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# Error Patterns from Alternative Cost Progress Models

## Moses, O. Douglas

Monterey, California. Naval Postgraduate School

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ERROR PATTERNS FROM ALTERNATIVE

COST PROGRESS MODELS

O. Douglas Moses
Associate Professor
Department of Administrative Sciences
Naval Postgraduate School
Monterey, CA 93943
408-656-3218

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# ERROR PATTERNS FROM ALTERNATIVE
# COST PROGRESS MODELS

## INTRODUCTION

Cost progress models have proven their value in estimating tasks encountered in production, purchasing and the management of other organizational operations. Going by various names (e.g. "experience curves", "learning curves", "cost improvement curves"), cost progress models have long been accepted as a useful tool for planning, estimating, and predicting the pattern of costs expected from a repetitive production or acquisition process. Various cost progress models exist but most such models are versions of the standard learning curve, perhaps with additional variables added to improve explanatory power and forecast accuracy.

How accurate are various cost progress models? Does their accuracy depend on the conditions surrounding their use? Are particular cost progress models more accurate in some circumstances and other models more accurate in other circumstances? The purpose of this paper is to document the accuracy of a set of common cost progress models under various circumstances, indicating variables that may impact model accuracy, and highlighting situations when model accuracy may be expected to improve or deteriorate.

## RELATED RESEARCH

The literature on cost progress models/learning curves is substantial.[1] Three branches of research are relevant to the current study. The first branch has to do with alternative forms of cost progress models and alternative variables suggested for

inclusion. Most cost progress models start (some end) with some version of the familiar learning curve. The premise of the learning curve is that cumulative quantity is the primary cause of changes in unit cost during a production or acquisition program. There is general acknowledgement that cumulative quantity is only a partial explanation and hence much prior research has attempted to augment learning models with other variables. Some attention has been paid to variables reflecting changes in fixed costs associated with capacity (e.g., Balut, 1981; Balut, et.al., 1989; Moses, 1990), but the greatest amount of attention has been paid to changes in production rate.[2]

Conceptually production rate is argued to affect unit cost due to economies (or diseconomies) of scale (e.g., Bemis, 1981; Boger and Liao, 1990; Large, et. al., 1974; Linder and Willbourn, 1973). Empirically, evidence on the benefit of including production rate variables in cost progress models is mixed. Various studies (e.g., Alchian, 1963; Cochran, 1960; Hirsh, 1952; Large, Campbell and Cates, 1976) found little or no significance for rate variables. Other studies did document significant rate/cost relationships (e.g., Bemis, 1981; Cox and Gansler, 1990). In reviewing the existing research on production rate, Smith (1980) concluded that a rate/cost relationship may exist but that the existence, strength and nature of the relationship varies with the item produced and the cost element examined.[3] Collectively, this branch of literature suggests that inclusion of variables, such as production rate, in cost progress models sometimes has improved cost

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explanation -- but not always. It is relevant here because the present research selects a representative number of cost progress models from the existing literature and investigates their accuracy under various conditions.

The second branch of literature has been concerned with identifying factors that cause or influence the nature of the learning or cost improvement phenomenon, with attention paid to a wide variety of behavioral, organizational and process variables. Conway and Schultz's (1959) classic paper is an early example. Dutton and Thomas (1984) provide a typology of factors causing learning, dividing these factors into categories based on origin and type. Adler and Clark (1991) provide a step toward modeling the links between selected causal factors and resultant learning. This branch of literature is relevant to the current paper because it documents how cost improvement patterns are inevitably influenced by a host of variables. It implicitly acknowledges that the ability of cost progress models to adequately describe cost/output relationships will depend on these factors. In short, this literature implies that model forecast accuracy (irrespective of the form of model selected) will be conditional on circumstances.

The third branch of literature is concerned with explicitly examining cost progress model accuracy under various conditions. Smunt (1986) compared learning curve models to naive and moving average models, finding that relative accuracy depended on such factors as learning rate and forecast horizon. Moses (1991, 1992)

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examined learning curve and rate adjustment models, concluding that relative forecast accuracy and bias were dependent on a collection of variables, including variations in production rate, in factory burden, in data availability, as well as other factors. These studies are relevant here because they explicitly identify situations where cost progress models can be expected to be comparatively more or less accurate, one question of interest in the present study. Some of the conditions examined in these studies, conditions expected to influence cost progress model accuracy, are re-examined here. However, each of these prior studies observed accuracy using simulated data under well-controlled experimental conditions. Their results should perhaps be seen as hypotheses about how model accuracy may behave in practice with actual data. Observing the accuracy of various cost progress models, under various conditions, when applied to data from actual programs is the objective of this study.

## ALTERNATIVE COST PROGRESS MODELS

Consider the central purpose of a cost progress model. It is not really a model that explains cost per se. (It says nothing about the absolute amount of cost.) Rather its purpose is to explain the relationship between costs at different points during a repetitive production/acquisition process. Every cost progress model rests on two assumptions: (1) that future cost depends on past cost, and (2) that future cost differs systematically from past cost as a function of changing conditions during the repetitive process. Alternative models differ primarily in which

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"changing conditions" the modeler sees as sufficiently important to be included in the model. The most common cost progress model is the learning curve, which assumes that future cost systematically differs from past cost as a function of "experience", measured by cumulative output. The most common modification of the learning curve is, as mentioned previously, the incorporation of a term to reflect production rate, which assumes additionally that future cost systematically differs from past cost as a function of output per period.

This study investigates the accuracy of cost progress models that include the three variables just mentioned: (1) past cost, (2) cumulative quantity, and (3) production rate. Selectively combining these variables, four possibilities exist:

a) Future cost= f (past cost)

b) Future cost= f (past cost, cumulative quantity)

c) Future cost= f (past cost, production rate)

d) Future cost= f (past cost, cumulative quantity, production rate)

One model each from groups a, b, and c, and two models from group d, are investigated.

1. Random Walk (RW) Model: The simplest of all, the random walk model assumes that future cost is equal to the most recent past cost:

$$C_t = C_{t-1} \qquad (1)$$

where

    C = unit cost
    t = sequencing subscript

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This naive model serves as a benchmark for assessing the accuracy gained by including additional variables.

2. Learning Curve (LC) Model:  The familiar learning curve[4] is the model used for incorporating "experience" into the prediction.

$$C_t = C_1 \, Q_t^{\,b} \qquad (2)$$

where

$$C_1 = \text{theoretical first unit cost}$$
$$Q = \text{cumulative quantity produced}$$
$$b = \text{a parameter, the learning curve exponent or slope}$$
$$C, t = \text{as before}$$

3.  Rate Adjustment (RA) Model:  The assumption of the rate adjustment model is that future cost is equal to past cost, adjusted for any change in production rate (production volume per period).

$$C_t = C_{t-1} \, A_t \qquad (3)$$

$A_t$ is an adjustment factor capturing the impact of production rate on the spreading of fixed costs.

$$A_t = \frac{R_{t-1}}{R_t} \; F + (1 - F) \qquad (3a)$$

where

$$A = \text{adjustment factor}$$
$$R = \text{production rate per period}$$
$$F = \text{proportion of cost represented by fixed overhead}[5]$$
$$C, t = \text{as before}$$

Unit cost is assumed to vary inversely with production rate due to the spreading of fixed overhead cost over differing volume.  Thus unit cost will change as production rate (R) changes -- and the

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degree of change will depend on the proportion of fixed overhead cost in total cost (F). The adjustment factor is a version of an "overhead redistribution" model developed by Balut (1981).[6]

    4.  Bemis (BE) Learning/Rate Model: This is the first model presented here which considers (1) past cost, (2) cumulative quantity and (3) production rate. It is the most widely used model incorporating these three variables and was developed by augmenting the traditional learning curve with an analogous production rate term.

$$C_t = C_1 \ Q_t^{\ b} R_t^{\ d} \qquad\qquad (4)$$

where

        $d$ = a parameter, the production rate exponent or slope

        $C$, $C_1$, $Q$, $R$, $b$, $t$ = as before

Work on production rate dates at least to the 1950s (e.g., Hirsh, 1952) and empirical work on this learning/rate model was first conducted by RAND (e.g., Large, et. al., 1974), but Bemis (1981) has been credited with popularizing the model (the reason the Bemis label is used here).

    5.  Balut (BA) Learning/Rate Model: This is a second model which considers past cost, cumulative quantity and production rate. It is a version of the original Balut (1981) model and combines the traditional learning curve (Model 2) and the rate adjustment model (Model 3) previously discussed. The basic premise is that, in the absence of production rate changes, cost would follow a traditional learning curve. The impact of production rate change is incorporated by adjusting the cost forecasts from the learning

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curve model by an overhead redistribution adjustment factor.

$$C_t = C_1 \ Q_t^{\ b} \ A_{at} \qquad\qquad (5)$$

where

$$A_{at} = \frac{R_a}{R_t} \ F + (1-F) \qquad\qquad (5a)$$

and

$R_a$ = reference production rate, average production rate for past lots.
C, $C_1$, Q, b, t, R, F = as before

## ASSESSING ACCURACY

The objective of the study is to investigate model accuracy under various conditions. The data for the study involved costs and quantities for successive production lots. Accuracy here is defined in terms of the ability of a model to correctly forecast the "next lot average unit cost." Accuracy in such near term cost forecasting is seen as being a relatively minimal requirement expected of a cost progress model. The basic process is quite simple:

(a) Models were fit to a series of cost points to estimate (when necessary) model parameters.[7]

(b) Estimated models were used to forecast future (next period) average unit cost.

