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# The determination of Navy graduate school quotas

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# NAVAL POSTGRADUATE SCHOOL
## Monterey, California

THE DETERMINATION OF NAVY GRADUATE SCHOOL QUOTAS

by

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This paper presents both steady state and transient models for the determination of graduate education inputs (quotas) to meet forecasted future P-code requirements. Methods of smoothing fluctuating quotas are given. The pitfalls in the use of inventory/billet ratios are discussed, and comments are made on P-code requirements determination. Interactive computer programs written in APL are fully described in an Appendix.

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## 1. Introduction.

The purpose of this paper is to present simple models of the flow of officers through P-coded billets in order to determine yearly flows into graduate education to meet future billet requirements. Recent significant reductions in P-code billets in certain disciplines have led to serious problems in managing current inventories, in determining future educational inputs, and in determining how educational institutions should adjust to severely reduced inputs in a short period of time.

This report is written in six sections of which this is the first. In sections 2 through 5 it is assumed that the requirements for officers with graduate degrees is known (in section 6 some comments are made regarding determination of these requirements). Section 2 describes a steady state flow model in detail. An interactive computer program based on this model is described in Appendix (i). Section 3 describes two models which deal with the transient problems created with billet reductions. The first model leads to unsatisfactory cyclic quotas which result from an unrealistic (but not obvious) assumption in the model. In the second model we modify this hidden assumption and obtain much smoother quotas. The mathematical formulation of these models are given in the Appendix in sections (ii) and (iii) together with details of APL functions based on the models.

Section 4 contains comments and reasons for smoothing beyond the narrow mathematical considerations of section 3. Some of the factors discussed are difficult to quantify and so little mathematical reasoning is given. However, further smoothing of the quota from past and current levels to the new steady state levels given by the model in section 1 may be very important to the quality of graduate education.

Section 5 discusses problems in using the current inventory/billet ratio for a given P-code as an indicator of the "health" of that P-code community. Modifications of the current method are suggested and examples are given to show how the current ratio can be misleading in a transient stage caused by billet reduction.

Finally, in section 6, some comments are made concerning P-code billet requirements. These requirements drive the whole quota system and the models of sections 1 through 3 are for nought unless the billet requirements given as input to the models truly represent the requirement. Although it is not the purpose of this report to determine requirements it would not be complete without mentioning some of the problems associated with this difficult subject.

## 2. The Steady State Model.

The model developed in this section equates input rates and output rates to determine the basic flows through the officer billets. Minimum steady state flow rates (officers per year) into graduate education are determined which will meet future billet requirements.

In order to simplify notation, consider the P-code and designator fixed. The subscript $i$ or $j$ on the variables indicates rank, as follows:

| Subscript | Rank |
|---|---|
| 1 | LT |
| 2 | LCDR |
| 3 | CDR |
| 4 | CAPT |

Let $B_i$ be the number of P-coded billets in rank $i$ (for the fixed P-code and designator), and $T_i$ the tour length in these billets. If the $B_i$ billets have varying tour lengths, then $T_i$ can be thought of as an average.

Current career paths call for a time lag from the time an officer graduates to the time he serves in a P-coded billet. Many different paths are possible in this period and much turbulance can take place. This turbulance is summarized in a single parameter for each P-code and designator:

$\alpha_i$ = Fraction of officers who enter graduate education to meet a future billet requirement in rank $i$, who are still in the Navy and eligible to meet that requirement when it occurs, $i = 1,2,3,4$.

Note that the officer student may not be in rank $i$ when he enters school. Typically his rank would be $(i-1)$ with the current intervening 2-3 year tour between graduation and entrance into a P-coded billet. Clearly $\alpha_i$ is a function of policy as well as rank, P-code and designator. If a student immediately entered a P-code billet on graduation then $\alpha_i$ would be 1. If a student entered school as a LT to meet a P-code billet requirement as LCDR in five years (the current BuPers Model assumes this), and selection from LT to LCDR was 0.75, then $\alpha_2$ would be closer to 0.75.

Because of the complexities and interactions between the promotion structure, career paths, and rotation dates, it is possible that not all P-coded officers will get to serve a P-coded tour in rank. Again, the numerous alternative paths which officers can take cannot (and should not) all be treated as separate possibilities; rather they are summarized by a single parameter:

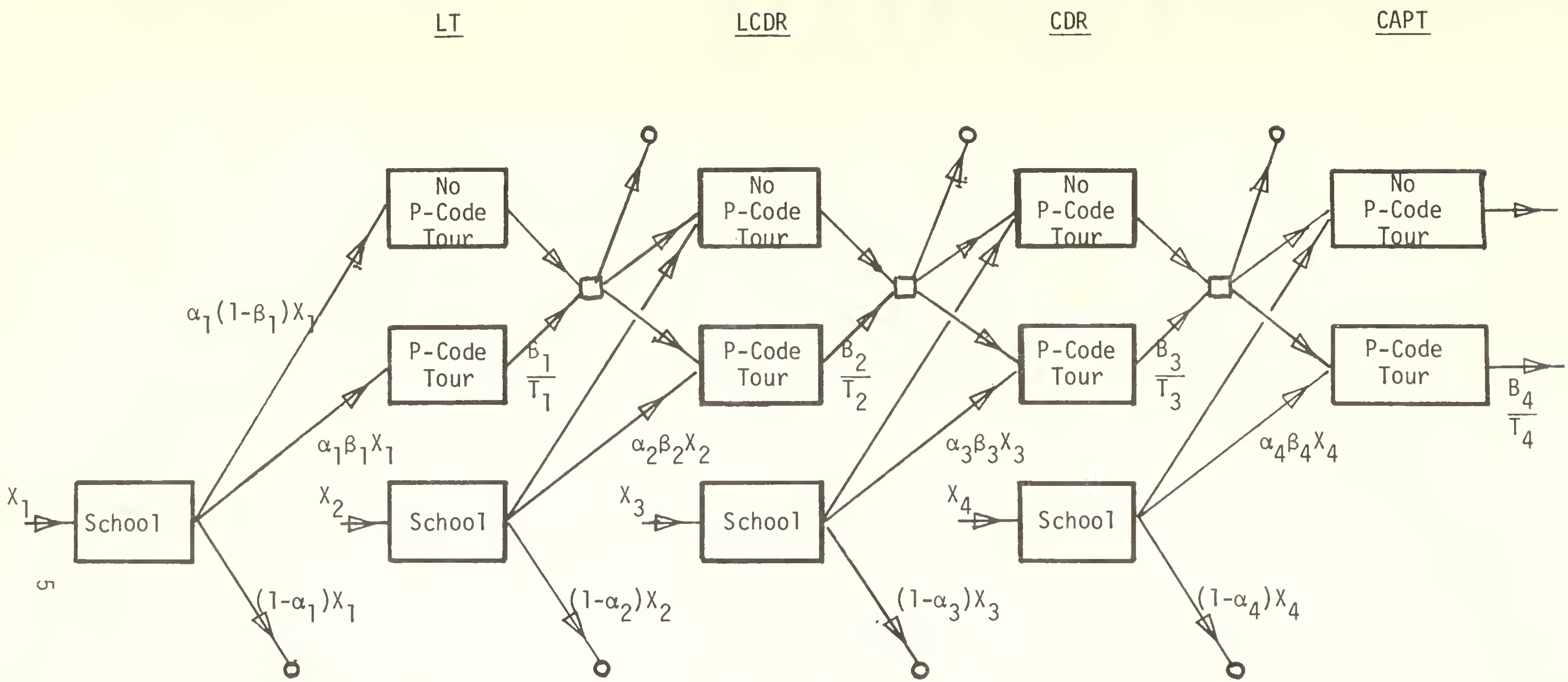
$\beta_i$ = Fraction of those available to serve a P-code tour in rank $i$ who get to serve such a tour.

In a perfect system $\beta_i$ would be 1. In practice it would be somewhat less than 1. Later we demonstrate how the parameters $\alpha_i$ and $\beta_i$ can be used to give bounds on the student input flow.

For ranks above LT it is possible (and desirable) that officers serve second or third tours in P-coded billets. We let

$\gamma_i$ = Fraction of those eligible to serve in a P-code billet in rank $i$ who are available to serve a P-code tour in rank $i + 1$.

The network of flows and inventories is shown in Figure 1. The basic flow rates for each rank are determined by the billets and tour lengths. Thus if in rank $i$ there are $B_i$ billets with an average tour length $T_i$, then $\frac{B_i}{T_i}$ of these billets become vacant each year (assuming steady state flows). Thus we must have an input flow equal to this number to meet the billet requirements.




Note: indicates "leaves the system."

Figure 1: Network of Steady State Flows Through Officer P-Code System.

Let $X_i$ be the input per year into graduate education necessary to meet the billet requirements in rank $i$, $i = 1,2,3,4$. Starting with $i = 1$ it is easy to see that the flow into graduate school to meet the LT billet requirements, $X_i$ is given by

$$X_1 = \frac{B_1}{\alpha_1 \beta_1 T_1} \cdot \quad (1)$$

PG Quota to meet LT Billet reqmts

The reader can check that with this input, if $\alpha_1$ remain in the system to fill a billet, and a fraction $\beta_1$ of these get to serve in the billet, then the flow into the LT billets is $\frac{B_1}{T_1}$, the outflow from these billets.

