1986-03

On inference and transient response for M/G/1 models

Gaver, Donald Paul; Jacobs, Patricia A.

Monterey, California. Naval Postgraduate School

https://hdl.handle.net/10945/30051

NPS55-86-006

NAVAL POSTGRADUATE SCHOOL

Monterey, California

ON INFERENCE AND TRANSIENT RESPONSE
FOR M/G/1 MODELS

DONALD P. GAVER
PATRICIA A. JACOBS

MARCH 1986

Approved for public release: distribution unlimited.

Prepared for:
Naval Postgraduate School
Monterey, CA 93943-5000

NAVAL POSTGRADUATE SCHOOL
Monterey, California

Rear Admiral R. H. Shumaker
Superintendent

D. A. Schrady
Provost

This report was prepared by:

URITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a REPORT SECURITY CLASSIFICATION: UNCLASSIFIED	1b RESTRICTIVE MARKINGS
2a SECURITY CLASSIFICATION AUTHORITY	3 DISTRIBUTION/AVAILABILITY OF REPORT: Approved for public release; distribution unlimited.
2b DECLASSIFICATION/DOWNGRADING SCHEDULE	
4 PERFORMING ORGANIZATION REPORT NUMBER(S): NPS55-86-006	5 MONITORING ORGANIZATION REPORT NUMBER(S)
6a NAME OF PERFORMING ORGANIZATION: Naval Postgraduate School; 6b OFFICE SYMBOL (If applicable): Code 55	7a NAME OF MONITORING ORGANIZATION
6c ADDRESS (City, State, and ZIP Code): Monterey, CA 93943-5000	7b. ADDRESS (City, State, and ZIP Code)
8a NAME OF FUNDING/SPONSORING ORGANIZATION; 8b OFFICE SYMBOL (If applicable)	9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c ADDRESS (City, State, and ZIP Code)	10 SOURCE OF FUNDING NUMBERS: PROGRAM ELEMENT NO / PROJECT NO / TASK NO. / WORK UNIT ACCESSION NO

11 TITLE (Include Security Classification)

ON INFERENCE AND TRANSIENT RESPONSE FOR M/G/1 MODELS

12 PERSONAL AUTHOR(S)
Gaver, Donald P. and Jacobs, Patricia A

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15 PAGE COUNT
Technical	FROM ______ TO ______	1986, March	16

16 SUPPLEMENTARY NOTATION

17 COSATI CODES			18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	estimation of virtual waiting time distribution, terminating renewal process, bootstrap, jacknife, transient behavior of queueing systems

19 ABSTRACT (Continue on reverse if necessary and identify by block number)

This paper addresses two problems of interest in service system analysis: (a) that of making statistical, data-driven estimates of the long-run probability of a long delay, and (b) the assessment of rate of approach to a long-run system performance measure such as expected delay, the rate being characterized by a simple exponential, at least initially. Both are illustrated by reference to M/G/1 and related systems.

20 DISTRIBUTION/AVAILABILITY OF ABSTRACT: ☐ UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS	21 ABSTRACT SECURITY CLASSIFICATION: UNCLASSIFIED
22a NAME OF RESPONSIBLE INDIVIDUAL: Donald P. Gaver	22b TELEPHONE (Include Area Code): (408)646-2605; 22c. OFFICE SYMBOL: Code 55Gv

DD FORM 1473, 84 MAR

83 APR edition may be used until exhausted
All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE

ON INFERENCE AND TRANSIENT RESPONSE FOR M/G/1 MODELS

Donald P. Gaver
Patricia A. Jacobs

Naval Postgraduate School
Monterey, California 93943
U.S.A.

This paper addresses two problems of interest in service system analysis: (a) that of making statistical, data-driven estimates of the long-run probability of a long delay, and (b) the assessment of rate of approach to a long-run system performance measure such as expected delay, the rate being characterized by a simple exponential, at least initially. Both are illustrated by reference to M/G/1 and related systems.

