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THESIS

CALCULATED NOISE PERFORMANCE OF A FREQUENCY HOP SEQUENCE SYSTEM WITH APPLICATIONS TO LOW ALTITUDE SATELLITE COMMUNICATIONS

by

Mehmet Tahir Ozden

December 1990

Thesis Advisor: Glen A. Myers

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This research considers communication amongst a community of earth terminals through a collection of low altitude earth oriented satellites (LASAT). In one proposed LASAT system, all satellites are identical and simultaneous transmission of near identical signals from several satellites creates interference at the earth receivers. Frequency hop sequence (FHS) is a new form of frequency hopping (FH) and is the proposed spread spectrum LASAT communications technique considered in this research. In military applications, the performance of the communications system in the presence of noise only, noise and jamming, noise and multipath or synchronization errors is of interest. FHS offers some immunity to jamming and multipath. This report represents the calculated noise performance of FHS in the presence of jamming, multipath and synchroniza-

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Calculated Noise Performance of a Frequency Hop Sequence System with Application to Low Altitude Satellite Communications

by

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Lieutenant Junior Grade Turkish Navy
B.S., Turkish Naval Academy, 1984

Submitted in partial fulfillment
of the requirements for the degree of

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ABSTRACT

This research considers communication amongst a community of earth terminals through a collection of low altitude earth oriented satellites (LASAT). In one proposed LASAT system, all satellites are identical and simultaneous transmission of near identical signals from several satellites creates interference at the earth receivers. Frequency hop sequence (FHS) is a new form of frequency hopping (FH) and is the proposed spread spectrum LASAT communications technique considered in this research. In military applications, the performance of the communications system in the presence of noise only, noise and jamming, noise and multipath or synchronization errors is of interest. FHS offers some immunity to jamming and multipath. This report presents the calculated noise performance of FHS in the presence of jamming, multipath and synchronization errors. For the noise only case, the calculated probability of bit error of a FHS system using eight symbols for a 10 dB value of the ratio of input bit energy to noise power density is 2×10^{-4}.

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I. INTRODUCTION

Radio communication is transitioning from analog to digital. The quality measure of a digital communication system is the probability of error, rather than output SNR as in analog communications. Military radio communication is concerned with the effects of jamming. This has produced the spread spectrum in which transmitted signals are digital. One form of spread spectrum is frequency hopping (FH).

Techniques and performance of FH are well documented [Ref.3]. FH provides immunity to interference from jamming and multipath effects [Ref.4]. This report is concerned with the application of a particular form of FH, called Frequency Hop Sequence (FHS), to a particular type of military communication system using a multitude of low-altitude satellites. Chapter II gives the background about FHS. In chapter III, the performance analysis of FHS is provided. Chapter IV contains conclusions and recommendations.

II. BACKGROUND

In a LASAT communication system proposed by Dr. Glen A. Myers, satellites provide over the horizon communication amongst the earth stations [Ref.5]. This research considers that proposal in which all satellites are identical with each consisting of a single wideband transponder (B Hz bandwidth). Simultaneous transmission of near identical signals from the several satellites creates interference at the earth station receivers. The interference is similar to that created by multipath signal reception. A well known method of eliminating the effects of multipath signal is frequency hopping (FH) [Ref.4]. FH also provides immunity to jamming in a hostile environment by requiring the jammer to spread the transmitted power over a much wider band (B Hz).

A new form of FH, called Frequency Hop Sequence (FHS), is the proposed communication technique for the LASAT communication system. In FHS, a group of bits is transmitted simultaneously and these groups or symbols are distinguished during transmission by assigning a unique FH sequence to each symbol. In all forms of FH, the data randomizes the hop pattern. With FHS, the data mixes the sequences rather than simply offseting the carrier hop by hop. The notion is that it may be more difficult to recognize and hence duplicate an entire sequence than it is to recognize the offset of a frequency cell. FHS also provides immunity to interference in the form of multipath and jamming.

The problem of interest in this report is the noise performance of FHS in the presence of interference in the form of multipath and jamming. The effect of synchronization error is also considered.

III. SYSTEM MODEL AND RESULTS

In FHS, a group of k bits is sent simultaneously. There are then, $M=2^k$ possible groups or symbols. These symbols are distinguished during transmission by assigning a unique FH sequence to each symbol. N_h is the number of hops per symbol, N_c is the number of frequency cells used, b is the bandwidth of each frequency cell and B is the total bandwidth used by the system. All of these system parameters are illustrated in Figure 1.

The receiver of the system has 2^k parallel dehop/detector subsystems as shown in Figure 2. The voltage of interest in each of these subsystems is represented by z_m where m = 1,2,3....M. Each of these subsystems has a synchonized hopping local oscillator (L.O.) at the front end with a unique hopping pattern. If the hopping pattern of L.O.in subsystem one is the same as the hopping pattern of symbol S_1, then the output of the intermediate frequency (IF) amplifier in subsystem one is a sinusoid at the IF with a duration T_s sec and amplitude A volts. This sinusoid is envelope-detected and, to improve the performance, the voltage at the output of the envelope detector is integrated for a symbol duration and the integrator is then dumped.

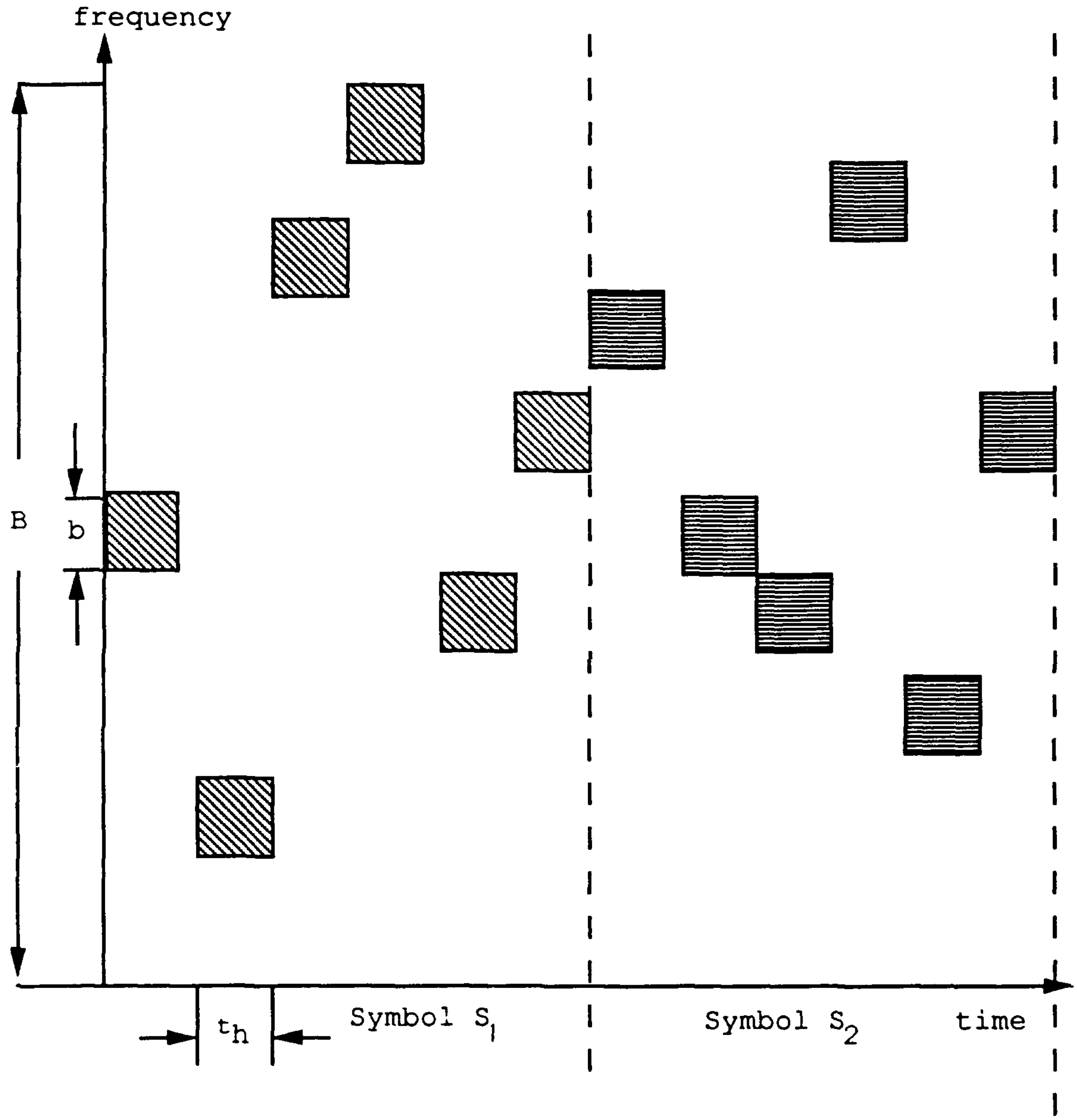

Figure 1. FHS Frequency versus Time Diagram

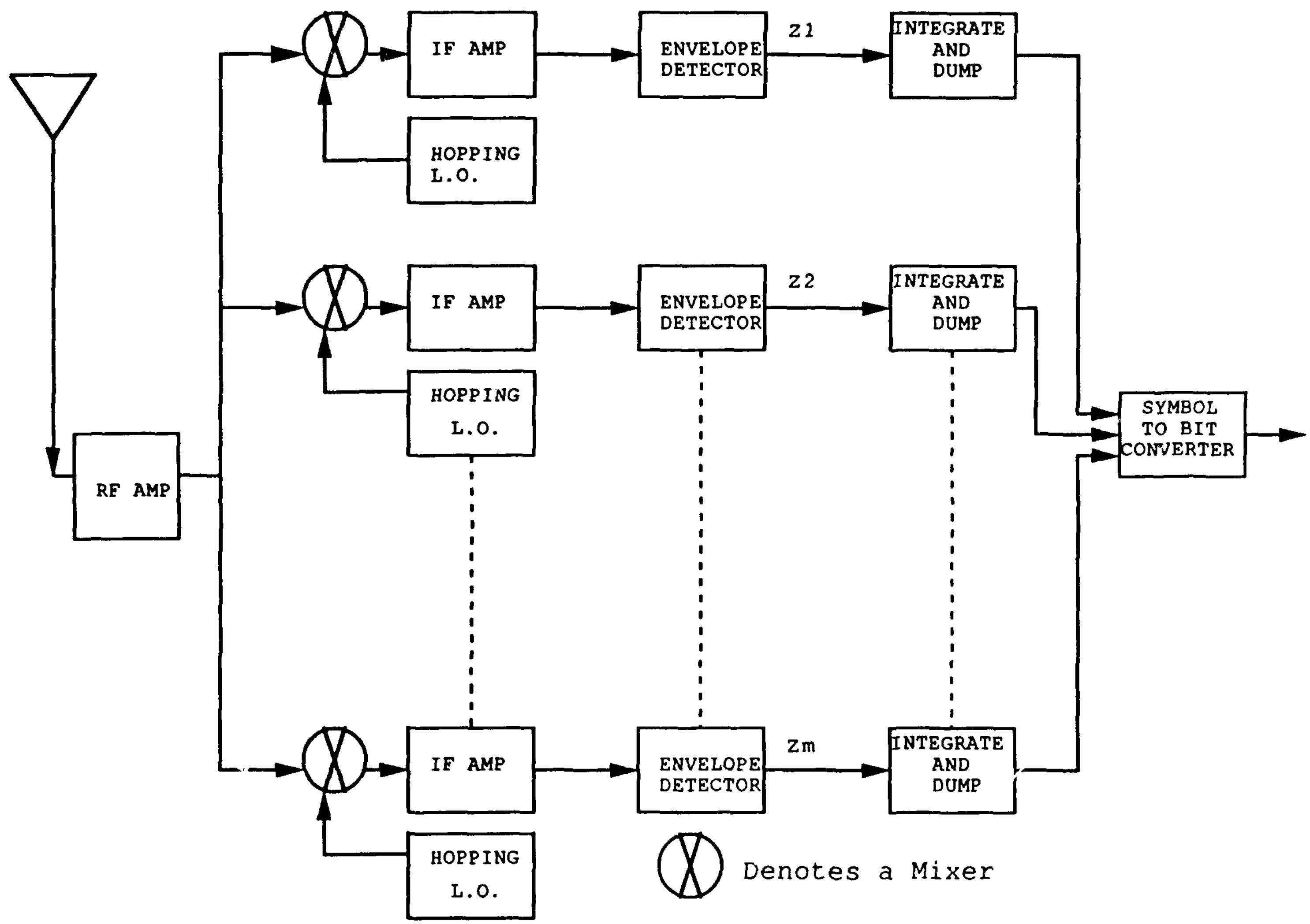

Figure 2. Block Diagram of a FHS Receiver

When the resulting output voltages of M subsystems are compared by a comparator, the decision will favor subsystem one or S_1. Then a symbol-to-bit converter produces the desired bit stream.

A. NOISE PERFORMANCE

The noise performance analysis of the system is done at the output of the envelope detector. It is assumed that the a priori probability of each symbol is equal. First, the probability of making a symbol error is found; then, with suitable conversions, the probability of bit error is obtained.

