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GOODNESS-OF-FIT TEST

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TABLES FOR A NEW MULTIVARIATE GOODNESS-OF-FIT TEST

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ABSTRACT

We present tables of critical values for a new multivariate goodness-of-fit test introduced by Foutz. Some details of our improved asymptotic approximation and evaluation of its accuracy are given.

1. Introduction

Foutz [1] proposed a new goodness-of-fit test for fitting univariate as well as multivariate distributions. He showed that the null distribution of the test statistic, F_n, does not depend on (1) the hypothesized distribution, or (2) the number of components in the random vector under study. An integral representation for the null CDF of F_n was provided. Closed form expressions for this null distribution are quite difficult to obtain, even for small sample sizes. The alternative has been to approximate the distribution by a normal distribution with mean e^{-1} and variance $(2e^{-1} - 5e^{-2})/n$; this, however, does not appear to provide a good approximation to the percentiles of the null distribution of F_n for moderate sample sizes.

The authors [2] compared the F_n-test with the Chi-squared test and the Kolmogorov-Smirnov test and found that the F_n-test does have higher power when fitting certain types of distributions. Another investigation by the authors and Linhart [3] examined the power of the F_n-test when fitting a multivariate normal distribution; the test did well in detecting mean shifts and variance shifts. We therefore believe that the F_n-test is a definite alternative to the Chi-squared and Kolmogorov-Smirnov tests when fitting univariate distributions and it is just about the only available test for fitting multivariate distributions. However, the test is not very convenient for applications due to the difficulty in obtaining accurate critical values. This paper fills the gap by providing tables of approximate pecentiles of the null distribution of F_n.

2. Description of the F_n-Test

The procedure for calculating the test statistic F_n is the following. Given a random sample $\underline{X}_1, \underline{X}_2, \ldots, \underline{X}_{n-1}$, from a continuous multivariate distribution, the sample space is partitioned into n statistically equivalent blocks. Let $h_1(\underline{X}), h_2(\underline{X}), \ldots, h_{n-1}(\underline{X})$ be any n-1 "cutting functions" such that $h_k(\underline{X})$ has a continuous distribution, k = 1, 2, ... , n-1, and let $k_1, k_2, \ldots, k_{n-1}$ be a permutation of 1, 2, ... , n-1. Let $\underline{X}(k_1)$ be the sample vector corresponding to the k_1^{th} order statistic of $h_{k_1}(\underline{X}_i)$, i = 1, 2, ... , n-1. The initial partition of the sample space into two blocks is defined by

$$B_1 = \{\underline{X} | h_{k_1}(\underline{X}) < h_{k_1}(\underline{X}(k_1))\}, \text{ and}$$

$$B_2 = B_1^c.$$

The cutting function $h_{k_2}(\underline{X})$ is then used to partition B_1 (if $k_2 < k_1$) or B_2 (if $k_2 > k_1$) into two subblocks in a similar fashion. When all the cutting functions are exhausted the sample space will have been partitioned into n statistically equivalent blocks, $\beta_1, \beta_2, \ldots, \beta_n$. A convenient choice for the cutting functions in the univariate case is the identity function. In the multivariate case letting $h_k(\underline{X}) = \underline{X}^{(j)}$, the j^{th} component of $\underline{X}$ (for various j), appears to work well. More details on partitioning the sample space into statistically equivalent blocks and some examples can be found in [3].

