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ON CHOOSING A FAMILY OF PROBABILITY DISTRIBUTIONS

FOR LEAD TIME DEMAND

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and

Marlin U. Thomas

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Technical Report

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The problem of selecting an appropriate family of probability distributions to be used in the procedures for forecasting future demands is examined. Using the principle of maximum entropy and other practical considerations, the gamma family is shown to result from conditions typical of those in the Naval Supply System. On considering other decision criteria based on measures of supply effectiveness, the gamma family is shown to be superior to the family of distribution presently in use.

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Executive Summary

Inventory managers are faced with the difficult problem of making decisions under the uncertainty of future needs. This problem is especially critical in the Navy where such important decisions as those concerning budgets, the range and depth of stock to be maintained and when to replenish stocks must all be made for hundreds of thousands of items on the basis of predictions about future demand. In order to provide a margin of safety to protect against random fluctuations, the inventory manager needs to make a probabilistic statement about the demand process for each item. But, because of the size of the Navy's inventory system, it is not feasible to give individual attention to determine "optimal" forecasts for each item. Consequently, the Navy has attempted to solve the forecasting problem by assuming a family (or families) of probability distributions and concentrating on the problem of estimating the parameters for the individual items. Several investigations have looked into the problem of how to estimate the parameters. In this report we focus attention on the problem of selecting an appropriate family of probability distributions.

Various decision criteria should be considered when selecting a family of distributions to describe demand. The family must be rich enough to be able to describe demand distributions for a large number of different items, and it must be mathematically tractable. Furthermore, the family should reflect the current knowledge of the characteristics of the demand, but, at the same time, it should also reflect the uncertainty. These considerations combined with previous experience with demand data and knowledge about the type of information available to the inventory manager led us to examine the family of gamma distributions.

Using the maximum entropy procedure, a decision criterion which leads one to select that family which maximizes the decision maker's uncertainty subject to his current information, we show that the gamma family results from information typically available to inventory managers. In addition, we used actual demand data to compare the gamma family to the normal family, which is currently employed almost exclusively in the Navy's forecasting procedures. Goodness-of-fit tests were performed and numerical evaluations of various measures of supply effectiveness were made. The experimental evidence gives strong support for the use of the gamma family as preferable to the normal family for describing demand.

1. Introduction

Large multi-item inventory systems have long been confronted with the problem of forecasting the occurrence of future demand. The stochastic nature of demand creates an uncertainty that has a significant impact on virtually every decision that an inventory manager must make. For example, such inventory decisions as

(1) what items should be stocked,

(2) at what depth should an item be stocked,

(3) how should budgets be allocated among items,

(4) when should resupply orders be placed, and

(5) how much should be ordered

all depend strongly on the forecasts of future demand. In order to provide a high level of service to the customer, the inventory manager must protect against stockouts as best possible within his limited resources. This can only be done by anticipating future demand.

In those cases where inventory levels are updated continuously and orders can be placed as soon as available stock reaches the reorder level, the time lag between the moment the order is placed and the time the order is received is an interval of uncertainty. Without incurring extra costs, the inventory manager is at the mercy of his customers and his supplier during the procurement lead time. He can only attempt to protect the inventory system against stockouts during those intervals of uncertainty by placing orders before the stock assets drop too low. Since he can never be certain that his on-hand stock will last throughout the lead time, he will usually assume that future demand will be similar to the past demand.

He will then estimate the probability distribution of lead time demand to guide him in making his decisions.

Nearly all mathematical models, from the very naive to the most sophisticated, require the probability distribution of lead time demand. For example, if $F_i(\cdot)$ represents the cumulative distribution of lead time demand for item i and r_i is the reorder level for item i, almost all models dictate, in some form, that the reorder level be determined by solving

$$1 - F_i(r_i) = C_i \tag{1}$$

The value C_i depends on the model; it is usually a function of the inventory costs or some specified measure of system performance. We are concerned with the problem of selecting an appropriate probability distribution $F_i(\cdot)$.