1. Introduction

The application of probability theory to a wide variety of congestion problems arising in communication systems has been well catalogued in many papers; the treatises by Borovkov (1984), Cohen (1969), Cooper (1972), Cox and Smith (1961), Feller (1966), Kleinrock (1975), Gross and Harris (1974), and many others attest to the attractiveness of the area for probability modelling. In this body of work, several features are noticeable. First, most of the elegant solutions obtained are in somewhat implicit form, being presented as functional equations, or, frequently, as integral (Laplace) transforms, generating functions, and sometimes as combinations of the above. Moments (particularly the first) of delays or number in queue under long-run conditions when steady state prevails are the most explicit and assessible performance summaries. Second, the results obtained are presented in terms of component distribution functions and stochastic processes (renewal, Poisson, etc.) that are taken as known; only rarely are issues addressed that arise when actual data is to be used as a basis for inference from the models; however, see Cox (1965). Third, only fragmentary comprehensible information concerning transient behavior, not to mention time-dependence of parameters is available; recently however work by Roth and Odoni (1983), Roth (1981), Abate and Whitt (1985), Lee (1985) has elucidated the simple exponential-like approach to the steady state displayed by a great many stochastic systems with stationary, time-homogeneous arrivals and service processes.

Instead of directly dealing with the above classes of problems much attention has been concentrated on modelling large networks of servers, particularly steady-state Markovian, cf. Kelly (1979) for an elegant treatment; see package programs created by AT&T (the D. Mitra PANACEA) and by IBM (work by Lavenberg, et al). Other numerical work by Neuts (1981) and associates has pioneered the exploitation of special structure of certain stochastic systems. And a considerable effort to create approximations, ranging from heavy-traffic and diffusion to, lately, WKB approaches, see Knessl, Matkowsky et al (1986) has been evident; see in particular Newell (1982) for pioneering work.

This paper deals specifically with approaches to the two classes of somewhat neglected problems referred to earlier: those of data-driven inference, and of assessing transient behavior. The approaches proposed are illustrated concretely in terms of the simple M/G/1 system, but apply more widely.

2. Inference Concerning Long Steady-State Delays in M/G/1-Like Systems

Consider a single service system approached by stationary Poisson (λ) traffic with λ known. Service times, or message lengths, X, are independently distributed as $F_X(x)$; assume $\lambda E[X] \equiv \rho < 1$. This is the M/G/1 system. Let observations of the service times be all that is known about F: denote these by $x_1, x_2, \ldots, x_n$. The objective is to supply estimates of long-run characteristics of the system: in particular, estimates, and error assessments thereof, of the probability of a long delay experienced by an arriving message.

2.1 Virtual Waiting Time Transform; The Long Tail.

It is well known that if W(t) is the virtual waiting time in the M/G/1 and $\rho < 1$, then the Laplace transform

$$E[e^{-sW}] = \lim_{t\to\infty} E[e^{-sW(t)}] = \frac{1-\rho}{1-\rho\left[\frac{1-E[e^{-sX}]}{sE[X]}\right]} = \frac{1-\rho}{1-\rho\tilde{A}(s)} \tag{2.1}$$

where A(s) is the Laplace-Stieltjes transform of a distribution. It can also be seen that in various interrupted and priority service systems, including those in which the server "takes vacations," that the above can be changed to

$$E[e^{-sW}] = \frac{1-\rho}{1-\rho\tilde{B}(s)} \cdot \tilde{C}(s) \tag{2.2}$$

where possibly $\tilde{B}(s)$ is the transform of a completion time (effective service time, i.e. X including durations of a random number of interruptions occurring therein), and $\tilde{C}(s)$ is the transform of an honest distribution that accounts for effects of busy period starts; see Gaver (1968).

If $\tilde{A}(s)$ (or $\tilde{B}(s)$) exists for $s > s_0$, $s_0 < 0$ then there will be a smallest real zero, $-s = \kappa > 0$, of the denominator of (2.1),((2.2)), which can be used to show that

$$P\{W > w\} \sim D(\kappa)e^{-\kappa w}, \quad w \to \infty \tag{2.3}$$

One way of establishing the above exponential tail property is to introduce

$$\psi(s) = \frac{1-E[e^{-sW}]}{s} = \int_0^\infty P\{W > w\}e^{-sw}dw$$

into (2.1) which leads to

$$\tilde{\psi}(s) = \rho\left[\frac{1-\tilde{A}(s)}{s}\right] + \rho\tilde{A}(s)\psi(s) \tag{2.4}$$

equivalent to the terminating renewal equation, see Feller (1966), p. 362,

$$\overline{F}_W(w) \equiv P\{W > w\} = \overline{H}(w) + \rho\int_0^w \overline{F}(w-x)A(dx) \tag{2.5}$$