We move now to $i = 2$, the LCDR billets. Input into these must be $\frac{B_2}{T_2}$ per year. Now $\frac{B_1}{\beta_1 T_1}$ officers are in service at the end of the LT P-code tour. Of these a fraction $\gamma_1$ are available to serve in LCDR billets some time later. Now if $\frac{\gamma_1 B_1}{\beta_1 T_1} \geq \frac{B_2}{T_2}$, then not all these get to serve a second tour and no new input to graduate school is required for future LCDR billets. However, if $\frac{\gamma_1 B_1}{\beta_1 T_1} < \frac{B_2}{T_2}$, then we need some entry into graduate school to make up the deficit in the flow rate. It follows that

$$X_2 = \frac{1}{\alpha_2 \beta_2} \operatorname{Max}\left(0, \frac{B_2}{T_2} - \frac{\gamma_1 B_1}{\beta_1 T_1}\right),$$

or, by substituting (1) in the right-hand-side,

$$X_2 = \frac{1}{\alpha_2 \beta_2} \operatorname{Max}\left(0, \frac{B_2}{T_2} - \alpha_1 \gamma_1 X_1\right). \quad (2)$$

Similar arguments show that

$$X_3 = \frac{1}{\alpha_3 \beta_3} \operatorname{Max}\left(0, \frac{B_3}{T_3} - (\alpha_1 \gamma_1 \gamma_2 X_1 + \alpha_2 \gamma_2 X_2)\right). \quad (3)$$

and

$$X_4 = \frac{1}{\alpha_4 \beta_4} \operatorname{Max}\left(0, \frac{B_4}{T_4} - (\alpha_1 \gamma_1 \gamma_2 \gamma_3 X_1 + \alpha_2 \gamma_2 \gamma_3 X_2 + \alpha_3 \gamma_3 X_3)\right). \quad (4)$$

Finally, the total input per year to graduate school for the particular P-code and designator is $X_1 + X_2 + X_3 + X_4$.

<u>Example</u>: The current (September 1974) billet requirements for code 8510P, unrestricted line are given below in Table 1 together with three year tour lengths which are assumed for this example. The values of $\alpha$, $\beta$ and $\gamma$ are chosen assuming three year tours between graduation and entering a P-code billet, 75% selection of LT's to LCDR's, 70% selection of LCDR's to CDR's, and 50% selection of CDR's to CAPT's. We have also assumed a 90% utilization of graduates in their first tour, and 100% re-utilization (this last assumption leads to $\gamma_{i-1} = \alpha_i$). The values of $X_1$ to $X_4$ calculated using (1) to (4) are shown in Table 1.

| Rank $i$ | Billets $B_i$ | Tour Length $T_i$ | $\alpha_i$ | $\beta_i$ | $\gamma_i$ | $X_i$ |
|---|---|---|---|---|---|---|
| 1 | 35 | 3 | .95 | .90 | .75 | 14 |
| 2 | 67 | 3 | .75 | .90 | .70 | 19 |
| 3 | 59 | 3 | .70 | .90 | .50 | 5 |
| 4 | 12 | 3 | .50 | .90 | -- | 0 |
| Total | 173 | - | -- | -- | -- | 38 |

<u>Table 1</u>: <u>Example of Quota Determination Using 8510P URL Data</u>.

The parameters $\alpha$, $\beta$, $\gamma$ are of course subject to interpretation and are affected by policy changes such as changes in career paths. Suppose we wish to find a <u>lower bound</u> on the input to meet the billets in this example. This is done by setting $\alpha_i = \beta_i = \gamma_i = 1$ for all $i$. What this says is that everyone sent to school as a LTjg or above will serve in a P-coded tour at every opportunity and there will be 100% selection all the way to CAPT. With these extremes we get $X_1 = 12$, $X_2 = 10$, $X_3 = X_4 = 0$ for a total input of 22 per year. This lower bound would never be sufficient to meet requirements in a real system, but might serve as useful information when trying to estimate the effects of uncertainty about the $\alpha$, $\beta$ and $\gamma$ parameters. Suppose now that we keep the $\alpha_i$ and $\beta_i$ fixed as in Table 1, but reduce the tour lengths to 2.5 years and have no one complete two or more P-coded tours. Thus $\gamma_i = 0$ and the new input would be: $X_1 = 16$, $X_2 = 40$, $X_3 = 37$, $X_4 = 11$, for a total of 104.

Clearly both these examples are extremes, but are given to illustrate how the model can be used to determine the effects of different policies. Using the same data as in Table 1, but with $\beta_i = 1.0 \quad i = 1,2,3,4$, we obtain Table 2. Thus by using all P-coded personnel at every opportunity the total quota is reduced from 38 to 36. Suppose in addition to all $\beta_i = 1$ we plan on two 3-year tours in the rank of LCDR. We then set $T_2 = 6$ and repeat the calculations. The results are shown in Table 3. Note that the total quota is decreased to 32, but the "mix" of the four inputs changes considerably.

An interactive computer program has been written which enables the user to change the various parameters for a given P-code and designator at a terminal. The program is written in APL and is given in the Appendix together with an explanation of how it is used and an example.

| Rank $i$ | Billets $B_i$ | Tour Length $T_i$ | $\alpha_i$ | $\beta_i$ | $\gamma_i$ | $X_i$ |
|---|---|---|---|---|---|---|
| 1 | 35 | 3 | .95 | 1.0 | .75 | 12 |
| 2 | 67 | 3 | .75 | 1.0 | .70 | 18 |
| 3 | 59 | 3 | .70 | 1.0 | .50 | 6 |
| 4 | 12 | 3 | .50 | 1.0 | -- | -- |
| Total | 173 | -- | -- | --- | -- | 36 |

Table 2: Quota calculation using Table 1 data, but with $\beta_i = 1$.

| Rank $i$ | Billets $B_i$ | Tour Length $T_i$ | $\alpha_i$ | $\beta_i$ | $\gamma_i$ | $X_i$ |
|---|---|---|---|---|---|---|
| 1 | 35 | 3 | .95 | 1.0 | .75 | 12 |
| 2 | 67 | 6 | .75 | 1.0 | .70 | 3 |
| 3 | 59 | 3 | .70 | 1.0 | .50 | 17 |
| 4 | 12 | 3 | .50 | 1.0 | -- | -- |
| Total | 173 | -- | -- | --- | -- | 32 |

Table 3: Quota calculation using Table 2 data, but with $T_2 = 6$.

3. Transient Problems and Quota Smoothing.

The model in section 1 assumes that the number of billets in a given rank, P-code, and designator are constant over some reasonable time period. The model does not have enough detailed structure to enable us to examine the transient effects of a change in the number of billets. Rather large decreases in P-coded billets have recently occurred and the effects of such changes on the quota are the subject of this section.

Before giving a mathematical flow formulation of the transient problem a simple example is used to illustrate the concepts. Consider a fictitious system which, up to and including the current planning period, had 90 billets requiring school education. For simplicity we assume that people are sent to school in year 1 of their career for a 1 year school program, and immediately follow this with three years in one of the 90 billets. We assume that everyone graduates and stays in the system at least four years.

Example 1: A Simple Illustrative Example.

This simple example is illustrated in Figure 2. Planning periods increase down the page and period 0 is the current period. Since we assume we have been running in steady state the current flowrate is 30 people per year out of the billets. Thus we have 30 people in school in period 0 ready to fill the vacated bilets in period 1. The four numbers in the top line boxes are a legacy of previous policy and cannot be changed.

Assume that a 40% reduction in billets occurs in period 1 and that the number of new billets will be constant at 54. Since there are 90 educated people in period 1 we have an unavoidable excess inventory of 36 people. Our problem now is to calculate the new school inputs in periods 1,2,3,... .

| Planning Period | School 1 | Tour 2 | Tour 3 | Tour 4 | Total Billets | Excess Inventory |
|---|---|---|---|---|---|---|
| 0 | 30 | 30 | 30 | 30 | 90 | 0 |
| 1 | 0 | 30 | 30 | 30 | 54 | 36 |
| 2 | 24 | 0 | 30 | 30 | 54 | 6 |
| 3 | 30 | 24 | 0 | 30 | 54 | 0 |
| 4 | 0 | 30 | 24 | 0 | 54 | 0 |
| 5 | 24 | 0 | 30 | 24 | 54 | 0 |
| 6 | 30 | 24 | 0 | 30 | 54 | 0 |
| 7 | | 30 | 24 | 0 | 54 | 0 |

Figure 2. Personnel Flows in a Simple Transient Example.

In period 2 the total inventory will be 60 plus the input from those trained in school in period 1. Since 60 already exceeds the 54 billets the school input in period 1 is zero. In period 3 the total inventory will be 30 plus the input from school in period 2. Since we need 54 the school input in period 2 must be 24.

It is easy to calculate the inputs for this example. For periods 1 through 6 they are 0, 24, 30, 0, 24, 30. The reader can see immediately that a cyclic input results. This is an extremely undesirable feature for planning purposes. Firstly, it adversely affects the school which must try to meet widely varying inputs with a relatively stable faculty. Secondly, it leads to gross inequities in educational openings between year groups.

This cyclic input feature is not restricted to our simple model, but is a result of the underlying arguments. These arguments hide some unwitting and unrealistic assumptions to which we return later. First we demonstrate the same cyclic feature of the quota derived for a realistic example.

Example 2: A Realistic Example Showing Cyclic Quotas.

