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STABILIZATION OF FREE-FLYING UNDERACTUATED MECHANISMS IN SPACE

Ranjan Mukherjee

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Under-actuated mechanisms provide low cost automation and can overcome actuator failures. These mechanisms are more suitable for space applications mainly because of their less weight and lower power consumption. Typical examples of useful under-actuated mechanisms in space would be large space structures and robot manipulators. Such mechanisms are however difficult to control because of the fewer number of actuators in the system. In this report we formulate the dynamics of an under-actuated mechanism using Hamilton's canonical equations. Next, we develop a theorem that provides us with some necessary ans some sufficient conditions for the asymptotic stability of autonomous systems. This theorem is more powerful than LaSalle's theorem when higher order derivatives of the Liapunov function can be easily computed. Finally, we use a Liapunov function approach to develop a control strategy to stabilize an under-actuated mechanism in space to an equilibrium manifold. The effectiveness of such control is verified using our asymptotic stability theorem.

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Abstract

Underactuated mechanisms provide low cost automation and can overcome actuator failures. These mechanisms are more suitable for space applications mainly because of their less weight and lower power consumption. Typical examples of useful underactuated mechanisms in space would be large space structures and robot manipulators. Such mechanisms are however difficult to control because of the fewer number of actuators in the system. In this paper we formulate the dynamics of an underactuated mechanism using Hamilton's canonical equations. Next, we develop a theorem that provides us with some necessary and some sufficient conditions for the asymptotic stability of autonomous systems. This theorem is more powerful than LaSalle's theorem when higher order derivatives of the Liapunov function can be easily computed. Finally, we use a Liapunov function approach to develop a control strategy that will stabilize an underactuated mechanism in space to an equilibrium manifold. The effectiveness of such control is verified using our asymptotic stability theorem.

1. Introduction

Structures in space are mostly required for high precision tasks, like in the case of the orbiting interferometer telescope, or in the case of the space station that needs to point its antenna in a specific direction. These space structures are made up of trusses that are designed to have a light weight. The motivation behind this is to minimize the payload of the rocket that sends it in orbit. Light weight members have lower structural rigidity. Therefore while designing structural elements for space, much attention is paid to the geometric shape of the members so as to maximize their structural rigidity. Nevertheless, these trusses still posses a significant amount of structural flexibility. This is a serious disadvantage for large space structures because they easily pick up vibrations due to their flexibility. Vibrations could be thermally induced by differential heating of the structure or could be induced by differential gravitational forces. In the case of structures like the space station where robots are expected to perform routine tasks, vibrations could be easily induced through dynamic interaction between the robot and the structure. A free-flying multidegree of freedom system in space is a non-holonomic system (Nakamura and Mukherjee, 1990, 1991). Such systems have a noninvolutive property and they will experience a change in orientation under periodic motion (Kane, Headrick and Yatteau, 1972; Vafa, 1987; Vafa and Dubowsky, 1987). Naturally, space structures will disorient themselves with time if vibrations persist. Though piezoelectric actuators may be used to damp out the vibrations in space structures, the system performs oscillations and undergoes an undesirable change in orientation over a prolonged period of time.

We now consider the prospect of replacing a single large flexible space structure with a chain of concatenated light weight members. Each of the members of the chain can be considered rigid due to their smaller dimension and can be assumed to be concatenated with revolute joints. In order to achieve control over the system, we intend to use motors at some of these joints (instead of piezoelectric actuators). The other joints would be left unactuated. We intend to control such an underactuated mechanism such that it would be possible to configure the system in any desired way. If such control can be established, underactuated mechanisms would provide a meaningful alternative to large flexible space structures.

Though the control of underactuated systems pose difficulties, in general they have a number of advantages. Underactuated manipulators have lower power consumption and also weigh less. Therefore these manipulators will be very suitable for space applications. Besides space, underactuated systems will find applications for low cost automation, hyper-redundant manipulators, and manipulators with actuator failures. The range of tasks that can be performed by underactuated manipulators are however limited since these systems are usually incapable of exerting forces. This limitation can be overcome by the use of brakes at the unactuated joints. These brakes need not be used to stop the motion of the unactuated joints. Instead they may be used as clamps to maintain a fixed configuration of the unactuated joints over certain periods of time. These brakes would then enable the manipulator to perform tasks like force control. They would also allow the manipulators to behave as reconfigurable actuated systems. In the absence of brakes, underactuated manipulators may be used with proper control to pick and place objects, and to perform non-contact tasks like spray painting, arc welding, etc.

Underactuated terrestrial robot manipulators were studied by Arai and Tachi (1990, 1991). In their studies, they assumed that the unactuated joints had brakes that could be used to stop the motion of the unactuated joints instantaneously. This simplification was used to eliminate the coupling between the actuated and unactuated links as and when desired. In 1991, Arai and Tachi proposed a PID control law to control the trajectory of the actuated joints only. They verified the effectiveness of their control law through experiments on a 2DOF manipulator with one passive joint. Jain and Rodriguez (1991) studied the kinematics and dynamics of underactuated manipulators. They adopted the spatial operator algebra to develop an algorithm for the inverse dynamics. Papadopoulos and Dubowsky (1991) proposed the failure recovery control of space robotic systems. They showed in their formulation that it may be possible to control the joint whose actuator has failed when there exists a dynamical coupling between this joint and a joint whose actuator is functioning properly. Furthermore, in order that the passive joint can be controlled, the system inertia has to be invariant with respect to the passive joint.