If it is assumed that symbol one is sent, then the probability that S_1 is detected correctly is the probability z_1 is greater than z_2 and is greater than z_3, etc. This can be expressed as $P_{sc}=Pr(z_2<z_1 \cap z_3<z_1 \cap \ldots\ldots z_M<z_1 | S_1)$ where $\cap$ is the and symbol and $Pr(A|B)$ is the conditional probability of A given B. Then the probability of making an error when detecting S_1 is $P_{se}=1-P_{sc}$.

$$P_{se}=1-\int_0^{\infty}\int_0^{z_1}\ldots\int_0^{z_1} p(z_1,z_2,z_3\ldots z_M | S_1)\,dz_1 dz_2\ldots dz_M \qquad (1)$$

where $p(A,B|C)$ is the joint probability of A and B given condition C. Since all subsystem voltages are statistically independent, the conditional probability function inside the integrals can be expressed as,

$$p(z_1,z_2,\ldots\ldots\ldots\ldots,z_M | S_1) = \prod_{m=1}^{M} P(z_m | S_1). \qquad (2)$$

If it is assumed that all subsystems except the one with the signal will have the same statistical behavior, then $\int_0^{z_1} p(z_m|S_1)\,dz_m$ are identical when m is not equal to one. Thus the probability of making an error when detecting S_1 can be written as,

$$P_{se}=\int_0^{\infty}[1-[\int_0^{z_1}p(z_m|S_1)\,dz_m]^{M-1}]\,p(z_1|S_1)\,dz_1 \ , \ m=2 \text{ or } 3 \ldots \text{or } M. \qquad (3)$$

Now, the probability density function of the voltage at the output of the envelope detector, given that the input is zero mean Gaussian, is the Rayleigh density function. So, the conditional density function inside the parenthesis can be expressed as,

$$p(z_m|S_1)=\frac{z_m}{\sigma_1^2}\exp(-\frac{z_m^2}{2\sigma_1^2}) \ , \ m=2 \text{ or } 3 \ldots \text{or } M. \qquad (4)$$

where $\sigma_1^2=(N_0/2)(2/T_s)(2)$ and $N_0/2$ is the two-sided power spectrum of the noise voltage at the input to the envelope detector. The integral of equation (4) is evaluated as

$$\int_0^{z_1}p(z_m|S_1)\,dz_m=\int_0^{z_1}\frac{z_m}{\sigma_1^2}\exp(-\frac{z_m}{2\sigma_1^2})\,dz_m=1-\exp(\frac{-z_1^2}{2\sigma_1^2}). \qquad (5)$$

Then, the expression inside the parenthesis in equation (3) can be written using the binomial expansion as

$$1-[1-\exp(-\frac{z_1}{2\sigma_1^2})]^{M-1}=\sum_{m=1}^{M-1}C_m^{M-1}(-1)^{m+1}\exp(-m\frac{z_1^2}{2\sigma_1^2}) \qquad (6)$$

where $C_m^{M-1}=\dfrac{(M-1)!}{m!\,(M-m-1)!}$.

Now, $p(z_1|S_1)$ is a Rician density function with the signal amplitude A. So,

$$P_{se}=\int_0^{\infty}\sum_{m=1}^{M-1} C_m^{M-1}(-1)^{m+1}\exp(-m\frac{z_1^2}{2\sigma_1^2})\ \frac{z_1}{\sigma_1^2}\exp-(\frac{z_1^2+A^2}{2\sigma_1^2})I_0(\frac{z_1A}{\sigma_1^2})dz_1 \quad (7)$$

Substituting this and equation (6) into equation (3) gives

$$P_{se}=\sum_{m=1}^{M-1} C_m^{M-1}\ (-1)^{m+1}\ \frac{\exp[\ -(\frac{m}{m+1})\ \frac{A^2}{2\sigma_1^2}\ ]}{m+1} \quad (8)$$

Since all symbols are equally likely, equation (8) is also the probability of error for all symbols.

To express P_{se} in terms of the ratio of input symbol energy to noise power density, define the symbol energy as $E_s=\frac{A^2T_s}{2}$. Then, the ratio of input symbol energy to noise power density becomes $E_s/N_0=A^2/\sigma_1^2$. When this expression is substituted in equation (8), the probability of symbol error becomes

$$P_{se}=\sum_{m=1}^{M-1}(-1)^{m+1}C_m^{M-1}\frac{\exp(-\frac{m}{m+1})\frac{E_s}{2N_0}}{(m+1)}\ . \quad (9)$$

Since a symbol consists of $k = \log_2 M$ bits, then mistaking one symbol for another can cause 1 or 2 or more up to k bits to be in error. In fact, the number of bits in error is just the Hamming distance of the two symbols. If all symbol errors are equally likely, $\frac{1}{M-1}\sum_{n=1}^{k}d_nC_n^{\ k}$ is the average number of bits in error for each symbol in error.

To convert the probability of symbol error to the probability P_{be} bit error, we use the following reasoning.

First, the transmission of, say, 1000 symbols is equivalent to the transmission of 1000 x k bits. Each symbol in error creates an average number $\frac{1}{M-1}\sum_{n=1}^{k} d_n C_n{}^k$ bits in error. So, P_{be} becomes $P_{be} = P_{se} \times (\frac{1}{M-1}\sum_{n=1}^{k} d_n C_n{}^k) \times \frac{1}{k}$. Using the equation (9) for P_{se} gives

$$P_{be}=(\frac{1}{(M-1)k})(\sum_{n=1}^{k} d_n C_n{}^k)\sum_{m=1}^{M-1}(-1)^{m+1}C_m^{M-1}\frac{\exp(-\frac{m}{m+1})\frac{E_b\log_2 M}{2N_0}}{(m+1)} \tag{10}$$

where we have let $E_s=E_b\log_2 M$. Figure 3 is plots of the probability of bit error versus the ratio of bit energy to noise power density for M = 2,4,8,16. For the two symbol case, the receiver was simulated and the probability density functions of the voltages at the outputs of the envelope detectors for signal and nonsignal subsystems were obtained from the simulation data. These probability density functions are the Rayleigh density function for the nonsignal subsystem and the Rician density function for the signal subsystem as shown in Figure 4. The probability of bit error for a particular value of the ratio of bit energy to noise power density is the total area under the tails of the probability density functions as calculated from equation (10).

The performance curves for the outputs of the envelope detector and the integrator were also obtained from the simulation data. As Figure 5 illustrates, the performance

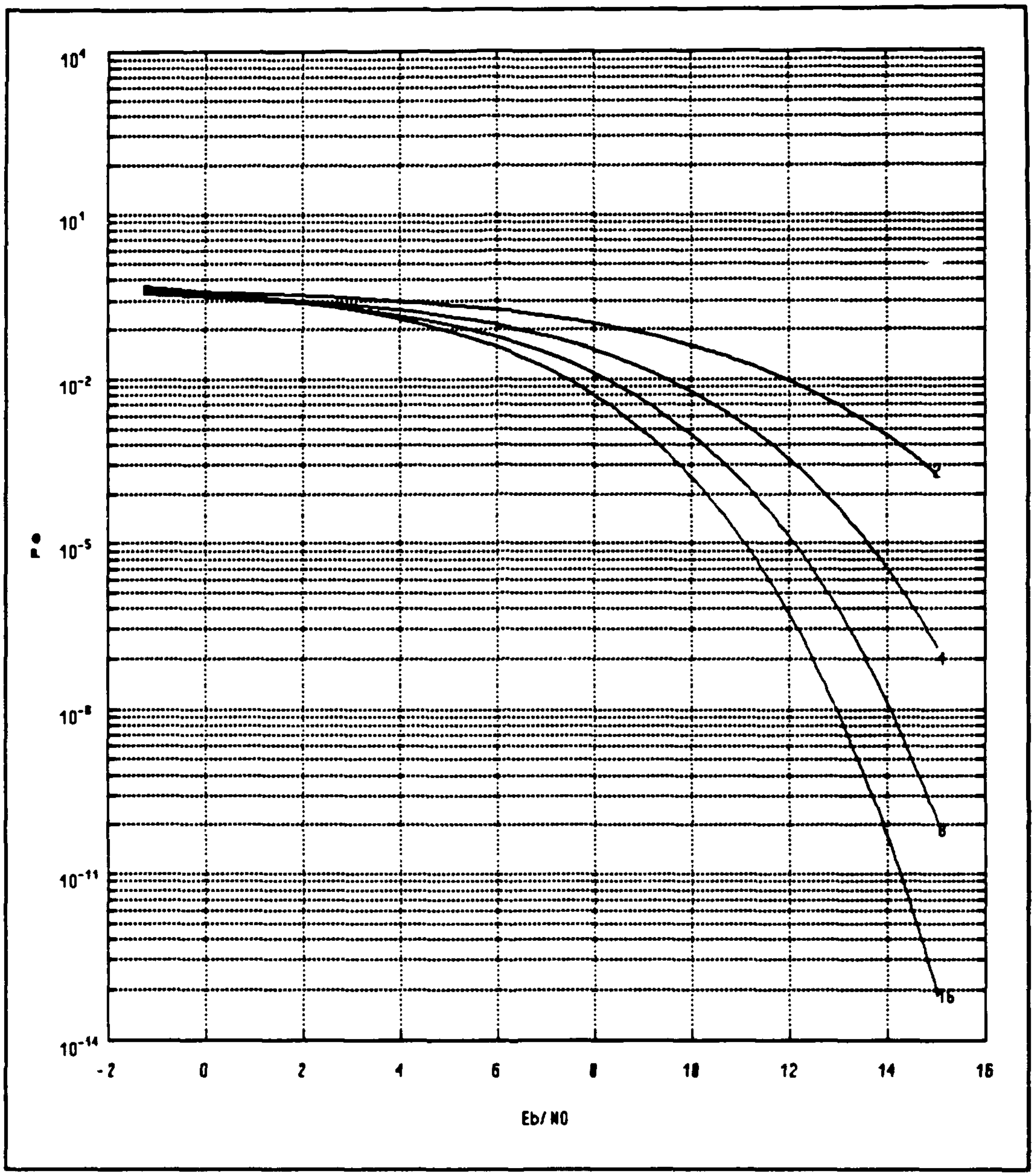

Figure 3. The Probability of Bit Error versus the Ratio of Bit Energy to Noise Power Density for M = 2,4,8,16.

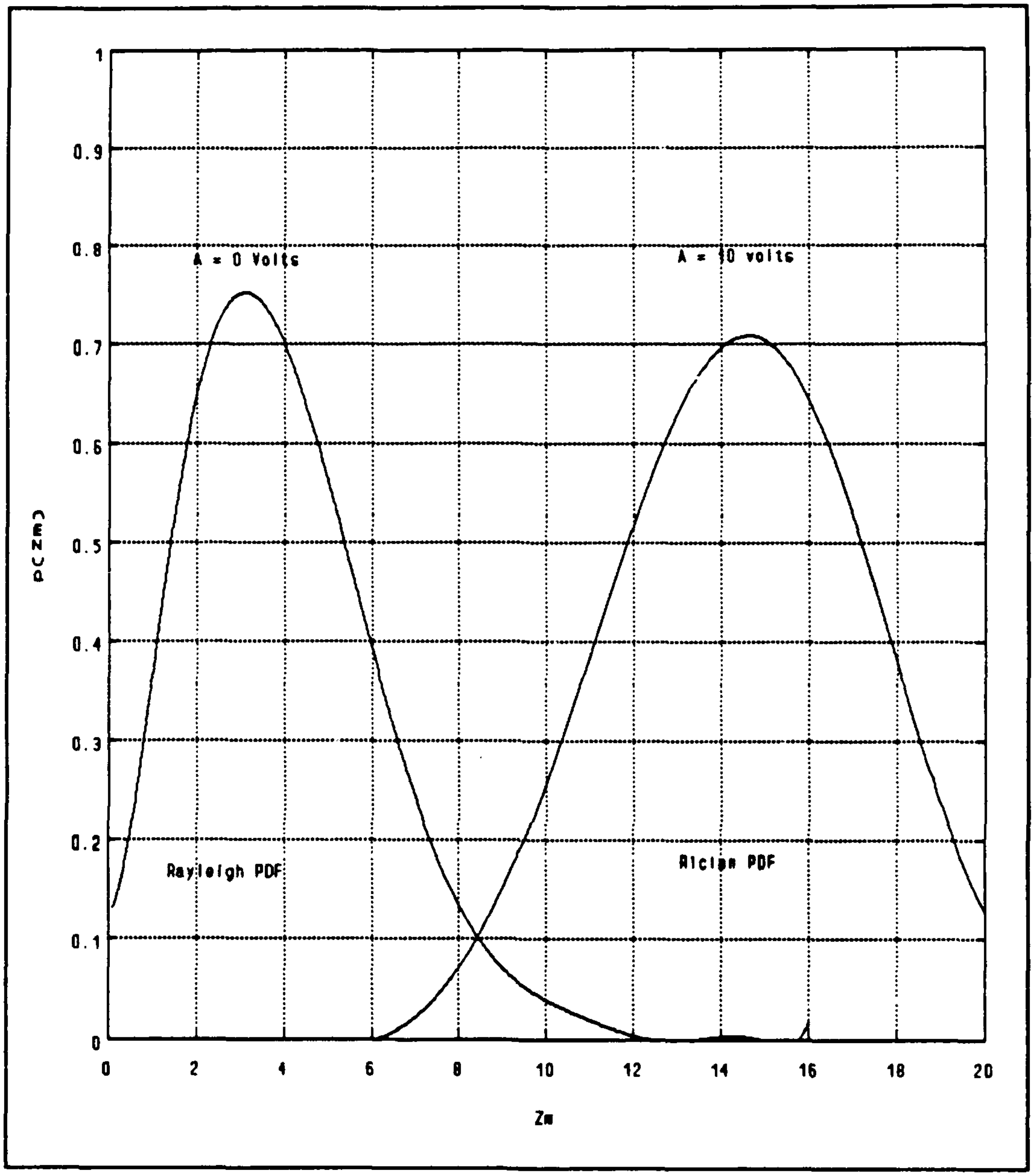

Figure 4. The Probability Density Functions of Voltage at the Output of the Envelope Detector of the Signal and Nonsignal Subsystems. M = 2.

