



Calhoun: The NPS Institutional Archive
DSpace Repository

Reports and Technical Reports

All Technical Reports Collection

1976-03

Nonsymmetric ballistic range, height,
time-of-flight and optimal flight path angle
computations with programs for a
Hewlett-Packard 65 calculator

Shudde, Rex H.

Monterey, California. Naval Postgraduate School

<https://hdl.handle.net/10945/29411>

This publication is a work of the U.S. Government as defined in Title 17, United States Code, Section 101. Copyright protection is not available for this work in the United States.

Downloaded from NPS Archive: Calhoun



Calhoun is the Naval Postgraduate School's public access digital repository for research materials and institutional publications created by the NPS community. Calhoun is named for Professor of Mathematics Guy K. Calhoun, NPS's first appointed -- and published -- scholarly author.

Dudley Knox Library / Naval Postgraduate School
411 Dyer Road / 1 University Circle
Monterey, California USA 93943

<http://www.nps.edu/library>

NPS55Su76031

NAVAL POSTGRADUATE SCHOOL

Monterey, California



NONSYMMETRIC BALLASTIC RANGE, HEIGHT, TIME-OF-FLIGHT
AND OPTIMAL FLIGHT PATH ANGLE COMPUTATIONS
WITH PROGRAMS FOR A HEWLETT-PACKARD 65 CALCULATOR

by

Rex H. Shudde

March 1976

Approved for public release; distribution unlimited.

NAVAL POSTGRADUATE SCHOOL
Monterey, California

Rear Admiral Isham Linder
Superintendent

Jack R. Borsting
Provost

Reproduction of all or part of this report is authorized.

Prepared by:

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NPS55Su76031	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Nonsymmetric Ballistic Range, Height, Time-of-Flight, and Optimal Flight Path Angle Computations with programs for a Hewlett-Packard Calculator.		5. TYPE OF REPORT & PERIOD COVERED Technical Report
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Rex H. Shudde		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, CA 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE March 1976
		13. NUMBER OF PAGES 17
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Ballistic Missile Range Maximization Ballistic Range Programmable Hand Calculator Flight Path Angle		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The purpose of this report is to provide the equations and HP-65 Programmable Calculator programs for computing ballistic range, height, time-of-flight, and particularly the flight path angle which maximizes ballistic ranges for non-symmetric launch and target positions. A no-atmosphere, non-rotating, spherical Earth is assumed.		

TABLE OF CONTENTS

I.	The Introduction and Apology	1
II.	The Problem	2
III.	The Computational Procedures	4
IV.	The Illustrative Example	9
V.	The Program Listings	13
VI.	The References	14
VII.	The Appendix	15

NONSYMMETRIC BALLASTIC RANGE, HEIGHT, TIME-OF-FLIGHT
AND OPTIMAL FLIGHT PATH ANGLE COMPUTATIONS
WITH PROGRAMS FOR A HEWLETT-PACKARD 65 CALCULATOR

I. The Introduction and Apology.

The range of a ballistic missile over the surface of the Earth is a topic which is discussed in almost every text on astronautics (Reference 1, for example). Unfortunately, none of the available texts contained a needed procedure for the exact solution of the unsymmetric ballistic problem (defined in Section II); all procedures found were approximate and were based on a symmetric approximation [See References 2 and 3, for example]. The author of this report faced the dilemma of either (1) taking the time to solve the problem afresh, or (2) taking an inordinate longer time to do a thorough search to find a solution which might not be the one desired. The former course of action was taken and all due apologies are hereby extended to those that have published more elegant solutions.

All of the computational methods are outlined in Section III. Instructions and a sample problem for the Hewlett-Packard 65 Calculator (HP-65) are given in Section IV. This HP-65 programs are in Section V, and the development of the β_{opt} procedure is in the Appendix. Familiarity with Newtonian two-body theory is assumed.