In this paper we model our underactuated space mechanism with an open link tree structure consisting of $(m+n)$ rigid links mounted on a free-flying space vehicle. n links of this mechanism are actuated while the rest m are unactuated. In the next section we use Hamilton's canonical equations (Goldstein, 1980; Nijmeijer and van der Schaft, 1990) to formulate the dynamics of this system. In section 4 we state and prove a theorem that provides us with some necessary and some sufficient conditions for the asymptotic stability of autonomous systems. This theorem is more powerful than LaSalle's theorem (LaSalle and Lefschetz, 1961) and gives us a systematic way to sort out the maximum invariant set from the set where the derivative of the Liapunov function (Liapunov, 1892) vanishes. Finally we use this theorem in section 5 to develop a control strategy for the stabilization of our underactuated mechanism in space to an equilibrium manifold.

2. Dynamics of free-flying underactuated systems - A Hamiltonian formulation

In this section we formulate the dynamical equations of free-floating underactuated multi-body systems in space. We assume without any loss of generality that the system is of the form of a manipulator mounted on a space vehicle, as shown in Fig.1. We assume that the manipulator has a total of $(m+n)$ joints, only n of which are actuated. The generalized coordinates of the system consist of $\boldsymbol{q}_1 \in R^6$ representing the position and orientation of the space vehicle, $\boldsymbol{q}_2 \in R^m$ representing the unactuated joint variables, and $\boldsymbol{q}_3 \in R^n$ representing the actuated joint variables. Due to the absence of gravitational potential energy in space, the Lagrangian $L_0(\boldsymbol{q}, \dot{\boldsymbol{q}})$ is equivalent to the kinetic energy of the system, and is given as

$$L_0(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \frac{1}{2}\dot{\boldsymbol{q}}^T \boldsymbol{M}\dot{\boldsymbol{q}}, \qquad \boldsymbol{q} \triangleq (\boldsymbol{q}_1^T \quad \boldsymbol{q}_2^T \quad \boldsymbol{q}_3^T)^T \in R^{(6+m+n)} \tag{1}$$

where, $\boldsymbol{M} \in R^{(6+m+n)\times(6+m+n)}$ is the inertia matrix of the system. It is a function of the joint variables $\boldsymbol{q}_2$ and $\boldsymbol{q}_3$, but not a function of the vehicle variables $\boldsymbol{q}_1$. This is true because the kinetic energy of the system is independent of the position and orientation of the space vehicle. Consequently, the Lagrangian is not a function of $\boldsymbol{q}_1$ and therefore the dynamics of the system is represented by the following vector equations:

$$\frac{d}{dt}\left(\frac{\partial L_0}{\partial \dot{\boldsymbol{q}}_1}\right) = 0 \tag{2}$$

$$\frac{d}{dt}\left(\frac{\partial L_0}{\partial \dot{\boldsymbol{q}}_2}\right) - \left(\frac{\partial L_0}{\partial \boldsymbol{q}_2}\right) = 0 \tag{3}$$

$$\frac{d}{dt}\left(\frac{\partial L_0}{\partial \dot{\boldsymbol{q}}_3}\right) - \left(\frac{\partial L_0}{\partial \boldsymbol{q}_3}\right) = \boldsymbol{\tau}^T \tag{4}$$

where, $\boldsymbol{\tau} \in R^n$ represents the vector of the joint torques at the actuated joints. The right hand side of Eq.(2) is zero because we do not use the reaction jets or momentum wheels of the space vehicle. By refraining from using reaction jets, we can minimize the usage of jet fuel on board the spacecraft which is limited in quantity, and therefore we can maximize the useful lifespan of the system.

When the Lagrangian is not a function of a set of generalized coordinates, like $\boldsymbol{q}_1$ in our case, we call these coordinates cyclic or ignorable coordinates. In the presence of cyclic coordinates, some physical quantity of the system is conserved. In our case, the linear and angular momentum of the whole system is conserved. This conservation law is expressed by Eq.(2) and can be simplified to the form

$$\boldsymbol{M}_1\dot{\boldsymbol{q}} = \boldsymbol{c} \tag{5}$$

where $\boldsymbol{M}_1 \in R^{6\times(6+m+n)}$ includes the top six rows of the inertia matrix in Eq.(1), and $\boldsymbol{c} \in R^6$ represents the initial linear and angular momentum of the system. The above equation represents six velocity constraints on the motion of the system; three of these are holonomic while the other three are nonholonomic (Nakamura and Mukherjee, 1990, 1991). On the other hand, Eq.(3) represents m nonintegrable constraints that include second order derivatives of the generalized coordinates, and are therefore second order nonholonomic constraints. The degrees of freedom of our system are n, and is equal to the dimension of the control variable $\boldsymbol{\tau}$.