Table 4 gives the basic data used in the example. It is based on real data for the 8510 P-code of URL officers and billets in November 1974. The current inventory (period 0) is given in column 2 by year group. Columns 3, 4 and 5 give the expected additions to the inventory in planning years 1, 2 and 3 from students currently enrolled in the 360 curriculum at Monterey. Column 6 gives the assumed continuation rates from the year given by the row, to the next year. Column 7 gives the total P-coded billets by rank, which are the same as those used in the example in section 1. The horizontal lines indicate that LT billets are held by officers with 8-10 years commissioned service, LCDR billets by officers with 14-16 years, CDR billets by officers with 19-21 years, and CAPT billets by officers with 24-26 years of service.

| Years Commissioned Service | Current Inventory | Student Additions Yr 1 | Yr 2 | | Continuation Rates | P-Coded Billets | |
|---|---|---|---|---|---|---|---|
| 1 | | | | | 1.00 | | |
| 2 | | | | | 1.00 | | |
| 3 | 4 | | | | 1.00 | | |
| 4 | 6 | 1 | 2 | 2 | 0.98 | | |
| 5 | 6 | 1 | 4 | 4 | 1.00 | | |
| 6 | 9 | 4 | 8 | 2 | 1.00 | | |
| 7 | 13 | 3 | 8 | | 1.00 | | |
| 8 | 21 | | 8 | | 1.00 | | |
| 9 | 36 | | 3 | 2 | 0.75 | 35 | LT Billets |
| 10 | 35 | 4 | 3 | 3 | 1.00 | | |
| 11 | 51 | 4 | 2 | | 1.00 | | |
| 12 | 43 | | 1 | | 1.00 | | |
| 13 | 42 | 1 | | | 1.00 | | |
| 14 | 33 | 1 | 1 | 1 | 1.00 | | |
| 15 | 37 | | 1 | | 1.00 | 67 | LCDR Billets |
| 16 | 30 | | | | 0.70 | | |
| 17 | 25 | | | | 1.00 | | |
| 18 | 25 | | | | 1.00 | | |
| 19 | 21 | | | | 1.00 | | |
| 20 | 16 | | | | 1.00 | 59 | CDR Billets |
| 21 | 11 | | | | 0.50 | | |
| 22 | 5 | | | | 1.00 | | |
| 23 | 15 | | | | 1.00 | | |
| 24 | 5 | | | | 1.00 | | |
| 25 | 6 | | | | 1.00 | 12 | CAPT Billets |
| 26 | 6 | | | | 1.00 | | |
| 27 | 2 | | | | 1.00 | | |
| 28 | 2 | | | | 1.00 | | |
| 29 | 2 | | | | 1.00 | | |
| 30 | 6 | | | | 0 | | |
| Total | 512 | | | | | | |

Table 4: Basic Data Input for Transient Quota Model Calculation.

A mathematical formulation and description of the model is given in Appendix (ii) together with a computer program listing in APL and a sample example of the input and output. The results of the quota calculations for 15 years using the data in Table 4 are shown in Table 5. We have assumed a 5-year lag between entering school and entering a P-coded billet. Thus group 1 input would enter school in their third year of service, group 2 in their ninth year, group 3 in their fourteenth year, and group 4 (if any) in their nineteenth year.

A cyclic trend is clearly evident in groups 2 and 3. The total quota is also cyclic, and fluctuates from about 20 to 45 after an initial period. The reader can check that the averages of the 3-period cylces give 13, 17, 6, 0 respectively, for the four groups, which agree closely with the results of the steady state model (section 1) in Table 2 with $\beta_i = 1.0$, $i = 1,2,3,4$. It is implicitly assumed in the transient model that all educated offices will serve in a P-coded tour at every opportunity.

Example 3: The Simple Example with Different Assumptions.

We now return to our simple example where the 90 billets are reduced to 54. Although Figure 2 is simple and seems to demonstrate the correct flows, they are correct only if a hidden assumption is valid. This assumption is illustrated in Figure 3. Each box in each planning period now contains two numbers. The lower number in each box is the number of educated people filling P-coded billets. The upper number is the number of people educated, but not holding a P-coded billet. By looking at these two different communities an unrealistic assumption which is present, but hidden, in Figure 2 is demonstrated.

In the current period 0, since there are 90 billets there is no excess inventory of people in their second, third or fourth year. In period 1, 36

| Planning Period | School Quotas | | | | |
|---|---|---|---|---|---|
| | GRP 1 | GRP 2 | GRP 3 | GRP 4 | TOTAL |
| 1 | 7 | | | | 7 |
| 2 | 12 | | | | 12 |
| 3 | 12 | 22 | | | 32 |
| 4 | 13 | 7 | | | 20 |
| 5 | 14 | 5 | | | 19 |
| 6 | 12 | 33 | | | 45 |
| 7 | 13 | 12 | 17 | | 43 |
| 8 | 13 | 6 | | | 20 |
| 9 | 12 | 33 | | | 45 |
| 10 | 13 | 12 | 17 | | 43 |
| 11 | 13 | 7 | | | 20 |
| 12 | 13 | 32 | | | 45 |
| 13 | 13 | 12 | 17 | | 43 |
| 14 | 13 | 7 | | | 20 |
| 15 | 13 | 32 | | | 45 |

Table 5: 15 Yr Quota Calculations Using the Data in Table 4 and Assumptions in Example 1.

| Planning Period | School 1 | Tour 2 | Tour 3 | Tour 4 | Total Billets | Excess Inventory |
|---|---|---|---|---|---|---|
| 0 | | 0 | 0 | 0 | | 0 |
| | 30 | 30 | 30 | 30 | 90 | |
| 1 | | 0 | 6 | 30 | | 36 |
| | 0 | 30 | 24 | 0 | 54 | |
| 2 | | 0 | 0 | 6 | | 6 |
| | 24 | 0 | 30 | 24 | 54 | |
| 3 | | 0 | 0 | 0 | | 0 |
| | 30 | 24 | 0 | 30 | 54 | |
| 4 | | 0 | 0 | 0 | | 0 |
| | 0 | 30 | 24 | 0 | 54 | |
| 5 | | 0 | 0 | 0 | | 0 |
| | 24 | 0 | 30 | 24 | 54 | |
| 6 | | 0 | 0 | 0 | | 0 |
| | 30 | 24 | 0 | 30 | 54 | |
| 7 | | 0 | 0 | 0 | | 0 |
| | | 30 | 24 | 0 | 54 | |

Figure 3: Simple Example Assuming all Billets Removed from People at the End of Their Tours.

billets are "removed." In reality these billets usually have their P-code removed so they are not counted in the total which is used to plan the future educational input. The billet still exists usually, but in an uncoded form. This is an important point as we shall see.

The important question is, of the 36 billets which are removed, how many are filled with people currently in their first, second or third year in the billet? In order for the flows in Figure 2 to be correct we have assumed that of the 36 billets removed, 30 had people in their last year in the billet, and the other 6 had people in their second year in the billet. This assumption leads to the second line in Figure 3. In tour year one there is no spare inventory and 30 people in P-coded billets. In tour year two there are 6 now in uncoded billets and 24 in P-coded billets, and in tour year three there are 30 now in uncoded billets. As these move through the system in successive planning periods the same school input is generated as in Figure 2.

The removal of P-codes from billets is done independently of the length of time that the person has been in the billet. If 36 billets have their P-code removed, it is more realistic to assume that these are equally spread over the tour years. Therefore, of the 36 billets removed we assume 12 are currently filled by people in their first year, 12 by people in their second year, and 12 by people in their third year. This assumption leads to the numbers in Figure 4.

In period 1 there are 12 in "excess inventory" and 18 in P-coded billets in tour years 1 through 3. In period 2, 18 P-coded billets become vacant. If these are to be filled with new school input then we need 18 as the quota in period 1. Following this argument the reader can easily see that the quotas obtained in Figure 4 are all equal to the stationary value of 18.

| | School | Tour | | | Total Billets | Excess Inventory |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | | |
| 0 | | 0 | 0 | 0 | | 0 |
| | 30 | 30 | 30 | 30 | 90 | |
| 1 | | 12 | 12 | 12 | | 36 |
| | 18 | 18 | 18 | 18 | 54 | |
| 2 | | 0 | 12 | 12 | | 24 |
| | 18 | 18 | 18 | 18 | 54 | |
| 3 | | 0 | 0 | 12 | | 12 |
| | 18 | 18 | 18 | 18 | 54 | |
| 4 | | 0 | 0 | 0 | | 0 |
| | 18 | 18 | 18 | 18 | 54 | |
| 5 | | 0 | 0 | 0 | | 0 |
| | 18 | 18 | 18 | 18 | 54 | |
| 6 | | 0 | 0 | 0 | | 0 |
| | 18 | 18 | 18 | 18 | 54 | |
| 7 | | 0 | 0 | 0 | | 0 |
| | | 18 | 18 | 18 | 54 | |

Figure 4: Simple Example Assuming Billets Removed Uniformly Across Tour Years.

Thus, by making the more realistic assumption that the billets removed are distributed uniformly over the tour years we eliminate the cyclic nature of the quota and produce a stationary school input.

The reader will notice that the total excess inventory in Figure 4 (62) exceeds that in Figure 3 (42). But the question is, how much of this increase is unavoidable? Suppose that the billets which have their P-code removed are distributed uniformly among tour years. It might be possible to use some of the excess inventory in P-coded billets as vacancies occur in future years. Let us look at the 12 excess in tour year one, period 1. Can these be used to offset new school input in period 1 by being moved to P-coded billets in period 2? These 12 people have just started a tour and are in their first year. To use them in a P-coded billet would mean transferring them after no more than one year in their current billet. They would also be one year off in their career path if they were to be kept in a P-coded billet for a full 3-year tour. Such movements can be made, but the costs can be high, both in the dollar cost of transfer and in morale and efficiency costs associated with broken tours.

The real system is, of course, more complex, with a two-year school period, a 2-3 year intermediate tour between school and P-coded billet, continuation rates sometimes less than 1.0 and multiple P-coded tours in a career. However, the fundamental problems shown in Figures 3 and 4 still apply. Although there appears to be considerable unused P-coded inventory, much more of it is unusable than is assumed in the simple model leading to Figure 2.

Example 4: Smoothing the Quotas in Example 2.

We return now to example 2 with the data for the 8510 URL P-code. Using the data in Table 4 but assuming billets are uniformly distributed over the tour years as in example 2, we obtain the quotas in Table 6. The mathematical description of this smoothing is described in Appendix (iii) together with an APL computer program and an example.