II. The Problem.

Basically, the problem is to compute the range, S , and time-of-flight, t_f , of a ballistic missile given the booster cutoff at height h_E after a vertical ascent, a cutoff velocity v_L , and a subsequent flight path angle β (Fig. 1). Given that a solution to the stated problem is available, the next natural question to ask is, "What flight path angle β_{opt} will obtain the maximum range S_{max} ?" The solution is fairly simple in the symmetric ballistic problem, i.e. when $h_E = 0$, and called symmetric because the launch point L and target point T are symmetric with the line of apses of the Keplerian orbit (i.e. $\phi_1 = \phi_2$ in Fig. 1). The unsymmetric ballistic problem is that in which $h_E \neq 0$. Here we assume that h_E is given, the Earth with radius r_E is spherical, non-rotating, and has no atmosphere. The effect of the Earth's rotation may be calculated figuring how far the target has moved during the time of flight of the missile. Explicitly, we assume that Keplerian orbit is obtained at height h_E with terminal booster cutoff velocity v_L and flight path angle β (or β_L), and the target T is at the Earth's surface.

The computation of range and time-of-flight for given h_E , r_E , v_L , and β can easily be obtained from the equations of elliptical orbits. The determination of β_{opt} is somewhat complicated, however, and requires an iterative procedure. In all of the developments we assume that L and T are separated by the line of apses (which implies that $0 < \beta \leq \frac{\pi}{2}$).

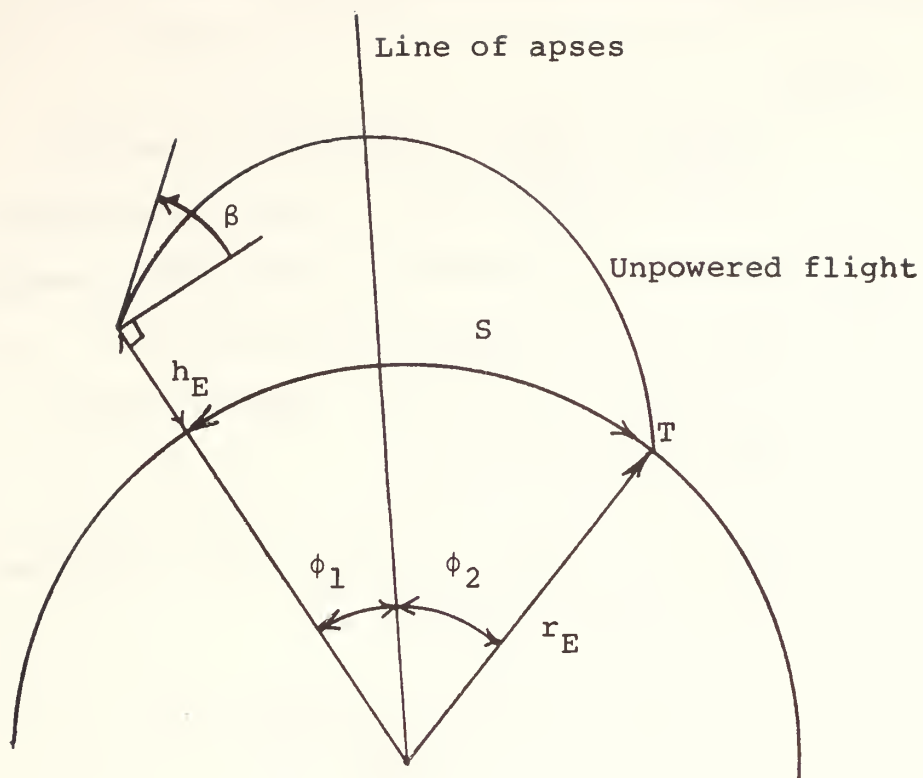


FIGURE 1

III. The Computational Procedures.

A. Range

1. Let r_L and r_T denote the distance of the launch point L and target point T from the center of the Earth. Usually we let $r_L = r_E + h_E$ and $r_T = r_E$ although r_L and r_T need not be so defined in general.
2. From r_L and v_L , compute the semimajor axis of the ellipse,

$$a = \frac{\mu}{\left(\frac{2\mu}{r_L}\right) - v_L^2},$$

where $\mu = k_E^2$ and k_E is the (Gaussian) gravitational constant for the Earth. We have assumed that the mass of the missile is negligible with respect to the mass of the Earth.