Once the statistically equivalent blocks are determined, a computational formula for the test statistic F_n for the hypothesis that the samples are from a specified distribution H is

$$F_n = \sum_{i=1}^{n} \max\left[0, \frac{1}{n} - D_i\right],$$

where $D_i = P[\underline{X} \varepsilon \beta_i | H]$

The integral representation for the null CDF of F_n results in the following closed form expressions for n = 3, 4, and 5.

$$P[F_3 \le x] = \begin{cases} 6x^2 & 0 < x \le \frac{1}{3} \\ 1 - 3\left(\frac{2}{3} - x\right)^2 & \frac{1}{3} < x \le \frac{2}{3} \\ 1 & x > \frac{2}{3} \end{cases}$$

$$P[F_4 \le x] = \begin{cases} 20x^3 & 0 \le x \le \frac{1}{4} \\ -20x^3 + 18x^2 - \frac{9}{4}x + \frac{1}{16} & \frac{1}{4} < x \le \frac{1}{2} \\ 1 - 4\left(\frac{3}{4} - x\right)^3 & \frac{1}{2} < x \le \frac{3}{4} \\ 1 & x > \frac{3}{4} \end{cases}$$

$$P[F_5 \le x] = \begin{cases} 70x^4 & 0 \le x \le \frac{1}{5} \\ -105x^4 + 80x^3 - 12x^2 + \frac{16}{25}x - \frac{1}{125} & \frac{1}{5} < x \le \frac{2}{5} \\ 45x^4 - 80x^3 + \frac{228}{5}x^2 - \frac{176}{25}x + \frac{31}{125} & \frac{2}{5} < x \le \frac{3}{5} \\ 1 - 5\left(\frac{4}{5} - x\right)^4 & \frac{3}{5} < x \le \frac{4}{5} \\ 1 & x > \frac{4}{5} \end{cases}$$

It does not appear to be possible to generate a closed form expression for the CDF of F_n in the general case. Foutz's large sample normal approximation is given by

$$P[F_n \le x] \doteq \phi \left[\frac{n(x - e^{-1})}{((2e^{-1} - 5e^{-2})n)^{1/2}} \right] \tag{1}$$

where ϕ is the standard normal CDF. To check the accuracy of this approximation in our earlier study [2], we generated samples of size n-1 = 20, 30, and 50 from a uniform distribution on [0,1] and tested the hypothesis that the the samples are in fact from that distribution. The empirical significance levels in 80,000 replications are given in Table 1.

Nominal Significance Level	n-1	20	30	50
0.10		0.0757	0.0800	0.0859
0.05		0.0372	0.0399	0.0428
0.01		0.0082	0.0083	0.0093

Table 1
Empirical Significance Level
(Based on 80,000 replications)

It can be seen that the observed significance levels are consistently smaller than the nominal values by about 10-20%. We therefore proposed the use of Monte Carlo critical values, which were based on 25,000 replications. These values are given in Table 2 and the corresponding observed significance levels, based on 225,000 subsequent repetitions, are given in Table 3.

Significance Level	n-1		
	20	30	50
0.10	0.42714	0.41903	0.40816
0.05	0.44865	0.43553	0.42116
0.01	0.48659	0.46579	0.44487

Table 2
Monte Carlo Critical Values
(Based on 25,000 replications)

Nominal Significance Level	n-1		
	20	30	50
0.10	0.1006	0.9700	0.1003
0.05	0.0486	0.0486	0.0498
0.01	0.0103	0.0101	0.0102

Table 3
Empirical Significance Level
(Based on 225,000 replications)

The above findings lead us into a search for an improved approximation for determining the percentiles of the null distribution of F_n. We found that allowing the mean and variance to be functions of the sample size leads to greatly improved approximations. While it is difficult to give precise error bounds on the percentile values, our computational experience indicates about a four decimal place accuracy. This leads to rejection rates with errors in the fourth decimal place, usually. Comparing the error in the rejection rates for the asymptotic approximation (1) given by Foutz, our approximation is better by a factor of 10 or more.

3. Modified Normal Approximation

The data for the approximation of the null distribution of the Foutz statistic was obtained by Monte Carlo methods. For a given sample size n-1, sequences of n-1 uniformly distributed numbers where generated using the IMSL* random number generator GGUBS. The Foutz statistic was then computed and tabulated into one of 200 equilength intervals. This process was replicated 25,000 times. The entire set consists of the empirical cumulative distribution functions obtained from this data for 60 sample sizes, n-1 = 2(1)40, 40(2)70, and 70(5)100. Potentially this yields as many as 12000 pieces of data, however if only intervals with nontrivial data in them are counted, this is reduced to about 4700.

