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Nonlinear transformation optics techniques in the design of counter-directed energy weapons shields for satellites

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NONLINEAR TRANSFORMATION OPTICS TECHNIQUES IN THE DESIGN OF COUNTER-DIRECTED ENERGY WEAPON SHIELDS FOR SATELLITES

by

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December 2012

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The purpose of this thesis is to examine the feasibility of using an emerging technique, called transformation optics (TO), in designing materials to be used as a defense against directed energy weapons for satellites. In order to do this, a method of determining the effectiveness of TO against high-intensity fields must be demonstrated. These high-intensity fields will cause a nonlinear response in the material and it is this nonlinear response that will be studied. TO has been shown to be effective when dealing with lower intensity fields and thus linear responses in matter[2]. This thesis will attempt to model the nonlinear response and solve for the fields due to this response.

The fields induced by the nonlinear response are considered an error field. To solve for the error field, a method to model the nonlinear response will be derived using Miller's Rule. Stemming from the Lorentz-Drude model of polarization, Miller's Rule serves as a model of the nonlinear response but has been shown experimentally to be approximately true[3]. Once the nonlinear response has been found, the error can be analyzed as an electrostatic problem to determine if the polarization or magnetization induces a field within the cloaked area.

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NONLINEAR TRANSFORMATION OPTICS TECHNIQUES IN THE DESIGN OF COUNTER-DIRECTED ENERGY WEAPONS SHIELDS FOR SATELLITES

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ABSTRACT

The purpose of this thesis is to examine the feasibility of using an emerging technique, called transformation optics (TO), for designing materials to be used as a defense against directed energy weapons for satellites. In order to do this, a method of determining the effectiveness of TO against high-intensity fields must be demonstrated. These high-intensity fields will cause a nonlinear response in the material and it is this nonlinear response that will be studied. TO has been shown to be effective when dealing with lower intensity fields and thus linear responses in matter. [1] This thesis will attempt to model the nonlinear response and solve for the fields due to this response.

The fields induced by the nonlinear response are considered an error field. To solve for the error field, a method to model the nonlinear response will be derived using Miller's Rule. Stemming from the Lorentz-Drude model of polarization, Miller's Rule serves as a model of the nonlinear response and has been shown experimentally to be approximately true. [2] Once the nonlinear response has been found, the error can be analyzed as an electrostatic problem to determine if the polarization or magnetization induces a field within the cloaked area.

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I. INTRODUCTION

A. THE PROBLEM DEFINED

The United States military's reliance on satellites is well known. Whether it is intelligence collection, communication, or precision navigation and timing, the United States relies heavily on its use of satellites in space to conduct effective and efficient operations around the globe. As potential adversaries look to exploit this reliance, antisatellite weapons in the form of directed energy beams are a potential future threat to our satellites that cannot be overlooked.

When directed energy weapons are mentioned, often the first things that come to mind are the laser beams of science fiction. In these stories, the usual defense is some sort of "energy shield." While it may be prominent only in science fiction, shielding from electromagnetic waves is possible and has been demonstrated using cloaking techniques. [1] Cloaking, in this sense, is the ability of certain designed materials to redirect electromagnetic waves around a region in space, thus cloaking the object. While the end goal of cloaking is invisibility, for a directed energy weapon simply redirecting the wave is sufficient. The fields (and thus energy) involved with current demonstrations so far has been weak, dealing strictly in the linear realm. [3] With directed energy weapons, it will be necessary to look at nonlinear effects. This thesis will deal with modeling nonlinear responses of materials specifically designed to shield against directed energy weapons.

Within the last decade, a technique for designing materials that exhibit the desired cloaking response has been developed called transformation optics. The field of transformation optics has introduced new methods of analysis and design of electromagnetic materials. The method simplifies many problems in electrodynamics by converting what would be a complicated analytical solution into a geometric coordinate transformation. [4] The trade-off is that the material needed to implement this transformation in real space is often complicated to manufacture. This hurdle has been overcome somewhat in the last decade and transformation optics is becoming a viable design technique for a whole host of new materials. [1]

Typically, transformation optic solutions are implemented with the use of metamaterials. Metamaterials are “artificially structured media whose effective constitutive parameters can be adjusted over a wide range of parameters through judicious design.” [5] For the purpose of designing a shield for a satellite, weight is one obvious concern. Luckily these metamaterials can be manufactured from lightweight alloys and do not require large scale structure. [1] In fact, the structures are often sub-wavelength in size. In this thesis, the effectiveness of the technique of transformation optics for designing metamaterials for use in space is evaluated and if the nonlinear effects of the high intensity fields associated with directed energy weapons can be approximated through Miller’s rule.

Miller’s rule is a technique to predict the second order nonlinear susceptibility of a material in a consistent way, in close analogy to a power or Taylor series expansion. Based on the first order linear response, an approximation is made for the second order response. Once the second order susceptibility is known, the second order polarization is calculated. The nonlinear response is then analyzed as an electrostatic problem to determine if the polarization or magnetization induces a field within the cloaked area.

B. ANTISATELLITE (ASAT) TECHNOLOGY

1. Definition

Antisatellite (ASAT) technology has gained attention in recent years due to two high profile tests of ASAT weapon systems by the Chinese and the United States. The sanctuary provided by the altitude and speed of low earth orbit (LEO) has been lost. The United States’ reliance on satellite systems cannot be over-stated, and the subsequent threat posed by antisatellite weapon systems cannot be ignored.

The purpose of a weapon system is to deliver a destructive amount of energy to a target to damage or destroy it. Conventionally, a projectile fired at great velocity, an explosive charge, or a combination of both transfers the energy to the target. The larger the energy transfer, the greater the destruction. Directed energy weapon systems (DEWS) use electromagnetic waves to transfer energy. They focus or direct large amounts of energy onto a relatively small spot. By concentrating the energy, DEWS require less total energy to inflict damage.

a. Conventional ASAT

The two ASAT systems recently tested are kinetic in nature, requiring launch by the host country and intercept capability by the ASAT. This requirement of highly advanced technology to build an ASAT has served to limit the proliferation of antisatellite systems. This could change in the future. "Robert Joseph, the State Department's point man for arms control and international security, said other nations and possibly terrorist groups were 'acquiring capabilities to counter, attack and defeat U.S. space systems.' " [6]

In addition to the technical hurdles to overcome, the destruction of a satellite in LEO poses risks to other satellites, including manned space flight, within the LEO belt. International ramifications from the Chinese test serve to support this. "A variety of regional and extra-regional states have expressed concern about a suspected, although not publicly confirmed, antisatellite weapon (ASAT) test by China on 11 January." [7] The debris cloud formed from the satellite and exploding missile is a problem for all nations wishing to utilize the LEO belt. "According to David Wright of the Cambridge, Mass.-based Union of Concerned Scientists, the satellite pulverized by China could have broken into nearly 40,000 fragments from 1 to 10 centimeters (a half-inch to 4 inches) in size, roughly half of which would stay in orbit for more than a decade." [8] If an ASAT system could be developed that limits debris and requires simpler technology to operate, then the above obstacles could be overcome.

b. Directed Energy

One class of such weapons are directed energy weapons. The best known example of a directed energy weapon would be a laser, but other forms exist, including directed microwave weapons. For the reasons mentioned above, the U.S. must develop a robust counter-directed energy weapon (CDEW) capability against directed energy weapons.

Directed energy weapons offer two distinct advantages over conventional, kinetic antisatellite weapons. The first is the ease of delivering energy to LEO. For a conventional system, the host nation needs to have developed a launch and rendezvous capability of extremely high precision. This is currently beyond the reach of many nations and not something easily acquired. The second advantage is the destruction of the satellite, or at least its capability, without the debris cloud, and thus without the international ramifications. A directed energy weapon could simply cause enough damage by burning through or creating fields within the electronics to damage them. The satellite would become inoperable yet remain in the same orbit. Directed energy weapons are currently being developed for these reasons. "The latest report follow claims in September, reported by Defense News, that China was aiming high-powered, ground-based lasers at U.S. spy satellites — apparently to test whether sensors on the satellites could be blinded." [6]

One defensive technology that could serve to protect U.S. satellites by bending electromagnetic fields away from a region of space is TO. The bending or cloaking effect is typically implemented through metamaterials. This cloaking would render the satellite invisible to the directed energy, causing it to pass harmlessly around or away from the asset requiring protection. While still highly theoretical, examples of cloaking a cylinder in two dimensions from microwave radiation have been demonstrated. [1] The fields involved in this experiment were of low intensity however, and did not demonstrate an effect for high intensity fields. For cloaking to be effective at high intensities, nonlinear effects need to be taken into account.

II. ELECTRODYNAMICS OF MATTER

A. MAXWELL'S EQUATIONS IN MATTER

1. Introduction

In order to understand transformation optics, it is necessary to understand the behavior of electromagnetic waves in media. The reader is assumed to be familiar with electrostatics, Maxwell's equations, and general electrodynamics. This section is included for completeness but does not serve as a complete derivation or proof, rather as a review and reference for subsequent sections. Brau's Modern Problems in Electrodynamics and Mill's Nonlinear Optics are both excellent sources for further study of the material in this section. [2] [9]

2. Maxwell's Equations in Matter

Maxwell's equations are

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_o} \quad (2.1)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (2.2)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (2.3)$$

$$\nabla \times \mathbf{B} = \mu_o \mathbf{j} + \mu_o \varepsilon_o \frac{\partial \mathbf{E}}{\partial t} \quad (2.4)$$

Here **E** is the electric field, **B** the magnetic induction field, ρ the charge density, and **j** the current density. Maxwell's equations are true everywhere, to include inside media. They are, however, difficult to apply due to a lack of knowledge of the microscopic structure of most media. Nonetheless, field averages can be examined.

When a material is immersed in an electric field, it becomes polarized. An additional field **P**, called the "polarization field," is then induced due to the molecular interactions with **E**. It can be shown that **P** is the dipole moment per unit volume. In this case, (2.1) becomes

$$\nabla \cdot (\mathbf{E} + \frac{1}{\varepsilon_o}\mathbf{P}) = \frac{\rho}{\varepsilon_o} \quad (2.5)$$

Define a new field **D**, called the “displacement field”

$$\mathbf{D} \equiv \varepsilon_o \mathbf{E} + \mathbf{P} \quad (2.6)$$

In this case (2.1) becomes

$$\nabla \cdot \mathbf{D} = \rho \quad (2.7)$$

In a similar way, matter responds to an imposed magnetic field through the magnetization response **M**. Define the magnetic field **H** as

$$\mathbf{H} = \frac{1}{\mu_o}\mathbf{B} - \mathbf{M} \quad (2.7)$$

in which case, (2.4) becomes

$$\nabla \times \mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t} \quad (2.8)$$

Combining the above results, Maxwell’s equations in matter are

$$\nabla \cdot \mathbf{D} = \rho \quad (2.9)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (2.10)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (2.11)$$

$$\nabla \times \mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t} \quad (2.12)$$

Of primary concern is the polarization field **P** induced by high intensity electric fields. The next section examines **P** more closely, in particular its response to an applied high intensity electric field.