Because of the sheer magnitude of large multi-item inventory systems, the inventory manager can rarely afford the luxury of examining the demand histories of the items individually and fitting the "best" distribution to each item. For the most part the data required for such an analysis is unavailable. Even if the data were available the dynamic nature of demand histories would make a simple distribution-fitting approach of limited value. These problems notwithstanding a cost/benefit analysis would probably suggest that some alternative method be employed.

2. Selecting A Family of Distributions

One feasible way to accomplish the task is to select a family of probability distributions in some rational manner. Given an individual item, a particular member of the chosen family would be selected by estimating the necessary parameters using statistics collected from the item's demand data. This approach is particularly attractive because of its computational simplicity. Indeed, most large inventory systems have taken this approach. However, it is evident from the bulk of the forecasting literature (see, for example, Brown[1], Harrison[3], and Winters[8]) that most attention has focused on the problem of estimating the parameters of the distribution. When placed in the proper perspective, it is hard to justify the use of a costly sophisticated forecast technique to estimate parameters for a poorly chosen family of probability distributions.

What is the appropriate family of distributions? Falling back on first principles, it can be argued in many cases that a Poisson process or a compound Poisson process generates demands. Nevertheless, the lead time itself may be a random variable so that the marginal distribution of lead time demand can easily become intractable even if the process generating demands were relatively simple like a compound Poisson process. What is needed is a simple distribution which lends itself easily to numerical calculations and which provides a good approximation in practical applications.

The quest for such a family of approximating distributions has led, in most cases, to the choice of a family of distributions for continous variables despite the fact that demands are normally integer valued. This is because it is usually easier to work analytically with variables that can be treated as continuous. Any problems caused by using a continuous distribution to approximate a discrete distribution can usually be ignored. The most frequently used family of distributions has been the normal family. The advocates of the normal family will support their choice by arguing that it is easy to work with and well tabulated. For theoretical support they will then make some appeal to the Central Limit Theorem to argue for the normality of leadtime demand. These arguments certainly support the choice of the normal family, but is that really the best family to use? The normal distribution has one obvious shortcoming--it does not have the proper domain for the inventory application. This may not be serious if the weight assigned to the negative half of the real line is small, but problems could arise if that weight is not negligible. A simple way to correct that deficiency is to truncate the left tail of the normal distribution at zero and normalize the resulting truncated distribution. We are then led in a natural way to consider the family of truncated normal distributions. This is certainly appealing, but a stronger justification is needed for selecting any family to be used. We must establish reasonable decision criteria upon which to base our choice of a family of distributions.

3. Maximum Entropy Criterion

The problem confronting the inventory manager is to select a probability distribution for the lead time demand given only limited knowledge about the random nature of the process. An appealing criterion which is often used to make such a decision is that of maximizing the entropy.

The inspiration for the method of maximum entropy is due to the work of Shannon and Weaver[5] in communication theory. The entropy of a probability function is defined to be the expectation of the logarithm of the probability function. Mathematically, if $f(x)$ is the probability density function associated with the continuous random variable X defined on the interval (a,b), the entropy of the probability function is given by

$$H(f) = -\int_a^b f(x) \ln f(x) dx \tag{2}$$

The entropy is taken as a measure of the amount of "uncertainty" contained in the distribution of X. The inventory manager usually has some knowledge of the characteristics of the random variable X through past history, his experience with the customers or perhaps known moments. His problem may then be viewed as one of finding a probability density $f(x)$ that maximizes the entropy subject to constraints that reflect the current knowledge of the characteristics of the random variable. Formally, the inventory manager wishes to

$$\max H(f)$$

subject to

$$\int_a^b f(x)dx = 1 \tag{3}$$

$$\int_a^b \xi_k(x)f(x)dx = \mu_k \ , \qquad k = 1,2,\ldots,n$$

$$f(x) \geq 0 \ , \qquad a \quad x < b$$

where μ_k is a constant for a given function $\xi_k(x)$. For example, if $\xi_k(x) = x^k$, μ_k is the $k\underline{th}$ moment.