A data fitting problem with 4700 points is not easily handled unless a linear model is accepted. We do not know the behavior of the distribution as the sample size gets large, so we were reluctant to impose a form with only linear parameters, especially in sample size. We decided on attempting a correction to the asymptotic approximation given by Foutz.

After some experimentation with various types of corrections, it was decided the most reasonable was to include correction terms in the argument of the asymptotic approximation. In order to make the computation feasible it was decided to fit the data in a two pass scheme; first the null distribution for each sample

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*International Mathematics and Statistical Libraries, 7500 Bellaire Drive, Houston, TX 77036 size was approximated as below, and then the parameters in these approximations were fit by functions of sample size.

The precise form of the approximation was through the argument of a normal distribution, which was taken to be of the form

$$(a + b\, n(x-e^{-1}) + c\, (x-e^{-1})^{2})/ \sqrt{(2\ ^{-1} - 5e^{-2})n}.$$

Because we are strongly interested in the inverse CDF, the data was weighted at each point by the centered difference from the Monte Carlo data, which then resulted in a greater weight on the part of the curve with a large slope. The results of this least squares process yielded a table of values of a, b, and c versus sample size (actually we consider them as functions of n = sample size + 1). We observe that the amount of scatter increases as n increases. There tends to be even more scatter with higher powers of $(x-e^{-1})$. For this reason it was decided to weight the smaller sample sizes more heavily, and a weight of 1/n was adopted. Since the data is more dense for smaller sample sizes this results in considerably less weight for the large sample sizes, although we feel the trend is still properly modelled and that our approximation is considerably better than the asymptotic approximation for very large sample sizes, say even up to 1000.

In the second stage of the process the coefficients a, b, and c was chosen to allow a rate of decay (or growth) of the coefficients to be dictated by the data. Thus we fit a, b and c with functions of the form $A + Bn^{C}$.

For the terms which are constant and linear in $(x-e^{-1})$ the exponent was negative, however, for C(n) the exponent was positive, indicating that the term grows (somewhat slower than linearly) with sample size. We do not consider this as bothersome, however, since the linear term in $(x-e^{-1})$ has already (due to the form of the asymptotic approximation) been included with a factor that grows linearly with sample size.

The overall result of this nonlinear least squares approximation is the approximate CDF involving the nine parameters,

$$(2) \qquad P[F_n < x] \quad \Phi\left[(g(x)/\sqrt{(2\,e^{-1} - 5\,e^{-2})n})\right],$$

$$\text{where } g(x) = a(n) + b(n)\,n\,(x-e^{-1}) + c(n)\,(x-e^{-1})^2, \text{ and}$$

$$a(n) = 0.2089 + 0.1876\,n^{-1.4416},$$

$$b(n) = 1.0015 - 0.05672\,n^{-0.7377},$$

$$c(n) = 0.3049 - 0.5912\,n^{0.8927}.$$

In order to test our results, two different approaches were taken. First, the number of rejections for previously run tests were available for sample sizes of n-1 = 20, 30, and 50, at (approximately) the 0.10, 0.05, and 0.01 levels. By computing the derivative of the approximate CDF, equation (2), and making a correction along the tangent line, we were able to estimate the anticipated rejection rate that would occur with our present approximation. This data was accumulated over 225,000 replications, and is given in Table 4. The main entry is the anticipated rejection rate when using the results of our approximation, above. As a point of comparision with Foutz's asymptotic approximation, we include the corresponding rates for it in parenthesis. Second, to test the approximation for a smaller, as well as an intermediate sample size, we computed the Foutz statistic for 300,000 uniformly distributed samples of sizes 10 and 40, and tabulated them at intervals of .0001 in the range of interest. The results of these calculations are shown in Table 5 for the 0.10, 0.05, and 0.01 levels.