B. POLARIZATION RESPONSE TO LARGE ELECTRIC FIELDS

1. Derivation [9]

If the applied electric field is of sufficient strength to invoke a measurable nonlinear response in the material yet not strong enough as to overcome the coulomb attraction between the electron and nucleus, then the polarization can be expanded as a Taylor series in powers of the macroscopic electric field such that

$$P_\alpha(\mathbf{r},t) = P_\alpha^{(0)} + \sum_\beta \left(\frac{\partial P_\alpha}{\partial E_\beta} \right) E_\beta + \frac{1}{2!} \sum_{\beta\gamma} \left(\frac{\partial^2 P_\alpha}{\partial E_\beta \partial E_\gamma} \right) E_\beta E_\gamma + \ldots \frac{n!}{r!(n-r)!} \quad (2.13)$$

Here $P_\alpha(\mathbf{r},t)$ is the αth Cartesian component of the dipole moment per unit volume with α ranging over x, y, and z.

In most cases, the first term of equation (2.14) vanishes identically. If it did not, the material would have an intrinsic polarization, analogous to a ferromagnetic material. This is equivalent to stating that any polarization present is due to the applied electric field. With this being the case, equation (2.14) becomes

$$P_\alpha(\mathbf{r},t) = \sum_\beta \left(\frac{\partial P_\alpha}{\partial E_\beta} \right) E_\beta(\mathbf{r},t) + \frac{1}{2!} \sum_{\beta\gamma} \left(\frac{\partial^2 P_\alpha}{\partial E_\beta \partial E_\gamma} \right) E_\beta(\mathbf{r},t) E_\gamma(\mathbf{r},t) + \ldots \quad (2.14)$$

Based on (2.15), the susceptibilities of the material are defined as the partial derivative terms. In other words,

$$\chi_{\alpha\beta}^{(1)} = \frac{\partial P_\alpha}{\partial E_\beta} \quad (2.15)$$

$$\chi_{\alpha\beta\gamma}^{(2)} = \frac{\partial^2 P_\alpha}{\partial E_\beta \partial E_\gamma} \quad (2.16)$$

are the first order and second order susceptibilities respectively. The quasi-static limit will be used in this analysis. It will be assumed that a steady state condition has been achieved to remove the time dependence from (2.15). Given the above, the polarization is now

$$P_\alpha(\mathbf{r}) = \sum_\beta \chi^{(1)}_{\alpha\beta} E_\beta(\mathbf{r}) + \sum_{\beta\gamma} \chi^{(2)}_{\alpha\beta\gamma} E_\beta(\mathbf{r}) E_\gamma(\mathbf{r}) + ... \qquad (2.17)$$

As can be seen, transformation optics requires tensor analysis. The metamaterials themselves are intrinsically inhomogeneous and anisotropic, which requires that the susceptibilities be tensors.

The polarization is now separated into a linear and nonlinear contribution.

$$P_\alpha(\mathbf{r}) = P^{(L)}_\alpha(\mathbf{r}) + P^{(NL)}_\alpha(\mathbf{r}) \qquad (2.18)$$

From (2.18), the linear response is

$$P^{(L)}_\alpha(\mathbf{r}) = \sum_\beta \chi^{(1)}_{\alpha\beta} E_\beta(\mathbf{r}) \qquad (2.19)$$

and the nonlinear response is

$$\boxed{P^{(NL)}_\alpha(\mathbf{r}) = \sum_{\beta\gamma} \chi^{(2)}_{\alpha\beta\gamma} E_\beta(\mathbf{r}) E_\gamma(\mathbf{r}) + ...} \qquad (2.20)$$

For this analysis, the second order response will be examined with higher order contributions ignored. E.g. (2.21) describes the nonlinear polarization. The next step is to determine the nonlinear susceptibility using Miller's rule.

C. LORENTZ-DRUDE MODEL OF POLARIZATION AND MILLER'S RULE DERIVATION FOR SECOND ORDER SUSCEPTIBILITY [2]

Miller's rule is a method that approximates the second order susceptibility in terms of the first order susceptibility. A simple and elegant derivation of Miller's Rule, paraphrased in this section from Brau's textbook Modern Problems in Electrodynamics, follows from the Lorentz-Drude Model for the polarization of the atom[2]. In this model, the electron is harmonically bound to the nucleus. In many cases, it may be difficult to derive the first order susceptibility of a material, but with transformation optics, the derivation is simple. As will be shown later, the first order susceptibility is simply a function of the transformation matrix used in transformation optics. The derivation of the second order susceptibility begins with a simple linear isotropic case, extends to the nonlinear case, and finally to anisotropic materials.

1. ISOTROPIC MATERIALS

a. Linear Materials

In the presence of an electric field **E**, the displacement **r** of the atom in the nonrelativistic limit is

$$\frac{d^2\mathbf{r}}{dt^2}+\gamma\frac{d\mathbf{r}}{dt}+\omega^2{}_r\mathbf{r}=\frac{q}{m}\mathbf{E} \tag{2.21}$$

Here q is the charge, m the mass, γ the damping coefficient, and ω_r the resonant frequency of the system. The polarization per unit volume is the dipole moment $q\mathbf{r}$ times the electron density n_e, or $\mathbf{P} = n_e q\mathbf{r}$. This leads to the equation

$$\frac{d^2\mathbf{P}}{dt^2}+\gamma\frac{d\mathbf{P}}{dt}+\omega^2{}_r\mathbf{P}=\varepsilon_o\omega^2{}_p\mathbf{E} \tag{2.22}$$

where

$$\omega_p=\sqrt{\frac{n_e q^2}{\varepsilon_0 m}} \tag{2.23}$$

is the plasma frequency of the material. Taking the Fourier transform of both sides of this equation, keeping in mind that d/dt transforms to $-i\omega$,

$$(\omega_r^2-\omega^2-i\gamma\omega)\tilde{P}=\varepsilon_0\omega_p^2\tilde{E} \tag{2.24}$$

Here the tilde indicates the transformed, or frequency domain, fields.

Comparing the above equation with the equation for polarization

$$P=\varepsilon_0\chi E \tag{2.25}$$

it can be seen that the dielectric susceptibility is

$$\chi(\omega) = \frac{\omega_p^2}{\omega_r^2 - \omega^2 - i\gamma\omega} \tag{2.26}$$

b. Nonlinear Materials [2]

Taking this result and extending it to the nonlinear realm requires an additional term in the inharmonic oscillator equation. The new equation describing the electron bound to the nucleus is

$$\frac{d^2P}{dt^2} + \gamma\frac{dP}{dt} + \omega^2{}_r P + K^{(2)}P^2 = \eta E \tag{2.27}$$

where $K^{(2)}$ represents the nonlinear contribution to the restoring force. The parameter η is introduced to act as an ordering parameter.

Assume that η is sufficiently small such that the solution can be written as a power series having the form

$$P(t) = \eta P^{(1)}(t) + \eta^2 P^{(2)}(t) + ... \tag{2.28}$$

Take the Fourier transform of this equation to obtain

$$\tilde{P}(\omega) = \eta\tilde{P}^{(1)}(\omega) + \eta^2\tilde{P}^{(2)}(\omega) + ... \tag{2.29}$$

In order to compute the polarization now, plug the above equation into (2.28) and collect like terms of η. To first order in η

$$\frac{d^2P^{(1)}}{dt^2} + \gamma\frac{dP^{(1)}}{dt} + \omega^2{}_r P^{(1)} = E \tag{2.30}$$

The Fourier transform is again taken and compared to the answer of the equation for polarization (2.26).

$$\eta \tilde{P}^{(1)}(\omega) = \frac{\eta}{\varepsilon_0} \frac{\varepsilon_0}{\omega_r^2 - \omega^2 - i\gamma\omega} \tilde{E}(\omega) \tag{2.31}$$

From this, it is seen that the first order, linear susceptibility is

$$\chi^{(1)}(\omega) = \frac{\eta}{\varepsilon_0} \frac{1}{\omega_r^2 - \omega^2 - i\gamma\omega} \tag{2.32}$$

Taking the exact same steps, but this time to the second order of η the equation obtained is

$$\frac{d^2 P^{(2)}}{dt^2} + \gamma \frac{dP^{(2)}}{dt} + \omega^2{}_r P^{(2)} + K^{(2)} (P^{(1)})^2 = 0 \tag{2.33}$$

Again taking the Fourier transform and using (2.32), it is found that

$$\begin{aligned} &(\omega_r^2 - \omega^2 - i\gamma\omega)\tilde{P}^{(2)}(\omega) = -\frac{K^{(2)}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt\ e^{i\omega t} (P^{(1)}(t))^2 = \\ &-\frac{\varepsilon_0^2 K^{(2)}}{(2\pi)^{3/2} \eta^2} \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} d\omega' \int_{-\infty}^{\infty} d\omega'' e^{i(\omega - \omega' - \omega'')t} \chi^{(1)}(\omega') \chi^{(1)}(\omega'') \tilde{E}(\omega') \tilde{E}(\omega'') \end{aligned} \tag{2.34}$$

The integral over *t* is a delta function. Using (2.33), the nonlinear polarization is

$$\eta^2 \tilde{P}^{(2)}(\omega) = \varepsilon_0 \int_{-\infty}^{\infty} d\omega' \int_{-\infty}^{\infty} d\omega'' \chi^{(2)}(\omega', \omega'') \tilde{E}(\omega') \tilde{E}(\omega'') \delta(\omega - \omega' - \omega'') \tag{2.35}$$

where the second order susceptibility is

$$\chi^{(2)}(\omega',\omega'') = -\frac{\varepsilon_0^2 K^{(2)}}{\sqrt{2\pi\eta}} \chi^{(1)}(\omega'+\omega'')\chi^{(1)}(\omega')\chi^{(1)}(\omega'') \qquad (2.36)$$

This is Miller's rule.

2. ANISOTROPIC MATERIALS

a. Nonlinear Materials [2]

The above derivation for Miller's rule applies to isotropic materials. Metamaterials are by their nature anisotropic. In order to derive the anisotropic Miller's Rule, proceed as before but now expand into three dimensions by including the appropriate indices. The following derivation includes a nonstandard notation, utilizing a semicolon to denote an index which does not exhibit symmetry. The indices on the right of the semicolon are permutable; the index on the left is not. This semicolon can be ignored for the most part, but offers a way to simplify the calculations. In other words $\chi_{i;jk} = \chi_{i;kj}$.

