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Monterey, California

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in Missile Combat

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NSWC, Carderock Division

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MONTEREY, CALIFORNIA

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THE VALUE OF WARSHIP ATTRIBUTES FOR MISSILE COMBAT

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1 October 1992

ABSTRACT

A methodology is introduced with which to compare the military worth of warship combat capabilities. It is based on two principles. First, a simple salvo model is necessary for exploratory analysis of modern characteristics. Second, the "fractional exchange ratio" is a robust way to compare equal-cost configurations of naval forces, even though we cannot know in advance how and where the warships will be fought.

The methodology is justified three ways: By tracing the evolution of elemental naval force-on-force models since 1902. By a summary of studies covering model exposition, verification and application, and the collection of battle data. And by illustration with important conclusions from parametric analysis, among which are:

(1) The risk of unstable circumstances grows as combat power of a force grows relative to its survivability. ("Stable" means the persistence of victory by the side with the greater combat potential.)

(2) Numerical advantage is the force attribute that is consistently the most advantageous. For example, if A's forces are twice B's, then for combat parity, B's unit striking power, staying power and defensive firepower must be twice that of A's units.

With regard to staying power, the well-known advantage of design balance has been lost in modern warships, and weak staying power is a root source of observable instability. Also, staying power is the ship design element least affected by the particulars of a battle, including poor tactics.

THE VALUE OF WARSHIP ATTRIBUTES IN MISSILE COMBAT

I. INTRODUCTION

Background

Staying power, the ability of a ship to absorb hits and continue fighting, is a major attribute of warships. Developing ways and means to enhance staying power is a matter of detailed engineering design. The naval architect is faced with two dilemmas. First, if the history of combat at sea is any guide, when similar quantities of ordnance strike similar warships, the variation in the amount of damage is quite large. Second, even if one could predict with a high degree of accuracy the effect of, say, hits by Exocet missiles on a DDG-51, the difficult question would still remain: What is the military worth of staying power to the DDG-51 relative to its other combat attributes? For both reasons, the warship designer who knows how to toughen a ship does not know whether doing so will pay off in battle and be worth the cost.

Knowing the value of staying power and other warship attributes, however, has always been of central importance to a navy. There was a time when the relative balance between warship firepower, staying power, speed and endurance was debated publicly, energetically and with the knowledge that:

> You cannot have everything. If you attempt it, you will lose everthing...On a given tonnage...there cannot be the highest speed, <u>and</u> the heaviest battery, <u>and</u> the thickest armor, <u>and</u> longest coal endurance. [Mahan, 1911, p. 44].
> A country can, or will, pay only so much for its war fleet. That amount of money means so much aggregate tonnage. How shall that tonnage be allotted? And especially, how shall the total tonnage invested in armored ships be divided? Will you have a very few big ships, or more numerous medium ships? [Mahan, 1898, p.37]

The case for staying power in the form of armor ended with the atomic bomb. We would have, we thought, one ship sunk with every hit; survivability would have to come from other means. As the threat of nuclear war wanes, corresponding interest in staying power has not been reborn. The U. S. Navy has enjoyed the luxury of contributing to decision on land while being itself relatively free from attacks from the land. But the sanctuary of the sea seems less secure today, along with the prospect of taking hits while fighting close against the littorals.

The problem is now, as it was when Mahan wrote at the turn of the century, what is the proper mix of attributes in a modern warship? We will wish to look briefly at the analysis of his day for ways to correct the lack of analysis in our own. We will see that the analysis then was framed by naval officers not merely as a single ship design question but one of balance in a fighting formation, for there was a trade-off then, as there is now, between warship strength and number of ships.

The methodology we propose for addressing the question of balance between warship attributes is deceptively simple. It rests on the same premise which guided naval officers who attempted to study warship attributes with combat models at the turn of the century: engineering detail adds no insight until the major attributes are settled by examining their military worth in a force-on-force context.

But the simple salvo model introduced in Section II does not look like the force-on-force models developed then because combat now with missiles is different from combat then with guns.

Study Objective

The primary purpose of this study is to offer a methodology to study modern surface warfare in a form suitable to help put the value of a modern surface warship's staying power in context of overall "military worth."

A salvo model of combat is developed to compare staying power with offensive firepower and various defensive measures that reduce susceptibilility to damage. The model and its attributes are first exhibited in Section II and further discussed in Section VII. The analytical role and value of other attributes are specified, most notably the capacity to acquire tactical information about the enemy. An embellished model is in Section VIII.

