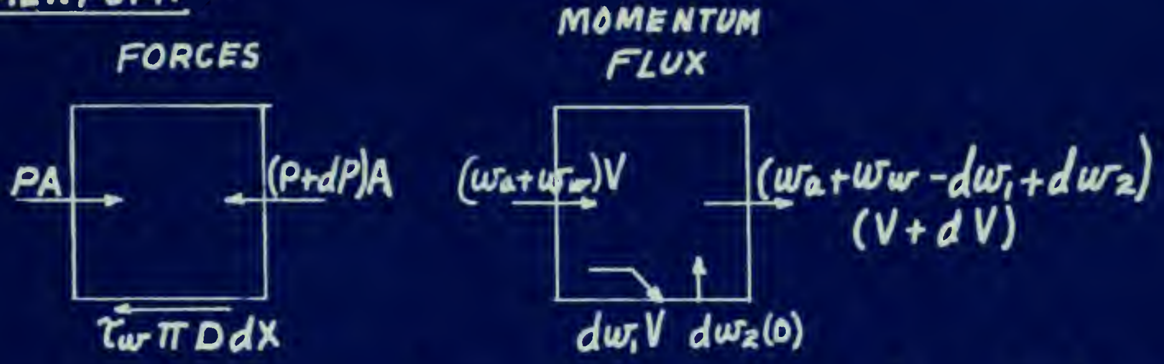
$$0 = \frac{dT}{T} + \frac{\frac{K-1}{2}\left(1 + \frac{w_w}{w_a}\right)M^2}{1 + \frac{K-1}{2}\left(1 + \frac{w_w}{w_a}\right)M^2}\,\frac{dM^2}{M^2} + \frac{\frac{K-1}{2}M^2\,d\left(\frac{w_w}{w_a}\right)}{1 + \frac{K-1}{2}\left(1 + \frac{w_w}{w_a}\right)M^2} \qquad (4)$$

IN STEADY STATE WITH WATER ON THE WALLS, ASSUME THE FILM THICKNESS IS CONSTANT WITH TIME AT ANY SECTION AND THE FILM VELOCITY IS NEGLIGIBLE.

$$\therefore\quad d\left(\frac{w_w}{w_a}\right) = 0$$

MODIFIED FANNO LINE ANALYSIS- CON'T

MOMENTUM:



$$-A\,dP - \tau_w \pi D\,dx = (w_a + w_w)\,dV + dw_2 V - (dw_1 - dw_2)\,dV$$

$$-\frac{dP}{P} - \frac{4\pi}{P}\frac{dx}{D} = \left(1+\frac{w_w}{w_a}\right)\frac{\rho V^2}{2P}\frac{dV^2}{V^2} + 2\frac{dw_2}{w_a}\frac{\rho V^2}{2P}$$

DEFINE:

$$f = \frac{\tau_w}{\frac{1}{2}\rho V^2} = \frac{\tau_w}{\frac{1}{2} K P M^2}$$

$$\therefore \frac{dP}{P} + \left(1+\frac{w_w}{w_a}\right)\frac{KM^2}{2}\frac{dV^2}{V^2} + \frac{KM^2}{2}\left[4f\frac{dx}{D} + 2\frac{dw_2}{w_a}\right] = 0 \qquad (5)$$

ASSUME THAT IN ANY SPECIFIC CASE THE AMOUNT OF WATER PICKED UP BY THE STREAM FROM THE WALLS IS PROPORTIONAL TO THE LENGTH.

$$\therefore\ 2\frac{dw_2}{w_a} = K\,dX$$

RESULTS OF SIMILTANEOUS SOLUTION OF EQUATIONS (1) THROUGH (5) ARE PRESENTED IN TABLE X

MODIFIED FANNO LINE ANALYSIS - CON'T

TABLE X

COEFFICIENTS OF FRICTION TERM, $\left[4\frac{f}{D}+K\right]dX$

$\frac{dM^2}{M^2}$	$\frac{KM^2\left[1+\frac{K-1}{2}\left(1+\frac{w_w}{w_a}\right)M^2\right]}{1-\left(1+\frac{w_w}{w_a}\right)M^2}$
$\frac{dT}{T}$	$-\frac{K\left(\frac{K-1}{2}\right)\left(1+\frac{w_w}{w_a}\right)M^4}{1-\left(1+\frac{w_w}{w_a}\right)M^2}$
$\frac{dV^2}{V^2}=-2\frac{d\rho}{\rho}$	$\frac{KM^2}{1-\left(1+\frac{w_w}{w_a}\right)M^2}$
$\frac{dP}{P}$	$-\frac{\frac{KM^2}{2}\left[1+(K-1)\left(1+\frac{w_w}{w_a}\right)M^2\right]}{1-\left(1+\frac{w_w}{w_a}\right)M^2}$