Nominal Significance Level	n-1		
	20	30	50
0.10	0.0994 (0.0764)	0.1002 (0.0801)	0.1007 (0.0840)
0.05	0.0496 (0.0385)	0.0500 (0.0402)	0.0505 (0.0420)
0.01	0.0098 (0.0085)	0.0095 (0.0086)	0.0098 (0.0088)

Table 4
Anticipated Rejection Rates
From Approximate Critical Values
(Based on 225,000 replications)

Nominal Significance Level	n-1	
	10	40
0.10	0.0989 (0.0687)	0.0998 (0.0824)
0.05	0.0481 (0.0349)	0.0491 (0.0087)
0.01	0.0086 (0.0069)	0.0098 (0.0087)

Table 5
Empirical Significance Levels
(Base on 300,000 replications)

As is shown by the tables, we expect the error in the rejection rates due to use of our approximate percentiles to be smaller by a factor of 10-20 for the 0.20 to 0.05 level than they are for Foutz's normal approximation. At the extreme tails, our approximation is not as good as at the more moderate levels, but is still a worthwhile improvement over the asymptotic approximation.

Table 6 lists some upper percentiles of the approximate CDF given by Equation (2) for sample sizes 5(1)100, 100(10)200, and 200(100)1000. The exact values are given for n-1 = 2, 3, and 4. Since we expect the entries to have about 4 digit accuracy, linear interpolation for intermediate sample sizes will have comparable accuracy. Linear interpolation in the percentiles is not accurate, and other percentiles should be calculated from equation (2). It is interesting to observe the "surface" of the null CDF in a perspective plot, as in Figure 1. Of course, only discrete slices exist; the cross section lines in the direction of sample size are an artifact of the plotting package. The convergence toward a sharp rise of the CDF in the vicinity of $x - e^{-1}$ as sample size increases is very apparent.

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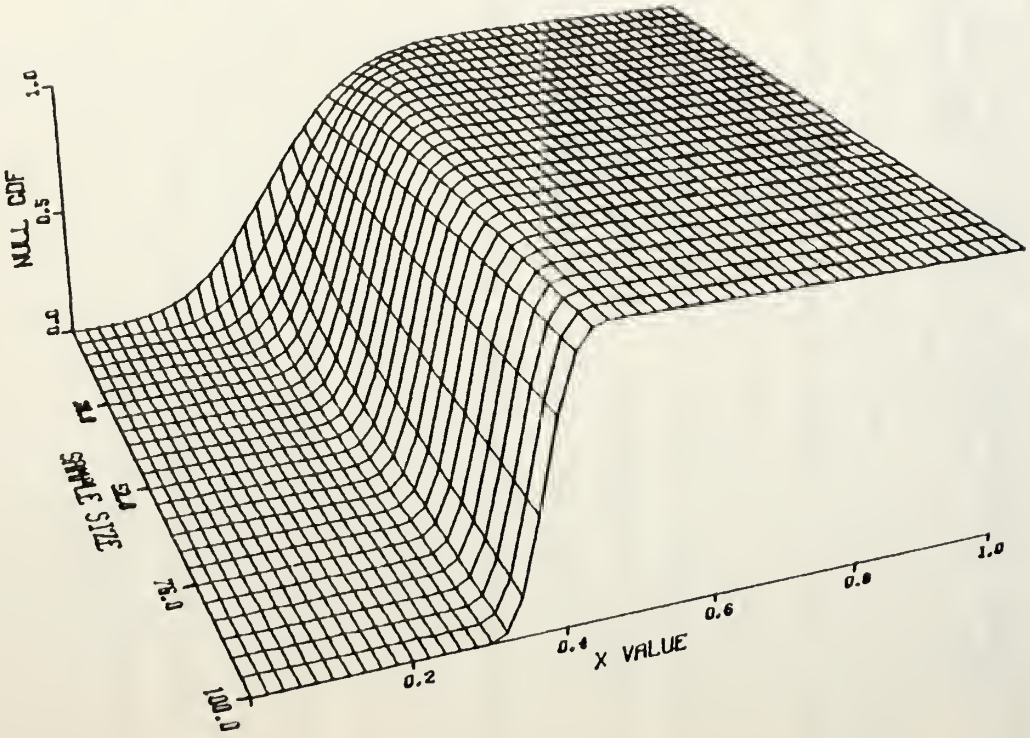