3. Using the flight path angle β , compute the elliptical semiparameter

$$p = (r_L v_L \cos \beta)^2 / \mu.$$

4. Compute the eccentricity

$$e = \sqrt{1 - \frac{p}{a}}.$$

5. Compute the true anomaly of the launch point

$$f_L = \cos^{-1} \left[\frac{1}{e} \left(\frac{p}{r_L} - 1 \right) \right]$$

where $0 \leq f_L \leq \pi$. Note that the line of apses can be said to have true anomaly $f = \pi$.

6. Compute the true anomaly of the target point

$$f_T = 2\pi - \cos^{-1} \left[\frac{1}{e} \left(\frac{p}{r_T} - 1 \right) \right]$$

7. The surface range of the missile is then

$$S = r_E (f_T - f_L) .$$

B. Time-of-Flight

1. Compute the eccentric anomaly of the launch point,

$$E_L = \cos^{-1} \left[\frac{1}{e} \left(1 - \frac{r_L}{a} \right) \right]$$

where $0 \leq E_L \leq \pi$.

2. Compute the eccentric anomaly of the target point,

$$E_T = 2\pi - \cos^{-1} \left[\frac{1}{e} \left(1 - \frac{r_T}{a} \right) \right]$$

where $\pi \leq E_T \leq 2\pi$.

3. Using Keplers' equation, compute the time-of-flight

$$t_f = \frac{a^{3/2}}{\sqrt{\mu}} \left[(E_T - E_L) - e(\sin E_T - \sin E_L) \right] .$$

C. Height of Trajectory & Circular Velocity

1. The maximum height of the trajectory above the Earth's surface for any h_E , v_L , and β is given by

$$h_{\max} = h = a(1 + e) - r_E$$

2. Circular Velocity

The velocity required for a circular orbit at r_L is

$$v_c = \sqrt{\mu/r_L} .$$

v_c will be used in Section III D.

D. Optimum Flight Path Angle & Maximum Range

For a specified h_E and v_L , a value of β can be found such that

$$S_{\max} = S(\beta_{\text{opt}}) = \max_{\beta} S(\beta) .$$

In this unconstrained optimization problem

we set $\frac{\partial S}{\partial \beta} \Big|_{\beta = \beta_{\text{opt}}} = 0$ and solve

for β_{opt} . The relationship between S and β is given by the computational formulas for the range in part A. Although an explicit formula for β is unobtainable, the following exact relationship is useful:

$$(1) \quad \cos^2 \beta_{\text{opt}} = \frac{1}{(2-\alpha^2)} \left\{ 1 + \frac{\left[1 - \left(\frac{r_L}{r_T} \right) (2-\alpha^2) \cos^2 \beta_{\text{opt}} - \frac{2\mu}{v_L^2} \left(\frac{1}{r_L} - \frac{1}{r_T} \right) \right] \sin \beta_{\text{opt}}}{\sqrt{1 - \left(\frac{r_L}{r_T} \right)^2 \cos^2 \beta_{\text{opt}} - \frac{2\mu}{v_L} \left(\frac{1}{r_L} - \frac{1}{r_T} \right)}} \right\}$$

where $\alpha = \frac{v_L}{v_c}$.

In the symmetric ballistic problem $r_L = r_T$; the equation for β_{opt} then reduces to

$$(2) \quad \cos^2 \beta_{\text{opt}} = \frac{1}{2-\alpha^2},$$

which is the result given in References 1, 2, and 3.