(c) Realized actual unit costs were compared to forecasted costs to assess accuracy.

It should be noted here that model accuracy centrally involves the ability to correctly forecast in advance, not the ability to explain a cost series ex post.[8] Two notions of accuracy apply.

One is the absolute magnitude of forecast error, regardless of whether the forecast is too high or too low. The second is the direction of the error, whether the model under or over-estimates future cost. Given two concepts, two measures were used:

$$\text{ERROR} = |PUC - AUC| \div AUC \quad (6)$$

$$\text{BIAS} = (PUC - AUC) \div AUC \quad (7)$$

where

PUC = predicted unit cost

AUC = actual unit cost

ERROR is a commonly used accuracy measure, the absolute percentage error. ERROR can take on only positive values and higher values, of course, signal poorer forecasts. BIAS takes on both positive and negative values. Positive (negative) values signal over (under) prediction of cost.

## CONDITIONS AFFECTING MODEL ACCURACY

The general research hypothesis is that the accuracy of models will depend on the circumstances in which they are used. What circumstances might impact accuracy? Research cited above (Smunt, 1986; Moses, 1991, 1992) suggested and discussed variables that might have an effect. Below such variables are listed, with a brief description and comment on how they were operationalized (measured) empirically. Collectively these variables will be referred to as the "condition" variables because they attempt to represent exogenous conditions which may affect model accuracy.

1. Fixed Cost Burden: Total unit cost must consist of both variable costs and a share of the total fixed cost burden

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associated with capacity. A major role of production rate is determining the volume of output over which fixed capacity costs will be spread. Hence, the importance of including a production rate variable in a cost model, and thus model accuracy, may depend on the degree to which total unit cost is made up of fixed costs. The following regression equation was fit to cost series data and the coefficient f used as a measure of fixed cost burden.

$$c_t = v + f \ \frac{1}{R_t}$$

This equation is consistent with seeing total unit cost per period ($c_t$) as the sum of variable cost per unit (v) plus a standard fixed cost per unit (f) adjusted for relative production rate per period ($R_t$). Higher values of f would be consistent with greater fixed cost burden, i.e., a greater proportion of fixed cost in total cost.

2. Learning Slope: Past simulation research (Smunt, 1986) shows that the importance of including a learning parameter in a cost model depends, not surprisingly, on the degree of learning that exists in the data. Hence, accuracy across the five models examined may depend on learning rate. Learning slopes were measured by using the b parameter estimated from model 2, transformed to learning rates (e.g., 90%, 80%, etc.). Higher values indicate less learning.

3. Cost Variability: Costs may vary from period to period due to unsystematic random factors. Such random factors influencing cost can be expected to obscure systematic relationships between cost and quantity or rate variables, reducing

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the chance that a cost model will be estimated correctly and forecast accurately (Smunt, 1986; Moses, 1991). Empirically, Cost Variability was measured by the average period-to-period (lot-to-lot) percentage change in average unit cost. Higher values indicate greater period-to-period variability in unit cost.

4. Quantity Variability: If production rate was highly stable across periods, there would be little need for a rate variable in a cost model (and little ability to correctly estimate a rate parameter by fitting a model to past data). Hence, the importance of incorporating a rate variable into a cost model, and model accuracy, may depend on the degree to which production rate/quantity varies. Empirically, Quantity Variability was measured by the average period-to-period (lot-to-lot) percentage change in production quantity. Higher values indicate greater quantity variability.

5. Quantity Trend: When initiating a production/acquisition program for a new item, does production rate (lot quantity) start at a low level and build up slowly to full capacity? Or is full capacity production achieved rapidly? Simulation results (Moses, 1991) have shown that the rate at which lot quantities grow when initiating a program affects cost model accuracy. Does a similar relationship exist when using real data? Empirically, the growth trend in lot quantity was operationalized by dividing first lot quantity by the average lot quantity over the (to date) life of a program. Hence, it is a measure of first lot size as a proportion of average lot size and a crude indicator of the trend in quantity.

Lower values indicate greater growth in quantity relative to initial quantity.

6.    Plot Points:   The number of data points available to estimate the parameters of a model may affect model accuracy.  Not surprisingly, simulation results (Moses, 1991) show that when comparing the relative accuracy of models, models with fewer (more) parameters tend to be relatively more accurate when the number of observations is smaller (greater).  One question is whether similar findings will come from real data.

7.   Future Production Rate:   Once a model is estimated using past data, it is used to forecast future cost.   Changes in production rate between the model estimation period and the future should alter future unit cost and hence a model's ability to forecast that future cost accurately.  Cost models incorporating production rate variables would be expected to have some advantage in such situations, and the degree of advantage would be expected to depend on how much future production rate differs from the past. Empirically, a variable measuring the change in production rate was constructed by dividing next (future) period's rate by last (most recent) period's rate.   (This ratio was then logged to make the distribution symmetrical.)   Positive (negative) values indicate increases (decreases) in production quantities.

SAMPLE AND DATA

The accuracy of the cost progress models was investigated using data for a sample of military aircraft and missile systems programs taken from the U. S. Military Aircraft Cost Handbook

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(DePuy, et. al., 1983) and the <u>U. S. Missile Cost Handbook</u> (Crawford, et. al., 1984). These handbooks contain data for virtually all military aircraft and missile programs from the early 1960s through the early 1980s. Two basic data items were collected from the handbooks for each program: annual lot quantities and average airframe unit costs per lot (in 1981 constant dollars). Programs were deleted from consideration if there were incomplete data or if the programs ran less than five years (a minimum number of data points was needed to fit the cost progress models). Based on these criteria, 46 programs (32 aircraft, 14 missile) were included in the final sample. These programs ranged in length from five years to thirteen years.

The original sample of 46 programs was "expanded" into 121 separate cost series. This was accomplished by dividing each program cost series into separate individual year-to-date cost series. For example, if a particular program had cost data available for six years, say 1970-1975, this single program cost series would be expanded into three separate series as follows:

Cost series #1: 1970-1973 data (used to forecast 1974 cost)

Cost series #2: 1970-1974 data (used to forecast 1975 cost)

Cost series #3: 1970-1975 data (used to forecast 1976 cost)

Thus the initial cost series for each program includes the first four years of data, while subsequent cost series were created by additionally including data from the next year in the cost series. This approach makes maximum use of data and approximates the actual process of a cost estimator who would update a forecast model each

period to incorporate the most recent data.

## ANALYSIS AND FINDINGS

The basic methodology used to assess cost model accuracy was as follows: Each of the five alternative models was estimated (when necessary) on each of the 121 cost series. Next-period data ( e.g. cumulative quantity and/or production rate) was input to each model to forecast next-period cost. Then next-period forecasted cost and next-period actual cost were compared. Thus the process produced 121 measures of error for each of the five models. The analysis primarily involves describing and explaining (when possible) the pattern of errors observed across the different models and across the different circumstances (i.e., across different values of the seven condition variables).

## General Error Patterns - Descriptive Statistics:

Table 1 provides selected descriptive statistics for both ERROR and BIAS for the five models. A general pattern is evident: Moving from the left to the right in the table, both magnitude of ERROR (mean and median) and the dispersion in ERROR (standard deviation and SIQR) tend to increase. Average magnitude of error ranges from about 13% to 25%. Note that this movement from left to right in the table coincides with increased complexity of the models: The random walk (RW) model considers only past cost in the forecast; the learning curve (LC) and rate adjustment (RA) models additionally consider either learning or production rate, but not both; while the Bemis (BE) and Balut (BA) models consider both

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Table 1
Error Statistics for Alternative Cost Progress Models

| Statistic | MODELS | | | | |
|---|---|---|---|---|---|
| | RW | LC | RA | BE | BA |
| Mean-absolute error | .125 | .169 | .160 | .208 | .245 |
| Median-absolute error | .074 | .124 | ..099 | .138 | .143 |
| Stnd. Dev.-absolute error | .129 | .153 | .173 | .211 | .296 |
| SIQR[1] -absolute error | .126 | .169 | .146 | .273 | .230 |
| Mean-bias | .049 | -.033 | .113 | .023 | .129 |
| Median-bias | .016 | -.061 | .059 | -.013 | .047 |

1. SIQR= Semi-interquartile range: (75th quantile - 25th quantile)

learning and production rate. One might have hypothesized in advance that accuracy would improve, not deteriorate, with the incorporation of additional variables; that of course is the point of using more complex models for forecasting.