Assume the solution has the form

$$P_i(t) = \eta P_i^{(1)}(t) + \eta^2 P_i^{(2)}(t) + ... \qquad (2.37)$$

with a Fourier transform

$$\tilde{P}_i(\omega) = \eta \tilde{P}_i^{(1)}(\omega) + \eta^2 \tilde{P}_i^{(2)}(\omega) + ... \qquad (2.38)$$

Inserting equation (2.38) into the equation of motion to first order η obtains

$$\frac{d^2 P_i^{(1)}}{dt^2} + \sum_j \gamma_{i;j} \frac{dP_j^{(1)}}{dt} + \sum_j \omega^2{}_{i;j} P_j^{(1)} = E_i^{(1)} \qquad (2.39)$$

The Fourier transform of (2.40) is

$$\sum_j \kappa_{i;j} \tilde{P}_j^{(1)} = \tilde{E}_i \tag{2.40}$$

where

$$\kappa_{i;j} = \omega_{i;j}^2 - i\omega\gamma_{i;j} - \omega^2\delta_{ij} \tag{2.41}$$

The solution to this equation is

$$\eta \tilde{P}_i^{(1)} = \varepsilon_0 \sum_j \chi_{i;j}^{(1)} \tilde{E}_j \tag{2.42}$$

where the linear susceptibility tensor

$$\chi_{i;j}^{(1)} = \frac{\eta}{\varepsilon_0} \kappa_{i;j}^{-1} \tag{2.43}$$

is also the inverse of the tensor $\kappa_{i;j}$ such that

$$\varepsilon_0 \sum_j \chi_{i;j}^{(1)} \kappa_{j;k} = \eta \delta_{ik} \tag{2.44}$$

Using the same arguments as the previous cases, the quadratic term in the polarization is

$$\eta^2 \tilde{P}_i^{(2)}(\omega) = \varepsilon_0 \sum_{j;k} \int_{-\infty}^{\infty} d\omega' \int_{-\infty}^{\infty} d\omega'' \chi_{i;jk}^{(2)}(\omega', \omega'') \tilde{E}_j(\omega') \tilde{E}_k(\omega'') \delta(\omega - \omega' - \omega'') \quad (2.45)$$

where the second order susceptibility tensor is

$$\chi_{i;jk}^{(2)}(\omega', \omega'') = -\sum_{p,q,r} \frac{\varepsilon_0^2 K_{p;qr}^{(2)}}{\sqrt{2\pi}\eta} \chi_{i;p}^{(1)}(\omega' + \omega'') \chi_{q;j}^{(1)}(\omega') \chi_{r;k}^{(1)}(\omega'') \quad (2.46)$$

Equation (2.47) is the Miller's rule determined second order nonlinear susceptibility. Since the quasi-static case is being examined, all frequency dependence is removed.

$$\boxed{\chi_{i;jk}^{(2)} = -\sum_{p,q,r} \frac{\varepsilon_0^2 K_{p;qr}^{(2)}}{\sqrt{2\pi}\eta} \chi_{i;p}^{(1)} \chi_{q;j}^{(1)} \chi_{r;k}^{(1)}} \quad (2.47)$$

Additionally, it can be shown [2] that there exists symmetry across the last two indices of the second order susceptibility, but not the first. This is a cross check of the calculation.

While the first order susceptibility is a unitless value, the second order susceptibility has the units of meters per Volt, or m/V, in the SI system. Also of note is the second order susceptibility involves the product of three first order susceptibilities. It then makes sense that a material with a strong first order susceptibility will have a relatively strong second order susceptibility as well. [2]

b. Miller's coefficient

As seen in (2.48), there is a collection of terms that must be accounted for when calculating the nonlinear susceptibility

$$\frac{\varepsilon_0^2 K_{p;qr}^{(2)}}{\sqrt{2\pi}\eta} \quad (2.47)$$

This collection of terms, sometimes referred to as Miller's coefficient when taken together, has been found to be approximately equal to 4.52 x 10^{-2} m^2C^{-1} for a wide range of materials studied. [10] The reason for this is not well understood, but it is found to be experimentally true to within an order of magnitude. [10] For anisotropic materials, this coefficient becomes a tensor, further complicating its calculation. Referring back to (2.48) and removing the directional dependence for simplicity, the coefficient is defined to be

$$\frac{\varepsilon_0^2 K^{(2)}}{\sqrt{2\pi\eta}} \tag{2.47}$$

Here, η is the ordering parameter, but it can also be shown to equal

$$\eta = \frac{Nq^2}{m} \tag{2.47}$$

where N is the number of oscillators, q the charge, and m the mass. One can see that to calculate this value directly for an anisotropic metamaterial would be extremely difficult. It will therefore be set to its empirical value.

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III. TRANSFORMATION OPTICS

A. BACKGROUND

TO uses coordinate transformations to determine material specifications that control electromagnetic fields in interesting and useful ways. [11] By applying a coordinate transformation to the constitutive relations, a material can be designed that mimics free space yet possesses a curved geometry. It is important to keep in mind that in the general case the constitutive relations are tensor quantities.

> TO allows the derivation of a linear constitutive relation directly from a desired trajectory of light – in a completely algebraic way. The key ingredient of TO is to define the trajectory in the specified medium as a result of a transformation applied on the trajectory in free space. Unlike normal coordinate transformations, under which physics is invariant, the field lines are considered to be attached to the coordinates during the transformation, thereby changing the physics and deriving the constitutive law of the desired medium. [12]

Analogous to general relativity, the medium in effect fools the electromagnetic wave into believing it is propagating in free-space.

B. LINEAR FORMULATION

Smith, Kundtz, and Pendry give a succinct explanation of the concepts behind transformation optics.

> It has been recognized for some time that Maxwell's equations can be written in a form-invariant manner under coordinate transformations, such that only the permittivity and permeability tensors are modified. With the coordinate transformation applied to the constitutive parameters, electromagnetic waves in one coordinate system can be described as if propagating in a different coordinate system. [5]

Therefore, the first task will be to write Maxwell's equations in this form-invariant manner.

1. Arbitrary Coordinates [4]

It is first necessary to introduce tensor notation and its associated concepts. This section is not intended to be a full discussion on the topic, but rather it serves as a review of the concepts necessary for TO. For a more thorough handling of the differential geometry involved, see Leonhardt and Philbin's article "Transformation Optics and the Geometry of Light" included in the bibliography. The discussion below borrows heavily from section three of the article. [4]

a. Einstein Summation Convention

The Einstein summation convention is defined as the summation over any repeated index, whether it be in a subscript or superscript, over its entire range. As an example

$$a_i x_i = a_1 x_1 + a_2 x_2 + a_3 x_3 \qquad (3.1)$$

Additionally, the location of the indices will have meaning. A superscript will be taken to be a contravariant vector (column vector) while a subscript is a covariant vector (row vector).

b. Coordinate Transformations

TO deals with general, curvilinear coordinates and it is necessary to have a way of expressing a generalized coordinate system and performing an arbitrary transformation to another set of coordinates. Define one coordinate system $\{x^i, i = 1,2,3\}$ that is transformed to another system, differentiated with a prime $\{x^{i'}, i' = 1,2,3\}$. Here $x^{i'} = [x']^i$ or the i-th component in the primed coordinate system. The differentials of this coordinate system are related by the chain rule.

$$dx^i = \frac{\partial x^i}{\partial x^{i'}} dx^{i'} \text{ , } dx^{i'} = \frac{\partial x^{i'}}{\partial x^i} dx^i \qquad (3.1)$$

In order to carry out the transformation, define a standard transformation matrix

$$\Lambda^{i}_{i'} = \frac{\partial x^{i}}{\partial x^{i'}} \tag{3.1}$$

such that

$$dx^{i} = \Lambda^{i}_{i'} dx^{i'} = \Lambda^{i}_{i'} \Lambda^{i'}_{j} dx^{j} \tag{3.2}$$

and

$$dx^{i'} = \Lambda^{i'}_{i} dx^{i} = \Lambda^{i'}_{i} \Lambda^{i}_{j'} dx^{j'} \tag{3.3}$$

Taken together, the above two equations imply

$$\begin{aligned} \Lambda^{i}_{i'} \Lambda^{i'}_{j} &= \delta^{i}_{j} \\ \Lambda^{i'}_{i} \Lambda^{i}_{j'} &= \delta^{i'}_{j'} \end{aligned} \tag{3.4}$$

Where δ^{i}_{j} is the Kronecker delta. In other words, $\Lambda^{i}_{i'}$ and $\Lambda^{i'}_{i}$ are inverses of each other.

c. The Metric Tensor

Using the above notation, any arbitrary coordinate system can be defined. One thing that must remain invariant however is the distance between two points, despite the coordinate system used to represent those points. In mathematics, the Pythagorean theorem is a relation in Euclidean geometry

$$ds^{2} = dx^{2} + dy^{2} + dz^{2} \tag{3.5}$$

which is expressed in the new notation as

$$ds^{2} = \delta_{ij} dx^{i} dx^{j} \tag{3.6}$$

where δ_{ij} is again the Kronecker delta. For arbitrary coordinates however, the distance between two points is

$$ds^{2} = g_{ij} dx^{i} dx^{j} \tag{3.7}$$

The new term in the above equation, g_{ij}, is known as the metric tensor. It is a symmetric tensor $\left(g_{ij} = g_{ji}\right)$, which allows the calculation of the distance between two points. Using the metric tensor and (3.6), rewrite (3.9) in a different coordinate system:

$$ds^2 = g_{i'j'}dx^{i'}dx^{j'} = g_{ij}dx^i dx^j = g_{ij}\Lambda^i_{i'}\Lambda^j_{j'}dx^{i'}dx^{j'} \qquad (3.8)$$

From the above equation, it can be seen that the metric tensor itself changes under a transformation:

$$g_{i'j'} = \Lambda^i_{i'}\Lambda^j_{j'}g_{ij} \qquad (3.9)$$

Typically the metric tensor is ignored in Cartesian coordinates because it equals the identity tensor, but this property will be important later on in determining the metric tensor of the transformed space.