Introducing Lagrange multipliers $\alpha,\lambda_1,\ldots,\lambda_n$, we convert (3) to the following:

$$\max \phi(x,f(x)) = -\int_a^b [\ell n \ f(x) + \alpha + \sum_{k=1}^{n} \lambda_k \xi_k(x)]f(x)dx$$
$$+ \alpha + \sum_{k=1}^{n} \lambda_k \mu_k \tag{4}$$

It follows from the calculus of variations that $\phi(x,f(x))$ is maximum when

$$\frac{\partial}{\partial f(x)} \{\ell n \ f(x) + \alpha + \sum_{k=1}^{n} \lambda_k \xi_k(x)\}f(x) = 0$$

This leads to the general maximum entropy density

$$f(x) = \exp\left(-\sum_{k=0}^{n} \lambda_k \xi_k(x)\right) , \quad a \leq x \leq b \tag{5}$$

where $\lambda_0 = 1 + \alpha$ and we take $\xi_0(x) = 1$. The $n + 1$ constraint equations in (3) must be solved in order to determine $\lambda_0, \lambda_1, \ldots, \lambda_n$ in (5). The method of maximum entropy yields many of the classical probability density functions under appropriate conditions. Some of the results are summarized in Table 1.

Table 1.

Some Classical Maximum Entropy Densities

Conditions	Density Function: $f(x)$
$\xi_1(x) = 0$ $a < x < b$	UNIFORM $1/(b-a)$
$\xi_1(x) = x$ $x \in (0,\infty)$	EXPONENTIAL $\lambda \exp(-\lambda x)$ $\lambda > 0$
$\xi_1(x) = x$ $\xi_2(x) = \ln x$ $x \in (0,\infty)$	GAMMA $\dfrac{x^{(\alpha-1)} \exp(-x/\beta)}{\Gamma(\alpha)\beta^{\alpha}}$ $\alpha ; \beta > 0$
$\xi_1(x) = x$ $\xi_2(x) = x^2$ $x \in (0,\infty)$	TRUNCATED NORMAL $\dfrac{K \exp(-(x-\mu)^2/2\sigma^2)}{\sigma\sqrt{2\pi}}$ $K > 0$, $\sigma^2 > 0$
$\xi_1(x) = x$ $\xi_2(x) = x^2$ $x \in (-\infty,\infty)$	NORMAL $\dfrac{\exp(-(x-\mu)^2/2\sigma^2)}{\sigma\sqrt{2\pi}}$ $\sigma^2 > 0$

It is intuitively appealing that classical probability densities result from the maximum entropy method with the specification of simple moments. For the inventory problem the distributions of interest are exponential, gamma and truncated normal.

The appropriate discrete analogs for the densities and conditions given in Table 1 follow directly. The details leading to this table as well as the details for other maximum entropy densities are found in the works by Tribus[6,7] and Clough[2]. Although the concept of maximizing entropy as a criterion for decision processes has only recently received wide acceptance in engineering, psychology and business and economic applications, it has long been known as a tool for statistical inference. For a rigorous mathematical treatment of this, the reader is directed to the work of Kullback[4].

4. Other Decision Criteria

Under appropriate conditions (see Table 1) the exponential, truncated normal and gamma were all seen to be maximum entropy distributions over the interval $(0,\infty)$. In the inventory problem the above moments may not be known, but estimates can probably be obtained by examining past demand data. It is the decision-maker's prerogative to collect whatever data he deems useful. Those statistics typically include estimates for only the mean and variance, but it would be a simple task to collect an estimate for other moments such as $E[\ln x]$. The maximum entropy distribution depends on the information that is available. Therefore, we are faced with another decision--what information should be collected?

The information that is collected should result in a family of maximum entropy distributions which is versatile (allowing for many shapes) and mathematically tractable. More importantly, the resulting family should perform well in view of the measures of supply effectiveness. For example, if the objective of the inventory system is to minimize the total number of stockouts subject to a given investment in stock, one should prefer that family which yields the fewest stockouts with a given inventory investment. Other measures of effectiveness such as the probability of a stockout or time-weighted backorders might also be appropriate as decision criteria for selecting among families of distributions.