In the unsymmetric case Equation (1) is used to find β_{opt} as follows:

1. Solve (2) for β_{opt} .
2. Substitute β_{opt} into the right hand side of (1) to obtain a new value for β_{opt} .

3. Average the last two values of β_{opt} to obtain a new value of β_{opt} and return this value of β_{opt} to step 2.
4. It has been found that five iterations are sufficient to obtain a reliable value for β_{opt} .
5. Using β_{opt} , compute the maximum range $S_{max} = S(\beta_{opt})$ using procedure A.

IV. The Illustrative Example.

The HP-65 program consists of four magnetic cards which are used as follows:

Card 1: This card must be entered first. It is used to set the two constants r_E and μ for any choice of four distance units and any choice of four time units. The distance units are kilometers, meters, statute miles, and nautical miles; the distance unit is entered by pressing key A, B, C, or D, respectively. The display shows the equatorial Earth radius r_E in the respective unit. The time units are seconds, minutes, hours, and days; the time unit is entered by pressing key A, B, C, or D, respectively. The display shows the constant μ in the appropriate (distance)³/(time)² unit. Note - after entering program Card 1, the distance unit must be entered before the time unit is entered since the keys A through D perform a dual function. The basic definitions used are:

$$1 \text{ statute mile} = 1.609344 \text{ kilometers(km)}$$

$$1 \text{ nautical mile} = 1.852 \text{ km}$$

$$r_E = 6378.160 \text{ km}$$

$$\mu = 398603 \text{ km}^3/\text{sec}^2$$

Card 2: This card computes S , h_{\max} , and v_C given h_E , v_L and β . It must be used after the

execution of Card 1 and it may be used before and/or after the execution of Cards 3 and 4. If it is used after Card 4 to compute S_{\max} , then β_{opt} as computed by Card 4 need not be re-entered. Otherwise, in any order: enter h_E and press A, enter v_L and press B, enter β and press C. To find the range S, press E. Then to find h_{\max} , press R/S; and then to find v_C , press R/S again. Note: If h_E , v_L , and β have been entered using Card 3, then they do not have to be reentered when using Card 2. Also, any new value of h_E , v_L , or β can be entered and the range computed without having to re-enter unchanged values.

Card 3: This card computes the time of flight, t_f . It must be used after the execution of Card 1 and it may be used before and/or after the execution of Cards 2 and 4. If it is used after Card 4, then β_{opt} as computed by Card 4 need not be re-entered; if it is used after Card 2, then only new quantities need be entered. Otherwise, in any order: enter h_E and press A, enter v_L and press B, enter β and press C. To find the time of flight, press E.

Card 4: This card computes β_{opt} . It must be used after h_E and v_L have been entered from either Card 2 or Card 3. No entries are required. To compute β_{opt} , press any key A through E; five iterations will be performed and the resulting value of β_{opt} will be displayed. To perform five more iterations, press R/S. To monitor the intermediate iterations press keys f SF2. Each intermediate result will be displayed. To continue, press R/S. To disable the monitoring, press keys f⁻¹ SF2.

Using the four program cards as described above find the range and time-of-flight for a ballistic missile that has a velocity of 10000 knots at a cut-off altitude of 50 nautical miles for a flight path angle of 30° and 40° . Also, find the maximum possible range and time-of-flight for the same missile.

Proceed as follows:

<u>Step</u>	<u>Instruction</u>	<u>Data</u>	<u>Key</u>	<u>Display</u>
1	Enter Card 1			
2	Distance unit is n.mi.		D	$r_E = 3443.93 \text{ n.mi.}$
3	Time unit is hours		C	$\mu = 8.13247 \times 10^{11}$
4	Enter Card 2			
5	Enter cutoff altitude	50	A	$r_L = 3493.93 \text{ n.mi.}$
6	Enter cutoff velocity	10000	B	
7	Enter flight path angle	30	C	
8	Compute range		E	$S = 1928.08 \text{ n.mi.}$