Writing (3.11) out in matrix notation and letting $g_{ij} = G$ and $g_{i'j'} = G'$ it is seen that

$$G' = \Lambda^T G \Lambda \qquad (3.10)$$

with Λ denoting the transformation matrix defined in (3.3). Additionally, it can be shown:

$$g_{i'j'} = \Lambda^i_{i'}\Lambda^j_{j'}\delta_{ij} = \delta_{i'j'} \qquad (3.11)$$

Also note that the inverse of the metric tensor matrix is denoted by placing the indices in the superscript position.

$$g^{ij} = \left(G^{-1}\right) \qquad (3.12)$$

The metric tensor not only characterizes the length in arbitrary coordinates but also volume. The standard Cartesian volume element is transformed using the transformation matrix determinant as indicated below:

$$dV = \left|\det \Lambda\right| dV' \qquad (3.13)$$

Using the matrix representation (3.12) it can be seen

$$g' = (\det \Lambda)^2 g \qquad (3.14)$$

with g and g' denoting the determinants of their respective metric tensors. This then implies

$$dV = \sqrt{g'}dV' \qquad (3.15)$$

The primes can then be dropped and the volume element is always described by $\sqrt{g}dV$ in either Cartesian or curved coordinates.

d. Vector Products

For describing electromagnetics utilizing the arbitrary representation from above, the vector product needs to be defined. Keeping in mind that the vector product is antisymmetric, or $U \times V = -V \times U$, and the vector products of the Cartesian basis vectors are cyclic, define the Levi-Civita tensor

$$\in^{ijk} = [ijk] \qquad (3.16)$$

with $[ijk]$ being the permutation symbol

$$[ijk] = \begin{cases} +1 \text{ if i,j,k is an even permutation of 1,2,3} \\ -1 \text{ if i,j,k is an odd permutation of 1,2,3} \\ \quad 0 \qquad \text{otherwise} \end{cases} \qquad (3.17)$$

A transformation can then be applied to (3.18) using the rules defined above. This gives a definition for the Levi-Civita tensor in arbitrary coordinates

$$\in^{i'j'k'} = \Lambda_i^{i'} \Lambda_j^{j'} \Lambda_k^{k'} [ijk] = \det(\Lambda_l^{l'})[i'j'k'] = \frac{[i'j'k']}{\det \Lambda} \qquad (3.18)$$

which utilizes the Leibniz formula for the determinant of $\Lambda_l^{l'}$. Also, $\det(\Lambda)$ is the determinant of the transformation matrix $\Lambda_{l'}^{l}$ which is the inverse of $\Lambda_l^{l'}$. Then from (3.16) the determinant is

$$\det(\Lambda) = \pm\sqrt{g'} \tag{3.19}$$

With the negative sign utilized if the transformation changes the handedness of the coordinate system and the positive sign otherwise.

Finally, the definition of the Levi-Civita tensor in arbitrary coordinates is

$$\in^{ijk} = \pm\frac{1}{\sqrt{g}}\left[ijk\right] \tag{3.20}$$

This symbol is required to compute the vector product in arbitrary coordinates:

$$U \times V = \in^{ijk} U_j V_k e_i \tag{3.21}$$

where e_i is the i-th basis vector in the coordinate system.

e. Divergence and Curl

Using the above notation, the divergence in three dimensions and curl can now be defined for an arbitrary coordinate system. While skipping a great deal of important mathematics in their derivations, it will suffice to simply show what they are and move on to Maxwell's equations.

The divergence of a vector field can be shown to be

$$\nabla \cdot \mathbf{V} = \frac{1}{\sqrt{g}} \frac{\partial \sqrt{g} V^i}{\partial x^i} \tag{3.22}$$

The curl of a vector field $\mathbf{V}$ is defined as

$$(\nabla \times \mathbf{V})_i = \left[ijk\right]\frac{\partial E_k}{\partial x_j} = \left[ijk\right]\partial_j E_k \tag{3.23}$$

where $\partial_j = \partial / \partial x_j$.

2. Maxwell's Equations and Transformation Optics

Since TO deals with curvilinear coordinates, introducing the above notation takes care of the covariance properties in such coordinate systems.

Using the above identities, Maxwell's equations in empty space (2.1) through (2.4) can be rewritten in an arbitrary coordinate system:

$$
\begin{aligned}
&\frac{1}{\sqrt{g}}\frac{\partial\sqrt{g}E^i}{\partial x^i}=\frac{\rho}{\varepsilon_o}\\
&\frac{1}{\sqrt{g}}\frac{\partial\sqrt{g}H^i}{\partial x^i}=0\\
&\in^{ijk}\partial_j E_k=-\frac{1}{\mu_o}\frac{\partial H^i}{\partial t}\\
&\in^{ijk}\partial_j B_k=\mu_o\varepsilon_o\frac{\partial E^i}{\partial t}+\mu_o j^i
\end{aligned}
\qquad (3.24)
$$

Rewriting (3.26) by placing all the indices in the lower position through use of the metric tensor $E_i = g_{ij}E^j$, utilizing (3.22), and rearranging slightly yields

$$
\begin{aligned}
&\frac{\partial\sqrt{g}g^{ij}E_j}{\partial x^i}=\frac{\sqrt{g}\rho}{\varepsilon_o}\\
&\frac{\partial\sqrt{g}g^{ij}H_j}{\partial x^i}=0\\
&[ijk]\partial_j E_k=-\frac{1}{\mu_o}\frac{\partial\left(\pm\sqrt{g}g^{ij}H_j\right)}{\partial t}\\
&[ijk]\partial_j B_k=\mu_o\varepsilon_o\frac{\partial\left(\pm\sqrt{g}g^{ij}E_j\right)}{\partial t}+\mu_o\sqrt{g}\,j^i
\end{aligned}
\qquad (3.25)
$$

Examining the above equations closely, it is seen that the form-invariant Maxwell's vacuum equations resemble the macroscopic Maxwell equations in dielectric media, keeping in mind that $\mathbf{B}=\frac{\mathbf{H}}{\mu_o}$

$$
\begin{aligned}
&\nabla \cdot \mathbf{D} = \rho \\
&\nabla \cdot \mathbf{B} = 0 \\
&\nabla \times \mathbf{E} = -\frac{1}{\mu_o}\frac{\partial \mathbf{H}}{\partial t} \\
&\nabla \times \mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t}
\end{aligned}
\tag{3.26}
$$

Taking the constitutive relations

$$
\begin{aligned}
D^i &= \varepsilon_o \varepsilon^{ij} E_j \\
B^i &= \mu_o \mu^{ij} H_j
\end{aligned}
\tag{3.27}
$$

and comparing them with the above equations, it becomes apparent that

$$
\boxed{\varepsilon^{ij} = \mu^{ij} = \pm\sqrt{g}\, g^{ij}} \tag{3.28}
$$

Essentially what has happened is that when the form-invariant Maxwell's equations are written out, the space typically occupied by ε and μ is instead occupied by the right side of (3.30), implying that geometries appear as dielectric media and vice-versa.

Remembering that

$$
\varepsilon = \varepsilon_o(1 + \chi_e) \tag{3.29}
$$

it can be seen that the transformed susceptibility tensors are related to the transformation itself. In other words

$$
\boxed{\chi_e^{ij} = \chi_m^{ij} = -\delta^{ij} \pm \sqrt{g}\, g^{ij}} \tag{3.30}
$$

The above equations allow a change in coordinates to be expressed through a fictitious variation of the constitutive parameters and source terms without needing to change the form of Maxwell's Equations with every different coordinate system. This transformation is implemented through transformation media.

a. *Transformation Media*

Transformation media implement coordinate transformations in Maxwell's equations. The way to think of this is to imagine two different spaces, a virtual space and a physical space. The virtual space is a flat Cartesian or electromagnetic space . A non-trivial coordinate transformation is then applied to the virtual space, which defines the transformation medium. This transformation is interpreted in the physical space with the medium implementing the coordinate transformation, showing the ray trajectories.

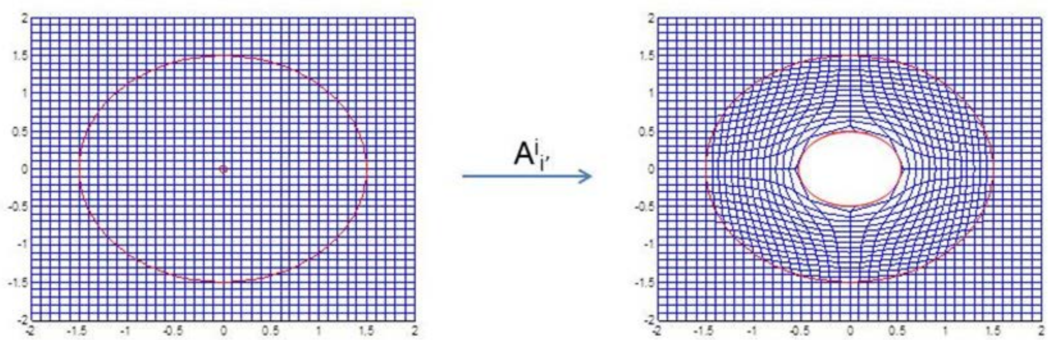

Figure 1. Transformation media implement coordinate transformations. The left figure shows the Cartesian grid of electromagnetic space that is mapped to the curved grid of physical space through the coordinate transformation implemented by $A^i_{i'}$ in the right figure. The physical coordinates enclose a hole that is made invisible in electromagnetic space (where it shrinks to the point indicated there). Consequently, a medium that performs this transformation acts as an invisibility device. [4]

The transformation is implemented by performing the coordinate transformation on the constitutive relations. Remembering (3.30), the transformed equation takes the form

$$\varepsilon^{ij} = \mu^{ij} = \pm\sqrt{g}\, g^{i'j'} A^i_{i'} A^j_{j'} \qquad (3.30)$$

As mentioned earlier, since both the virtual and physical spaces are Cartesian, the metric tensor is the identity matrix and it's determinant equals one. Equation (3.33) is rewritten in matrix notation.

$$\varepsilon = \mu = \frac{\mathbf{A}\mathbf{A}^T}{\det \mathbf{A}} \tag{3.30}$$

The above equation nicely deals with the +/- in the tensor equation with the determinant. Now, using (3.32) as the guide, the first order susceptibility tensor is calculated.

$$\chi = \varepsilon - \mathbf{I} = \frac{\mathbf{A}\mathbf{A}^T}{\det \mathbf{A}} - \mathbf{I} \tag{3.30}$$

b. Impedance Matching

The assumption in the analysis is that the original medium is free space. Since it is important that the metamaterial be impedance matched to free space to eliminate reflections, the relative permittivity and permeability must be equal. The impedance of free space is

$$\eta_0 = \sqrt{\frac{\mu_o}{\varepsilon_o}} = 120\pi \approx 377\Omega \tag{3.30}$$

Impedance in a dielectric is defined as

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0(1+\chi_m)}{\varepsilon_0(1+\chi_e)}} = 120\pi\sqrt{\frac{(1+\chi_m)}{(1+\chi_e)}} \tag{3.30}$$

Since materials with matched impedance are desired, this implies that the radical on the right side of (3.37) equals 1 or

$$(1+\chi_e) = (1+\chi_m) \tag{3.30}$$

The above equation defines the condition that the magnetic susceptibility equals the electric susceptibility, at least to first order. Since the second order, nonlinear electric susceptibility is found through Miller’s Rule, as an ansatz the second order, nonlinear magnetic susceptibility will be taken to be equal to its electric counterpart as well.

C. NONLINEAR FORMULATION

Based on the above arguments, a formulation for transformed first order susceptibility is now found. From this approximation, a second order susceptibility using Miller's Rule is derived. With this second order susceptibility, the nonlinear polarization field and magnetization of the dielectric metamaterial is found and are used as source terms for a standard electrostatic problem with the appropriate susceptibility.

$$\begin{aligned} P_i^{(NL)}(\mathbf{r}) &= \chi^{(2)}_{[e]ijk} E_j(\mathbf{r}) E_k(\mathbf{r}) \\ M_i^{(NL)}(\mathbf{r}) &= \chi^{(2)}_{[m]ijk} H_j(\mathbf{r}) H_k(\mathbf{r}) \end{aligned} \tag{3.30}$$

Adding the polarization field and the applied electric field together, the displacement field is found and analyzed to determine the radial component at the inner boundary of the medium. A radial component implies that there are fields within the cavity. The field caused by these nonlinear response terms is the "error field."