Much of the study is intended to trace the pedigree of the salvo model, including its deep roots in analysis by naval officers. The salvo model is only new and original because naval combat with missiles is new, different and untested in fleet actions. Complex computer simulations tend to obscure the essential structure of a modern sea battle, and so mask some important implications for warship design and the significance of staying power.

While this paper's central purpose is methodological, nevertheless there are substantive conclusions. At this juncture the reader is well advised to turn directly to Sections X and XI, the Conclusions and Recommendations, for the following reasons:

o If the model is sound, then Sections X and XI serve as an Executive Summary of implications. Some of these suggest the need for new directions in U. S. warship construction.

o Like every analytical tool, the methodology has advantages and limitations. The reader may wish to navigate the study more attentively in light of the destination.

o The reader will find that the conclusions are based on exercising the model with parametric inputs. Analysis with "real," specific warship design characteristics lies in the future. Full value accrues when a reader wishes to experiment with the salvo model himself. That will take a grasp of detail.

Organization

The remainder of the Introduction justifies the methodology used in the study and denies the suitability of more detailed simulations.

After the Introduction the paper is organized as follows:

Section II: States the Basic Equation of modern salvo warfare between surface warships employing missiles, or between warships and attacking aircraft. Defines Fractional Exchange Ratio, or FER, which is used to compare the relative military worth of warship attributes.

Section III: A section for clearing underbrush. Describes the original motivation behind the salvo equations as a tactical, rather than a design, planning aid. Confronts the model validation issue as it applies to descriptive models for exploratory analysis. Points out some salient reasons why force-on-force models of ground combat have validated poorly in order to show that these reasons do not apply to naval combat.

Section IV: Definitions and Symbology. Largely follows the nomenclature of aircraft survivability developed by R. E. Ball [1985] and ship survivability now in preparation by Ball and Calvano [1992]. Their terminology is thought to be the best blend of many in use, but I have had to include a few additions. Not for casual reading, Section IV is essential for understanding and profitable manipulation of the salvo equations. At the end is a summary of most symbols used in Sections V through IX.

Section V: The basic equation for continuous fire. Developed first in 1902; still appropriate for rifled naval ordnance.

Section VI: The evolution of "salvo fire" in the 20th Century. We see a single equation form, having changing interpretations and applications.

Section VII: The modern missile combat equation itself.

Section VIII: The important embellishments to the salvo equation; they are simple in mathematical structure and profound in their operational significance. In most instances the manipulation of the basic equation is mathematically tantamount to the manipulation of the embellished equation. The embellished model's primary value is to show some seldom-treated tactical factors that vastly alter combat results.

Section XI: A catalogue and summary of fourteen pertinent studies of naval combat that have influenced this paper.

Section X: Conclusions

Section XI: Recommendations

Design Criteria for Exploratory Analysis

We start with these premises:

o The best measure of a warship's productivity, or military worth, is the quantity of accurately delivered lethality, or ordnance, over the combat life of the warship.

o The best measure of naval force productivity is similar. It is the quantity of accurately delivered lethality over the combat life of a group of warships fighting in a concerted way.

The measures are similar to the U. S. Arms Control and Disarmament Agency's nuclear weapon exchange models which seek means of stable deterrence by avoiding forces that are cost-efficient in terms of throw weight but are susceptible to an enemy first strike.

Both measures of effectiveness involve facts about the enemy that are inherently unknowable. It seems self-evident that a combat simulation, no matter how comprehensive and rich in detail, has little or no predictive power, because one does not know in advance what inputs to use. In such circumstances, some form of what has been called exploratory modeling [The RAND Corporation, e. g., Bankes, 1992] is appropriate. The modern way of analysis, including RAND's, is to use computer power in rather complicated simulations. The approach herein is the opposite. In the spirit of naval officer analysts like Chase, Fiske, and Baudry, some of whose models will be introduced in Section IV, this study employs the simplest mathematical model that appears to capture the essential dynamics of modern force-on-force war at sea. This approach is also in keeping with the highly utilitarian methods espoused by Morse and Kimball [1951, pp. 9-10, 77-80, 110-121], among others.

Since a return to simplicity in this study runs counter to a trend toward more and more complicated simulations, and since it is contrary to much engineering analysis, including detailed models to study warship vulnerability, we will take space to elaborate with one of the studies summarized in Section IX, by T. R. Beall [1990].

Beall shows that a simple naval combat model can be validated from historical battles, but <u>only if one knows and applies inputs that were observed in the battles</u>. Besides actions by the enemy and damage effects that are unpredictable, other critical values noted by Beall were:

o Hit probabilities of ordnance delivered (unexpectedly low in many cases).

o When and how many warships in the force opened fire and for how long.

o The distribution of fire among targets.

o Whether a force was surprised or otherwise placed at a tactical disadvantage by the enemy.

