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A psychometric method for determining optimum

Shaw, Charles Houston

Monterey, California. Naval Postgraduate School

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THESIS

A PSYCHOMETRIC METHOD
FOR DETERMINING OPTIMUM, TACTICAL
PATHS
IN COMBAT DECISION MAKING AND ANALYSIS

by

Charles Houston Shaw, III

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Thesis Advisor Samuel H. Parry

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19 Abstract *(continue on reverse if necessary and identify by block number)*

This thesis examines and demonstrates a methodology for determining optimum, tactical movement paths for a specified vehicle and or small unit based on the operator's cognitive decision processes, as well as the physical effects of terrain and environment on mobility. The approach specifically uses psychometric techniques inherent to the Generalized Value System (GVS) in order to determine a "Power Function" based on the specific tactical scenario and given equipment configuration which provides a means to determine the Tactical Movement Potential (TMP) for each terrain cell. This cognitive value in an interval scale can then be translated into the same scale as the physical continuum using techniques proposed by L.L. Thurstone and W.S. Torgerson. The cognitive time value based on the user's decision process is then added to the physical traversal times for each cell computed from output provided by The Condensed Army Mobility Management System (CAMMS). This renders a value mapping which can be optimized using one of several existing algorithms. The Dijkstra Algorithm is used in this demonstration model. The resulting sets of path points are optimized for speed time and the cognitive tactical considerations evaluated using these psychometric methods thereby rendering a truly optimum, tactical movement path and optimum traversal time. The movement path and resulting times can be used in combat planning and modelling. This output is also particularly important in determining the time values needed to compute the Situationally Inherent Power (SIP) of the GVS. In actuality, this methodology could be applied to almost any tactical decision process in the development of expert systems and models.

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A Psychometric Method for Determining Optimum, Tactical Paths in Combat Decision Making and Analysis

by

Charles Houston Shaw, III
Captain, United States Army
B.S., United States Military Academy, 1979

Submitted in partial fulfillment of the
requirements for the degree of

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ABSTRACT

This thesis demonstrates a method to determine optimum, tactical movement paths for a specified vehicle and/or small unit based on the operator's cognitive decision processes, as well as the physical effects of terrain and environment on mobility. The approach uses psychometric techniques inherent to the Generalized Value System (GVS) to determine a "Power Function" based on a specific tactical scenario and given equipment configuration and provides a means to determine the Tactical Movement Potential (TMP) for each terrain cell. This cognitive value in an interval scale can then be translated into the same scale as the physical continuum using techniques proposed by L.L. Thurstone and W.S. Torgerson. The cognitive time value based on the user's decision process is then added to the physical traversal times for each cell computed from output provided by The Condensed Army Mobility Management System (CAMMS). This renders a value mapping which can be optimized using one of several existing algorithms. The Dijkstra Algorithm is used in this demonstration model. The resulting sets of path points are optimized for speed time and the cognitive tactical considerations evaluated using these psychometric methods. The movement path and resulting times can be used in combat planning and modelling. This output is also particularly important in determining the time values needed to compute the Situationally Inherent Power (SIP) of the GVS. This methodology could be applied to almost any tactical decision process in the development of expert systems and models.

THESIS DISCLAIMER

The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

The reader is further cautioned that certain vehicle system input parameters used and portions of the computer program developed in this research are not valid for all scenarios of interest. While every effort has been made, within the time available, to verify all computer programs; they cannot be considered validated. Any application of the methodology using these programs is done so at the user's own risk.

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I. INTRODUCTION

A. GENERAL

The ability to accurately portray the battlefield environment is a key requirement in combat model development. This same ability is required in the construction of tactical decision aids and tools. One major factor to be considered in any of these applications is the effect of terrain, vegetation, and weather on the mobility trafficability of weapons systems and units. Another major factor is the effect of these same parameters on visibility, detection, and acquisition by various weapons systems. While the preceding factors are objective for the most part and can be accurately measured; other factors are not so clearly defined and may possess large variances. This is certainly the case where a human decision process is required such as the decision logic used by various combat unit and vehicle commanders in tactical movement. A clear distinction must be made between mobility as modelled in relation to the physical constraints of the battlefield and movement which requires interaction with the user decision maker or must be modelled taking the pertinent cognitive considerations as well as the physical constraints of the battlefield into account.

In any case, all of these factors and more must be accounted for in our combat models and tactical decision tools. There are a number of existing high resolution combat models and tactical decision aids which incorporate many of these functions. Most of these packages use one of several available versions of digitized terrain data and either actual, historical, or user input weather data. Most combat models also incorporate appropriate tactical decision logic algorithms to control some key functions such as detection, acquisition, and other essential combat actions of player units. Some tactical decision tools also do this. However, at the present time there is no combat model or tactical decision tool which combines the appropriate decision logic to determine the optimum path a unit or vehicle should traverse in order to simultaneously maximize speed and trafficability, minimize inter-visibility and detection, and appropriately balance their effects in terms of the decision logic.

The need and desire of commanders and combat planners to have and use such tactical decision tools and to incorporate them into combat models is of definite benefit. Some commanders presently have a tactical decision aid in the form of the Army Mobility Model (AMM) and the Condensed Army Mobility Management System

(CAMMS) which provide trafficability, speed, and time contour mappings for use in route selection and other tactical decisions. There are also any number of combat models available for study where a commander or combat analyst can select appropriate terrain data bases and manually input desired unit or vehicle path points and or positions. This is the case with present JANUS based models, the Battalion Combat Outcome Model (BCOM), and others. However, there is no tactical decision tool or combat model that can systemically determine sets of optimum, tactical path points for the commander or for use in analysis. This is also true of almost all models where tactical decision processes are represented.

The first requirement in developing such a model is to properly identify those variables which could and or would affect the tactical movement or other decision process such as the mission, threat, corresponding locations, ranges to various threats, vehicular agility or localized speed, allowable completion time, and others. It is soon obvious that a large number of variables, many having probable interactions with other variables, exist. The possible combinations or instances is quite large and each variable may vary significantly between instances. Structuring this maze in order to be able to extract usable data on the cognitive process is no small task.

Another requirement is finding a theoretically sound method for evaluating large groups of variables simultaneously and then being able to compute a specific value which appropriately measures their combined effect on the process. This field of study has a name all its own, Psychophysics.

A third requirement is to find a theoretically sound method that allows for the mapping of these psychophysical values into an appropriate physical continuum so that they may be used concurrently with physical attributes and variables in the model. This field of study also has a name all its own, Psychometry.

The art of combining and applying these techniques is known as Psychometrics. We will use these techniques in conjunction with an existing mobility model, CAMMS (Version 2.0), to provide total movement time values based on physical engineering traversal times and the translated psychometric times for each terrain cell and subsequent traverse segment on the battlefield. These values will then be used as network cost values in a form of Dijkstra's Algorithm which renders the set of minimum cost optimum path points for that scenario.

The organization, purpose, scope, and assumptions of this research are explained in this Introduction. Background information on previous route selection methodologies, terrain representations, physical attributes of terrain considered in mobility, cognitive considerations which impact on movement, and the desire to place such technologies in the hands of the commander in the field is presented in Chapter II. The development of the "Psychometric Function" using psychometric procedures inherent to the Generalized Value System, GVS, is shown in Chapter III. Application of these techniques and construction of the model algorithm is detailed in Chapter IV. Results of the algorithm implementation on an AT-386 personal computer and limited verification of these results are presented in Chapter V. Proposed enhancements to the existing model and areas requiring additional study are presented in Chapter VI. Finally, Chapter VII discusses possible utilizations for this methodology and proposed applications with respect to tactical decision and training tools, research and analysis tools, combat models, and the Generalized Value System.

It is of significant interest to note that prominent Soviet military analysts, Nikita Moiseyev in particular, have proposed a model framework based on decision theoretic developments using such state variables and decision functions since the 1970's. Two specific areas discussed have been tactical decision processes and movement of the line of contact. [Ref. 1: pp. 21-26]

B. PROBLEM STATEMENT

1. Purpose

This thesis examines and demonstrates a methodology for determining optimum, tactical movement paths for a specified vehicle and/or small unit based on the operator's cognitive decision processes, as well as the physical effects of terrain and environment on mobility. The resulting sets of path points are optimized for speed/time and the cognitive tactical considerations evaluated using psychometric methods thereby rendering a truly optimum, tactical movement path and optimum traversal time. The movement path and resulting times are to be used in combat planning and modelling. This output is also particularly important in determing the time values needed to compute the Situationally Inherent Power (SIP) of the GVS.

2. Scope

This research will be limited to one specific scenario of interest in order to demonstrate the methodology without undue involvement in various scenarios with different sets of fixed state variables. The specific scenario used is discussed in detail in Chapter III. This research will not involve detailed studies of relative effective range used in the algorithm and discussed in Chapter IV. There will also not be any extensive verification and validation of the results of the computer application of the algorithm due to a lack of time and resources.

This research will examine the use of a movement model based on the methodology for a single vehicle unit of specified type and mission against a threat array of specified types performing a certain mission while in a selected area of operations and known terrain.

3. Assumptions

1. The mathematical forms of range dependency equations studied by Bonder and Farrell and used to portray perceived lethality as relative effective range curves related to those found in JANUS are correct.
2. The survey results, interval scales, and regression analysis are valid for the range of values surveyed only. However, these should encompass the vast majority of instances.
3. The translated cognitive time scale and traversal time scales are additive and not multiplicative.
4. This methodology is applicable to all types of tactical decision processes when the appropriate state variables can be identified.

II. BACKGROUND

A. PREVIOUS ROUTE SELECTION AND TACTICAL MOVEMENT MODELS

1. The Dynamic Tactical Simulation (DYNTACS) Model

The DYNTACS Model was developed by the Systems Research Group at Ohio State University from 1964 to 1969. The model is extremely high resolution in order to predict individual unit performance in armored combat engagements from both the design and operational perspectives [Ref. 2]. Dynamic programming is used to run and solve the simulation which consists of a driver program and 34 subroutines. At least 15 of these algorithms are related to or affect tactical movement in the model [Ref. 3]. An explicit surface terrain representation of the battlefield is used in order to determine plane departure points for a predetermined grid size [Ref. 3: pp. 57-66]. This representation is closely related to and can be derived from explicit grid terrain data available from a number of sources [Ref. 4: pp. 3.3-3.15].

The model determines an optimum route for an advancing element or unit by optimization where "Tactical Difficulty" is minimized. Tactical difficulty is computed using a heuristic of the form, $TD = (1 + E)T$, where E is the difficulty computed due to a set of factors along each route segment and T is the travel time for that segment [Ref. 3: pp. 94-116]. Such heuristic procedures are computationally efficient but do not guarantee an optimal solution. The set of factors and corresponding values for the difficulty, E, were obtained using comparative judgement techniques. Travel times are computed using engineering models for each weapon system. It is interesting to note that difficulty is a function of the effective range, actual range, and threat disposition [Ref. 3: pp. 97-100] and that psychometric methods similar to many we will examine were used to obtain these values. The major shortcoming with the DYNTACS approach is that the cognitive and physical scales are related multiplicatively using a heuristic due to existing computer limitations rather than developing a truly optimal technique.

2. The Simulation of Tactical Alternative Responses (STAR) Model

The STAR Model is similar to DYNTACS in most respects except that dynamic programming is discarded and the optimization is performed in FORTRAN applying Dijkstra's Algorithm. The additional concept of the "Sliding Pattern" is used to restrict the search for the path [Ref. 5: pp. 23-34]. The route selection is performed sequentially from start point, to a horizon or horizons, to an objective by sliding the optimization along the network. Unfortunately, the identical heuristic equation found in DYNTACS is also applied in this model [Ref. 5: p. 33] rendering it suspect.

3. Other Route Selection Methodologies

There are any number of similar movement models proposed over the past 25 years and certainly all of them cannot be mentioned. However, it is necessary to introduce another type of model which is probabilistic in nature. One such model was investigated and formulated by Sitmourang [Ref. 6] in 1981. This dynamic route selection model performs an optimization in order to minimize the probability of being attrited enroute to the destination. The attrition probabilities along each route segment are derived from two sources. These are enemy elements capable of destroying the tactical element and terrain factors which end movement. Speed reduces the probability of attrition due to the enemy but increases the probability of attrition due to terrain, especially rough terrain [Ref. 6: pp. 21-22]. The total probability of being attrited, P, is taken to be the sum of the probability of being killed by the enemy, PKFOE, and the probability of being killed due to terrain/speed, PKVEL, which is minimized using Dijkstra's Algorithm [Ref. 6: p. 34].

There are a number of problems with this methodology. The model assumes perfect information in computing probability of kill due to enemy. This is certainly not the case in most tactical decisions. The probability of attrition due to terrain speed is computed using a heuristic equation with a power factor or constant for all combinations of conditions which is applied to a ratio of recommended speed to actual speed. This value is then used multiplicatively with normal expectancy in order to determine a probability of attrition due to speed in given terrain [Ref. 6: pp. 24-26]. No reference or theoretical support for this technique is provided. This certainly makes such a methodology suspect.

B. CURRENT MOBILITY MODELS

Mobility models provide comprehensive mappings by terrain cell or other construct of the measure or ability of weapons systems to physically traverse types of terrain in varying conditions. Mobility models should not be confused with movement models which attempt to describe how tactical elements traverse the battlefield based on both physical and cognitive considerations. The two primary mobility models used today are The NATO Research Mobility Model (NRMM) and The Condensed Army Mobility Management System (CAMMS). Most mobility routines used in models today are based on this family of models. The NRMM and CAMMS are both derived from the Army

Mobility Model (AMM) developed in the 1970's. The CAMMS is currently used as the base model for:

1. The Planning Analysis Work Station (PAWS)
2. The Terrain Analysis Work Station (TAWS)
3. The Airland Battlefield Environment (ALBE)
4. The Digital Terrain Support System (DTSS)

The model is also used by numerous Army, Defense, and other government activities to include The Defense Mapping Agency, The U.S. Army Command and General Staff College, The United States Military Academy, The Army Research Institute, The U.S. Army Engineer School, The U.S. Army Training and Doctrine Command (TRADOC), and The TRADOC Analysis Command (TRAC). The CAMMS is used as the base mobility model in this research due to its applicability and acceptance in the modelling community.[Ref. 7]

1. The Condensed Army Mobility Management System (CAMMS)

The CAMMS vehicle performance model was originally developed by simplifying the existing AMM through addition of a vehicle preprocessor and restricting the model to one vehicle type at a time. Obstacle performance is also determined statistically based on years of testing and data collection. This allows the CAMMS to operate on personal computers versus larger mainframe computers for which the AMM was developed. The model is much too large and detailed to be discussed fully here; however, an examination of the physical aspects considered in the CAMMS, output available, and current uses of the model is worthwhile.

a. Physical Considerations Affecting Mobility

The CAMMS off road prediction model evaluates three general categories of terrain data. These are surface geometry data, surface composition data, and vegetation data. Surface geometry data includes slope information, surface roughness factors, and all obstacle data. Surface composition data primarily relates to the type of soil and its strength which are greatly affected by weather. The vegetation data includes stem diameter, type, and spacing data along with visibility information. These data are used in a number of routines to compute surface traction, resistance, obstacle traction, interference, gap crossing capability, vegetation effect, and ride dynamics which are combined to produce a highly accurate speed prediction for the vehicle.[Ref. 7]

b. CAMMS Output Mappings

The CAMMS is widely used throughout the Army and particularly in the European Theater of Operations. A range of mappings or Tactical Decision Aids, TDA's, are currently available in the CAMMS. These include:

1. Vehicle Predictions
   a. Speed
   b. Time
   c. Reason Limited
   d. Comparison of Two Vehicles
2. Foot Soldier Prediction
3. Maneuver Damage Prediction
4. Soil Strength Prediction
5. Route Analysis Prediction

The vehicle predictions and especially speed and time mappings for the digitized terrain grid are of most significance to this research [Ref. 7]. These will be addressed concerning their potential use in subsequent chapters.

c. Technology to the Field

The CAMMS takes advantage of recent advancements in the personal computer field to provide an extremely powerful yet portable and inexpensive tool for the commander in the field. The most current CAMMS requires only an IBM compatible AT personal computer with a math coprocessor, DOS 3.2, EGA graphics, 1 megabyte of RAM, and sufficient hard disk drive storage in order to operate. All the TDA's mentioned previously are available in a matter of minutes provided the appropriate digitized terrain data bases are available. The only shortcoming in the current system is the lack of standarization in terrain data bases and lack of data in general for some regions. The CAMMS is being used extensively as an aid on REFORGER exercises and as an operational planning aid in some tactical units.

The methodology developed during this research will also be derived so as to make it conformable for use on a personal computer. This will allow the model to be used as a portable tactical decision tool in the hands of the commander.

C. COGNITIVE CONSIDERATIONS IMPACTING ON MOVEMENT

The discussion up to this point makes it quite apparent that tactical movement modelling is much more involved than mobility modelling. The major question becomes, how do we model the cognitive decision process? There is no definitive answer to this question and numerous techniques are available and feasible depending on the application. It is apparent that the mission, terrain effects, and threat all influence the combatant or tactical decision maker to some degree. The decision concerning where and when to move is made based on the combatant's evaluation of how the enemy can affect his mission accomplishment and/or survival as he moves to each possible location. There is also a question of the trade-off made by most humans between surviving and accomplishing a mission within a certain time. Some people are more risk averse than others.

The process is synonomous to shopping for fruit or vegetables of different types and deciding which items to purchase based on their size, shape, color, texture, and price. The shopper makes a determination when comparing different items as to which is best in terms of overall quality relative to all others and what he will purchase with a certain amount of funds just as a combatant makes a determination of the potential lethality facing him based on the types of enemy, their range, and their capabilities based on range and disposition in order to decide what risk he can accept when moving on the battlefield.

An enumeration of the variables identified as relative to this tactical movement decision process and how they are structured for use in this research is provided in detail in Chapter III.

III. PSYCHOMETRICS AND PSYCHOPHYSICAL ANALYSIS IN THE GVS

A. GENERAL

The Generalized Value System, GVS, is an axiomatic value system used as a tool for evaluating the power or potential of entities at present and future times. The formal structure of GVS was first proposed by Professor Arthur Schoenstadt of the Naval Postgraduate School in 1985 [Ref. 8: p. 3]. This procedure outlines a method of determining the state of an entity over time, allows us to compare the values of entities relative to one another, and is consistent for all entities on the battlefield. The values assigned to each entity are referred to as "power" in previous GVS documentation.

Power is a subjective value given to each element on the battlefield and combines both inherent and derived power. Derived power consists of the power an element possesses due to its ability to influence support other friendly elements [Ref. 9: p. 33]. Inherent power is the ability of an element to directly affect the outcome of a battle [Ref. 9: p. 32]. Inherent power may be categorized as Basic Inherent Power (BIP) or Situational Inherent Power (SIP) which are the power an element has at full strength and the power an element is predicted to have at some future time, respectively. The SIP is highly dependent on the available time, t_a , which is determined by the time domain networks for the respective battlefield [Ref. 8: p. 5].

The methodology proposed in this research directly supports use of the GVS by providing a means of determining actual optimum time values for the terrain grid network representing the battlefield. The methodology will also provide actual optimum time values and sets of path points for use in other combat models using conventional Lanchester attrition or firepower scores as long as they utilize explicit terrain grid networks for representing the battlefield. It will also provide excellent tactical movement planning and decision tools when incorporated into the appropriate mobility models.

Modelling the cognitive process in order to determine appropriate values for a given tactical movement scenario and instance and to then be able to map these values into the physical continuum, time, on the battlefield requires an understanding of how attitudes are measured, how to develop respective interval scales, how to convert cognitive values in interval scale into physical values in a ratio scale, and how to solve for the

"Psychometric Power Function" which allows the user to solve for this value given any set of realistic variables at different levels.

B. THE MEASUREMENT OF VALUES

The correct measurement of psychological values is fundamental to evaluating instances or combinations of variables and subsequently being able to use them in any physical construct. However, there are numerous ways to develop measurements and each results in different scales with varying characteristics. The three major characteristics of measurement are order, distance, and origin. Distinctions between types of measurement and types of scale depends on what combinations of these characteristics exist [Ref. 10: p. 15].

A nominal scale exhibits none of these characteristics and will therefore not be evaluated in terms of potential use [Ref. 10: p. 17]. An ordinal scale does possess order and may have an origin; however, the scale does not exhibit distance which results in no ability to discriminate between instances in terms of a value [Ref. 10: p. 16]. This limits potential types of usable scales in psychophysical analysis to interval scales.

1. Interval Scales

Interval scales always possess the properties of order and distance. It is also often possible to determine a rational origin for such scales [Ref. 11: p. 196]. Interval scales which exhibit all three major characteristics are known as ratio scales. These scales are the most important scales used in math and science and the term "measurement" is often restricted to these types of scales [Ref. 10: p. 31]. Interval scales can be determined in a variety of ways for sets of instances and scenarios. Three powerful methods are Continuous Response Scale Judgements, Comparative Judgements, and Categorical Judgements.

a. Continuous Response Scale Judgements

A continuous response scale allows a judge to indicate the value of an instance on a continuous scale between a minimum and maximum bound, normally 0 to 100. This method lends itself to Analysis of Variance (ANOVA) testing commonly used to determine relationships. The scale value of each instance is easily obtained and the summary statistics along with the distribution of responses for each instance are also easily obtained. However, one problem with this approach is that judges have difficulty actually determining values to assign between various instances [Ref. 8: p. 10]. This is magnified the larger the number of instances becomes. For example, only five variables broken down into five distinct levels would result in 5^5 or 3125 unique combinations of variables requiring responses. It is extremely difficult for a judge to realistically place a value on each with this number of instances and this approach was rejected due to this factor.

b. Comparative Judgements

The Law of Comparative Judgements presents the judge with all possible pairs of instances in order to determine which has the greater value in each pair. This discriminal process relies on the principle of "just noticeable differences" developed by L.L. Thurstone. The results of these judgements can be tabulated to provide a discriminal difference and deviation [Ref. 11: pp. 39-47]. Application of this theory is done using the method of paired comparisons. A matrix solution based on the frequencies of the responses is used to compute an interval scale. However, with n instances it requires $\frac{n(n-1)}{2}$ comparisons [Ref. 10: pp. 166-179]. For the five variables broken down into five levels, this results in 3125 instances and 4,881,250 separate comparisons. This is far more comparisons than is feasible in most studies and too labor intensive in terms of data collection.

c. Categorical Judgements

The Law of Categorical Judgements parallels the Law of Comparative Judgements in many respects. The judges evaluate each instance independently and select a rating category they feel best represents the general value of that instance. The categories are understood to be a mutually exclusive set of successive intervals which collectively exhaust all possible values of the continuum. Each rating category has descriptors and explanations which serve to help the judge with his task [Ref. 12: p. 1]. Normally distributed responses for each instance across the sampled population is a key assumption of this method. This is easily observed by examining the frequency matrix in Appendix B. Computing the interval and or ratio scale is relatively straight forward using a matrix solution [Ref. 10: pp. 205-239]. A clear, concise method is given by Professor Glenn Lindsay. Discussions with Drs. Parry and Lindsay at The U.S. Naval Postgraduate School assured that such an approach would be theoretically sound and yield usable results without requiring excessive survey and data collection effort and time once the scenarios were properly structured.

C. THE EXPERIMENTAL DESIGN

As previously addressed in the Problem Statement, the purpose of this study is to develop and demonstrate a methodology for determining an optimum, tactical movement path based on both the cognitive and physical considerations or constraints of the battlefield. As was also stated in the Introduction, devising and structuring tactical scenarios which insure the correct combinations of state variables or instances impacting on movement logic are addressed to the appropriate sample population can be an enormous and complex task. Administering the data collection effort once scenarios are developed is another major step in the experiment.

The process becomes more structured once the data are obtained. The Law of Categorical Judgements is used in this case and a well defined method for obtaining and translating the interval scale from the responses is available. Multiple regression analysis can then be applied to determine the functional relationship of the scale values for each instance to the set of state variables. The resulting regression equation becomes the "Psychometric Power Function" needed to determine values for use in GVS and other models.

1. Scenario Development

Identification of all possible state variables or those items needed to describe the specific tactical situation is the first step. This was accomplished in several ways by examining previous and ongoing tactical movement studies, reviewing doctrinal information on movement, and through discussions with groups of selected officers and combat modellers familiar with tactical movement. The following 14 variables are identified as the significant factors affecting decisions on tactical movement:

1. Mission
2. Time available
3. Equipment and resources available
4. Threat equipment and capability
5. Threat intention or mission
6. Range to the threat
7. Cover
8. Concealment
9. Environment, both weather and obscurrants
10. Area of operations or theater
11. Speed or vehicular agility
12. Range or distance to the objective
13. Obstacles, both natural and man-made
14. Artillery

The problem now becomes how to organize these variables into scenarios containing organized combinations or instances in order to provide a means of measuring certain attitudes concerning tactical movement. The number of state variables and their respective levels must be structured to reduce the required number of responses while maintaining the desired degree of fidelity in the experiment.

One technique is to eliminate any variables not significant to a majority of sources. This was the case with the variable artillery since the major impact doctrinally concerned obscurrants which fall under environment. In addition, other sources considered artillery to influence movement only through the use of improved or special munitions which usually fall under obstacles.

A second technique is to fix selected state variables for a given scenario, especially those which are categorical variables and not suited to a regression analysis. The respondent or judge is made aware of these in the definition of the scenario and the resulting scale and function are only valid for that specified combination of fixed variables. The following state variables are fixed when using this methodology.

1. Mission
2. Threat intention or mission
3. Area of operations or theater
4. Equipment and resources available
5. Threat equipment and capability

A third technique is to identify which variables may be combined when actually considered by the decision maker. This is the case when evaluating cover, concealment, environment, and range. All of these combine to determine the degree of inter-visibility or "line-of-sight" between the element and the threat. This line-of-sight or LOS and the number of lines-of-sight are what become critical to an element manuevering on the battlefield. There are also a number of excellent line of sight algorithms available for use in combat modelling.

This same technique is applicable in the case of localized speed or vehicular agility. Obstacles, environment, and vegetation combine with the terrain itself to determine speed. An element moving over the terrain is not concerned with soil moisture or tree stem spacing but with the total effect all these variables have on his speed. The combatant evaluates movement based on speed because he knows that the greater his speed, the less likely it is he may be engaged and the less time he will require to reach his objective in general. The Condensed Army Mobility Management System, CAMMS, provides excellent and highly accurate speed mappings for digitized terrain data.

The variable "range to threat" also requires further investigation. A combatant must evaluate many threats simultaneously and they are often different types with varying capabilities. The combatant makes his tactical movement decision based upon the greatest known or suspected enemy capability which can influence his mission at that moment. This becomes his greatest perceived threat. As mentioned previously, the combatant makes a determination of the potential lethality facing him based on the types of enemy, their range, and their capabilities based on range and disposition. A variation of the range dependency equations used in many Lanchester models will be used to calculate the "Relative Effective Range" or RER of each enemy system and allow the analyst to compare the lethality of various systems at different ranges much the same as the combatant. These have several mathematical forms studied by Seth Bonder and Bob Farrell from 1969 through 1974 [Ref. 13: No page]. The minimum RER of all threat systems with line-of-sight at a particular location then becomes the range which equates to that range given in the survey and used in the model.

One last observation concerns the variable time. Time actually becomes the dependent variable for which we wish to determine a relationship. The range to the objective is also directly related to and affects the time domain network inherent to GVS. These two variables are not included in the set of state variables given for each instance.

This now reduces the number of state variables to the five fixed variables identified previously and the following three key variables.

1. Localized speed
2. Number of lines of sight
3. Range to the greatest threat

The three key variables must now be broken down into distinct levels in order to construct the set of instances needed for the opinion survey. An extract of the actual survey used is located at Appendix A. A conscious decision was made to keep the required number of responses at a level which allowed the the survey to be completed in an hour or less by a respondent. The variables are each divided into four levels which cover the majority of conceivable occurrances for each variable. This results in 4^3 or 64 instances to be evaluated and which must be appropriately organized for the survey.

2. Survey Construction and Execution

This is only a pilot survey given to a limited population of U.S. Army and Marine Corps officers assigned to the U.S. Naval Postgraduate School, TRAC-Monterey, and Fort Ord. A total of 62 surveys were distributed during a one week period allowing two weeks for a response. A minimum of 25 surveys was needed and it was hoped that 50 usable surveys could be obtained. A total of 42 surveys were received at the end of three weeks of which 40 usable surveys were compiled to provide the data set located at Appendix B.

The actual survey used to gather responses is located at Appendix A. The state variables, divided into contiguous levels as listed, are organized into the 64 combinations or instances. These are then randomized to alleviate any possible influence on the respondents or judges due to ordering of the variables as each instance is evaluated. The number of levels and resulting number of instances has been kept as small as possible while still providing sufficient fidelity in the data in order to demonstrate the validity of this method without undue effort in the data collection. The cross-country or localized speed is broken down into levels of 0 to 5 KMPH, 6 to 15 KMPH, 16 to 30 KMPH, and 31 to 45 KMPH. The number or degree of LOS is divided into no lines-of-sight, 1 line-of-sight, 2 or 3 lines-of-sight, and 4 or more lines-of-sight. The range is divided into one kilometer bands out to 4000 meters. The exact value used for each variable within each band was also determined randomly in the case of speed and range using a pseudo-normal random number generator with the median value of the variable in that level as the mean. This provides further assurance that judges are not influenced by ordering or repeating of values.