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IV. CYLINDRICAL CLOAK APPLICATION

A cylindrical cloak is modeled below to demonstrate a potential CDEW shield design. The cylindrical cloak is chosen because it is relatively straightforward mathematically while still demonstrating the concepts effectively. This design applies a coordinate transformation where the new coordinate system has had the origin stretched out to a radius $a = 1m$, with the outer radius $b = 1.5m$ providing finite spatial dimension. Only the electrical field is examined. The magnetic field would exhibit identical behavior due to the impedance matching requirement.

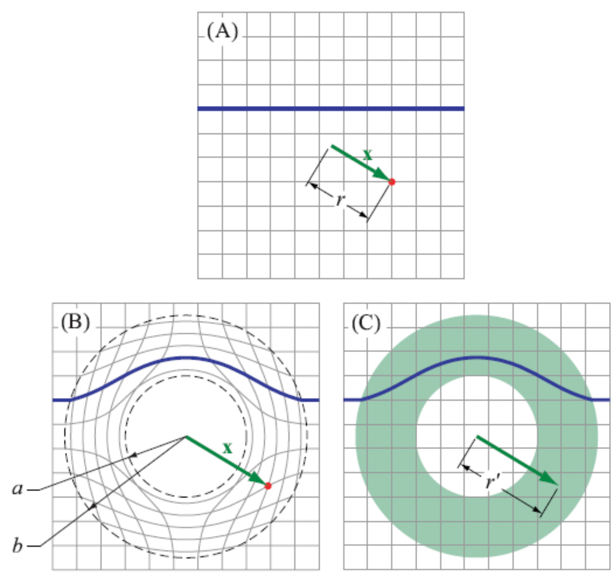

Figure 2. Cylindrical cloak example. The thick blue line shows the path of the same ray in (A) the original Cartesian space, and under two different interpretations of the electromagnetic equations, (B) the topological interpretation and (C) the materials interpretation. The position vector **x** is shown in both the original and transformed spaces, and the length of the vector where the transformed components are interpreted as Cartesian components is shown in (C). [11]

The MATLAB script file included in the appendix calculates the second order nonlinear susceptibility for a point contained within the transformation medium. Since by design, the transformation and susceptibility are position dependent, there exists a tensor field with a different tensor for every point within the medium. It would be tedious to list every result outputted by the script file. Instead, the calculations are outlined in broad terms and the results are then shown and analyzed.

A. CALCULATIONS

The tools outlined above now allow the calculation of the second order susceptibility directly from the first order, transformed susceptibility. Below are listed the steps, in order, to perform the calculation.

1. Transformation Matrix

The first step is to determine the transformation matrix needed to move from the primed, standard Cartesian coordinate system in electromagnetic or virtual space, to the unprimed system in physical space which contains the medium. In the unprimed system, the origin is expanded to a radius a. An outer radius b gives the transformation finite spatial extent. The designed material transforms the radial component yet preserves the angle and vertical components according to

$$\begin{aligned} r &= \frac{b-a}{b} r' + a \\ \phi &= \phi' \\ z &= z' \end{aligned} \tag{3.30}$$

When $r' = 0$ in the primed virtual coordinate system, the radius is “a” in the unprimed system, as expected. Additionally, when $r' = b$, the radius now equals “b” in the unprimed system.

Utilizing the standard equations relating Cartesian to cylindrical coordinates, it is found that

$$x = \frac{rx'}{\sqrt{x'^2 + y'^2}}$$
$$y = \frac{ry'}{\sqrt{x'^2 + y'^2}} \tag{3.31}$$
$$z = z'$$

By expressing both the primed and unprimed system in Cartesian systems, the metric tensor goes to the identity tensor and the calculations are simplified.

Using the standard formulation for the transformation matrix (3.3), the Cartesian transformation matrix from the unprimed to the primed system is [13]

$$A_{i'}^{i} = \begin{bmatrix} R\cos^2\phi + \frac{r}{r'}\sin^2\phi & \left(R - \frac{r}{r'}\right)\cos\phi\sin\phi & 0 \\ \left(R - \frac{r}{r'}\right)\cos\phi\sin\phi & \frac{r}{r'}\cos^2\phi + R\sin^2\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.32}$$

The above is written in cylindrical coordinate notation to save space with $R = \frac{dr}{dr'} = \frac{b-a}{b}$.

2. First Order Response

The linear displacement field is found using

$$D^{ij} = \varepsilon_o \varepsilon^{ij} E^{ij} \tag{3.32}$$

utilizing (3.34) for the permittivity. The permittivity of free space is included to provide units. The first order susceptibility, necessary for finding the second order susceptibility, is found using (3.35).

3. Second Order Response

Plugging the first order susceptibility into Miller's Rule (2.48) and performing the sum will yield a second order susceptibility for any given point within the confines of the medium, i.e. $r \leq b$. This is a rank 3 tensor and thus has 27 elements.

For the nonlinear case, the nonlinear displacement field is then found by using the second order susceptibility and applying it to (2.21), obtaining the nonlinear permittivity. This permittivity is then inserted into the equation for the displacement field (2.6) to obtain the nonlinear response.

B. RESULTS

In order for the cavity to be free of electric fields, the normal component of the displacement field must equal zero at the inner surface of the dielectric media. Strictly speaking, the difference between the normal components on the inner dielectric media boundary equals the surface charge density sigma.

$$\mathbf{D}_{1\perp} - \mathbf{D}_{2\perp} = \sigma \tag{3.32}$$

Since it is a dielectric and not a conductor, the surface charge density is taken to be zero. This defines the boundary conditions necessary to shield the cavity against incoming radiation fields. Another way of expressing this is that the radial component of D at the inner surface falls to zero, or that the dot product of the vector with its position vector equals zero, implying they are orthogonal.

For the below examples, the applied electric field equals 100,000 V/m or 100kV/m, which is a large electric field strength. While the intensity of the applied field certainly affects the field strengths in the solution, it does not affect the qualitative behavior of the results. The area of interest is in the nonlinear realm, but the linear response is included for completeness and to verify that the method of calculation is correct.

1. Linear Response

The linear response obtained from the MATLAB script falls in line with what is to be expected from a standard transformation optics design technique.

Below is a plot of the coordinate system and how it deforms under the transformation.

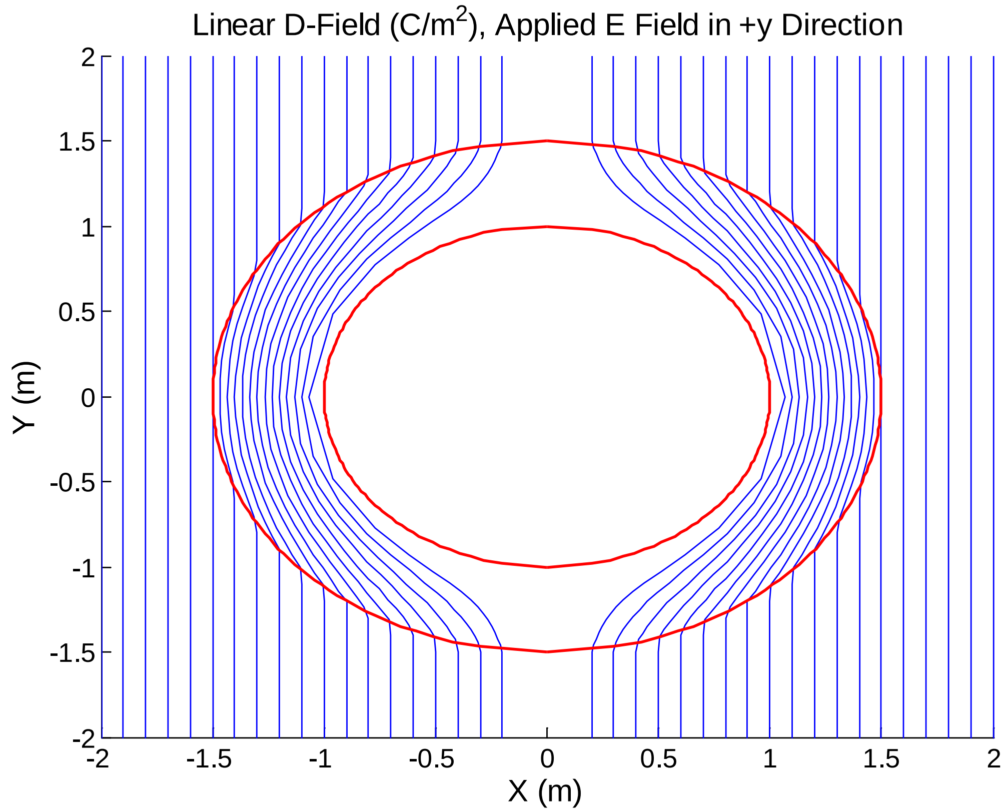

Figure 3. Linear response to an applied electric field oriented in the +y direction. The tangential behavior of the displacement field, shown in blue, implies shielding of the cavity. The streamlines are added to illustrate the geometry of the solution.

Each streamline is a line of constant x and demonstrates how the medium transforms space. As mentioned previously, the electromagnetic fields are taken to be attached to the coordinate system, and therefore it is expected that the displacement field vectors follow these lines.

Figure 4 shows the displacement vector field coincident with the transformed coordinate lines from the previous figure. The displacement field exhibits tangential behavior with the inner surface of the medium, implying zero radial components and thus shielding of the linear fields within the cavity. A close up view in Figure 5 shows that the displacement field vector near the boundary is indeed tangential.