The purpose of Beall's thesis was to assess the validatity of two simple combat equations for continuous and salvo fire. His results show two contrasting things, both of which are nearly always true about naval combat models.

On one hand, when the appropriate combat model describes only a small number of essential features in an engagement, it will be validated by historical engagements <u>a posteriori</u>, if the small number of inputs correspond to what transpired in the battle: namely, the ships that actually fought, the correct open and cease fire times of fighting units, the correct targeting of warships, and the actual hit probabilities and expected-value damage of the ordnance. If one knows what happened, then one can reproduce the important features of the battle with a simple, but appropriate, model.

On the other hand, Beall's analysis shows that if one does not know what happened he cannot reproduce a battle, even if he uses a model of any complexity and level of detail whatsoever. Its predictive power <u>a priori</u> is nil, since few of the vital inputs to the model can be known in advance.

The analytical approach of this study therefore rests on two pillars. First, we are free to use a simple, but appropriate exploratory model of modern naval salvo warfare that is sufficient to examine and compare the combat value of warship attributes. By appropriate is meant that the most essential phenomena are modeled in a fashion similar to the combat activities themselves. Second, we must find a utilitarian way to compare attributes even though we cannot know in advance how and where the warships will fight or the competence of the tacticians who will employ them in combat.

<u>Other Approaches Using Simple Mathematics of Combat</u>

Two common analytic approaches to exploratory, or descriptive, force-on-force analysis that serve as precedents are Lanchester equations and stochastic duels, both of which are well developed and have extensive literature. See for example, J. G. Taylor [1983] for the former and DARCOM Pamphlet 706-101 [1977], C. J. Thomas [1966], or C. J. Ancker [1982] for the latter. Neither approach is suitable for modern naval combat because neither captures the essential elements of modern naval combat with salvoes of missiles as the principal ordnance.

A third, less well known, approach is by Theodore C. Taylor. His unpublished article, "A Salvo Exchange Model for Modern Tactical Conditions" [1990], is interesting, very much to the subject, and insightful in many respects. Taylor treats Offensive and Defensive combat power as a fraction of enemy capability. He defines:

E_{OB} as the fraction of side **A**'s tactical potential destroyed by **B**'s salvo in the absence of defensive measures by **A**.

E_{DA} as the fraction of E_{OB} eliminated by **A**'s defense, so that the fraction of **A**'s combat power remaining after **B**'s salvo is

$$F_{AR} = 1 - [E_{OB}(1 - E_{DA})]$$

F_{BR} is defined symmetrically for **A**'s salvo effectiveness against **B**.

The Taylor formulation can be manipulated with some interesting analytical results, but the use of fractions sometimes conceals important effects and is not trustworthy for the objectives of this study.

<u>REFERENCES</u> [Complete citations are in the Bibliography]

Ancker, C. J., Jr., <u>One-on-One Stochastic Duels</u>, 1982

Ball, R. E., <u>The Fundamentals of Aircraft Combat Survivability Analysis and Design</u>, 1985

----- and C. N. Calvano, <u>The Fundamentals of Surface Ship Combat Survivability Analysis and Design</u>, 1992 Draft

Bankes, S. C., "Exploratory Modeling and the Use of Simulation for Policy Analysis," 1992

Beall, T. R., "The Development of a Naval Battle Model and Its Validation Using Historical Data," 1990

Mahan, A. T., <u>The Lessons of the War With Spain</u>, 1899

-----, <u>Naval Strategy, Compared and Contrasted With the Principles and Practices of Military Operations</u>, 1911

Morse, P. M., and G. E. Kimball, <u>Methods of Operations Research</u>, 1950

Taylor, J. G., <u>Lanchester Models of Warfare</u>, 1983

Taylor, T. C., "A Salvo Exchange Model for Modern Tactical Conditions," Revision 5, 1990

Thomas, C. J., <u>Military Operations Research Course: Selected Topics of Military Operations</u>, 1966

U. S. Army DARCOM, <u>Engineering Design Handbook</u>, 1977

II. THE BASIC EQUATIONS

The Salvo Model of Naval Combat

The basic salvo equations are:

$$\Delta B = \frac{\alpha A - b_3 B}{b_1} \qquad \Delta A = \frac{\beta B - a_3 A}{a_1}$$

where:

- A = number of units in force **A**.
- B = number of units in force **B**.
- α = number of well aimed missiles fired by each A unit.
- β = number of well aimed missiles fired by each B unit.
- a_1 = number of hits by B's missiles needed to put one A OOA.
- b_1 = number of hits by A's missiles needed to put one B OOA.
- a_3 = number of well aimed missiles destroyed by each A.
- b_3 = number of well aimed missiles destroyed by each B.
- ΔA = number of units in force A OOA from B's salvo
- ΔB = number of units in force B OOA from A's salvo