The fixed variables are defined in an operations order given as part of the survey. The Central Army Group (CENTAG) area of Germany is the setting for an Army '92 balanced team with M1 Tanks, M2 IFV's, and attachments to conduct a deliberate attack against T80 and T72 Tank, BMP IFV, and BRDM2 ATV equipped Soviet units in defensive positions. Information on aircraft, helicopter, and artillery support is also provided.

The introduction of the survey also contains detailed explanations of the value being rated and the rating categories with examples of instances that would normally fall into that category. The five categories are :

1. No Movement Potential
2. Marginal Movement Potential
3. Effective Movement Potential
4. Good Movement Potential
5. Excellent Movement Potential

This rating of the state of a tactical element with respect to possible tactical movement based on the combination of variables or the instance is then combined with the corresponding ratings provided by all other judges in order to establish a data set from which an interval scale can be determined. This comprises the raw frequency matrix located at Appendix B, Section A.

D. RELATING THE COGNITVE AND PHYSICAL CONTINUUMS

The ability to accurately relate the cognitive and physical aspects of the battlefield varies immensely depending on the action being studied, the scenario, and the target population. Numerous methods can be applied allowing for complete translation of the cognitive continuum into the physical or structuring the resultant scales in order to perform a combinatorial optimization. Two specific methods and the underlying theory associated with them are examined in this study.

1. Computing Interval Scales from Categorical Responses

The theory involved in deriving interval scales using the Law of Categorical Judgements is fully explained by Professor Glenn Lindsay [Ref. 12: pp. 6-12] and W.S. Torgerson [Ref. 10: pp. 207-227]. However, a review of the major points and underlying assumptions is worthwhile. There are 6 key assumptions when using this technique.

1. Judges' feelings about the scale value of an instance are normally distributed across the population with mean μ_i and variance σ_i^2.
2. The categories must be a mutually exclusive set of successive intervals which collectively exhaust the property continuum.
3. Five categories are required for rating instances.
4. Judges' feelings about a category upper bound are normally distributed so that for category j, the upper bound would have mean μ_j and variance σ_j^2.
5. This also implies that judges feelings about the distance between an instance value and category bound are normally distributed.
6. Category bounds have the same variances for all j categories.

The technique takes the raw frequency matrix for all instances as mentioned and first divides these frequencies by the total number of respondents in order to determine the relative frequencies for each instance and category. These relative frequencies are then summed across the categories for each instance to derive cumulative relative frequencies which are related to cumulative values under a normal curve. These cumulative relative frequencies for category j of instance i become the normal probabilities, $p_{i,j}$, used in computing the interval scale. The $p_{i,j}$ are grouped based on which instances have categories with similar ratings and all $p_{i,j}$ greater than 0.98 and less than 0.02 are rolled over into the adjacent category in order to reduce the effect and possible influence of outlying judges. Treating the $p_{i,j}$ as the area under a Normal (0,1) Curve, the Z matrix of normal values can be constructed. Row, column, and grand averages are computed from this matrix along with an estimate of the standard deviation. The interval scale values of all instances are computed as the grand average minus the value of the row average times the standard deviation. A linear transform is then used to obtain the correct distance for the scale.

The 10 step method of interval scale development given by Lindsay is shown below:

1. Arrange the raw frequency data in tablular form where the rows are instances and the columns represent categories. Columns should be sorted in a rank order from least to most favorable as left to right columns respectively.

2. Compute the cumulative relative frequencies for each row and record these values in a new table. This table is referred to as the P array and all values of $p_{i,j}$ greater than 0.98 or less than 0.02 are combined with the following category eliminating extreme cases. This creates an n by (m-k) array where k is the number of columns removed.

3. Treating the $p_{i,j}$ values as leftward areas under a Normal (0,1) Curve, find the corresponding value of Z from a normal distribution table. Record these values as a new table which is the $Z = \|\, z_{i,j} \,\|$ array.

4. For each instance, i, in the Z array; compute the row average, $\bar{z}_i$.

5. For each column or category, j, in the Z array; compute the column average, $\bar{b}_j$. Note that $\bar{b}_j$ is the value of category j's upper bound on the interval scale.

6. Compute the grand average, $\bar{b}$, of all Z values.

7. Compute $B = \sum_{j=1}^{m-k} (b_j - \bar{b})^2$ which is the sum of the squared column differences.

8. For each row compute $A_i = \sum_{j=1}^{m-k} (z_{i,j} - \bar{z}_i)^2$, the sum of the squared individual differences.

9. For each instance compute $\sqrt{\dfrac{B}{A_i}}$, an estimate of $\sqrt{\sigma_i^2 + c}$.

10. Finally, for each row or instance compute $S_i = \bar{b} - \bar{z}_i \times \sqrt{\frac{B}{A_i}}$, for all i.

The S_i values are the interval scale values of the instances which are on the same scale as the category bounds, b_j . This now enables us to impart distance and a rational origin to the scale through anchoring and linear transformations. The above procedure is performed on the raw data at Appendix B. The data is segregated into 7 groups corresponding to how instances were rated by like categories. The first and last groups cannot be utilized in determing interval scales since they represent extreme instances in the survey, have incomplete $Z_{i,j}$ arrays, and only contain one boundry point. All groups have incomplete $Z_{i,j}$ arrays which is why they are segregated. A tactic which enables us to continue with the scale development requires performing the scaling procedure on each group and then performing a linear transformation between groups with two or more common boundries [Ref. 12: pp. 18-20]. Each group of data is then normalized and an interval scale computed utilizing the A Programming Lanquage (APL) program at Appendix B, Section G. The APL program "NORM" was originally developed by Paul M. Crawford during his thesis research [Ref. 8: p. 89]. The results of the normalization and interval scale for each group are located at Appendix B, Section H.

2. Converting Interval Scales to Usable Forms

The major requirement at this point is to transform the interval scale values for all groups into one contiguous scale by establishing initial boundry values for one group and then performing the linear transformations on the remaining groups. This provides consistent values for all instances evaluated on one usable scale. Another necessity in order to use the scale in models is to convert the interval scale into a ratio scale by establishing a rational origin and anchor point or points which intersect on the physical and cognitive scales. This allows us to directly map the two continuums together simultaneously. The boundry conditions applied will begin with Group II where a rational origin can be applied at the boundry in common with the extreme points shown at Appendix B, Section E. A method to anchor the other extreme end of the scale will also be discussed.

a. The Natural Origin and Anchoring in Computing Interval Scales

The extreme cases reflecting excellent potential correspond to cases which reflect little or no anti-potential. We can reverse the rating categories if desired (which was done in this case) so that excellent TMP or no anti-potential is the worst rating and then perform the normalization procedure and develop the scales. This is done in order to correctly place the rational origin. The physical scale of the battlefield based on traversal times between cells or nodes reflects only the effects of constraints on the physical mobility of a given system. At long ranges when there is no inter-visibility with opposing forces and the system can move with great speed, there is no effect due to the cognitive considerations of the battlefield. This zero point or rational origin on the cognitive scale corresponds to the effect any cognitive rating of these extreme cases would have and also the respective value of this boundry on the scale. An arbitrary value of 100 was selected for the upper boundry of Group II. This now allows for the use of the APL program "TRANS" shown at Appendix B, Section G. This program was also originally developed by Paul Crawford [REF. 8: p. 90].

b. The Linear Transformation

The linear transformation performed by this program solves a set of simultaneous equations on each group of the form:

$$\text{Lower Bound} = \alpha + (\beta \times \text{Best Column Average})$$
$$\text{Upper Bound} = \alpha + (\beta \times \text{Worst Column Average})$$

where the column averages are already computed and the bounds are input. Once α and β are known, the instance values and other intermediate boundry values can be easily computed. These other boundry values along with those input are then used to perform linear transformations on all other groups with two or more boundries in common. All groups can then be combined in order to form one contiguous scale and the value of the extreme cases located at the rational origin may be added to the set of scale values of the instances. The set of transformed instance values is shown at Appendix B, Section I in the last column. The set of instance values along with its corresponding set of variable values are combined to form the regression matrix shown at Appendix B, Section I.

3. Regression Analysis of the Interval Scale Values

Multiple regression techniques are used on the data and derived scale values. Two different regression packages, Grafstat and SAS, are used applying backward elimination procedures. Plots of the independent variables taken two at a time with the instance values of TMP are done and then surface fitting procedures applied in Grafstat. These plots are similar to scatter plots and used in much the same way. Relationships between independent variables are of particular interest. The example plots in Appendix C, Section A reflect the linear relationships between pairs of variables. The fitted surfaces appear as planes. Due to this result, multiple linear regression is used to include product terms of all pairs of independent variables. Appendix C, Section B gives the results of backward elimination using Grafstat and Appendix C, Section C gives the corresponding results of backward elimination using SAS. Each section gives the best 4 and 5 variable models based on the largest R^2 with the best acceptable levels of significance.

The respective best regression equations for the four variable case and the five variable case are:

$$TMP = 118.24 - 0.015(RNG) + 12.0(LOS) - 2.0(SPD) + 0.2(LOS \times SPD) \qquad \{1\}$$

$$\begin{aligned} TMP = 112.32 - 0.012(RNG) + 11.4(LOS) - 1.7(SPD) \\ + 0.21(LOS \times SPD) - 0.00015(RNG \times SPD) \end{aligned} \qquad \{2\}$$

These were the only two equations found acceptable based on the multiple regression analysis.

4. The Psychometric Power Function

The "Psychometric Power Function" is the selected best regression equation from the regression analysis. The scale can be fully determined by taking the set of maximum and minimum values for the independent variables and using these to compute the corresponding maximum and minimum values of TMP from the regression equation. The user must be aware that use of this equation or power function is only valid for the initial range of values surveyed and used in the regression. The minimum value reflects the rational origin or highest TMP which imparts zero additional time to the physical scale as discussed previously. The problem now becomes how to anchor the other extreme of the scale which reflects the maximum scale value or no TMP.

5. Translating Values of the Interval Scale to the Physical Scale

The essential step proposed by this methodology requires translating the cognitive values obtained using psychometrics into the same scale as the physical traversal time scale of the battlefield or to be able to somehow perform a combinatorial optimization between the two scales. There is a fortunate structure in the data resulting from the survey. Referring to Appendix B, Section E; there is only one extreme instance occuring on the end of the scale opposite the rational origin. It is therefore possible to prompt the user or survey a selected population to determine the amount of additional time in mission completion a combatant would be willing to accept in order to avert such a tactical instance. It is only possible to construct such an anchor due to the fact that there is only one or are a relatively small number of related instances placed at this extreme end of the continuum. The scale value of this combination of variables or instance is easily calculated. It becomes quite simple to then translate the cognitive scale to the time scale by dividing the scale or TMP values by the maximum value computed from the power function for the extreme instance which results in a (0,1) scale. Then multiply through by the given additional time value from a population mean or as provided by an individual respondent. This time scale obtained using the cognitive data can be used additively with the physical traversal time scale. Values for each individual user or a mean value for a selected population could be preferred depending on the application. This translation allows for speed and simplicity when performing the optimization using Dijkstra's algorithm which is discussed in Chapter IV. Such an approach is highly preferred if at all possible. However, it is often not possible to relate the cognitive and physical scales as in this research. Other optimization techniques which work in combinatorial optimization applications are also discussed in Chapter IV.

IV. MODEL CONSTRUCTION

A. GENERAL

This chapter outlines overall structure of the model and how the methodology already discussed is controlled and implemented. There is also some detailed examination of algorithms and methodologies; particularly concerning line-of-sight, relative effective range, and minimum cost optimization. This study follows a logical order beginning with required inputs, continues with algorithms and theory used to process data in the model, examines optimization techniques, and concludes with the model output.

The research version of this model, called the Tactical Optimum Path or TOP Model, is located at TRAC; Monterey, California and at the Mobility Systems Division, U.S. Geotechnical Laboratory; Vicksburg, Mississippi. The current model contains approximately 80,000 lines of code with most routines done in FORTRAN and the user interface, graphics, and data management executed in the C Programming Language which is used as a shell. More detailed documentation and a draft users manual for the model will be published as a joint technical report through the U.S. Army Waterways Experiment Station in late 1989.

B. TERRAIN DATA BASES

The digitized terrain data bases used for this model are the identical terrain data bases used for the CAMMS (Version 2.0). This is 100 meter grid explicit terrain data translated specifically for this version of the CAMMS [Ref. 7]. The model allows for selection of a geographic region, country, area, and specified "Quad" sheet. This selection corresponds to one of the fixed scenario variables, area of operations, identified previously.

A detailed explanation of explicit terrain data and its use can be found in *Lecture Notes In High Resolution Combat Modelling* by James K. Hartman [Ref. 4]. We will therefore dispense with any major discussion of these data. It is important to note that many of the terrain parameters provided in the data set are used in other algorithms. This is true of the line-of-sight (LOS) algorithms used. Portions of these algorithms have been altered to conform to this specific terrain data base.

C. REQUIRED INPUTS

The remaining required inputs primarily relate to the scenario development or fixed variables as well. Two of these variables needed as inputs are the appropriate missions of each force. Two other variables comprise the equipment mix available to each force. These variables are input to the model by the user during initialization of the program. The variables listed up to this point encompass all the fixed scenario variables for the methodology. It is important to stress that only the psychometric function determined for that specific combination of variables or scenario is valid for that combination in the model.

There are presently three other inputs to this model which must be addressed in our discussion. These are the element locations for each force, current environment, and the parameter related to the extreme scale value selection for translating the cognitive scale which was discussed in Chapter III, Section D. The necessity of this last input is evident. The locations and environment are necessary in determing path start, intermediate, and end points as well as range and line-of-sight results. The environment also affects mobility calculations and may be played using real time data once the Airland Battlefield Environment (ALBE) system is incorporated in the CAMMS.

The user inputs these data to include season, current weather, light condition, and soil-moisture condition for the environment. The user must input initial locations and any required subsequent locations depending on the mission as each type of combat element is entered. The user is then prompted as the last step prior to initiation of the program to provide the time value corresponding to the extreme case or anchor cited previously or to select the expected value derived from the sample population. This concludes all required inputs necessary to run the model.

D. THE MOBILITY MODEL, CAMMS (VERSION 2.0)

The CAMMS has evolved from research, testing, and analysis conducted over the past 15 years. The CAMMS was initiated in 1983 in order provide a capability for predicting mobility in a field environment in real time [Ref. 7]. There are numerous outputs available from the CAMMS relating to mobility as listed in Chapter II. This model will utilize two specific output mappings and files from the CAMMS Route Analysis Prediction: the Vehicle Speed Prediction Map and a version of the Vehicle Time Contour Map. These mappings provide the localized cross-country speed or vehicular agility which is one of the independent variables necessary for the psychometric function and the traversal times along each segment between nodes which comprises the set of arc traversal values for the physical continuum. These output values are valid for the specified item or weapon system on that terrain under the specified environmental conditions. The weather and environment are incorporated in all calculations through the Soil Moisture Strength Prediction which is one of the first routines performed by the CAMMS [Ref. 7]. A speed value is stored for all vehicle types for each grid cell or node to be used in later calculations. The arc traversal times for segments between all adjacent grid cells or nodes are also stored for later use.

The output provided up to this point by the CAMMS are all mobility modelling calculations to be used in the model. The remaining areas we will examine pertain to the cognitive considerations affecting movement and performing the optimization.

E. LINE-OF-SIGHT MODELS

The model also determines if lines-of-sight exist and to what degree or how many in order to obtain the next independent variable, LOS. There are several excellent means by which to determine LOS. A detailed explanation of line-of-sight calculations based on all types of terrain data can be found in *Lecture Notes In High Resolution Combat Modelling* by James K. Hartman [Ref. 4]. We will therefore dispense with any major discussion of LOS modelling here.

The LOS algorithms used in this model are derivations of two LOS subroutines, IVSCAN and VISTA, developed and used by the BDM Corporation in the Battalion Combat Outcome Model (BCOM) and U.S. Army TRAC respectively. The subroutines allow for point to point LOS determinations or inter-visibility fan determinations depending on the desired application and which routine is used. The subroutines are modified to use the explicit grid terrain data parameters from the CAMMS terrain data base.

The number of LOS for a given grid cell or node is determined by using the inter-visibility fan from each known or suspected threat location to obtain a frequency count of the total number of lines-of-sight for each cell, FLOS. This value is also one of the independent variables stored for use later in the model.

F. RELATIVE EFFECTIVE RANGE METHODOLOGIES

The value of range used in the model requires a more detailed discussion than do the other variables. One of the assumptions given initially addressed the fact that the forms of the range dependency equations studied and proposed by Seth Bonder and Bob Farrell are correct [Ref. 13]. Several such forms and their use are expounded by James Taylor [Ref. 14 : pp. 93-99]. One similar form was used by Gordon Clark in DYNTACS in determining the "Difficulty", E, from his set of comparative judgements [Ref. 3: p. 98]. While there appears to have been a great deal of past interest in such a methodology; there has not been any significant or sufficient analysis of how lethality and particularly perceived lethality on the part of a combatant is affected by range. Such a study is well beyond the scope of this thesis and is not attempted. This is the primary reason the stated assumption is made. However, an explanation of the equation forms and why they are used in this research is necessary.

The basic approach concerning range in this model is to assume the combatant has sufficient technical knowledge of the systems on the battlefield in order to accurately assess their potential effectiveness against other systems. It is possible to use Probability of Hit, Probability of Kill|Hit, and Total Probability of Kill values from classified AMSAA JANUS model scenarios along with the classified effective range data for a weapon system type to fit one of the equation forms previously proposed. This becomes the perceived as well as actual lethality curve for that system. The systems all have various maximum effective ranges with differing curves and equations. The systems are equated through the use of a baseline range which is assumed to be the range of the battlefield against which the actual range to maximum effective range ratio or difference is adjusted. This results in a Relative Effective Range, RER, which reflects the range given in the survey and used in the model.

Some technical data from unclassified reports, periodicals, and the general population's knowledge files were easily obtained. These sources provided some idea as to the shape and resulting type of equations the lethality curves for various systems possessed. Ranges were obtained from current military periodicals [Ref. 15]. The next step is to find a set of fictitious yet properly structured and ordered parameters for the equation forms identified from these sources. A number of potential forms could be evaluated. Actual ranges and parameters examined in JANUS data files or other sources are not used in this research.

The following four equations are examined based on discussions with and previous analysis performed by the aforementioned combat modellers:

$$RER = RG\left\{\frac{RA_i}{RM_i}\right\}^{\mu_i} \qquad \{3\}$$

$$RER = RG\left\{e^{-\alpha_i(RM_i - RA_i)}\right\} \qquad \{4\}$$

$$RER = \frac{RG}{(\gamma_i + 1)}\left\{\gamma_i - \cos\left(\frac{\pi \times RA_i}{RM_i}\right)\right\} \qquad \{5\}$$

$$RER = \frac{RG}{(\gamma_i + 1)}\left\{\gamma_i + \cos\left(\frac{2 \times \pi \times RA_i}{RM_i}\right)\right\} \qquad \{6\}$$

where,

RER is the Relative Effective Range used in the model

RG is the baseline range used for this tactical scenario

RA_i is the actual range to system i

RM_i is the maximum effective range of system i

μ, α, and γ are the respective shape scale parameters for each respective equation form

Equation number 3 is a basic Power form of the range dependency equation similar to the actual versus maximum range ratio attrition equation used in Lanchester models [Ref. 14: p.98]. Equation number 4 is an Exponential form which works well when fitting non-ballistic weapon systems with fairly constant lethality over range such as current generation anti-armor missle systems. Equation 5 is a Half-Cosine form which works well for systems that tend to have much smaller lethalities in comparison to most other systems such as small arms. Equation 6 is a Full-Cosine form applicable for systems which may have minimum as well as maximum effective ranges. [Ref. 14: pp. 96-99]

Sample graphical results of the plots performed is useful at this point. Figure 1 shows a comparison of the Soviet T72 main Battle Tank versus an AT5 Spandrael Anti-Tank Missle.

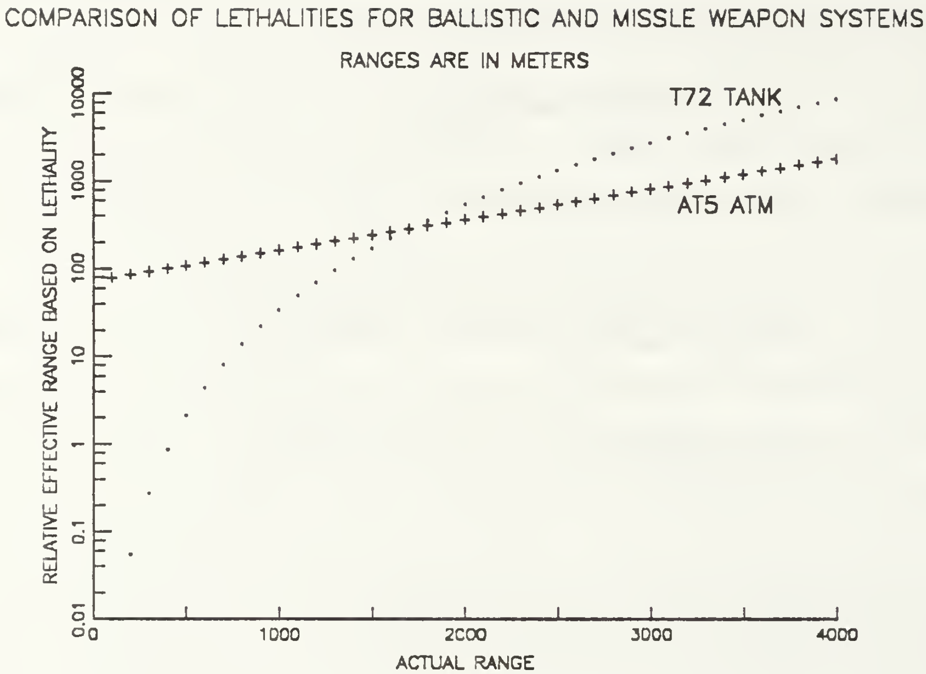


Figure 1. Comparison of Relative Effective Range for Soviet MBT vs. ATGM

The tank is represented by a Power equation with a μ of 4.0 and RM of 3200 meters and the ATGM is represented by an Exponential equation with an α of 0.00080 and RM of 5000 meters. The RER is plotted on a logrithmic scale due to the extremely low values of RER at shorter ranges. Low RER implies greater lethality. Note the significant difference in Relative Effective Range out to approximately 1200 meters. The tank is perceived to have much greater lethality at these shorter ranges. No major difference is evident between 1400 and 2000 meters; but, the lethality of the ATGM remains more constant over range and Relative Effective Range changes in favor of the missle from 2500 meters out to the limit of the battlefield.

A similar comparison exists for the M1A1 Main Battle Tank and the M2/3 Infantry/Cavalry Fighting Vehicle where the tank is a ballistic system and the IFV is a missle system based on the anti-tank missle system in addition to a 25 millimeter chain gun. This comparison is shown in Figure 2.

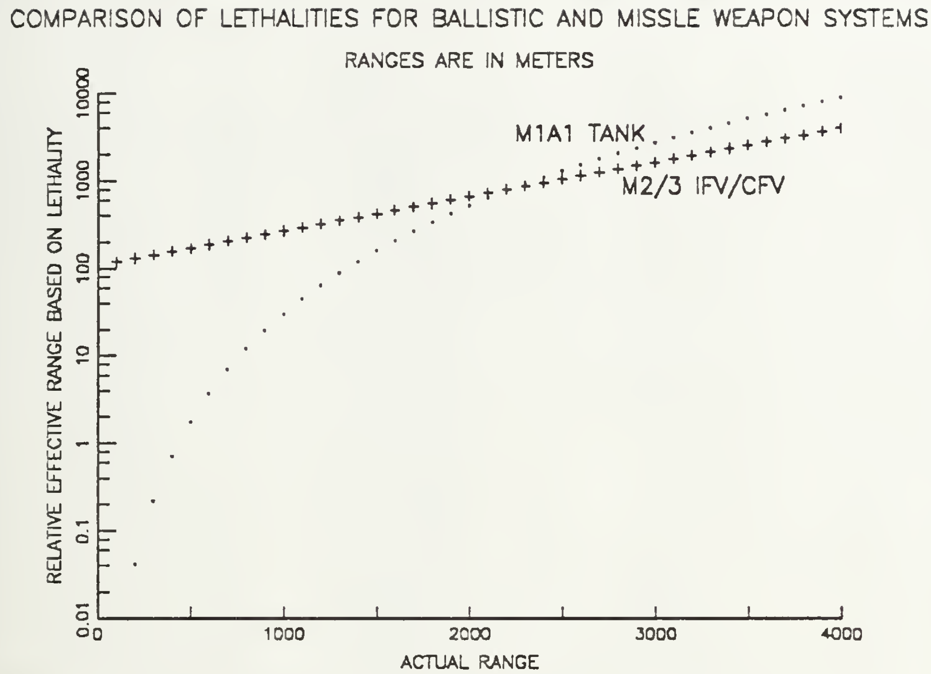


Figure 2. Comparison of Relative Effective Range for U.S. MBT vs. IFV

The tank is represented by a Power equation with a μ of 4.25 and RM of 3500 meters and the IFV is represented by an Exponential equation with an α of 0.00090 and RM of 4000 meters. Note the significant difference in Relative Effective Range signifying the tank is the greatest perceived threat out to 2000 meters and the IFV equipped with the TOW-2 missle is obviously greater past approximately 2800 meters.

Complete listings of the systems included, the equation forms used, and resulting parameters are shown in Tables 1 and 2. Only the Power and Exponential forms are used in this analysis and in the model. Table 1 lists the values of the maximum range and scale parameter, α , for systems represented using the Exponential form equation and Table 2 lists the values of the maximum range and shape parameter, μ , for systems represented using the Power form equation.

NOMENCLATURE	COUNTRY	PARAMETER, α	MAXIMUM RANGE
AT4 ATGM	USSR	0.00085	4000 m.
AT5 ATGM	USSR	0.00080	5000 m.
AT3 ATGM	USSR	0.0030	1500 m.
BMP-1 IFV	USSR	0.00275	2000 m.
BMP-2 IFV	USSR	0.00080	4000 m.
BRDM-2 ATV	USSR	0.00075	5000 m.
HIP HELO	USSR	0.00180	4000 m.
HIND HELO	USSR	0.0010	4000 m.
M901 ITV	USA	0.0010	4000 m.
M2 3 IFV CFV	USA	0.00090	4000 m.
TOW ATGM	USA	0.00125	4000 m.
DRAGON ATGM	USA	0.00250	1000 m.
AH1S HELO	USA	0.0010	4000 m.
APACHE HELO	USA	0.00075	4000 m.

Table 1. TECHNICAL DATA FOR MISSLE SYSTEM EQUATIONS

It is also important to note that we can compare NATO and Soviet systems using these curves. Note that the T72 Main Battle Tank has a lower μ value and is not as effective as the M1A1 Main Battle Tank nor does it possesss as long a maximum effective range. The difference is noticeable when the plots shown in Figures 1 and 2 are compared.

It is also interesting to note how the lethality for various systems change with respect to range. This gives some idea as to whether or not the RER calculated makes sense or is verifiable. The small change in slope of approximately 0.1 observed within the first 1000 meters of actual range for both tank weapons systems is one example. This range is within the boresight range of each system and there is little or no change in the trajectory of tank cannon munitions at such short ranges. The slope and subsequent RER support these facts. The slope between 2000 and 3000 meters actual range increases to approximately 1.7 indicating perceived lethality decreases more quickly. There is almost no difference in the RER of 1 at 500 meters range and the RER of 14 at 1000 meters range for the M1A1 Tank. However, there is a significant difference in the RER of 2600 at 3500 meters for the tank. The TOW-2 missle on the IFV has an RER of only 1200 at 3500 meters range. While these equations and parameters are fictitious, their application and results correspond to the manner in which experienced military personnel tend to compare these systems.

NOMENCLATURE	COUNTRY	PARAMETER, μ	MAXIMUM RANGE
T54 55 MBT	USSR	3.50	2000 m.
T62 64A MBT	USSR	3.75	2800 m.
T64B 72 MBT	USSR	4.00	3200 m.
T80 MBT	USSR	4.25	3200 m.
BTR60 60P PC	USSR	0.50	1500 m.
DIS. INF.	USSR	0.40	1000 m.
UNK. VEH.	USSR	1.50	4000 m.
UNK. OTHER	USSR	1.00	4000 m.
M48 60 MBT	USA	3.80	2800 m.
M60A1 MBT	USA	3.90	3200 m.
M60A3 MBT	USA	4.00	3200 m.
M1P1 MBT	USA	4.10	3200 m.
M1A1 MBT	USA	4.25	3500 m.
M113 PC	USA	0.50	1200 m.
M60CEV	USA	0.80	1000 m.
DIS. INF.	USA	0.40	1000 m.
UNK. VEH.	USA	1.50	4000 m.
UNK. OTHER	USA	1.00	4000 m.