<u>Step</u>	<u>Instruction</u>	<u>Data</u>	<u>Key</u>	<u>Display</u>
8a	Optional: Display h_{\max}		R/S	$h_{\max} = 344.74 \text{ n.mi.}$
8b	Optional: Display v_c		R/S	$v_c = 15256.47 \text{ knot}$
9	Enter flight path angle	40	C	
10	Compute range		E	$S = 1956.60 \text{ n.mi.}$
11	Enter Card 3			
12	Compute time-of-flight Note: $h_E = 50$, $v_L = 10000$, and $\beta = 40^\circ$ since no further entries have been made.		E	$t_f = 0.3043 \text{ hrs.}$
13	Enter flight path angle	30	C	
14	Compute time-of-flight		E	0.2504 hrs.
15	Enter Card 4			
16	Compute β_{opt} (5 iterations)		E	$\beta_{\text{opt}} = 36.0895$
17	Make 5 more iterations to be sure		R/S	$\beta_{\text{opt}} = 36.0895$
18	Enter Card 2			
19	Compute S_{\max}		E	$S_{\max} = 1973.95 \text{ n.m.}$
20	Enter Card 3			
21	Compute time-of-flight for maximum range.		E	$t_f(\beta_{\text{opt}}) = 0.2846$

V. The Program Listings.

CARD 1

Set units of distance and time display r_E and μ

Key Entry	Code Shown	Key Entry	Code Shown
LBL	23	3	03
B	12	7	07
EEX	43	8	08
3	03	.	83
CHS	42	1	01
GTO	22	6	06
0	00	X	71
LBL	23	STO 4	3304
C	13	R/S	84
10 1	01	60 LBL	23
.	83	B	12
6	06	6	06
0	00	0	00
9	09	GTO	22
3	03	1	01
4	04	LBL	23
4	04	C	13
GTO	22	3	03
0	00	6	06
20 LBL	23	70 0	00
D	14	0	00
1	01	GTO	22
.	83	1	01
8	08	LBL	23
5	05	D	14
2	02	8	08
GTO	22	6	06
0	00	4	04
LBL	23	0	00
30 A	11	80 0	00
1	01	GTO	22
LBL	23	1	01
0	00	LBL	23
g	35	A	11
1/x	04	1	01
ENTER	41	LBL	23
ENTER	41	1	01
ENTER	41	ENTER	41
X	71	X	71
40 X	71	90 RCL 1	3401
3	03	X	71
9	09	STO 1	3301
8	08	R/S	84
6	06	g NOP	3501
0	00	g NOP	3501
3	03	"	3501
X	71	"	3501
STO 1	3301	"	3501
CL x	44	"	3501
50 6	06	100 "	3501

CARD 2

Compute range, h_{max} , v_c

Key Entry	Code Shown	Key Entry	Code Shown
RCL 4	3404	X	71
+	61	1	01
STO 3	3303	-	51
R/S	84	CHS	42
LBL	23	f	31
B	12	$\sqrt{\quad}$	09
STO 2	3302	STO 6	3306
R/S	84	RCL 8	3408
LBL	23	RCL 3	3403
10 C	13	60 D	14
STO 7	3307	RCL 8	3408
R/S	84	RCL 4	3404
LBL	23	D	14
D	14	+	61
g	35	g	35
RAD	42	π	02
\div	81	ENTER	41
1	01	+	61
-	51	g x \leftrightarrow y	3507
20 RCL 6	3406	70 -	51
\div	81	RCL 4	3404
f-1	32	x	71
COS	05	R/S	84
RTN	24	RCL 6	3406
LBL	23	1	01
E	15	+	61
g	35	RCL 5	3405
DEG	41	\div	81
30 RCL 7	3407	80 RCL 4	3404
f	31	-	51
COS	05	R/S	84
RCL 2	3402	RCL 1	3401
X	71	RCL 3	3403
RCL 3	3403	\div	81
X	71	f	31
ENTER	41	$\sqrt{\quad}$	09
X	71	R/S	84
RCL 1	3401	GTO	22
\div	81	E	15
40 STO 8	3308	90 g NOP	3501
2	02	g NOP	3501
RCL 3	3403	"	3501
\div	81	"	3501
RCL 2	3402	"	3501
ENTER	41	"	3501
X	71	"	3501
RCL 1	3401	"	3501
\div	81	"	3501
-	51	"	3501
50 STO 5	3305	100 "	3501