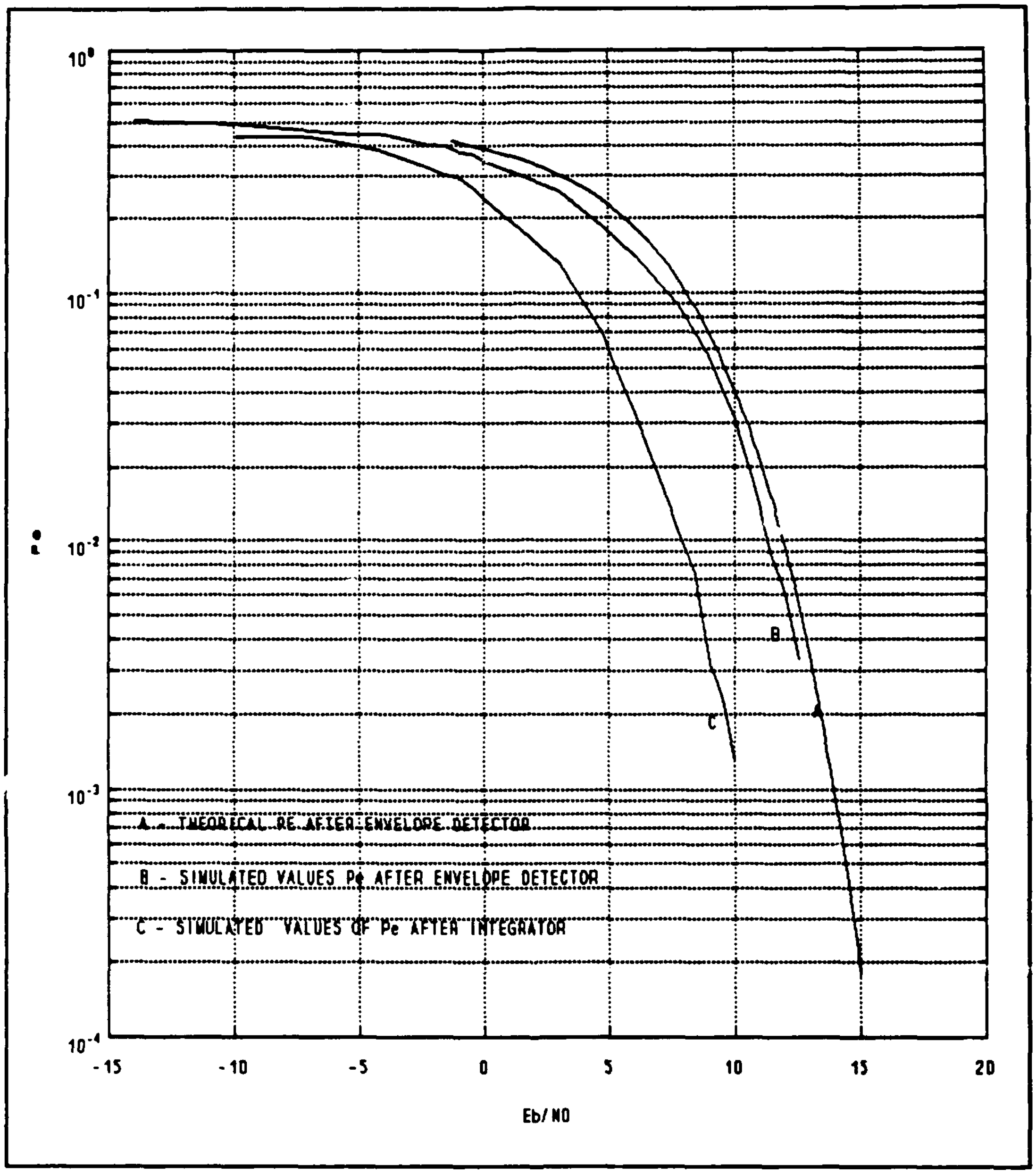

Figure 5. The Simulated Values of Probability of Bit Error versus Bit Energy to Noise Power Density Ratio.

improvement factor of the integrator is about 3 dB for high ratios of bit energy to noise power density, but it is less than 3 dB for low ratios of bit energy to noise power density. By extrapolation, this same performance improvement can be obtained for the 4,8 and 16 symbol cases.

B. THE EFFECT OF SYNCHRONIZATION ERROR

If the hopping L.O. of Figure 2 is not exactly time synchronized with the received FHS, then the output of the IF amplifier is not a sinusoid for the entire symbol duration (T_s seconds). So, a synchronization error of ϵ seconds per hop (early or late) causes the voltage amplitude at the output of the integrator drop from AT_s volts to BT_s volts as shown in Figure 6. That is, the voltage at the output of the envelope detector drops from A volts to $B=A(1-\frac{\epsilon N_h}{T_s})$ for $0<\epsilon<\frac{T_s}{N_h}$. Then, the energy of the symbol at the output of the integrator decreases from E_s to $\hat{E}_s=(1-\frac{\epsilon N_h}{T_s})^2 E_s$. This effectively moves the probability of bit error versus the ratio of bit energy to noise power density curves of Figure 3 to the right depending on the value of the ratio $e=\frac{\epsilon N_h}{T_s}$.

Figures 7,8,9,10 illustrate the probability of bit error curves for a possible range of e when M = 2,4,8,16 respectively. All of these curves should be moved to the left in accordance with Figure 5 to include the improvement effect of the integrator. For the two symbol case, the

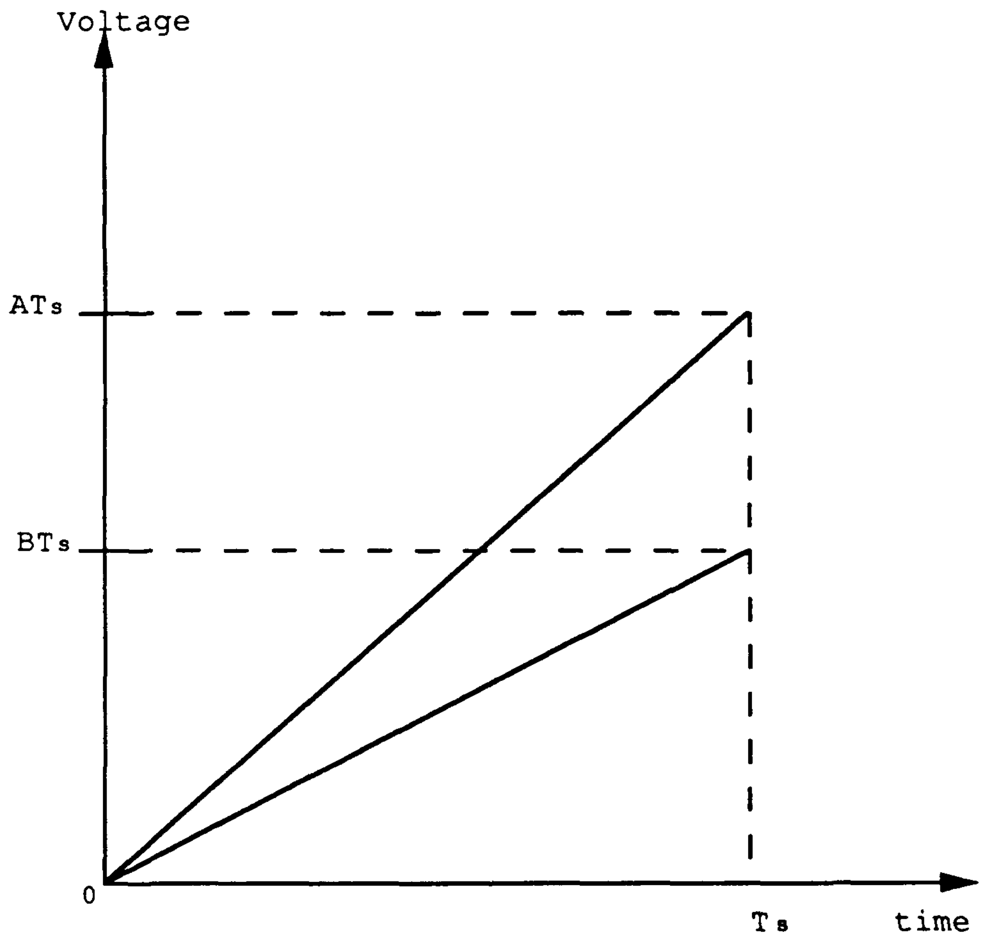

Figure 6. The Voltage Drop at the Output of the Integrator Due to Synchronization Error in the Signal Subsystem.

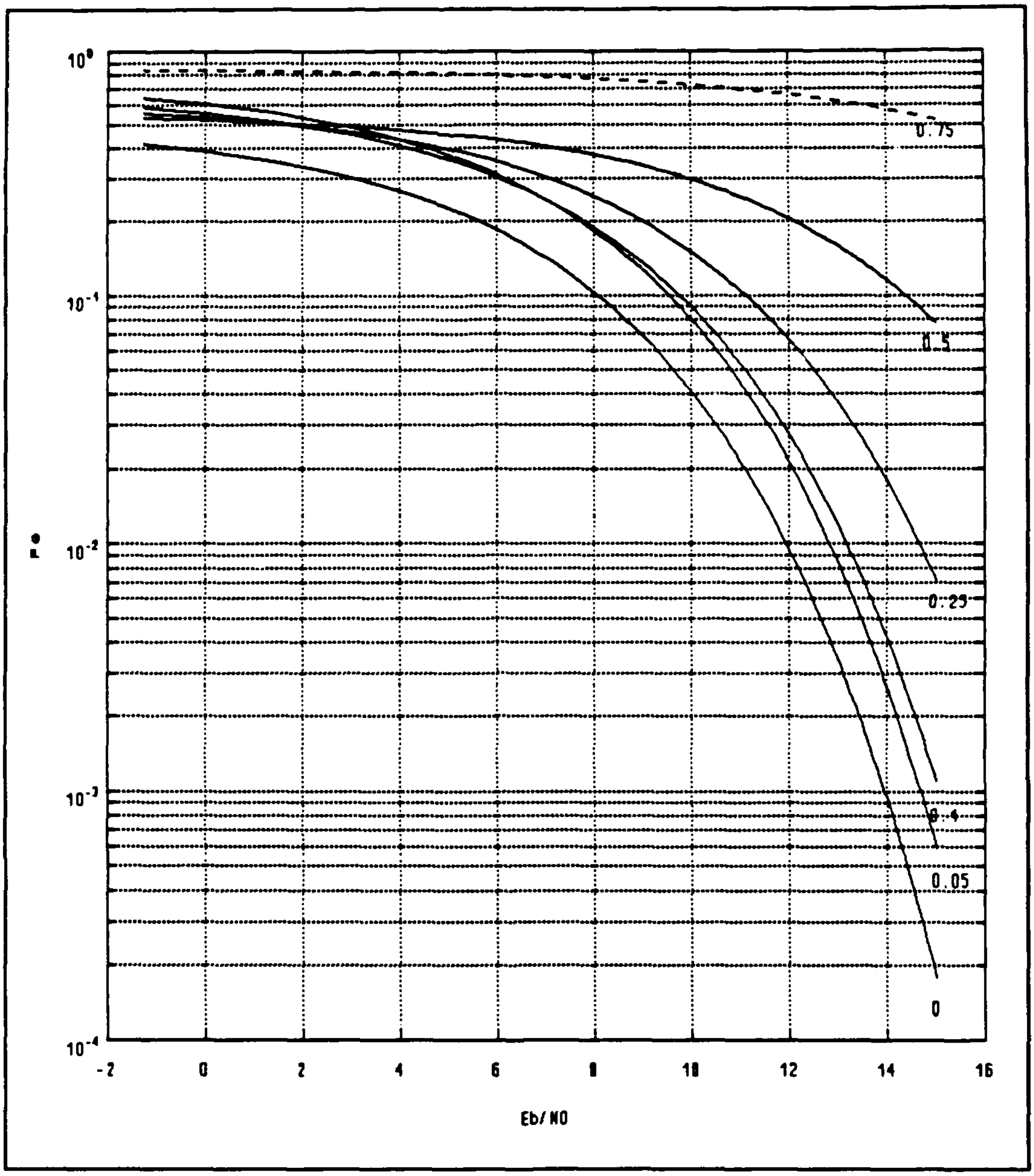

Figure 7. The Effect of Synchronization Error on System Performance for M = 2 and for Various Values of e.

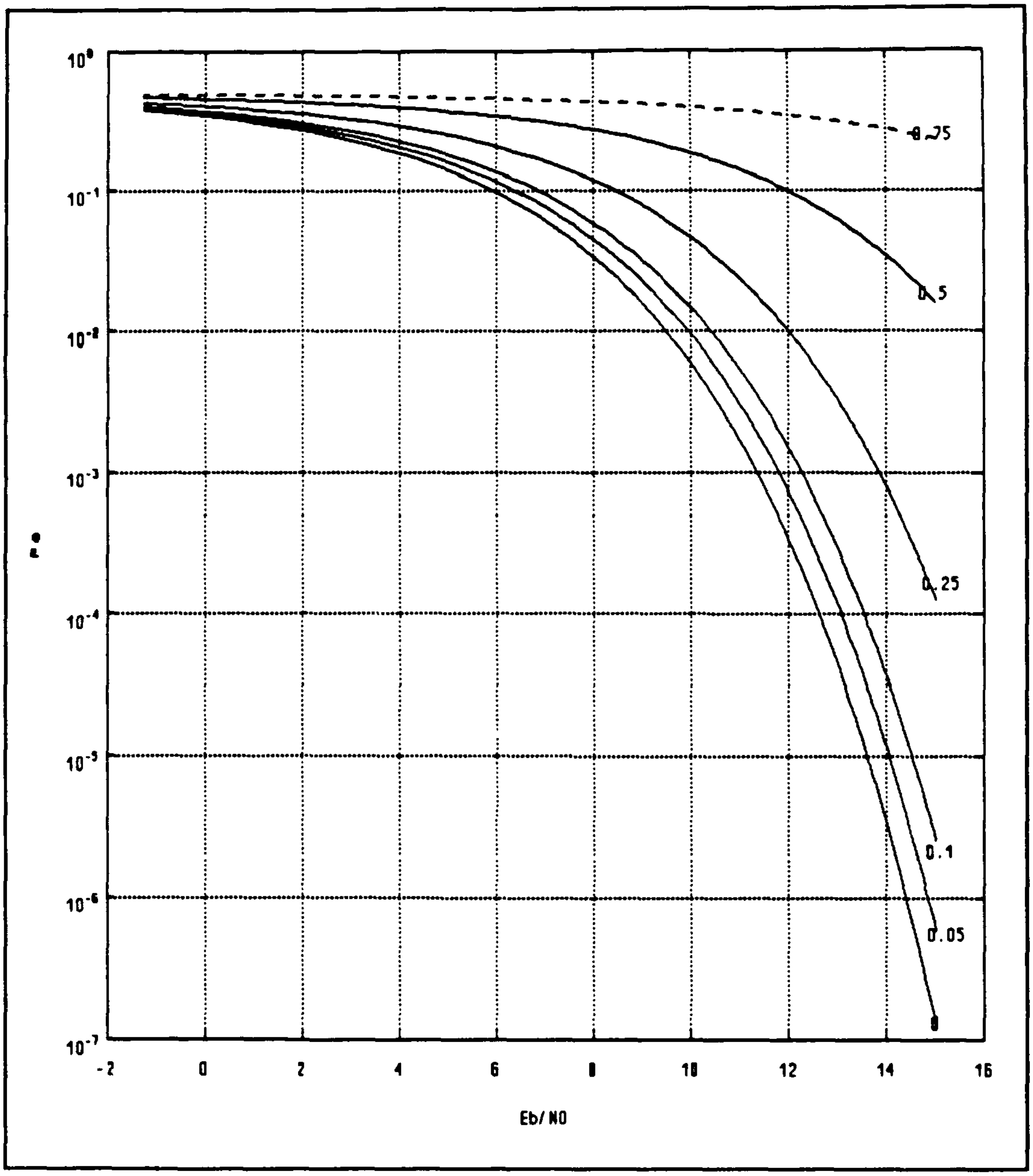

Figure 8. The Effect of Sychronization Error on System Performance for M = 4 and for Various Values of e.