Table 2. TECHNICAL DATA FOR BALLISTIC SYSTEM EQUATIONS

For this model, an RER value for each cell or node is determined by taking the actual range, *RA*, for each system with line-of-sight to that cell and using this range in the appropriate RER equation form for that system. The minimum value of the RER for all systems with line-of-sight, if more than one exists, is retained as the RER value for a cell.

The data and equation forms identified and derived from this short study do need to be evaluated more thoroughly. However, such an effort is beyond the scope of this thesis and would require development of an extensive experiment in order to sufficiently determine how perceived lethality is affected by range for various weapons systems. Equation forms with exact parameters derived from existing AMSAA data would almost certainly be classified as would the results of such an experiment. This is certainly a proposal for future study and enhancements; but, the assumption stated is used at this point in order to continue development of methodology based on the use of psychometric techniques without undue requirements due to an analysis of perceived lethality and/or inclusion of classified data.

All required inputs to the model are complete once the CAMMS has been run; the LOS calculations are performed; the SPD, FLOS, and RER values have been obtained for each cell; and the arc traversal times between all adjacent cells are stored. The application of the cognitive portion of the model and transformation of the values to be used in the optimization can now be done.

G. APPLYING THE PSYCHOMETRIC FUNCTION

The physical time scale is already determined from the CAMMS but we must utilize the other data as inputs to the psychometric or decision function derived from the regression analysis in order to obtain the cognitive or tactical time scale. The regression equation or decision function uses FLOS, RER, and SPD as independent variables in order to output a dependent variable, Tactical Movement Potential or TMP. This value is computed as a ratio scale value for each cell through application of a rational origin and then translated into the corresponding physical time scale made up of the traversal times through the use of an anchor as outlined in Chapter III, Section D. We now have two sets of corresponding time values.

The cognitive or tactical time values represent the additional cost in units of time, equivalent to those time units on the physical scale, for a combat element to enter that cell based on the existing set or combination of state variables. The two sets of time values are additive. Consequently, the tactical time computed for each cell is then added to the physical traversal times stored for each arc which enters or has a tail node in that particular cell. This set of time values becomes the total time along each arc and the set of values desired for use in the optimization.

H. THE OPTIMIZATION

Optimization of this set of translated network values is performed using Dijkstra's algorithm which is relatively quick and efficient in this application. However, it is not always possible to determine a rational origin and particularly one or more anchors in order to translate the cognitive scale. Dijkstra's algorithm can solve the optimization problem very simply if the requirement is to either reach the objective by minimizing tactical difficulty which is equivalent to maximizing TMP, to reach the objective by minimizing only the traversal time, or to reach the objective by minimizing some total translated time as in this case. However, the problem is one of combinatorial optimization when it is not possible to determine a relationship between the two scales. We will first conduct a quick review of minimum cost optimization using Dijkstra's algorithm and then examine one method for performing the same optimization combinatorially on separate physical and cognitive scales.

1. Dijkstra's Algorithm

Dijkstra's algorithm uses a label setting procedure on a network graph with non-negative costs for all arcs in the network. This network structure must be maintained in order to avoid possible negative cycle lengths. Two sets of cells are maintained. The first set are those cells whose shortest distance or minimum cost path is already known. The second set consists of all other non-traversed cells. The known set is initially comprised of only the start or source node. At each iteration, a new cell is added to the known set which has the shortest distance or minimum cost path to the start point. The minimum cost value or shortest distance can be recorded or a predecessor array maintained. Eventually, all possible cells on the battlefield are included in the known set. At this point, the minimum cost path from the source to any other cell is available and the value can be determined. [Ref. 16: pp. 203-208]

Dijkstra's algorithm can be made even more efficient by limiting the nodes searched at each iteration. This is applicable in the model since the only permissible arcs are those between adjacent cells. The algorithm can also be used to perform a minimum cost optimization along a set of intermediate objective points or cells through application of the "Sliding Pattern Concept" [Ref. 5: pp. 22-26]. The optimization is performed locally between the ordered set of cells n times, where n is the number of intermediate cells when the order is specified and must be maintained. The optimization is performed in total until all intermediate cells are labeled or in the known set and all are contained along one unique path. This can be accomplished using any number of data structures. The set of objective points or cells is input by the user when the model is being initialized. In summation, Dijkstra's algorithm is quite useful for almost any network and can be adapted to make it even faster and more efficient with little work.

2. Combinatorial Optimization

The optimization problem can be solved very simply if the requirement is to either reach the objective by minimizing some cost along one unique scale when the cognitive and physical scales can be related as demonstrated in the previous sections. However, the problem is one of combinatorial optimization when it is not possible to determine a relationship between the two scales. This requires the analyst to find a method by which it is possible to minimize tactical difficulty while also optimizing the traversal time. The optimized time must be less than time $\overline{T}$ and the tactical difficulty must also be minimized within this optimization. Time $\overline{T}$ should be given by the user or obtained from the mission statement of the scenario. The model should select a route which satifies all constraints while solving one equation for both tactical difficulty, td_x, and the traversal time, t_x. A method that combines both constraints into one equation is the use of a Lagrangian multiplier, $\lambda \geq 0$, which can be used in the following equation:

$$\min_{\text{all x}}(td + \lambda \times t)x - \lambda \times \overline{T} \qquad \{7\}$$

where,

x is a vector of the arcs which make up a network solution

td is the tactical difficulty based on exposure associated with each x

t is the travel time associated with each x

λ is the Lagrangrain multiplier

$\overline{T}$ is the maximum time limit to traverse the network

a. Assumptions

1. $\overline{T}$ is larger than the time it takes an element to reach its objective using a shortest time network solution.
2. The network is undirected. This allows an element at destination B to backtrack along its route if that route offers a less difficult route and still meets the $\overline{T}$ time limit to reach the final objective.

b. Methodology

The equation is used to calculate the shortest path based on λ using Dijkstra's algorithm. The algorithm shown at Appendix I uses a hierarchial adjacency list to search for the shortest path between the start point and the intermediate point and then from the intermediate point to the final objective. Other data structures could be used.

To calculate the maximum possible value, λ_{max} , the largest t_x from all the edges in the network is multiplied by the number of vertices in the network. If λ_{max} is used as λ in the equation, the objective function minimizes the time traveled throughout the network with no regard to tactical difficulty. When λ is zero, the minimum possible value of tactical difficulty is found. There exists a λ such that $0 \leq \lambda \leq \lambda_{max}$ gives the optimum tradeoff in minimizing the tactical difficulty while ensuring the traversal time constraint is not violated.

The best lower bound for the equation is determined by solving for $\max_{\lambda \geq 0} L(\lambda)$ which can be solved by doing a binary search from λ_{min} to λ_{max}. As long as the slope of $L(\lambda)$ is non-positive then the route selected is a feasible solution with regard to $\overline{T}$. A binary search is conducted on the interval between λ_{min} and λ_{max} until a specified interval, ε , is obtained between λ_{max} and λ_{min}., This λ with ε sufficiently small indicates the search for the optimum path minimizing tactical difficulty while arriving at the objective within $\overline{T}$ traversal time has been accomplished.

c. *The Combinatorial Optimization Algorithm*

A VS-Fortran program was developed to solve the tactical movement problem (see Appendix I) and consists of the following steps:

1. The data set containing the edges and their associated time and difficulty values are read into a hierarchial adjacency list. The value of the maximum t_x and TDMAX in the network are found.

2. Inititially Dijkstra's algorithm is called to find the slope, objective function value and path for $\lambda = 0$ and $\lambda_{max} = TDMAX \times (Nodes - 1)$

3. A check is conducted between λ_{min} and λ_{max} and if $\lambda_{max} - \lambda_{min} \leq \varepsilon$ then the program is completed and the shortest path with the lowest tactical difficulty has been found.

4. λ_{mid} is the midpoint between λ_{min} and λ_{max}

5. Dijkstra's algorithm is called to compute a slope, objective value and path based on λ_{mid}

6. If the slope is positive then λ_{mid} becomes λ_{min} and Step 3 is repeated. Otherwise the slope is non-positive, indicating a feasible path and λ_{mid} becomes the new λ_{max}. The objective function value of λ_{mid} is compared to the previous best objective function value, OBJMAX. If λ_{mid} objective value is larger than OBJMAX, λ_{mid} objective value becomes the new OBJMAX and the optimal path, OPTX, is updated. Go to Step 3.

Once the ε value has been reached the optimum path and associated optimal values for time and tactical difficulty are outputted to a file.

d. *Conclusions*

The complexity of Dijkstra's algorithm is $O(\mid V \mid^2)$ and the complexity of the binary search is $O(\log_2 TDMAX)$. The Dijkstra algorithm is utilized with each iteration of the binary search; therefore, the total complexity of the program is $O(\mid V \mid^2 \log_2 TDMAX)$

To calculate the required number of iterations for a set of nodes, use the equation $\frac{1}{|V|^2 t_{max}^2}$ to determine the interval required and then solve for the number of iterations, k, using the equation

$$k > \frac{3\log_2|V| + \log_2 TDMAX + 2\log_2 T_{max}}{\log_2 2} \qquad \{8\}$$

or use an ε of sufficiently small value.

Changes to the program can be made to decrease the run time if more information is known about the nature of the network. In a directed network the Dijkstra's algorithm can be changed to stop the search once point B has been found and then continue from point B until point C is found. The search could also be restricted over certain portions of the network based on the network's structure, greatly reducing the number of searches. Another method that could be incorporated is changing the binary search to a slope intercept search over the interval between λ_{min} and λ_{max}. These methods were not incorporated in this model since it could be used to model several types of networks with varying numbers of intermediate points and structures.

I. MODEL OUTPUT

The output for the model is designed to be presented graphically in addition to the data files for use in models. Some output is available interactively during model initialization. All output is available upon completion of a model run for use in planning, analysis, and to provide an audit trail. The key output is certainly the set of optimum path points and total route traversal time.

Interactive output consists of LOS fans displayed to the screen, a selection of base maps which include terrain elevation or vehicle speed maps, element locations, area of operation outline, and path point locations. Upon initiation of the model, cells are labelled in accordance with Dijkstra's algorithm and shown on the screen until completion of the routine, at which time the optimum path is displayed. The data are also stored for later use. Elimination of the interactive graphics capability when the model is interfaced with a combat model will greatly reduce run time.

Final output is comprised of a selection of mappings which are the vehicle speed map or SPD map, FLOS map, RER map, TMP map, and total time map. The locations, path points, area of operations outline, and actual optimum path can be imposed on any one of the selected base maps. Base maps for hard copy output may be selected in scales from 1 to 25,000 up to and including 1 to 250,000. Actual cell values for each state variable can be accessed by selecting a point interactively with a bit mapping utility. The set of optimum path points are easily obtained from a data file.

V. CONCLUSIONS AND RECOMMENDATIONS

A. GENERAL

The working model is resident in the warlab of The TRADOC Analysis Command; Monterey, California. Initial verification runs were conducted on 17 through 25 July 1989. The model is specifically tailored so as to optimize speed and efficiency on a PC-AT 386 computer. A Dell System 325 with VGA-Plus graphics and a high speed, wide carriage color printer for hard copy output was purchased to support the model. Results and subsequent conclusions and recommendations based on the methodology implementation and model runs are addressed in the following two sections.

B. CONCLUSIONS

1. Model Verification

The ability to perform a validation of the model within the scope of this thesis is impractical if validation is possible at all. However, verification in terms of the general behavior exhibited by the model and the sensibility of the results must certainly be examined. A very strong verification is not even possible without additional support and experimentation. Certainly, a more stringent verification or validation should be the focus of additional study; but, we must rely on the knowledge of experienced military officers and combat modellers in order to accomplish this limited verification.

There were 16 different variations of fixed state variables initiated for the test scenario, each with a different set of beginning path points and objectives. The expected value of 5.5 minutes for the scale time value is used in 10 of the model runs. A value of 0.0 minutes is used in 2 of the model runs in order to compare the results to the base CAMMS model runs where there is no effect due to TMP. There are no discrepancies in these comparisons. There are 4 other model runs which use times varying from 2.0 to 100.0 minutes in order to determine some idea of the sensitivity of the model with respect to the scale time value. This is related to allowing the user to input this value based on their own risk posture where a large value implies risk aversity. Some output from these verification runs are located at Appendix E. A full scale demonstration of the model was presented to a selected audience of military officers and professors on 19 July 1989. Impressions concerning the model results and and proposed enhancements were solicited with an enthusiastic response to the results.

2. Output Analysis

The model appears to work extremely well in terms of the route selection process using the expected value and in terms of the sensitivity analysis for varying risk postures. The first four pairs of 1 to 25,000 scale plots shown in Appendix E reflect movement based strictly on mobility versus tactical movement using the expected scale time value. The last two pairs reflect a change in risk posture where the expected time scale value is compared to greater times signifying a more risk averse posture.

The results are quite evident from these output mappings. The methodology and use of a psychometric decision function does provide a set of path points which appear to optimize TMP during movement in addition to mobility considerations. The model can also be applied deterministically, stochastically, or individually using different values for the scale time value. These output also appear to support sensitivity of the model to varying risk postures.

C. RECOMMENDATIONS

1. Enhancements and Areas for Research

a. Approved Scenario List

The scenarios discussed in the development of this methodology are a key element in further applications and research concerning the methodology. A complete set of scenarios of interest detailing all possible combinations of the fixed state variables must be determined in order to structure a data collection effort and target appropriate sample populations. There will eventually be a decision function derived for each of the identified combinations if the methodology is to be used. This effort would be best performed through the efforts of the U.S. Army Center for Army Lessons Learned (CALL) using information from throughout the military.

b. Expanded Survey and Regression Capability

The fidelity of this survey and corresponding regression were discussed previously. Continuing data collection and analysis efforts should be more detailed in terms of categorization and the number of levels or groups used for the independent variables. There is certainly a balance needed between the amount of fidelity desired in the model and the magnitude of the collection and analysis effort. The populations used for these efforts should also be expanded and examined closely to insure the appropriate sample is surveyed. Larger samples would certainly lower the error and increase the power of any such test. These actions should be a cooperative effort between the Army Research Institute (ARI) and TRADOC Analysis Command (TRAC).

c. *Multiple Vehicle Paths and Formation Structure*

The need to model units using a formation structure or by defining a path width for each arc on the battlefield which can be used to control movement will be a necessary requirement prior to interfacing this model with other combat models. This area of research alone is practically unbounded. A decision should be made by the Army Model Board concerning which approach to use. A formation structure could be applied similar to that found in DYNTACS [Ref. 3: pp. 150-155] or developing path parameters for the digitized terrain data and then limiting the optimization to only those arcs with a minimum acceptable width [Ref. 17: pp. 33-36] are both feasible. It is also possible to perform a combinatorial optimization using LaGrangian relaxation on three variables to include path width. This effort would be best accomplished by the TRADOC Analysis Command (TRAC).

d. *Relative Effective Range Determinations*

A similar type of pyschophysical analysis needs to be conducted in order to determine if combatants' perceived lethality sufficiently corresponds to actual lethality curves. The same method of categorical judgements could be used to derive exact regression equations or lethality curves for desired weapons systems in the model or actual classified lethality curves could be instituted if there is strong correlation between the perceived and actual results. The data and curves would certainly be classified in either case. These actions should also be a cooperative effort between the ARI and TRAC or the Army Material Systems Analysis Activity (AMSAA) and TRAC.

e. *Symmetric Applications*

There is also a need to determine if psychometric analysis and functions done with U.S. and allied sample populations can be extrapolated to and/or directly applied to Soviet or other forces and if so, at what levels and for which actions. This analysis is necessary unless some drastic assumptions are made concerning symmetry between opposing force decision processes and tactics in general. This effort will certainly be classified and require the assistance of all major intelligence agencies.

2. Methodology Utilization and Future Applications

a. *Tactical Decision Tool*

A major advantage of this methodology used in a model versus a Tactical Decision Aid (TDA) such as the CAMMS is the ability of this model to provide or make a tactical decision in real time. The CAMMS and other models only provide pertinent information to the combat decision maker to assist him in choosing a course of action. The model can make tactical decisions in a laboratory or analytical environment and could be used in the field environment given the appropriate terrain data bases and intelligence input. The system and model have been developed to be easily transportable.

b. Tactical Training Tool

The model would make an excellent Tactical Training Tool allowing personnel to compare their decisions in a laboratory or classroom environment to computer simulations based on the model. This application could be performed using a variety of mediums. This application could even be done as a field exercise given the flexibility of current personal computers.

c. Combat Model Preprocessor / Subroutine

The methodology could be used as a pre-processor to provide sets of optimum path points during the initialization of any number of current combat models and then be used interactively as an event driven subroutine to recompute path changes during the course of a battle. This is highly desirable in development of systemic models where we wish to remove the human from the process and decrease variability in the battle calculus. The major problems will be to determine which events would drive an update in the path evaluation and whether to use the paths at an individual vehicle or unit level depending on resolution.

The model has been designed and structured to make it compatible with several existing combat models in terms of the terrain data bases used, the path output, and program lanquages used. These include the Battalion Combat Outcome Model (BCOM) developed by the BDM Corporation, both versions of JANUS combat models, CASTFOREM, and others.

d. Research and Analysis Tool

The methodology has numerous applications and great potential as an analysis tool in current form or as part of a systemic research model. The methodology can be applied in evaluating any type of decision process when the appropriate state variables can be determined and a proper sample population identified. Similar techniques can be used to include these decision functions in models leading to true systemic models and expert systems.

This specific model, once scenarios are determined and analysis performed to fully expand the model, can be used to analyze alternative General Defense Plans concerning movement. It may be incorporated into models such as the Obstacle Planner System (OPS) to assist in analyzing optimum placement of available engineer resources in countermobility or mobility roles. These are only a couple of possible research and analysis applications.

e. The Generalized Value System, GVS

The methodology also provides a theoretically sound method of determining total traversal times for the time domain network of the battlefield which is a necessary input to the Generalized Value System. There is additional work to be done in applying the methodology to the GVS. This specifically relates to the decision level at which a future state decision is made in the GVS versus the level at which paths are computed in the model. The problem does not exist at lower levels such as section or platoon. However, it may not be feasible to extrapolate values of the time domain network at a higher level from a number of smaller units comprising a large unit. The use of the path width discussed previously may be one solution. However, there would need to be a method of determining traversal speeds and other data for aggregated units with combinations of vehicles which the CAMMS and this model can not accomplish.

D. SUMMARY

The methodology demonstrates a way to determine optimum, tactical movement paths for a specified vehicle and/or small unit based on the operator's cognitive decision processes, as well as the physical effects of terrain and environment on mobility. The approach uses psychometric techniques inherent to the Generalized Value System (GVS) in order to determine a decision function based on the specific tactical scenario and given equipment configuration which provides a means to determine the Tactical Movement Potential (TMP) for each terrain cell. This cognitive value in an interval scale can then be translated into the same scale as the physical continuum. The cognitive time value based on the user's decision process is then added to the physical traversal times for each cell computed from output provided by The Condensed Army Mobility Management System (CAMMS). This renders a value mapping which can be optimized using Dijkstra's Algorithm or a combinatorial method could be instituted. The resulting sets of path points are optimized for speed/time and the cognitive tactical considerations evaluated using these psychometric methods thereby rendering a truly optimum, tactical movement path and optimum traversal time. The movement path and resulting times can be used in a multitude of modelling or analytical applications and particularly in determining the time values needed to compute the Situationally Inherent Power (SIP) of the GVS. In actuality, this methodology could be applied to almost any tactical decision process in the development of expert systems and models and shows great potential for future use.

APPENDIX A. A SURVEY OF FACTORS INFLUENCING TACTICAL / COMBAT MOVEMENT.

A. INTRODUCTION.

1. Purpose.

The purpose of this survey is to obtain an estimate of the degradation to a system's Tactical Movement Potential (TMP) based on changes in 3 key variables and combinations of these variables. The results of analysis to be performed on these survey responses will then be used to develop input to a larger tactical decision tool designed to provide sets of optimum path points to commanders in the field. Your response to each question will reflect the value you place on a given combination of the 3 key variables with respect to Tactical Movement Potential (TMP). Your responses as a tactical decision maker / commander will assist in determining accurate representations of how changes in each key variable affect the rating of its relative importance to tactical movement. They may also enable us to determine how the key variables are correlated.

2. Structure.

a. Key Variables.

There are 3 key variables which have been identified as critical in determining a system's TMP. These 3 key variables are:

1. System Speed based on the cross-country movement rate.
2. Cover and Concealment based on enemy line-of-sight (LOS).
3. Range based on the distance to known or suspected enemy positions.

b. Rating Categories.

There are 5 rating categories from which to choose in evaluating TMP. These are a mutually exclusive set of successive intervals that collectively exhaust all possible values of the TMP. These 5 rating categories are:

1. No Tactical Movement Potential.
2. Marginal Tactical Movement Potential.
3. Effective Tactical Movement Potential.
4. Good Tactical Movement Potential.
5. Excellent Tactical Movement Potential.

c. Discussion.

A short discussion to insure complete understanding of the variables and the rating category you should select based on your tactical experience and judgement regarding each scenario is required. We will first examine the key variables and then define / discuss the rating categories.

The 3 key variables represent the primary factors considered by a vehicle or unit commander when selecting an Axis-of-Advance or Route-of-Movement in operations such as an Attack, Movement-to-Contact, Withdrawal, or other operation. In each given scenario; which is simply a combination of the 3 key variables; the system employed will have a varying TMP. For example, a system travelling over rolling, open terrain at a range exceeding 4 km. from the nearest opposing force with a formation of low hills preventing enemy observation based on LOS would be capable of moving at great speed and in relative safety. This would result in high values for each key variable and exhibit "Excellent Tactical Movement Potential". On the other hand, a system moving through a peat bog and becoming virtually mired within 800 m. of several known enemy positions with little or no cover and concealment would almost certainly be destroyed or at least badly damaged and become combat ineffective. This would result in low values for each key variable and suggest the system has "No Tactical Movement Potential". Varying combinations of the 3 key variables render scenarios which will be rated between these two extremes. This leads to the second portion of our discussion.

In order to insure consistent responses between the respondents when evaluating each scenario, all respondents must thoroughly understand the definition of each rating category. These categories are:

- "No Tactical Movement Potential" implies the system cannot move through an area, a No-Go area such as a swamp, or that the system has practically no probability of surviving such a scenario in the judgement of the respondent. This may be due to a slow speed, numerous enemy observations at a close range, etc.. This represents a tactical situation any prudent commander would never select if possible.

- "Marginal Tactical Movement Potential" implies the system can move at a marginal rate and has some probability of surviving the scenario which is low. However, this is a situation where although the system can survive; the simple act of survival would render the system combat ineffective for a period of time. This represents a tactical situation which is only acceptable as a last resort for brief periods of time.

- "Effective Tactical Movement Potential" implies the system can move reasonably well and / or has a moderate probability of survival and mission success all things considered. This represents a tactical situation most commanders would consider acceptable in most cases although not preferable.

- "Good Tactical Movement Potential" implies the system can move at better than average cross-country movement rates and or has a relatively high probability of survival and subsequent mission success. This is certainly a preferable tactical situation.
- "Excellent Tactical Movement Potential" implies the system can move at or near the maximum cross-country speed with relatively little or no probability of being engaged. This is the optimum tactical situation.

3. Instructions.

The survey consists of three sections. This 'Introduction' is the first section followed by a 'Personal History ' section used to collect population data. The actual tactical situation and survey compose the 'Tactical Movement Survey' section which is the last but most important section.

Please insure you feel comfortable with the variables and especially the rating categories. Fill in the required personal data as completely as possible and proceed to the survey.

You will begin by reading a synopsized OPORD giving the Task Organization, Situation, and Mission. Once you fully understand the tactical situation you will continue by answering or rating the TMP for 64 questions scenarios. Please rate ALL scenarios in accordance with your initial judgement in as timely a manner as possible. This is done in order to replicate the manner in which such decisions are made in fast-paced, tactical operations. Do not change answers once you have recorded your initial response unless the change is due to an inadvertent error. Answer ALL scenarios. Thank you for your cooperation.

B. PERSONAL HISTORY DATA.

1. Present Rank:

- LTC, O-5 or above ..... _____
- MAJ, O-4 .... _____
- CPT, O-3 .... _____
- LT, O-1 & O-2 .... _____
- E8 - E9 .... _____
- E6 - E7 .... _____
- E5 or below .... _____

2. Branch or MOS:

- Armor / Cavalry .... _____
- Infantry .... _____
- Aviation .... _____
- Air Defense .... _____
- Field Artillery .... _____
- Engineer .... _____
- Other .... _____

3. Experience:

- Time on active duty .... _____ Years _____ Months
- Time spent in Combat Arms units ..... _____ Years _____ Months
- Time attached to Combat Arms units ... _____ Years _____ Months

4. Opinion:

Do you feel any other variables should be included in the scenarios which could significantly affect the tactical decision logic?

- Yes _____ No _____

If so, what are these variables and how do they impact tactical movement?

C. TACTICAL MOVEMENT SURVEY.

1. OPORD 1-89, HQ/TF 1-15 XXX

Task Organization :

Team Yankee 1-15 XXX (-)

2 Mech. Plt. (M2) 2 Tank Plt. (M1) Team HQ's

1 Engineer Sqd. w/ CEV 1 Stinger 1 Fist

1. Situation.

a. Enemy Forces:

Unidentified forces of the Soviet 111TH MRR are defending in sector. The enemy is estimated to be at 72% strength in men, equipment, and supplies. They are preparing defensive positions. The enemy is equipped with organic BMP IFV's and BRDM ATV's as well as being reinforced by a T80 Guards Tank Battalion and what is left of the Regimental T72 Tank Battalion. Expect enemy artillery support from the RAG using 122 SP's. Other artillery support is possible. The enemy may receive Helo support in the form of HIP or HIND-D units if our operations are successful. The status of enemy fixed-wing support is not known at this time. The enemy has positions about 5 Kilometers away. Some enemy forces have been located and identified (assume you have an Intelligence Overlay). Other larger enemy forces are known to be preparing positions behind their security zone.

b. Friendly Forces:

Team Yankee has been conducting a Movement-to-Contact as part of TF 1-15 XXX and the 10TH Brigade. The Scouts and Team X-Ray are to our front and have encountered enemy forces in a security zone forward of their main defensive belt. Team X-Ray lost 3 vehicles today. Team Whiskey and Team Zulu are on are left and right respectively. The TF has a Battalion of 155 SP's in DS. Teams Whiskey and Zulu will attack abreast to secure intermediate Objectives Frick and Frack. Team X-Ray will provide supporting fires from their present positions.

2. Mission.

Team Yankee TF 1-15 XXX will attack NLT 1000 hours passing through Team X-Ray vicinity CP66 and between teams Whiskey and Zulu vicinity CP99 in order to sieze high ground vicinity Objective Sam. Team Yankee will then defend in place until relieved.

3. *

4. *

5. *

Based on the above given tactical situation; place yourself in the position of a vehicle, platoon, or unit commander and give your rating of the Tactical Movement Potential for each of the following scenarios.

2. **Example Survey Scenarios for Questions 34 through 41.**

- 34. You are moving cross-country as a part of Team Yankee attacking to seize Objective Sam under the following conditions:
  - Range to the nearest known or suspected enemy position is about 1475 meters.
  - Your cross-country speed is less than 5 KMPH or 3 MPH.
  - You are within the LOS of 1 known or suspected enemy location.

  What do you rate the Tactical Movement Potential of this scenario as -

  (_)No TMP (_)Marginal TMP (_)Effective TMP (_)Good TMP (_)Excellent TMP

- 35. You are moving cross-country as a part of Team Yankee attacking to seize Objective Sam under the following conditions:
  - Range to the nearest known or suspected enemy position is 3740 meters
  - Your cross-country speed is greater than 45 KMPH or 27 MPH.
  - You are within the LOS of 4 or more known or suspected enemy locations.

  What do you rate the Tactical Movement Potential of this scenario as -

  (_)No TMP (_)Marginal TMP (_)Effective TMP (_)Good TMP (_)Excellent TMP

- 36. You are moving cross-country as a part of Team Yankee attacking to seize Objective Sam under the following conditions:
  - Range to the nearest known or suspected enemy position is about 750 to 770 meters.
  - Your cross-country speed is approximately 20 KMPH or 12.5 MPH.
  - You are not within the LOS of any known or suspected enemy locations.

  What do you rate the Tactical Movement Potential of this scenario as -

  (_)No TMP (_)Marginal TMP (_)Effective TMP (_)Good TMP (_)Excellent TMP

- 37. You are moving cross-country as a part of Team Yankee attacking to seize Objective Sam under the following conditions:
  - Range to the nearest known or suspected enemy position is 2575 meters
  - Your cross-country speed is approximately 42 KMPH or 26 MPH.
  - You are not within the LOS of any known or suspected enemy locations.