At least three possibilities perhaps explain the contrary finding. First, the more complex models could simply be mis-specified in that the relations implied between cost, quantity and rate do not adequately describe reality. Forecasts from theoretically incorrect models would be expected to perform poorly. Second, the models could be correctly specified, but the amount of "noise" in the cost data relative to the proportion of variance in cost explainable by the learning or rate variables may be too high. Hence, parameter estimates are unreliable and forecasts poor. Third, the more complex models could be correctly specified but, because they incorporate more variables, the data in general are too lean (too few observations in the cost series) to estimate the model parameters. This is a problem of degrees of freedom. If this is the case, then the more complex models should perform better as the data become richer. This particular possibility will be addressed later.

It should also be noted that more complex models incorporating more variables typically have greater ability to explain, ex post, a cost series (i.e., $r^2$ goes up as the number of explanatory variables does). Thus, the results here suggest that ex post explanation and ex ante forecasting need not be strongly related. This is consistent with previous findings for cost models from

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simulation studies (Moses, 1993).

Another general result from table 1 concerns bias. Values for BIAS tend to be positive, except for the LC model. Thus, the models tend to over-estimate future cost, providing forecasts that on average are too high. This tendency is strongest for the RA and BA models. In contrast, the traditional learning curve (LC) tends to under-estimate future cost. This finding for the learning curve is also consistent with previous conclusions from simulation studies (Moses, 1992).

Relationship Between Accuracy and Conditions:

Is the accuracy of the models dependent on the circumstances in which they are used? Do models perform well in some circumstances, less well in others? To get a first-cut answer to these questions, three tests of the relationship between ERROR (from each of the five models separately) and the condition variables were conducted:

1. Pairwise Correlations: This is a univariate test of association, where measurement errors in other variables do not intrude.

2. Multiple Regression of ERROR on the Condition Variables together: This is a test of association for each variable while controlling for the others.

3. Stepwise Regression of ERROR on the Condition Variables: This permits variables that maximally explain ERROR to be identified. (The stepwise procedure was stopped when no additional variable would significantly (alpha < .05) enter the regression

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## Table 2
## Test of Relationship Between Cost Progress
## Model Errors and Explanatory Conditions

| Conditions | Test Statistics | RW | LC | RA | BE | BA |
|---|---|---|---|---|---|---|
| | | | | | | |
| Burden: | Corr. | .13 | .19* | .22* | .21* | .12 |
| | Reg. Coef. | .05 | .01 | .14 | -.02 | .14 |
| | Reg. t | 1.11 | .20 | 2.40* | -.30 | 1.40 |
| | Step. Coef. | - - | - - | .16 | - - | - - |
| | Step. t | - - | - - | 3.60*** | - - | - - |
| | | | | | | |
| Learning Slope: | Corr. | .12 | .01 | .13 | -.11 | .20* |
| | Reg. Coef. | .32 | .48 | .44 | .05 | 1.21 |
| | Reg. t | 2.20* | 2.89** | 2.36* | .20 | 3.79*** |
| | Step. Coef. | - - | .29 | .39 | - - | .89 |
| | Step. l | - - | 2.29* | 2.29** | - - | 3.68*** |
| | | | | | | |
| Cost Variability | Corr. | .09 | .35*** | .06 | .26** | .07 |
| | Reg. Coef. | .06 | .35 | .04 | .38 | .27 |
| | Reg. t | .76 | 3.58*** | .42 | 2.87** | 1.43 |
| | Step. Coef. | - - | .35 | - - | .32 | .42 |
| | Step. t | - - | 4.68*** | - - | 3.48*** | 2.87** |
| | | | | | | |
| Quantity Variability | Corr. | -.10 | .03 | -.15 | -.15 | -.09 |
| | Reg. Coef. | -.07 | -.03 | -.01 | -.14 | -.01 |
| | Reg. t | -1.31 | -.54 | -.23 | -1.77 | -.13 |
| | Step. Coef. | - - | - - | - - | - - | - - |
| | Step. t | - - | - - | - - | - - | - - |

Table 2 Continued

| Quantity Trend | Corr. | -.05 | .09 | -.09 | .06 | -.04 |
|---|---|---|---|---|---|---|
| | Reg. Coef. | .04 | .06 | .01 | -.01 | .09 |
| | Reg. t | 1.31 | 1.88 | .31 | -.16 | 1.50 |
| | Step. Coef. | - - | - - | - - | - - | - - |
| | Step. t | - - | - - | - - | - - | - - |
| | | | | | | |
| Plot Points | Corr. | .06 | .01 | -.08 | -.15 | -.05 |
| | Reg. Coef. | .00 | .00 | -.01 | -.02 | -.01 |
| | Reg. t | .64 | .25 | .89 | -1.80 | -.77 |
| | Step. Coef. | - - | - - | - - | - - | - - |
| | Step. t | - - | - - | - - | - - | - - |
| | | | | | | |
| Future Production Rate | Corr. | .21* | .09 | -.30*** | -.25** | -.27** |
| | Reg. Coef. | .04 | .01 | -.07 | -.07 | .11 |
| | Reg. t | 2.50* | .68 | -3.66*** | -3.02** | -3.33** |
| | Step. Coef. | .03 | - - | -.07 | -.08 | -.12 |
| | Step. t | 2.30* | - - | -3.93*** | -3.38** | -3.89*** |

\*   Significant at .05
\*\*   Significant at .01
\*\*\* Significant at .001

model.)

Correlations, regression coefficients and t values from these three approaches are provided in Table 2. Several observations concerning Table 2 follow.

First, where results are strong (significant at a higher level of probability) in one of the three tests, they tend to be corroborated in the other two tests. So there is at least some convergence across the tests.

Second, for three of the seven conditions (Quantity Variability, Quantity Trend and Plot Points) there are no significant results and thus no indication that model accuracy depends on these factors. This is of interest simply because all of the factors in this study have been shown to impact accuracy in at least one of the simulation studies cited previously. Of particular interest is the non-result for Plot Points. For none of the five models does the magnitude of forecast error depend on the number of observations in the cost series used to estimate the model. This suggests that the degrees of freedom problem in model estimation mentioned earlier is not the likely explanation for some models performing better or worse than others.

Third, significant results are found for the other four condition variables, and these results are not limited to single models. Rather, the accuracy of several of the models (at least three of the five) are related to these conditions.

How these conditions affect the accuracy of individual models differs from model to model, however. What follows is a model-by-

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model look at the impact of the conditions.  The approach used was to partition the sample into three subsamples depending on whether the values for a condition variable were low (bottom quartile), medium (middle 50%), or high (top quartile) and then, for each model, observe and plot average values for ERROR for these three subsamples.  This approach is followed below for variables found significant in the Table 2 tests.

## Error Analysis for Each Model

1.  Random Walk Model:  The Table 2 tests showed that RW model accuracy depended on two conditions -- Learning Curve Slope and Future Production Rate -- so the sample was partitioned (separately) on each of these two variables and average ERROR from the RW model determined for each of the three subsamples.  Plots showing RW ERROR as a function of these two condition variables are in Figure 1.  A horizontal line in the plot marks the overall average RW ERROR, so movement above and below this line indicates the impact of differing conditions.