INTEGRATION OF EQUATIONS:

WHEN: $X=0$; $M^2=M^2$

$X=L_{MAX}$; $M^2=\frac{1}{1+\frac{w_w}{w_a}}$ [CHOKING (*) CONDITION]

$$\int_0^{L_{MAX}}\left[4\frac{f}{D}+K\right]dX=\int_{M^2}^{\frac{1}{1+\frac{w_w}{w_a}}}\frac{1-\left(1+\frac{w_w}{w_a}\right)M^2}{KM^4\left[1+\frac{K-1}{2}\left(1+\frac{w_w}{w_a}\right)M^2\right]}\,dM^2$$

RESULTS OF INTEGRATION AND INTEGRAL RELATIONS ARE GIVEN IN TABLE XI

MODIFIED FANNO LINE ANALYSIS - CON'T

TABLE XI

FORMULAS IN INTEGRAL FORM

$$\left[4\frac{f}{D}+K\right]L_{MAX} = \left(1+\frac{\omega_w}{\omega_a}\right)\left[\frac{1-\left(1+\frac{\omega_w}{\omega_a}\right)M^2}{K\left(1+\frac{\omega_w}{\omega_a}\right)M^2}+\left(\frac{K+1}{2K}\right)LN\frac{(K+1)\left(1+\frac{\omega_w}{\omega_a}\right)M^2}{2\left[1+\frac{K-1}{2}\left(1+\frac{\omega_w}{\omega_a}\right)M^2\right]}\right]$$

$$\frac{T}{T^*} = \frac{K+1}{2\left[1+\frac{K-1}{2}\left(1+\frac{\omega_w}{\omega_a}\right)M^2\right]}$$

$$\frac{V}{V^*}=\frac{\rho^*}{\rho} = M\sqrt{1+\frac{\omega_w}{\omega_a}}\sqrt{\frac{K+1}{2\left[1+\frac{K-1}{2}\left(1+\frac{\omega_w}{\omega_a}\right)M^2\right]}}$$

$$\frac{P}{P^*} = \frac{1}{M\sqrt{1+\frac{\omega_w}{\omega_a}}}\sqrt{\frac{K+1}{2\left[1+\frac{K-1}{2}\left(1+\frac{\omega_w}{\omega_a}\right)M^2\right]}}$$

$$\frac{F}{F^*} = \frac{1}{M\sqrt{1+\frac{\omega_w}{\omega_a}}}\frac{\left[1+K\left(1+\frac{\omega_w}{\omega_a}\right)M^2\right]}{\sqrt{2(K+1)\left[1+\frac{K-1}{2}\left(1+\frac{\omega_w}{\omega_a}\right)M^2\right]}}$$

ASSUMING DROPLETS DECELERATE ISENTROPICALLY:

$$\frac{P_o}{P_o^*} = \frac{1}{M\sqrt{1+\frac{\omega_w}{\omega_a}}}\left[\frac{2\left[1+\frac{K-1}{2}\left(1+\frac{\omega_w}{\omega_a}\right)M^2\right]}{(K+1)}\right]^{\frac{K+1}{2(K-1)}}$$

ASSUMING DROPLETS DO NOT DECELERATE:

$$\frac{P_o}{P_o} = \frac{\left(\frac{2}{K+1}\right)^{\frac{K+1}{2(K-1)}}}{M\sqrt{1+\frac{\omega_w}{\omega_a}}}\frac{\left[1+\frac{K-1}{2}M^2\right]^{\frac{K}{K-1}}}{\sqrt{1+\frac{K-1}{2}\left(1+\frac{\omega_w}{\omega_a}\right)M^2}}$$

MODIFIED FANNO LINE ANALYSIS - CON'T

VALUES OF $\left[4\frac{f}{D}+K\right]L_{MAX}$; $\frac{P_0}{P_0^*}$; $\frac{\rho_0}{\rho_0^*}$; $\frac{P}{P^*}$; $\frac{T}{T^*}$; $\frac{V}{V^*}$; $\frac{F}{F^*}$;

ARE GIVEN FOR VALUES OF M FROM 0 TO 2;

$\frac{w_w}{w_a}$ = .1, .2, .3, .4; K=1.4; IN TABLES XII THROUGH XV AND PLOTTED IN FIGURES XI THROUGH XVII.