NOTE: Before putting a program into memory, after switching to W/PGM mode, press f and then PRGM to clear memory.

CARD 3

Compute time-of-flight

Key Entry	Code Shown		Key Entry	Code Shown
RCL 4	3404		-	51
+	61		STO 5	3305
STO 3	3303		X	71
R/S	84		1	01
LBL	23		-	51
B	12		CHS	42
STO 2	3302		f	31
R/S	84		$\sqrt{\quad}$	09
LBL	23		STO 6	3306
C	13	60	g	35
STO 7	3307		π	02
R/S	84		ENTER	41
LBL	23		+	61
D	14		RCL 4	3404
g	35		D	14
RAD	42		-	51
RCL 5	3405		ENTER	41
X	71		f	31
1	01		SIN	04
-	51	70	RCL 6	3406
CHS	42		X	71
RCL 6	3406		-	51
\div	81		RCL 3	3403
f^{-1}	32		D	14
COS	05		ENTER	41
RTN	24		f	31
LBL	23		SIN	04
E	15		RCL 6	3406
g	35		X	71
DEG	41	80	-	51
RCL 7	3407		-	51
f	31		RCL 5	3405
COS	05		\div	81
RCL 2	3402		RCL 1	3401
X	71		RCL 5	3405
RCL 3	3403		X	71
X	71		f	31
ENTER	41		$\sqrt{\quad}$	09
X	71		\div	81
RCL 1	3401	90	R/S	84
\div	81		GTO	22
STO 8	3308		E	15
2	02		g NOP	3501
RCL 3	3403		g NOP	3501
\div	81		g NOP	3501
RCL 2	3402		"	3501
ENTER	41		"	3501
X	71		"	3501
RCL 1	3401		"	3501
\div	81	100	"	3501

CARD 4

Compute maximizing flight path angle β_{opt}

Key Entry	Code Shown		Key Entry	Code Shown
2	02		SIN	04
RCL 2	3402		X	71
ENTER	41		RCL 5	3405
X	71		RCL 3	3403
STO 5	3305		RCL 4	3404
RCL 3	3403		\div	81
X	71		RCL 7	3407
RCL 1	3401		f	31
\div	81		COS	05
-	51	60	X	71
STO 6	3306		ENTER	41
RCL 4	3404		X	71
g	35		-	51
1/x	04		f	31
RCL 3	3403		$\sqrt{\quad}$	09
g	35		\div	81
1/x	04		1	01
-	51		+	61
RCL 5	3405		RCL 6	3406
\div	81	70	\div	81
RCL 1	3401		f	31
X	71		$\sqrt{\quad}$	09
ENTER	41		f^{-1}	32
+	61		COS	05
1	01		f	31
+	61		TF 2	81
STO 5	3305		R/S	84
g	35		g NOP	3501
DEG	41		RCL 7	3407
0	00	80	gx \leftrightarrow y	3507
STO 7	3307		STO 7	3307
5	05		0	00
STO 8	3308		g x=y	3523
LBL	23		GTO	22
0	00		0	00
RCL 5	3405		+	61
RCL 3	3403		+	61
RCL 4	3404		2	02
\div	81		\div	81
RCL 6	3406	90	STO 7	3307
X	71		g	35
RCL 7	3407		DSZ	83
f	31		GTO	22
COS	05		0	00
X	71		R/S	84
gLSTx	3500		5	05
X	71		STO 8	3308
-	51		GTO	22
RCL 7	3407		0	00
f	31	100	g NOP	3501

NOTE: Before putting a program into memory, after switching to W/PGM mode, press f and then PRGM to clear memory.