Figure 1: Null CDF of F_n

n-1	0.80	0.85	0.90	0.95	0.975	0.99	0.995
* 2	0.40847	0.44306	0.48409	0.53757	0.57538	0.60893	0.62584
* 3	0.41528	0.43838	0.46673	0.51792	0.56580	0.61428	0.64228
* 4	0.41639	0.43975	0.46850	0.50954	0.54437	0.58870	0.62217
5	0.41588	0.43654	0.46314	0.50386	0.54058	0.58514	0.61677
6	0.41529	0.43436	0.45886	0.49626	0.52987	0.57048	0.59918
7	0.41436	0.43215	0.45496	0.48971	0.52085	0.55834	0.58474
8	0.41332	0.43004	0.45146	0.48403	0.51314	0.54810	0.57264
9	0.41224	0.42807	0.44832	0.47906	0.50548	0.53933	0.56234
10	0.41119	0.42625	0.44549	0.47467	0.50065	0.53171	0.55343
11	0.41018	0.42456	0.44294	0.47075	0.49549	0.52502	0.54563
12	0.40921	0.42301	0.44061	0.46724	0.49089	0.51908	0.53873
13	0.40829	0.42156	0.43849	0.46407	0.48576	0.51375	0.53257
14	0.40743	0.42023	0.43655	0.46118	0.48301	0.50897	0.52703
15	0.40661	0.41898	0.43475	0.45854	0.47960	0.50463	0.52201
16	0.40583	0.41782	0.43310	0.45612	0.47548	0.50066	0.51744
17	0.40510	0.41674	0.43156	0.45388	0.47351	0.49702	0.51325
18	0.40441	0.41573	0.43012	0.45181	0.47096	0.49366	0.50939
19	0.40376	0.41477	0.42879	0.44988	0.46850	0.49055	0.50583
20	0.40313	0.41387	0.42753	0.44808	0.46521	0.48767	0.50252
21	0.40254	0.41303	0.42635	0.44639	0.46407	0.48497	0.49944
22	0.40198	0.41223	0.42524	0.44481	0.46206	0.48246	0.49656
23	0.40144	0.41147	0.42420	0.44332	0.46018	0.48010	0.49387
24	0.40093	0.41074	0.42321	0.44192	0.45841	0.47788	0.49133
25	0.40045	0.41006	0.42226	0.44059	0.45673	0.47578	0.48894
26	0.39998	0.40941	0.42137	0.43934	0.45515	0.47381	0.48669
27	0.39953	0.40878	0.42052	0.43814	0.45365	0.47193	0.48456
28	0.39911	0.40819	0.41972	0.43701	0.45222	0.47016	0.48254
29	0.39870	0.40762	0.41894	0.43593	0.45086	0.46847	0.48062
30	0.39830	0.40708	0.41821	0.43490	0.44957	0.46687	0.47879
31	0.39792	0.40655	0.41750	0.43392	0.44834	0.46533	0.47705
32	0.39755	0.40605	0.41683	0.43298	0.44716	0.46387	0.47539
33	0.39720	0.40557	0.41618	0.43207	0.44604	0.46243	0.47381
34	0.39687	0.40511	0.41556	0.43121	0.44496	0.46114	0.47229
35	0.39654	0.40466	0.41496	0.43038	0.44392	0.45986	0.47084
36	0.39622	0.40423	0.41438	0.42958	0.44292	0.45863	0.46944
37	0.39592	0.40382	0.41383	0.42882	0.44197	0.45744	0.46810
38	0.39562	0.40341	0.41329	0.42808	0.44105	0.45631	0.46681
39	0.39534	0.40303	0.41277	0.42736	0.44016	0.45521	0.46557
40	0.39506	0.40265	0.41228	0.42668	0.43931	0.45416	0.46438
41	0.39479	0.40229	0.41179	0.42601	0.43848	0.45314	0.46323
42	0.39453	0.40194	0.41133	0.42537	0.43769	0.45216	0.46212
43	0.39428	0.40160	0.41088	0.42475	0.43692	0.45121	0.46105
44	0.39403	0.40127	0.41044	0.42415	0.43617	0.45030	0.46001
45	0.39379	0.40095	0.41001	0.42357	0.43545	0.44941	0.45901
46	0.39356	0.40064	0.40960	0.42301	0.43475	0.44855	0.45804
47	0.39333	0.40034	0.40920	0.42246	0.43407	0.44772	0.45710
48	0.39311	0.40004	0.40882	0.42193	0.43342	0.44691	0.45618
49	0.39290	0.39976	0.40844	0.42142	0.43278	0.44613	0.45530
50	0.39269	0.39948	0.40807	0.42092	0.43216	0.44537	0.45444
51	0.39249	0.39921	0.40772	0.42043	0.43156	0.44463	0.45361
52	0.39229	0.39895	0.40737	0.41996	0.43098	0.44391	0.45280
53	0.39210	0.39869	0.40703	0.41950	0.43041	0.44322	0.45201
54	0.39191	0.39844	0.40670	0.41905	0.42986	0.44254	0.45125
55	0.39172	0.39819	0.40638	0.41861	0.42932	0.44188	0.45051
56	0.39155	0.39796	0.40607	0.41819	0.42879	0.44124	0.44978
57	0.39137	0.39772	0.40576	0.41777	0.42828	0.44061	0.44907
58	0.39120	0.39750	0.40547	0.41737	0.42778	0.44000	0.44839
59	0.39103	0.39727	0.40517	0.41697	0.42730	0.43940	0.44772
60	0.39087	0.39706	0.40489	0.41659	0.42682	0.43882	0.44706