We now use the transformation

$$L(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\tau}) = L_0(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{q}_3^T\boldsymbol{\tau} \tag{6}$$

to define the input dependent Lagrangian function $L(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\tau})$ (Nijmeijer and van der Schaft, 1990). Under this transformation, we have the following relations

$$\left(\frac{\partial L}{\partial \dot{\boldsymbol{q}}_3}\right) = \left(\frac{\partial L_0}{\partial \dot{\boldsymbol{q}}_3}\right), \qquad \left(\frac{\partial L}{\partial \boldsymbol{q}_3}\right) = \left(\frac{\partial L_0}{\partial \boldsymbol{q}_3}\right) + \boldsymbol{\tau}^T \tag{7}$$

By substituting Eqs.(6) and (7) into Eqs.(2), (3), and (4), we obtain the following homogeneous dynamical equations

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\boldsymbol{q}}_1}\right) = 0 \tag{8}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\boldsymbol{q}}_2}\right) - \left(\frac{\partial L}{\partial \boldsymbol{q}_2}\right) = 0 \tag{9}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\boldsymbol{q}}_3}\right) - \left(\frac{\partial L}{\partial \boldsymbol{q}_3}\right) = 0 \tag{10}$$

We define the generalized momentum $\boldsymbol{p} \in R^{(6+m+n)}$ corresponding to the generalized coordinates $\boldsymbol{q}$ by the relation

$$\boldsymbol{p} = \left(\frac{\partial L}{\partial \dot{\boldsymbol{q}}}\right)^T \in R^{(6+m+n)} \tag{11}$$

We next define the input dependent *Hamiltonian function* $H(\boldsymbol{q}, \boldsymbol{p}, \boldsymbol{\tau})$ for the system, with the help of a Legendre transformation, as follows

$$H(\boldsymbol{q}, \boldsymbol{p}, \boldsymbol{\tau}) = \boldsymbol{p}^T \dot{\boldsymbol{q}} - L(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\tau}) \tag{12}$$

Using the Legendre transformation in Eq.(12), the homogeneous dynamical equations given by Eqs.(8), (9), and (10) can be simply represented by Hamilton's canonical equations

$$\dot{\boldsymbol{q}} = \left(\frac{\partial H}{\partial \boldsymbol{p}}\right)^T, \qquad \dot{\boldsymbol{p}} = -\left(\frac{\partial H}{\partial \boldsymbol{q}}\right)^T \tag{13}$$

Additionally, by substituting Eq.(6) in Eq.(12) we get the relation

$$H(\boldsymbol{q}, \boldsymbol{p}, \boldsymbol{\tau}) = H_0(\boldsymbol{q}, \boldsymbol{p}) - \boldsymbol{q}_3^T \boldsymbol{\tau}, \qquad H_0(\boldsymbol{q}, \boldsymbol{p}) \triangleq \boldsymbol{p}^T \dot{\boldsymbol{q}} - L_0(\boldsymbol{q}, \dot{\boldsymbol{q}}) \tag{14}$$

which on differentiation yields

$$\dot{H}(\boldsymbol{q}, \boldsymbol{p}, \boldsymbol{\tau}) = \dot{H}_0(\boldsymbol{q}, \boldsymbol{p}) - \dot{\boldsymbol{q}}_3^T \boldsymbol{\tau} - \boldsymbol{q}_3^T \dot{\boldsymbol{\tau}}$$

$$\text{or,} \qquad \left(\frac{\partial H}{\partial \boldsymbol{q}}\right) \dot{\boldsymbol{q}} + \left(\frac{\partial H}{\partial \boldsymbol{p}}\right) \dot{\boldsymbol{p}} + \left(\frac{\partial H}{\partial \boldsymbol{\tau}}\right) \dot{\boldsymbol{\tau}} = \dot{H}_0 - \dot{\boldsymbol{q}}_3^T \boldsymbol{\tau} - \boldsymbol{q}_3^T \dot{\boldsymbol{\tau}} \tag{15}$$

By substituting the relation $(\partial H / \partial \boldsymbol{\tau})^T = \boldsymbol{q}_3$, and the canonical expressions of Eq.(13) in the above equation, we finally get

$$\dot{H}_0(\boldsymbol{q}, \boldsymbol{p}) = \dot{\boldsymbol{q}}_3^T \boldsymbol{\tau} \tag{16}$$

We now go back to Eq.(14) for the definition of the Hamiltonian function H_0. Using Eqs.(1), (6), and (11) we can show that

$$\boldsymbol{p} = \left(\frac{\partial L}{\partial \dot{\boldsymbol{q}}}\right)^T = \left(\frac{\partial L_0}{\partial \dot{\boldsymbol{q}}}\right)^T = \boldsymbol{M} \dot{\boldsymbol{q}} \tag{17}$$

Therefore, from the definition of H_0,

$$H_0 = \dot{\boldsymbol{q}}^T \boldsymbol{M} \dot{\boldsymbol{q}} - L_0 = \frac{1}{2} \dot{\boldsymbol{q}}^T \boldsymbol{M} \dot{\boldsymbol{q}} = L_0 \tag{18}$$

In other words, the Hamiltonian function H_0 of our system represents the kinetic energy or equivalently the total internal energy of the system. Although H_0 is equivalent to L_0, it is a function of $\boldsymbol{q}$ and $\boldsymbol{p}$ only, and therefore the correct expression for H_0 would be

$$H_0 = \frac{1}{2} \boldsymbol{p}^T \boldsymbol{M}^{-1} \boldsymbol{p} \tag{19}$$

which was obtained by substituting the relation $\dot{\boldsymbol{q}} = \boldsymbol{M}^{-1}\boldsymbol{p}$ from Eq.(17) into Eq.(18).