  What do you rate the Tactical Movement Potential of this scenario as -

  (_)No TMP (_)Marginal TMP (_)Effective TMP (_)Good TMP (_)Excellent TMP

- 38. You are moving cross-country as a part of Team Yankee attacking to seize Objective Sam under the following conditions:
  - Range to the nearest known or suspected enemy position is 1440 to 1515 meters.
  - Your cross-country speed is less than 5 KMPH or 3 MPH.
  - You are within the LOS of 4 or more known or suspected enemy locations.

  What do you rate the Tactical Movement Potential of this scenario as -

  (_)No TMP (_)Marginal TMP (_)Effective TMP (_)Good TMP (_)Excellent TMP

- 39. You are moving cross-country as a part of Team Yankee attacking to seize Objective Sam under the following conditions:
  - Range to the nearest known or suspected enemy position just more than 3500 meters.
  - Your cross-country speed is less than 5 KMPH or 3 MPH.
  - You are within the LOS of 1 known or suspected enemy location.

  What do you rate the Tactical Movement Potential of this scenario as -

  (_)No TMP (_)Marginal TMP (_)Effective TMP (_)Good TMP (_)Excellent TMP

- 40. You are moving cross-country as a part of Team Yankee attacking to seize Objective Sam under the following conditions:
  - Range to the nearest known or suspected enemy position is slightly less than 650 meters.
  - Your cross-country speed is less than 5 KMPH or 3 MPH.
  - You are within the LOS of 4 or more known or suspected enemy locations.

  What do you rate the Tactical Movement Potential of this scenario as -

  (_)No TMP (_)Marginal TMP (_)Effective TMP (_)Good TMP (_)Excellent TMP

- 41. You are moving cross-country as a part of Team Yankee attacking to seize Objective Sam under the following conditions:
  - Range to the nearest known or suspected enemy position is between 2280 and 2320 meters.
  - Your cross-country speed is approximately 40 KMPH or 25 MPH.
  - You are within the LOS of 4 or more known or suspected enemy locations.

  What do you rate the Tactical Movement Potential of this scenario as -

  (_)No TMP (_)Marginal TMP (_)Effective TMP (_)Good TMP (_)Excellent TMP

APPENDIX B. TRANSFORMING THE CATEGORICAL RESPONSES TO AN INTERVAL SCALE

A. RAW FREQUENCIES

	STATE VARIABLES						RESPONSE FREQUENCIES				
QUES NUM	RANGE (METERS)		LOS (QTY)		SPEED (KMPH)		NO TMP	MARG TMP	EFF TMP	GOOD TMP	EXC TMP
01	3	2400	2	2.5	3	22.0	0	9	20	11	0
02	2	1600	4	0.0	1	5.0	0	12	19	9	0
03	4	3200	2	2.5	4	40.0	0	2	6	20	12
04	1	750	2	2.5	1	5.0	17	18	5	0	0
05	3	2500	3	1.0	1	5.0	3	13	20	4	0
06	2	1400	1	4.5	2	11.0	5	30	5	0	0
07	4	3400	3	1.0	3	24.0	0	0	4	22	14
08	1	640	2	2.5	3	24.0	3	22	12	3	0
09	3	2500	1	4.5	1	5.0	3	27	9	1	0
10	2	1500	3	1.0	3	25.0	0	5	20	14	1
11	4	3300	2	2.5	2	12.0	0	9	17	12	2
12	1	800	4	0.0	1	5.0	2	13	22	3	0
13	3	2350	3	1.0	4	40.0	0	1	8	19	12
14	2	1575	2	2.5	1	5.0	3	30	7	0	0
15	4	3450	3	1.0	4	35.0	0	0	5	19	16
16	1	700	1	4.5	2	11.0	16	18	6	0	0
17	3	2450	3	1.0	2	10.0	0	3	30	7	0
18	2	1450	2	2.5	4	38.0	0	11	23	5	1
19	4	3400	1	4.5	1	5.0	5	20	13	1	1
20	1	575	4	0.0	4	42.0	0	3	9	12	16
21	3	2600	4	0.0	3	25.0	0	0	6	19	15
22	2	1525	2	2.5	2	11.0	2	22	16	0	0
23	4	3600	4	0.0	1	5.0	2	6	16	10	6
24	1	550	2	2.5	4	42.0	2	14	16	7	1
25	3	2625	1	4.5	3	22.0	2	19	16	3	0
26	2	1480	4	0.0	4	39.0	0	1	8	16	15
27	4	3300	1	4.5	3	25.0	0	10	24	5	1
28	1	660	3	1.0	2	10.0	5	18	14	3	0
29	3	2600	4	0.0	1	5.0	0	13	19	8	0
30	2	1585	1	4.5	3	23.0	2	30	8	0	0
31	4	3550	4	0.0	3	26.0	0	0	3	20	17
32	1	400	1	4.5	3	23.0	14	17	7	2	0
33	3	2640	2	2.5	2	11.0	0	19	19	2	0
34	2	1475	3	1.0	1	5.0	2	20	18	0	0
35	4	3740	1	4.5	4	45.0	1	5	15	17	2
36	1	760	4	0.0	3	20.0	0	3	12	22	3
37	3	2575	4	0.0	4	42.0	0	0	0	10	30
38	2	1480	1	4.5	1	5.0	16	22	2	0	0
39	4	3550	3	1.0	1	5.0	1	16	13	9	1
40	1	600	1	4.5	1	5.0	25	15	0	0	0
41	3	2305	1	4.5	4	40.0	0	15	19	6	0
42	2	1620	4	0.0	2	12.0	0	2	23	15	0

43	4	3300	3	1.0	2	10.0	0	8	22	9	1
44	1	575	3	1.0	4	42.0	0	7	15	13	5
45	3	2375	2	2.5	1	5.0	5	27	8	0	0
46	2	1565	3	1.0	4	40.0	0	6	17	15	2
47	4	3500	1	4.5	2	12.0	3	14	20	2	1
48	1	500	4	0.0	2	10.0	0	8	15	14	3
49	3	2480	4	0.0	2	11.0	0	4	13	22	1
50	2	1500	1	4.5	4	40.0	0	25	13	2	0
51	4	3450	2	2.5	3	21.0	0	10	19	9	2
52	1	700	2	2.5	2	10.0	12	22	6	0	0
53	3	2600	3	1.0	3	22.0	0	5	22	13	0
54	2	1390	4	0.0	3	24.0	0	0	7	28	5
55	4	3475	2	2.5	1	5.0	5	19	13	2	1
56	1	500	1	4.5	4	40.0	10	15	10	5	0
57	3	2475	2	2.5	4	42.0	0	9	20	10	1
58	2	1550	3	1.0	2	11.0	0	12	25	3	0
59	4	3600	4	0.0	4	40.0	0	0	0	5	35
60	1	540	3	1.0	1	5.0	9	20	11	0	0
61	3	2500	1	4.5	2	10.0	9	27	4	0	0
62	2	1500	2	2.5	3	24.0	2	18	19	1	0
63	4	3650	4	0.0	2	11.0	0	2	13	16	9
64	1	625	3	1.0	3	22.0	3	11	17	8	1

B. GROUPED FREQUENCIES

STATE VARIABLES							RESPONSE FREQUENCIES				
QUES NUM	RANGE (METERS)		LOS (QTY)		SPEED (KMPH)		NO TMP	MARG TMP	EFF TMP	GOOD TMP	EXC TMP
59	4	3600	4	0.0	4	40.0	0	0	0	5	35
37	3	2575	4	0.0	4	42.0	0	0	0	10	30
31	4	3550	4	0.0	3	26.0	0	0	3	20	17
07	4	3400	3	1.0	3	24.0	0	0	4	22	14
15	4	3450	3	1.0	4	35.0	0	0	5	19	16
21	3	2600	4	0.0	3	25.0	0	0	6	19	15
54	2	1390	4	0.0	3	24.0	0	0	7	28	5
13	3	2350	3	1.0	4	40.0	0	0	9	19	12
26	2	1480	4	0.0	4	39.0	0	0	9	16	15
20	1	575	4	0.0	4	42.0	0	3	9	12	16
03	4	3200	2	2.5	4	40.0	0	2	6	20	12
36	1	760	4	0.0	3	20.0	0	3	12	22	3
63	4	3650	4	0.0	2	11.0	0	2	13	16	9
44	1	575	3	1.0	4	42.0	0	7	15	13	5
11	4	3300	2	2.5	2	12.0	0	9	17	12	2
46	2	1565	3	1.0	4	40.0	0	6	17	15	2
48	1	500	4	0.0	2	10.0	0	8	15	14	3
51	4	3450	2	2.5	3	21.0	0	10	19	9	2
35	4	3740	1	4.5	4	45.0	0	6	15	17	2
23	4	3600	4	0.0	1	5.0	0	8	16	10	6
10	2	1500	3	1.0	3	25.0	0	5	20	15	0
43	4	3300	3	1.0	2	10.0	0	8	22	10	0
18	2	1450	2	2.5	4	38.0	0	11	23	6	0
27	4	3300	1	4.5	3	25.0	0	10	24	6	0
49	3	2480	4	0.0	2	11.0	0	4	13	23	0
57	3	2475	2	2.5	4	42.0	0	9	20	11	0
39	4	3550	3	1.0	1	5.0	0	17	13	10	0
42	2	1620	4	0.0	2	12.0	0	2	23	15	0
53	3	2600	3	1.0	3	22.0	0	5	22	13	0
01	3	2400	2	2.5	3	22.0	0	9	20	11	0
29	3	2600	4	0.0	1	5.0	0	13	19	8	0
02	2	1600	4	0.0	1	5.0	0	12	19	9	0
17	3	2450	3	1.0	2	10.0	0	3	30	7	0
41	3	2305	1	4.5	4	40.0	0	15	19	6	0
50	2	1500	1	4.5	4	40.0	0	25	13	2	0
58	2	1550	3	1.0	2	11.0	0	12	25	3	0
33	3	2640	2	2.5	2	11.0	0	19	19	2	0
24	1	550	2	2.5	4	42.0	2	14	16	8	0
64	1	625	3	1.0	3	22.0	3	11	17	9	0
47	4	3500	1	4.5	2	12.0	3	14	20	3	0
55	4	3475	2	2.5	1	5.0	5	19	13	3	0
19	4	3400	1	4.5	1	5.0	5	20	13	2	0
05	3	2500	3	1.0	1	5.0	3	13	20	4	0
56	1	500	1	4.5	4	40.0	10	15	10	5	0
32	1	400	1	4.5	3	23.0	14	17	7	2	0
25	3	2625	1	4.5	3	22.0	2	19	16	3	0
08	1	640	2	2.5	3	24.0	3	22	12	3	0
12	1	800	4	0.0	1	5.0	2	13	22	3	0
28	1	660	3	1.0	2	10.0	5	18	14	3	0

62	2	1500	2	2.5	3	24.0	2	18	20	0	0
09	3	2500	1	4.5	1	5.0	3	27	10	0	0
34	2	1475	3	1.0	1	5.0	2	20	18	0	0
22	2	1525	2	2.5	2	11.0	2	22	16	0	0
14	2	1575	2	2.5	1	5.0	3	30	7	0	0
60	1	540	3	1.0	1	5.0	9	20	11	0	0
52	1	700	2	2.5	2	10.0	12	22	6	0	0
45	3	2375	2	2.5	1	5.0	5	27	8	0	0
30	2	1585	1	4.5	3	23.0	2	30	8	0	0
16	1	700	1	4.5	2	11.0	16	18	6	0	0
06	2	1400	1	4.5	2	11.0	5	30	5	0	0
04	1	750	2	2.5	1	5.0	17	18	5	0	0
61	3	2500	1	4.5	2	10.0	9	27	4	0	0
38	2	1480	1	4.5	1	5.0	16	22	2	0	0
40	1	600	1	4.5	1	5.0	25	15	0	0	0

C. RELATIVE FREQUENCIES

RESPONSE FREQUENCIES				
EXC TMP	GOOD TMP	EFF TMP	MARG TMP	NO TMP
0.875	0.125	0	0	0
0.75	0.25	0	0	0
0.425	0.5	0.075	0	0
0.35	0.55	0.1	0	0
0.4	0.475	0.125	0	0
0.375	0.475	0.15	0	0
0.125	0.7	0.175	0	0
0.3	0.475	0.225	0	0
0.375	0.4	0.225	0	0
0.4	0.3	0.225	0.075	0
0.3	0.5	0.15	0.05	0
0.075	0.55	0.3	0.075	0
0.225	0.4	0.325	0.05	0
0.125	0.325	0.375	0.175	0
0.05	0.3	0.425	0.225	0
0.05	0.375	0.425	0.15	0
0.075	0.35	0.375	0.2	0
0.05	0.225	0.475	0.25	0
0.05	0.425	0.375	0.15	0
0.15	0.25	0.4	0.2	0
0	0.375	0.5	0.125	0
0	0.25	0.55	0.2	0
0	0.15	0.575	0.275	0
0	0.15	0.6	0.25	0
0	0.575	0.325	0.1	0
0	0.275	0.5	0.225	0
0	0.25	0.325	0.425	0
0	0.375	0.575	0.05	0
0	0.325	0.55	0.125	0
0	0.275	0.5	0.225	0
0	0.2	0.475	0.325	0
0	0.225	0.475	0.3	0
0	0.175	0.75	0.075	0
0	0.15	0.475	0.375	0
0	0.05	0.325	0.625	0
0	0.075	0.625	0.3	0
0	0.05	0.475	0.475	0
0	0.2	0.4	0.35	0.05
0	0.225	0.425	0.275	0.075
0	0.075	0.5	0.35	0.075
0	0.075	0.325	0.475	0.125
0	0.05	0.325	0.5	0.125
0	0.1	0.5	0.325	0.075
0	0.125	0.25	0.375	0.25
0	0.05	0.175	0.425	0.35
0	0.075	0.4	0.475	0.05
0	0.075	0.3	0.55	0.075
0	0.075	0.55	0.325	0.05
0	0.075	0.35	0.45	0.125

0	0	0.5	0.45	0.05
0	0	0.25	0.675	0.075
0	0	0.45	0.5	0.05
0	0	0.4	0.55	0.05
0	0	0.175	0.75	0.075
0	0	0.275	0.5	0.225
0	0	0.15	0.55	0.3
0	0	0.2	0.675	0.125
0	0	0.2	0.75	0.05
0	0	0.15	0.45	0.4
0	0	0.125	0.75	0.125
0	0	0.125	0.45	0.425
0	0	0.1	0.675	0.225
0	0	0.05	0.55	0.4
0	0	0	0.375	0.625

D. CUMULATIVE RELATIVE FREQUENCIES

RESPONSE FREQUENCIES				
EXC TMP	*GOOD TMP*	*EFF TMP*	*MARG TMP*	*NO TMP*
0.875	1	1	1	1
0.75	1	1	1	1
0.425	0.925	1	1	1
0.35	0.9	1	1	1
0.4	0.875	1	1	1
0.375	0.85	1	1	1
0.125	0.825	1	1	1
0.3	0.775	1	1	1
0.375	0.775	1	1	1
0.4	0.7	0.925	1	1
0.3	0.8	0.95	1	1
0.075	0.625	0.925	1	1
0.225	0.625	0.95	1	1
0.125	0.45	0.825	1	1
0.05	0.35	0.775	1	1
0.05	0.425	0.85	1	1
0.075	0.425	0.8	1	1
0.05	0.275	0.75	1	1
0.05	0.475	0.85	1	1
0.15	0.4	0.8	1	1
0	0.375	0.875	1	1
0	0.25	0.8	1	1
0	0.15	0.725	1	1
0	0.15	0.75	1	1
0	0.575	0.9	1	1
0	0.275	0.775	1	1
0	0.25	0.575	1	1
0	0.375	0.95	1	1
0	0.325	0.875	1	1
0	0.275	0.775	1	1
0	0.2	0.675	1	1
0	0.225	0.7	1	1
0	0.175	0.925	1	1
0	0.15	0.625	1	1
0	0.05	0.375	1	1
0	0.075	0.7	1	1
0	0.05	0.525	1	1
0	0.2	0.6	0.95	1
0	0.225	0.65	0.925	1
0	0.075	0.575	0.925	1
0	0.075	0.4	0.875	1
0	0.05	0.375	0.875	1
0	0.1	0.6	0.925	1
0	0.125	0.375	0.75	1
0	0.05	0.225	0.65	1
0	0.075	0.475	0.95	1
0	0.075	0.375	0.925	1
0	0.075	0.625	0.95	1
0	0.075	0.425	0.875	1

0	0	0.5	0.95	1
0	0	0.25	0.925	1
0	0	0.45	0.95	1
0	0	0.4	0.95	1
0	0	0.175	0.925	1
0	0	0.275	0.775	1
0	0	0.15	0.7	1
0	0	0.2	0.875	1
0	0	0.2	0.95	1
0	0	0.15	0.6	1
0	0	0.125	0.875	1
0	0	0.125	0.575	1
0	0	0.1	0.775	1
0	0	0.05	0.6	1
0	0	0	0.375	1

E. EXTREME INSTANCES

EXCELLENT CASES

0.875	1	1	1	1
0.75	1	1	1	1

POOR CASES

0	0	0	0.375	1

F. GROUPED CUMULATIVE RELATIVE FREQUENCY MATRICES

GROUPI

0.425	0.925	1	1	1
0.35	0.9	1	1	1
0.4	0.875	1	1	1
0.375	0.85	1	1	1
0.125	0.825	1	1	1
0.3	0.775	1	1	1
0.375	0.775	1	1	1

TGROUPI

0.425	0.925
0.35	0.9
0.4	0.875
0.375	0.85
0.125	0.825
0.3	0.775
0.375	0.775

GROUPII

0.4	0.7	0.925	1	1
0.3	0.8	0.95	1	1
0.075	0.625	0.925	1	1
0.225	0.625	0.95	1	1
0.125	0.45	0.825	1	1
0.05	0.35	0.775	1	1
0.05	0.425	0.85	1	1
0.075	0.425	0.8	1	1
0.05	0.275	0.75	1	1
0.05	0.475	0.85	1	1
0.15	0.4	0.8	1	1

TGROUPII

0.4	0.7	0.925
0.3	0.8	0.95
0.075	0.625	0.925
0.225	0.625	0.95
0.125	0.45	0.825
0.05	0.35	0.775
0.05	0.425	0.85
0.075	0.425	0.8
0.05	0.275	0.75
0.05	0.475	0.85
0.15	0.4	0.8

GROUPIII

0	0.375	0.875	1	1
0	0.25	0.8	1	1
0	0.15	0.725	1	1
0	0.15	0.75	1	1
0	0.575	0.9	1	1
0	0.275	0.775	1	1
0	0.25	0.575	1	1
0	0.375	0.95	1	1
0	0.325	0.875	1	1
0	0.275	0.775	1	1
0	0.2	0.675	1	1
0	0.225	0.7	1	1
0	0.175	0.925	1	1
0	0.15	0.625	1	1
0	0.05	0.375	1	1
0	0.075	0.7	1	1
0	0.05	0.525	1	1

TGROUPIII

0.375	0.875
0.25	0.8
0.15	0.725
0.15	0.75
0.575	0.9
0.275	0.775
0.25	0.575
0.375	0.95
0.325	0.875
0.275	0.775
0.2	0.675
0.225	0.7
0.175	0.925
0.15	0.625
0.05	0.375
0.075	0.7
0.05	0.525

GROUPIV

0	0.2	0.6	0.95	1
0	0.225	0.65	0.925	1
0	0.075	0.575	0.925	1
0	0.075	0.4	0.875	1
0	0.05	0.375	0.875	1
0	0.1	0.6	0.925	1
0	0.125	0.375	0.75	1
0	0.05	0.225	0.65	1
0	0.075	0.475	0.95	1
0	0.075	0.375	0.925	1
0	0.075	0.625	0.95	1
0	0.075	0.425	0.875	1

TGROUPIV

0.2	0.6	0.95
0.225	0.65	0.925
0.075	0.575	0.925
0.075	0.4	0.875
0.05	0.375	0.875
0.1	0.6	0.925
0.125	0.375	0.75
0.05	0.225	0.65
0.075	0.475	0.95
0.075	0.375	0.925
0.075	0.625	0.95
0.075	0.425	0.875

GROUPV

0	0	0.5	0.95	1
0	0	0.25	0.925	1
0	0	0.45	0.95	1
0	0	0.4	0.95	1
0	0	0.175	0.925	1
0	0	0.275	0.775	1
0	0	0.15	0.7	1
0	0	0.2	0.875	1
0	0	0.2	0.95	1
0	0	0.15	0.6	1
0	0	0.125	0.875	1
0	0	0.125	0.575	1
0	0	0.1	0.775	1
0	0	0.05	0.6	1

TGROUPV

0.5	0.95
0.25	0.925
0.45	0.95
0.4	0.95
0.175	0.925
0.275	0.775
0.15	0.7
0.2	0.875
0.2	0.95
0.15	0.6
0.125	0.875
0.125	0.575
0.1	0.775
0.05	0.6

G. COMPUTER PROGRAMS USED

THE APL COMPUTER PROGRAM NORM

```
    ∇NORM[⎕]∇
    ∇ NORM
   ⍝∇NORM;MTX;NOR;ROWAV;GRAV;S;AA;B;AAI;II
   ⍝∇NORM;MTX;NOR;ROWAV;GRAV;S;AA;B;AAI;II
    ⎕←'INPUT THE CUMREL FREQUENCY MATRIX'
    MTX←⎕
    NOR←NQUAN MTX
    S←⍴MTX
    ROWAV←(+/NOR)÷(S[2])
    ROWAV←(S[1],1)⍴ROWAV
    COLAV←(+⌿NOR)÷(S[1])
    GRAV←(+/(+/NOR))÷((S[1])×(S[2]))
    ⎕←'NORMALIZED VALUES              ROW AVERAGE'
    ⎕←'----------------------------------------------'
    ⎕←NOR,ROWAV
    ⎕←'  '
    ⎕←'   COLUMN AVERAGES '
    ⎕←'----------------------'
    ⎕←COLAV
    ⎕←'  '
    ⎕←'GRAND AVERAGE'
    ⎕←'-------------'
    ⎕←GRAV
    ⎕←'  '
    AAI←S⍴0
    II←0
   L2:II←II+1
    AA←(NOR[;(II)]-,ROWAV)*2
    AAI[;II]←AA
    →(II<S[2])/L2
    AAI←(S)⍴AAI
    AAI←+/AAI
    B←+/((COLAV-GRAV)*2)
    SQR←(B÷AAI)*0.5
    SQR←((S[1]),1)⍴SQR
    SSI←GRAV-(ROWAV×SQR)
    AAI←(S[1],1)⍴AAI
    GRAV←(S[1],1)⍴GRAV
    ⎕←'   B    '
    ⎕←'-------'
    ⎕←B
    ⎕←'  '
    ⎕←'    AI     '
    ⎕←'----------'
    ⎕←AAI
    ⎕←'  '
    ⎕←'SCALE VALUES = GRAND AVERAGE - (ROW AVERAGE × (B÷AI) * .
    ⎕←'-------------------------------------------------------------
    ⎕←SSI,GRAV,ROWAV,SQR
    ⎕←'  '
    ⎕←' COLUMN AVERAGES '
    ⎕←'------------------'
    ⎕←COLAV
    ∇
```

```
      ∇NQUAN[⎕]∇
      ∇ Z←NQUAN P;A;B;C;D
[1]     →((+/(A←(P≤0)∨(P≥1)))>0)/L1
[2]     C← 2.515517 0.802853 0.010328
[3]     D← 1.432788 0.189269 0.001308
[4]     P←((A←(P≤0.5))×P)+((P>0.5)×(1-P))
[5]     B←(⍟P*¯2)*0.5
[6]     Z←((2×A)-1)×-B-((B∘.* 0 1 2)+.×C)÷(1+((B∘.* 1 2 3)+.×D))
[7]     →0
[8]   L1:⎕←'THERE IS NO QUANTILE FOR P = ',⍕A/P
      ∇
```

THE APL COMPUTER PROGRAM TRANS

```
      ∇TRANS[⎕]∇
      ∇ TRANS
[1]   ⍝∇TRANS;UP;LOW;BETA;ALPHA;MX;COLUP;COLLOW;TRCOL
[2]     ⎕←'INPUT COL AVERAGES'
[3]     COL←⎕
[4]     S←⍴COL
[5]     COLUP←COL[S]
[6]     COLLOW←COL[1]
[7]     ⎕←'INPUT THE VECTOR OF BOUNDRY VALUES TO BE TRANSFORMED'
[8]     MX←⎕
[9]     ⎕←'INPUT UPPER BOUNDRY VALUE'
[10]    UP←⎕
[11]    ⎕←'INPUT LOWER BOUNDRY VALUE'
[12]    LOW←⎕
[13]    BETA←(UP-LOW)÷(COLUP-COLLOW)
[14]    ALPHA←UP-BETA×COLUP
[15]    TR←ALPHA+BETA×MX
[16]    TRCOL←ALPHA+BETA×COL
[17]    ⎕←'TRANSFORMED UPPER BOUNDS ARE'
[18]    ⎕←TRCOL
[19]    ⎕←'  '
[20]    ⎕←' TRANSFORMED DATA '
[21]    ⎕←TR
      ∇
```

H. THE NORMALIZED AND TRANSFORMED VALUES OF THE INSTANCES

TGROUPII

NORMALIZED VALUES			ROW AVERAGE
¯0.2529332678	0.5240018704	1.43980047	0.5702896907
¯0.5240018704	0.8414567174	1.64521144	0.6542220957
¯1.43980047	0.3181998625	1.43980047	0.1060666208
¯0.7551784916	0.3181998625	1.64521144	0.4027442703
¯1.150435626	¯0.1253809931	0.9345033954	¯0.1137710747
¯1.64521144	¯0.384877085	0.7551784916	¯0.4249700112
¯1.64521144	¯0.1887560404	1.036431485	¯0.2658453318
¯1.43980047	¯0.1887560404	0.8414567174	¯0.2623665976
¯1.64521144	¯0.5974048985	0.67418914	¯0.5228090662
¯1.64521144	¯0.06254483635	1.036431485	¯0.2237749304
¯1.036431485	¯0.2529332678	0.8414567174	¯0.1493026786

COLUMN AVERAGES

¯1.198129767 0.01829137737 1.117242841

GRAND AVERAGE

¯0.02086518298

B

2.682775012

AI
1.435887596
2.405328399
4.213551553
2.891657548
2.173687547
2.883347076
3.604518528
2.610194948
2.698156333
3.634597104
1.779340999

*SCALE VALUES = GRAND AVERAGE - (ROW AVERAGE × (B÷AI) * .5)*

¯0.8003854753	¯0.02086518298	0.5702896907	1.366884769
¯0.7117889701	¯0.02086518298	0.6542220957	1.056099743
¯0.1054995408	¯0.02086518298	0.1060666208	0.7979358365
¯0.4087904496	¯0.02086518298	0.4027442703	0.9632049299
0.1055285172	¯0.02086518298	¯0.1137710747	1.110947581
0.3890574987	¯0.02086518298	¯0.4249700112	0.9645920205
0.2084841776	¯0.02086518298	¯0.2658453318	0.8627172766
0.2451241358	¯0.02086518298	¯0.2623665976	1.01380786
0.5004515703	¯0.02086518298	¯0.5228090662	0.9971455871
0.1713888323	¯0.02086518298	¯0.2237749304	0.8591400964
0.1624633793	¯0.02086518298	¯0.1493026786	1.227898683

```
 COLUMN AVERAGES
-----------------
¯1.198129767 0.01829137737 1.117242841
      TRANS
INPUT UPPER BOUNDRY VALUE
⎕:
      100.0
INPUT LOWER BOUNDRY VALUE
⎕:
      0.0
TRANSFORMED UPPER BOUNDS ARE
7.105427358E¯15 53.53673384 100

 TRANSFORMED DATA
 17.17841399
 21.00486097
 47.1902545
 34.09124367
 56.30447038
 68.54997163
 60.75108343
 62.3335483
 73.36103621
 59.14895057
 58.76346389



      TGROUPI
NORMALIZED VALUES            ROW AVERAGE
---------------------------------------
 ¯0.1887560404  1.43980047     0.6255222146
 ¯0.384877085   1.281728757    0.4484258358
 ¯0.2529332678  1.150435626    0.4487511792
 ¯0.3181998625  1.036431485    0.3591158114
 ¯1.150435626   0.9345033954  ¯0.1079661155
 ¯0.5240018704  0.7551784916   0.1155883106
 ¯0.3181998625  0.7551784916   0.2184893146

   COLUMN AVERAGES
---------------------
¯0.4482005164 1.050465245

GRAND AVERAGE
-------------
0.3011323644

   B
-------
1.122999532

    AI
----------
 1.326098153
 1.388787515
```

```
 0.9847221265
 0.917513044
 2.173485362
 0.8181511992
 0.5760705455

SCALE VALUES = GRAND AVERAGE - (ROW AVERAGE × (B÷AI) * .5)
-----------------------------------------------------------------
 ¯0.2744993195    0.3011323644    0.6255222146    0.9202417922
 ¯0.1021066199    0.3011323644    0.4484258358    0.8992322745
 ¯0.1780916163    0.3011323644    0.4487511792    1.067905786
 ¯0.09616729347   0.3011323644    0.3591158114    1.106327389
  0.3787390085    0.3011323644   ¯0.1079661155    0.7188055603
  0.1657110832    0.3011323644    0.1155883106    1.17158284
 ¯0.003925450303  0.3011323644    0.2184893146    1.396213885

 COLUMN AVERAGES
-----------------
¯0.4482005164 1.050465245
TRANS
INPUT UPPER BOUNDRY VALUE
⎕:
     53.536734
INPUT LOWER BOUNDRY VALUE
⎕:
     0.0
TRANSFORMED UPPER BOUNDS ARE
3.552713679E¯15 53.536734

 TRANSFORMED DATA
  6.20511592
 12.36348848
  9.649081675
 12.57565863
 29.54070381
 21.93072188
 15.87080765



      TGROUPIII
NORMALIZED VALUES           ROW AVERAGE
-----------------------------------------
 ¯0.3181998625   1.150435626    0.4161178819
 ¯0.67418914     0.8414567174   0.08363378866
 ¯1.036431485    0.5974048985  ¯0.2195132933
 ¯1.036431485    0.67418914    ¯0.1811211726
  0.1887560404   1.281728757    0.7352423985
 ¯0.5974048985   0.7551784916   0.07888679654
 ¯0.67418914     0.1887560404  ¯0.2427165498
 ¯0.3181998625   1.64521144     0.6635057888
 ¯0.4533333679   1.150435626    0.3485511292
 ¯0.5974048985   0.7551784916   0.07888679654
 ¯0.8414567174   0.4533333679  ¯0.1940616747
 ¯0.7551784916   0.5240018704  ¯0.1155883106
 ¯0.9345033954   1.43980047     0.2526485371
```