First, RW ERROR depends somewhat on the Learning Curve Slope exhibited in the cost data, with greater ERROR experienced when learning slopes are high -- i.e., when little learning apparently has occurred.  The fact that RW ERROR depends on the degree of learning is not surprising; the RW model ignores learning and hence the degree to which it mis-forecasts cost ought to depend on the degree of learning occurring in the cost series.  But the observed pattern is the opposite of the expected one.  One would expect the RW ERROR to be greater when more learning was taking place, not
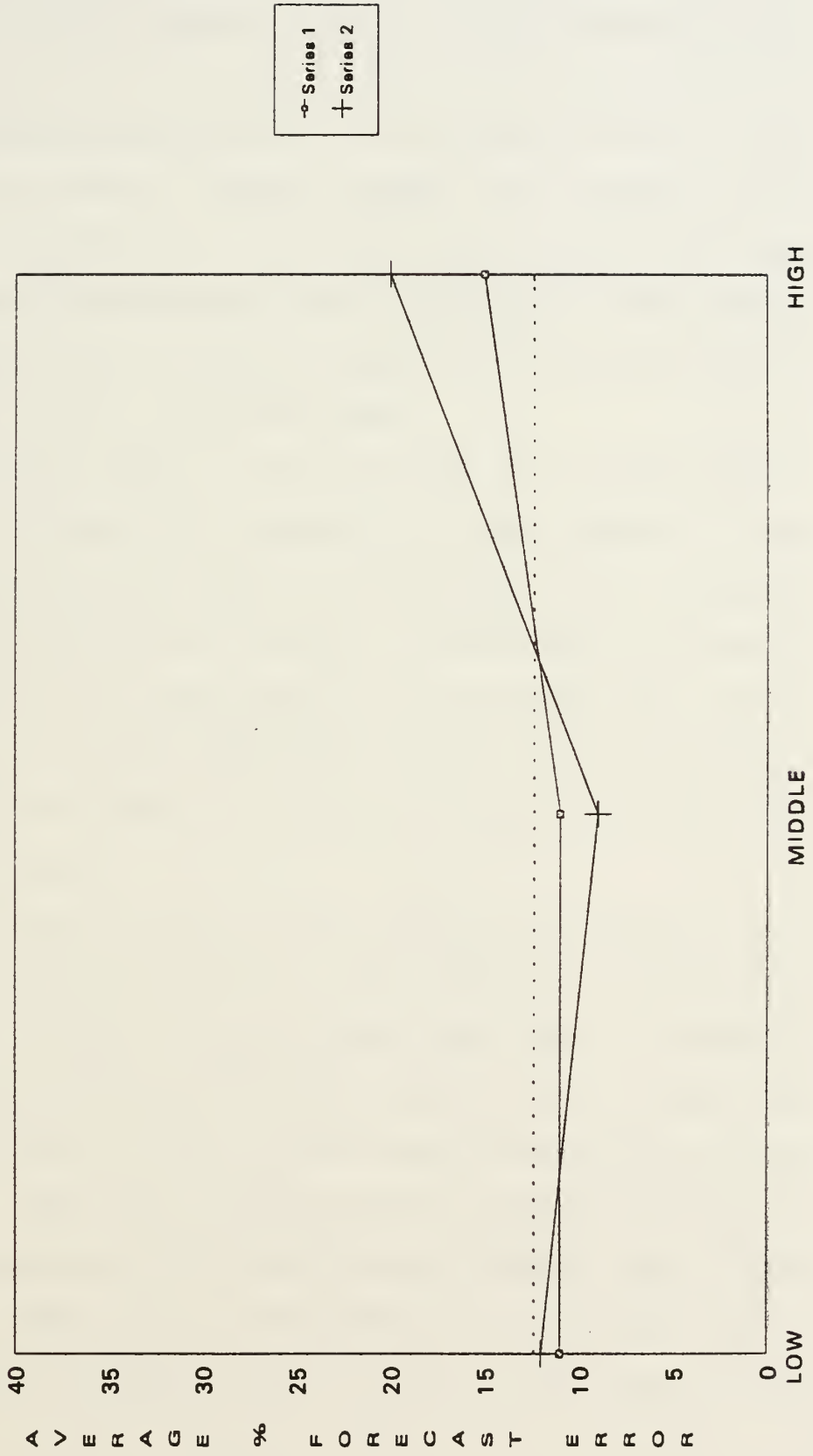
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# RANDOM WALK MODEL

## PLOT OF FORECAST ERROR BY LEVELS OF CONDITION VARIABLES

LEGEND:
ERROR PLOTTED BY LEVEL OF LEARNING CURVE SLOPE = SERIES 1
ERROR PLOTTED BY LEVEL OF FUTURE PRODUCTION RATE = SERIES 2

LEVEL OF LEARNING CURVE SLOPE
LEVEL OF FUTURE PRODUCTION RATE

FIGURE 1

less. The degree to which ERROR depends on learning slope is admittedly small, but the reason for the particular pattern is not obvious.

Second, RW ERROR depends on the Future Production Rate. Note the pattern is not monotonic; ERROR is higher than average for low values of future production, dips below average for mid-range values, and increases substantially for high values. This pattern is quite interesting but, as will be seen, it is repeated for all of the models and will be discussed later.

2. Learning Curve Model: Figure 2 shows how the accuracy of the traditional learning curve depends on Burden, Learning Curve Slope and Cost Variability. The role of Burden seem straight-forward: The LC model does not include a production rate variable, and one of the roles of a rate variable is to deal with the effect of spreading fixed overhead burden over varying levels of output. The LC model should be expected to perform more poorly when the level of burden is high.

That the accuracy of the LC model should also depend on the degree of learning estimated by the model is somewhat interesting. The effect shown in Figure 2 is mild but shows that LC ERROR is slightly higher when estimated learning rates are in either the bottom or top quartiles. A fuller story comes from observing BIAS rather than ERROR. When much learning appears to be occurring, the LC model under-estimates future cost (average BIAS of -15%). When little learning appears to be occurring, the LC model over-estimates future cost (average BIAS of +12%). What seems to be

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happening is a "regression to the mean" effect. A high (low) rate of past cost reduction causes the model to forecast a high (low) rate of future cost reduction and, in each case, the high (low) rate regresses to a more average rate, causing consistent over-or under-estimation of future cost.
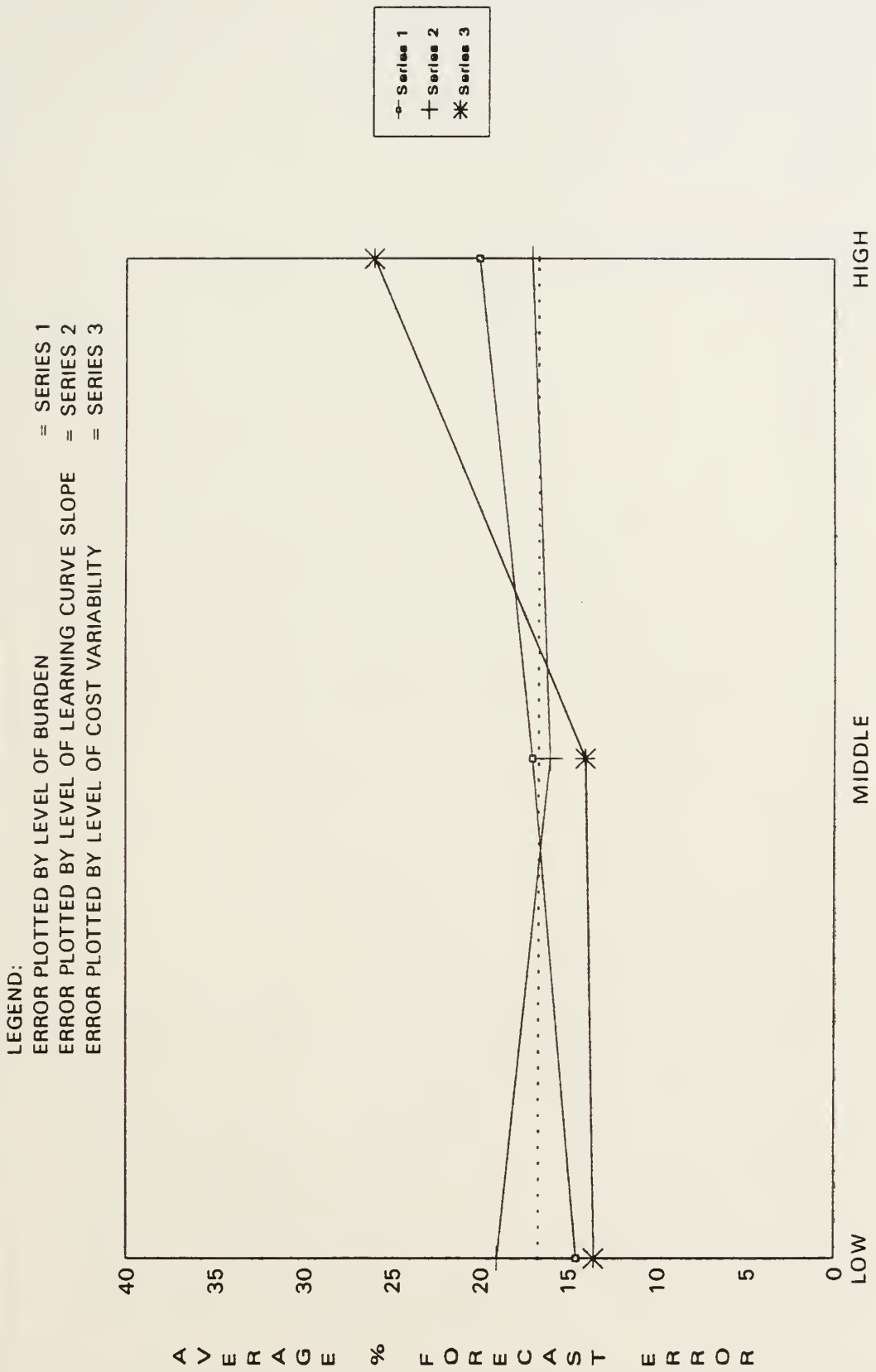
A more pronounced effect occurs for Cost Variability, with a sharp increase in LC ERROR when past costs have varied greatly from period to period. This finding is consistent with past simulation results suggesting that LC models try to explain all variability in cost through the estimation of the single learning parameter and, when there is considerable period to period "noise" in the cost series, end up erroneously "interpreting" that noise in the estimated learning rate.