Introduce $\overline{F}_W^{\#}(w) = \overline{F}_W(w)e^{\theta w}$, θ real and positive, into (2.5)

$$\overline{F}_W^{\#}(w) = \overline{H}^{\#}(w) + \int_0^w \overline{F}^{\#}(w-x)\rho e^{\theta x}A(dx), \tag{2.6}$$

and choose $\theta = \kappa$ so that

$$\int_0^\infty \rho e^{\theta x}A(dx) = \int_0^\infty A^{\#}(dx) = 1, \tag{2.7}$$

yielding a standard renewal equation for $\overline{F}^{\#}$, to which the key renewal theorem applies, yielding as $w \to \infty$

$$\lim_{w\to\infty} \overline{F}_W^{\#}(w) = \lim_{w\to\infty} \overline{F}_W(w)e^{\kappa w} = D(\kappa) = \frac{\int_0^\infty \overline{H}^{\#}(w)dw}{\int_0^\infty xA^{\#}(dx)} , \tag{2.8}$$

equivalent to (2.3). A similar expression is available for (2.2).

2.3 Statistical Estimation of Probability of Long Wait.

If data are available on service times (message lengths) then an estimate of the requisite transform is

$$\hat{E}[e^{-sX}] = \frac{1}{n}\sum_{i=1}^{n} e^{-sx_i} \tag{2.9}$$

and $\hat{\kappa}$ is the unique positive solution of this sample equivalent of (2.7):

$$\lambda\{\frac{1}{\theta}(\frac{1}{n}\sum_{i=1}^{n} e^{\theta x_i} - 1) = 1 . \tag{2.10}$$

A three-term Taylors' series expansion around $\theta = 0$ gives an initial estimate

$$\hat{\kappa} \simeq 2(\frac{1-\lambda\overline{x}}{\overline{x^2}}) \tag{2.11}$$

where $\overline{x^k} = (x_1^k + \ldots + x_n^k)/n$, the k<u>th</u> sample moment; (2.11) is recognized as being the non-parametric sample version of the exponential distribution parameter obtained by normalizing to $W/E[W]$ and letting $\rho \uparrow 1$ (the heavy traffic approximation). A search or Newton-Raphson iteration starting from (2.11) quickly yields $\hat{\kappa}$ as accurately as is necessary.

Some theoretical asymptotic results concerning modes of behavior of $\hat{\kappa}$ will now be sketched; more detail will be presented elsewhere. First, it is essential that $E[e^{\kappa X}] < \infty$ in order for (2.7) to hold, and so by continuity $\hat{f}^{(m)}(\kappa) = E[X^m e^{\kappa X}] < \infty$ for any integer m. Now express (2.10) as

$$\hat{f}(\infty) = \lambda\{\frac{1}{\theta}(\frac{1}{n}\sum_{i=1}^{n} e^{\theta x_i} - 1)\} - 1 \tag{2.12}$$

and expand in Taylor's series around κ: since $\hat{f}(\hat{\kappa}) = 0$, for some ϕ

$$0 = \hat{f}(\kappa) + (\hat{\kappa} - \kappa)\hat{f}'(\kappa) + \frac{1}{2}(\hat{\kappa} - \kappa)^2\hat{f}''(\phi\kappa) , \tag{2.13}$$

so

$$\hat{\kappa} - \kappa = \frac{\hat{f}(\kappa)}{\hat{f}'(\kappa) + \frac{1}{2}(\hat{\kappa}-\kappa)\hat{f}''(\phi\kappa)} \tag{2.14}$$

Note that if $E[e^{2\kappa X}] < \infty$ then $Var[e^{\kappa X}] < \infty$ and

$$Var[\hat{f}(\kappa)] = \frac{\lambda^2}{\kappa^2}\cdot\frac{1}{n} Var[e^{\kappa X}] , \tag{2.15}$$

while

$$E[\hat{f}(\kappa)] = 0 .$$

Additionally, $\hat{f}'(\kappa) \to E[\hat{f}'(\kappa)]$ by the weak law of large numbers and $\hat{\kappa}-\kappa \to 0$ in probability, so by the central limit theorem,

$$\sqrt{n}(\hat{\kappa}-\kappa) \cdot \frac{E[\hat{f}'(\kappa)]}{\sqrt{\frac{\lambda^2}{\kappa^2}\mathrm{Var}[e^{\kappa X}]}} \sim N(0,1) \tag{2.16}$$