3. Issues of stability and controllability

In this section we first consider the stability of our nonlinear system from a linearization of the dynamics in the neighborhood of an equilibrium point. From Eqs.(13) and (14), our affine nonlinear system can be expressed by the form

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{B}\boldsymbol{\tau} \tag{20}$$

$$\boldsymbol{x} \triangleq \begin{pmatrix} \boldsymbol{q} \\ \boldsymbol{p} \end{pmatrix}, \qquad \boldsymbol{f} \triangleq (\,\partial H_0/\partial \boldsymbol{p} \quad \partial H_0/\partial \boldsymbol{q}\,)^T, \qquad \boldsymbol{B} \triangleq \begin{pmatrix} 0 \\ \boldsymbol{E}_n \end{pmatrix} \tag{21}$$

where $\boldsymbol{x} \in R^{2(6+m+n)}$ is the state vector, $\boldsymbol{f} \in R^{2(6+m+n)}$, $\boldsymbol{B} \in R^{2(6+m+n)\times n}$, and $\boldsymbol{E}_n$ represents the identity matrix of size n. Therefore a linearization of Eq.(20) around the equilibrium point $(\boldsymbol{q}_0, \boldsymbol{p}_0, \boldsymbol{\tau}_0) \equiv (0, 0, 0)$ gives

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{\tau} \tag{22}$$

$$\boldsymbol{A} \triangleq \begin{pmatrix} (\partial^2 H/\partial \boldsymbol{p}\partial \boldsymbol{q}) & \boldsymbol{M}^{-1} \\ -(\partial^2 H/\partial \boldsymbol{q}^2) & -(\partial^2 H/\partial \boldsymbol{q}\partial \boldsymbol{p}) \end{pmatrix} \in R^{2(6+m+n)\times 2(6+m+n)}$$

From the definition of the matrix $\boldsymbol{A}$ it is clear that

$$\sum_{i}^{2(6+m+n)} (\partial \dot{x}_i/\partial x_i) = tr(\boldsymbol{A}) = 0 \tag{23}$$

Equation (23) is the mathematical statement of *Liouville's Theorem* (Goldstein, 1980). The above equation implies that the linearized system has as many eigenvalues in the open left half plane as those in the open right half plane. Therefore in the absence of the control vector $\boldsymbol{\tau}$, we can conclude that the actual system is not exponentially stable.

The simplest approach to study the controllability of a nonlinear system as in Eq.(20) is to consider its linearization. If the linearized system is found to be controllable, the nonlinear system is controllable in the neighborhood of the equilibrium point. However the linearization approach is often unsatisfactory. In the process of linearization the nonlinear system may loose much of its structure. Therefore a nonlinear system may be controllable though its linearization may not. In our case, it can be easily verified that the rank of the matrix $(\boldsymbol{A} - s\boldsymbol{E}_{2(m+n+6)} \quad \boldsymbol{B})$, where $\boldsymbol{E}_i$ is the identity matrix of size i, is at most $2(m+n)+6$. Therefore the linearization of our system is not completely controllable.

The controllability of a number of simple nonholonomic systems like the rolling contact (Li and Canny, 1990) and the single and multibody car systems (Laumond, 1987) have been individually studied by constructing the control Lie algebra. The control Lie algebra is defined as the smallest involutive distribution containing the span of the vector fields of the system and closed under Lie bracket operations. For these systems the local controllability was ascertained by showing that the rank of the control Lie algebra is equal to the dimension of the state space. It should be emphasized that unlike most of these nonholonomic systems, our system has a drift term ($\boldsymbol{f}$ in Eq.(20)) due to the formulation of the problem at a dynamical level. Therefore the analysis based on the control Lie algebra cannot be performed on our system.

In general our system may be asymptotically stabilizable by means of a linear or a nonlinear feedback. However, Brockett (1983) has established some necessary conditions for the existence of smooth (infinitely continuously differentiable) stabilizing feedback laws for the general nonlinear system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}), \qquad \boldsymbol{x} \in R^N,\ \boldsymbol{u} \in R^M,\ \boldsymbol{f}(\boldsymbol{x}_e, 0) = 0 \tag{24}$$

with $\boldsymbol{f}(.,.)$ continuously differentiable in the neighborhood of the equilibrium point $(\boldsymbol{x}_e, 0)$. One of the three conditions require the mapping

$$\boldsymbol{\gamma} : R^N \times R^M \to R^N \qquad \text{defined by} \qquad \boldsymbol{\gamma} : (\boldsymbol{x}, \boldsymbol{u}) \mapsto \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u})$$

to be onto an open set containing the origin, where $\dot{\boldsymbol{x}} = 0$. When $\dot{\boldsymbol{q}} = 0$, $(\partial H_0 / \partial \boldsymbol{q}) = 0$ since H_0 is quadratic in $\dot{\boldsymbol{q}}$, and therefore from Eqs. (20) and (21) we arrive at

$$\dot{\boldsymbol{p}} = \begin{pmatrix} 0 \\ \boldsymbol{E}_n \end{pmatrix} \boldsymbol{\tau} \tag{25}$$