The reader can see immediately how the quotas have been smoothed and reach steady state (12, 18, 6, 0) in six years.

| Planning Period | School Quotas | | | | |
|---|---|---|---|---|---|
| | GRP 1 | GRP 2 | GRP 3 | GRP 4 | TOTAL |
| 1 | 6 | 5 | | | 11 |
| 2 | 12 | 10 | | | 22 |
| 3 | 12 | 8 | | | 20 |
| 4 | 13 | 12 | | | 25 |
| 5 | 14 | 15 | | | 29 |
| 6 | 12 | 18 | 6 | | 36 |
| 7 | 13 | 18 | 6 | | 36 |
| 8 | 13 | 16 | 6 | | 35 |
| 9 | 12 | 18 | 6 | | 36 |
| 10 | 13 | 17 | 6 | | 36 |
| 11 | 13 | 16 | 6 | | 35 |
| 12 | 13 | 18 | 6 | | 36 |
| 13 | 13 | 17 | 6 | | 36 |
| 14 | 13 | 17 | 6 | | 35 |
| 15 | 13 | 18 | 6 | | 36 |

Table 6: 15 Yr Quota Calculations Using Data in Table 4 and Assumption in Example 3.

4. Further Smoothing Arguments.

Table 6 in section 2 illustrates the quotas for the 8510P URL code when the smoothing model is used. Although there is much less fluctuation between years than is seen in Table 5, a comparison of the quotas in Table 6 with past school inputs illustrates dramatically the effect of a large billet reduction made in one time period with no thought given to its effect on the system.

Past billet requirements have fluctuated, and growth factors have been applied to them to forecast future billet requirements. Recall that there is typically a five year period between school entrance and P-coded billet entrance. Therefore, the quota for next year (FY76) is aimed at meeting requirements in FY 81. Again using the 8510P code as an example, growth rates as high as 23% per year were applied to current billet levels to project ahead five years. At the time these growth rates were considered reasonable, given the results of a delphi-technique method used by the Navy to determine them. Today the growth rates are considered to be zero, and the billet base to which the growth rate applied has dropped from about 280 to below 200 (see Figure 7, page 35).

Table 7 shows past school inputs (at Monterey) for the 8510P code from FY 70 through FY 75, together with the forecasted future quotas for FY 76 through FY 80 from Table 6. Percentage changes from year to year are given also. This data is plotted in Figures 5 and 6.

The curves in Figures 5 and 6 clearly demonstrate what typically happens to future quota predictions when large billet reductions occur in a single planning period. We see that the system "overreacts" causing severe percentage changes from year to year as seen in Figure 6. This phenomenon has been widely observed in industrial production processes which try to adjust to changing demand. There is a large literature in "Production Smoothing" where the aim

| Fiscal Year | USN Input/Quota | Percent Change | |
|---|---|---|---|
| 1970 | 96 | -- | |
| 71 | 82 | -14.6 | |
| 72 | 74 | - 9.8 | ↑ |
| 73 | 55 | -25.7 | |
| 74 | 45 | -18.2 | Past |
| 75 | 38* | -15.6 | Inputs |
| 76 | 11 | -71.1 | Projected |
| 77 | 22 | +100.0 | Quota |
| 78 | 20 | - 9.1 | |
| 79 | 25 | +25.0 | ↓ |
| 80 | 29 | +26.0 | |

Table 7: Past Inputs and Future Quotas for the 8510P Code.

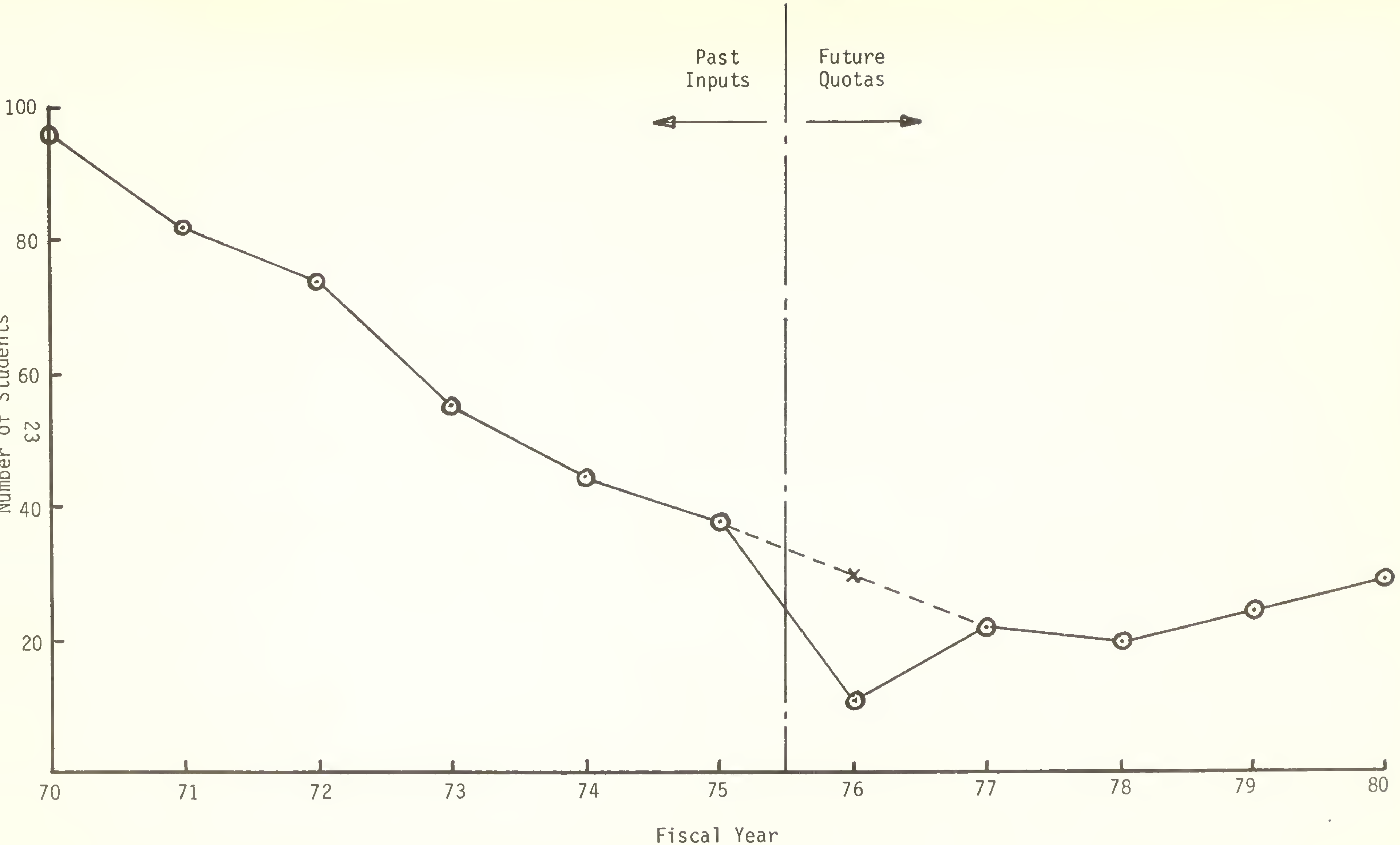



Figure 5: Past Input and Future Quotas for the 8510P Code.

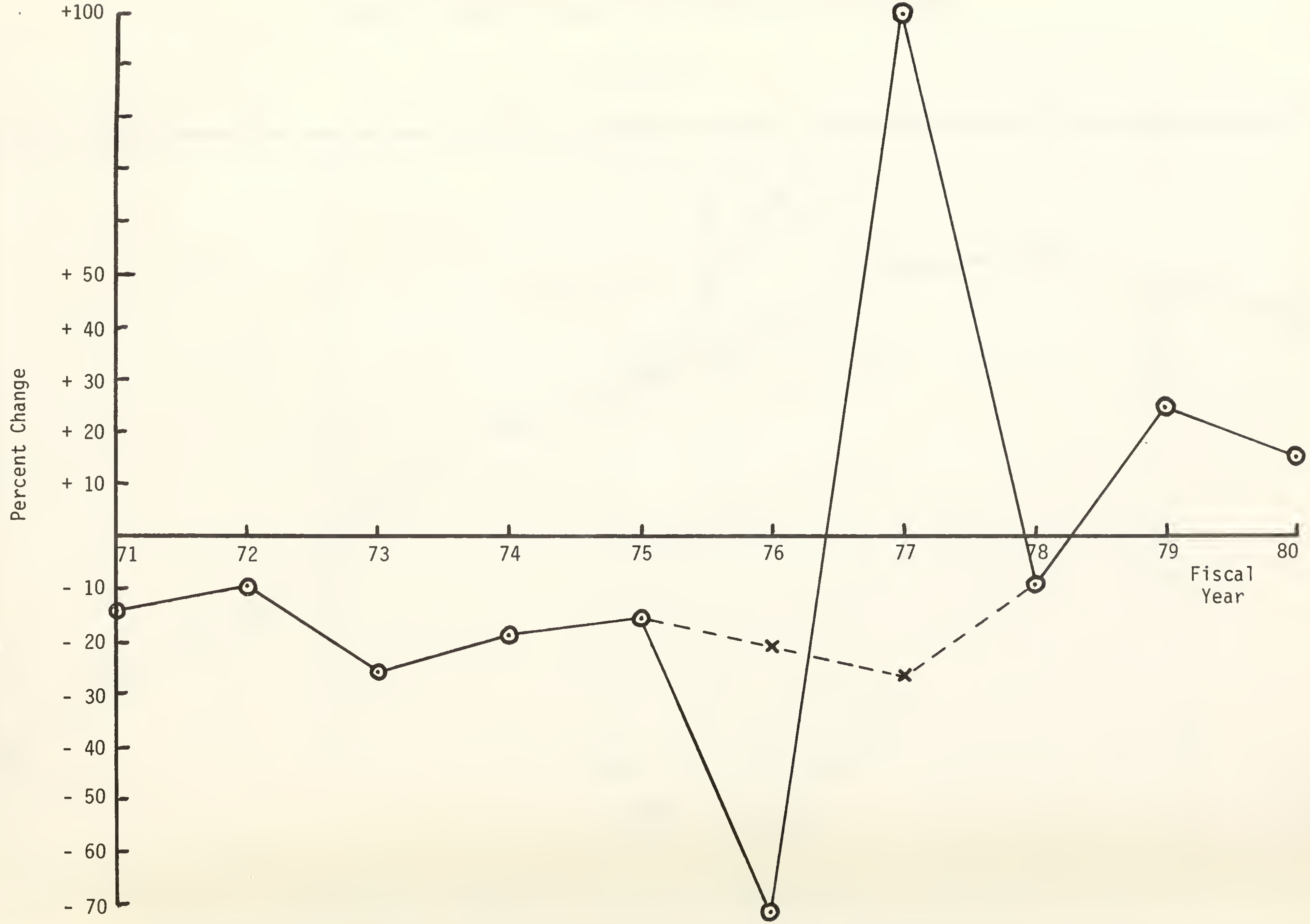



Figure 6. Percent Change in Past Inputs and Future 8510P Quotas.

is to even out production so that serious disruptions in the production process are avoided.