the standard Normal/Gaussian distribution. If, however, $E[e^{2\kappa X}] = \infty$, then if any kind of limiting distribution is to exist, the components

$$e^{\kappa X_i}$$

of the sum $\hat{f}(\kappa)$ must be in the domain of attraction of a stable law, which will be assumed. In any case $\hat{f}'(\kappa) \to E[\hat{f}'(\kappa)]$ as before, so a proper normalization by a power of n shows that $(\hat{\kappa}-\kappa)$ has a stable law as limiting distribution. Fortunately, the relatively well-behaved Normal/Gauss limit prevails when traffic intensity is relatively high ($\rho > 1/2$ in case $X \sim \mathrm{Exp}(\mu)$, for example).

2.4 Assessing Variability of the Estimates.

Having obtained an estimate of the parameter κ, and of the probability of a waiting time exceeding large t, which involves κ, it is desirable to appraise the errors involved. Two error sources are: the systematic error (bias) resulting from fitting an incorrect model, and the effect of finite sample size (random error). The latter is the easiest to evaluate, provided the underlying model is nearly correct. The bias issue is more difficult to deal with; one approach is to begin by fitting an elaborate, multiparameter model and then to check for the contribution of extra parameters in a prediction context. Since this paper takes a non-parametric approach to estimation, the bias resulting from an incorrect specification of the service time distribution is presumably absent, although the assumptions of stationary Poisson arrivals, and independence are still reflected in the basic transform (2.1), and hence in the estimates. Unfortunately, classical procedures that, for example, avail themselves of the asymptotic approximations of properties of maximum likelihood parameter estimates to estimate sampling errors are no longer available in the present environment. We have chosen instead to investigate the performance of several modern procedures often recommended for obtaining standard errors of estimate and confidence intervals: the jackknife (Quenouille, Tukey, Miller, Hinkley, and others) and the bootstrap (Efron and associates and others). These methods, particularly the bootstrap, which involves simulation, are computer intensive, but are the only alternatives currently known for dealing with complex situations such as that at hand.

The Jackknife

The above name refers to a procedure originally introduced by Quenouille for bias reduction (1956), and adapted by Tukey (1958) to obtain approximate confidence intervals. Suppose interest is on a parameter θ (e.g. our κ, or some function thereof) that is estimated by θ, using a complex calculation from data $(x_1, x_2, \ldots, x_n)$, just as $\hat{\kappa}$ is. The idea is that of assessing variability by recomputing κ after removing independent subgroups of data of equal size, and then using the recomputed $\hat{\kappa}$ values to estimate a variance, which is in turn applied to state a standard error or a two-sided confidence interval that contains the true θ with specified confidence. A few details follow; for more, see Efron (1982) and his more recent work, or Mosteller and Tukey (1977). The actual calculation involves splitting n into g disjoint groups of size m; n = mg. Then calculate $\theta_{(-j)}$, j = 1,2,...,g: the estimate of θ that omits the jth group. Now Tukey (but not Efron) computes pseudo-values

$$y_j = g\hat{\theta} - (g-1)\hat{\theta}_{(-j)}$$

which are then treated as independent: use $\bar{y} = \hat{\theta}_{JK}$ or the point estimate of θ, and its approximate variance as

$$s_y^2/g = \sum_{j=1}^{g} (y_j - \bar{y})^2/(g-1)(g) = \sum_{j=1}^{g} (\hat{\theta}_{(-j)} - \bar{\hat{\theta}}_{(-j)})^2 (g-1)/g,$$

as it turns out. Tukey recommends referring $\bar{y}$ to Student's t with g-1 degrees of freedom to obtain confidence limits. As the name "jackknife" is intended to imply, the tool is inexact and a bit crude for small samples--just as a true jackknife is not well-adapted for delicate surgery. It is a handy non-parametric option.