¯1.036431485	0.3181998625	¯0.3591158114
¯1.64521144	¯0.3181998625	¯0.9817056513
¯1.43980047	0.5240018704	¯0.4578992996
¯1.64521144	0.06254483635	¯0.7913333019

COLUMN AVERAGES

¯0.8126365611 0.7084504496

GRAND AVERAGE

¯0.05209305576

B

1.156852847

AI

1.078445099
1.148591183
1.334710664
1.463111462
0.5972946791
0.9147409136
0.3723371922
1.927491972
1.286037493
0.9147409136
0.8382406825
0.8181511992
2.818659422
0.917513044
0.8804798637
1.928259815
1.45821575

*SCALE VALUES = GRAND AVERAGE - (ROW AVERAGE × (B÷AI) * .5)*

¯0.4830723627	¯0.05209305576	0.4161178819	1.035714459
¯0.1360270889	¯0.05209305576	0.08363378866	1.00358999
0.1522718504	¯0.05209305576	¯0.2195132933	0.9309910257
0.1089602245	¯0.05209305576	¯0.1811211726	0.8892018419
¯1.075327804	¯0.05209305576	0.7352423985	1.391697147
¯0.1408074984	¯0.05209305576	0.07888679654	1.124579099
0.3757359646	¯0.05209305576	¯0.2427165498	1.762669339
¯0.5661218971	¯0.05209305576	0.6635057888	0.774716438
¯0.3826747026	¯0.05209305576	0.3485511292	0.9484452039
¯0.1408074984	¯0.05209305576	0.07888679654	1.124579099
0.1758857464	¯0.05209305576	¯0.1940616747	1.174774991
0.08535423712	¯0.05209305576	¯0.1155883106	1.189110665
¯0.2139511961	¯0.05209305576	0.2526485371	0.640645468
0.3511505256	¯0.05209305576	¯0.3591158114	1.122878939
1.07318704	¯0.05209305576	¯0.9817056513	1.146249992
0.3025784214	¯0.05209305576	¯0.4578992996	0.7745621745
0.6527421852	¯0.05209305576	¯0.7913333019	0.8906932632

```
 COLUMN AVERAGES
-----------------
¯0.8126365611 0.7084504496
      TRANS
INPUT UPPER BOUNDRY VALUE
⎕:
      100.0
INPUT LOWER BOUNDRY VALUE
⎕:
      53.536734
TRANSFORMED UPPER BOUNDS ARE
53.536734 100

 TRANSFORMED DATA
  63.60363281
  74.20451018
  83.01091652
  81.68791555
  45.51254274
  74.05848734
  89.83687224
  61.06679397
  66.67038912
  74.05848734
  83.7322288
  80.96684485
  71.82423312
  89.08588313
 111.141278
  87.60219509
  98.29833015



      BET←(100-53.536734)÷(1.329835441-.0680260583 4)
      BET
36.82272983
      ALP←100+(BET×.0680260583 4)
      ALP
102.5049052
      UPE←ALP+(BET×1.267100775)
      UPE
149.1630147



      TGROUPIV
NORMALIZED VALUES           ROW AVERAGE
---------------------------------------
 ¯0.8414567174  0.2529332678  1.64521144    0.3522293302
 ¯0.7551784916  0.384877085   1.43980047    0.3564996877
 ¯1.43980047   ¯0.1887560404  1.43980047   ¯0.06291868014
 ¯1.43980047   ¯0.2529332678  1.150435626  ¯0.1807660371
```

¯1.64521144	¯0.3181998625	1.150435626	¯0.2709918921
¯1.281728757	_0.2529332678	1.43980047	_0.1370016603
¯1.150435626	¯0.3181998625	0.67418914	¯0.2648154496
¯1.64521144	¯0.7551784916	0.384877085	¯0.6718376156
¯1.43980047	¯0.06254483635	1.64521144	_0.04762204472
¯1.43980047	¯0.3181998625	1.43980047	¯0.1060666208
¯1.43980047	_0.3181998625	1.64521144	_0.1745369443
¯1.43980047	¯0.1887560404	1.150435626	¯0.1593736279

COLUMN AVERAGES

¯1.329835441 ¯0.06802605834 1.267100775

GRAND AVERAGE

¯0.04358690797

B

3.372934763

AI

3.106548825
2.410174235
4.169803347
3.36247368
3.911164149
3.723520871
1.668902612
2.071048262
4.776854354
4.213551553
4.789607793
3.355956505

SCALE VALUES = GRAND AVERAGE - (ROW AVERAGE × (B÷AI) ⋆ .5)

¯0.4106074699	¯0.04358690797	0.3522293302	1.041993186
¯0.4653211892	¯0.04358690797	0.3564996877	1.182986398
¯0.100175096	¯0.04358690797	_0.06291868014	0.8993861263
0.1374601038	¯0.04358690797	¯0.1807660371	1.001554356
_0.2080690671	¯0.04358690797	¯0.2709918921	0.9286476179
¯0.1739794959	¯0.04358690797	_0.1370016603	0.9517591802
0.3328843213	¯0.04358690797	¯0.2648154496	1.4216362
_0.8137927532	¯0.04358690797	¯0.6718376156	1.276170969
¯0.08360358382	¯0.04358690797	_0.04762204472	0.8402973052
_0.05131141717	¯0.04358690797	¯0.1060666208	0.8947048978
¯0.1900544398	¯0.04358690797	_0.1745369443	0.8391778165
0.1161893582	¯0.04358690797	¯0.1593736279	1.002526379

COLUMN AVERAGES

¯1.329835441 ¯0.06802605834 1.267100775
TRANS

```
INPUT UPPER BOUNDRY VALUE
⎕:
      UPE
INPUT LOWER BOUNDRY VALUE
⎕:
      53.536734
TRANSFORMED UPPER BOUNDS ARE
53.536734 99.99999999 149.1630147

 TRANSFORMED DATA
  87.38521723
  85.37050872
  98.81618467
 107.5665614
 110.1665762
  96.09850519
 114.7626146
 132.4709759
  99.42639298
 104.3943316
  95.50658187
 106.7833145



      TGROUPV
NORMALIZED VALUES           ROW AVERAGE
---------------------------------------
  1.010066757E¯7   1.645211440E0     8.226057706E¯1
 ¯6.741891400E¯1   1.439800470E0     3.828056648E¯1
 ¯1.253809931E¯1   1.645211440E0     7.599152235E¯1
 ¯2.529332678E¯1   1.645211440E0     6.961390862E¯1
 ¯9.345033954E¯1   1.439800470E0     2.526485371E¯1
 ¯5.974048985E¯1   7.551784916E¯1    7.888679654E¯2
 ¯1.036431485E0    5.240018704E¯1   ¯2.562148074E¯1
 ¯8.414567174E¯1   1.150435626E0     1.544894545E¯1
 ¯8.414567174E¯1   1.645211440E0     4.018773614E¯1
 ¯1.036431485E0    2.529332678E¯1   ¯3.917491087E¯1
 ¯1.150435626E0    1.150435626E0     0.000000000E0
 ¯1.150435626E0    1.887560404E¯1   ¯4.808397929E¯1
 ¯1.281728757E0    7.551784916E¯1   ¯2.632751325E¯1
 ¯1.645211440E0    2.529332678E¯1   ¯6.961390862E¯1

   COLUMN AVERAGES
---------------------
¯0.8262856749 1.035021384

GRAND AVERAGE
-------------
0.1043678548

   B
-------
1.732231984
```

```
    AI
----------
 1.353360175
 2.234476035
 1.567498782
 1.801476666
 2.818659422
 0.9147409136
 1.217476129
 1.983817554
 3.091759263
 0.8312307332
 2.64700426
 0.8967171601
 2.074495569
 1.801476666

SCALE VALUES = GRAND AVERAGE - (ROW AVERAGE × (B÷AI) * .5)
-------------------------------------------------------------
 ¯0.8262857891    0.1043678548    0.8226057706    1.131348305
 ¯0.2326815247    0.1043678548    0.3828056648    0.880471243
 ¯0.6944809054    0.1043678548    0.7599152235    1.051234053
 ¯0.5782611302    0.1043678548    0.6961390862    0.9805928133
 ¯0.09369293795   0.1043678548    0.2526485371    0.7839380152
 ¯0.004189260708  0.1043678548    0.07888679654   1.37611261
  0.4099845144    0.1043678548   ¯0.2562148074    1.192814197
 ¯0.0399935175    0.1043678548    0.1544894545    0.9344415953
 ¯0.1964431591    0.1043678548    0.4018773614    0.7485144544
  0.6698908224    0.1043678548   ¯0.3917491087    1.443584567
  0.1043678548    0.1043678548    0               0.8089575013
  0.7726743588    0.1043678548   ¯0.4808397929    1.389873538
  0.3449462429    0.1043678548   ¯0.2632751325    0.9137907782
  0.7869968397    0.1043678548   ¯0.6961390862    0.9805928133

 COLUMN AVERAGES
----------------
¯0.8262856749 1.035021384
      TRANS
INPUT UPPER BOUNDRY VALUE
⎕:
      UPE
INPUT LOWER BOUNDRY VALUE
⎕:
      100.0
TRANSFORMED UPPER BOUNDS ARE
100 149.1630147

 TRANSFORMED DATA
  99.99999698
 115.6789657
 103.4813814
 106.5511138
 119.3500945
 121.7141701
 132.6538113
 120.7684663
```

116.6361357
139.5187601
124.5815073
142.2335988
130.9359445
142.6119009

I. THE REGRESSION MATRIX

STATE VARIABLES				SCALE VALUE
QUES NUM	RANGE (METERS)	LOS X 1000(QTY)	SPEED X 100(KMPH)	TMP RATING
31	3550	0	2600	6.20511592
7	3400	1000	2400	12.36348848
15	3450	1000	3500	9.64908167
21	2600	0	2500	12.57565863
54	1390	0	2400	29.54070381
13	2350	1000	4000	21.93072188
26	1480	0	3900	15.87080765
20	575	0	4200	17.17841399
3	3200	2500	4000	21.00486097
36	760	0	2000	47.1902545
63	3650	0	1100	34.09124367
44	575	1000	4200	56.30447038
11	3300	2500	1200	68.54997163
46	1565	1000	4000	60.75108343
48	500	0	1000	62.3335483
51	3450	2500	2100	73.36103621
35	3740	4500	4500	59.14895057
23	3600	0	500	58.76346389
10	1500	1000	2500	63.60363281
43	3300	1000	1000	74.20451018
18	1450	2500	3800	83.01091652
27	3300	4500	2500	81.68791555
49	2480	0	1100	45.51254274
57	2475	2500	4200	74.05848734
39	3550	1000	500	89.83687224
42	1620	0	1200	61.06679397
53	2600	1000	2200	66.67038912
1	2400	2500	2200	74.05848734
29	2600	0	500	83.7322288
2	1600	0	500	80.96684485
17	2450	1000	1000	71.82423312
41	2305	4500	4000	89.08588313
50	1500	4500	4000	111.141278
58	1550	1000	1100	87.60219509
33	2640	2500	1100	98.29833015
24	550	2500	4200	87.38521723
64	625	1000	2200	85.37050872
47	3500	4500	1200	98.81618467
55	3475	2500	500	107.5665614
19	3400	4500	500	110.1665762
5	2500	1000	500	96.09850519
56	500	4500	4000	114.7626146
32	400	4500	2300	132.4709759
25	2625	4500	2200	99.42639298
8	640	2500	2400	104.3943316
12	800	0	500	95.50658187
28	660	1000	1000	106.7833145
62	1500	2500	2400	99.99999698

9	2500	4500	500	115.6789657
34	1475	1000	500	103.4813814
22	1525	2500	1100	106.5511138
14	1575	2500	500	119.3500945
60	540	1000	500	121.7141701
52	700	2500	1000	132.6538113
45	2375	2500	500	120.7684663
30	1585	4500	2300	116.6361357
16	700	4500	1100	139.5187601
6	1400	4500	1100	124.5815073
4	750	2500	500	142.2335988
61	2500	4500	1000	130.9359445
38	1480	4500	500	142.6119009

APPENDIX C. REGRESSION ANALYSIS OF COGNITIVE FACTORS AFFECTING MOVEMENT

A. SURFACE PLOTS

Surface plots of all possible combinations of independent variables were done in order to determine if there were any particular relationships between variables with respect to TMP which needed to be accounted for in the regression analysis. These are used in a fashion similar to scatter plots in simple regression. Note the linear or planar relationship between most combinations shown.

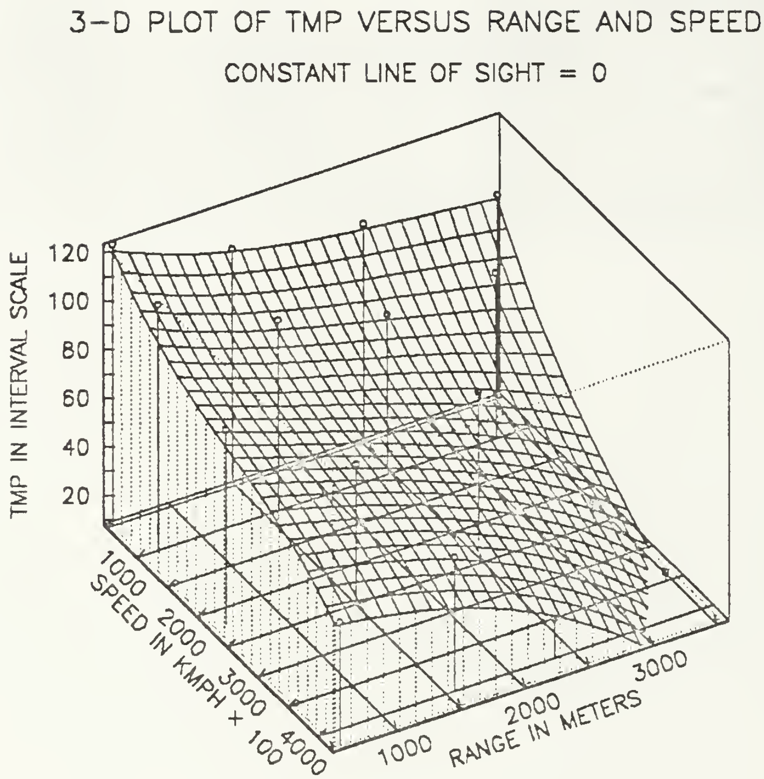


Figure 3. Surface Plot of TMP Versus SPD and RNG for No LOS

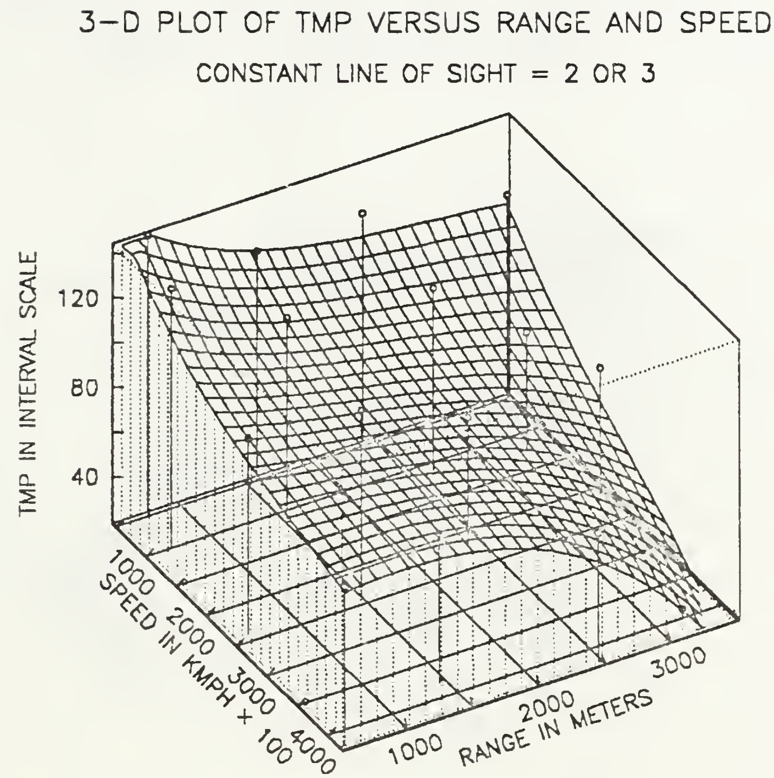


Figure 4. Surface Plot of TMP Versus SPD and RNG for 2-3 LOS

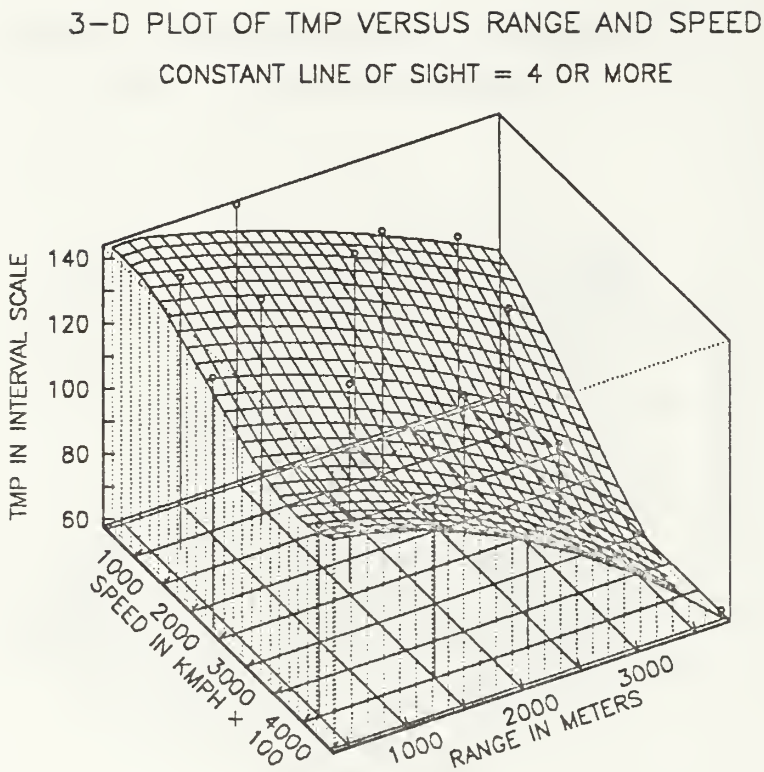


Figure 5. Surface Plot of TMP Versus SPD and RNG for 4- LOS

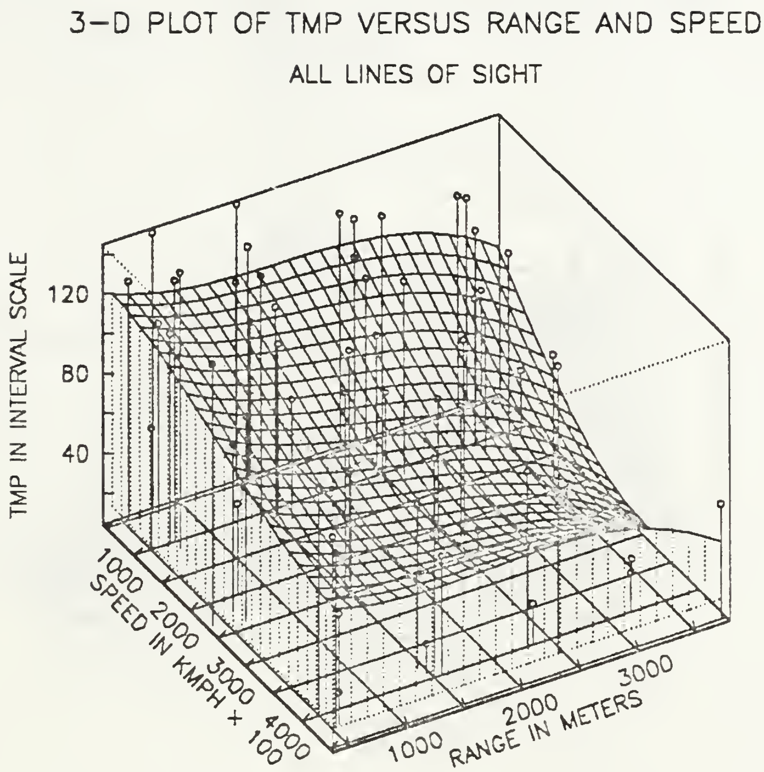


Figure 6. Surface Plot of TMP Versus SPD and RNG for All LOS

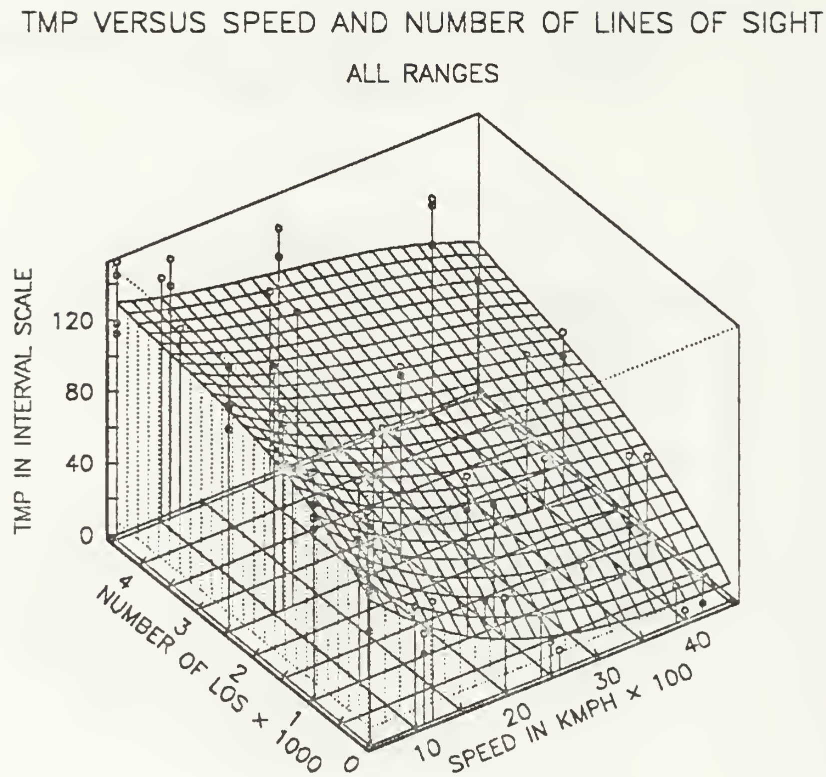


Figure 7. Surface Plot of TMP Versus SPD and LOS for All RNG

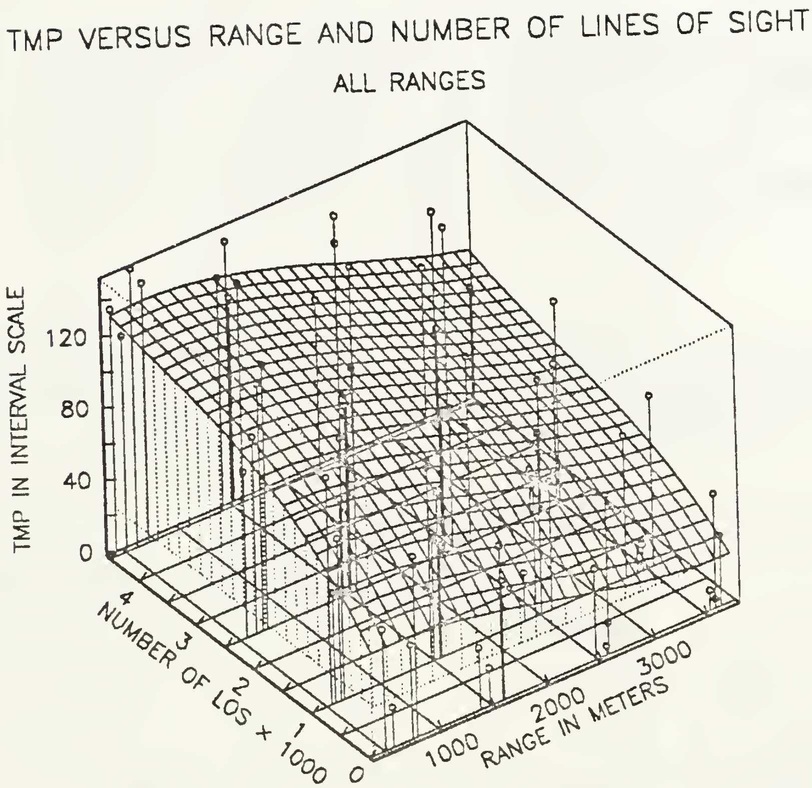


Figure 8. Surface Plot of TMP Versus RNG and LOS for All SPD

B. REGRESSION USING GRAFSTAT

MULTIPLE REGRESSION USING BACKWARD ELIMINATION

TABLE OF COEFFICIENTS

63 *OBSERVATIONS* *R-SQUARED* = 0.9048 *STANDARD ERROR* = 12.486
5 *VARIABLES* *ADJ R-SQUARED* = 0.8983

COEF	*ESTIMATE*	*STD ERR*	*T STAT*	*SIG LEVEL*
INTERCEPT	1.1824*E*2	5.2563*E*0	22.495	1.6653*E*-16
*RNG*3	-1.4744*E*-2	1.4786*E*-3	-9.9715	3.4722*E*-14
*LOS*3	1.1545*E*-2	1.6888*E*-3	6.8366	5.5107*E*-9
*SPD*3	-1.9871*E*-2	1.7745*E*-3	-11.198	6.3838*E*-16
(*LOS*3-*SPD*3)	1.9839*E*-6	6.8170*E*-7	2.9102	5.1154*E*-3

The assumptions of normality in the residuals and homogeneity of variance in the residuals are supported. The model is valid based on the standard error of the estimate and the extremely high R^2.