3. Rate Adjustment Model: Figure 3 shows how the accuracy of the rate adjustment model depends on Burden, Learning Curve Slope and Future Production Rate. The figure shows that RA model ERROR increases as the fixed overhead burden increases. Although statistically significant, the effect is mild. It is also not obvious why this should occur. The approach of the RA model is to adjust unit cost for the effect of spreading fixed cost burden over varying output volume. The evidence here indicates that the ability of this model to properly adjust depends on how much fixed overhead there is.

The finding that RA ERROR (mildly) depends on Learning Curve Slope, or at least the direction of the finding, is unexpected. Since the RA model ignores learning, one would expect ERROR to be

# LEARNING CURVE MODEL
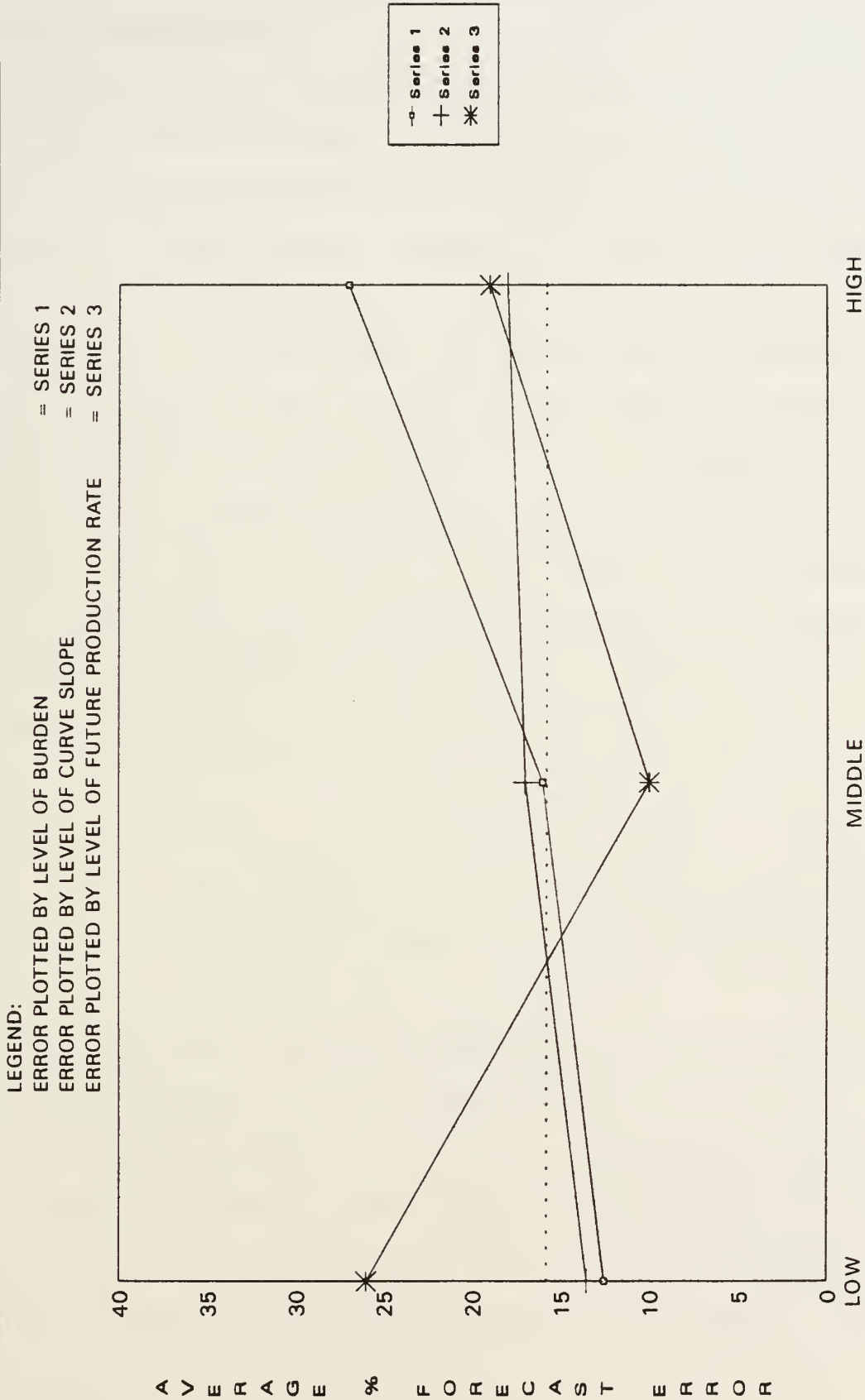## PLOT OF FORECAST ERROR BY LEVELS OF CONDITION VARIABLES

LEGEND:
ERROR PLOTTED BY LEVEL OF BURDEN                    = SERIES 1
ERROR PLOTTED BY LEVEL OF LEARNING CURVE SLOPE      = SERIES 2
ERROR PLOTTED BY LEVEL OF COST VARIABILITY          = SERIES 3

FIGURE 2

RATE ADJUSTMENT MODEL

PLOT OF FORECAST ERROR BY LEVELS OF CONDITION VARIABLES

LEGEND:
ERROR PLOTTED BY LEVEL OF BURDEN                    = SERIES 1
ERROR PLOTTED BY LEVEL OF CURVE SLOPE               = SERIES 2
ERROR PLOTTED BY LEVEL OF FUTURE PRODUCTION RATE     = SERIES 3

LEVEL OF BURDEN
LEVEL OF LEARNING CURVE SLOPE
LEVEL OF FUTURE PRODUCTION RATE

FIGURE 3

greatest when learning was greatest (lowest slope values). The opposite effect is exhibited.

The biggest impact on RA ERROR is due to differences in Future Production Rate. As noted when discussing the RW model, a "V" shaped pattern occurs, with ERROR growing as Future Production Rate diverges from the middle range. Again, this will be discussed later.

4. Bemis Model: Figure 4 shows how the accuracy of the Bemis model depends on Burden, Cost Variability and Future Production Rate. BE model ERROR increases with increases in Burden. This positive relationship is the same as just noted for the RA model, as is the interpretation. In both cases, the model includes a rate term which is designed in part to capture the effect of spreading fixed cost burden over differing output volume. In both cases, the model's accuracy declines as the amount of Burden increases.

As Figure 4 shows, BE ERROR also is larger when there is relatively greater period-to-period variation in cost. The BE model is the same as the LC model, with a rate term tacked on, and this finding is shared with the LC model (and a similar explanation may apply).

Lastly, BE ERROR also depends on Future Production Rate, with the same "V" shaped pattern to be discussed later.

5. Balut Model: Figure 5 shows that the accuracy of the Balut model depends on Learning Curve Slope, Cost Variability and Future Production Rate. BE ERROR tends to be considerably smaller when learning is great (lower Learning Curve Slope values). Two

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offsetting effects may explain this.  First, models with learning variables tend to be biased toward under-forecasting of future cost when the apparent learning is great (because of the regression-to-the-mean effect, previously discussed).  Second, overall, the BA model tends to be biased toward over-forecasting of future cost (as seen in Table 1).  These two effects offset, resulting in more accurate forecasts for the subsample where learning is great. (BIAS turned out to be essentially zero for this subsample and about +5-8% for the other two).