This clearly implies that the mapping $\boldsymbol{\gamma}$ is not onto an open set containing the origin. Hence there does not exist a smooth feedback law that can stabilize the system to an equilibrium point. This fact should however not perturb us for we can always consider the problem of stabilizing the system to an equilibrium manifold, or stabilizing the system to an equilibrium point via a non-smooth feedback. In this paper we consider only the problem of stabilizing the system to an equilibrium manifold. In our next paper we shall address the problem of stabilizing our system to an equilibrium point via a non-smooth feedback.

4. Theorem on Asymptotic Stability

The Liapunov stability theorems provide sufficient conditions for proving the asymptotic stability of dynamical systems. For autonomous systems these theorems are easy to apply when we can show that the derivative of the Liapunov function is negative definite. When the derivative of the Liapunov function is negative semidefinite, we often face problems. In such situations it may be possible to conclude the asymptotic stability of the system using LaSalle's theorem provided we can show that the maximum invariant set contains only the equilibrium point. It is always possible and easy to identify the set of points where the derivative of the Liapunov function vanishes but the maximum invariant set is only a subset of this set. The main challenge of LaSalle's theorem is therefore to sort out the maximum invariant set. More importantly, LaSalle's theorem is inapplicable to nonautonomous systems. In the event where the derivative of the Liapunov function vanishes, there exists no readily applicable result for proving the asymptotic stability of nonautonomous systems.

In this section we develop sufficient conditions for proving the asymptotic stability of autonomous systems when the first derivative of the Liapunov function is negative semidefinite. These sufficient conditions involve higher order derivatives of the Liapunov function that contain information of the higher order dynamics of the system. Consequently, it becomes easier to identify the maximum invariant set. In this section we also provide some necessary conditions for the asymptotic stability of autonomous and nonautonomous systems. Before stating our asymptotic stability theorem, we state the following Lemmas.

Lemma 1. A real function $f(t) \in C^2$ defined in (a, b) is concave iff $f''(t) \leq 0$, $\forall t \in (a, b)$.
Lemma 2. Let $f(t)$ be a nonpositive function such that $f(t_0) = 0$ and $f(t) < 0$ for some values of t. If the function $f(t)$ is analytic, then $f(t)$ is concave is some open neighborhood of t_0.

The proofs of the two Lemmas stated above have been provided in the Appendix for reference. Using these two lemmas we can conclude that if $f(t)$ is a smooth nonpositive function and $f(t_0) = 0$, then $f'(t_0) = 0$ because $f(t)$ is locally maximum at t_0, and $f''(t) \leq 0$ in some open neighborhood of t_0. If $f''(t_0) = 0$ also, then we can apply our lemmas to $f''(t)$. In such a case $f'''(t_0) = 0$, and $f''''(t) \leq 0$ in some open neighborhood of t_0. Our lemmas can therefore be applied recursively. When some even derivative of $f(t)$ vanishes at t_0, the next higher derivative which is an odd derivative also vanishes at t_0, and the second next derivative is nonpositive in some open neighborhood of t_0.

Let us now consider the nonautonomous system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(t, \boldsymbol{x}) \tag{26}$$

where $\boldsymbol{f} : R_+ \times D \rightarrow R^n$ is a smooth vector field on $R_+ \times D$, $D \subset R^n$ is a neighborhood of the origin $\boldsymbol{x} = 0$. Let $\boldsymbol{x} = 0$ be an equilibrium point for the system described by Eq.(26). We then have

$$\boldsymbol{f}(t, 0) = 0, \qquad \forall t \geq 0 \tag{27}$$

We next state and prove our theorem on asymptotic stability.

Theorem 1. Let $V(t, \boldsymbol{x}) : R_+ \times D \rightarrow R_+$ be locally positive definite and smooth on $R_+ \times D$, such that

$$\dot{V}(t, \boldsymbol{x}) \triangleq \partial V/\partial t + (\partial V/\partial \boldsymbol{x})\, \boldsymbol{f}(t, \boldsymbol{x}) \tag{28}$$

is locally negative semidefinite. Then whenever an odd derivative of V vanishes, the next derivative necessarily vanishes and the second next derivative is necessarily negative semidefinite. Furthermore, a sufficient condition for an autonomous system to be asymptotically stable is that there exists a positive integer k such that

$$\begin{cases} V^{(2k+1)}(\boldsymbol{x}) < 0 & \forall \boldsymbol{x} : \dot{V}(\boldsymbol{x}) = 0 \\ V^{(i)}(\boldsymbol{x}) = 0 & \text{for } i = 2, 3, \cdots, 2k \end{cases} \tag{29}$$

where $V^{(*)}(\boldsymbol{x})$ denotes the $(*) - th$ time derivative of V with respect to time.