ANOVA RESULTING FROM THE REGRESSION

ANALYSIS OF NORMAL DISTRIBUTION FIT

DATA : *RESIDUAL*
SELECTION : *ALL*
X AXIS LABEL: *RESIDUAL*
SAMPLE SIZE : 63
CENSORING : *NONE*
FREQUENCIES : 1
EST. METHOD : *MAXIMUM LIKELIHOOD*
CONF METHOD : *ASYMPTOTIC NORMAL APPROXIMATION*

		CONF. INTERVALS (95 PERCENT)		*COVARIANCE MATRIX OF PARAMETER ESTIMATES*	
PARAMETER	*ESTIMATE*	*LOWER*	*UPPER*	*MU*	*SIGMA*
MU	2.0510*E*¯13	¯2.9589	2.9589	2.2781	0
SIGMA	1.1980*E*1	9.8878	14.072	0	1.1391

	SAMPLE	*FITTED*
MEAN :	1.9331*E*¯13	2.0510*E*¯13
STD DEV :	1.2076*E*1	1.1980*E*1
SKEWNESS:	-3.1684*E*-2	0.0000*E*0
KURTOSIS:	2.6120*E*0	3.0000*E*0

PERCENTILES	*SAMPLE*	*FITTED*
5:	-20.102	-1.9710*E*1
10:	-14.305	-1.5355*E*1
25:	-8.0305	-8.0768*E*0
50:	-1.0027	1.2101*E*-6
75:	10.122	8.0768*E*0
90:	14.321	1.5355*E*1

GOODNESS OF FIT

CHI-SQUARE :	2.9566
DEG FREED:	3
SIGNIF :	0.39836
KOLM-SMIRN :	0.060943
SIGNIF :	0.97341
CRAMER-V M :	0.037255
SIGNIF :	> .15
ANDER-DARL :	0.22113
SIGNIF :	> .15

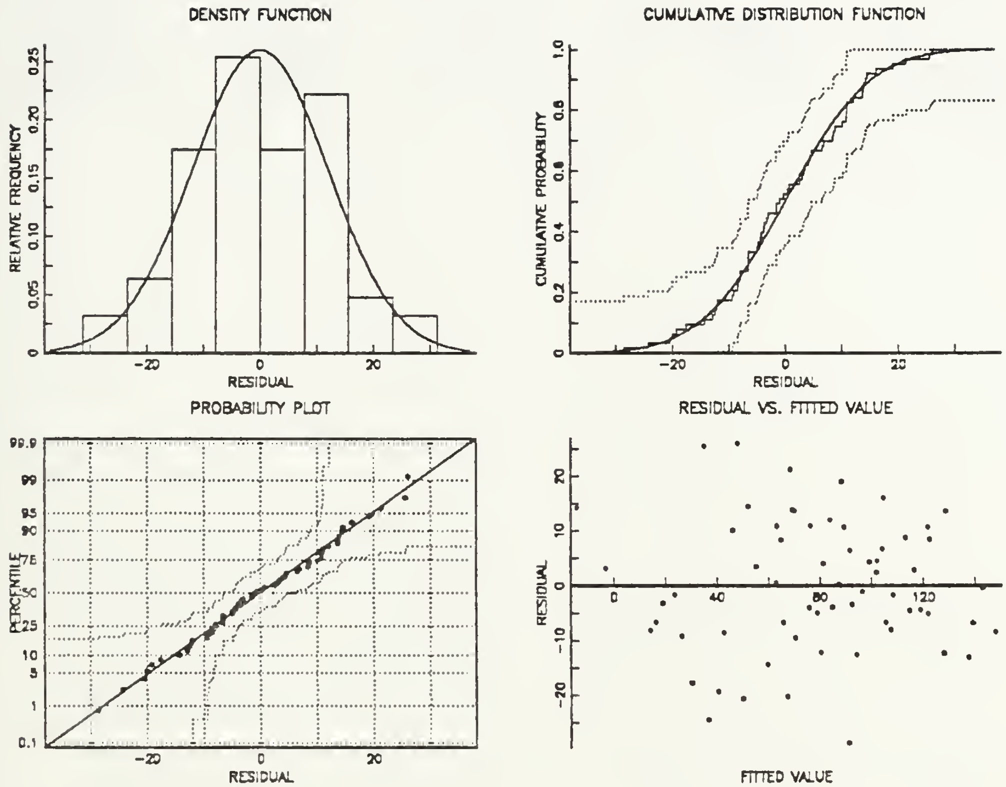


Figure 9. Residual Analysis of the Four Variable Regression

95: 19.153 1.9710E1

GOODNESS OF FIT ON THE RESIDUALS FROM THE REGRESSION

CHI-SQUARE GOODNESS OF FIT TABLE

LOWER	*UPPER*	*OBS*	*EXP*	*O-E*	*((O-E)*2)÷E*
-INF.	¯15.639	6	6.0403	¯0.040285	0.00026868
¯15.639	¯7.8194	11	10.149	0.85093	0.071344
¯7.8194	0	16	15.311	0.68937	0.031039
0	7.8194	11	15.311	¯4.3107	1.2136
7.8194	15.639	14	10.149	3.8509	1.4612
15.639	*+INF.*	5	6.0403	¯1.0403	0.17916
TOTAL		63	63		2.9566

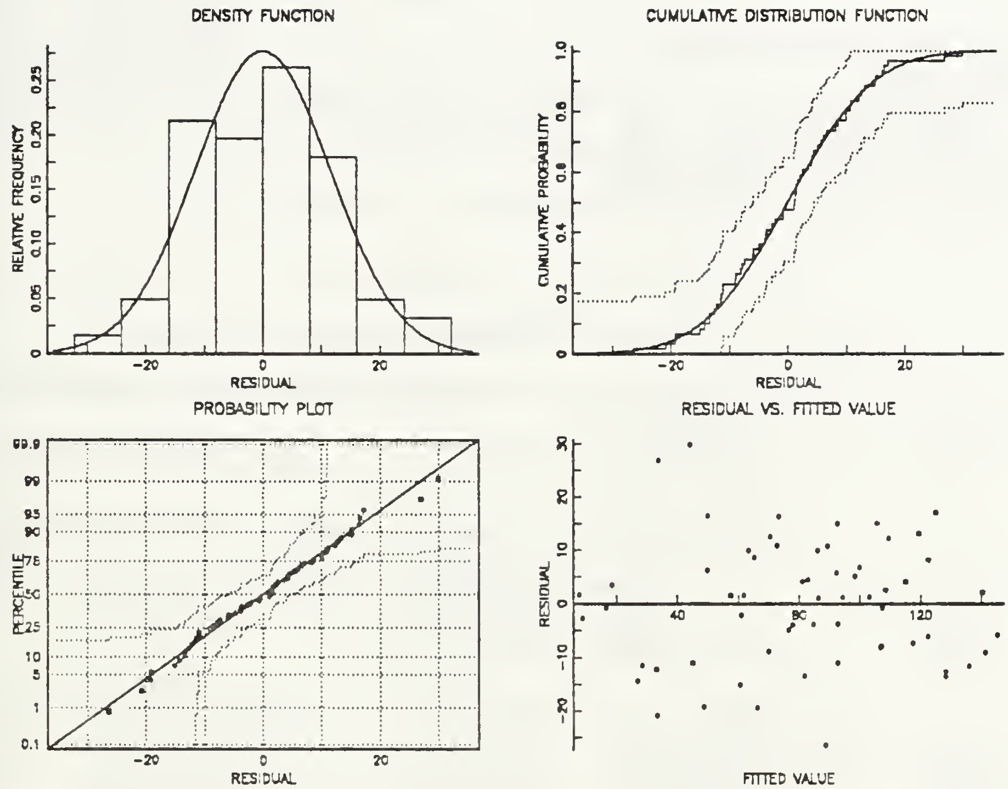


Figure 10. Residual Analysis of the Five Variable Regression

C. REGRESSION USING SAS

1. THE FIRST SAS MULTIPLE REGRESSION

```
OPTIONS LINESIZE=80;
DATA ONE;
  CMS FILEDEF DD1 DISK GDATA22 DATA A;
  INFILE DD1;
  INPUT RANG LOSS SPED TAMP;
PROC STEPWISE DATA=ONE;
  MODEL TAMP = RANG LOSS SPED / BACKWARD MAXR;
```

```
         SAS        9:54 TUESDAY, JUNE 13, 1989

         BACKWARD ELIMINATION PROCEDURE FOR DEPENDENT VARIABLE TAM

 STEP 0  ALL VARIABLES ENTERED          R SQUARE = 0.89099524
                                        C(P) =     4.00000000

              DF    SUM OF SQUARES   MEAN SQUARE         F        PROB>F

 REGRESSION    3    84699.83472023  28233.27824008    160.75     0.0001
 ERROR        59    10362.21574916    175.63077541
 TOTAL        62    95062.05046940

          B VALUE          STD ERROR     TYPE II SS          F        PROB>F

 INTCPT 110.47735149
 RANG     -0.01482470     0.00156910  15677.32260814     89.26     0.0001
 LOSS      0.01563357     0.00099485  43371.36043090    246.95     0.0001
 SPED     -0.01595209     0.00122666  29702.31143979    169.12     0.0001

 BOUNDS ON CONDITION NUMBER:      1.002224,      9.013743
 -------------------------------------------------------------------------

 ALL VARIABLES IN THE MODEL ARE SIGNIFICANT AT THE 0.1000 LEVEL.
```

2. THE SECOND SAS MULTIPLE REGRESSION

```
OPTIONS LINESIZE=80;
DATA ONE;
  CMS FILEDEF DD1 DISK GDATA22 DATA A;
  INFILE DD1;
  INPUT RANG LOSS SPED TAMP;
  RS = RANG*SPED;
  LS = LOSS*SPED;
  RL = RANG*LOSS;
PROC STEPWISE DATA=ONE;
  MODEL TAMP = RANG LOSS SPED RS LS RL / BACKWARD MAXR;
```

```
        SAS        9:55 TUESDAY, JUNE 13, 1989

        BACKWARD ELIMINATION PROCEDURE FOR DEPENDENT VARIABLE TAMP

STEP 0  ALL VARIABLES ENTERED          R SQUARE = 0.90984757
                                       C(P) =     7.00000000

            DF   SUM OF SQUARES   MEAN SQUARE         F      PROB>F

REGRESSION   6   86491.97593599  14415.32932267     94.20    0.0001
ERROR       56    8570.07453341    153.03704524
TOTAL       62   95062.05046940

        B VALUE          STD ERROR       TYPE II SS       F      PROB>F

INTCPT 109.07985496
RANG    -0.01017627     0.00299509    1766.66050270    11.54    0.0013
LOSS     0.01337164     0.00249723    4387.82533431    28.67    0.0001
SPED    -0.01728175     0.00270126    6263.83410262    40.93    0.0001
RS      -0.00000133     0.00000106     240.98633319     1.57    0.2147
LS       0.00000203     0.00000068    1373.72045367     8.98    0.0041
RL      -0.00000093     0.00000086     179.42161660     1.17    0.2835

BOUNDS ON CONDITION NUMBER:      7.239146,      206.9791
----------------------------------------------------------------
```

STEP 1 VARIABLE RL REMOVED   R SQUARE = 0.90796016
C(P) = 6.17240643

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	5	86312.55431939	17262.51086388	112.46	0.0001
ERROR	57	8749.49615001	153.49993246		
TOTAL	62	95062.05046940			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTCPT	112.32206019				
RANG	-0.01170732	0.00264429	3008.87281840	19.60	0.0001
LOSS	0.01136915	0.00168059	7024.89975456	45.76	0.0001
SPED	-0.01705147	0.00269694	6136.04920158	39.97	0.0001
RS	-0.00000146	0.00000105	292.39366190	1.90	0.1729
LS	0.00000206	0.00000068	1411.56803480	9.20	0.0036

BOUNDS ON CONDITION NUMBER: 6.576496, 116.9728

STEP 2 VARIABLE RS REMOVED   R SQUARE = 0.90488434
C(P) = 6.08301351

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	4	86020.16065748	21505.04016437	137.95	0.0001
ERROR	58	9041.88981191	155.89465193		
TOTAL	62	95062.05046940			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTCPT	118.23954426				
RANG	-0.01474358	0.00147857	15500.71093605	99.43	0.0001
LOSS	0.01154539	0.00168875	7286.43553484	46.74	0.0001
SPED	-0.01987087	0.00177449	19548.66357433	125.40	0.0001
LS	0.00000198	0.00000068	1320.32593725	8.47	0.0051

BOUNDS ON CONDITION NUMBER: 4.72315, 45.35

ALL VARIABLES IN THE MODEL ARE SIGNIFICANT AT THE 0.1000 LEVEL.

SUMMARY OF BACKWARD ELIMINATION PROCEDURE FOR DEPENDENT VARIABLE

STEP	VARIABLE RMVED	NUMBER IN	PARTIAL R**2	MODEL R**2	C(P)	F	PROB>F
1	RL	5	0.0019	0.9080	6.17241	1.1724	0.2835
2	RS	4	0.0031	0.9049	6.08301	1.9048	0.1729

MAXIMUM R-SQUARE IMPROVEMENT FOR DEPENDENT VARIABLE TAMP

STEPS 1 THROUGH 3 OMITTED

STEP 4 VARIABLE LS ENTERED R SQUARE = 0.90488434
C(P) = 6.08301351

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	4	86020.16065748	21505.04016437	137.95	0.0001
ERROR	58	9041.88981191	155.89465193		
TOTAL	62	95062.05046940			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTCPT	118.23954426				
RANG	-0.01474358	0.00147857	15500.71093605	99.43	0.0001
LOSS	0.01154539	0.00168875	7286.43553484	46.74	0.0001
SPED	-0.01987087	0.00177449	19548.66357433	125.40	0.0001
LS	0.00000198	0.00000068	1320.32593725	8.47	0.0051

BOUNDS ON CONDITION NUMBER: 4.72315, 45.35

THE ABOVE MODEL IS THE BEST 4 VARIABLE MODEL FOUND.

STEP 5 VARIABLE RS ENTERED R SQUARE = 0.90796016
C(P) = 6.17240643

	DF	SUM OF SQUARES	MEAN SQUARE	F	PROB>F
REGRESSION	5	86312.55431939	17262.51086388	112.46	0.0001
ERROR	57	8749.49615001	153.49993246		
TOTAL	62	95062.05046940			

	B VALUE	STD ERROR	TYPE II SS	F	PROB>F
INTCPT	112.32206019				
RANG	-0.01170732	0.00264429	3008.87281840	19.60	0.0001
LOSS	0.01136915	0.00168059	7024.89975456	45.76	0.0001
SPED	-0.01705147	0.00269694	6136.04920158	39.97	0.0001
RS	-0.00000146	0.00000105	292.39366190	1.90	0.1729
LS	0.00000206	0.00000068	1411.56803480	9.20	0.0036

BOUNDS ON CONDITION NUMBER: 6.576496, 116.9728

THE ABOVE MODEL IS THE BEST 5 VARIABLE MODEL FOUND.

STEP 6 OMITTED

APPENDIX D. COMBINATORIAL OPTIMIZATION PROGRAM USING LAGRANGIAN RELAXATION

```
      PROGRAM NTWRKS
*************************************************************************
* PROBLEM:      A TACTICAL UNIT MUST MOVE FROM START POINT A, THROUGH   *
*               AN INTERMEDIATE POINT B, TO OBJECTIVE C IN NO MORE THAN *
*               TBAR TIME UNITS.  EACH ARC/EDGE (U,V) IN THE NETWORK    *
*               HAS A TRAVERSAL TIME TC(E) AND EXPOSURE/TACTICAL DIFFI- *
*               CULTY MOVEMENT ANTI-POTENTIAL OF TD(E). WE WISH TO FIND *
*               A PATH FROM A THROUGH B TO C WHICH HAS AN ACCEPTABLE    *
*               TRAVERSAL TIME AND TACTICAL DIFFICULTY OR TO MINIMIZE   *
*               TD SUBJECT TO TBAR.  IT IS ASSUMED THAT THERE EXISTS AT *
*               LEAST 1 FEASIBLE PATH FROM A THROUGH B TO C.            *
* AUTHORS:      RICHARD MILLER, MAJOR, USMC                             *
*               CHARLES H. SHAW, CAPTAIN, USA                           *
*               IAN KEITH, CAPTAIN, USMC                                *
* SUBROUTINE:   DYKSTR - DIJKSTRA'S ALGORITHM                           *
* VARIABLES:  UVCD - INPUT ARRAY OF EDGES (U,V),TIME COST TC(E), AND    *
*                    TACTICAL DIFFICULTY TD(E).                         *
*             EDGES - NUMBER OF ARCS/EDGES (10000 MAXIMUM).             *
*             X1,X2 - BOOLEAN VECTORS FROM X IN SUBROUTINE DIJKSTRA.    *
*             OPTX - BEST VECTOR X1,X2,X3 FOUND THUS FAR.               *
*             NODES - NUMBER OF NODES, I.E., VERTICES (1100 MAXIMUM).   *
*             S, T - START/STOP VERTICES FOR DIJKSTRA'S ALGORITHM.      *
*             U, V - ALL ARBITRARY VERTICES.                            *
*             EPS - EPSILON, A SMALL NUMBER OR DIFFERENTIAL.            *
*             TBAR - MAXIMUM TIME ALLOWED IN NETWORK  (INPUT).          *
*             PRED - PREDECESSOR OF A NODE.                             *
*             D - D(V) 'LABEL' ARRAY FOR DIJKSTRA'S ALGORITHM.          *
*             DMIN - DMIN FOR DIJKSTRA'S ALGORITHM.                     *
*             EP - ENTRY POINT ARRAY.                                   *
*             OUTDEG - OUTDEGREE OF NODE TO COMPUTE ENTRY POINT ARRAY.  *
*             ADJ - ADJACENT VERTICES ARRAY.                            *
*             LENGTH - EDGE LENGTH (TIME) ARRAY.                        *
*             TACDIF - EXPOSURE/TACTICAL DIFFICULTY ARRAY.              *
*             TDMAX - LARGEST VALUE OF TACDIF().                        *
*             OBJ - OBJECTIVE FUNCTION VALUE OF P1 IN DIJKSTRA.         *
*             OBJMAX - OBJECTIVE FUNCTION MAXIMUM FOR P3.               *
*             LAM - LAGRANGIAN MULTIPLIER IN SUBROUTINE DIJKSTRA.       *
*             LAMMIN - TEMPORARY MINIMUM OF LAM.                        *
*             LAMMID - MIDPOINT OF LAMMIN AND LAMMAX.                   *
*             LAMMAX - TEMPORARY MAXIMUM OF LAM.                        *
*             ENDLAM - BEST VALUE OF LAM.                               *
*             TDLAMT - (TD + LAMBDA*T) ARRAY.                           *
*             SLOPE - SLOPE (TC*X - TBAR) IN SUBROUTINE DIJKSTRA        *
*************************************************************************
```

```
      INTEGER UVCD(10000,4),X1(10000),X2(10000),OPTX(10000)
      INTEGER EP(10001),OUTDEG(10001)
      INTEGER ADJ(10001),LENGTH(10001),TACDIF(10001)
      INTEGER EDGES,I,J,K,NODES,A,B,C,TDMAX,L,TIME,U,V

      REAL TBAR,LAMMIN,LAMMID,LAMMAX,OBJMAX,EPS,P1,ENDLAM
      REAL SLOPE1,SLOPE2,SLOPE3,OBJ1,OBJ2,OBJ3

      COMMON UVCD,EP,ADJ,LENGTH,TACDIF,EDGES,NODES,TBAR,A,B,C

      PARAMETER (EPS = 0.0001)

* SET FILEDEFS

      CALL EXCMS('FILEDEF 01 DISK PROJNET DATA B')
      CALL EXCMS('FILEDEF 02 DISK NETWORKS OUTPUT A')

* INITIALIZE ARRAYS TO ZERO

      DO 5 I = 1, 10001
           OUTDEG(I) = 0
           EP(I) = 0
           ADJ(I) = 0
           LENGTH(I) = 0
           TACDIF(I) = 0
   5  CONTINUE

* READ IN THE DATA ONE ROW AT A TIME AND FIND
* THE NUMBER OF NODES AND MAXIMUM VALUE OF TD(E)

      READ(1,*) NODES,A,B,C,TBAR
      I = 1
      TDMAX = 0
  10  READ(1,*,END=20) (UVCD(I,J), J=1,4)
      TDMAX = MAX(UVCD(I,4), TDMAX)
      I = I + 1
      GO TO 10
  20  IF (NODES .GT. 1100) THEN
           STOP '*** ERROR, NUMBER OF NODES EXCEEDS 1000 ***'
      END IF

* COMPUTE NUMBER OF EDGES IN THE NETWORK

      EDGES = I - 1
      IF (EDGES .GT. 10000) THEN
           STOP '*** NUMBER OF EDGES EXCEEDS SIZE DEFINED ***'
      END IF

* COMPUTE OUTDEGREE FOR EACH NODE
```

```
      DO 30 I = 1, EDGES
           OUTDEG(UVCD(I,1)) = OUTDEG(UVCD(I,1)) + 1
   30 CONTINUE

* INITIALIZE ENTRY POINT ARRAY EP()

      EP(1) = 1
      DO 40 I = 1, NODES
           EP(I+1) = OUTDEG(I)
           EP(I+1) = EP(I+1) + EP(I)
   40 CONTINUE

* COMPUTE THE HIERARCHICAL ADJACENCY LISTS FOR EACH NODE, EP() TO
* ADJ() AND LIST ALL THE NODE'S ADJACENT VERTICES AND CORRESPONDING
* LENGTH OR TIME COST AND EXPOSURE/TACTICAL DIFFICULTY

      K = 1
      DO 50 I = 1, NODES
           DO 51 J = 1, EDGES
                IF (UVCD(J,1) .EQ. I) THEN
                     ADJ(K) = UVCD(J,2)
                     LENGTH(K) = UVCD(J,3)
                     TACDIF(K) = UVCD(J,4)
                     K = K + 1
                END IF
                IF (K .GT. 10000) THEN
                   STOP '*** ADJACENCY LISTS EXCEEDS SIZE DEFINED ***'
                END IF
   51      CONTINUE
   50 CONTINUE

      PRINT *, 'INITIALIZATION COMPLETE AND BEGINNING SEARCH'

* FIND BOUNDS ON LAM

      LAMMIN = 0.0
      LAMMAX = (NODES - 1) * TDMAX

      L = 0
      CALL DYKSTR(LAMMIN,X1,SLOPE1,OBJ1)
      WRITE(2,*) 'LAMMIN=',LAMMIN,' SLOPE1=',SLOPE1,' OBJ1=',OBJ1
      CALL DYKSTR(LAMMAX,OPTX,SLOPE3,OBJ3)
      OBJMAX = OBJ3
      WRITE(2,*) 'LAMMAX=',LAMMAX,' SLOPE3=',SLOPE3,' OBJ3=',OBJ3
      PRINT *,'INITIAL DIJKSTRA COMPLETE'

   70 LAMMID = (LAMMIN + LAMMAX)/2.0
      IF (ABS(LAMMAX - LAMMIN) .LE. EPS) GO TO 500
      L = L + 1

      PRINT *, 'ITERATION',L,'STARTING'
```

```
      CALL DYKSTR(LAMMID,X2,SLOPE2,OBJ2)
      WRITE(2,*) 'LAMMID=',LAMMID,' SLOPE2=',SLOPE2,' OBJ2=',OBJ2

      IF (SLOPE2 .GT. 0) THEN
           LAMMIN = LAMMID
      END IF

      IF (SLOPE2 .LE. 0) THEN
           LAMMAX = LAMMID
           IF (OBJ2 .GT. OBJMAX) THEN
                OBJMAX = OBJ2
                ENDLAM = LAMMID
                DO 80 I = 1, EDGES
                     OPTX(I) = X2(I)
 80             CONTINUE
           END IF
      END IF
      GO TO 70

500   WRITE(2,*) 'WITH',L,'ITERATIONS THE '
      WRITE(2,85) OBJMAX
  85  FORMAT(' MAXIMUM OBJECTIVE FUNCTION VALUE FOR P2 IS ',F10.3)
      TIME = 0
      P1 = 0.0
      DO 90 I = 1, EDGES
           TIME = OPTX(I) * UVCD(I,3) + TIME
           P1 = OPTX(I) * UVCD(I,4) + P1
  90  CONTINUE
      WRITE(2,*) 'THE SOLUTION FOR (TD*X) IS ',P1
      WRITE(2,*) 'EPSILON IS ',P1 - OBJMAX,', TIME ON THE PATH IS ',TIME
      WRITE(2,*) 'AND THE LAST VALUE OF LAM IS ',ENDLAM
      WRITE(2,*) 'THE BEST PATH FROM NODE ',A,', THROUGH NODE ',B,','
      WRITE(2,*) 'TO OBJECTIVE NODE ',C,'IS ALONG PATH:'
      WRITE(2,95) A
  95  FORMAT('NODE ',I5)
      U = A
 550  I = 1
 600  IF ((U .EQ. UVCD(I,1)) .AND. (OPTX(I) .EQ. 1)) THEN
           WRITE(2,95) UVCD(I,2)
           IF (C .EQ. UVCD(I,2)) STOP 'THE END'
           U = UVCD(I,2)
           GO TO 550
      ELSE
           I = I + 1
           GO TO 600
      END IF

      END
```

****

```
      SUBROUTINE DYKSTR(LAM,X,SLOPE,OBJ)

      INTEGER PRED(1100),I,J,K,S,T,U,V,VMIN,X(10000)
      INTEGER UVCD(10000,4),EP(10001),ADJ(10001),LENGTH(10001)
      INTEGER TACDIF(10001),EDGES,NODES,A,B,C

      REAL D(1100),DMIN,SLOPE,OBJ,LAM,TDLAMT(10001),TINF,TBAR

      COMMON UVCD,EP,ADJ,LENGTH,TACDIF,EDGES,NODES,TBAR,A,B,C

      PARAMETER (TINF = 1.0E15)

*  SET TDLAMTC() FOR NEW LAM

      DO 300 I = 1, EDGES + 1
           TDLAMT(I) = TACDIF(I) + LAM * LENGTH(I)
  300 CONTINUE

* RESET OPTIMAL PATH FOR NEXT SEGMENT

      DO 305 I = 1, EDGES
           X(I) = 0
  305 CONTINUE

* DIJKSTRA'S ALGORITHM FOR SHORTEST PATH FROM S TO ALL NODES

      S = A
      T = B

* INITIALIZE THE SHORTEST PATH AND PREDECESSOR ARRAYS

  400 DO 310 I = 1, NODES
           PRED(I) = 0
           D(I) = TINF
  310 CONTINUE

* INITIALIZE START VERTEX, S

      D(S) = 0.0
      PRED(S) = S

      DO 320 I = 1, NODES
           DMIN = TINF
           DO 330 J = 1, NODES
                IF ((PRED(J).GE.0) .AND. (D(J).LT.DMIN)) THEN
                     DMIN = D(J)
                     VMIN = J
                END IF
  330      CONTINUE
           PRED(VMIN) = -PRED(VMIN)
```

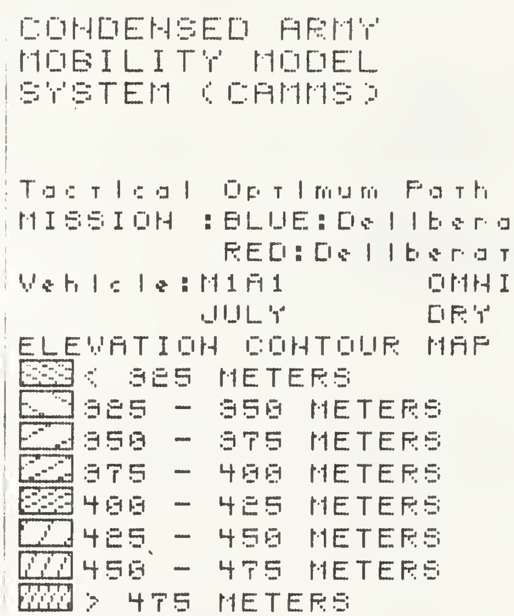
```
          DO 340 K = EP(VMIN), EP(VMIN+1)-1
               IF ((D(VMIN) + TDLAMT(K)) .LT. D(ADJ(K))) THEN
                    D(ADJ(K)) = D(VMIN) + TDLAMT(K)
                    PRED(ADJ(K)) = VMIN
               END IF
  340     CONTINUE
  320 CONTINUE

* CONVERT PRED() TO PATH X()

      V = T
  360 I = 1
  370 IF ((-PRED(V) .EQ. UVCD(I,1)) .AND. (V .EQ. UVCD(I,2))) THEN
          X(I) = 1
          V = -PRED(V)
          GO TO 360
      ELSE
          IF (V .EQ. -PRED(S)) GO TO 380
          I = I + 1
          GO TO 370
      END IF

  380 IF (S .EQ. B) GO TO 390

* NOW GET SHORTEST PATH FROM B TO C

      S = B
      T = C
      GO TO 400

* COMPUTE OBJECTIVE FUNCTION VALUE AND SLOPE FOR LAM

  390 OBJ = 0.0
      SLOPE = 0.0
      DO 450 I = 1, EDGES
          OBJ = (UVCD(I,4) + LAM * UVCD(I,3)) * X(I) + OBJ
          SLOPE = UVCD(I,3) * X(I) + SLOPE
  450 CONTINUE
      OBJ = OBJ - LAM * TBAR
      SLOPE = SLOPE - TBAR

      END
```

APPENDIX E. EXAMPLE MODEL OUTPUT

The purpose of these example output plots is to demonstrate proper functioning of the model in terms of tactical movement decision logic and some sensitivity analysis of the cognitive time scale values. Plots are appropriately labelled with all normal Tactical Optimum Path (TOP) output plots being expected value model runs unless otherwise noted. High risk averse plots use an input value of 30 minutes and extreme risk averse plots use an input value of 60 minutes rather than the expected value of 5.5 minutes.

These verification model runs are based on the Lauterbach map sheet near Fulda in the central region of The Federal Republic of Germany. The base map is one version of the terrain relief map available in the CAMMS. The NATO vehicle represented is an M1A1 Main Battle Tank moving as part of a deliberate attack against the specified Warsaw Pact systems shown in a deliberate defense.