BA accuracy is also dependent on Cost Variability, although the finding is only mildly significant.  Figure 5 shows greater ERROR when Cost Variability is in the middle range; there is no obvious explanation for this non-monotonic inverted "V" pattern.
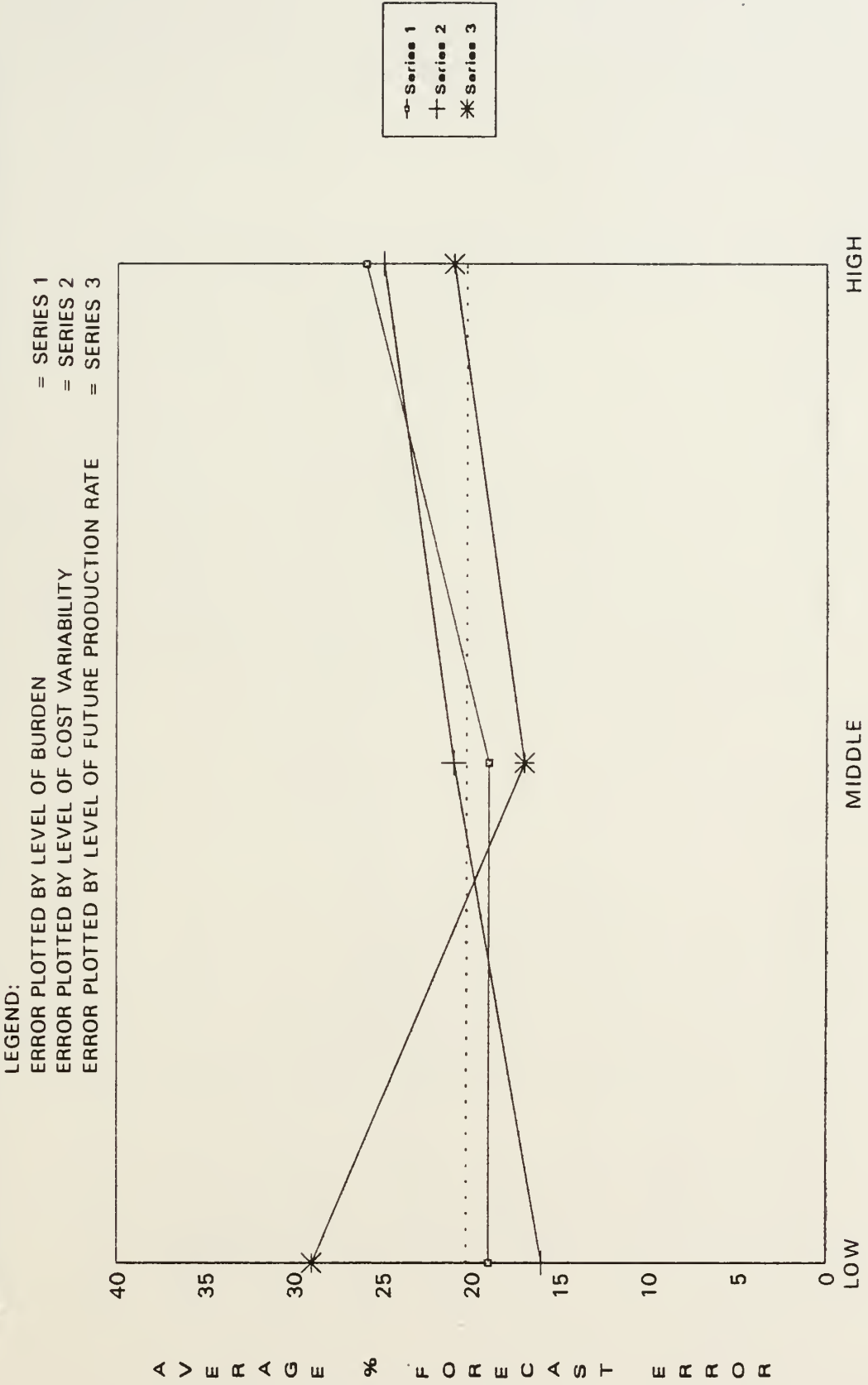
Lastly, BA ERROR is also dependent on the Future Production Rate, with the now familiar "V" pattern.  This general result will be discussed next.

## The Impact of Future Production Rate

Of the seven condition variables, Future Production Rate is special for four reasons.  First, conceptually it is distinct.  The other six variables describe conditions existing during the periods over which the models are estimated -- i.e., the past.  In contrast, Future Production Rate describes a condition (the level of production) expected to exist during the period for which cost is being forecast.  Second, how models perform in situations where production rates are changing is of particular importance for today's cost analyst, facing cost forecasting problems in an

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# BEMIS MODEL

## PLOT OF FORECAST ERROR BY LEVELS OF CONDITION VARIABLES

LEGEND:
ERROR PLOTTED BY LEVEL OF BURDEN                         = SERIES 1
ERROR PLOTTED BY LEVEL OF COST VARIABILITY              = SERIES 2
ERROR PLOTTED BY LEVEL OF FUTURE PRODUCTION RATE       = SERIES 3

-□- Series 1
-+- Series 2
-*- Series 3

LEVEL OF BURDEN
LEVEL OF COST VARIABILITY
LEVEL OF FUTURE PRODUCTION RATE

LOW                    MIDDLE                    HIGH

AVERAGE % FORECAST ERROR

FIGURE 4

# BALUT MODEL

## PLOT OF FORECAST ERROR BY LEVELS OF CONDITION VARIABLES

LEGEND:
ERROR PLOTTED BY LEVEL OF LEARNING CURVE SLOPE      = SERIES 1
ERROR PLOTTED BY LEVEL OF COST VARIABILITY          = SERIES 2
ERROR PLOTTED BY LEVEL OF FUTURE PRODUCTION RATE    = SERIES 3
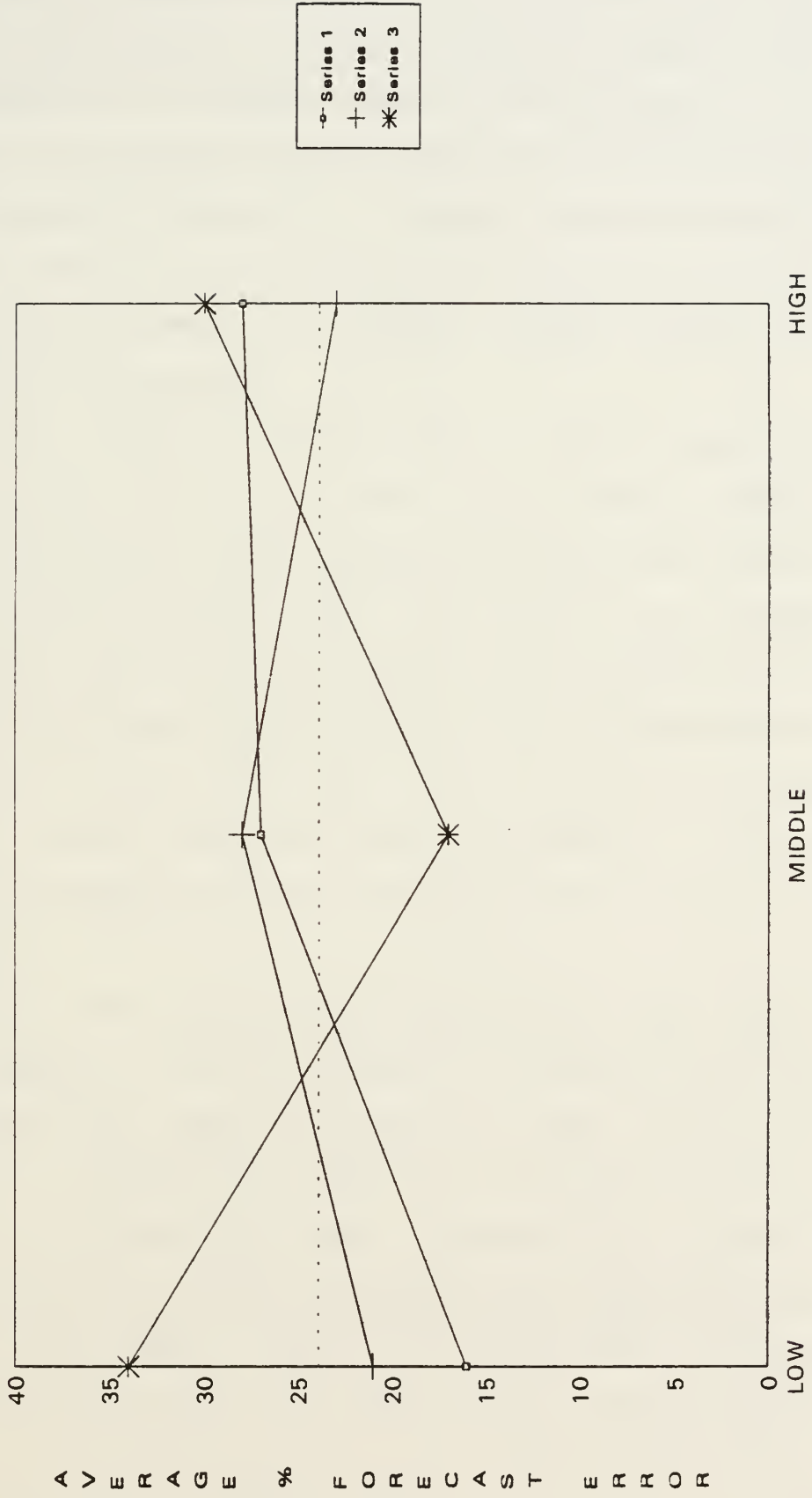
LEVEL OF LEARNING CURVE SLOPE
LEVEL OF COST VARIABILITY
LEVEL OF FUTURE PRODUCTION RATE

FIGURE 5

environment of rapid industrial change, such as production rate cutbacks in the defense industry. Third, the previous results have shown that in general the largest swings in average ERROR occur when moving across the subsamples partitioned on Future Production Rate. Last, the pattern of errors is consistent and non-monotonic, a V-shaped pattern with top and bottom quartile values for Future Production Rate associated with larger ERROR. Figure 6 summarizes this finding for all five models.

What does the V-shaped pattern mean. Simply put, if production rate in the period for which cost is being forecast diverges much from the recent past, either up or down, the accuracy of all five of the models deteriorates. This is not a surprising finding for models 1 and 2, the random walk (RW) and traditional learning curve (LC), because neither model incorporates production rate as a variable. But the fact that the RA, BE and BA models exhibit the same pattern indicates that the attempts of these models to explicitly capture production rate effects have not been fully successful.