The gamma family of distributions becomes particularly attractive when one considers its versatility and tractability. This two parameter family is so rich that it can approximate virtually any nonnegative unimodal distribution. In fact, many of the classical distributions such as the exponential, chi-square and the Erlang are special cases of the gamma family. Furthermore, a member of this family can usually be found that provides a good approximation to such distributions as the Weibull, log normal and the truncated normal (having domain $(0,\infty)$). These facts, combined with the knowledge that it is a maximum entropy distribution, weigh heavily toward the selection of the gamma family. However, the crucial test is how well the inventory system performs in actual use with the gamma family.

5. Data Comparisons: Gamma vs. Normal

We have previously commented that the normal family enjoys widespread use in the inventory community for describing the distribution of lead time demand. We have presented supporting arguments for using the gamma family. We now seek to make the case for the gamma family even stronger by presenting a posteriori comparisons with the normal family using actual demand data. The comparisons reveal which family gives the better "fits" as well as which family does the better job of maximizing the effectiveness of the inventory system.

In order to compare the normal and gamma families, a random sample of 50 items was selected from a data base of 1000 items in the inventory of the United States Air Force. For each item there were 57 observations of lead time demand. The first test examined the "goodness of fit" provided by the normal and gamma families. Next, the stockout risks were examined. Finally, the two families were compared in an aggregate manner using the average overall system risk per dollar of stockage cost as a measure of effectiveness.

5.1 Tests of Goodness of Fit

For each item, estimates of the mean μ, the variance σ^2 and $E[\ell n\ X]$ were calculated as follows:

$$\hat{\mu} = \frac{1}{n} \sum_{i=1}^{n} x_i = \bar{x}$$

$$\hat{\sigma}^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 = S^2 \tag{6}$$

$$\widehat{E[\ell n\ X]} = \frac{1}{n} \ell n \prod_{i=1}^{n} x_i = \overline{\ell n\ x}$$

Using the gamma density $f(x;\alpha,\beta)$ given in Table 1, a particular member was selected for each item by determining the parameter pair (α,β) by the method of moments. That procedure gives the following estimates:

$$\begin{aligned} \psi(\alpha) - \ell n\ \alpha &= \overline{\ell n\ x} - \ell n\ \bar{x} \\ \beta &= \bar{x}/\alpha \end{aligned} \tag{7}$$

where $\psi(\alpha) = d\ \ell n\ \Gamma(\alpha)/d\alpha$ is the digamma function. Similarly, the normal distribution with parameters $\mu = \bar{x}$ and $\sigma^2 = S^2$ was selected from the normal family. The Kolmogorov-Smirnov test was then applied for each of the 50 items to test first the hypothesis

H_N: The data are from the selected normal distribution.

and then the hypothesis

H_G: The data are from the selected gamma distribution.

The tests were conducted at a significance level of .10 with the results as summarized in Table 2.

Table 2.

K-S Test Results

Distribution	Times Accepted	Times Rejected	Acceptance Percentage
Normal	0	50	0%
Gamma	31	19	62%

The goodness-of-fit tests, alone, present a strong argument in favor of the gamma family. Whereas the normal family never provided a sufficiently good fit, the gamma family gave a satisfactory fit in 62% of the cases. Furthermore, the gamma beat the normal in all cases (using the K-S statistic as the test criterion).

5.2 Stockout Risks

As discussed earlier, a probability distribution for lead time demand is used primarily to determine reorder levels. The higher the reorder level r_i, the lower will be the probability that the lead time demand will exceed r_i. Define the stockout risk to be this probability. One cost-independent method of determining the reorder level is to specify a value for the risk which is acceptable, say ρ_i, and solve (1) for r_i with $C_i = \rho_i$. Since $F_i(\cdot)$ is really unknown, the actual risk may differ from the desired risk. The better the distribution $F_i(\cdot)$ approximates the true underlying distribution, the smaller will be the difference between the actual and desired values for stockout risk.