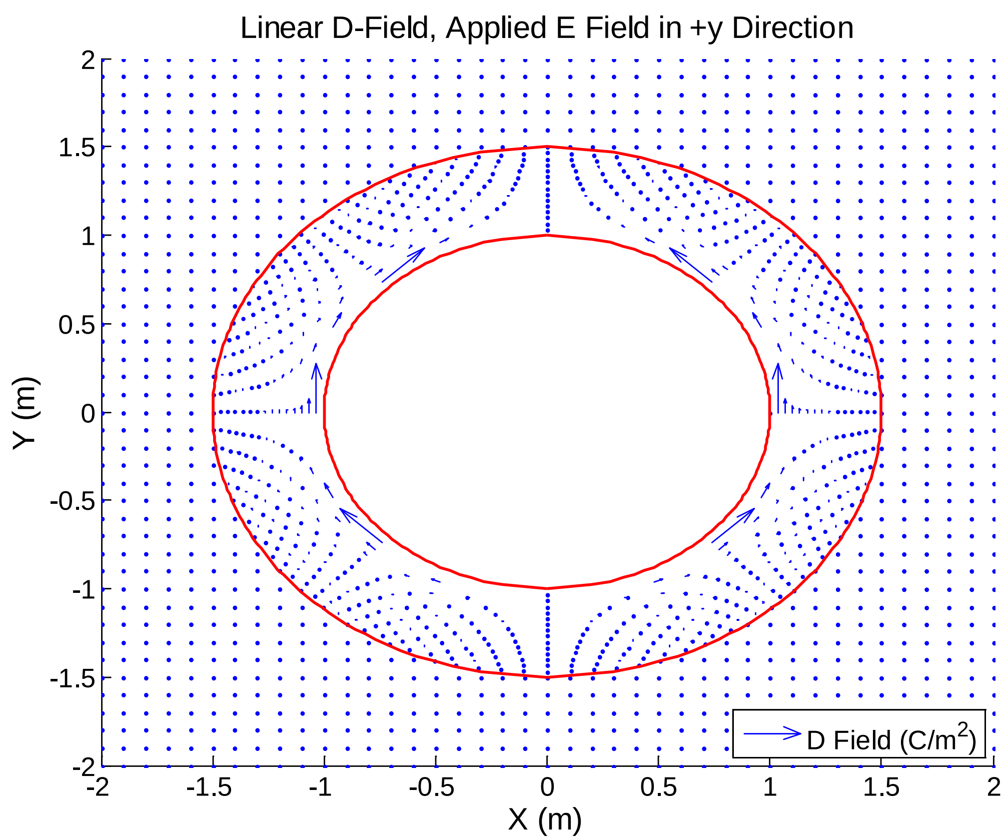

Figure 4. Displacement vector field transformed by the linear response of the medium. The tangential behavior of the D field is apparent.

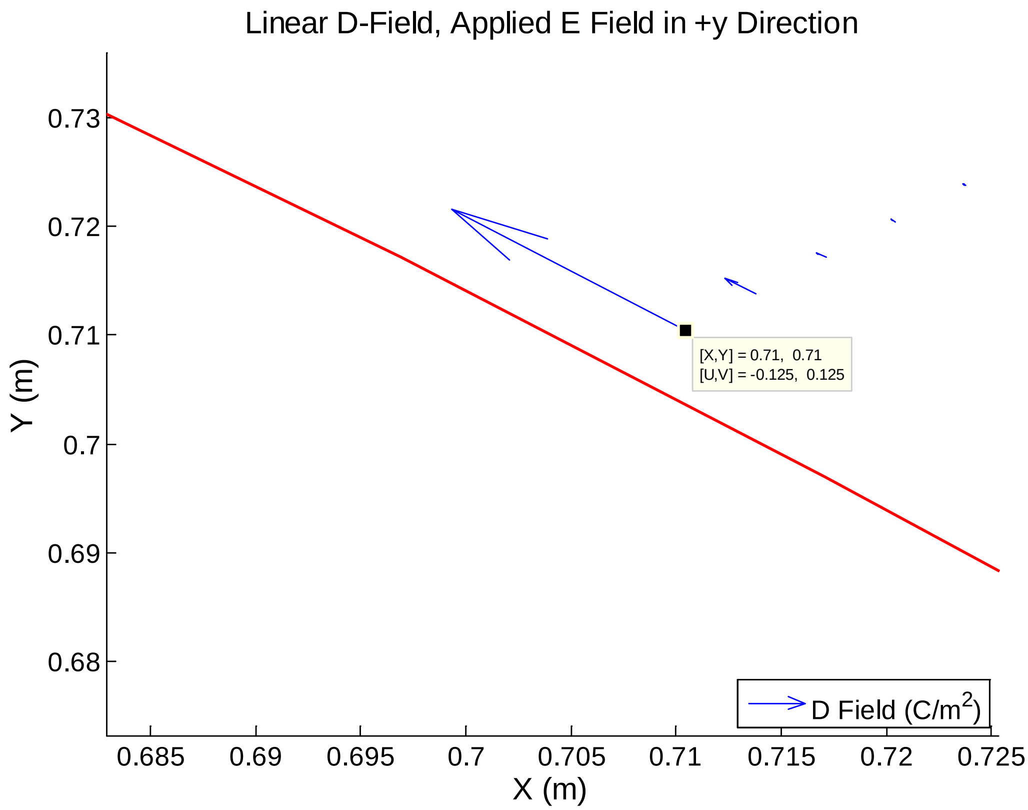

Figure 5. Close up of the linear displacement field as it approaches the boundary of the medium. As can be easily verified, the dot product of the field vector with its position vector is zero. Which means it is tangent to the surface of the medium. The [X,Y] values are the position components, the [U,V] values are the field components. X*U + Y*V = 0.

The radial component of the field line is zero as it approaches the inner medium boundary. The above linear response is expected and validates the methods used to calculate the fields.

Of note is the field strength as it approaches the inner boundary. Since the inner boundary is the origin in EM space blown out to finite spatial dimension in physical space, it is in effect a singularity. As the fields approach this inner boundary, the transformed field strength grows large. This is not unexpected however, as the first and second susceptibilities have inverse radial dependence. As this radius approaches “a” in physical space, or zero in primed virtual space, the fields increase. In the linear case, the radial components remain zero however and the cavity remains shielded.

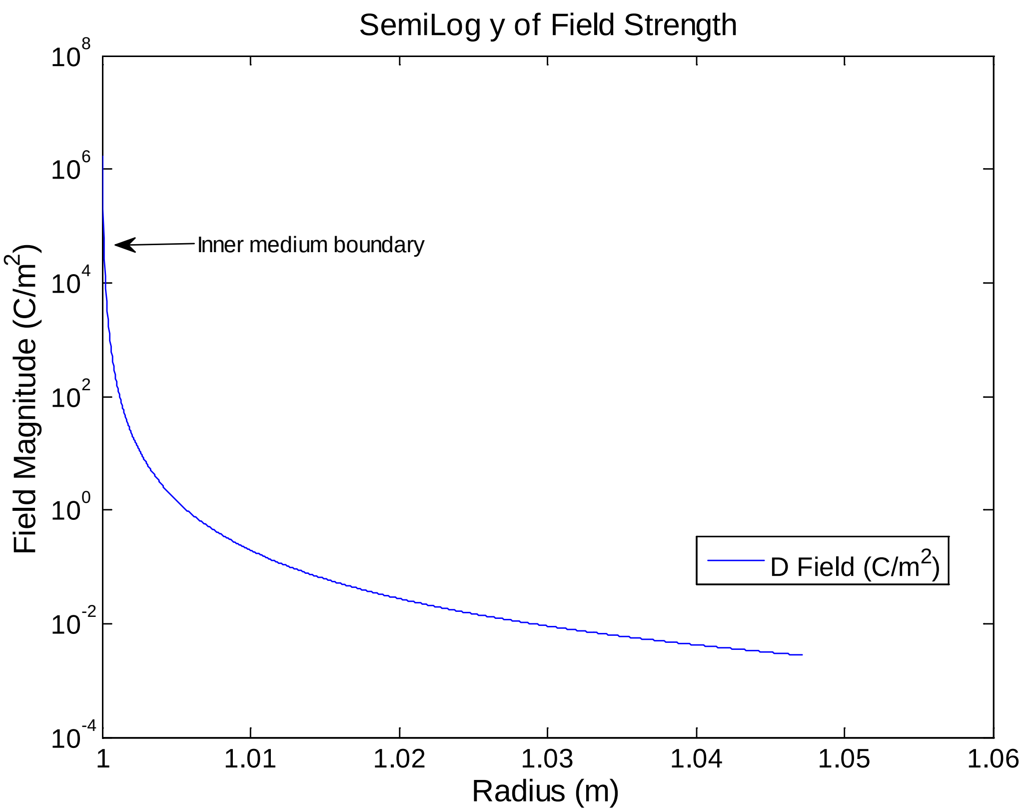

Figure 6. SemiLog plot of the field strength as a function of radius. The exponential growth of the field strength as it approaches the inner boundary is shown.

While the fields grow large, the energy remains finite, and thus does not violate conservation of energy. Consider the energy contained within a volume, which includes the origin, in virtual space. The energy density is mapped to physical space by the transformation medium through the coordinate transformation process. The coordinate transformation process however does not induce an energy flux across the boundary. Therefore, while the fields grow large in physical space, the energy remains constant and finite.

2. Nonlinear Response

The nonlinear response of the medium is now calculated from Miller's rule. In order to calculate the nonlinear displacement field, (2.6) is used with the nonlinear polarization field found from (2.21). Miller's coefficient is set at the experimentally derived average value of $4.52\text{x}10^{-2}\ m^2C^{-1}$. [10]

a. Miller's Coefficient = $4.52\text{x}10^{-2}\ m^2C^{-1}$

Below is the result of the calculations with Miller's coefficient equal to the accepted average, experimentally derived value. [10] At first, it is difficult to ascertain the radial component of the entire medium since the symmetry is odd. The symmetry differs from that of the linear case, but can be understood when one considers that the second order susceptibility is a cubic function rather than a direct product. The vectors in the second and fourth quadrants that are scaled to a point which can be seen appear to be tangential and in the opposite direction to the linear case.

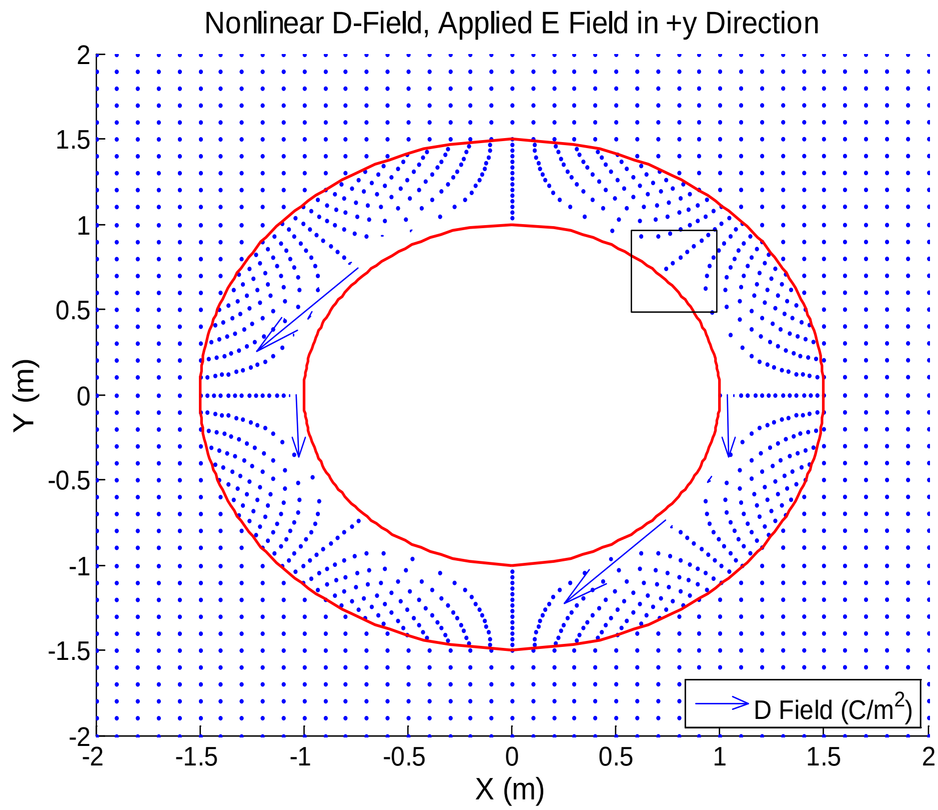

Figure 7. Displacement field with Miller's coefficient equal to 4.52x10–^{2} m^{2}C^{-1}. There appears to be a slight radial component as the medium inner boundary is approached in the first quadrant. The box is the area displayed in the following figure.