Given that all the models mis-forecast cost when future production rate changes, a related question is: In what direction? This can be answered by observing values for BIAS, which are plotted in Figure 7. Some patterns from Figure 7 are of interest: First, the RW and LC models both under-estimate cost (negative BIAS) when future production rate falls and over-estimate cost when future production rate rises. This is not surprising. Falling rate should increase actual unit cost, because fixed capacity costs

are spread over less output. The RW and LC models "miss" this effect and thus consistently under-estimate unit cost. The opposite effect occurs when production rate increases, leading to over-estimates of unit cost.

BIAS for the BE model goes from slightly positive to slightly negative as Future Production Rate increases, but the effect is mild and insignificant. This is consistent with other investigations of this model which showed that, although the magnitude of error may vary across conditions, the BE model is consistently unbiased (Moses, 1992).

The impact of Future Production Rate on BIAS from the RA and BA models is more dramatic and significant -- and difficult to explain. Changing Future Production Rate in either direction, up or down, causes these models to over-estimate cost (positive BIAS). Both the RA and BA models "handle" rate changes in the same way, using the rate adjustment factor developed by Balut. But why this factor might lead to consistent over-estimation of cost, regardless of whether future production rate rises or falls, is not obvious.

## Comparisons of Model Accuracy

Given that the accuracy of the five models depends on the conditions under which they are used, an inevitable question arises: Which model appears to perform "best" under which conditions? Table 3 ranks the models by median ERROR, both overall (full sample) and by subsamples partitioned on values of the seven condition variables. Several observations seem noteworthy from these comparisons.

# PLOT OF MODEL FORECAST ERRORS BY LEVEL OF FUTURE PRODUCTION RATE



LEGEND:
RANDOM WALK MODEL          (MODEL 1) = SERIES 1
LEARNING CURVE MODEL       (MODEL 2) = SERIES 2
RATE ADJUSTMENT MODEL      (MODEL 3) = SERIES 3
BEMIS MODEL                (MODEL 4) = SERIES 4
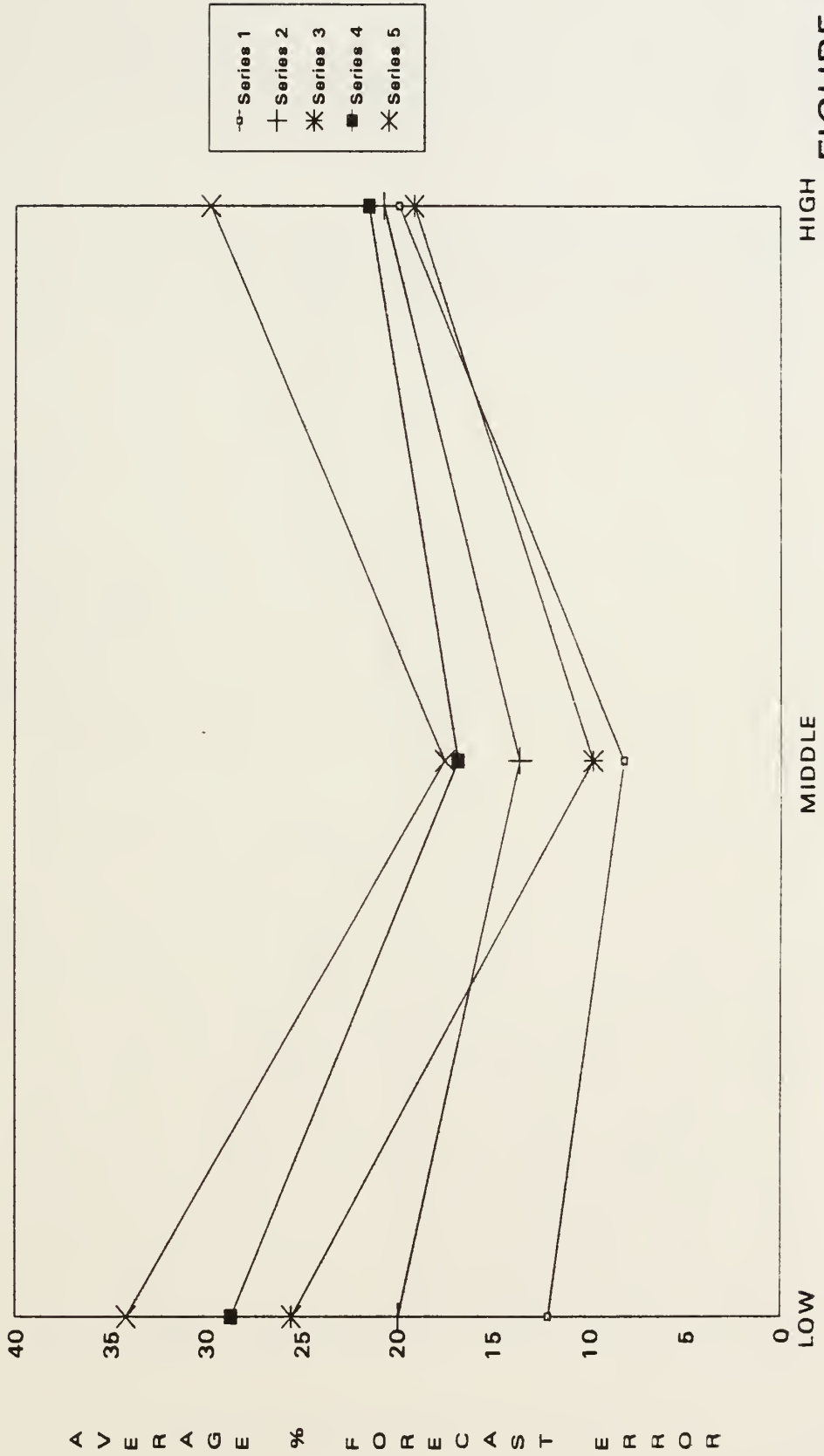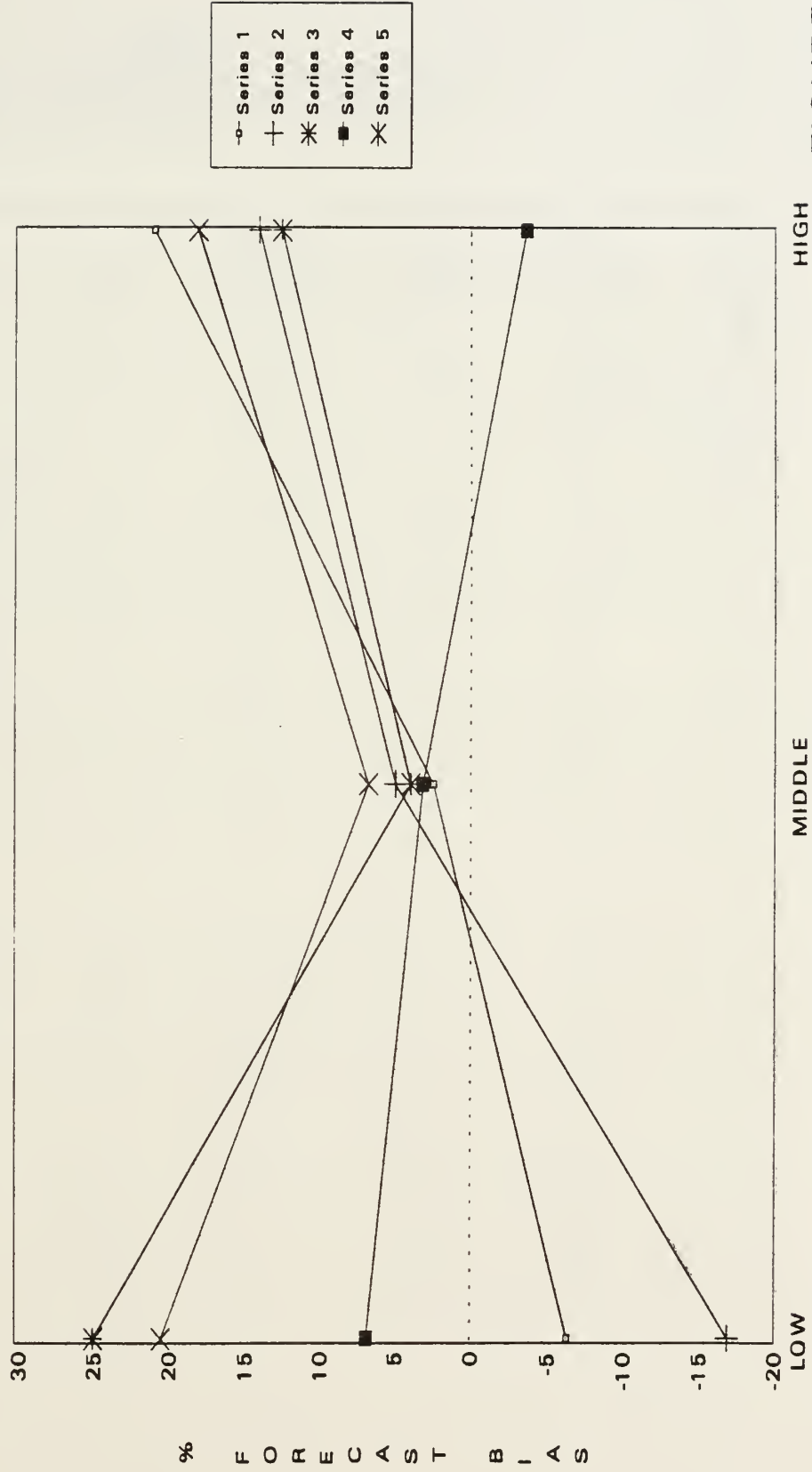BALUT MODEL                (MODEL 5) = SERIES 5