A second comparison of the two families was undertaken to determine how well each performed in determining reorder levels. With a selected value of $\rho_i = 0.20$, the actual empirical distribution $F_E(\cdot)$ for each item was used to determine the reorder level from (1). (The actual risks were slightly less than 0.20 because the reorder levels were rounded to the next higher integer.) In the same manner, both the normal and the gamma distributions, $F_N(\cdot)$ and $F_G(\cdot)$, were substituted into (1) to determine r_N and r_G respectively. These values were then used with the empirical distribution to estimate the actual risks yielded by the normal and gamma families. That is, the values

$$\rho_N = 1 - F_E(r_N)$$

and (8)

$$\rho_G = 1 - F_E(r_G)$$

were determined. For example, if $F_E(10) = 0.75$, $F_E(15) = 0.80$ and $F_E(20) = 0.90$, then the reorder level should be 15 to get a risk of 0.20. Now, if $r_N = 20$ and $r_G = 10$, then the actual risks are $\rho_N = 0.10$ and $\rho_G = .25$.

In general, these comparisons revealed that the actual risks given by the gamma family were closer to the desired levels than were those obtained from the normal family. The reorder levels determined from the normal family tended to overprotect against stockouts; that is, the values of r_N were usually much higher than those needed to give the desired protection. Three typical comparisons are illustrated in Table 3.

Table 3.

Comparison of Risks ($\rho = 0.20$)

Distribution	Item 1		Item 2		Item 3	
	r	Risk	r	Risk	r	Risk
Empirical	4	.193	22	.176	171	.193
Gamma	4	.193	26	.142	181	.175
Normal	10	.090	28	.123	368	.070

Judging the two families on the basis of how close the reorder levels r_N and r_G are to the reorder levels determined by the empirical distributions, the gamma family showed to be superior to the normal family.

5.3 System Synthesis: Average Stockout Risk per Dollar

The comparison of stockout risks revealed that the normal family tended to overprotect against stockouts. While it is desirable to reduce the stockout risks as low as possible, this cannot be accomplished without paying the price of carrying larger safety stocks. For example, Table 3 shows that the reorder level for item 3 as calculated by the normal distribution to give a risk of 0.20 was 368 when a level of 171 would have been sufficient. The inventory system would certainly provide better service (in terms of reduced stockouts) with the higher reorder level, but it must pay for that extra performance through higher investment and holding costs. With the limited budgets of large multi-item inventory systems, such excesses cannot generally be tolerated. When one item is given too great a protection in a system with limited resources, the risks for other items must increase.

The final comparison looked at overall system risk by synthesizing the operation of the inventory system (50 items) for 57 months with actual demand data using reorder levels determined in one case by the normal family and then repeated using the gamma family. A startup period of 21 months was used to give initial estimates of the parameters of the demand distributions. Those parameters, and consequently the reorder levels, were updated continually as more data became available. Assuming a holding cost rate of one per cent per month, total holding costs were accumulated.

The fraction of those lead times in which the lead time demand exceeded the reorder level (stockouts occurred) was used to estimate "total system risk." The system syntheses were repeated for different values of ρ with results as depicted in Figure 1.

