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Doctors on ships?

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by

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ABSTRACT

The decision as to whether a medical doctor, or other expensive specialist, should be carried aboard ship depends upon demand for service, consequences of not providing this service, and cost of providing the service. We supply a simple preliminary mathematical model to aid in making this decision wisely.

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DOCTORS ON SHIPS?

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1. Introduction.

The purpose of this paper is to initiate study of the question: should expensive specialists, e.g. medical doctors, be assigned to ships or submarines on patrol? The issues that arise are as follows.

(1) If a man is injured or becomes seriously ill while a mission is in progress, and if no doctor is present, it may be necessary to transfer the individual from the ship to a hospital for treatment. Presence of a doctor will, at least in some cases, permit the treatment to take place on the ship. Thus the cost of the transfer may be eliminated--at, of course, the expense of maintaining the doctor.

Similarly, but not entirely analogously, breakdown of key equipment entails loss of military effectiveness and may require that the mission be prematurely terminated. If a skilled specialist is aboard such events may be forestalled.

(2) The cost of retaining doctors or other trained specialists is very high. The cost for such a specialist may rightfully include some of the expense of his original recruitment and training.

One subsidiary purpose of this paper is to show that the use of simulation methods is not always essential when dealing with problems of some complexity, and, in particular, those that involve chance effects.

2. The Occurrence of Demands.

Basic to the question of whether a doctor or highly trained and expensive repairman should be added to a ship's complement is the likelihood of demand for services that he alone can supply. We propose some probability models for this question. Actually, a complex variety of sources may conspire to cause demand.

Model 1. Simple Chance Demand, Caused by Accidents or Sudden Disease.

Imagine that a ship has n individuals aboard when it sets out on a mission of duration M. Each individual is thought to have a constant probability λdt of experiencing an accident or sudden severe illness between t and $t + dt$, dt being a small number. The occurrence of accidents or illness is first assumed to be independent from individual to individual. Then our simple model implies that each individual experiences his demanding event at time T_i $(i = 1,2,\ldots,n)$ measured from the start of the mission, T_i being distributed in accordance with an exponential distribution with rate parameter λ:

$$
\begin{aligned}
P\{T_i \le t\} &= 1 - e^{-\lambda t} && 0 \le t < \infty \\
&= 0 && t < 0.
\end{aligned}
\tag{2.1}
$$

Next, the occurrence of the smallest T_i on the ship of crew size n is the distribution of the minimum of a sample of n independent

T_i's, which is exponential with parameter n . Let us call this time τ_n; then

$$P\{\tau_n > t\} = e^{-n\lambda t} \qquad 0 \le t < \infty$$
$$= 1 \qquad t < 0 \tag{2.2}$$

According to this model, if no doctor (or repairman) is present:

(A) The probability that the mission <u>does not</u> terminate during M for the cause associated with λ is

$$P\{\tau_n > M\} = e^{-n\lambda M} \tag{2.3}$$

(B) The expected time to the scheduled end of a mission that involves a rescue (or terminates early) is

$$E[M - \tau_n \mid \tau_n < M] = \frac{\int_0^M (M-x)e^{-n\lambda x} \, n\lambda dx}{1 - e^{-n\lambda M}} \tag{2.4}$$

$$= \frac{M}{1 - e^{-n\lambda M}} - \frac{1}{n\lambda}$$

It is of interest to see what this formula approaches as λ becomes small, a condition likely to be true in practice. Write (2.4) as

$$E[M - \tau_n \mid \tau_n < M] = \frac{e^{-n\lambda M} - 1 + n\lambda M}{(n\lambda)(1-e^{-n\lambda M})}$$

and then expand in Taylor's series to obtain

$$P[M - \tau_n \mid \tau_n < M] = \frac{\frac{(n\lambda M)^2}{2!} + o(n\lambda M)}{n\lambda(n\lambda M + o(n\lambda M))} \tag{2.5}$$

$$\rightarrow \frac{M}{2}$$

This states that in the limit of vanishingly small accident or disease rate and average of one-half the mission time will be lost, provided that the probability is very small of an accident or breakdown of the sort envisioned (e.g. a heart attack, stroke, severe injury, in the medical case).

Model 2. Chance Demand, Differing Demand Rates.

We can realistically generalize Model 1 to the situation for which each individual has a different characteristic rate or probability of requiring the vital service: $\lambda_j dt$ is essentially the demand probability for individual j $(j = 1,2,...,n)$. Then the overall demand rate is, assuming independence, equal to

$$\lambda(n) = \lambda_1 + \lambda_2 + \dots \quad \lambda_n \tag{2.6}$$

and the distribution of τ_n is still exponential, with $\lambda(n)$ replacing $n\lambda$ in (2.2). All this is well-known; see Feller [1].

Practically speaking, one might consider classes of individuals who are more or less susceptible to disease or accident, and who can be characterized by the same failure or catastrophe rates within the class. For example, the older officers (ship captain, executive officer, etc.) are probably less prone to incur an appendicitis outbreak than would a younger man; on the other hand, an older person might be more prone to heart attack. This sort of consideration would compel us to put individuals into classes, with characteristic rates $\{\lambda'_u, \ u = 1,2,...,c\}$; then

$$\lambda(n) = \sum_{u=1}^{c} n(u)\lambda'_u \qquad (2.7)$$

where $n(u)$ is the number of individuals in the u^{th} class.