FIGURE 6

PLOT OF MODEL FORECAST BIAS BY LEVEL OF FUTURE PRODUCTION RATE

FIGURE 7

# Table 3
## Ranking of Alternative Cost Progress
## Models in Terms of Median Error
### (Most accurate= 1, least= 5)

| Conditions: | RW | LC | RA | BE | BA |
|---|---|---|---|---|---|
| | | | | | |
| Overall | 1 | 3 | 2 | 4 | 5 |
| | | | | | |
| Burden | | | | | |
|   Low | 1 | 4 | 3 | 2 | 5 |
|   Moderate | 1 | 2 | 3 | 2 | 5 |
|   High | 1 | 3 | 2 | 5 | 4 |
| Learning Slope | | | | | |
|   Steep | 1 | 4 | 3 | 5 | 2 |
|   Moderate | 1 | 4 | 3 | 5 | 2 |
|   Slight | 1 | 4 | 3 | 2 | 5 |
| Cost Variability | | | | | |
|   Little | 2 | 4 | 3 | 1 | 5 |
|   Moderate | 1 | 2 | 3 | 4 | 5 |
|   Great | 1 | 4 | 2 | 5 | 3 |
| Quantity Variability | | | | | |
|   Little | 1 | 3 | 2 | 4 | 5 |
|   Moderate | 1 | 5 | 2 | 4 | 3 |
|   Great | 1 | 4 | 2 | 3 | 5 |

Table 3 Continued

| Quantity Trend | | | | | |
|---|---|---|---|---|---|
| Little Growth | 1 | 2 | 3 | 4 | 5 |
| Moderate Growth | 1 | 3 | 2 | 5 | 4 |
| High Growth | 1 | 5 | 2 | 3 | 4 |
| Plot Points | | | | | |
| Few | 1 | 3 | 2 | 5 | 2 |
| More | 1 | 4 | 2 | 3 | 5 |
| Many | 2 | 4 | 1 | 3 | 5 |
| Future Prod. Rate | | | | | |
| Down | 1 | 3 | 2 | 4 | 5 |
| Little ch. | 1 | 5 | 2 | 3 | 4 |
| Up | 4 | 2 | 1 | 5 | 3 |

First is the consistent domination of the RW model, ranking most accurate overall and in all but a couple of the subsamples.

Next, is the "second place" showing for the RA model. It is second most accurate overall (and in a majority of the subsamples) and tends to be the model that outperforms the RW when the RW is not most accurate. This showing for the RA model is a bit surprising. The model is an abbreviated (no learning) version of the Balut (1981) model, and was created for this study simply to include, and test, a model incorporating rate changes but not learning. This model easily outperformed the "full" Balut model (#5) suggesting that Balut's contribution to modeling, the rate adjustment factor, may be even more useful when left "unattached" to the learning curve.

Third is the tendency for the models that required estimation of a learning rate (the LC, BE and BA models) to perform less well.

Last, there is a general pattern: An inverse relationship between accuracy and the number of variables in a model: The LC and RA models incorporate one variable more than the RW model (either cumulative quantity or production rate) and accuracy declines. The BE and BA models incorporate two additional variables (both cumulative quantity and production rate) and accuracy declines some more.

CONCLUSIONS AND FINAL COMMENTS

The objective of this paper has been to document the accuracy of five familiar cost progress models under varying conditions, using cost data from real world programs. Accuracy was evaluated

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in terms of ability to forecast next-period unit cost. Data consisted of annual lot costs from 46 military aerospace programs, arranged so that models were used to forecast 121 next-period costs. The five cost progress models forecasted future cost using some combination of variables reflecting (a) past costs, (b) cumulative quantity, and (c) production rate. Specific findings and error patterns have been presented; broader conclusions follow:

1. The accuracy of all cost progress models (tested) does depend on the circumstances or conditions in which they are used. Those conditions can be identified in advance. Thus a cost estimator using a particular model may be able to assess the risk of forecast error depending on the conditions.

2. Which conditions affect accuracy, and by how much, varies somewhat from model to model. But the results suggest that the amount of fixed cost burden, the degree of apparent learning, the degree of past variability in period-to-period cost and, particularly, the nature and degree of change in the future production rate provide information that can inform a cost estimator about the risk of forecast error from using a particular model.

3. It is not obvious that more sophisticated cost progress models improve forecasting. Quite the contrary for the sample here; forecast accuracy declined as additional variables were included.

4. Attempts by the models in this study to deal with the effects of changing production rate (of particular interest

currently, given the changing industrial picture) do not appear to have been very successful. This conclusion follows from the relatively poorer accuracy of the BE and BA models and from the fact that error for all the models increased when future production rates varied from the past. The model that did the best at (explicitly) adjusting forecasts for rate changes seems to be the simpler RA model and further study of the usefulness of this model seems warranted.

5. Although a relatively large sample of aerospace programs was included, all of the findings and conclusions should be tempered by the acknowledgement that they came from tests on one set of data -- cost data that was at a high level of aggregation (annual lot costs) and reasonably lean (the maximum data points for fitting a model was 13). Results would likely be most generalizable to similar cost forecasting situations. On the other hand, many of the error patterns observed in this study have also been observed in previous studies evaluating models on simulated data, so it is unlikely that the error patterns observed can be discounted as simply sample specific. Perhaps some of the findings may be viewed as tentative -- as hypotheses to be additionally supported (or contradicted) by future research. Given the findings of this study, one direction such research might take would be to start with the following question: Under what circumstances can more complex cost progress models outperform the simple random walk model?

1.   Yelle (1979) reviews the literature, with an emphasis on applications of the learning curve approach.   Dutton and Thomas (1984) provide a more recent review, identifying and categorizing the factors that cause the learning phenomenon.   Teplitz (1991) provides a comprehensive practical introduction to using learning curves, including a discussion of modeling problems and curve forms.

2.   One review of the literature pertaining to learning curves (Cheney, 1977) found that 36% of the articles reviewed attempted to augment the learning curve model in some manner by the inclusion of production related variables.

3.   Several explanations for these varying, inconclusive empirical results can be offered:   (a)   Varying results are to be expected because rate changes can lead to both economies and diseconomies of scale.   (b)   Production rate effects are difficult to isolate empirically because of colinearity with cumulative quantity (Gulledge and Womer, 1986).   (c)   Researchers have usually used inappropriate measures of production rate leading to misspecified models (Boger and Liao, 1990).   (d)   The impact of a production rate change is dominated by other uncertainties (Large, Hoffmayer, and Kontrovich, 1974), particularly by cumulative quantity (Asher, 1956).   Alchian (1963), for example, was unable to find results for rate adjustment models that improved on the traditional learning curve without a rate parameter.

4.   Note that this is an incremental unit cost model rather than a cumulative average cost model. Liao (1988) discusses the differences between the two approaches and discusses why the incremental model has become dominant in practice. One reason is that the cumulative model weights early observations more heavily and, in effect, "smooths" away period-to-period changes in average cost.

5.   Empirically a value for F of 14.7% was used.   This figure comes from Balut (1981) and is an average derived from aerospace industry data during the late 1970s.

6.   Readers familiar with the Balut modeling approach will recall that cost estimates were made using a learning curve and then adjust using the overhead redistribution model.   Learning is ignored here by design.   The intent is to present a model which reflects changes in production rate only (i.e., a model from category 3 listed previously).   Model 5 will reincorporate learning.

7.   There was no need to estimate parameters for models 1 and 3. Variables can just be plugged in to create a cost forecast. Models 2 and 4 were estimated using standard linear regression on logged variables.   Estimating model 5 on real data and making a cost forecast using the model involved several steps:   (1)   An average

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production rate ($R_a$) for all past lots was calculated as a reference. (2) Adjustment factors ($A_{at}$) for each lot were calculated as a function of differences between lot production rate ($R_t$) and the average rate ($R_a$). (3) Actual past unit costs were transformed using the adjustment factors to the unit costs they "would have been" if the production rate had not differed from the average. (4) Traditional learning curves were fit to these transformed costs to estimate learning curve parameters. (5) The learning curve was used to forecast future cost, assuming future production rate would be average. (6) Future unit cost was adjusted if the production rate in the future period differed from the average.

8. Some research (e.g., Moses, 1993) has shown there may be little association between a cost model's ability to explain past costs and its ability to forecast future costs. Higher $R^2$ (better explanation) can always be achieved by adding variables to a model but high $R^2$ may be a poor indicator of forecast accuracy.

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