The Bootstrap

In simplest form the bootstrap suggests creating from observations (x_i, i = 1,...,n) the empirical distribution

$$\hat{F}_n(x) = \frac{1}{n} \sum_{i=1}^{n} I(x_i, x); \qquad I(x_i, x) = \begin{cases} 1, & x_i \leq x \\ 0, & x_i > x \end{cases} .$$

Then one re-samples with replacement from F_n to obtain a bootstrap sample of size n, from which $\hat{\theta}$ (e.g. $\kappa(1)$) is calculated. This process is repeated B (e.g. 300-500) times, and the empirical distribution, $F_{\hat{\kappa}}$, of the resulting estimates, $\{\hat{\kappa}(b), b = 1,2,..,B\}$ is employed as an approximation to the sampling distribution of $\hat{\kappa}$: the upper and lower 10% points of $F_{\hat{\kappa}}$ approximate two-sided 80% confidence limits for $\hat{\kappa}$, for instance.

3. Transient Response: Exponential Characterizations

Applications of queueing theory frequently require assessment of service system transient behavior, particularly when the system is initially idle and heavy traffic conditions prevail, for then approach to steady-state conditions may be slow. Odoni and Roth (1983) have reviewed work on the nature of the approach to steady-state conditions, and have investigated characterizations of that approach that are simply stated and comprehensible, being exponential. In this section we elaborate upon the theme of exponential approach. Work by Abate and Whitt (1985) has similar aims, but proceeds somewhat differently.

3.1 From Transform to Exponential

Suppose $\{X(t), t \geq 0\}$ is the state variable of a process, and that expressions for

(a) $\tilde{\psi}_X(s) = \int_0^\infty e^{-st} E[\psi(X(t)) | X(0)] dt = \int_0^\infty e^{-st} \psi_X(t) dt$ (3.1)

(b) $\lim_{t\to\infty} E[\psi(X(t) | X(0)] < \infty$

are known. An approximate representation of

(c) $\psi_X(t) = E[\psi(X(t)) | X(0)]$

is desired. The proposed representation is a simple exponential interpolation of the form

$$\tilde{\psi}_X(t) = \psi_X(0) e^{-\beta t} + \psi_X(\infty)(1 - e^{-\beta t}) , \qquad (3.2)$$

where β governs the rate of approach to the steady-state value. The proposal is to identify β by minimizing the integrated squared error

$$\Delta(\beta) = \int_0^\infty \{\psi_X(t) - \psi_X(0)e^{-\beta t} + \psi_X(\infty)(1 - e^{-\beta t})\}^2 w(t)dt . \tag{3.3}$$

The weight function w(t) may be made to focus upon values of β appropriate for t-regions of interest. To optimize on β, study

$$\frac{d\Delta(\beta)}{d\beta} = K \int_0^\infty \{\psi_X(t) - \psi_X(0)e^{-\beta t} - \psi_X(\infty)(1 - e^{-\beta t})\} t e^{-\beta t} w(t)dt \tag{3.4}$$

$$= 0 .$$

This is equivalent to

$$\int_0^\infty \psi_X(t)w(t)te^{-\beta t}dt = \psi_X(0)\int_0^\infty e^{-2\beta t}tw(t)dt + \psi_X(\infty)\int_0^\infty (1-e^{-\beta t})e^{-\beta t}tw(t)dt \tag{3.5}$$

If $w(t) \equiv 1$ the above can be expressed in terms of the derivative of the Laplace transform $\tilde{\psi}_X(s)$:

$$-\frac{d}{d\beta}\tilde{\psi}_X(\beta) = \frac{1}{\beta^2}[\frac{1}{4}\psi_X(0) + \frac{3}{4}\psi_X(\infty)] , \tag{3.6}$$

often a transcendental equation that must be solved numerically for β. Clearly, if no solution or multiple solutions exist then the simple universal form (3.2) is called into question, but in such cases weight functions are usefully invoked. If interest centers on ultimate approach $(t \to \infty)$ to $\psi_X(\infty)$ then the smallest β satisfying (3.6) is of interest.

3.2 The M/G/1 Example.

As an important illustration of the above ideas, let $X(t) = W(t)$ be the virtual waiting time in an M/G/1 system, and consider $\psi_W(t) = E[W(t)|W(0)=0]$. We know that

$$\tilde{\psi}_W(s) = \frac{\rho-1}{s^2} + p_{oo}(s)\cdot\frac{1}{s} \tag{3.7}$$

where

$$p_{oo}(s) = \frac{1}{s+\lambda[1-b(s)]} = \frac{1}{s}\frac{1}{1+\lambda[\frac{1-b(s)}{s}]} = \frac{1}{s}h \tag{3.8}$$

b(s) being the transform of a busy period duration, satisfying

$$b(s) = \hat{F}_X(s + \lambda(1-b(s))) ;$$

$\hat{F}_X$ is the Laplace-Stieltjes transform of the service time or message length distribution F_X. Numerical solution of (3.6) is now possible. An approximation to the (smallest) value of β governing the ultimate approach $(t \to \infty)$ when $\rho \uparrow 1$, i.e. in the heavy-traffic situation, is

$$\beta \simeq \sqrt{\frac{3}{4}\frac{\rho E[X^2]}{1-\rho}\{\frac{1}{-h'''(0)}\}} . \tag{3.9}$$