BuMed data might well be available to provide estimates for the above rates. For equipments, 3M data might well be interpreted for the same purpose.

Model 3. Simple Chance Demand, Demand Rates Randomly Selected from a Population.

An interesting and realistic generalization of Model 2 involves the assumption that each individual's "failure" or demand rate is itself randomly and independently drawn from some fixed population. An immediate generalization, in the spirit of Model 2, is that certain classes of individuals' demand rates come from different populations.

To explore this sort of assumption we first write down the survival probability of the group (crew), given their failure rates. This turns out to be, by independence, equal to

$$P\{\tau_n > x \mid \lambda_1, \lambda_2, \ldots, \lambda_n\} = e^{-\left(\sum_{j=1}^{n} \lambda_j\right)x}$$

Now by assumption each λ_j is independently drawn from a population with distribution $H(y)$, and density $h(y)$. Consequently, removal of the condition on the λ's amounts to integrating:

$$P\{\tau_n > x\} = \prod_{j=1}^{n} \int_0^\infty e^{-yx}\, h(y)dy = [\hat{h}(x)]^n$$

where $\hat{h}(x)$ represents the Laplace transform of the density h.

Example. Let h be a Gamma density:

$$h(y) = \frac{e^{-\alpha y}(\alpha y)^{\beta-1}\alpha}{\Gamma(\beta)} \tag{2.8}$$

Then

$$\hat{h}(x) = \left(\frac{\alpha}{\alpha+x}\right)^{\beta} \tag{2.9}$$

and the chance that a mission proceeds with no demand is

$$P\{\tau_n > M\} = \left(\frac{\alpha}{\alpha+M}\right)^{\beta n} \tag{2.10}$$

It will also be interesting to derive the expected lost mission time under this model. It is

$$E[(M-\tau_n)^+ \mid \lambda_1, \lambda_2, \ldots, \lambda_n] = \int_0^M (M-x)e^{-\lambda(n)x} \lambda(n)dx$$

$$= M[1 - e^{-\lambda(n)M}] - \frac{1}{\lambda(n)} [1 - (1+\lambda(n)M)e^{-\lambda(n)M}]$$

$$= M - \frac{1}{\lambda(n)} [1 - e^{-\lambda(n)M}] \qquad (2.11)$$

Next the conditions on the λ's must be removed. This can be done easily in terms of our gamma density illustration.

Example. h is Gamma.

Then $\lambda(n)$ is Gamma with parameters α and βn. Consequently, we need

$$E\left[\frac{1}{\lambda(n)}\right] = \int_0^\infty \frac{1}{y} e^{-\alpha y} \frac{(\alpha y)^{\beta n-1}}{\Gamma(\beta n)} \alpha \, dy$$

$$= \alpha \int_0^\infty e^{-\alpha y} \frac{(\alpha y)^{\beta n-2}}{\Gamma(\beta n)} \alpha \, dy$$

$$= \frac{\alpha}{\beta n - 1} \qquad (2.12)$$

and

$$E\left[\frac{1}{\lambda(n)} e^{-\lambda(n)M}\right] = \int_0^\infty \frac{1}{y} e^{-yM} e^{-\alpha y} \frac{(\alpha y)^{\beta n-1}}{\Gamma(\beta n)} \alpha \, dy =$$

$$(\alpha+M)\left(\frac{\alpha}{\alpha+M}\right)^{\beta n} \int_0^\infty \frac{1}{y(\alpha+M)} \frac{e^{-y(\alpha+M)}}{\Gamma(\beta n)} [(\alpha+M)y]^{\beta n-1} dy = \frac{\alpha+M}{\beta n-1}\left(\frac{\alpha}{\alpha+M}\right)^{\beta n} \qquad (2.13)$$

Assembling the expression for the expectation of (2.11) we find that

$$E[(M-\tau_n)^+] = M - \frac{\alpha}{\beta n-1} + \frac{1}{\beta n-1} (\alpha+M) \left[\left(\frac{\alpha}{\alpha+M}\right)\right]^{\beta n} \qquad (2.14)$$

A generalization can be carried out for the case in which several subgroups of crew members are described by their specific gamma distributions, but this step will be postponed.

3. Costs and Decisions.

Armed with various models that describe demand for service by a medical doctor or other specialist we can formulate decision analyses. Our demand models provide inputs to these analyses, as do certain costs.

Decision Model 1.

Suppose ship missions are of approximately constant duration M. Let D be the (dollar) cost per unit time of maintaining a medical doctor aboard ship. Then MD is the dollar cost per mission of keeping the doctor aboard ship while the ship is engaged in an active mission.

Let R be the cost of the evacuation or rescue operation necessary when an emergency arises and no doctor is present. Think of R as being an average cost; clearly this cost will vary with the location of the ship, and hence the individual, that is the recipient of the rescue attempt.