Proof: The necessary conditions of this theorem can be proven very easily with the help of Lemmas 1 and 2.

To prove that Eq.(29) provides sufficient conditions for asymptotic stability, we first realize that $\boldsymbol{x} = 0$ is stable by standard argument since V is locally positive definite and $\dot{V} \leq 0$.

Next, since V is bounded from below by zero and V is nonincreasing ($\dot{V} \leq 0$), $V \rightarrow \alpha$, $\alpha \geq 0$, as $t \rightarrow \infty$.

Since V is smooth, $\dot{V}$ is uniformly continuous. Hence when $V \rightarrow \alpha$, $\dot{V} \rightarrow 0$ as $t \rightarrow \infty$, by Barbalat's lemma (Slotine and Li, 1991).

Since V is locally positive definite, $V \rightarrow 0 \quad \Rightarrow \quad \boldsymbol{x} \rightarrow 0$ as $t \rightarrow \infty$. Therefore if we can show that $\alpha = 0$ we can conclude asymptotic stability. We prove $\alpha = 0$ by contradiction. Since

$V \to \alpha \neq 0$ and V is locally positive definite, $\exists$ an open neighborhood $\mathcal{N}$ of $\boldsymbol{x} = 0$ such that the trajectory of $\boldsymbol{x}(t)$ lies outside $\mathcal{N}$ $\forall t > T$, and for some $T \geq 0$.

Let $S = \left\{ \boldsymbol{x} : \dot{V}(\boldsymbol{x}) = 0 \right\}$. Since $\boldsymbol{x}(t)$ converges to S but lies outside $\mathcal{N}$ for large t, the set $W = S - \mathcal{N}$ is nonempty and is the limit set for $\boldsymbol{x}(t)$. Then let U be an open neighborhood of W whose closure U^C does not contain $\boldsymbol{x} = 0$. Now let us denote

$$-\gamma = \max_{x \in U^C} V^{(2k+1)}(\boldsymbol{x}) \tag{30}$$

Then $-\gamma < 0$. Since $\boldsymbol{x}(t) \to W$ as $t \to \infty$, $\exists$ T_1 such that $\boldsymbol{x}(t) \in U^C$ $\forall t \geq T_1$. Now integrating $V^{(2k+1)}(t)$ with respect to time to get V, we have

$$\begin{aligned} V(t) - V(T_1) &= \underbrace{\int_{T_1}^{t} \cdots \int_{T_1}^{t}}_{(2k+1)} V^{(2k+1)}(t)dt \\ &\leq \underbrace{\int_{T_1}^{t} \cdots \int_{T_1}^{t}}_{(2k+1)} -\gamma dt \\ &= -\gamma \frac{(t - T_1)^{2k+1}}{(2k+1)!} \end{aligned} \tag{31}$$

Hence $V(t) \leq V(T_1) - \gamma(t - T_1)^{2k+1}/(2k+1)!$. Since $V(T_1) \leq V(t = 0)$, $V(t) \to -\infty$ as $t \to \infty$. This contradicts the fact that $V \geq 0$. Hence $\alpha = 0$ and that implies that the system is asymptotically stable.

5. Stabilization to an equilibrium manifold

The state variables of a free-flying underactuated mechanism in space was shown in Eq.(21) as $\boldsymbol{x} \triangleq (\, \boldsymbol{q}^T \quad \boldsymbol{p}^T \,)^T$, where $\boldsymbol{q} \triangleq (\, \boldsymbol{q}_1^T \quad \boldsymbol{q}_2^T \quad \boldsymbol{q}_3^T \,)^T \in R^{6+m+n}$ denote the generalized coordinates and $\boldsymbol{p} \in R^{6+m+n}$ denote the generalized momentum corresponding to these generalized coordinates. We intend to control the system in such a way that $\boldsymbol{p} \to 0$ and $\boldsymbol{q}_3 \to \boldsymbol{q}_{3d}$ at the final point of time. $\boldsymbol{q}_{3d}$ denotes the desired configuration of the actuated joints of the system. If such a control can be established, the underactuated system would come to a complete rest and the actuated joints of the system would converge to their desired values simultaneously. We therefore define a Liapunov function (Liapunov, 1892) v as

$$v = H_0 + \frac{1}{2}\Delta \boldsymbol{q}_3^T \Delta \boldsymbol{q}_3, \qquad \Delta \boldsymbol{q}_3 \triangleq (\boldsymbol{q}_{3d} - \boldsymbol{q}_3) \tag{32}$$

where H_0 is the Hamiltonian of the system defined by Eqs.(18) and (19). Since the Hamiltonian H_0 represents the total kinetic energy of the system, $H_0 = 0$ is attained only when $\boldsymbol{p} = 0$ or alternatively $\dot{q} = 0$. If we now define an equilibrium manifold $M_e = \{x : q_3 = q_{3d}, p = 0\}$, then the Liapunov function v defined by Eq.(32) is zero only on the equilibrium manifold and positive everywhere else. The derivative of v is next computed as