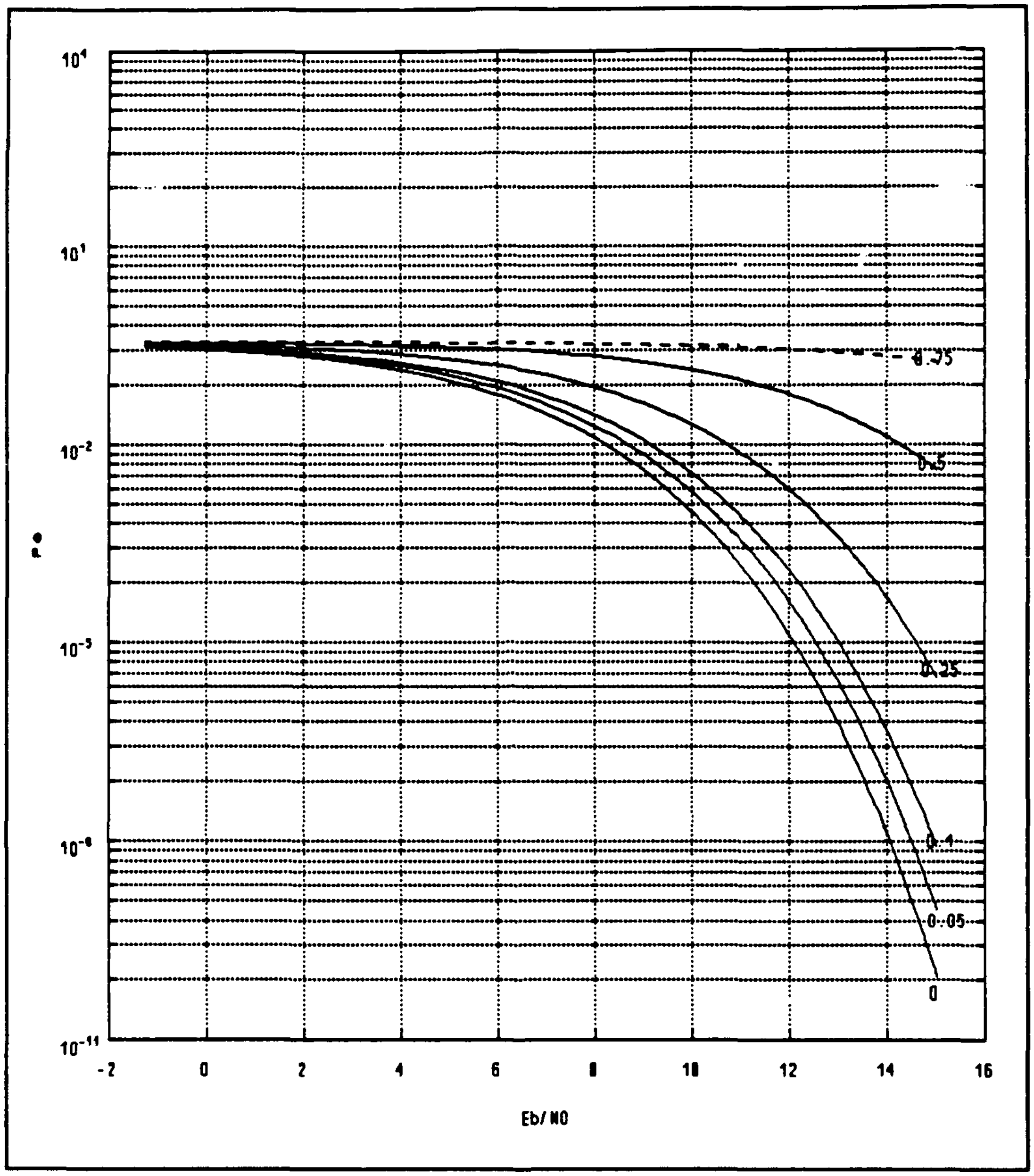

Figure 9. The Effect of Synchronization Error on System Performance for M = 8 and for Various Values of e.

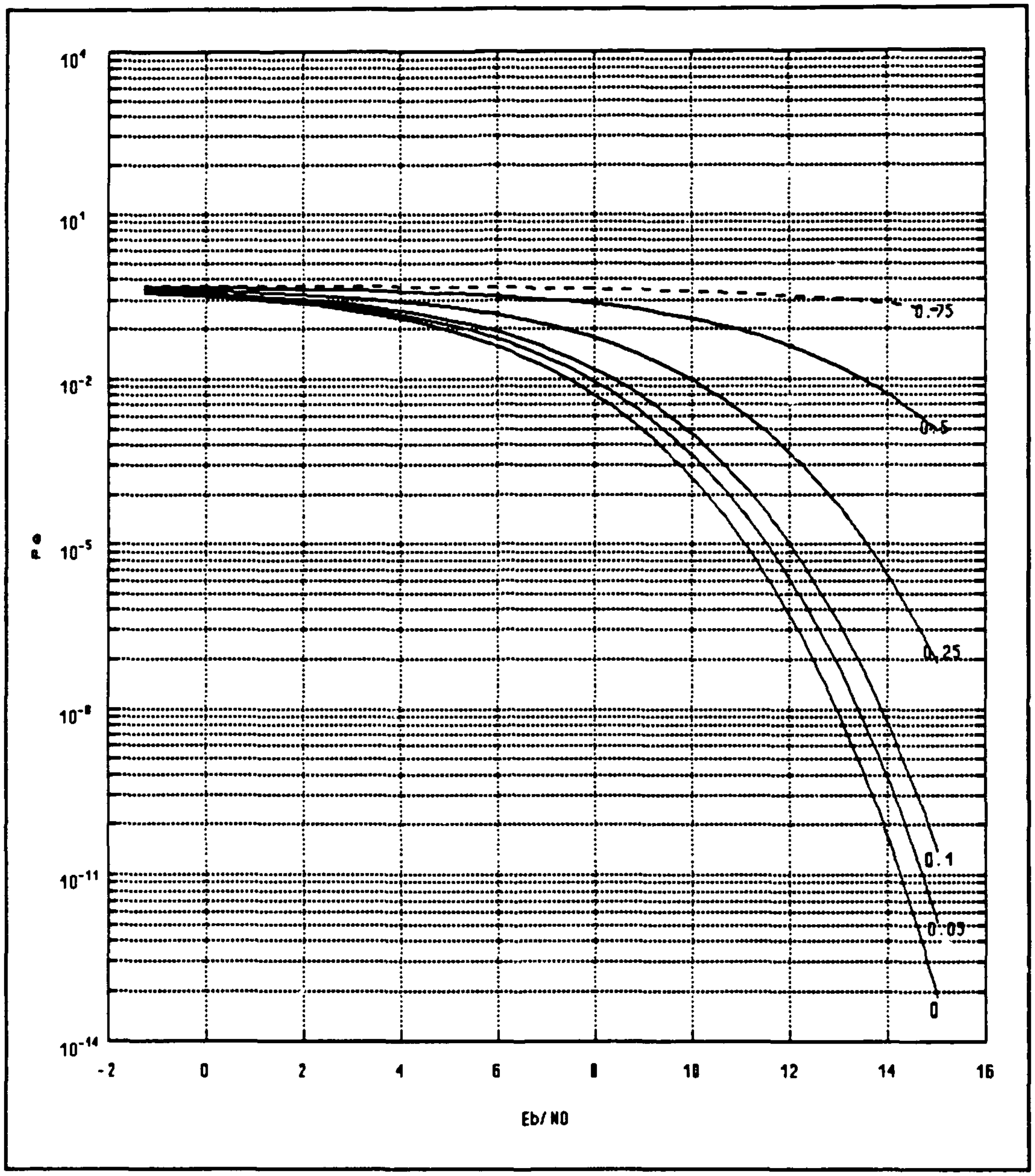

Figure 10. The Effect of Synchronization Error on System Performance for M = 16 and for Various Values of e.

receiver with a synchronization error in the signal subsystem was simulated.

The probability density functions of voltage at the output of the envelope detector for the signal and the nonsignal subsystem were obtained from the simulation data and are shown in Figure 11. The Rician density function, which is the density function in the signal subsystem, moves to the left due to the synchronization error. This increases the area under the tails and so also the probability of error.

C. THE EFFECT OF INTERFERENCE

Interference can occur because of multipath effects or jamming. In the case of interference, the nonsignal subsystems have an interference signal in addition to the noise input to the system. So, the probability density functions of the voltages at the outputs of the envelope detectors become the Rician density functions as shown in Figure 12 for the two symbol case. The probability of error is the area under the tails as in the previous cases. Now, assume that the effect of the interference is to increase the mean of the Rayleigh distribution at the output of the envelope detector of the nonsignal subsystems. The Rician density function of the voltage at the output of the envelope detector in the nonsignal subsystem is then approximated by the Rayleigh density function such that the area under the tails of the probability density functions or

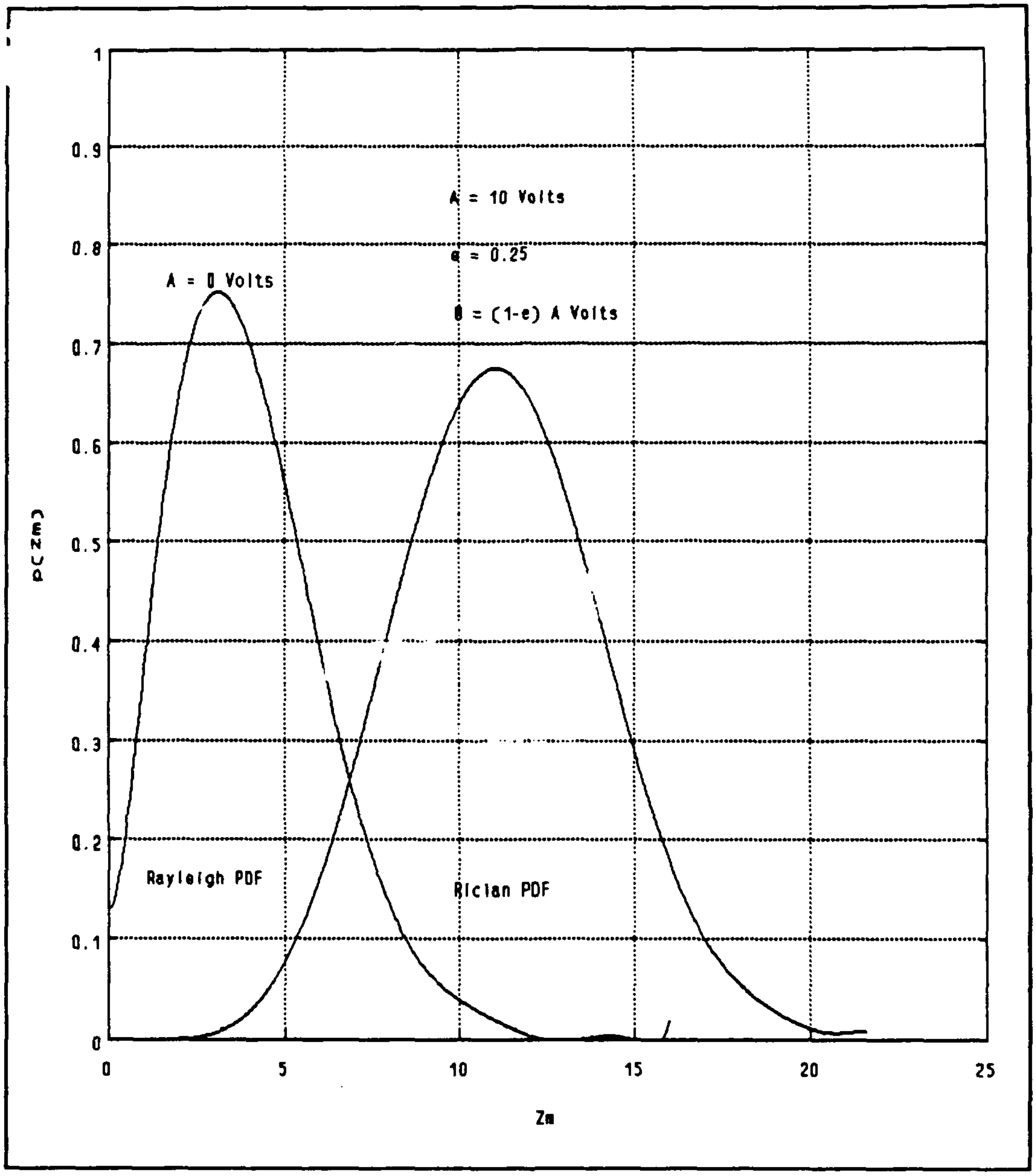

Figure 11. The Probability Density Functions of Voltage at the Output of the Envelope Detector of the Signal and Nonsignal Subsystems in Case of Synchronization Error. M = 2.