The sequence of numbers shown on the mapping represents the path points in the order specified by the user. The continuous line connecting these points is the resulting optimum path from the optimization. The other irregular polygon shapes represent the user input area of operations which appropriately restricts the network searched during the optimization. The set of optimum path points and large amounts of audit data are available in data files for use directly in combat models and analysis rather than output to a screen or map.



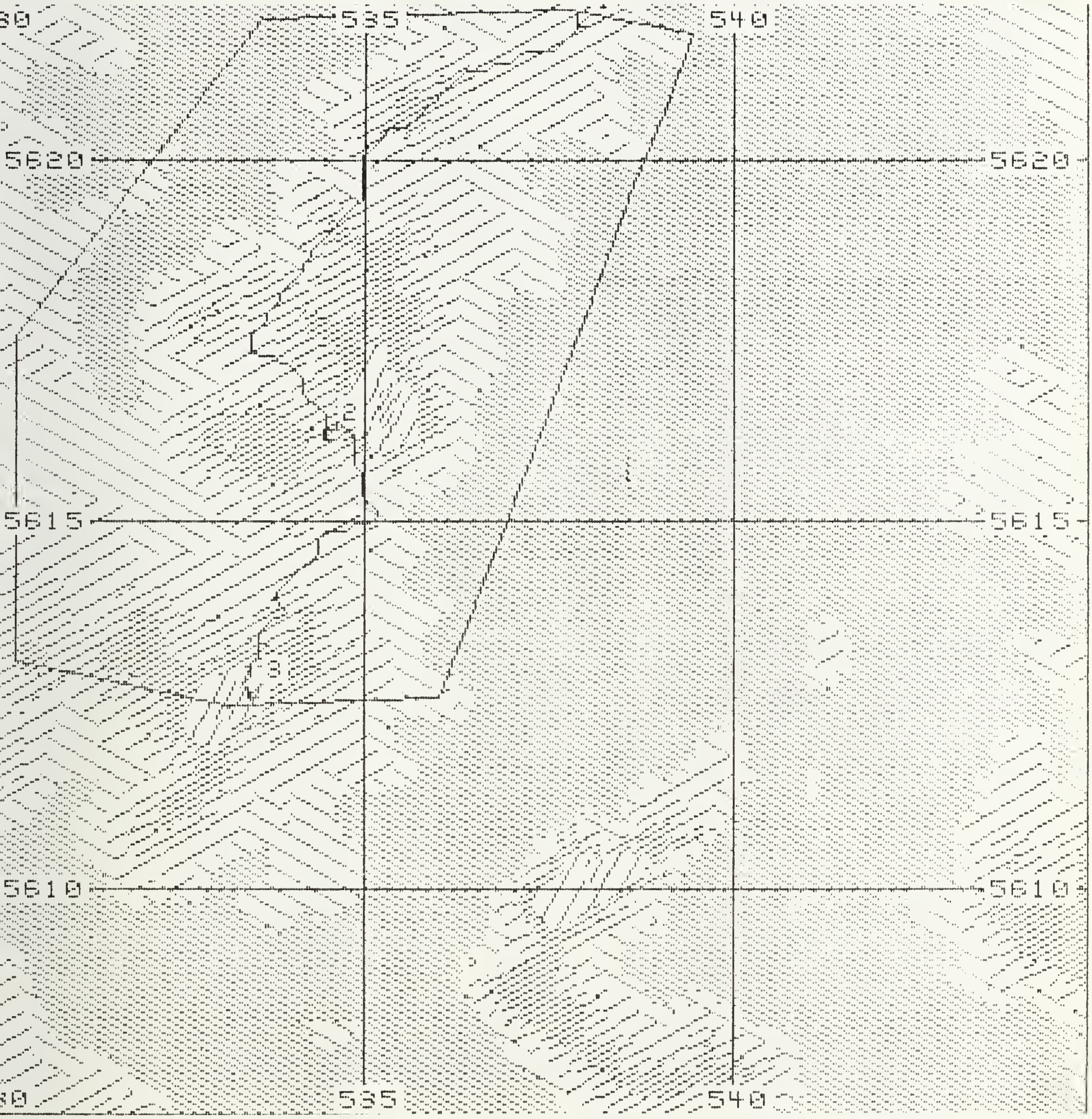


Figure 11. CAMMS Output Mapping For Set 3

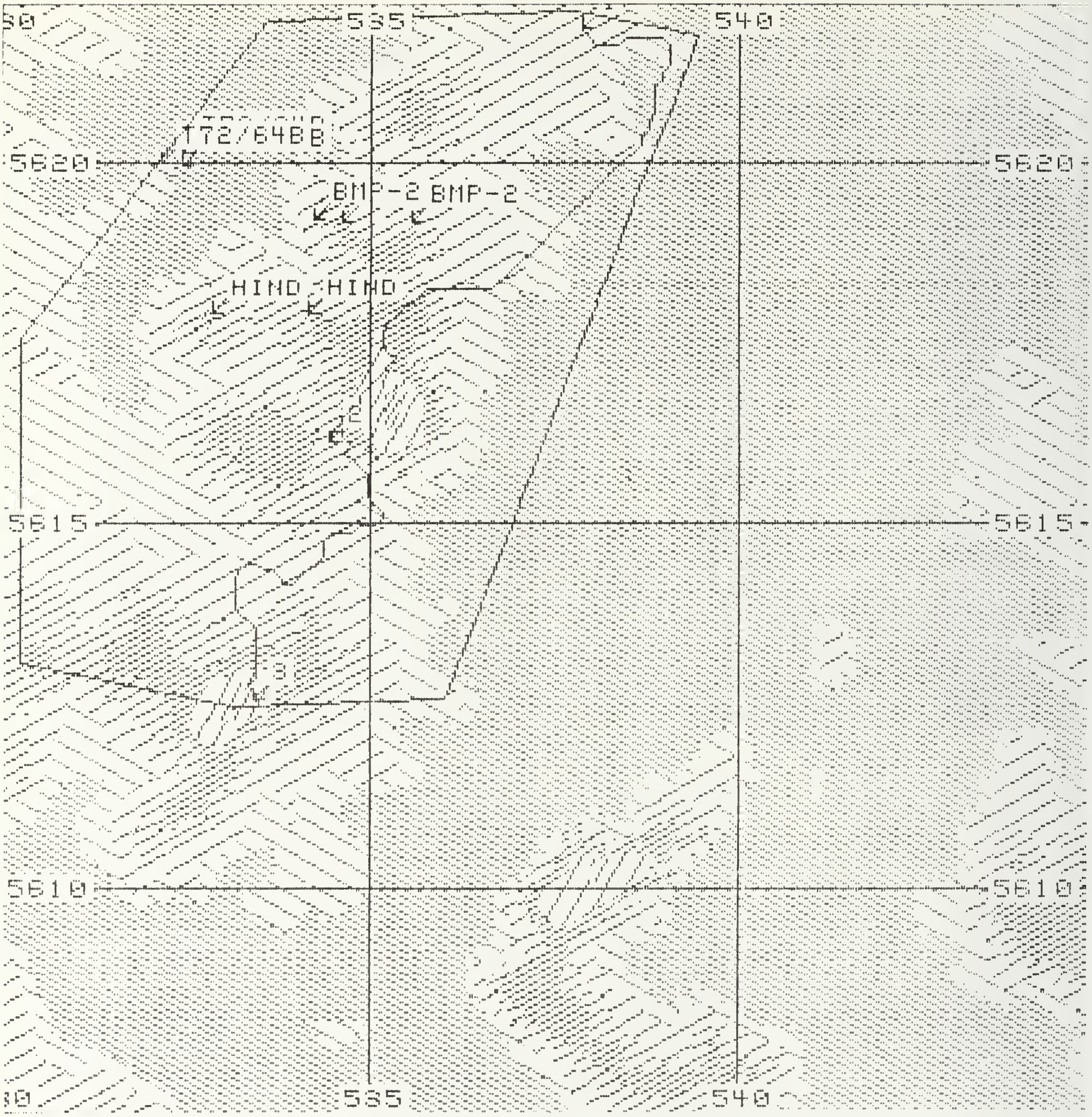


Figure 12. TOP Output Mapping For Set 3

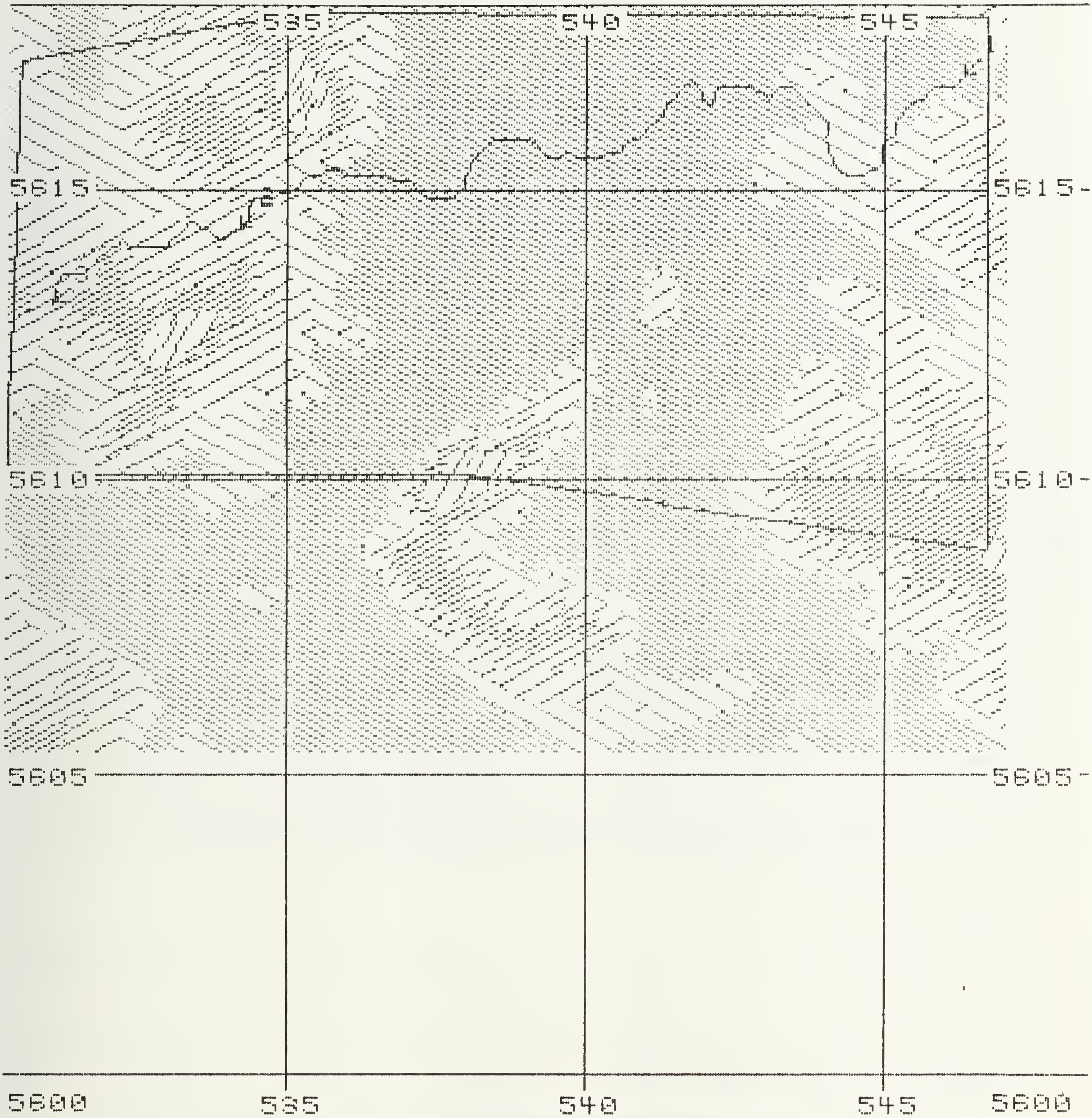


Figure 13. CAMMS Output Mapping For Set 4

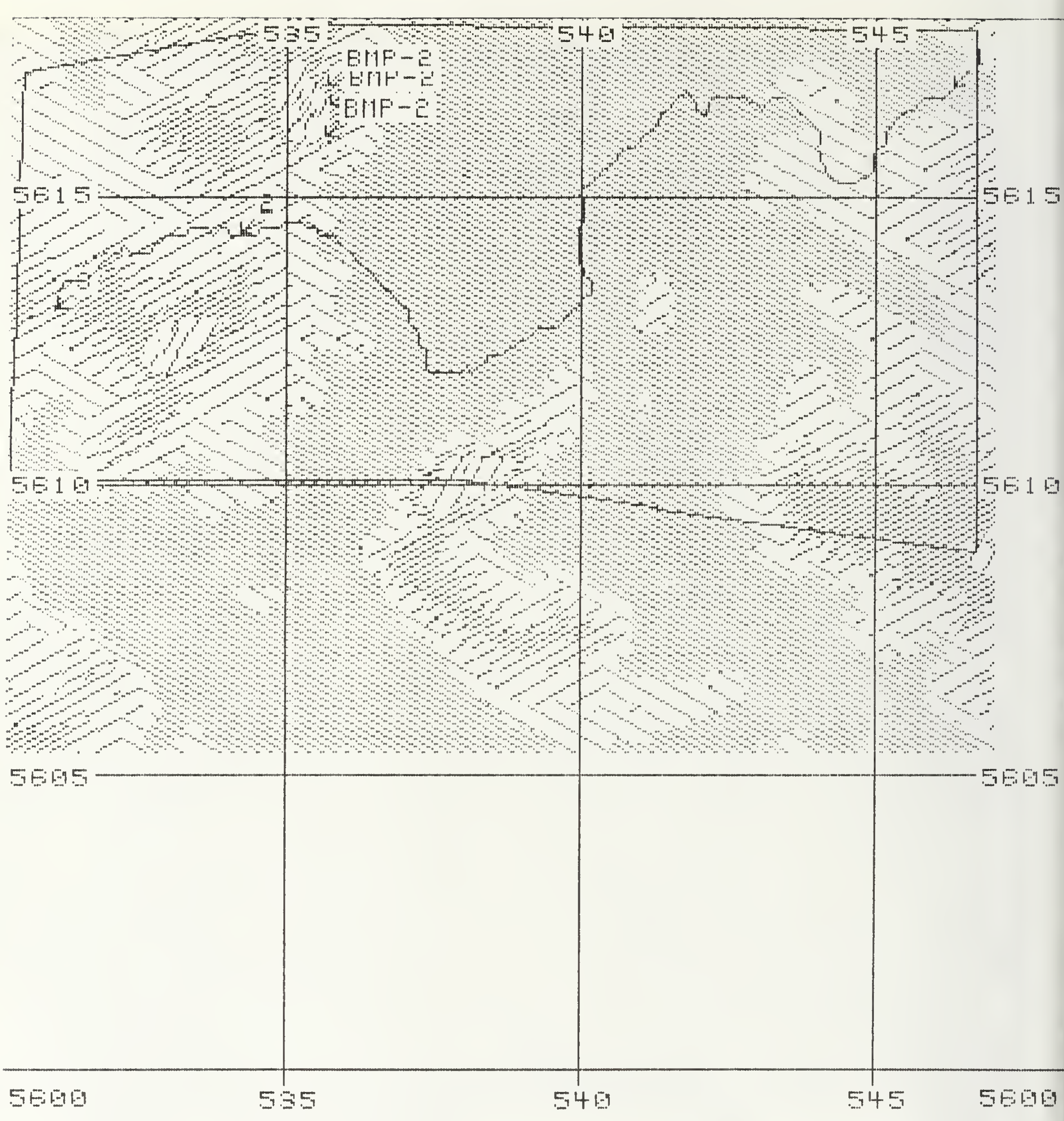


Figure 14. TOP Output Mapping For Set 4

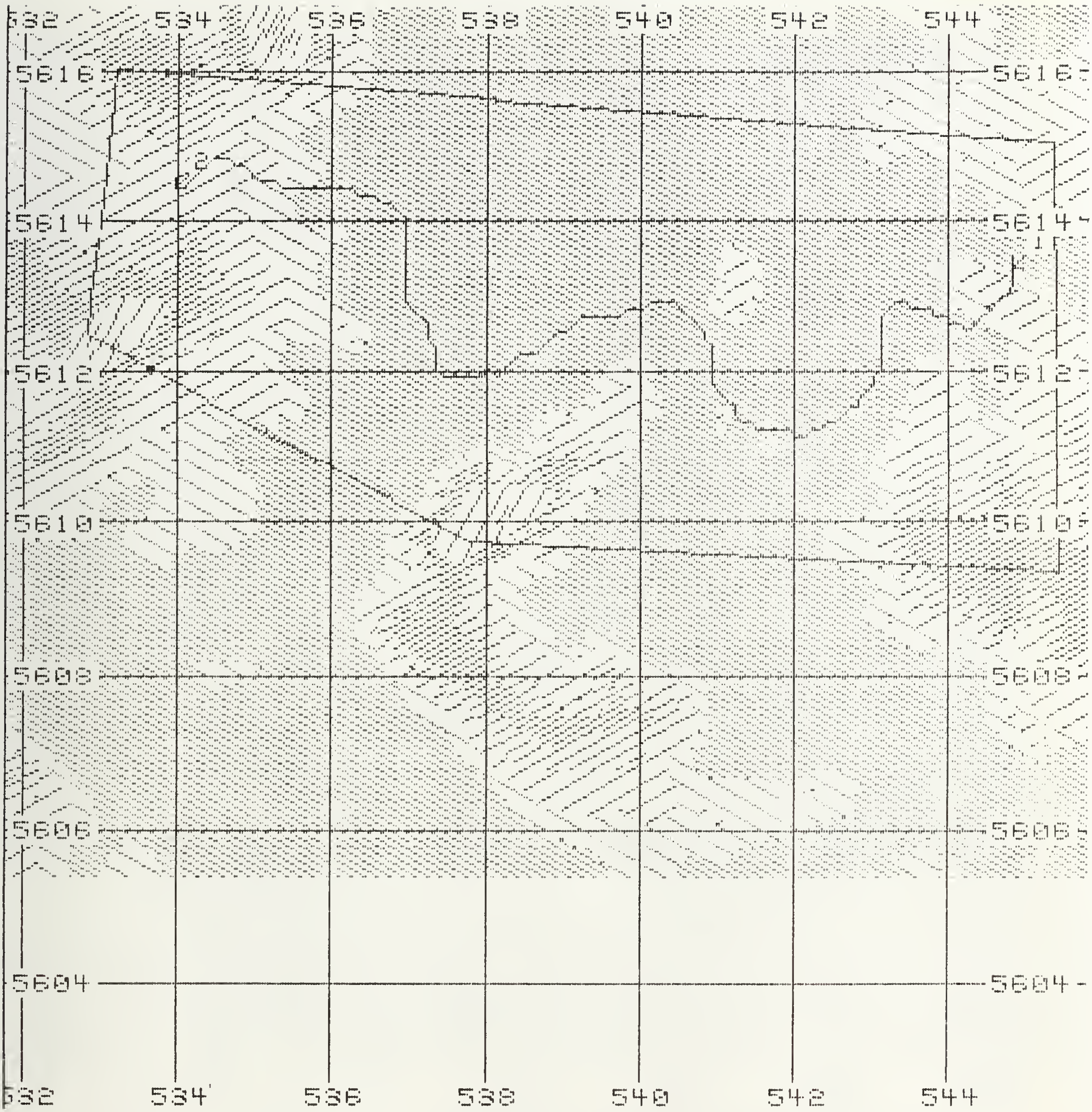


Figure 15. CAMMS Output Mapping For Set 5