$$\begin{aligned} \dot{v} &= \dot{H}_0 - \Delta q_3^T \dot{q}_3 \\ &= \dot{q}_3^T \tau - \Delta q_3^T \dot{q}_3 = \dot{q}_3^T (\tau - \Delta q_3) \end{aligned} \tag{33}$$

where $\dot{H}_0 = \dot{q}_3^T \tau$ was substituted from Eq.(16). We now choose τ in Eq.(33) as

$$\tau = \Delta q_3 - \beta \dot{q}_3 \tag{34}$$

where β is a positive scalar quantity. Substitution of Eq.(34) in Eq.(33) yields

$$\dot{v} = -\beta \| \dot{q}_3 \|^2 \tag{35}$$

Clearly, $\dot{v}$ is negative semidefinite and is equal to zero when $\dot{q}_3 = 0$. At this point LaSalle's theorem (LaSalle and Lefschetz, 1961) could be used to conclude the asymptotic stability of the system to the equilibrium manifold provided we could show that $\dot{q}_3 = 0$ is attained only when $q_3 = q_{3d}$ and $p = 0$. Since LaSalle's theorem does not provide us with any systematic way to sort out the maximum invariant set from the set of all $x : \dot{v} = 0$, we refer to our theorem that was stated and proved in the earlier section.

By computing the second and the third derivatives of the Liapunov function v from Eq.(35) we can show that when $\dot{v} = 0$ or equivalently $\dot{q}_3 = 0$, $\ddot{v} \triangleq v^{(2)} = 0$ and $v^{(3)} = -2\beta \| \ddot{q}_3 \|^2 \leq 0$. Additionally if $v^{(3)} = 0$ then $\ddot{q}_3 = 0$. Then we can show by computing the higher order derivatives of v that $v^{(4)} = 0$ and $v^{(5)} = -6\beta \| q_3^{(3)} \|^2 \leq 0$, where $q_3^{(3)}$ is the third derivative of q_3 with respect to time. In other words whenever an odd derivative of the Liapunov function v vanishes, the next derivative also vanishes and the second next derivative is found to be negative semidefinite. This is in complete agreement with Lemmas 1 and 2. Furthermore this satisfies the necessary conditions of our asymptotic stability theorem.

From the above discussion it follows that the choice of the control vector τ in Eq.(34) results in

$$v^{(2k+1)} = -\beta_k \| q_3^{(k+1)} \|^2 \quad \text{for } \beta_k > 0, \ \text{andfor } k = 1, 2, \cdots, \infty \tag{36}$$

when $v^{(i)} = 0$ for $i = 1, 2, \cdots, 2k$. Therefore when $\dot{v} = 0$ or equivalently $\dot{q}_3 = 0$, if $q_3^{(k+1)} \neq 0$ for some positive integer k, then the sufficient conditions of our theorem given by Eq.(29) are satisfied and we can conclude asymptotic stability of our system to the equilibrium manifold M_e.

6. Conclusion

We have discussed in our paper the dynamics and control of underactuated mechanisms in space. The dynamics of the system was formulated using Hamilton's canonical equations. To prove the stability of our system we have developed a general asymptotic stability theorem. It is an elaboration of LaSalle's theorem and it provides us with a systematic way to sort out the maximum invariant set from the set where the dervative of the Liapunov function vanishes. Similar to LaSalle's theorem, the limitation of our theorem is that it is applicable only to autonomous systems. Using a Liapunov function approach we have developed in this paper a control strategy that brings an underactuated mechanism to rest and converges the actuated joints to their desired configuration simultaneously. We show that our control law is effective provided all the derivatives of the actuated joint velocities are not zero simultaneously. Hence the observability of the system, when we take the velocity of the actuated joints as the output, is going to play an important role in the stabilization. This part is going to be worked out using a geometric nonlinear control approach and will be appended to the paper by the time it is due. We will also include examples of situations where observability is lost, and provide the physical meaning of such situations.

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7. Appendix

Lemma 1. A real function $f(t) \in C^2$ defined in (a, b) is concave iff $f(t)'' \leq 0, \ \forall x \in (a, b)$.

Proof:

(a) Necessity

Let $x \in (a, b)$. Then for h small enough, $x - h,\ x + h \in (a, b)$. From the definition of concavity (Rudin), $f(x) \geq \frac{1}{2}(f(x-h) + f(x+h))$. Therefore, since $f \in C^2$,

$$f''(x) = lim_{h \to 0} \frac{f(x-h) + f(x+h) - 2f(x)}{h^2} \leq 0 \qquad (A-1)$$

(b) Sufficiency

Let $x, y \in (a, b)$, and $x < y$. For $\lambda \in [0, 1]$, and $t = \lambda x + (1 - \lambda) y$, the first order Taylor's series approximation of $f(x)$ and $f(y)$ are respectively

$$f(x) = f(t) + f'(t)(x - t) + f''(\xi_1)(x - t)^2, \qquad \xi_1 \in [x, t] \qquad (A-2)$$

$$f(y) = f(t) + f'(t)(y - t) + f''(\xi_2)(y - t)^2, \qquad \xi_2 \in [t, y] \qquad (A-3)$$

Therefore it follows that

$$\begin{aligned} \lambda f(x) + (1-\lambda) f(y) &= f(t) + \lambda f''(\xi_1)(x-t)^2 + (1-\lambda) f''(\xi_2)(y-t)^2 \\ &\leq f(t) \qquad \text{since } f''(\xi_1) \leq 0, \ f''(\xi_2) \leq 0 \end{aligned} \qquad (A-4)$$

Therefore the function is concave by definition.