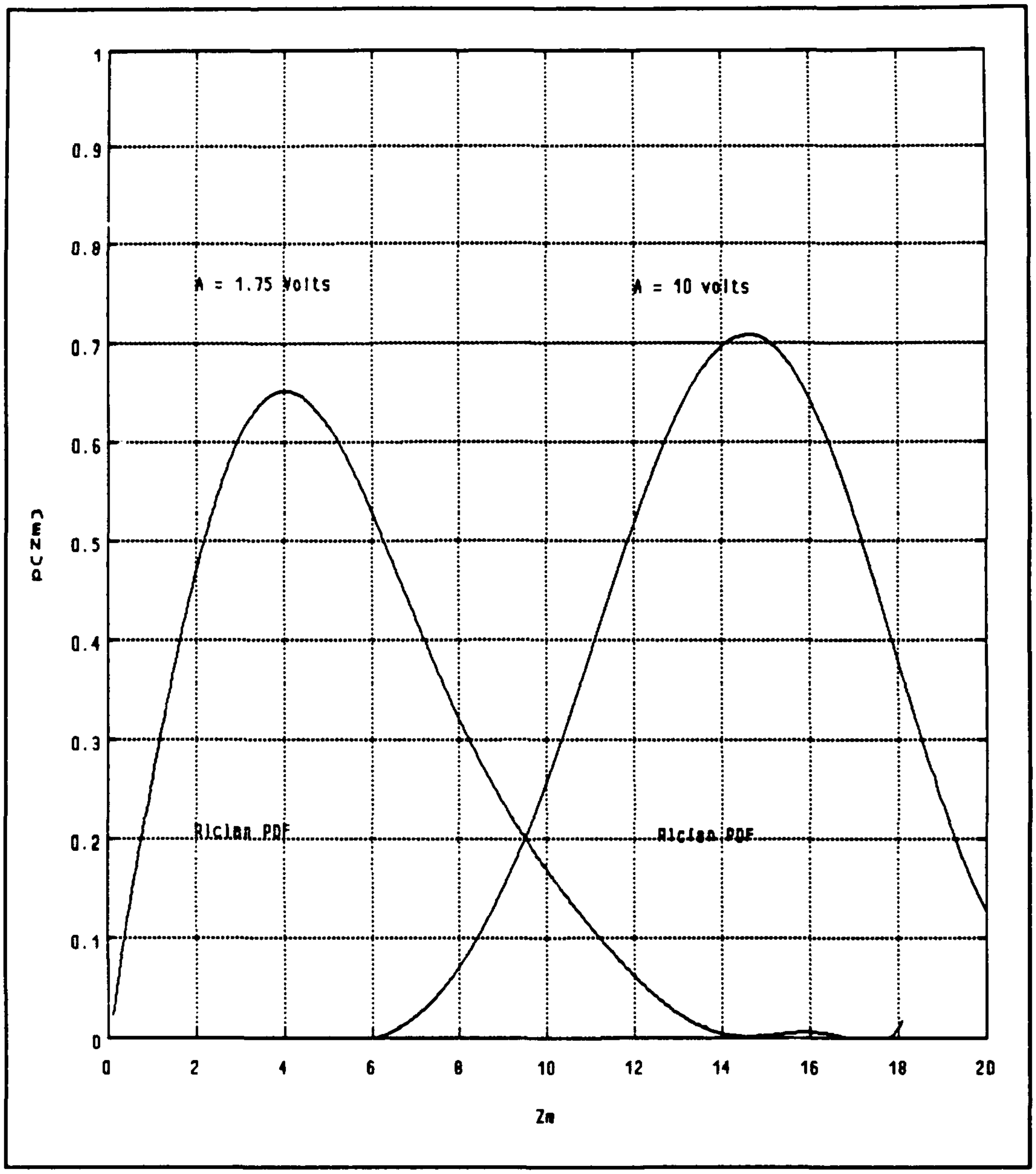

Figure 12. The Probability Density Functions of Voltage at the Output of the Envelope Detector of the Signal and Nonsignal Subsystems in Case of Interference. M = 2.

the probability of error does not change as shown in Figure 13. With these approximations, the analysis can be done using the equations developed in section A of this chapter. Two cases of interference are considered: interference in all nonsignal subsystems and interference in only one nonsignal subsystem.

1. Interference in all Nonsignal Subsystems

When there is no interference, the probability density function at the output of the envelope detector in all nonsignal subsystems is the Rayleigh density function with a mean of $\overline{z_m}=\sqrt{\pi/2}\,\sigma_1$ where σ_1^2 is the variance of the input Gaussian noise. In the case of interference, the mean of the Rayleigh distribution at the output of the envelope detector of the interfered subsystems is defined as $\overline{y}=\overline{z_m}+\overline{\mu}$ where $\overline{\mu}$ is the additional dc voltage at the output of the envelope detector created by the interference. Since the mean of the Rayleigh distribution is related to its standard deviation of the input noise by $\overline{y}=\sqrt{\pi/2}\,\sigma_2$, then the same performance analysis approach of section A can be used by taking different variances for the signal subsystem (σ_1^2) and for the nonsignal subsystems (σ_2^2) when interference occurs. The equation (4) can now be rewritten as,

$$P(z_m|S_1)=\frac{z_m}{\sigma_2^2}\exp\left(-\frac{z_m^2}{2\sigma_2^2}\right),\ m=2,3,4,\ldots,M \tag{11}$$

where $\sigma_2=\sqrt{2/\pi}\,\overline{y}=\sigma_1+\sigma_I$ and since $\overline{y}=\overline{z_m}+\overline{\mu}$, $\overline{z_m}=\sqrt{\pi/2}\,\sigma_1$ $\overline{\mu}=\sqrt{\pi/2}\,\sigma_I$.

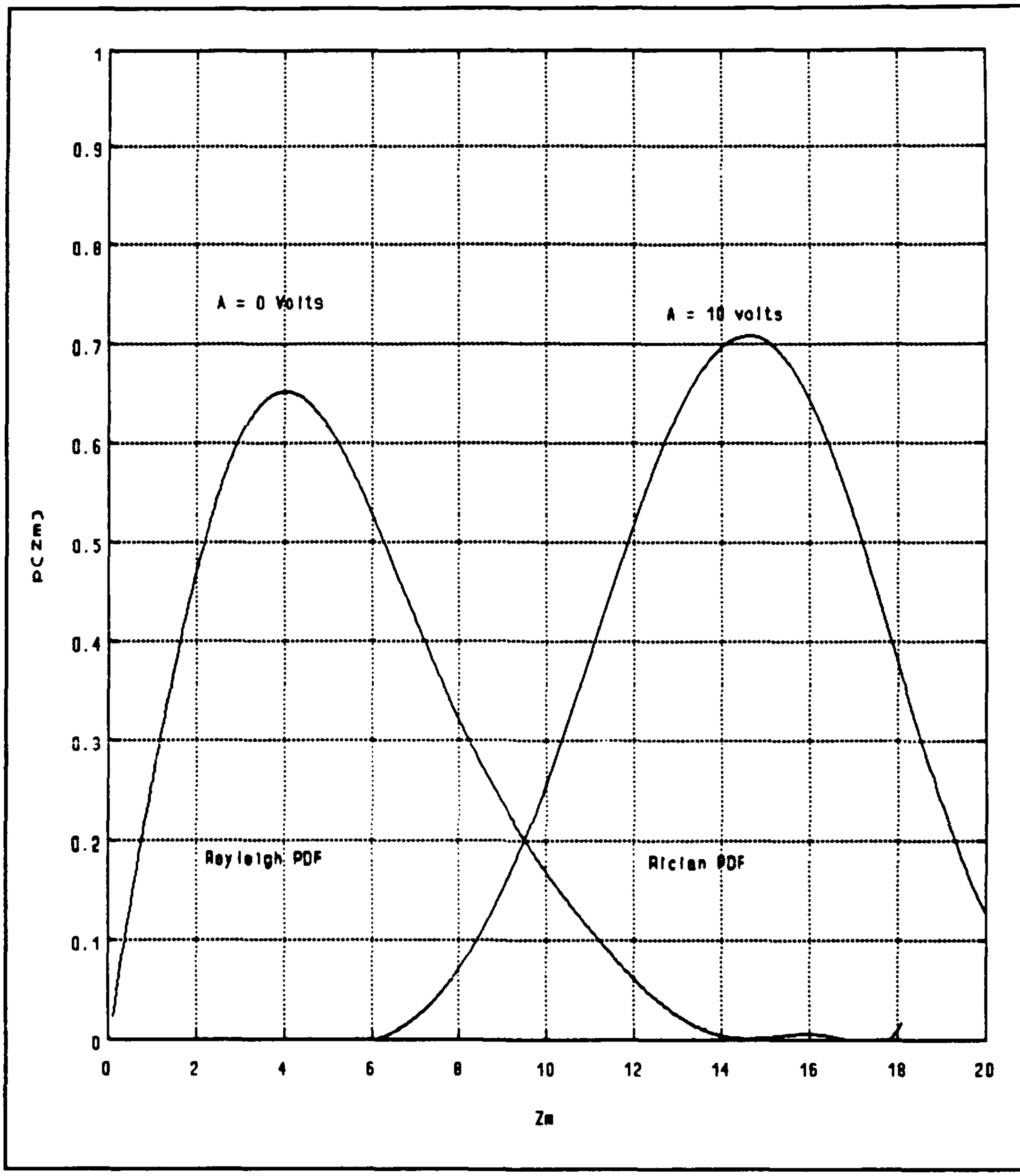

Figure 13. The Probability Density Functions of Voltage at the Output of the Envelope Detector in Case of Interference When the Rician Density Function is approximated by the Rayleigh Density. M = 2.

If the integral of the equation (11) is evaluated,

$$\int_0^{z_1} P(z_m|S_1)\,dz_m=\int_0^{z_1}\frac{z_m}{\sigma_2^2}\exp(-\frac{z_m}{2\sigma_2^2})\,dz_m=1-\exp(\frac{-z_1^2}{2\sigma_2^2}) \qquad (12)$$

then the equation (7) can be rewritten as

$$P_{se}=\int_0^{\infty}\sum_{m=1}^{M-1} C_m^{M-1}(-1)^{m+1}\exp(-m\frac{z_1^2}{2\sigma_2^2})\,\frac{z_1}{\sigma_1^2}\exp-(\frac{z_1^2+A^2}{2\sigma_1^2})\,I_0(\frac{z_1 A}{\sigma_1^2})\,dz_1. \qquad (13)$$

When this integral is rearranged and evaluated, the probability of symbol error becomes

$$P_{se}=\sum_{m=1}^{M-1} C_m^{M-1}\ (-1)^{m+1}\ [\frac{\sigma_2^2/\sigma_1^2}{m+\sigma_2^2/\sigma_1^2}]\ \exp[\frac{-(m\ A^2/2\sigma_1^2)}{(m+\sigma_2^2/\sigma_1^2)}]. \qquad (14)$$

Substituting $E_s/N_0 = A^2/\sigma_1^2$, $E_s = E_b\log_2 M$ and defining

$z = \sigma_2^2/\sigma_1^2$ and including the factor $L=\frac{1}{(M-1)\,k}\sum_{n=1}^{k} d_n C_n^{\,k}$, the probability of bit error becomes

$$P_{be}=(L)\sum_{m=1}^{M-1} C_m^{M-1}\ (-1)^{m+1}\ [\frac{z}{m+z}]\ \exp[\frac{-m(E_b\log_2 M)}{(m+z)\,2N_0}]. \qquad (15)$$

To obtain the probability of bit error as a function of the ratio of bit energy to noise power density and signal to interference ratio, (SIR), the following relationships are defined: $SIR=\frac{A^2}{\mu^2}$, $\sigma_2=\sigma_1+\sigma_I$, $\sigma_1=\frac{A}{\sqrt{E_s/N_0}}$. Using these relationships, a new variable r is defined as $(E_s/N_0)/SIR$. Then z is found to be related to r by the following quadratic equation: $z=1+2\sqrt{r}+r$.

Figure 14 is curve of z versus r for the range of interest of r. Figures 15,16,17,18 show the family of probability of bit error versus the ratio of bit energy to noise power density curves for different values of r.