In general, h'''(0) will involve higher moments of the service time; in case $X \sim Exp(\mu)$ considerable simplification occurs and

$$\beta \simeq \mu(1-\rho)^2 \{8(1+\rho)\}^{-0.5} \tag{3.10}$$

which will not, of course agree exactly with formulas or numerical results given previously by other authors, e.g. see Odoni and Roth (1983) being a compromise ("Procrustean") interpolation of a single exponential over the infinite t-range. In order to focus upon a β-value relevant to ultimate approach it has been found that a weight procedure of the form $w_{k+1}(t) = [1-\exp(-\beta(k)t)]$ is workable and tractable; start with $\beta(0) = 0$ (the unweighted procedure described), then introduce w_{k+1} to determine β_{k+1} and iterate to convergence, which occurs rapidly. A similar approach should apply to find a β appropriate for small t.

The procedures described above should be applicable in the data-driven context of the problem of Section 1: a first step is to introduce the empirical transform $\hat{F}_X$ and from it proceed to $\hat{b}(s)$, to $\hat{\hat{\psi}}_W(s)$, and then to $\hat{\beta}$. Progress in this area, and details of the above investigations will be reported elsewhere.

4. Acknowledgement

The authors gratefully acknowledge the research support by the Probability and Statistics Program of the Mathematical Science Division of the Office of Naval Research.

REFERENCES:

[1] Abate, J. and Whitt, W. personal communication

[2] Borovkov, A.A., Asymptotic Methods in Queueing Theory (John Wiley, 1984).

[3] Cohen, J. The Single Server Queue (North-Holland, 1969).

[4] Cooper, R.B. Introduction to Queueing Theory (MacMillan, 1972).

[5] Cox, D.R., and Smith, W.L. Queues (John Wiley, 1961).

[6] Cox, D.R. Some problems of statistical analysis connected with congestion, Proc. of the Symposium on Congestion Theory, ed. W.L. Smith and W.E. Wilkinson (Univ. of No. Carolina Press, 1965), pp. 289-310.

[7] Cox, D.R., and Hinkley, D. Theoretical Statistics (Chapman and Hall; John Wiley, 1974).

[8] Efron, B. The Jackknife, The Bootstrap, and Other Resampling Plans (S.I.A.M.-NSF, 1982).

[9] Feller, W. An Introduction to Probability Theory and Its Applications, Vols. I and II (John Wiley, 1966).

[10] Gaver, D.P. Diffusion approximations and models for certain congestion problems, J. Applied Prob., 5 (1968), pp. 607-623.

[11] Gross, D. and Harris, C.M. Fundamentals of Queueing Theory (John Wiley, 1974).

[12] Kelly, F.P. Reversibility and Stochastic Networks (John Wiley, 1979).

[13] Kleinrock, L. Queueing Systems, Vols. 1 and 2 (John Wiley, 1975).

[14] Knessl, C., Matkowsky, B.J., Schuss, Z., and Tier, C. Asymptotic analysis of a state-dependent M/G/1 queueing system, S.I.A.M. J. of Applied Math. (to appear).

[15] Lee, I-Jen, Stationary Markovian systems: an approximation for the transient expected length. M.I.T. MS Thesis (1985).

[16] Miller, R.G. Jr. The jackknife--a review. Biometrika, 61 (1974), pp. 1-15.

[17] Neuts, M.F. Matrix-Geometric Solutions in Stochastic Models: An Algorithmic Approach (John Hopkins Press, 1981).

[18] Newell, G.F. Applications of Queueing Theory, 2nd Edition (Chapman and Hall, 1982).

[19] Odoni, A.R., and Roth, E. An empirical investigation of the transient behavior of stationary queueing systems. Operations Research, 31 (1983), pp. 432-455.

[20] Quenouille, M.H. Notes on bias in estimation, Biometrika, 4 (1956). pp. 353-360.