In order to use the mathematical concepts of production smoothing to an institute of higher education one must know the "costs" of certain disruptions in the institution. These costs are in terms of factors such as quality of education, morale, stability and long-term goals of the faculty, and the ability of the faculty to make short term (1-3 years) commitments to research. Unlike machinery, which can be shut down and re-started for reasonably well predicted costs, we currently cannot measure quantitatively shut-down and start-up costs in a graduate school faculty. Any mathematical formulation at this stage using production function smoothing in graduate education would be sterile. Rather, we proceed with ad hoc approach based on the quota results of the models in sections 1 and 2.

Looking again at Figure 5 we see that if we ignore for the moment period FY 76, the quota continues to decrease, "bottoms-out" at 20 in FY 78, then gradually increases to a steady-state value of about 36. If we draw a straight line from the point for FY 75 to the point for FY 77 we obtain a quota of 30 for FY 76 which is about 83% of the steady state value. The effect of a quota of 30 in FY 76 on the yearly percentage change curve in Figure 6 is shown with the dashed line. Yearly percentage decreases remain in the region of 15%-25% through FY 77 (2 years), and then decrease, turn positive for a few years, and eventually go to zero.

What we have done is to use the smoothed transient model in section 2 to determine quotas for a future 5 year period (or longer if necessary), plot the quota together with past school inputs, and use "eye-ball" smoothing, to eliminate wild oscillations in the year percentage change curve. In our example a change to the quota model figures is necessary only in the first year (FY76). This will usually be the case due to the dampening effects of numerous stochastic elements in the real system. Such smoothing is not possible with a model which determines the quota for only one future period. It is <u>essential</u> for good planning that not only the immediate year's quota is determined, but also the quotas for at least a five year period so that magnitudes and direction can influence the current quota. It is almost certain that the current billet requirements will <u>not</u> stay constant over the next 5 years. But this does not mean that we should not try to calculate or be influenced by future quotas based on these current requirements. When decisions on factors such as academic tenure and departmental research programs affect academic planning over very long periods it is vital that we consider future projected quotas.

5. Inventory/Billet Ratios.

A common indicator of the "health" of a given P-code community is the ratio of total inventory to billets for a given rank. The purpose of this section is to point out some of the problems with this indicator and how certain simple modifications might improve its usefulness.

Before considering the real system let us look back to the simple transient problem in Figure 2. Let $K(t)$ be the ratio of inventory to billets in period $t$. In our simple example $K$ should be 1.0, which is the case for $t = 0$. Using the simple ratio of total inventory to billets we obtain

$$K(0) = 1.0, \quad K(1) = 1.7, \quad K(2) = 1.1, \quad K(t) = 1.0, \quad t \geq 3.$$

Consider now the arguments leading to Figure 4. We should really be interested in the ratio of usable inventory to billets. Let us assume that any inventory in Tour year 1 is usable (even though there are not enough billets available in which to use them). Then the ratio would be

$$K(0) = 1.0, \quad K(1) = \frac{66}{54} = 1.2, \quad K(1) = 1.0, \quad t \geq 2.$$

These ratios are much more indicative of the availabilities of P-code inventory.

The first point then is that usable inventory should replace total inventory in the numerator of the ratio. But why are we interested in such a ratio? Usually it is used as an indicator of future graduate education requirements. In this case we should not be interested in the current ratio since any new input to graduate school cannot be used in billets for 5 years. A more meaningful ratio is

$$\frac{\text{Usable inventory predicted in 5 years}}{\text{Billets predicted in 5 years}}$$

for the given P-code, designator and rank. In a perfect system this ratio should be 1.0, and any deviation from it would indicate that either the real system cannot attain this ideal figure because of possibly unavoidable factors, or that temporary deficits or excesses exist.

In order to use this ratio we must be able to determine the usable inventory predicted in a given future year. But the effort needed to do this is the same effort needed to calculate the quota; in fact the ratio can easily be printed out using either *QUOTA* or *SMQUOTA* (see Appendix, sections (ii) and (iii)). The main advantage of using such a ratio is, of course, that it be simple to calculate.

The simplest modification to the ratio currently used is simply to look at the total inventory predicted in 5 years to billets predicted in 5 years. Although much simpler to calculate than the ratio above its usefulness in a transient situation following a large reduction in billets is not clear.

Table 8 shows the ratios of predicted usable inventories to billets and predicted total inventory *SMQUOTA* (see Appendix (ii)). Usable inventory is assumed to be those in a billet or in their 14th year of service and not in a billet. It is assumed that a person is a LCDR only when he has years of service between 11 and 16 inclusive.

The only ratios in this table which can be affected by quota input for LCDR are for periods exceeding 5. The first column shows there is no excess usable inventory using the quota in Table 6. The second column shows how total inventory to billet ratio decreases from 2.40 to 1.97. The figures in parenthesis show how the ratios would change if the FY 76 quota were 15 LT's and 15 LCDR's, given a total of 30 as shown by the dotted lines in Figures 5 and 6 of section 4.

| Planning Period | Usable Inventory/ Billets | | Total Inventory/ Billets | |
|---|---|---|---|---|
| 1 | 1.30 | | 3.69 | |
| 2 | 1.33 | | 3.67 | |
| 3 | 1.43 | | 3.54 | |
| 4 | 1.25 | | 3.21 | |
| 5 | 1.15 | | 2.87 | |
| 6 | 1.0 | (1.11) | 2.40 | (2.52) |
| 7 | 1.0 | (1.0) | 2.13 | (2.25) |
| 8 | 1.0 | (1.0) | 1.97 | (2.09) |
| 9 | 1.0 | (1.0) | 1.97 | (1.97) |
| 10 | 1.0 | (1.0) | 1.97 | (1.97) |

Table 8: Predicted LCDR Inventory/Billet Ratios Using the Data in Table 4.

A problem with the use of ratios is knowing what a reasonable ratio should be. The steady state model in section 1 can be used to determine the long range ratio for a given policy. Recall that $B_i$ is the number of P-coded billets in rank $i$ (for the given P-code and designator), $T_i$ is the tour length, and $\beta_i$ is the fraction who get to serve a tour. Let $L_i$ be the "lifetime," or total expected time an officer spends in rank $i$. Then in steady state

$$K_i^t = \left(\sum_{j=1}^{i} X_j \prod_{k=j}^{i} \alpha_k\right) L_i/B_i, \qquad i = 1,2,3,4. \tag{5}$$

gives the ratio of <u>total</u> inventory to billets. The quotas $\{X_i\}$ are given in equations (1)-(4) of section 1. Using equation (5) and the data in Table 2 of section 1 we find that

$$K_2^t = 1.97.$$

This agrees with the long-run ratio (after at least 8 years) found in Table 8 using the smoothed transient model.

Now let $K_i^u$ be the steady state usable inventory to billet ratio in rank $i$. Then one can show that

$$K_1^u = 1 + \frac{\alpha_1(1-\beta_1)X_1}{B_1},$$

$$K_2^u = \begin{cases} 1 + \alpha_2(1-\beta_2)X_2/B_2 & \text{if} \quad X_2 > 0 \\ 1 - \frac{1}{B_2}\left(\frac{B_2}{T_2} - \alpha_1\gamma_1X_1\right) & \text{if} \quad X_2 = 0, \end{cases}$$

$$K_3^u = \begin{cases} 1 + \alpha_3(1-\beta_3)X_3/B_3 & \text{if} \quad X_3 > 0 \\ 1 - \frac{1}{B_3}\left(\frac{B_3}{T_3} - \alpha_1\gamma_1\gamma_2X_1 - \alpha_2\gamma_2X_2\right) & \text{if} \quad X_3 = 0, \end{cases}$$

and

$$K_4^u = \begin{cases} 1 + \alpha_4(1-\beta_4)X_4/B_4 & \text{if} \quad X_4 > 0 \\ 1 - \frac{1}{B_4}\left(\frac{B_4}{T_4} - \alpha_1\gamma_1\gamma_2\gamma_3X_1 - \alpha_2\gamma_2\gamma_3X_2 - \alpha_3\gamma_3X_3\right) & \text{if} \quad X_4 = 0. \end{cases}$$

For the data in Table 2 we find that

$$K_1^u = 1, \quad K_2^u = 1, \quad K_3^u = 1, \quad \text{and} \quad K_4^u = 2.95.$$

Note that $K_2^u$ agrees with the ratio in Table 8 after 6 years.

It is easy to add these ratios $K_i^t$ and $K_i^u$ as outputs to the steady state quota model.

6. Determination of Requirements.

The models discussed in this paper produce quotas to meet given P-coded billet requirements. The only exception is the smoothing discussed in section 4 and illustrated by the dotted lines on Figures 5 and 6. Thus it is crucial that accurate estimates of future requirements can be made. The 5 year lag time from school entry to billet entry adds to the forecasting problem. It is not the purpose of this paper to discuss in detail the determination of graduate education requirements, but some observations are necessary in order to see the quota model in perspective.