MODIFIED FANNO LINE - LOW VELOCITY ANALYSIS

ASSUME: $M^2 << 1 \; ; \; \frac{P_2}{P_1} \cong 1$

$$\left[4\frac{f}{D} + K\right] L_{1,2} \cong \frac{1}{KM_1^2} - \frac{1}{KM_2^2} + \left(\frac{K+1}{2K}\right)\left(1 + \frac{\omega_w}{\omega_a}\right) LN \frac{M_1^2}{M_2^2}$$

$$\frac{P_1}{P_2} \cong \frac{M_2}{M_1}$$

$$\therefore \left[4\frac{f}{D} + K\right] L_{1,2} \cong \frac{1}{KM_1^2}\left[1 - \left(\frac{P_2}{P_1}\right)^2\right] - \left(1 + \frac{\omega_w}{\omega_a}\right)\left(\frac{K+1}{2K}\right)\left[1 - \left(\frac{P_2}{P_1}\right)^2 + \cdots\right]$$

$$1 - \left(\frac{P_2}{P_1}\right)^2 \cong \frac{K\left[4\frac{f}{D} + K\right] L_{1,2}}{\left[\frac{1}{M_1^2} - \left(1 + \frac{\omega_w}{\omega_a}\right)\left(\frac{K+1}{2}\right)\right]} \cong M_1^2 K\left[4\frac{f}{D} + K\right] L_{1,2}$$

$$\frac{P_2}{P_1} \cong 1 - \frac{1}{2} M_1^2 K\left[4\frac{f}{D} + K\right] L_{1,2} + \cdots$$

$$\therefore \frac{P_1 - P_2}{P_1} \cong \frac{1}{2} M_1^2 K\left[4\frac{f}{D} + K\right] L_{1,2}$$

ALSO $\frac{P_{01} - P_{02}}{P_{01}} \cong \frac{1}{2} M_1^2 K\left[4\frac{f}{D} + K\right] L_{1,2}$

Table XII

Frictional, Adiabatic, Constant Area Flow

Perfect Gas, $K = 1.4$, $\frac{W_w}{W_a} = .1$

M	$\left[\frac{4f}{D} + \text{[illegible]}\right]_{max}$	$\frac{P_o}{P_o^*}$	$\frac{p_o}{p_o^*}$	$\frac{p}{p^*}$	$\frac{T}{T^*}$	$\frac{V}{V^*} = \frac{\rho^*}{\rho}$
0	∞	∞	∞	∞	1.200	0
.1	66.0	5.84	5.83	10.40	1.197	.1147
.2	14.24	2.91	2.82	5.19	1.190	.2286
.3	5.15	1.966	1.940	3.45	1.177	.340
.4	2.136	1.531	1.511	2.563	1.159	.450
.5	.978	1.298	1.271	2.030	1.137	.559
.6	.425	1.140	1.127	1.675	1.112	.663
.7	.1651	1.057	1.040	1.419	1.085	.764
.8	.0472	1.020	.983	1.221	1.052	.860
.9	.00485	1.001	.955	1.070	1.019	.951
1.0	.00286	1.001	.946	.946	.984	1.041
1.1	.0231	1.017	.951	.844	.948	1.122
1.2	.0565	1.049	.970	.758	.911	1.201
1.3	.0945	1.097	1.003	.687	.875	1.275
1.4	.1370	1.152	1.049	.624	.839	1.348
1.5	.1784	1.229	1.103	.570	.803	1.411
1.6	.2201	1.315	1.171	.522	.768	1.476
1.7	.260	1.420	1.253	.481	.734	1.530
1.8	.297	1.536	1.345	.444	.701	1.561
1.9	.333	1.670	1.448	.410	.669	1.630
2.0	.366	1.834	1.572	.381	.638	1.676