VI. The References.

1. Berman, Arthur I., The Physical Principles of Astronautics, John Wiley and Sons, Inc. 1961.
2. Seifert, Howard (editor), Space Technology, John Wiley and Sons, Inc. 1959.
3. Wyckoff, Robert C., Private Communication, 1975.

VII. The Appendix.

As indicated in section IIID, the maximum range for a specified cutoff altitude h_E and cutoff velocity v_L is found by the following unconstrained optimization:

$$S_{\max} = S(\beta_{\text{opt}}) = \max S(\beta)$$

$$0 < \beta \leq \frac{\pi}{2}$$

Using the equations in section IIIA, we want to find β_{opt} such that

$$\left. \frac{\partial S}{\partial \beta} \right|_{\beta = \beta_{\text{opt}}} = r_E \left. \frac{\partial}{\partial \beta} (f_T - f_L) \right|_{\beta = \beta_{\text{opt}}} = 0 .$$

We find that

$$\frac{\partial}{\partial \beta} (f_T - f_L) = (\gamma_T - \gamma_L) \frac{\partial p}{\partial \beta} ,$$

where

$$\gamma_i = \frac{1}{\sqrt{1 - \frac{1}{e^2} \left(\frac{p}{r_i}\right)^2 - 1}} \left[\frac{1}{er_i} + \frac{1}{2ae^3} \left(\frac{p}{r_i} - 1\right) \right], \quad i=T, L .$$

For the derivative of $f_T - f_L$ with respect to β to vanish at $\beta = \beta_{\text{opt}}$, we must have either:

$$1. \quad (\gamma_T - \gamma_L)_{\beta = \beta_{\text{opt}}} = 0$$

or

$$2. \quad \left. \frac{\partial p}{\partial \beta} \right|_{\beta = \beta_{\text{opt}}} = 0$$

We find that

$$\frac{\partial p}{\partial \beta} = - \frac{r_L^2 v_L^2 \sin 2\beta}{\mu} = 0$$

for $\beta = 0, \pm \pi/2, \pm \pi, \text{ etc.}$ Since we require that the launch point and the target point be separated by the line of apsides, we require that $\theta < \beta \leq \frac{\pi}{2}$. This requirement is imposed because we must properly interface the discontinuity caused by the principal angle in the arc cosine used in steps 5 and 6 of section III A. Thus $\beta = \pi/2$ is the only angle of interest for which $\partial p / \partial \beta = 0$, and this angle clearly gives rise to a minimum in range since the trajectory of the missile is vertical (straight up, and then straight down).

The value of β which maximizes S is found from the requirement that

$$(\gamma_T - \gamma_L)_{\beta = \beta_{\text{opt}}} = 0 .$$

This equation has defied attempts to find an analytic solution, but it can be rearranged and displayed as equation 1 in section III D, where the solution procedure is detailed.

INITIAL DISTRIBUTION LIST

	Copies
Dean of Research, Code 023 Naval Postgraduate School Monterey, California 93940	1
Defense Documentation Center Cameron Station Alexandria, Virginia 22314	2
Library, Code 0212 Naval Postgraduate School Monterey, California 93940	2
Library, Code 55 Department of Operations Research and Administrative Sciences Naval Postgraduate School Monterey, California 93940	2
Code 55Su Naval Postgraduate School Monterey, California 93940	100

30 NOV 76

S10501-

30 NOV 76

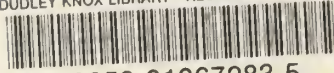
S10501-

U-172,215

Naval Postgraduate School.
NPS-55Su76031.

U 172215

DUDLEY KNOX LIBRARY - RESEARCH REPORTS



5 6853 01067283 5

U 1722

NPS