Zooming in on the area boxed in the figure demonstrates that there are radial fields present. Below is a close up view of a vector in the 1st quadrant closest to the boundary with phi equal to 45 degrees.

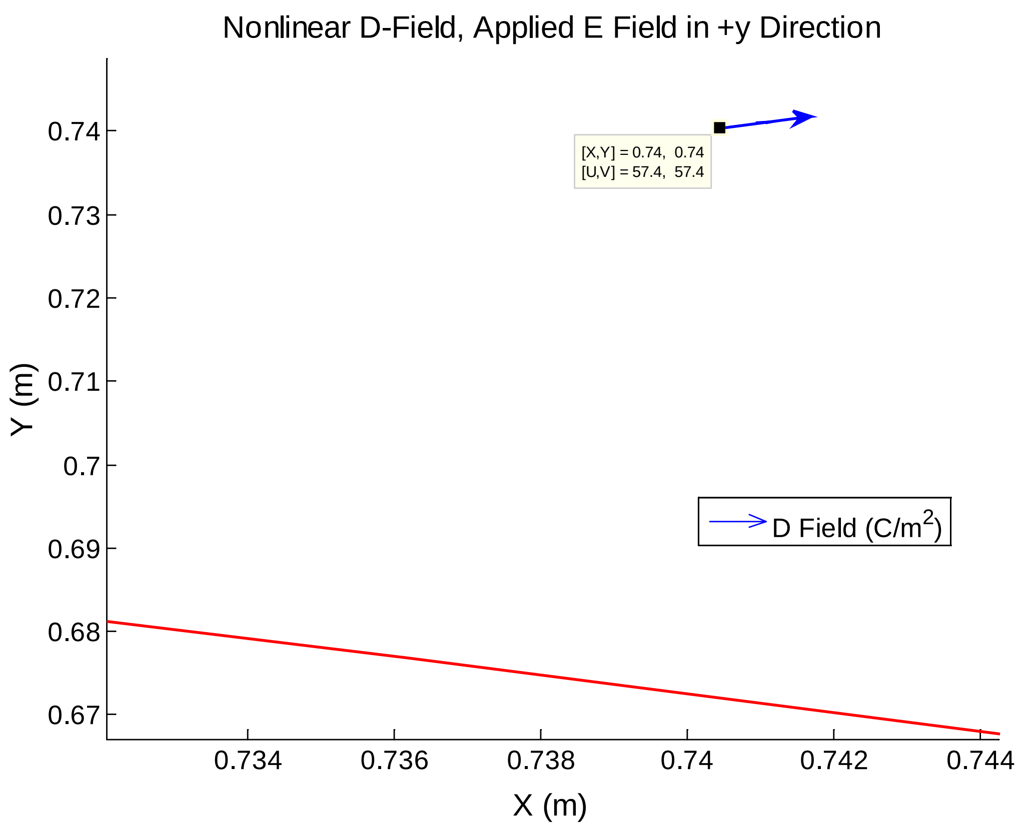

Figure 8. Close up of media boundary with Miller's coefficient = $4.52\text{x}10^{-2}$ m^2C^{-1}. The radial component of the displacement field is apparent. Taking the dot product of the field with its position vector yields a radial component of 84.95 C/m^2.

It appears that as the inner boundary is approached in the first and third quadrant, the displacement field magnitude increases and the field lines rotate to a non-tangential orientation. The strength of the radial component in the region increases as the inner medium boundary is approached, but using the same energy density arguments as in the linear case, the energy remains finite. It is also apparent from the below figure that the radial component varies as a function of phi for a constant radius as a result of the symmetry.

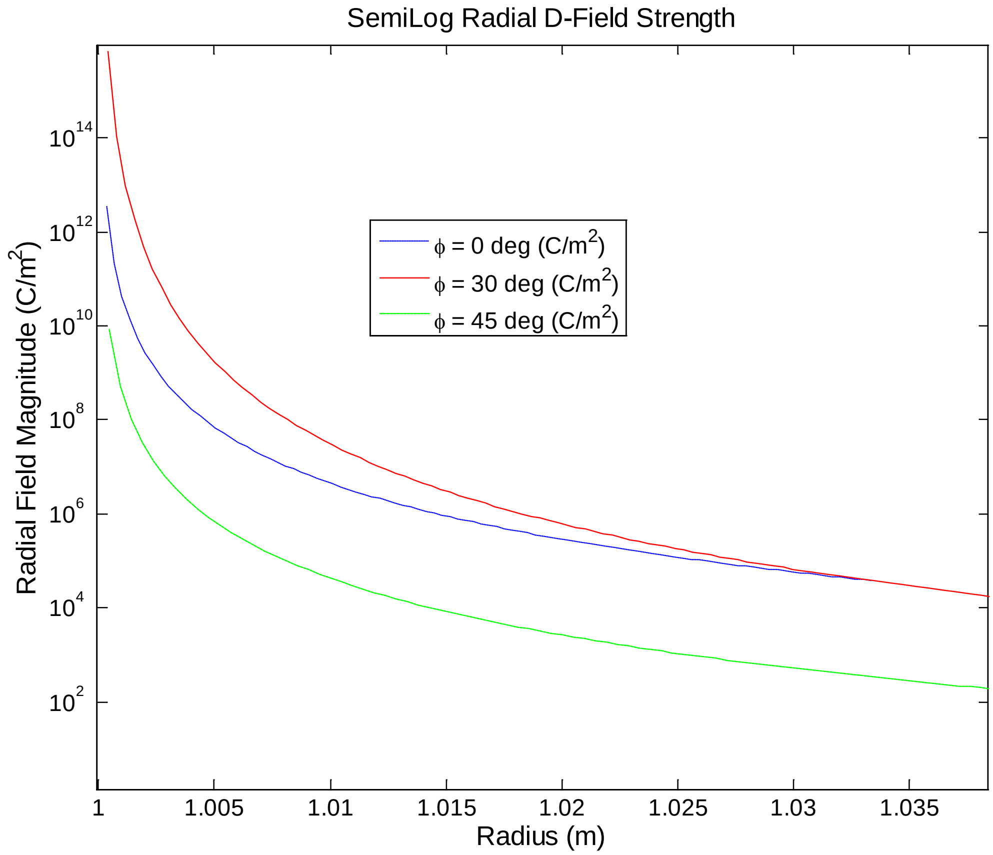

Figure 9. D field strength as a function of radius for several different values of phi. The radial component of the field increases by orders of magnitude in each case, demonstrating an inability of the medium to shield the cavity from nonlinear effects.

The two plots in Figure 9 demonstrate that the radial component of the displacement field at the inner medium boundary is not zero, implying a field within the cavity. This radial component is due to the polarization field blowing up near the singularity and dominating the vector sum in (2.6), or

$$\mathbf{D} \equiv \varepsilon_o \mathbf{E} + \mathbf{P} \qquad (3.33)$$

A closer examination of the P field in this case illustrates the behavior. As the inner boundary is approached, the polarization field grows at a much greater rate than the applied electric field. Thus, the polarization field dominates in the above sum and the radial component grows larger. This implies that the inner cavity is not shielded.

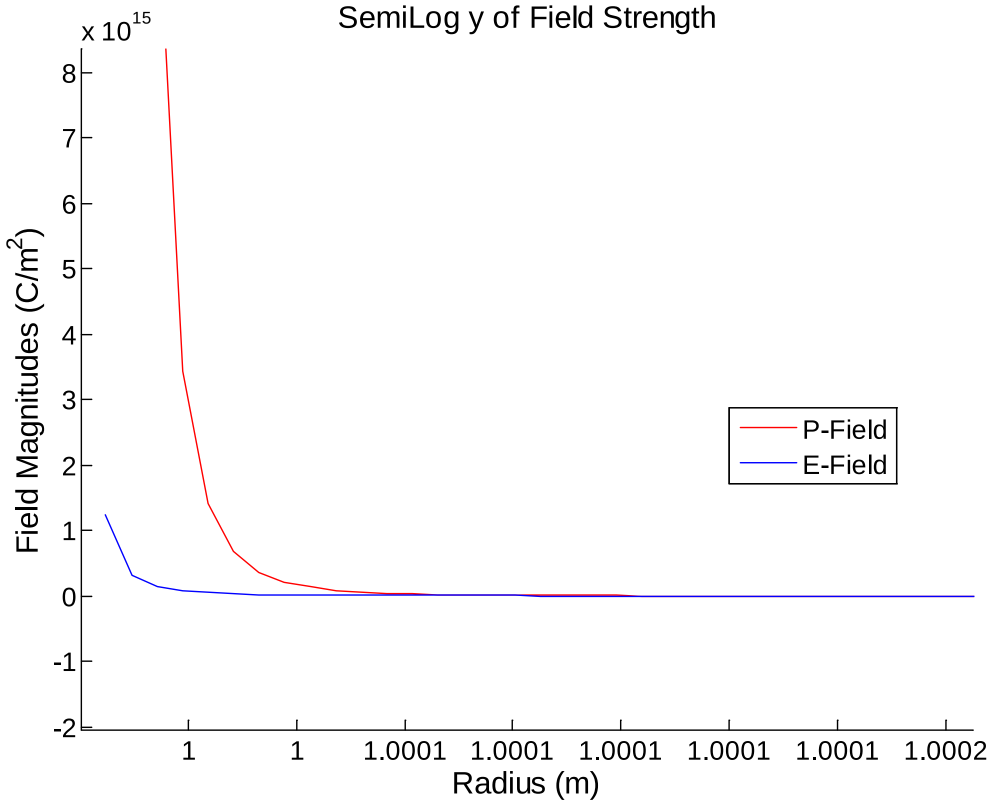

Figure 10. A semilog plot of the polarization and electric fields. The polarization field grows at a much greater rate as it approaches the singularity, causing the D field calculation to be dominated by the P field.

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V. CONCLUSIONS

A. FINDINGS

The above analysis demonstrates a radial component of the displacement vector near the inner boundary of the medium when dealing with the nonlinear response as described by Miller's rule. This implies the presence of a field within the cavity. Whether this field is strong enough to cause damage will of course be dependent on what is contained within the shield and its radiation tolerance, as well as the intensity of the incoming fields.