There is no doubt that precise estimates of P-coded billets five years in the future cannot be made. The uncertainties in the system introduce large variances which can easily be seen in past attempts at forecasting (see Figure 7). However, some reasons for the variance can be discovered and to some extent these can be controlled. For example, all past attempts at forecasting have assumed a constant change in billets from year to year (i.e. straight-line projection). Extrapolating a growth rate for the immediate year out to five years often leads to unreasonable, if not unbelievable, numbers of billets. No other models for forecasting requirements have been used. Even now, because past forecasts have been so much in error, the method of forecasting future billets is to assume no change from current billets over the next five years. One possible improvement over this might be to take the current billets in a given specialty, multiplied by the forecasted future officer strength (from the five year defense plan for example), and divided by the current officer strength. This calculation would help to correct for fluctuations in total officer strength. These numbers could be further refined by using factors which indicate how a given specialty is changing in the Navy over the years.

An important point to remember in determining requirements is that it should be done independently of the current ability to fill the requirement. It is easy to argue for an increase in billets if an excess inventory of educated people are available. It is even easier (and currently more convincing) to argue that if not enough inventory exists (or will exist) in a field, then the billet requirements must be cut because quotas cannot be filled. Such arguments lead to neat bookkeeping but avoid the difficult real problems.

One could argue that no fixed actual requirement exists, and so much effort is spent trying to determine non-existing numbers. Recently many billets were removed because it was felt that graduate education was not essential to the filling of the billet. Such arguments could probably be made on even more billets, and the results are probably more a result of the relative obstinacies of the two sides, billet removers, or billet keepers, than on any real requirement. At a lower level, if a man has to be able to read numbers off a chart and write them down, then clearly it is essential that he can read or write. But as educational level increases beyond repetitive trade-type skills, the minimum level of education requirements becomes quite fuzzy. This is especially so when one realizes that in graduate education the emphasis should be primarily on ways of thinking about complex systems, and not simply on learning skills at a more advanced level. Not to appreciate this vital difference is to miss the point of graduate education.

The point of this argument is to show that it might be possible to continue a given billet without a P-code with little expectancy for improvements or changes. Alternatively, it might be P-coded because it is planned that the billet in future years should change from its present scope to one requiring graduate education. Thus both viewpoints would be correct for differing objectives concerning the billet. It is the reason for the billet's existance which should be analyzed in deciding whether it should be P-coded. If agreement cannot be reached on the reason's for the billet, then the requirement for P-coding remains uncertain.

In the past it has been common to ask the person currently in a billet as to whether or not it should be P-coded. But how can this person be expected to keep separate the current work he has to do in the billet, with what the objectives for the billet are five years in the future? He may currently feel that most of his time is spent in "fire-fighting" mode for which he does not need an advanced degree. Two points should be made here. Firstly, if he does not have an advanced degree, he cannot possibly know if it would help him or not; and secondly, what is <u>planned</u> for the billet in five years may be very different from what is currently being done. In short, by asking current billet holders to ascertain requirements tends to continue past policy mistakes into the future. Perhaps a more healthy approach is to have a separate body look at the <u>objectives</u> of each community and their billets, and determine from these whether or not a graduate education is desirable.

Finally, increasing the complexity of the quota model to better imitate real-life personnel movements is wasted effort when the results are so sensitive to the unknown billet requirements. When uncertainty exists it is much better to aggregate where possible to take advantage of the "law of large numbers." Variances in forecasts with aggregation tend to be smaller than in forecasts without aggregation. The tendency toward even more finely divided specialty codes will only make quota determination even more difficult, with even less chance of having a good match of people to billets in five years. It is this last match that is important, but only if the specialty code on the billet accurately describes the requirement for the billet. A move toward more general coding, with substitution among P-codes, would lead to a more flexible system, would indicate a greater understanding of graduate education, and would probably have beneficial psychological effects in job satisfaction.

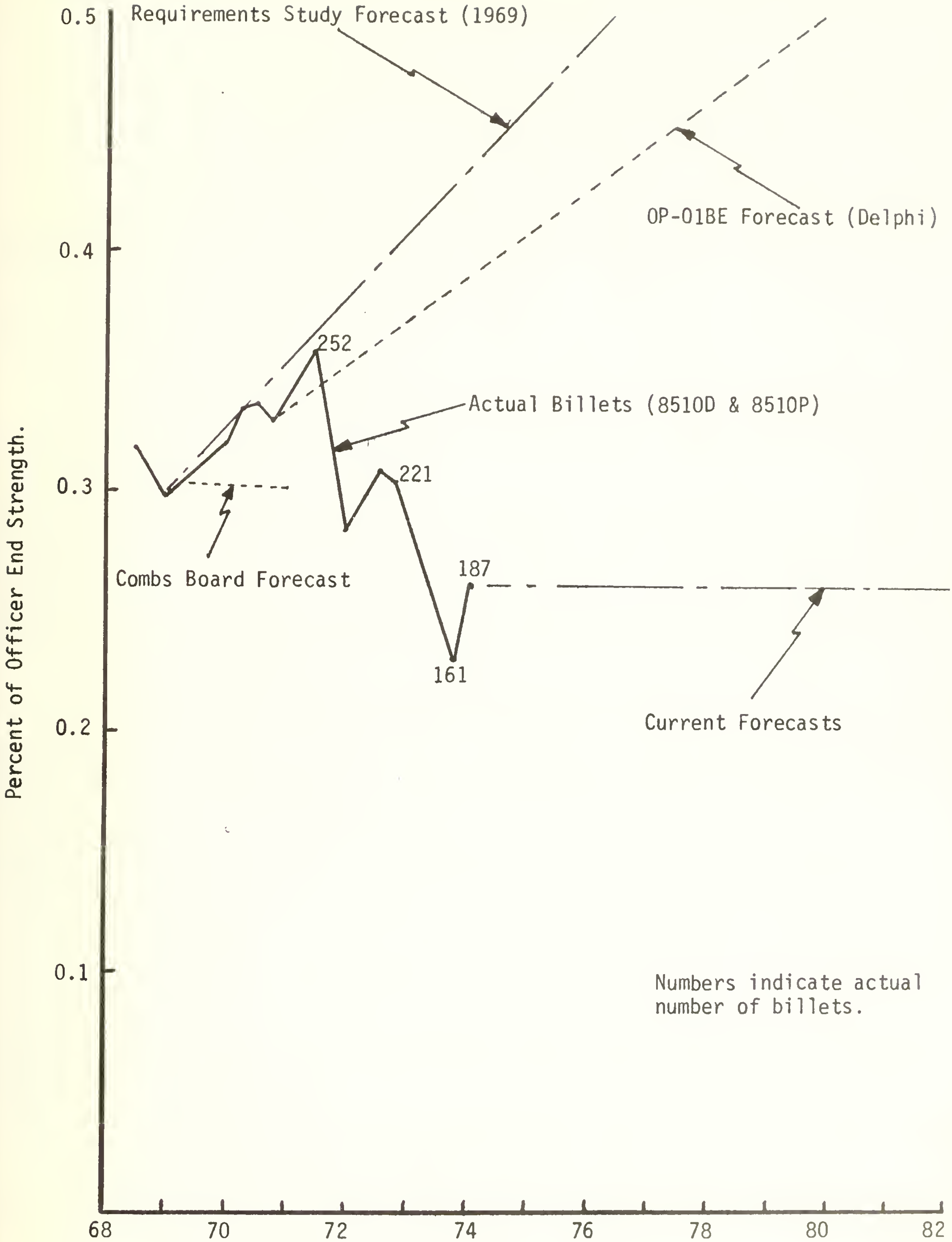



Figure 7: Actual and Forecasted 8510P and D Code Billets.

## APPENDIX

Terminology of the *APL* programming language is used frequently in this appendix. Readers unfamiliar with this terminology are referred to Katzan [1], or Gilman and Rose [2].

(i) The Steady State Model.

The steady state flowrates (officers per year) given by equations (1)-(4) in section 1 are calculated by an *APL* function called *SSQUOTA*, which is listed in Figure A1.

Syntax: *SSQUOTA* is monadic function which takes a vector as its right-hand argument. The elements of the vector are the numerical parts of the P-codes for which one would like to calculate the steady state quota. For example,

*SSQUOTA* 9210 8530 9410 8110

would result in calculations for computer management, computer science, financial management and aeronautical engineering.

Global Variables: The single subscripted notation in section 1 must now be double subscripted. So we now have $B_{ki}$, $T_{ki}$, $\alpha_{ki}$, $\beta_{ki}$ and $\gamma_{ki}$, where $k$ indexes a particular P-code/designator. For simplicity in what follows it is assumed that only the P-code is considered. However, with a slightly different interpretation of $k$ the model can be used for any P-code/designator combination.

The global variables required by *SSQUOTA* are:

*PCV* - a vector of the numerical parts of all relevant P-codes, the $k^{\underline{th}}$ element being the $k^{\underline{th}}$ P-code. Thus *PCV* has an many elements as there are P-codes. Let this be $m$.

*B* - an $m \times 4$ matrix of billets with $(k,i)^{\underline{th}}$ element $B_{ki}$.

*T* - an $m \times 4$ matrix of tour lengths with $(k,i)\underline{th}$ element $T_{ki}$.

*CF* - an $m \times 4$ matrix of continuation fractions with $(k-i)\underline{th}$ element $\alpha_{ki}$.

*UF* - an $m \times 4$ matrix of utilization fractions with $(k-i)\underline{th}$ element $\beta_{ki}$.

*RF* - an $m \times 4$ matrix of reutilization fractions with $(k-i)\underline{th}$ element $\gamma_{ki}$.

Function Description: Line 1 checks that all codes in the right-hand argument of *SSQUOTA* are valid (i.e. they are contained in *PCV*). Lines 2-6 calculate the four quota numbers $X_{k1},\ldots,X_{k4}$ for all $k$ corresponding to the right-hand argument of the function. Note that these calculations are made simultaneously for all $k$. There is no looping through lines 2-6. Lines 7-10 format and print the output for each $k$. Thus the program loops using line 11. The formatted output uses the *APL+* formatting function *ΔFMT*.