TABLE 6
Approximate percentage points for the null distribution of the Foutz statistic
(Note: sample size is n-1)

* from exact distribution

n-1	0.80	0.85	0.90	0.95	0.975	0.99	0.995
61	0.39071	0.39685	0.40461	0.41621	0.42636	0.43826	0.44642
62	0.39055	0.39664	0.40434	0.41584	0.42591	0.43770	0.44580
63	0.39039	0.39644	0.40408	0.41549	0.42546	0.43716	0.44519
64	0.39024	0.39624	0.40382	0.41513	0.42503	0.43664	0.44460
65	0.39010	0.39604	0.40356	0.41479	0.42461	0.43612	0.44402
66	0.38995	0.39585	0.40332	0.41446	0.42420	0.43562	0.44345
67	0.38981	0.39567	0.40307	0.41413	0.42379	0.43512	0.44289
68	0.38967	0.39548	0.40283	0.41381	0.42340	0.43464	0.44235
69	0.38954	0.39531	0.40260	0.41349	0.42301	0.43417	0.44182
70	0.38940	0.39513	0.40237	0.41318	0.42263	0.43370	0.44130
71	0.38927	0.39496	0.40215	0.41288	0.42226	0.43325	0.44079
72	0.38914	0.39479	0.40193	0.41258	0.42190	0.43281	0.44029
73	0.38902	0.39462	0.40171	0.41229	0.42154	0.43237	0.43980
74	0.38889	0.39446	0.40150	0.41201	0.42119	0.43195	0.43932
75	0.38877	0.39430	0.40130	0.41173	0.42085	0.43153	0.43886
76	0.38865	0.39415	0.40109	0.41146	0.42051	0.43112	0.43840
77	0.38853	0.39399	0.40089	0.41119	0.42018	0.43072	0.43794
78	0.38842	0.39384	0.40070	0.41092	0.41986	0.43033	0.43750
79	0.38830	0.39369	0.40050	0.41067	0.41954	0.42994	0.43707
80	0.38819	0.39355	0.40032	0.41041	0.41923	0.42956	0.43664
81	0.38808	0.39340	0.40013	0.41016	0.41892	0.42919	0.43622
82	0.38797	0.39326	0.39995	0.40992	0.41862	0.42882	0.43581
83	0.38787	0.39312	0.39977	0.40968	0.41833	0.42846	0.43541
84	0.38776	0.39299	0.39959	0.40944	0.41804	0.42811	0.43501
85	0.38766	0.39285	0.39942	0.40921	0.41775	0.42776	0.43462
86	0.38756	0.39272	0.39925	0.40898	0.41747	0.42742	0.43424
87	0.38745	0.39259	0.39908	0.40875	0.41720	0.42709	0.43386
88	0.38736	0.39247	0.39891	0.40853	0.41693	0.42676	0.43349
89	0.38726	0.39234	0.39875	0.40831	0.41666	0.42643	0.43313