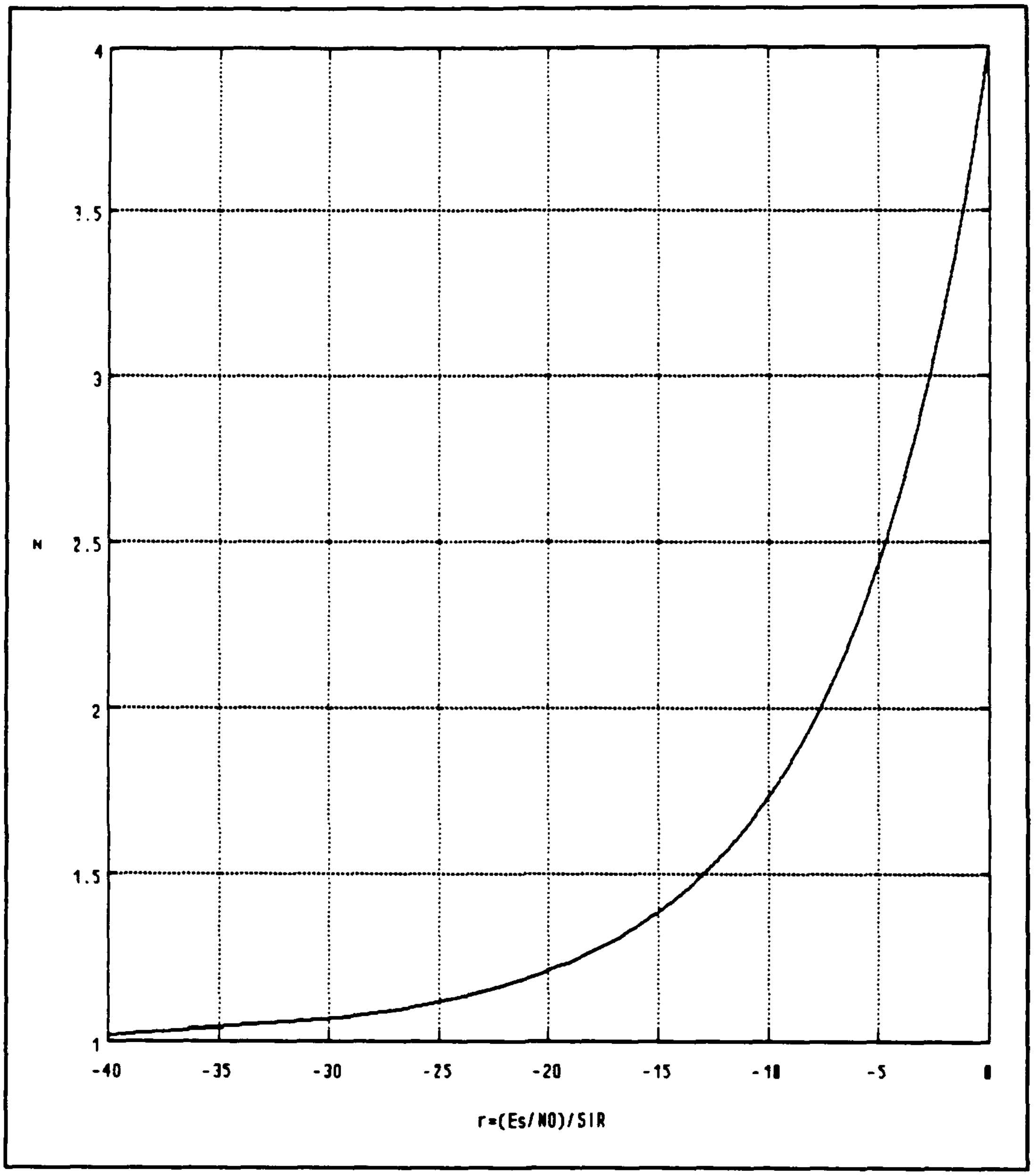

Figure 14. $z = \sigma_2^2/\sigma_1^2$ versus $r = (E_s/N_0)/SIR$

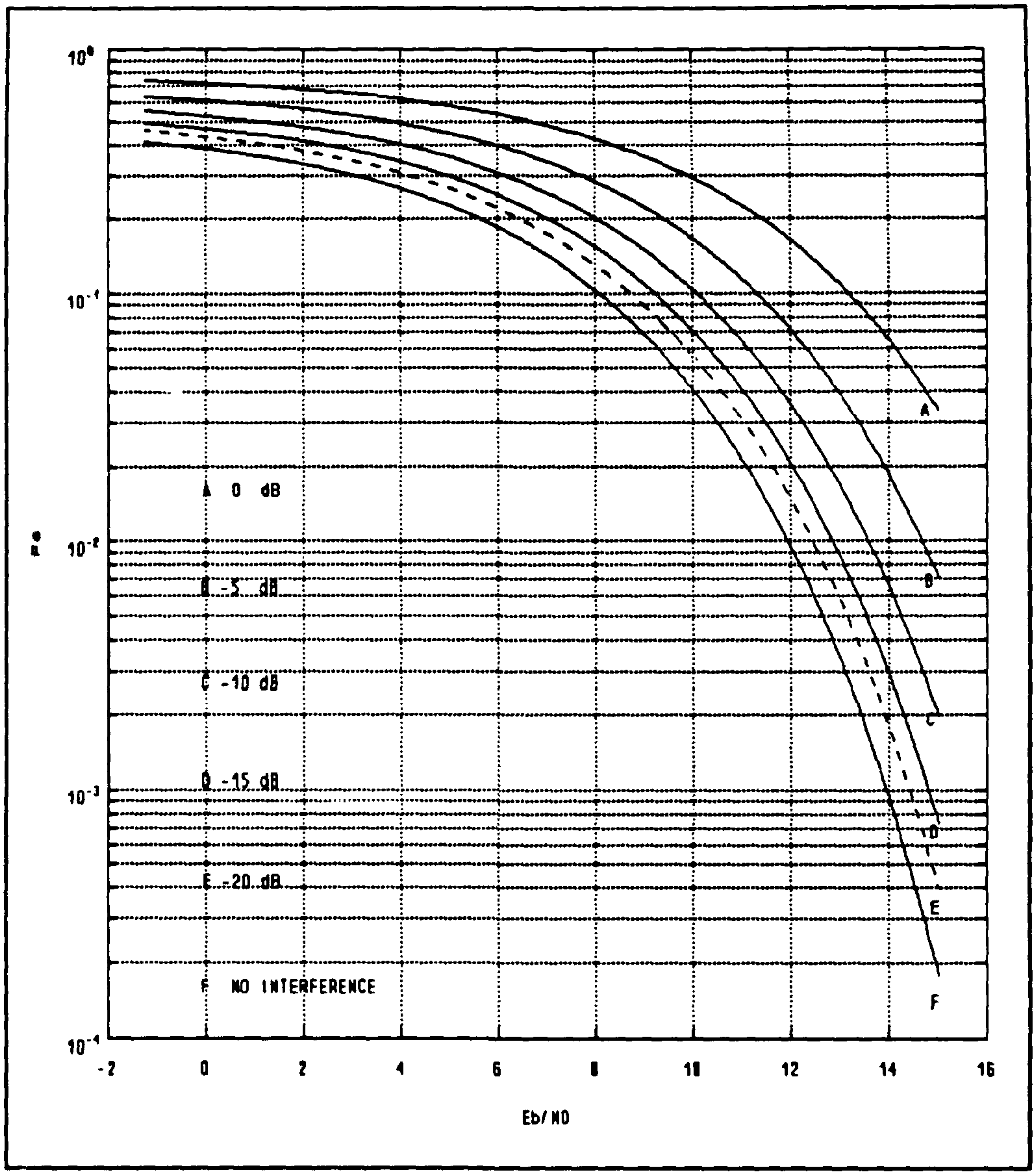

Figure 15. The Effect of Interference on System Performance when All Nonsignal Subsystems have Interference for M = 2. $(E_s/N_0)/SIR$ is Marked on each Curve.

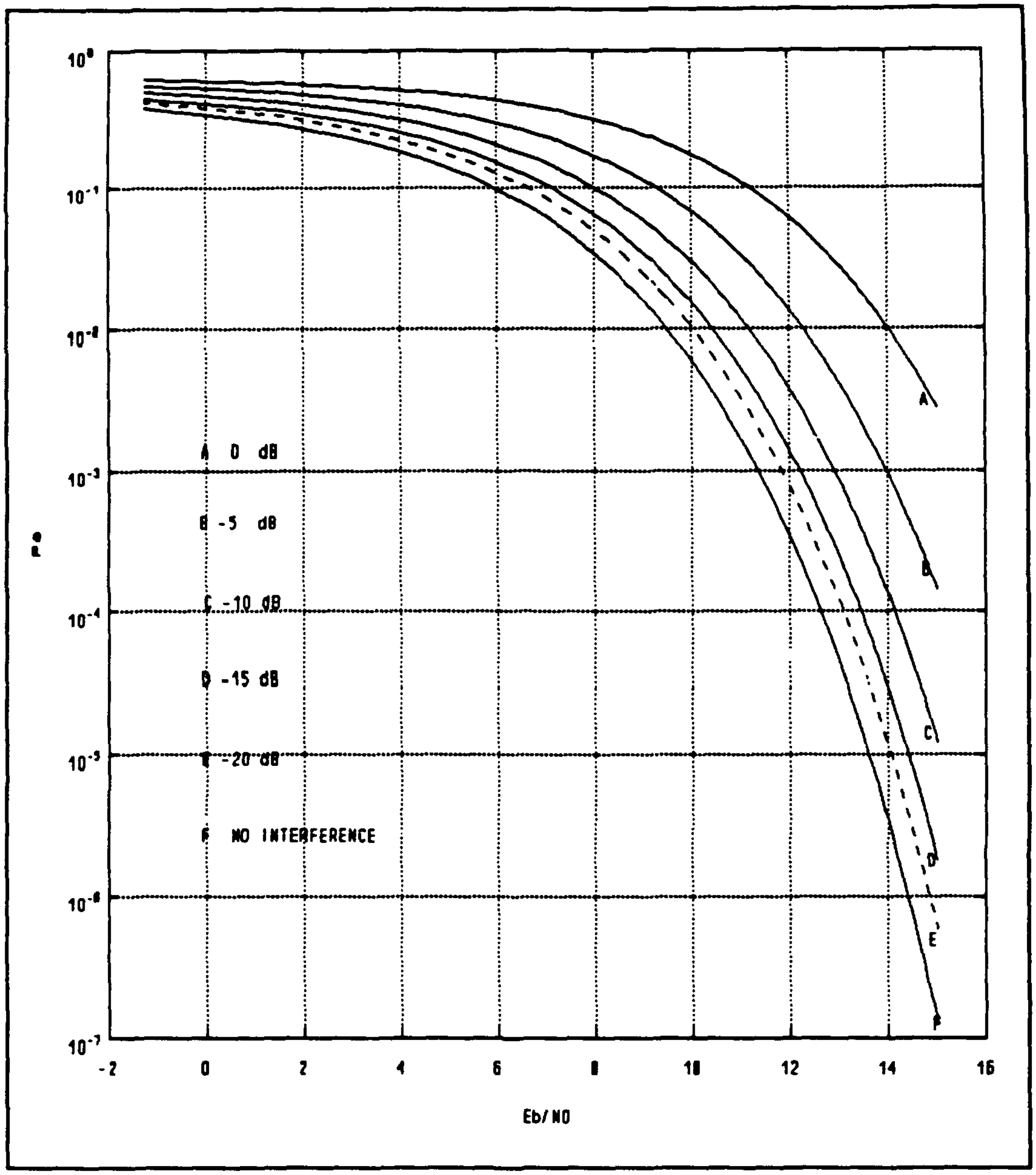

Figure 16. The Effect of Interference on System Performance when All Nonsignal Subsystems have Interference for M = 4. (E_s/N_0)/SIR is Marked on each Curve.

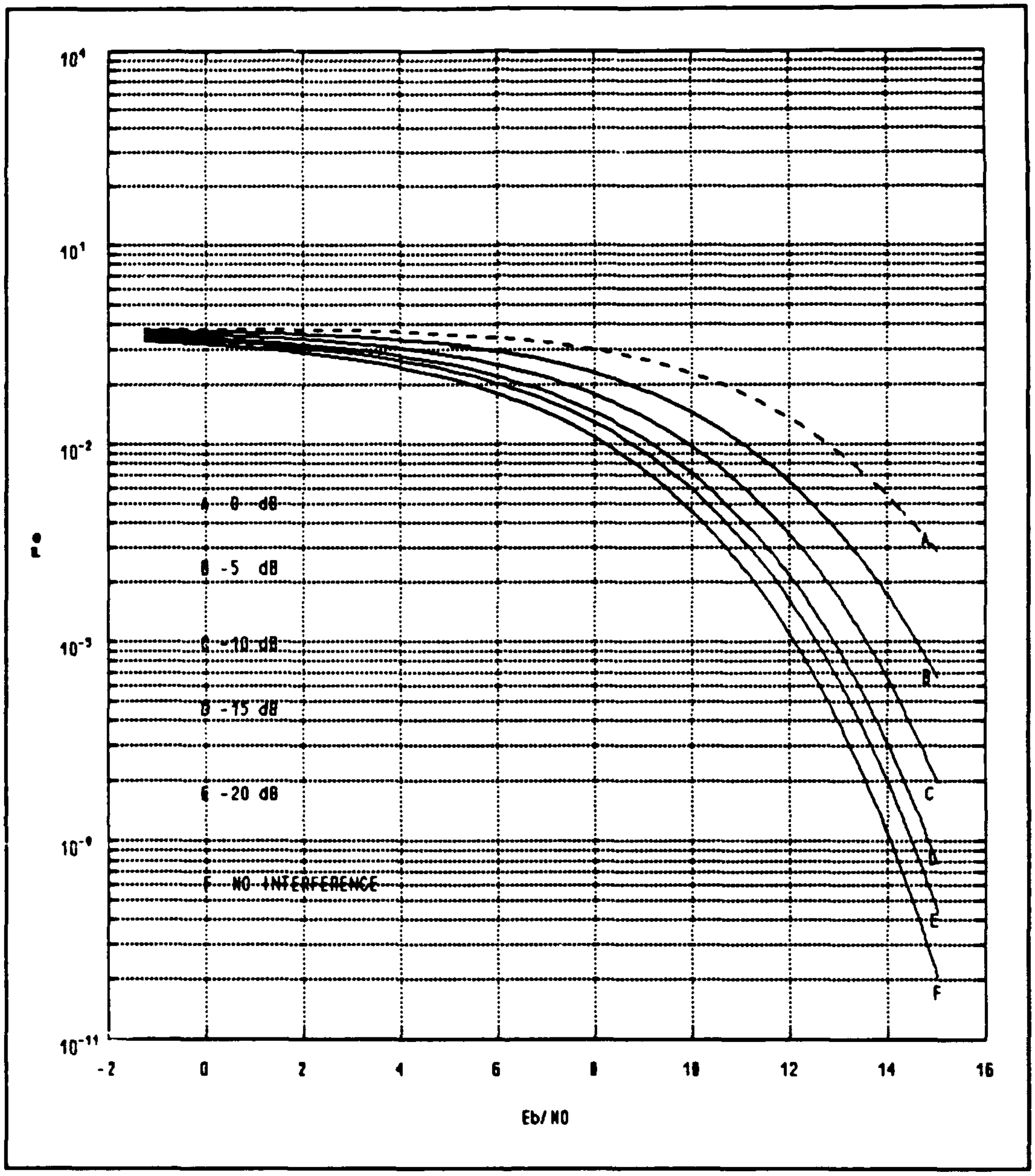

Figure 17. The Effect of Interference on System Performance when All Nonsignal Subsystems have Interference for M = 8. (E_s/N_0)/SIR is Marked on each Curve.