Table XIII

Frictional, Adiabatic, Constant-Area Flow

Perfect Gas, K = 1.4, $\frac{W_W}{W_a} = .2$

M	$\left[4\frac{f}{D} + K\right]L_{max}$	$\frac{P_o}{P_o{}^*}$	$\frac{p_o}{p_o{}^*}$	$\frac{P}{P^*}$	$\frac{T}{T^*}$	$\frac{V}{V^*} = \frac{\rho^*}{\rho}$
0	∞	∞	∞	∞	1.200	0
.1	66.2	5.29	5.28	9.92	1.197	.1201
.2	14.10	2.70	2.69	4.97	1.189	.2388
.3	4.95	1.871	1.850	3.29	1.175	.356
.4	2.062	1.471	1.442	2.450	1.156	.470
.5	.888	1.252	1.215	1.941	1.132	.583
.6	.367	1.123	1.076	1.600	1.105	.691
.7	.1276	1.047	.989	1.350	1.074	.794
.8	.0282	1.010	.936	1.164	1.040	.893
.9	.000323	1.001	.907	1.017	1.005	.989
1.0	.01108	1.008	.898	.898	.968	1.078
1.1	.0421	1.029	.902	.800	.930	1.162
1.2	.0842	1.071	.919	.718	.892	1.243
1.3	.1304	1.124	.949	.649	.854	1.318
1.4	.1781	1.199	.988	.588	.816	1.386
1.5	.2252	1.287	1.040	.537	.779	1.451
1.6	.271	1.392	1.105	.493	.743	1.513
1.7	.314	1.502	1.178	.452	.709	1.569
1.8	.355	1.649	1.263	.417	.675	1.621
1.9	.395	1.810	1.362	.386	.643	1.672
2.0	.431	1.989	1.479	.358	.612	1.719

Table XIV

Frictional, Adiabatic, Constant-Area Flow

Perfect Gas, $K = 1.4$, $\frac{w_w}{w_a} = .3$

M	$\left[4\frac{f}{D} + K\right]L_{max}$	$\frac{p_o}{p_o^*}$	$\frac{p_o}{p_o^*}$	$\frac{p}{p^*}$	$\frac{T}{T^*}$	$\frac{V}{V^*} = \frac{\rho^*}{\rho}$
0	∞	∞	∞	∞	1.200	0
.1	65.8	5.12	5.10	9.58	1.197	.1246
.2	13.84	2.61	2.59	4.78	1.188	.2486
.3	4.80	1.807	1.782	3.17	1.173	.371
.4	1.943	1.430	1.384	2.352	1.152	.489
.5	.810	1.228	1.165	1.861	1.127	.605
.6	.313	1.107	1.029	1.530	1.097	.716
.7	.0964	1.034	.947	1.292	1.064	.823
.8	.01430	1.006	.895	1.112	1.029	.925
.9	.001172	1.001	.866	.970	.991	1.022
1.0	.0236	1.016	.856	.856	.952	1.112
1.1	.0647	1.048	.858	.761	.913	1.199
1.2	.1152	1.097	.874	.683	.873	1.279
1.3	.1682	1.163	.900	.616	.834	1.355
1.4	.222	1.248	.940	.559	.795	1.422
1.5	.274	1.346	.986	.509	.757	1.489
1.6	.325	1.458	1.043	.466	.721	1.551
1.7	.372	1.603	1.112	.427	.685	1.608
1.8	.416	1.765	1.191	.393	.651	1.658
1.9	.457	1.938	1.281	.363	.619	1.704
2.0	.495	2.153	1.388	.336	.588	1.750

Table XV

Frictional, Adiabatic, Constant-Area Flow

Perfect Gas, $K = 1.4$, $\frac{w_w}{w_a} = .4$

M	$\left[4\frac{f}{D} + K\right]L_{max}$	$\frac{P_o}{P_o^*}$	$\frac{p_o}{p_o^*}$	$\frac{p}{p^*}$	$\frac{T}{T^*}$	$\frac{V}{V^*} = \frac{\rho^*}{\rho}$
0	∞	∞	∞	∞	1.200	0
.1	65.3	4.96	4.92	9.25	1.197	.1290
.2	13.50	2.53	2.49	4.60	1.187	.2583
.3	4.65	1.747	1.716	3.05	1.171	.384
.4	1.840	1.393	1.333	2.263	1.149	.507
.5	.734	1.188	1.113	1.788	1.122	.627
.6	.265	1.084	.989	1.470	1.090	.741
.7	.0703	1.027	.909	1.240	1.055	.850
.8	.00524	1.001	.856	1.065	1.018	.956
.9	.00645	1.003	.827	.927	.978	1.053
1.0	.0406	1.021	.818	.818	.938	1.148
1.1	.0917	1.062	.819	.726	.896	1.232
1.2	.1496	1.127	.833	.651	.855	1.313
1.3	.2105	1.199	.856	.586	.815	1.389
1.4	.270	1.295	.893	.531	.775	1.461
1.5	.328	1.409	.935	.483	.736	1.522
1.6	.381	1.542	.990	.442	.699	1.583
1.7	.431	1.700	1.059	.408	.663	1.641
1.8	.479	1.880	1.130	.373	.629	1.691
1.9	.522	2.080	1.210	.343	.597	1.739
2.0	.563	2.32	1.312	.318	.566	1.782

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