90	0.38717	0.39222	0.39859	0.40810	0.41640	0.42611	0.43277
91	0.38708	0.39209	0.39844	0.40789	0.41614	0.42580	0.43242
92	0.38698	0.39198	0.39828	0.40768	0.41589	0.42549	0.43207
93	0.38689	0.39186	0.39813	0.40748	0.41564	0.42519	0.43173
94	0.38680	0.39174	0.39798	0.40727	0.41539	0.42489	0.43139
95	0.38672	0.39163	0.39783	0.40708	0.41515	0.42459	0.43106
96	0.38663	0.39151	0.39768	0.40688	0.41491	0.42431	0.43074
97	0.38654	0.39140	0.39754	0.40669	0.41467	0.42402	0.43042
98	0.38646	0.39129	0.39740	0.40650	0.41444	0.42374	0.43010
99	0.38637	0.39118	0.39726	0.40631	0.41422	0.42346	0.42979
100	0.38629	0.39108	0.39712	0.40613	0.41399	0.42319	0.42949
110	0.38553	0.39009	0.39585	0.40443	0.41191	0.42067	0.42666
120	0.38485	0.38922	0.39473	0.40293	0.41009	0.41845	0.42419
130	0.38425	0.38844	0.39373	0.40161	0.40848	0.41651	0.42201
140	0.38371	0.38775	0.39285	0.40043	0.40705	0.41478	0.42006
150	0.38322	0.38712	0.39204	0.39937	0.40575	0.41321	0.41831
160	0.38278	0.38655	0.39131	0.39840	0.40458	0.41180	0.41673
170	0.38237	0.38603	0.39065	0.39752	0.40351	0.41051	0.41529
180	0.38199	0.38555	0.39004	0.39671	0.40253	0.40932	0.41397
190	0.38165	0.38511	0.38947	0.39597	0.40163	0.40823	0.41275
200	0.38132	0.38470	0.38895	0.39528	0.40079	0.40723	0.41162
300	0.37901	0.38176	0.38522	0.39038	0.39486	0.40009	0.40366
400	0.37760	0.37997	0.38297	0.38743	0.39130	0.39582	0.39890
500	0.37662	0.37874	0.38142	0.38540	0.38886	0.39289	0.39564
600	0.37589	0.37783	0.38027	0.38390	0.38706	0.39073	0.39324
700	0.37532	0.37711	0.37937	0.38273	0.38565	0.38905	0.39137
800	0.37485	0.37653	0.37865	0.38179	0.38451	0.38769	0.38986
900	0.37447	0.37605	0.37804	0.38100	0.38357	0.38657	0.38861
1000	0.37414	0.37564	0.37753	0.38034	0.38277	0.38561	0.38755

TABLE 6 (Continued)
Approximate percentage points for the null distribution of the Foutz statistic
(Note: sample size is n-1)

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