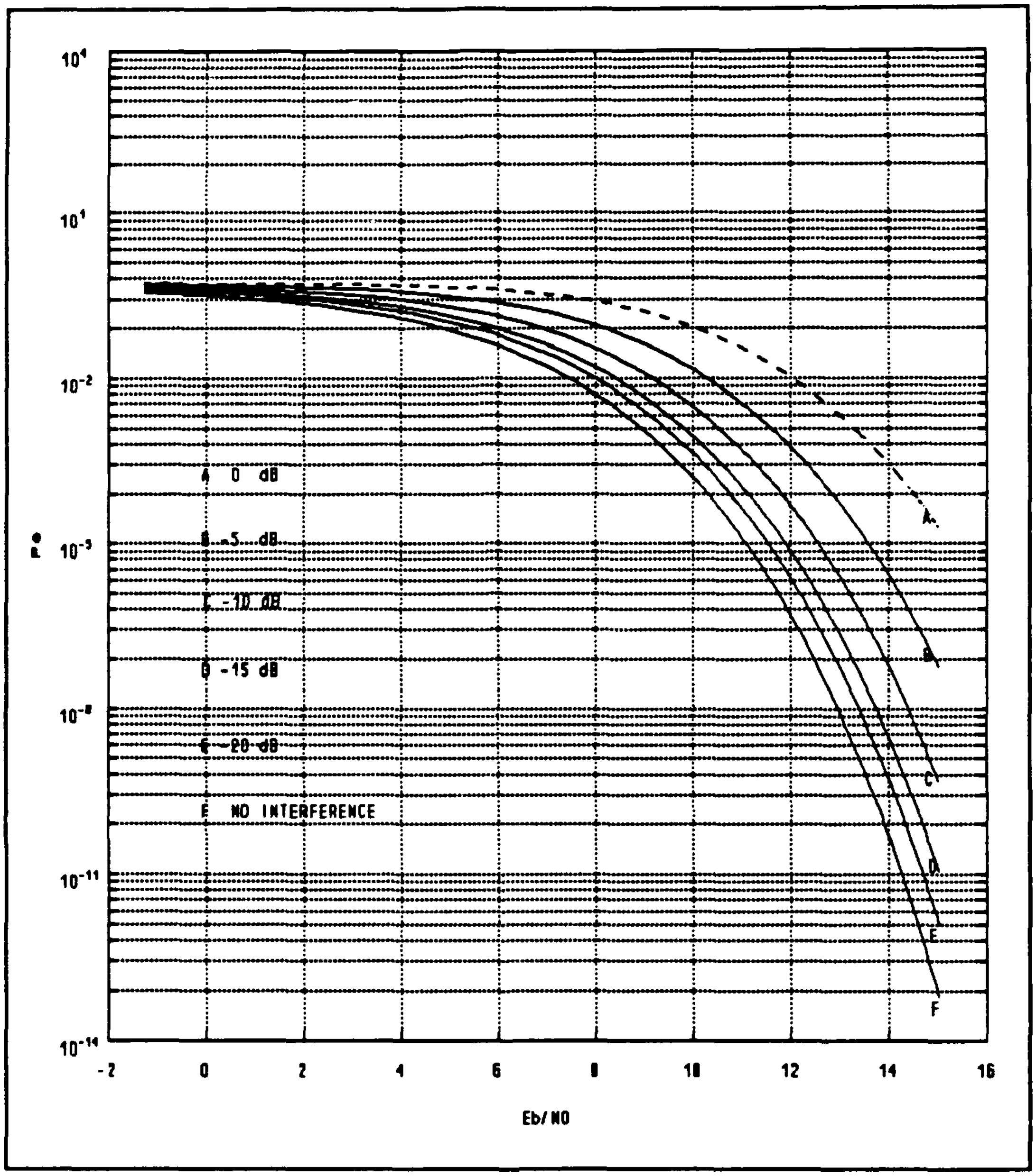

Figure 18. The Effect of Interference on System Performance when All Nonsignal Subsystems have Interference for M = 16. $(E_s/N_0)/SIR$ is Marked on each Curve.

All of these curves should be moved to the left in accordance with Figure 5 to include the performance improvement effect of the integrator.

Once z is determined from Figure 14, the probability of bit error versus the ratio of bit energy to noise power density curve can be obtained by equation (15). In section D, the use of Figure 14 for determining the probability of bit error will be clarified with an example.

2. Interference in One Nonsignal Subsystem

When only one nonsignal subsystem has interference, the equation (3) of section A, assuming subsystem + ‹ had interference, can be modified as

$$P_{se}=1-\int_0^{\infty} [\int_0^{} P(z_m|S_1)\,dz_m]^{M-2} [\int_0^{z_1} P(z_2|S_1)\,dz_2]\ P(z_1|S_1)\,dz_1 \qquad (16)$$

where m = 3 or 4...or M. Then equation (8) of section A can be rewritten as,

$$P_{se}=1-[\int_0^{\infty} \frac{z_1}{\sigma_1^2}\exp-(\frac{z_1^2+A^2}{2\sigma_1^2})I_0(\frac{z_1 A}{\sigma_1^2}dz_1]\,(f_1(m,z_1)-f_2(m,z_1)) \qquad (17)$$

where

$$f_1(m,z_1)=\sum_{m=0}^{M-2} C_m^{M-2}\ (-1)^m \exp-(\frac{mz_1^2}{2\sigma_1^2}) \qquad \text{and}$$

$$f_2(m,z_1)=\sum_{m=0}^{M-2} C_m^{M-2}\ (-1)^m \exp[-(\frac{m}{\sigma_1^2}+\frac{1}{\sigma_2^2})\frac{z_1^2}{2}]\ .$$

The probability of symbol error, after evaluating the integral and carrying out the multiplication in equation (17), has the form

$$P_{se}=1-(f_1(m)-f_2(m)) \qquad (18)$$

where

$$g_1(m)=\sum_{m=0}^{M-2} C_m^{M-2}\,(-1)^m\,\frac{\exp[\,-(\frac{m}{m+1})\,\frac{A^2}{2\sigma_1{}^2}\,]}{m+1} \qquad \text{and}$$

$$g_2(m)=\sum_{m=0}^{M-2} C_m^{M-2}(-1)^m\,[\frac{1}{(m+1)+\sigma_1^2/\sigma_2^2}]\,\exp[-\frac{A^2}{2\sigma_1^2}(\frac{m+\sigma_1^2/\sigma_2^2}{(m+1)+\sigma_1^2/\sigma_2^2})]\;.$$

After carrying out the substitutions for the symbol energy the probability of bit error as a function of the ratio of bit energy to noise power density and $z=\sigma_2^2/\sigma_1^2$ is

$$P_{be}=(\frac{1}{(M-1)\,k})\,(\sum_{n=1}^{k} d_n C_n^k)\,[1-(f_1(m)-f_2(m))] \qquad (19)$$

where

$$g_1(m)=\sum_{m=0}^{M-2} C_m^{M-2}\,(-1)^m\,\frac{\exp[\,-(\frac{m}{m+1})\,\frac{E_b\log_2 M}{2N_0}\,]}{m+1} \qquad \text{and}$$

$$g_2(m)=\sum_{m=0}^{M-2} C_m^{M-2}(-1)^m\,[\frac{1}{(m+1)+1/z}]\,\exp[-\frac{E_b\log_2 M}{2N_0}(\frac{m+1/z}{(m+1)+1/z})]\;.$$

Figures 19,20,21,22 show the family of probability of bit error versus the ratio of bit energy to noise power density for different values of $r = E_s/N_0/SIR$ when M = 2,4,8,16 respectively. All of these curves should be moved to the left in accordance with Figure 5 to include the improvement effect of the integrator. As before, equation (19) should be used by determining z from Figure 14.

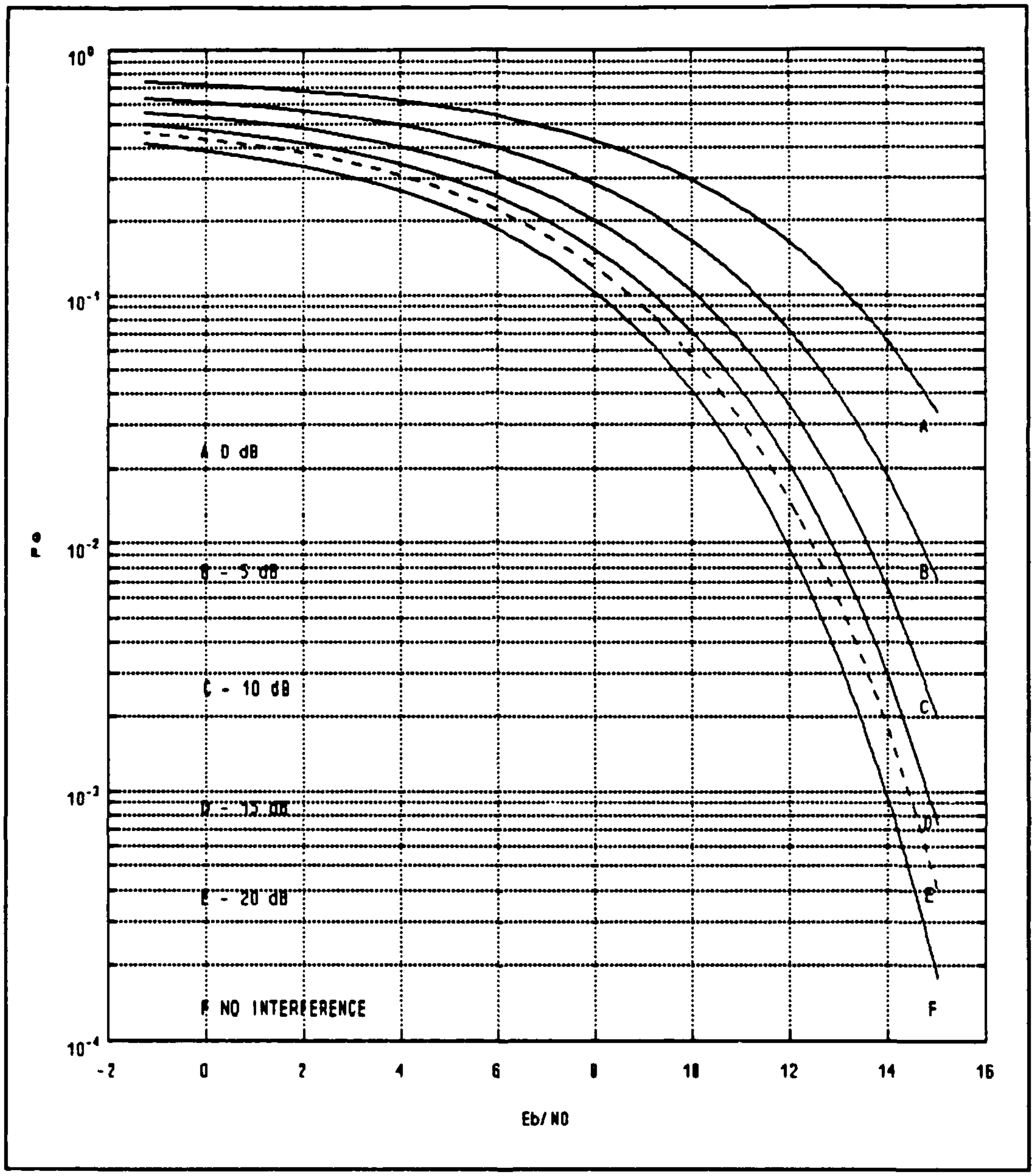

Figure 19. The Effect of Interference on System Performance when One Nonsignal Subsystem has Interference for M = 2. $r=(E_s/N_0)/SIR$ is Marked on each Curve.

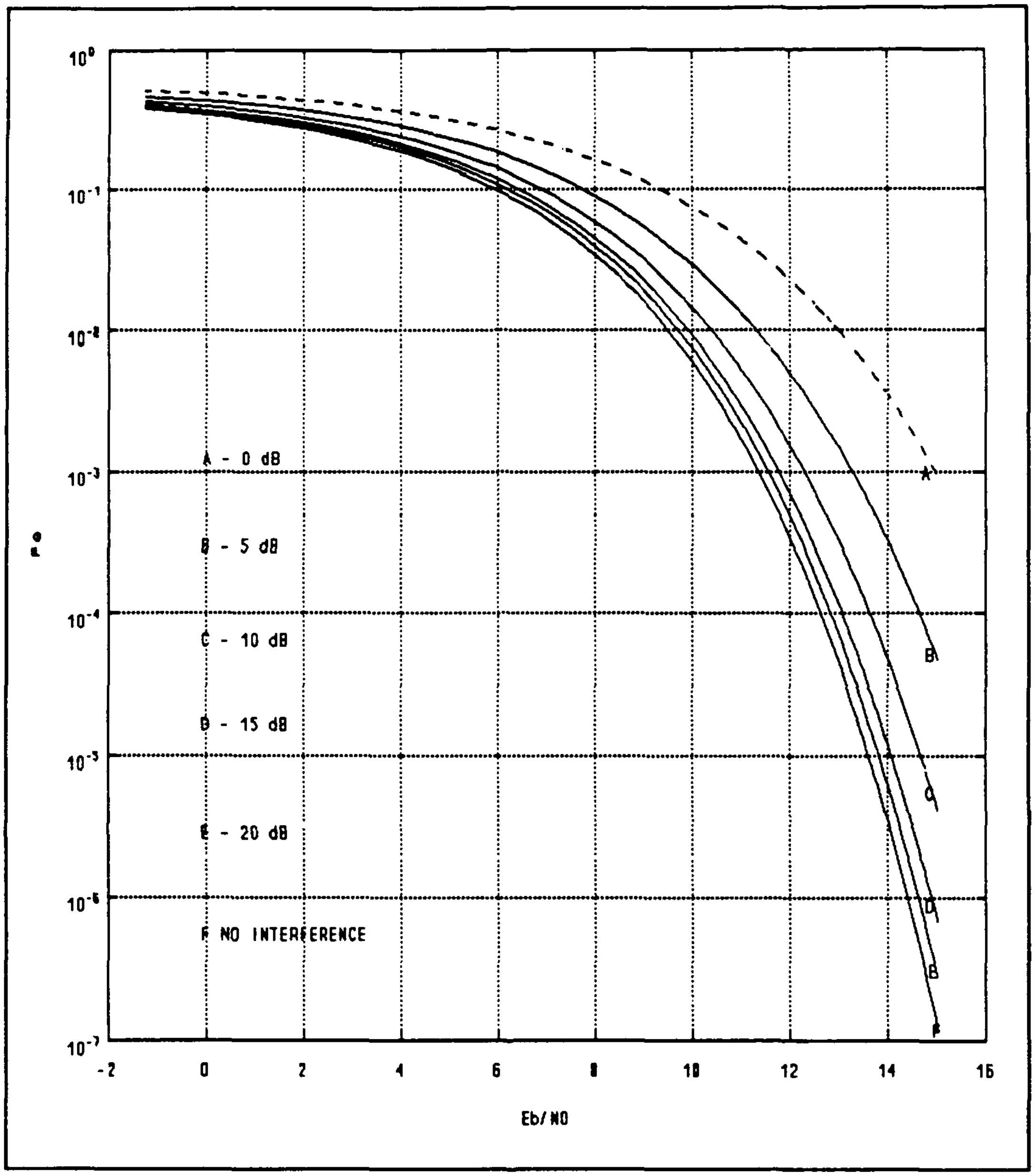

Figure 20. The Effect of Interference on System Performance when One Nonsignal Subsystem has Interference for M = 4. $r=(E_s/N_0)/SIR$ is Marked on each Curve.