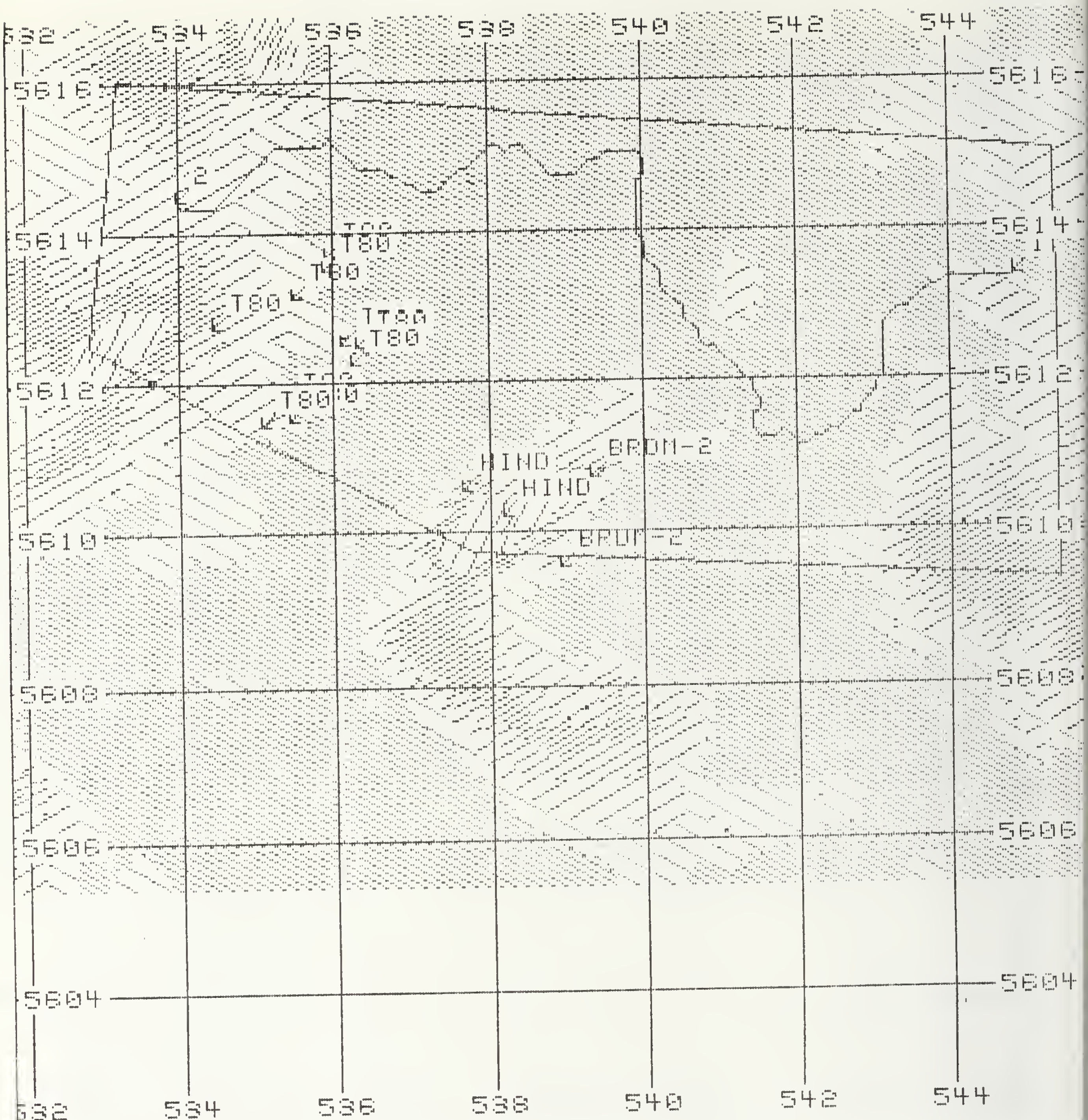


Figure 16. TOP Output Mapping For Set 5

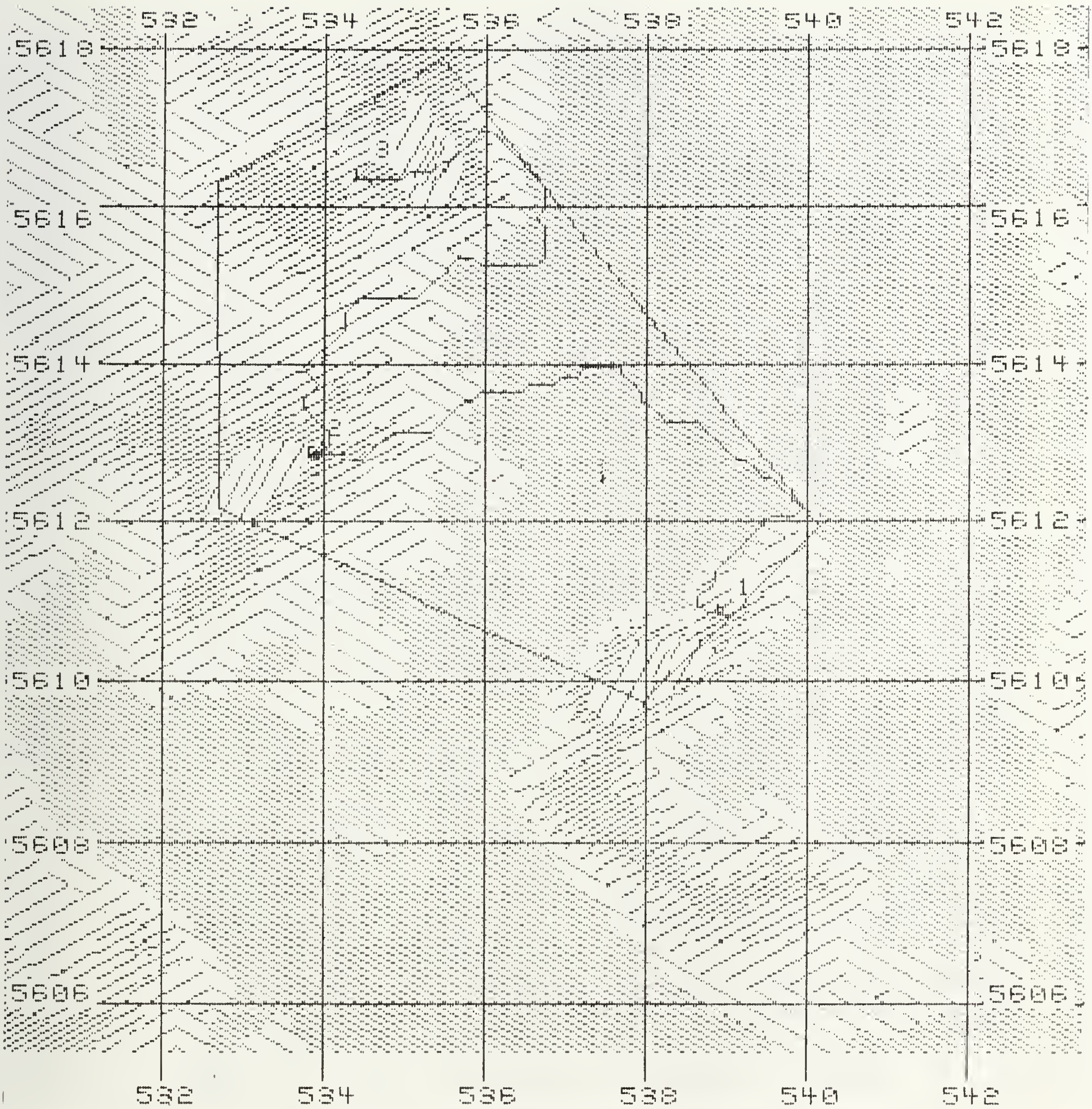


Figure 17. CAMMS Output Mapping For Set 6

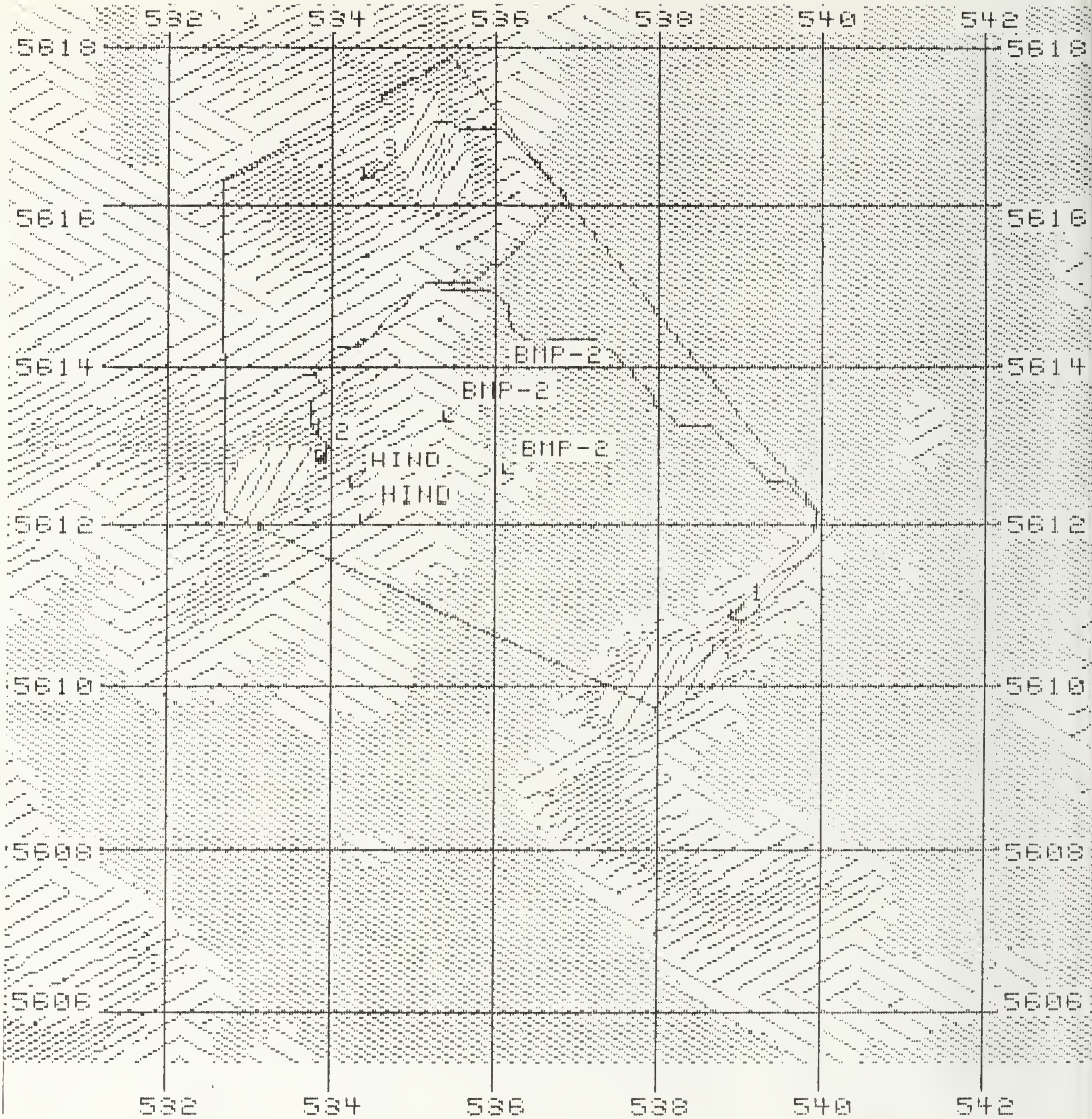


Figure 18. TOP Output Mapping For Set 6

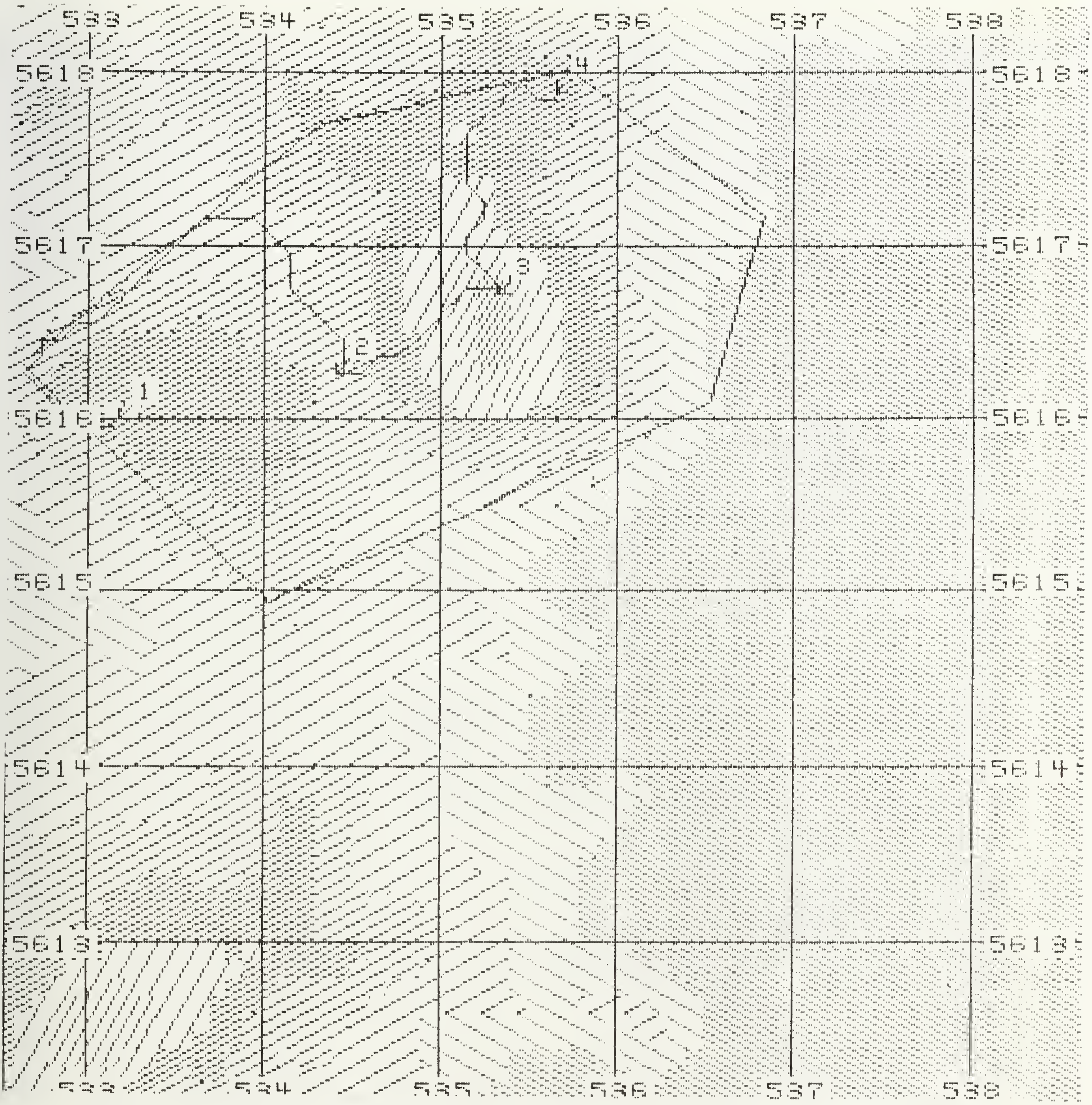


Figure 19. CAMMS Output Mapping For Set 15

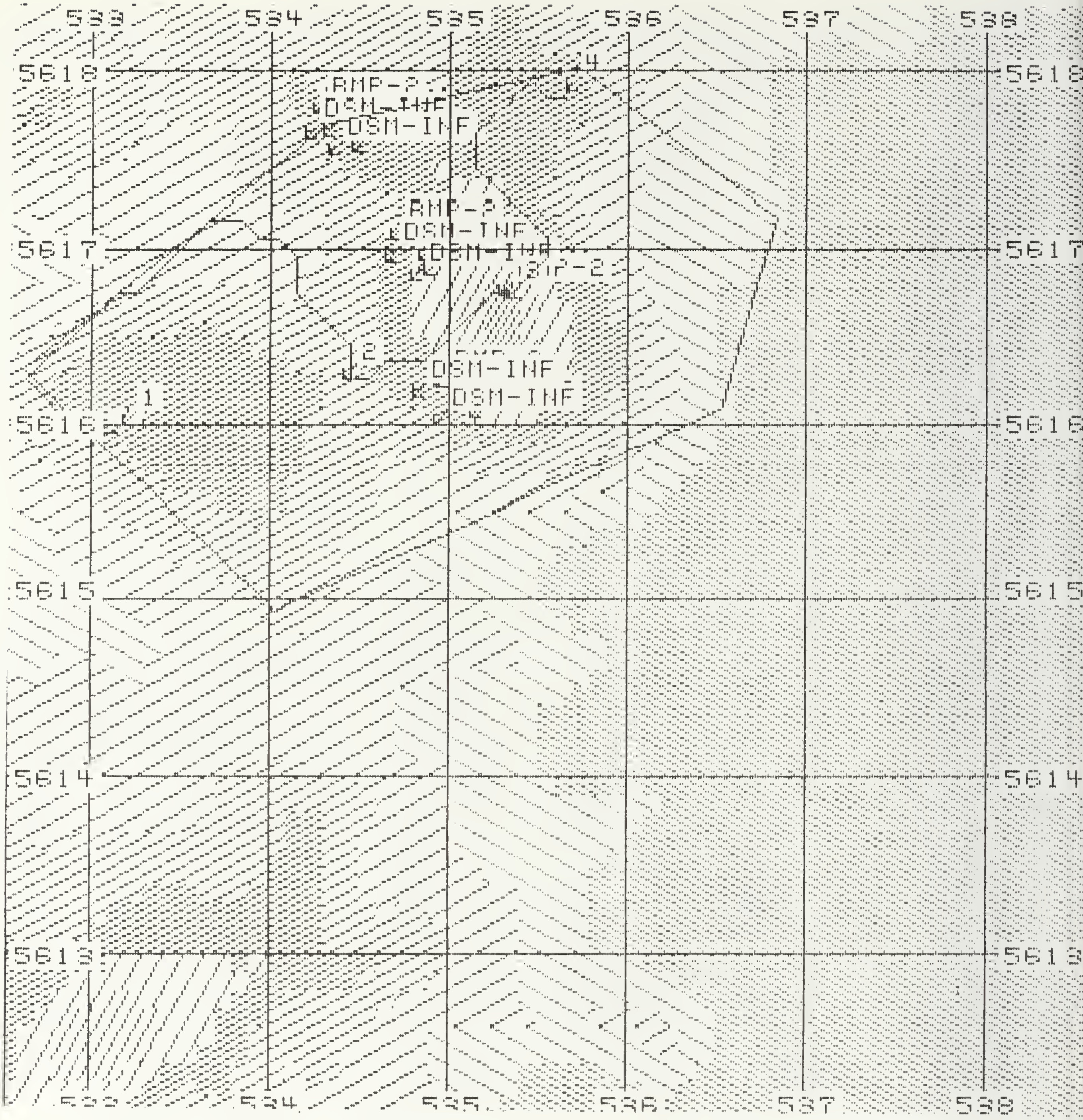


Figure 20. TOP Output Mapping For Set 15

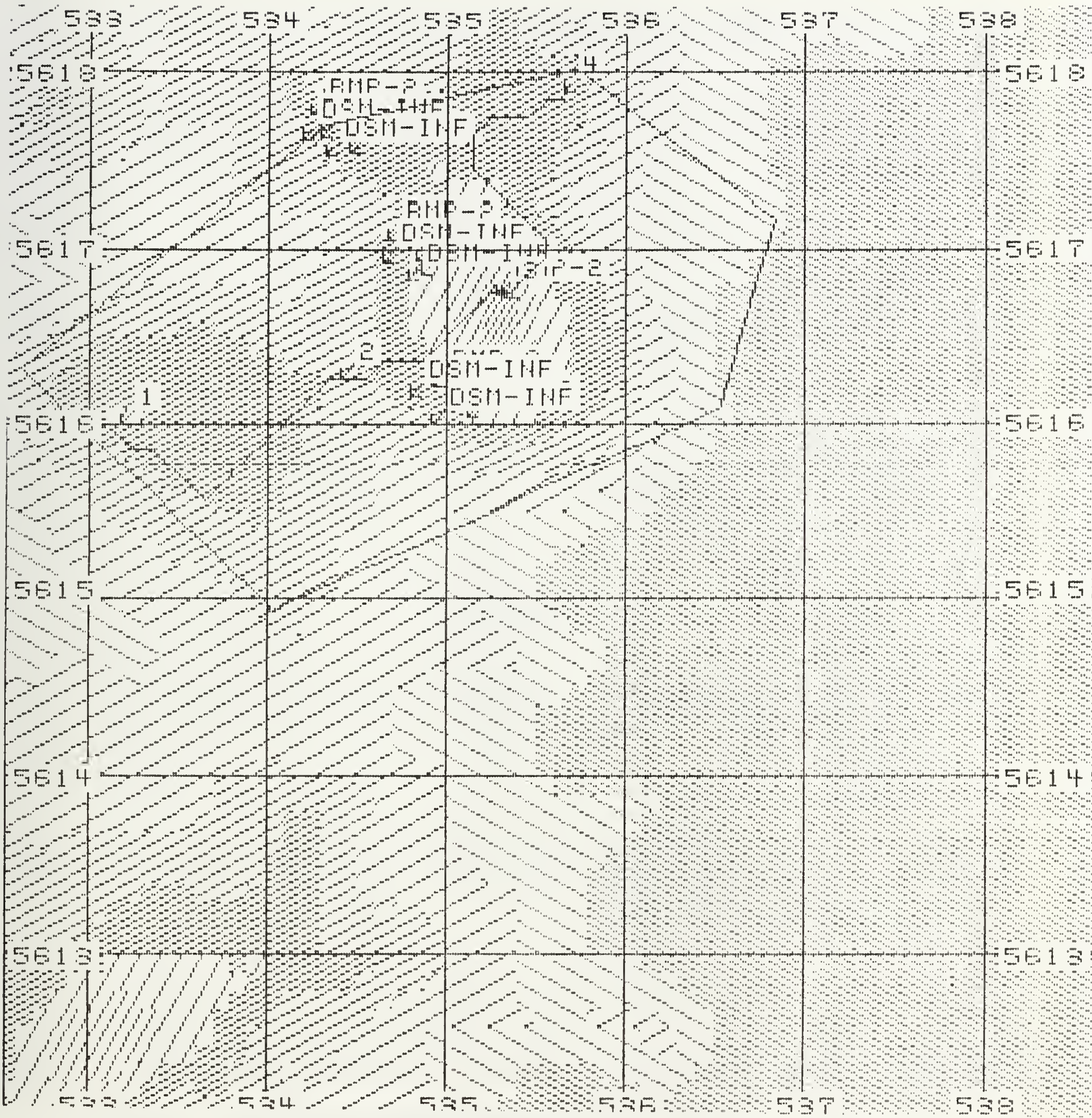


Figure 21. TOP Mapping With A High Risk Averse Posture For Set 15

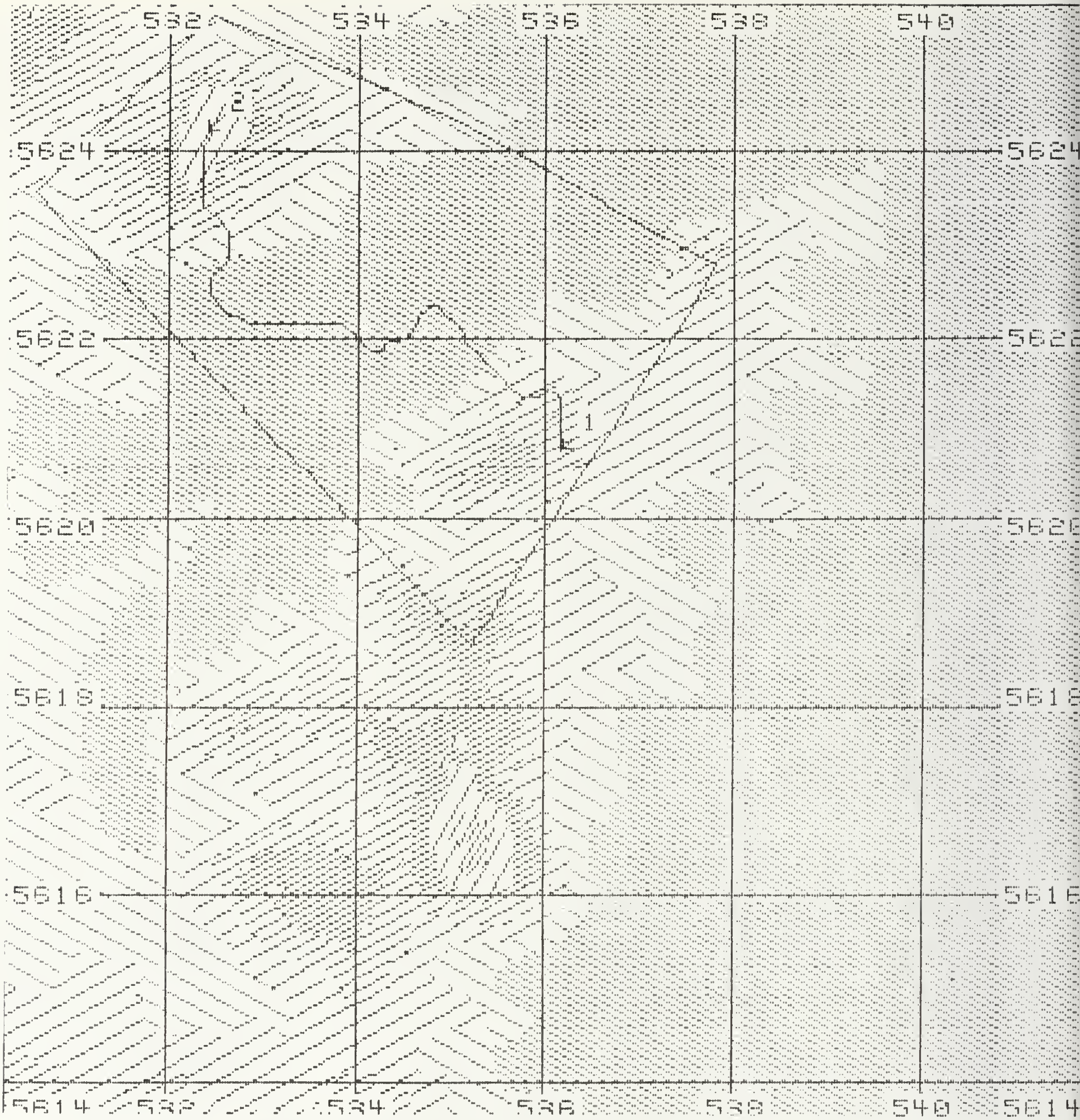


Figure 22. CAMMS Output Mapping For Set 16

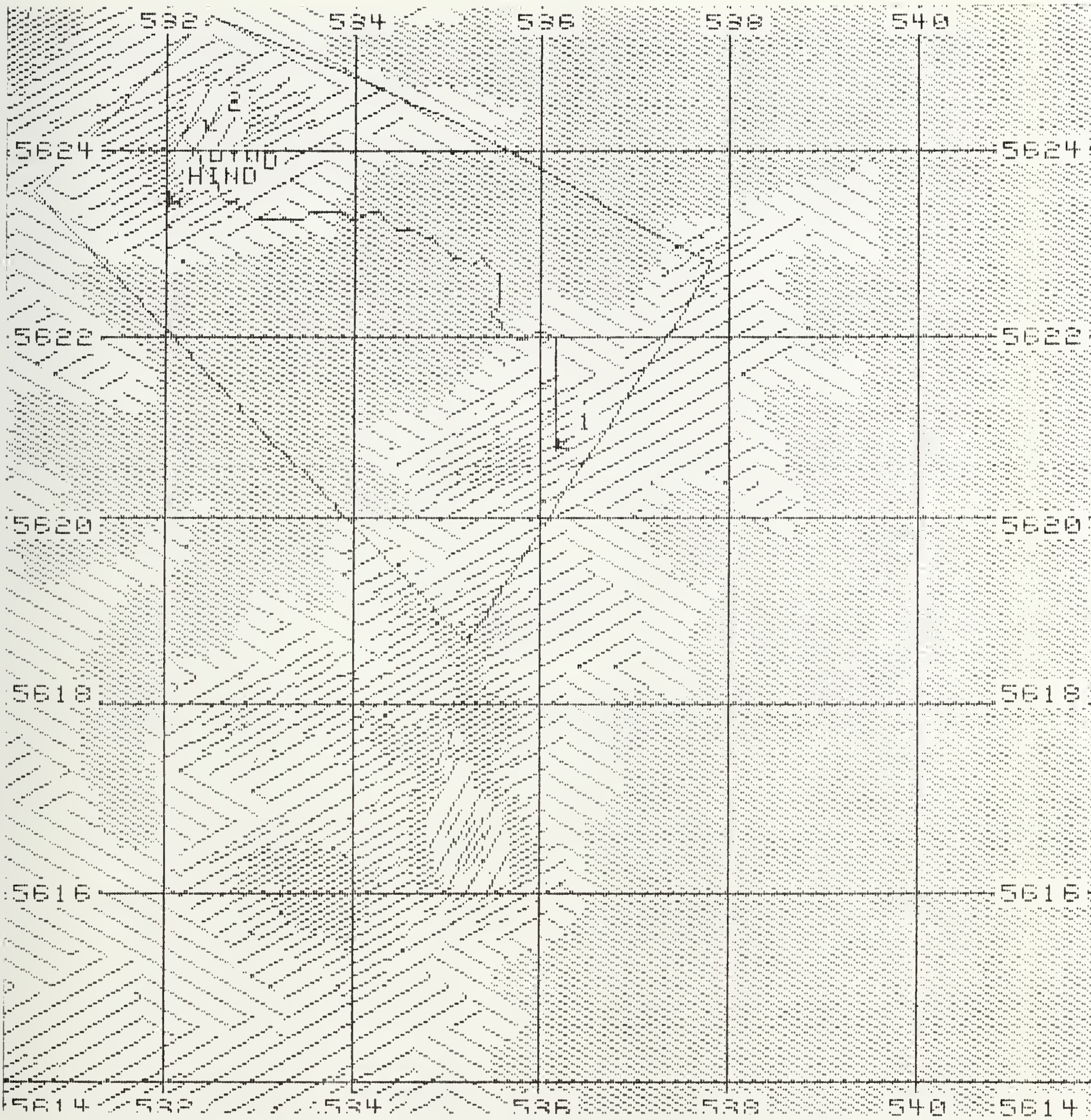


Figure 23. TOP Output Mapping For Set 16

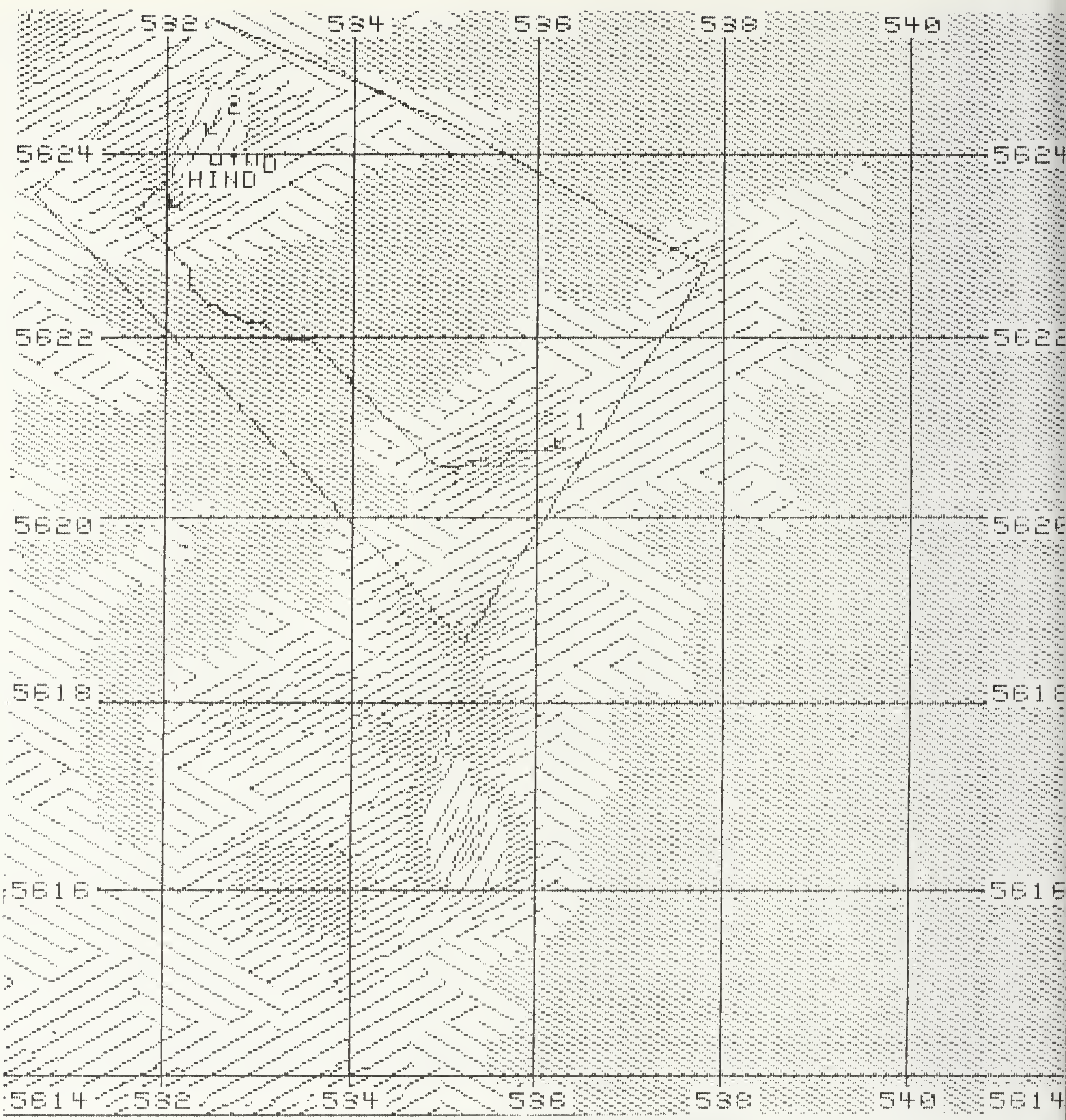


Figure 24. TOP Mapping With A High Risk Averse Posture For Set 16

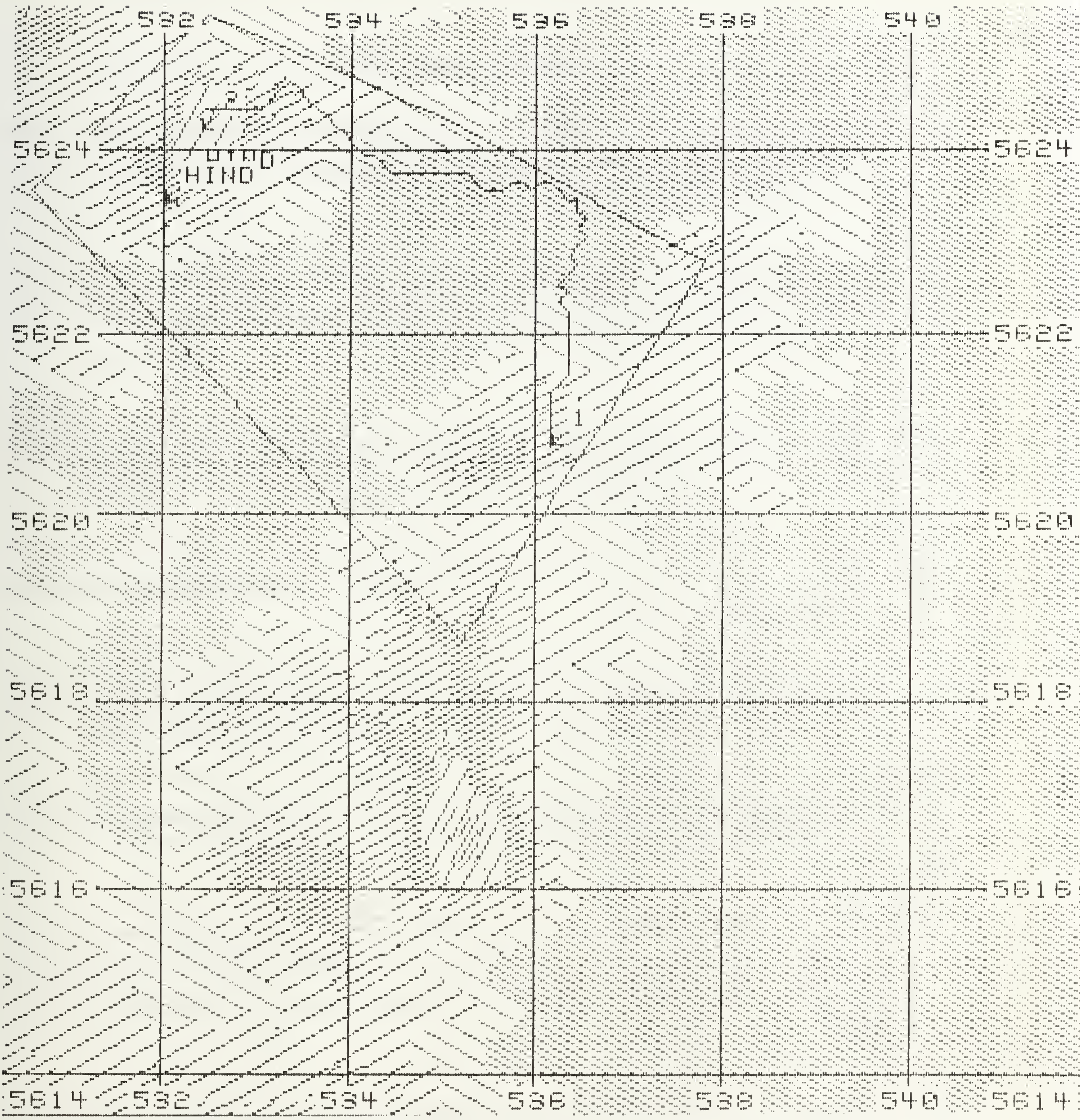


Figure 25. TOP Mapping With An Extreme Risk Averse Posture For Set 16

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