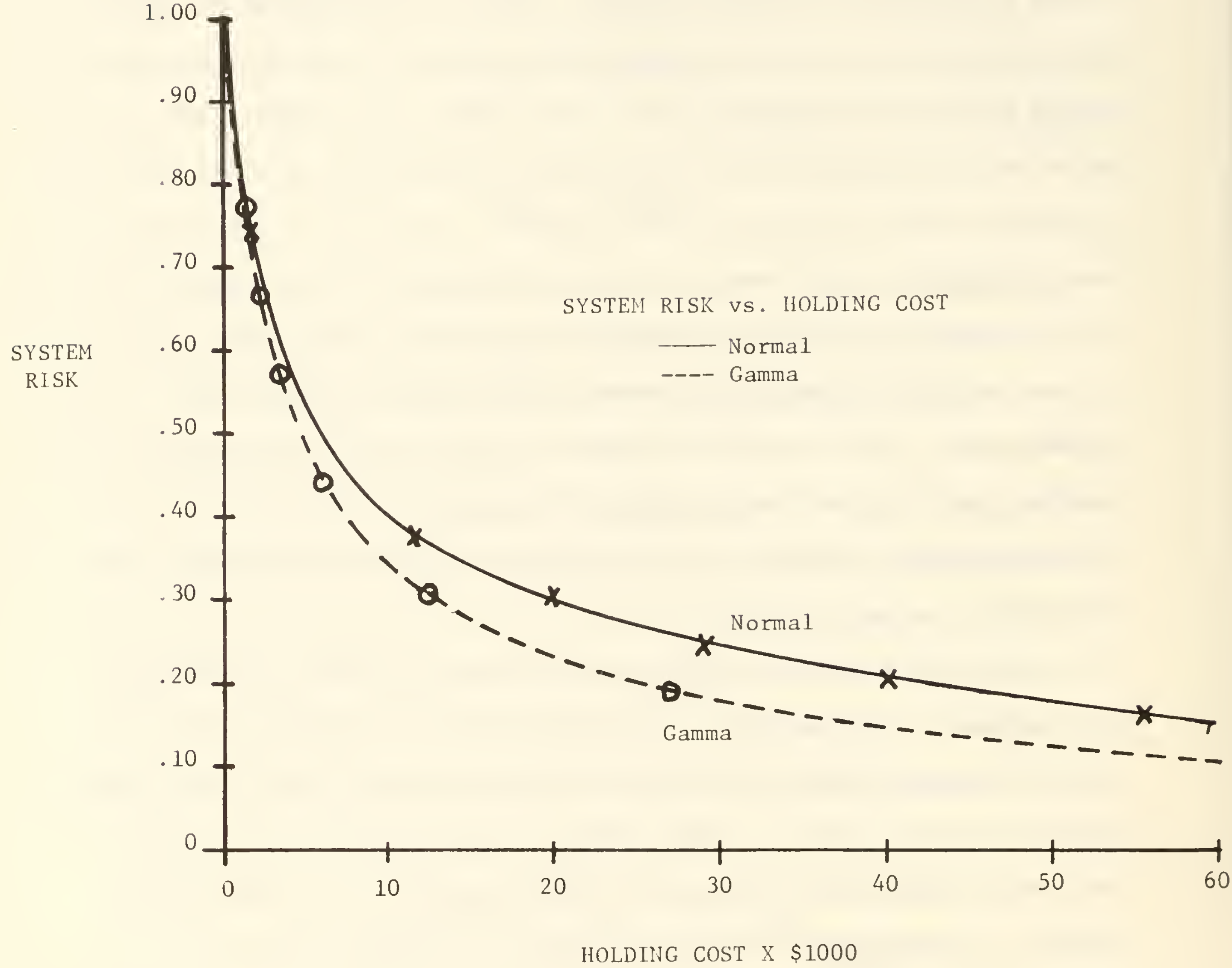

FIGURE 1

One observes from Figure 1 that, for any fixed value of total holding cost, the gamma family gave a system risk lower than that given by the normal family. Hence, use of the gamma family appears to be the more cost-effective alternative.

6. Conclusions

We have presented intuitive and theoretical arguments for using the gamma family to describe the distribution of lead time demand. We have also attempted to evaluate the gamma family as compared to the normal family, which is a widely used competitor. Three reasonable decision criteria based on measures of inventory effectiveness were used in those comparisons. The tests were conducted with a given set of real world demand data.

One may certainly argue that the numerical comparisons might not have favored the gamma family so strongly with a different set of actual demand data. Perhaps special characteristics of the processes generating demands in a given inventory system could be described better with the normal family or some other family.

Nevertheless, we have shown with the available data that the gamma family easily out performs the normal family with respect to (1) goodness of fit, (2) the determination of reorder levels and (3) total system risk per dollar of holding cost.

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