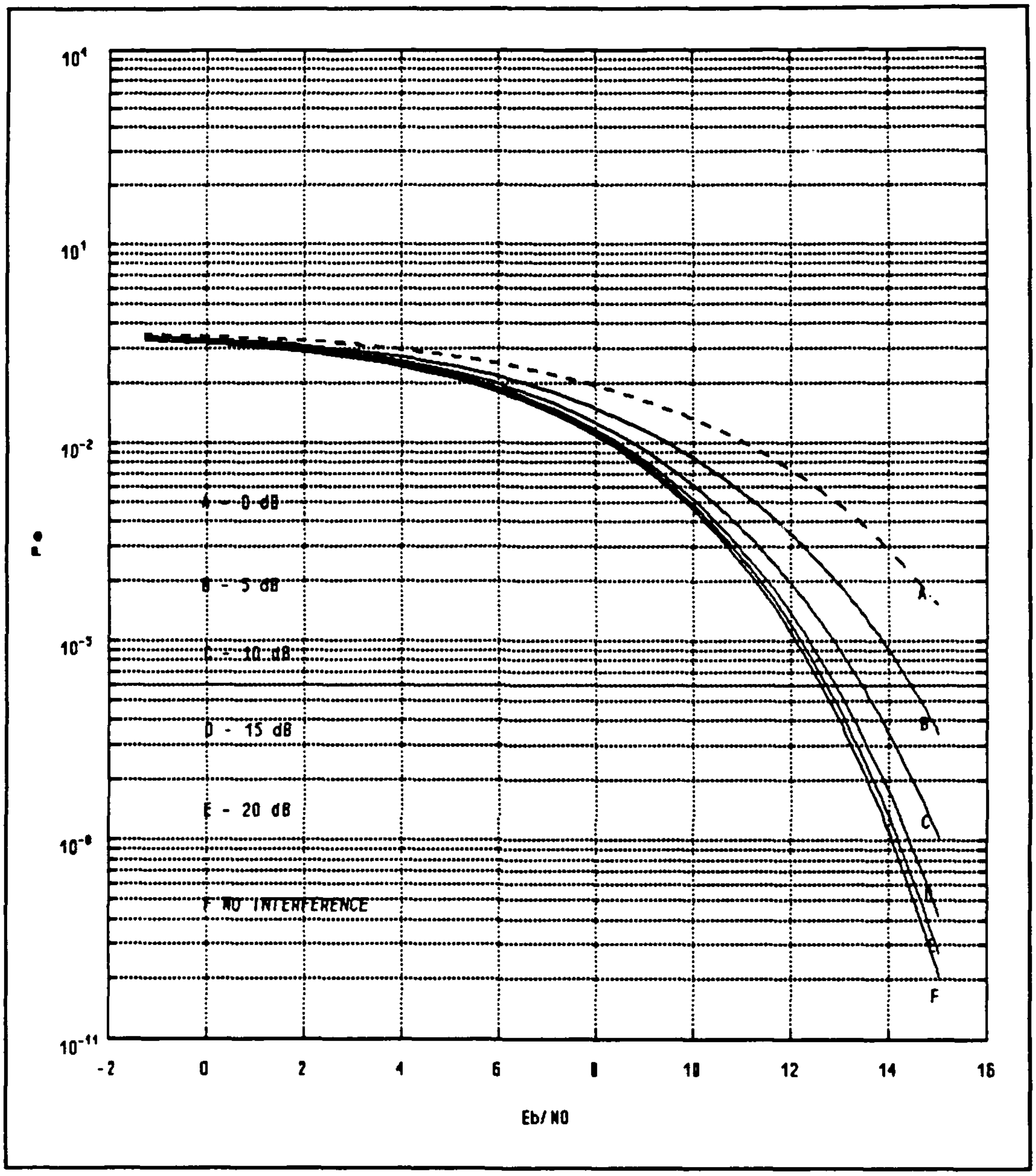

Figure 21. The Effect of Interference on System Performance when One Nonsignal Subsystem has Interference for M = 8. $r=(E_s/N_0)/SIR$ is Marked on each Curve.

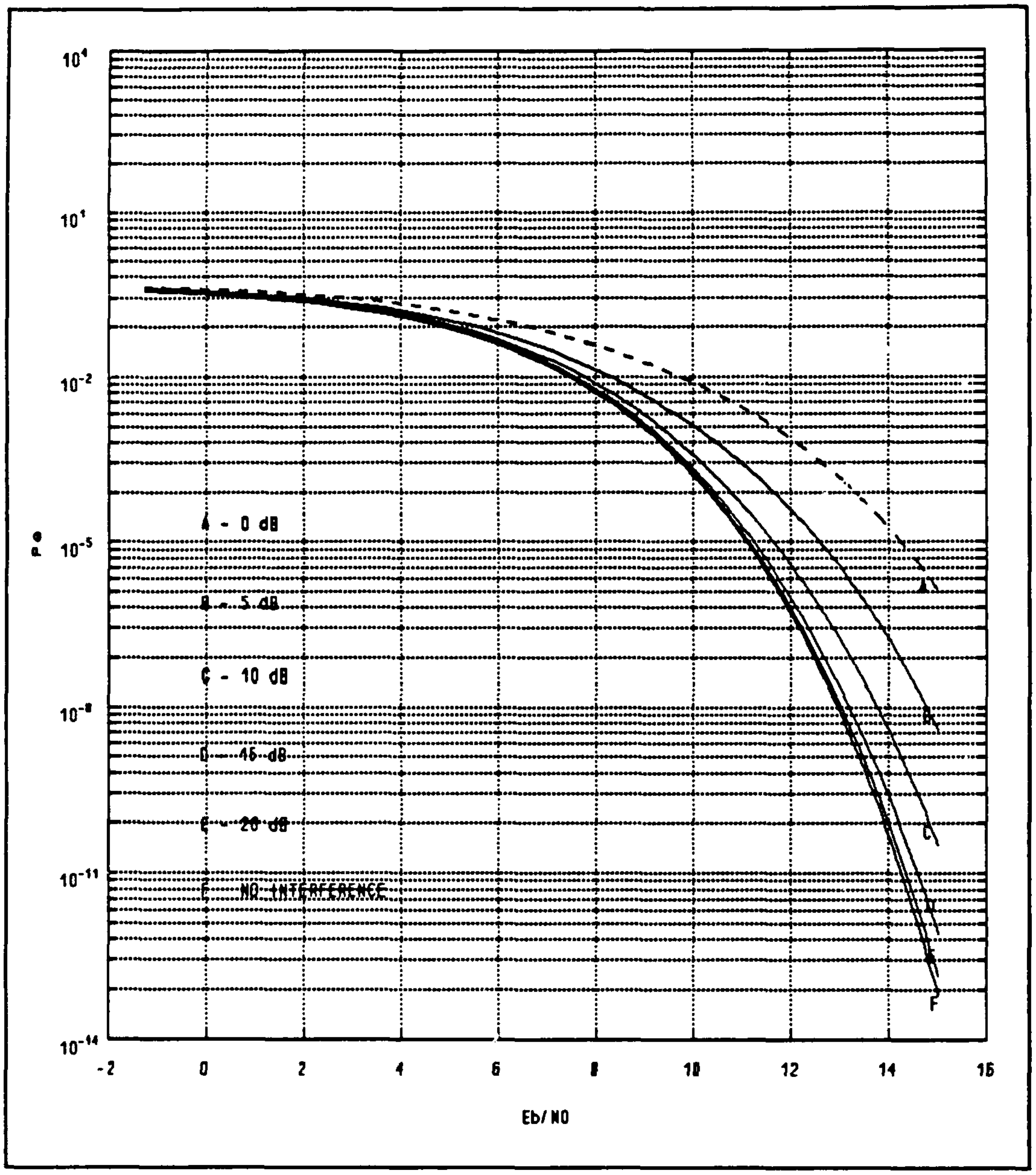

Figure 22. The Effect of Interference on System Performance when One Nonsignal Subsystem has Interference for M = 16. $r=(E_s/N_0)/SIR$ is Marked on each Curve.

D. EXAMPLES

The use of equations for the synchronization error and the interference cases are illustrated with the following examples.

The system parameters used in the examples are:

bit rate = 2000 bits/sec.

number M of symbols = 4

number (N_h) of hops per symbol = 10

Example 1: Determine the probability of bit error for a 10 dB ratio of bit energy to noise power density in the case of 25 micro seconds synchronization error and no interference.

To find the probability of bit error, the parameter e needs to be calculated. From the system parameters, e is 0.25. Then $\hat{E_b}/N_0=(1-e)^2\ E_b/N_0$ is substituted in equation (10) and the probability of bit error is 0.06. Including the effect of the integrator in accordance with Figure 5, the probability of bit error becomes 0.01.

Example 2: Determine the probability of bit error for a 14 dB ratio of bit energy to noise power density if all nonsignal subsystems had interference with r equal -5 dB and there was no synchronization error.

To find the probability of bit error, the parameter z as read from Figure 14, which is 1.7. Then, substituting this value of z, the value of the ratio of bit energy to noise power density, the values of k and M in equation (15), the

the integrator in accordance with Figure 5, the probability of bit error becomes 5×10^{-5}.

Instead of directly using the equations, the same results could also be obtained by using the probability of bit error curves in both cases since the curves for e = 0.25 and r = -10 dB are provided as Figures 8 and 16 respectively.

IV. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

System performance improves as the number of subsystems or symbols increases, at the expense of more hardware. The performance improvement due to an increase in the number of symbols is larger in value for larger values of the ratio of bit energy to noise power density. The integrator in the receiver improves the performance by about 3 dB for large values of the ratio of bit energy to noise power density, but the improvement gets less than 3 dB for small values.

The synchronization error degrades the system performance regardless of the number of symbols the system uses. For the same amount of synchronization error, the performance degradation worsens as the number of hops per symbol increases, as expected. If large synchronization errors are expected, the number of hops per symbol should be chosen carefully.

Interference in all nonsignal subsystems or in one subsystem, degrades system performance. When only one nonsignal subsystem has interference, the degradation worsens as the number of subsystems decreases. For the same amount of interference power, the interference in all nonsignal subsystems degrades the system performance more than the interference in only one nonsignal subsystem.

The composite effect of interference and synchronization error is to further degrade the system performance. When one nonsignal subsystem has interference and there is synchronization error, both the number of hops per symbol and the number of subsystems that the system uses become important design parameters.

B. RECOMMENDATIONS

It is recommended that the results obtained for the performance of the FHS system considered in this research be verified by a hardware realization.

APPENDIX

MATLAB SIMULATION CODE

```
%This program simulates the receiver of the FHS system for %
%two symbol case.  For a given input bit-energy-to-noise   %
%density ratio, the output of the program is the           %
%probability of bit error for that bit-energy-to-noise     %
%density ratio.                                            %


function n0 = noisevec(c)
%Forms the band limited white noise vector with variance 10%
rand('normal')          ;
M=rand(c,4000)          ;
B=(sqrt(10)/std(A)) * M ;
f=[0 .01 .02 .03 .04 .05..
.06 .07 .08 .09  .1 .2  ..
.3 .4 .5 .6 .7 .8 .9 1]  ;
k=[1 1 1 1 1 1 1 1 1 1 1..
0 0 0 0 0 0 0 0 0]      ;
[b,a]=yulewalk(15,f,k)  ;
C=filter(b,a,B)         ;
n0=reshape((sqrt(10)/std(C)) * C,20,200);
function F=sigenv(m)    ;
A=sqrt(10*m)            ;%Form the Eb/N0
```

```
for n=1:20                ;
n0=noisevec(1)            ;%Get the noise vector
%Form the envelope of signal+noise%
Es=sqrt((A+n0(:,1:100)).^2+(A+n0(:,101:200)).^2);
%Integrate for a symbol duration and then dump%
Is(n,:)=sum(Es)./20       ;
end                       ;
F=reshape(Is,2000,1)      ;
function E=noiseenv(N)    ;
for d=1:20                ;
n0=noisevec(1)            ;
%Form the envelope of the noise%
En=sqrt(n0(:,1:100).^2+n0(:,101:200).^2);
%Integrate for a symbol duration and dump%
In(d,:)=sum(En)./20       ;
end                       ;
E=reshape(In,2000,1)      ;
% Main Program
F=sigenv(m)             ;% Signal+noise  vector
E=noiseenv(20)          ;% Noise only vector
ms=mean(F)              ;% Find the mean of signal+noise
mn=mean(E)              ;% Find the mean of noise only
Vt=(ms+mn)/2            ;% Find the ML threshold
i=length(find(F<Vt))    ;
j=length(find(E>Vt))    ;
Pm=i/2000               ;% Find probability of miss
```

```
Pfa=j/2000              ;% Find probability of false alarm
Pe=0.5*(Pm+Pfa)         ;% Find probability of error
```

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