[21] Roth, E. An investigation of the transient behavior of stationary queueing systems. M.I.T. Ph.D. dissertation (1981).

[22] Mosteller, F., and Tukey, J.W. Data Analysis and Regression (Addison-Wesley, 1977).

[23] Tukey, J.W. Bias and confidence in not-quite large samples (abstract), Annuals of Math. Stat. 29 (1958), p. 614.

Distribution List

	NO. OF COPIES
Dr. D. F. Daley Statistics Dept. (I.A.S) Australian National University Canberra A.C.T. 2606 AUSTRALIA	1
Prof. F. J. Anscombe Department of Statistics Yale University, Box 2179 New Haven, CT 06520	1
Dr. David Brillinger Statistics Department University of California Berkeley, CA 94720	1
Dr. R. Gnanadesikan Bell Core 435 South Street Morris Township, NJ 07960	1
Prof. Bernard Harris Dept. of Statistics University of Wisconsin 610 Walnut Street Madison, WI 53706	1
Dr. D. R. Cox Department of Mathematics Imperial College London SW7 ENGLAND	1
Dr. A. J. Lawrance Dept. of Mathematical Statistics University of Birmingham P. O. Box 363 Birmingham B15 2TT ENGLAND	1
Professor W. M. Hinich University of Texas Austin, TX 78712	1

Dr. John Copas 1
Dept. of Mathematical Statistics
University of Birmingham
P. O. Box 363
Birmingham B15 2TT
ENGLAND

P. Heidelberger 1
IBM Research Laboratory
Yorktown Heights
New York, NY 10598

Prof. M. Leadbetter 1
Department of Statistics
University of North Carolina
Chapel Hill, NC 27514

Dr. D. L. Iglehart 1
Dept. of Operations Research
Stanford University
Stanford, CA 94350

Dr. D. Vere-Jones 1
Dept. of Mathematics
Victoria University of Wellington
P. O. Box 196
Wellington
NEW ZEALAND

Prof. J. B. Kadane 1
Dept. of Statistics
Carnegie-Mellon
Pittsburgh, PA 15213

Prof. J. Lehoczky 1
Department of Statistics
Carnegie-Mellon University
Pittsburgh, PA 15213

Dr. J. Maar (R51) 1
National Security Agency
Fort Meade, MD 20755

Dr. M. Mazumdar 1
Dept. of Industrial Engineering
University of Pittsburgh
Pittsburgh, PA 15235

Prof. M. Rosenblatt 1
Department of Mathematics
University of California-San Diego
La Jolla, CA 92093

Prof. Rupert G. Miller, Jr. 1
Statistics Department
Sequoia Hall
Stanford University
Stanford, CA 94305

Prof. I. R. Savage 1
Dept. of Statistics
Yale University
New Haven, CT 06520

Dr. Paul Shaman 1
National Science Foundation
Mathematical Sciences Section
1800 G. Street, NW
Washington, D. C. 20550

Prof. W. R. Schucany 1
Dept. of Statistics
Southern Methodist University
Dallas, TX 75222

Prof. D. C. Siegmund 1
Dept. of Statistics
Sequoia Hall
Stanford University
Stanford, CA 94305

Prof. H. Solomon 1
Department of Statistics
Sequoia Hall
Stanford University
Stanford, CA 94305

Dr. Ed Wegman 1
Statistics & Probability Program
Code 411(SP)
Office of Naval Research
Arlington, VA 22217

Dr. P. Welch 1
IBM Research Laboratory
Yorktown Heights, NY 10598

Dr. Marvin Moss 1
Office of Naval Research
Arlington, VA 22217

Dr. Roy Welsch 1
Sloan School
M. I. T.
Cambridge, MA 02139

Pat Welsh 1
Head, Polar Oceanography Branch
Code 332
Naval Ocean Research & Dev. Activity
NSTL Station, MS 39529

Dr. Douglas de Priest 1
Statistics & Probability Program
Code 411(SP)
Office of Naval Research
Arlington, VA 22217

Dr. Morris DeGroot 1
Statistics Department
Carnegie-Mellon University
Pittsburgh, PA 15235

Prof. J. R. Thompson 1
Dept. of Mathematical Science
Rice University
Houston, TX 77001

Prof. J. W. Tukey 1
Statistics Department
Princeton University
Princeton, NJ 08540