If Miller's rule is taken to be correct, then the process of determining the first order susceptibility through TO in the cylindrical cloaking case necessarily implies an inability to shield the cavity from nonlinear effects. In other words, the process of using Miller's rule to derive the susceptibility from a first order susceptibility found through a coordinate transformation destroys that transformation and therefore the geometry of the cloak for the second order response.

It is also possible that Miller's rule cannot derive the correct second order susceptibility when dealing with TO. As mentioned earlier, Miller's rule gives qualitative, not quantitative results. It is possible that Miller's rule is simply inapplicable when dealing with TO and that the second order susceptibility is not accurately described by it. If this is the case, then an alternate method needs to be derived in order to define the second order susceptibility. Perhaps defining the second order susceptibility through TO and then working backwards through Miller's rule to obtain the first order susceptibility would be a good technique. This is left for future work.

1. Inner Boundary as a Singularity

The inner boundary becomes a singularity in both the linear and nonlinear cases. In the linear case, while the field still blows up, it remains tangential. This is not true for the nonlinear case. In the nonlinear case, the polarization field blows up as it approaches the boundary, causing the displacement field to exhibit a radial component in certain regions. The nonlinear second order susceptibility causes a rotation of the polarization vector field. In turn, when that vector field blows up approaching the medium the radial component of the displacement field blows up. A boundary condition setting the field to zero at the inner medium is not applicable because the field at that point is desired. As mentioned previously however, the energy density remains finite.

This condition has a similar result documented in the literature when the problem is taken out of the quasi-static limit. [13] For perfect invisibility, rays traversing the medium need to traverse the transformed physical space in the same amount of time as they traverse the flat Cartesian electromagnetic virtual space, otherwise distortions would be introduced. This means that rays straddling the inner boundary need to traverse this distance in the same amount of time as it takes for them to traverse the point in electromagnetic virtual space. But the point in electromagnetic space has zero spatial extent, meaning that the rays need to be travelling infinitely fast. The conclusion drawn from this in the literature is that complete invisibility is not possible. [4] Complete invisibility is not necessary in this application however, and further study would need to be done to determine if the fact that the rays are unable to traverse infinitely fast implies that the rays are not completely shielded from the inner cavity, in both the linear and nonlinear cases.

B. SPACE APPLICATION FEASIBILITY

Metamaterials themselves have no intrinsic reason why they cannot be deployed on spacecraft. The materials thus far experimented with have been constructed of metals and plastics, but the designed electrical properties of the materials are what is important and not the actual materials. Ceramics are also being investigated. In fact, research is currently underway using metamaterials and TO to design antennas and electrical routing equipment that would require far less volume and power to operate in space. [14] [15]

The requirement of complete cloaking is also not of concern for space-based applications. A semicircular or flat plate placed in the expected direction of a directed energy weapon design to divert the fields away would prove to be sufficient. In addition, shielding of only critical components as a design philosophy is also feasible.

In general, there are no outright reasons why metamaterials and TO design techniques would not be suitable for spacecraft deployment. There are no power requirements, weight is determined by the materials themselves but is not limited to massive or fragile materials, and the spatial extent or size is not necessarily a concern depending on the design.

C. CONCLUSIONS AND FUTURE WORK

Whether or not TO offers a way to shield against nonlinear electromagnetic fields remains to be seen, but this work is a first step in that direction. Taking the problem out of the quasi-static arena and experimentation with high strength fields is necessary to increase understanding. This is left as future work.

Additional future work would include a more rigorous derivation of Miller's coefficient. While the coefficient has been experimentally determined, very little progress has been made in deriving a theoretical framework for why this is so. [10] A quantum mechanical or statistical mechanical approach may help to shed light of this problem.

Finally, as mentioned previously however, perfect cloaking is not the desired end state of this research, rather the elimination or diminution of the fields internal to the cavity to shield it from damage. The presence of normal field components demonstrated above implies that the nonlinear fields will "leak" into the cavity. The strength of these fields is what next needs to be determined. The calculations above say it would be an infinite field, but this is obviously nonphysical. Future work needs to be conducted to determine the strength of these fields.

Transformation optics and the use of metamaterials as transformation media is a nascent yet exciting development in electrodynamics. While the field of linear transformation optics has been developed to a point where real materials are producing results in the lab, the nonlinear effects are still a wide-open subject for theoretical as well as experimental studies.

APPENDIX. MATLAB CODE USED FOR CALCULATIONS

```
%{
THESIS SCRIPT
LCDR MATTHEW DEMARTINO
14DEC2012, NAVAL POSTGRADUATE SCHOOL

NOTES
THIS SCRIPT MAKES SEVERAL FUNCTION CALLS AND CALCULATIONS NECESSARY TO
PLOT THE LINEAR AND NONLINEAR DISPLACEMENT FIELDS FOR A CYCLINDRICAL
CLOAK OF OUTER RADIUS 1.5 M AND INNER RADIUS 1M. THE APPLIED ELECTRIC
FIELD IS 100kV/m ORIENTED IN THE POSITIVE Y DIRECTION
%}

clear all
close all
clc

I = [1 0 0;0 1 0;0 0 1];  %Identity matrix

%Define the virtual prime space extent and discrete step
step = .01;
minstart = -1.5;
maxstart = 1.5;
rng = (maxstart-minstart)/step;

[X,Y] = meshgrid(minstart:step:maxstart); %X’ and Y’ in flat EM space

%Define the transformation medium
a = 1;      %inner radius (m)
b = 1.5;    %outer radius (m)
z = 0;

eo = 8.854e-12; %permittivity of free space, SI units

%Initialize electric field vectors for each point x,y -- P = X Eo(jhat)
EX = zeros(size(X));
EY = zeros(size(Y))+1e6; %E = 0[x] + 100000[y]

%Initialize other data structures
%P fields, “2” => 2nd order, “N” => normalized
PX = zeros(size(X));
PY = zeros(size(Y));
PX2 = zeros(size(X));
PY2 = zeros(size(Y));
PX2N = PX2;
PY2N = PY2;

Z = zeros(size(X));  %Used for contour plots if desired

%D fields, “2” => 2nd order, “N” => normalized
DX = zeros(size(X));
```

```
DY = zeros(size(X));
DXN = zeros(size(X));
DYN = zeros(size(X));
DX2 = zeros(size(X));
DY2 = zeros(size(X));
DX2N = zeros(size(X));
DY2N = zeros(size(X));

%loops to step through each x and y coordinate in virtual space and
transform it and its fields to physical space
for(i = 1:rng)
    for(j = 1:rng)

        rprime = sqrt(X(i,j)^2+Y(i,j)^2);
        r = a + rprime*(b-a)/b;
        R =(b-a)/b;  %(dr/dr')
        phi = atan2(Y(i,j),X(i,j));

        if(r < b)
            %Calculate transformation matrix for point [X(i,j),Y(i,j)]
            A(1,1) = a/(rprime) - (a*X(i,j)^2)/(rprime^3) + (b-a)/b;
            A(1,2) = (-a*X(i,j)*Y(i,j))/(rprime^3);
            A(1,3) = 0;
            A(2,1) = (-a*X(i,j)*Y(i,j))/(rprime^3);
            A(2,2) = a/(rprime) - (a*Y(i,j)^2)/(rprime^3) + (b-a)/b;
            A(2,3) = 0;
            A(3,1) = 0;
            A(3,2) = 0;
            A(3,3) = 1;

            %calculate the first order susceptibility
            eps = eo*(A*A')./((r/rprime)*R);%first order trans epsilon
            Xi = eps/eo-I;                  %first order susceptibility

            %transform the electrical fields to their physical space
            %(unprimed) values
            EX(i,j) = (EX(i,j)*A(1,1)+EY(i,j)*A(1,2));
            EY(i,j) = (EX(i,j)*A(2,1)+EY(i,j)*A(2,2));

            %explicit matrix vector product for first order polariation
            PX(i,j) = eo*(Xi(1,1) * EX(i,j) + Xi(1,2) * EY(i,j));
            PY(i,j) = eo*(Xi(2,1) * EX(i,j) + Xi(2,2) * EY(i,j));

            %second order susceptibility
            Xi2 = SecOrdSuscept2(Xi);
            %2nd order polarization explicit calculation(2D)
            PX2(i,j) = eo*((Xi2(1,1,1)*EX(i,j)*EX(i,j)) +
(Xi2(1,1,2)*EX(i,j)*EY(i,j)) + (Xi2(1,2,1)*EY(i,j)*EX(i,j)) +
(Xi2(1,2,2)*EY(i,j)*EY(i,j)));
            PY2(i,j) = eo*((Xi2(2,1,1)*EX(i,j)*EX(i,j)) +
(Xi2(2,1,2)*EX(i,j)*EY(i,j)) + (Xi2(2,2,1)*EY(i,j)*EX(i,j)) +
(Xi2(2,2,2)*EY(i,j)*EY(i,j)));
            PX2N(i,j) = PX2(i,j)/sqrt(PX2(i,j)^2+PY2(i,j)^2);
            PY2N(i,j) = PY2(i,j)/sqrt(PX2(i,j)^2+PY2(i,j)^2);
```

```
            %calculate D fields
            %"N" = normalized, "2" = 2nd order
            DX(i,j) = eps(1,1)*EX(i,j)+ eps(1,2)*EY(i,j);
            DY(i,j) = eps(2,1)*EX(i,j)+ eps(2,2)*EY(i,j);
            DXN(i,j) = DX(i,j)/sqrt(DX(i,j)^2+DY(i,j)^2);
            DYN(i,j) = DY(i,j)/sqrt(DX(i,j)^2+DY(i,j)^2);
            DX2(i,j) = (eo*EX(i,j)+PX2(i,j));
            DY2(i,j) = (eo*EY(i,j)+PY2(i,j));
            DX2N(i,j) = DX2(i,j)/sqrt(DX2(i,j)^2+DY2(i,j)^2);
            DY2N(i,j) = DY2(i,j)/sqrt(DX2(i,j)^2+DY2(i,j)^2);

            %transform the points
            X(i,j) = r*cos(phi);
            Y(i,j) = r*sin(phi);

        end
    end
end


%Plotting functions
%PlotLinear;
%PlotNonLinear;

function [X2] = SecOrdSuscept(Xi)
%This function takes as input a first order susceptibility
%and returns a second order susceptibility based upon Miller's Rule.

%Calculate the second order susceptibity
X2 = zeros(3,3,3);  %rank 3 tensor => 27 elements

for(i = 1:3);

for(j = 1:3)
    for(k = 1:3)
        %Perform Miller's rule sum
        for(p = 1:3)
            for(q = 1:3)
                for(r = 1:3)
                    X2(i,j,k) = X2(i,j,k) + Xi(i,p)*Xi(q,j)*Xi(r,k);
                end
            end
        end
    end
end
end

K = 4.52e-2; %Miller's coefficient in m^2/C

X2 = -K*X2;
```

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