Line 12 uses a function *AYN* (answer yes or no) to ask if the used would like to make changes in the data and recalculate. A "*NO*" answer terminates the function. A "*YES*" answer results in a question asking which P-codes would the user like to investigate further (line 13). Lines 15 through 22 allow the user to input new data. When this is complete the function returns to line 3 and repeats the calculations.

A number of error checking devices have been inserted to prompt the user of errors in input. The variable *LF* which appears results in a "line feed" to make the terminal input/output easier to read.

Output: The output of the calculations in *SSQUOTA* is a table similar to Tables 1-3 in section 1. An example of the input/output is given in Figure A.3, with the global variable values displayed in Figure A.2. In this example $m = 2$.

```
 ∇SSQUOTA[⎕]∇
∇ SSQUOTA X;BM;TM;UFM;RFM;N;X1;X2;X3;CFM;I;Q;V;J;K;TCV;TV;IM
  →(∧/(1+ρPCV)≠V←,PCV⍳X)ρ2 ⋄ 'INCORRECT CODES' ⋄ →0
  QM←((N←ρV),0)ρ0 ⋄ BM←B[V;] ⋄ TM←T[V;] ⋄ UFM←UF[V;] ⋄ CFM←CF[V;] ⋄ RFM←RF[V;]
 L1:QM←QM,(X1←BM[;1]÷UFM[;1]×TM[;1])÷CFM[;1]
  QM←QM,(X2←(0⌈(BM[;2]÷TM[;2])-X1←RFM[;1]×X1)÷UFM[;2])÷CFM[;2]
  QM←QM,(X3←(0⌈(BM[;3]÷TM[;3])-X2←RFM[;2]×(X1+X2))÷UFM[;3])÷CFM[;3]
  QM←QM,(0⌈(BM[;4]÷TM[;4])-RFM[;3]×(X2+X3))÷UFM[;4]×CFM[;4] ⋄ LF ⋄ I←1
 L2:'CODE';PCV[V[I]] ⋄ LF
  'RANK   BILLETS   TOUR     CF       UF       RUF   QUOTA'
  'I3,I9,F8.1,3BF8.2,BI7' ΔFMT(⍳4;,BM[I;];,TM[I;];,CFM[I;];,UFM[I;];,RFM[I;];,QM[I;])
  'TOTAL',,'I7,X31,I8' ΔFMT(+/BM[I;];+/QM[I;])
  3ρLF ⋄ →(N≥I←I+1)ρL2
  →(~AYN 'ANY CHANGES?')ρ0
 L3:'P-CODE(S)?' .≥< →(∧/(1+ρPCV)≠V←PCV⍳TCV←,⎕)ρL4
  'INCORRECT CODE' .>< →L3
 L4:QM←((N←ρTCV),0)ρ0 ⋄ I←1 ⋄ BM←TM←CFM←UFM←RFM← 0 4 ρ0
 L5:LF ⋄ 'CODE';TCV[I] ⋄ J←V[I] ⋄ K←1 ⋄ IM← 5 0 ρ0
 L6:'RANK';K
  '     BILLETS   TOUR     CF      UF      RUF'
  'X5,I5,F8.1,3F7.2' ΔFMT(B[J;K];T[J;K];CF[J;K];UF[J;K];RF[J;K])
 L7:→(5≠ρTV←⎕)ρL8 ⋄ IM←IM,TV ⋄ →(4≥K←K+1)ρL6
  BM←BM,[1] IM[1;] ⋄ TM←TM,[1] IM[2;] ⋄ CFM←CFM,[1] IM[3;] ⋄ UFM←UFM,[1] IM[4;] ⋄ RFM←RFM,[1] IM[5;]
  →(N≥I←I+1)ρL5 ⋄ →L1
 L8:'INCORRECT INPUT' ⋄ →L7
∇
```

Figure A1. Program Listing of the Steady State Quota Model.

$PCV$

$$\begin{pmatrix} 8510 & 9111 & 8610 & 8710 \end{pmatrix}$$

$B$

$$\begin{bmatrix} 35 & 67 & 59 & 12 \\ 8 & 41 & 56 & 50 \\ 56 & 107 & 34 & 13 \\ 31 & 49 & 25 & 14 \end{bmatrix}$$

$T$

$$\begin{bmatrix} 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 \end{bmatrix}$$

$CF$

$$\begin{bmatrix} 0.95 & 0.75 & 0.7 & 0.5 \\ 0.95 & 0.75 & 0.7 & 0.5 \\ 0.95 & 0.75 & 0.7 & 0.5 \\ 0.95 & 0.75 & 0.7 & 0.5 \end{bmatrix}$$

$RF$

$$\begin{bmatrix} 0.75 & 0.7 & 0.5 & 0 \\ 0.75 & 0.7 & 0.5 & 0 \\ 0.75 & 0.7 & 0.5 & 0 \\ 0.75 & 0.7 & 0.5 & 0 \end{bmatrix}$$

$UF$

$$\begin{bmatrix} 0.9 & 0.9 & 0.9 & 0.9 \\ 0.9 & 0.9 & 0.9 & 0.9 \\ 0.9 & 0.9 & 0.9 & 0.9 \\ 0.9 & 0.9 & 0.9 & 0.9 \end{bmatrix}$$

Figure A2. Global Variable Values Used in Example.

```
    SSQUOTA 8510

CODE8510

RANK    BILLETS    TOUR    CF      UF      RUF     QUOTA
  1        35      3.0    0.95    0.90    0.75       14
  2        67      3.0    0.75    0.90    0.70       19
  3        59      3.0    0.70    0.90    0.40        5
  4        12      3.0    0.50    0.90
TOTAL     173                                        37



ANY CHANGES? YES
P-CODE(S)?
⎕:
      8510


CODE8510
RANK1
     BILLETS    TOUR     CF      UF      RUF
        35      3.0    0.95    0.90    0.75
⎕:
        35      3.0     .95     1       .75
RANK2
     BILLETS    TOUR     CF      UF      RUF
        67      3.0    0.75    0.90    0.70
⎕:
        67      6       .75     1       .7
RANK3
     BILLETS    TOUR     CF      UF      RUF
        59      3.0    0.70    0.90    0.50
⎕:
        59      3       .7      1       .5
RANK4
     BILLETS    TOUR     CF      UF      RUF
        12      3.0    0.50    0.90
⎕:
        12      3       .5      1       0

CODE8510


RANK    BILLETS    TOUR    CF      UF      RUF     QUOTA
  1        35      3.0    0.95    1.00    0.75       12
  2        67      6.0    0.75    1.00    0.70        3
  3        59      3.0    0.70    1.00    0.50       17
  4        12      3.0    0.50    1.00
TOTAL     173                                        32


ANY CHANGES? NO
```

Figure A3. Sample Input/Output Using Steady State Quota Model.

(ii) <u>The Basic Transient Model</u>.

Consider the P-code and designator as fixed, and let:

$I_k(t)$ = inventory in period $t$ with $k$ years of service, $k = 1,2,\ldots,30$, $t = 0,1,2,\ldots$

Period $t = 0$ is the current period.

$c_k$ = fraction of those with $k$ years of service who remain to have $(k+1)$ years of service, $k = 1,2,\ldots,30$.

$S_k(t)$ = additions to inventory in period $t$ with $k$ years of service from students in an earlier period.

$d$ = "delay" from entering school to entering the P-coded billet. ($d$ is assumed to be at least 4 years.)

$\ell_i$ = year of service when rank $i$ P-coded billets entered, $i = 1,2,3,4$.

$u_i$ = last year of service for rank $i$ P-coded billets, $i = 1,2,3,4$.

$B_i$ = number of P-coded billets in rank $i$.

$q_i(t)$ = school quota in period $t$ for rank $i$, $i = 1,2,3,4$.

$w_i$ = tour length of rank $i$ billets

$= u_i - \ell_i + 1.$

We assume that anyone entering a P-coded billet stays in the billet for the full tour length.

The inventories in the planning period $t = 0$, $\{I_k(0)\}$ are given. Then future inventories are given by:

$$I_{k+1}(t+1) = c_k I_k(t) + S_{k+1}(t+1), \quad k = 0,1,2,\ldots,29, \quad t \geq 0, \tag{A1}$$

where $I_0(t) = 0$. We define the "legacy" of past inventories to be $I'_k(t)$. Thus

$$I'_{k+1}(t+1) = c_k I_k(t). \tag{A2}$$

The $\{S_k(t)\}$ are given for $t = 1,2,3$, since these are from students currently enrolled in graduate school. The $\{S_k(t)\}$, $t \geq 4$ will determine future school input quotas. Consider future period $t + d$ for fixed $t > 0$. From our assumptions,

$$\text{Total Inventory Legacy in Rank } i = \sum_{k=\ell_i}^{u_i} I'_k(t+d).$$

The addition to inventory in period $(t+d)$ with $k$ years of service is

$$S_k(r+d) = \text{Max } [0, B_i - \sum_{j=\ell_i}^{u_i} I'_j(t+d)], \quad k = \ell_1, \ell_2, \ell_3, \ell_4, \tag{A3}$$

$$= 0 \qquad\qquad \text{otherwise.}$$

Thus the quota for rank $i$ in period $t$ is

$$q_i(t) = S_{\ell_i}(t+d) / \prod_{k=\ell_i-d}^{\ell_i-1} c_k, \quad i = 1,2,3,4 \quad t \geq 1. \tag{A4}$$

The procedure starts with $\{I_k(0)\}$ and $\{S_k(t)\}$ $t = 1,2,3$. Equation (A1) is used to calculate $\{I_k(t)\}$, $t = 1,2,\ldots,d$.

Equation (A2) is then used to calculate $I'_k(d+1)$ and these are used in (A3) to find $S_k(1+d)$. These are converted to quotas $q_i(1)$, $i = 1,2,3,4$, using (A4). The values of $S_k(1+d)$ are now used in (A1) to give $I_k(d+1)$, which in turn give $I'_k(d+2)$ using (A2). Use of these in (A3) give $S_k(2+d)$, which are used in (A4) to give $q_i(2)$. This procedure continues until all quotas are determined for the planning period.