Lemma 2. Let $f(t)$ be a nonpositive function such that $f(t_0) = 0$ and $f(t) < 0$ for some values of t. If the function $f(t)$ is analytic, then $f(t)$ is concave is some open neighborhood of t_0.

Proof: Since the function $f(t)$ is analytic, all derivatives of the function exist and the function can be expanded using Taylor's series as

$$f(t) = \sum_{n=0}^{\infty} \frac{f^{(n)}(t_0)}{n!}(t - t_0)^n \qquad (A-12)$$

Let us next assume that our function $f(t)$ is not concave in any open neighborhood of t_0. This implies from Lemma 1 that the condition $f''(t) \leq 0$ does not hold good in any open neighborhood of t_0. Therefore either $f''(t) \geq 0$, or $f''(t)$ changes sign in every open neighborhood of t_0. If $f''(t) \geq 0$ in every open neighborhood of t_0, then we can show from the corollary of Lemma 1 that $f(t)$ is convex everywhere. This is not true because $f(t)$ is nonpositive and has a maximum value at $t = t_0$. The other possibility is that $f''(t)$ changes sign in every open neighborhood of t_0. Then $f^{(n)}(t)$ for $n = 2, 3, \cdots, \infty$ changes sign in every open neighborhood of t_0. This implies that $f^{(n)}(t_0) = 0$ for $n = 2, 3, \cdots, \infty$. Additionally, since $f(t)$ is nonpositive and $f(t_0) = 0$, $f(t)$ achieves a local maximum at t_0. Therefore $f'(t_0) = 0$. Substituting these results in Eq.(A-12), we have $f(t) = 0$. This cannot be true because $f(t)$ is strictly negative for some values of t. We have therefore proved by contradiction that $f(t)$ is concave in some open neighborhood of t_0.

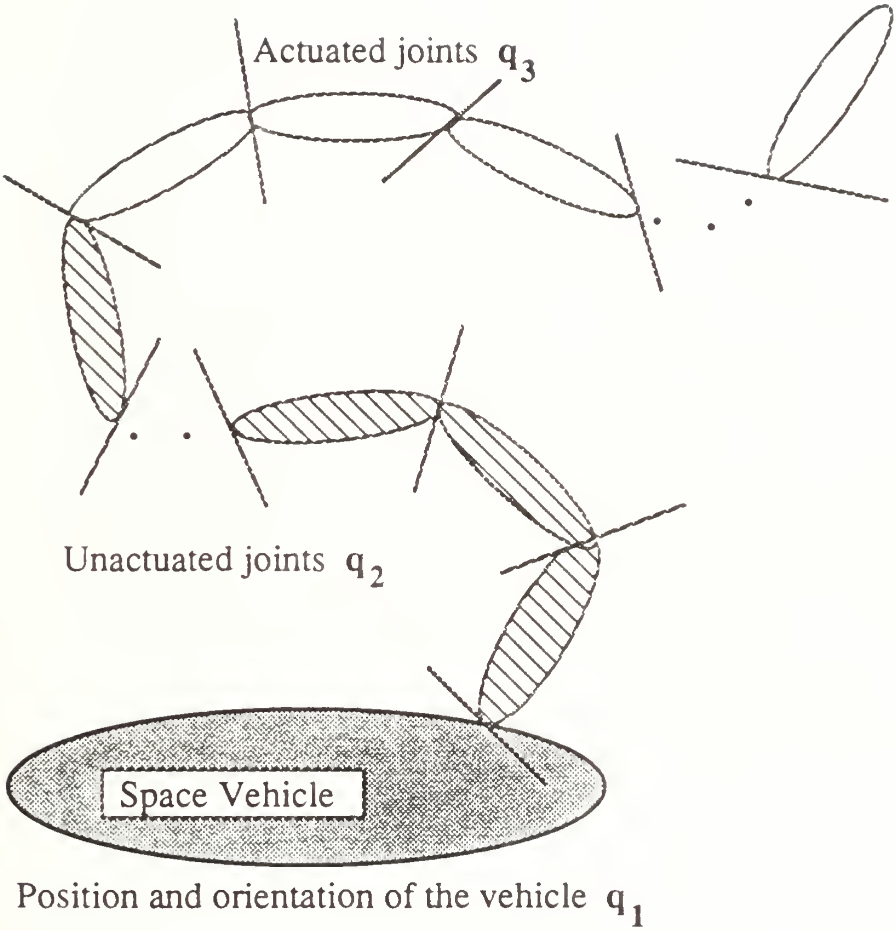

Fig.1. A free-flying under-actuated mechanism in space

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