Let $p(n;M)$ denote the expected number of emergencies that arise when a crew of n individuals embark on a mission of duration M. Our models of Section 2 provide various bases for evaluating $p(n;M)$.

Model 1 implies that the number of demands during a mission is binomially distributed with mean $p(n;M) = n(1-e^{-\lambda M})$. Suppose that each emergency requires a separate rescue or evacuation operation. Then the expected cost of rescues or evacuations in $p(n;M)R$ per mission if a doctor is not aboard. Presume that if a doctor is aboard all of these can be avoided, but at cost MD. The optimal decision rule is then

$$\begin{array}{ll} \text{Carry a doctor if} & MD < n(1-e^{-\lambda M})R \\ \text{Do not carry a doctor if} & MD > n(1-e^{-\lambda M})R \end{array} \qquad (3.1)$$

If λ is quite small, as should often be true, this becomes a good approximation:

$$\begin{array}{ll} \text{Carry a doctor if} & D < n\,\lambda\,R \\ \text{Do not carry a doctor if} & D > n\,\lambda\,R. \end{array} \qquad (3.2)$$

Of course if there is equality ($D = n\lambda R$, for example) then other considerations must settle the matter.

Decision Model 2.

This model simply recognizes the differences between demand (injury, accident, or sickness) rates between individuals, as in Demand Model 2. For that model the expected number of demands is

$$p(n;M) = \sum_{j=1}^{n} (1-e^{-\lambda_j M})$$

Hence our decision rule becomes

$$\begin{array}{ll} \text{Carry a doctor if} & MD < R \sum_{j=1}^{n} (1-e^{-\lambda_j M}). \\ \text{Do not carry a doctor if} & MD > R \sum_{j=1}^{n} (1-e^{-\lambda_j M}). \end{array} \qquad (3.3)$$

Some additional comments may be made on these models.

(A) The models tacitly assume that R, the cost of a rescue operation, is the same regardless of mission. In fact, one can assign a cost that depends upon the mission and then decide on the basis of our various decision models whether a doctor can be justified.

(B) The same comment as in (A) above holds for the rates or λ-values likely to prevail on different missions.

(C) The above decision rules, derived for ships, can apply also to groups of ships. The doctor can be located on one ship of the group, and emergencies will then be transferred to that ship when they occur.

(D) In the above discussion the λ-values are taken to be known. To make the decision we must obtain estimates, and then treat these estimates as equal to the parameter values actually prevailing. A more sophisticated approach explicitly recognizes that estimates are uncertain; one standard way of handling that situation is by means of Bayesian decision theory. We shall apply these ideas in a later report.

The above decision models assume that emergencies generate rescue costs, but do not shorten missions. In other situations, perhaps having to do with the failure of a major weapon system, this might not be the case. It may well be that if a major system goes out on, say, a submarine, the latter must return to port prematurely. We set up a simple and tentative model for this situation, anticipating that refinements in the model may suggest themselves.

Decision Model 3.

Suppose the initial cost for a copy of the ship in question is S (dollars), and that the anticipated life is equivalent to L missions of length M. It is reasonable to assess a penalty of $\frac{S}{LM}$ dollars per unit time that the ship is not carrying out its assigned task during a mission, owing to lack of specialized repair personnel or spare parts.

The expected lost time per mission of length M is obtained for Demand Model 1 by multiplying (2.4) by the probability of at least one demand during M, namely $1 - e^{-n\lambda M}$. Thus

$$E[\max(M-\tau_n,0)] = M - \left[\frac{1-e^{-n\lambda M}}{n\lambda}\right] . \tag{3.4}$$

Thus the expected cost of lost service if a specialist, or requisite spares, are not carried over the life of the ship is

$$\begin{aligned} \text{Expected cost} &= \left(\frac{S}{LM}\right) L \cdot E[\max(M-\tau_n,0)) \\ &= S\left\{1 - \left[\frac{1-e^{-n\lambda M}}{n\lambda M}\right]\right\} . \end{aligned} \tag{3.5}$$

The optimal decision rule is then derived from the principle that one should carry the specialist if his total cost over the life of the ship, MLD, is less than the expected cost of curtailed missions:

$$\begin{aligned} &\text{Carry specialist if} && MLD < S\left\{1 - \left[\frac{1-e^{-n\lambda M}}{n\lambda M}\right]\right\} . \\ &\text{Do not carry specialist if} && MLD > S\left\{1 - \left[\frac{1-e^{-n\lambda M}}{n\lambda M}\right]\right\} . \end{aligned} \tag{3.6}$$

A very similar formula can be written down if Demand Model 2 is invoked. Note that n now refers to the number of failure-prone equipments to be serviced by the specialist.

One qualitative fact that emerges from (3.6) is that for fixed total mission time $ML = T$ one can reduce the need for a specialist by shortening mission time, M, (and correspondingly increasing L). By indefinitely shortening M the right-hand side of (3.6) can be brought very close to zero, which guarantees that our decision rule will recommend that the specialist be left ashore. Of course, indefinite shortening of the mission time is impractical, but the tendency is of interest and can be quantitatively assessed by use of formulas like (3.6).

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