The basic transient model uses an *APL* function called *QUOTA* which is listed in Figure A4.

```
      ∇QUOTA[⎕]∇
    ∇ DES QUOTA N;BV;I;J;K;V;L;F;W
[1]     INV← 1 30 ρ,INV74[I←PCVιDES;] ⍝ K←J←1 ⍝ BV←,B[I;] ⍝ L←W←F←V←4ρ,Q←((N←N+D̲),4)ρ0
[2]   L3:F[K]←×/D̲↑((-D̲)+L[K]←BW[I;K;1]-1)↓,CR[I;] ⍝ W[K]←BW[I;K;2]-L[K]
[3]     →(4≥K←K+1)ρL3
[4]   L1:INV←INV,[1] STUD[I;;J⌊4]+0,¯1↓,CR[I;]×INV[J;]
[5]     K←1
[6]   L2:V[K]←+/W[K]↑L[K]↓,INV[J+1;]
[7]     →(4≥K←K+1)ρL2
[8]     Q[J;]←0⌈BV-V
[9]     INV[J+1;BW[I;;1]]←INV[J+1;,BW[I;;1]]+Q[J;]
[10]    →(N≥J←J+1)ρL1
[11]    Q←((D̲,0)↓Q)÷((N-D̲),4)ρF
[12]    'YEAR    GRP1     GRP2     GRP3     GRP4    TOTAL'
[13]    'I3,5BI8' ΔFMT(ιN-D̲;Q,+/Q)
    ∇
```

Figure A4. Program Listing of the Basic Transient Model.

Syntax: *QUOTA* is a dyadic function taking scalars for both left and right hand arguments. The left argument is the P-code number and the right argument is the planning period in years. For example

9211 *QUOTA* 12

will calculate the quotas for P-code 9211 for the next 12 years.

Global Variables: The global variables required by *QUOTA* are:

*PCV* - P-code vector (see (i) above).

*INV74* - an $m \times 30$ matrix of current (1974) inventories, where $(INV74)_{ij}$ is the number currently in service with P-code $i$ and $j$ years of service.

*B* - an $m \times 4$ billet matrix (see (i) above).

*D* - a scalar giving the delay between school entrance and billet entrance (lead time).

*BW* - an $m \times 4 \times 2$ three dimensional array. Element $(BW)_{ij1}$ gives the lowest years of service for a billet with P-code $i$ and rank $j$ (i..e $\ell_j$ for the particular P-code). $(BW)_{ij2}$ gives the highest years of service for a billet with P-code $i$ and rank $j$ (i.e. $u_j$ for the particular P-code).

*CR* - an $m \times 30$ matrix of continuation rates. $(CR)_{ij}$ is the fraction of officers with P-code $i$ with $j$ years of service who stay in to have $(j+1)$ years of service.

*STUD* - an $m \times 30 \times 4$ three dimensional array, where $(STUD)_{ijk}$ is the additions to inventory in year $k$ in P-code $i$ with $j$ years of service from currently enrolled students (for $k = 4$ all elements are zero).

Function Description: Lines 1-3 set up various arrays to be used in the function. Line 2-8 essentially calculate the $S_k(t+d)$ in (A3). Line 9 calculates the new inventory using (A1) and line 11 calculate the quota using (A4). Lines 12 and 13 format and print the output.

Output: The output is a table with 6 columns. Column 1 gives the planning year, columns 2-5 give the four quota numbers and column 6 the total quota. There is a row for each planning period. An example is shown in Figure A6, and the values of the global variables are given in Figure A5.

(iii) The Smoothed Transient Model.

Table 5 in section 2 shows a sample output of the basic transient model. Clearly this model leads to undesirable cycles in the quota. To smooth out these cycles we modify the basic model.

The same notation as section (ii) is used. In steady state, the number of billets per year of rank $i$ (for the given P-code and designator) which become vacant is $B_i/w_i$.

We now modify (A3) to

$$\text{(A3a)} \quad S_k(t+d) = \mathrm{Max}\Big[0, \Big(B_i - \sum_{j=\ell_i}^{u_i} I'_j(t=d)\Big), \Big(\frac{B_i}{w_i} - I'_k(t+d)\Big)\Big], \quad k = \ell_1, \ell_2, \ell_3, \ell_4$$

$$= 0 \qquad \text{otherwise.}$$

The remaining equations stay the same. The underlying assumption which leads to (A3a) is discussed in section 2 and is not repeated here.

The smoothed transient model uses an *APL* function called *SMQUOTA* which is listed in Figure A7.

Syntax: *SMQUOTA* has the same syntax as *QUOTA*. (See (ii).)

Global Variables: *SMQUOTA* uses the same global variables as *QUOTA*.

Function Description: Essentially the only difference between *SMQUOTA* and *QUOTA* is in line 8, which now uses equation (A3a) in place of (A3).

Output: A sample output is shown in Figure A8 using the values of the global variables in Figure A5.

```
      PCV[1]

8510

      INV74[1;]

0  0  4  6  6  9  13  21  36  35  51  43  42  33  37  30  25  25  21  16  11
      5  15  5  6  6  2  2  2  6

      B[1;]
35  67  59  12

      D
5

      BW[1;;]
   8  10
  14  16
  19  21
  24  26

      CR[1;]
1  1  1  0.98  1  1  1  1  0.75  1  1  1  1  1  1  0.7  1  1  1  1  0.5  1
     1  1  1  1  14  1  1  0
```

```
      STUD[1;;]
0  0  0  0
0  0  0  0
0  0  0  0
1  2  2  0
1  4  4  0
4  8  2  0
3  8  0  0
0  8  0  0
0  3  2  0
4  3  3  0
4  2  0  0
0  1  0  0
1  0  0  0
1  1  1  0
0  1  0  0
0  0  0  0
0  0  0  0
0  0  0  0
0  0  0  0
0  0  0  0
0  0  0  0
0  0  0  0
0  0  0  0
0  0  0  0
0  0  0  0
0  0  0  0
0  0  0  0
0  0  0  0
0  0  0  0
0  0  0  0
```

Figure A5. Global Variable Values Used in Example.

8510 QUOTA 15

| YEAR | GRP1 | GRP2 | GRP3 | GRP4 | TOTAL |
|---|---|---|---|---|---|
| 1 | 7 | | | | 7 |
| 2 | 12 | | | | 13 |
| 3 | 12 | 22 | | | 34 |
| 4 | 13 | 7 | | | 20 |
| 5 | 14 | 5 | | | 19 |
| 6 | 12 | 33 | | | 45 |
| 7 | 13 | 12 | 17 | | 43 |
| 8 | 13 | 6 | | | 20 |
| 9 | 12 | 33 | | | 45 |
| 10 | 13 | 12 | 17 | | 43 |
| 11 | 13 | 7 | | | 20 |
| 12 | 13 | 32 | | | 45 |
| 13 | 13 | 12 | 17 | | 42 |
| 14 | 13 | 7 | | | 20 |
| 15 | 13 | 32 | | | 45 |

Figure A6. Sample Input/Output Using Basic Transient Model.

```
     ∇SMQUOTA[⎕]∇
    ∇ DES SMQUOTA N;BV;I;J;K;V;L;F;W
[1]   INV← 1 30 ρ,INV74[I←PCV⍳DES;] ⍝ K←J←1 ⍝ BV←,B[I;] ⍝ L←W←F←V←4ρ,Q←((N←N+D̲),4)ρ0
[2]  L3:F[K]←×/D̲↑((-D̲)+L[K]←BW[I;K;1]-1)↓,CR[I;] ⍝ W[K]←BW[I;K;2]-L[K]
[3]   →(4≥K←K+1)ρL3
[4]  L1:INV←INV,[1] STUD[I;;J⌊4]+0,¯1↓,CR[I;]×INV[J;]
[5]   →(D̲≥J←1+J)ρL1 ⍝ K←1 ⍝ J←J-1
[6]  L2:V[K]←+/W[K]↑L[K]↓,INV[J+1;]
[7]   →(4≥K←K+1)ρL2
[8]   Q[J;]←0⌈(BV-V)⌈(BV÷W)-INV[J+1;BW[I;;1]]
[9]   INV[J+1;BW[I;;1]]←INV[J+1;,BW[I;;1]]+Q[J;]
[10]  →(N≥J←J+1)ρL1
[11]  Q←((D̲,0)↓Q)÷((N-D̲),4)ρF
[12]  'YEAR   GRP1    GRP2    GRP3    GRP4   TOTAL'
[13]  'I3,5BI8' ΔFMT(⍳N-D̲;Q,+/Q)
    ∇
```

Figure A7. Program Listing of the Smoothed Quota Model.

8510 *SMQUOTA* 15

| *YEAR* | *GRP1* | *GRP2* | *GRP3* | *GRP4* | *TOTAL* |
|---|---|---|---|---|---|
| 1 | 6 | 5 | | | 11 |
| 2 | 12 | 10 | | | 22 |
| 3 | 12 | 8 | | | 20 |
| 4 | 13 | 12 | | | 25 |
| 5 | 14 | 15 | | | 29 |
| 6 | 12 | 18 | 6 | | 36 |
| 7 | 13 | 18 | 6 | | 36 |
| 8 | 13 | 16 | 6 | | 35 |
| 9 | 12 | 18 | 6 | | 36 |
| 10 | 13 | 17 | 6 | | 36 |
| 11 | 13 | 16 | 6 | | 35 |
| 12 | 13 | 18 | 6 | | 36 |
| 13 | 13 | 17 | 6 | | 36 |
| 14 | 13 | 17 | 6 | | 35 |
| 15 | 13 | 18 | 6 | | 36 |

Figure A8. Sample Input/Output Using the Smoothed Quota Model.

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