Dr. Daniel H. Wagner 1
Station Square One
Paoli, PA 19301

Dr. Colin Mallows 1
Bell Telephone Laboratories
Murray Hill, NJ 07974

Dr. D. Pregibon 1
Bell Telephone Laboratories - AT&T
Murray Hill, NJ 07974

Dr. Jon Kettenring 1
Bell Core
435 South Street
Morris Township, NJ 07960

Dr. David L. Wallace 1
Statistics Dept.
University of Chicago
5734 S. University Ave.
Chicago, IL 60637

Dr. F. Mosteller 1
Dept. of Statistics
Harvard University
Cambridge, MA 02138

Dr. S. R. Dalal 1
Bell Laboratories - AT&T
Mountain Avenue
Murray Hill, NJ 07974

Prof. Donald P. Gaver 20
Code 55Gv
Naval Postgraduate School
Monterey, California 93943-5000

Prof. Patricia Jacobs 1
Code 55Jc
Naval Postgraduate School
Monterey, California 93943-5000

Dr. Guy Fayolle 1
I.N.R.I.A.
Dom de Voluceau-Rocquencourt
78150 Le Chesnay Cedex
FRANCE

Dr. M. J. Fischer 1
Defense Communications Agency
1860 Wiehle Avenue
Reston, VA 22070

Prof. George S. Fishman 1
Curr. in OR & Systems Analysis
University of North Carolina
Chapel Hill, NC 20742

Prof. Guy Latouche 1
University Libre Bruxelles
C. P. 212
Blvd De Triomphe
B-1050 Bruxelles
BELGIUM

Library 4
Code 1424
Naval Postgraduate School
Monterey, CA 93943-5000

Dr. Alan F. Petty 1
Code 7930
Naval Research Laboratory
Washington, DC 20375

Prof. Bradley Efron 1
Statistics Department
Sequoia Hall
Stanford University
Stanford, CA 94305

Prof. Carl N. Morris 1
Dept. of Mathematics
University of Texas
Austin, TX 78712

Dr. John E. Rolph 1
RAND Corporation
1700 Main Street
Santa Monica, CA 90406

Prof. Linda V. Green 1
Graduate School of Business
Columbia University
New York, NY 10027

Dr. David Burman 1
Bell Laboratories - AT&T
Mountain Avenue
Murray Hill, NJ 07974

Dr. Ed Coffman 1
Bell Laboratories - AT&T
Mountain Avenue
Murray Hill, NJ 07974

Prof. William Jewell 1
Operations Research Department
University of California, Berkeley
Berkeley, CA 94720

Dr. Tom A. Louis 1
Biostatics Department
Harvard School of Public Health
677 Huntington Avenue
Boston, MA 02115

Dr. Nan Laird 1
Biostatics Department
Harvard School of Public Health
677 Huntington Avenue
Boston, MA 02115

Dr. Marvin Zelen 1
Biostatics Department
Harvard School of Public Health
677 Huntington Avenue
Boston, MA 02115

Dr. John Orav 1
Biostatics Department
Harvard School of Public Health
677 Huntington Avenue
Boston, MA 02115

Prof. R. Douglas Martin 1
Chairman
Department of Statistics, GN-22
University of Washington
Seattle, WA 98195

Prof. W. Stuetzle 1
Department of Statistics
University of Washington
Seattle, WA 98195

Prof. F. W. Mosteller 1
Department of Statistics
Harvard University
1 Oxford Street
Cambridge, MA 02138

Dr. D. C. Hoaglin 1
Department of Statistics
Harvard University
1 Oxford Street
Cambridge, MA 02138

Prof. N. D. Singpurwalla 1
George Washington University
Washington, D. C. 20052

Center for Naval Analyses 1
2000 Beauregard Street
Alexandria, VA 22311

Prof. H. Chernoff 1
Department of Mathematics
M. I. T.
Cambridge, MA 02139

Dr. T. J. Ott 1
Bell Core
435 South Street
Morris Township, NJ 07960

Alan Weiss 1
AT&T Bell Laboratories
Murray Hill, NJ

Operations Research Center, Room E40-164 1
Massachusetts Institute of Technology
Attn: R. C. Larson and J. F. Shapiro
Cambridge, MA 02139

Office of Research Administration 1
Code 012
Naval Postgraduate School
Monterey, CA 93943