Observe a fundamental assumption that ΔA and ΔB are measured in warships put out of action, not ships sunk. The choice is basic because the amount of ordnance required to sink a ship is on the order of two to four times as much as would be required to achieve a firepower kill. We take the proper tactical aim to be to put all enemy ships out of action so that they pose no threat, after which the ships may be sink without risk. Ships sunk is a measure of strategic success, in that a ship sunk cannot be repaired to become a threat later.

The Measure of Effectiveness

To overcome the problem of the lack of a priori information about the employment in battle of a force, we offer as a suitable measure of comparative mission effectiveness what is sometimes called the Fractional Exchange Ratio (FER). It compares the fraction of two equal-cost forces destroyed by the other under the supposition that they exchange salvoes. Mathematically the ratio of fractional losses after **A** and **B** exchange salvoes is:

$$FER = \frac{\Delta B/B}{\Delta A/A}$$

When the FER is greater than one, side **A** has reduced **B** by a greater fraction that **B** has reduced **A** and so **A** has won in the sense that it will have surviving units when **B** is eliminated. When the FER is less than one, side **B** has the advantage of the exchange.

Value of Salvo Analysis in Context

Results using realistic values in the salvo equations are merely indicative. Other modeling, tests, discussions and interpretations must follow before design decisions are reached.

As an example, any conclusions about staying power of a warship in missile combat against enemy ships or aircraft can be expected to understate its value, because unlike missile striking power and SAM defensive power, staying power also contributes to survivability against mines, torpedoes and gunfire.

As a second example, the salvo model does not explicitly account for weapon range and scouting coverage. A warship whose missiles outrange the enemy's, when supported by reliable means to detect, track and target every approaching enemy, is unassailable as long as it can carry out its mission without closing the enemy in offshore, blue water operations. As will be evident, scouting is a crucial ingredient of success. The salvo equations cannot be used effectively without realistic inputs for the value of σ_A and σ_B, the scouting processes of both sides. Scouting effectiveness is added to the equations explicitly in Section VIII, Salvo Equation Embellishments.

Exploratory analysis using simple force-on-force models such as the salvo model is no substitute for the usual design analysis, but exploratory analysis is a necessary antecedent at the beginning and a supplement near the end of the design process. At the outset it establishes relationships that will not otherwise be grasped--as for example the special value of staying power when tactics are faulty, the value of scouting when conditions are unstable, and the value of numbers under all circumstances. At the conclusion, the simple model will serve as a highly transparent way of seeing why select features seem to provide balance. Presently some exploratory analysis of the DDV designs may be enlightening and confirmatory.

III. APPLICATION, INVESTIGATION AND CORROBORATION

Prior Model Application For Study of Operations

The salvo model was developed to explore and explain the tactical interactions and relationships of modern missile combat. It was first introduced in stark form in Chapter 10 of Fleet Tactics: Theory and Practice [1986]. The motivation was the same as that which lay behind the developments by J. V. Chase, B. A. Fiske, M. Osipov, A. Baudry, and F. W. Lanchester himself of force-on-force gunfire equations, often referred to collectively as Lanchester equations. It was apparent that continuous gunfire models were inapt for missile warfare. When the suitable model form is used, the equations serve their expository purpose well, communicating a better understanding of the value and limitations of combat power in different forms and under various tactical conditions.

This was the same motivation that lay behind the theory and development of stochastic duels (essentially one-on-one combat) in World War II. The various mathematical constructions enlightened wartime tactics without any baggage of formal validation. That is not to say that wartime analysts had no concern for accuracy. They understood that though precision would be elusive, analysis could still be used to make major improvements in sensor and weapon employment. Accuracy and predictive power were relative things. What was useful was usually apparent and what was useful was put to work.

Recent Salvo Model Application and Validation

The salvo equation form was initially developed in the same spirit: the aim was to improve tactical understanding. Utilitarian investigation and model corroboration were pursued concurrently by thesis students at the Naval Postgraduate School without much regard for any distinction between the two objectives. Some of these studies are summarized in Section IX. I have no hesitation to endorse the salvo formulation to clarify tactical thought. Whether it will be equally useful for exploratory analysis as the first step in ship design and procurement cannot be proven until it is tried, but that is my conjecture and hope.

Model validation is not a very useful concept, for combat model "validation" is never conclusive. It is better to substitute the notion of "corroboration," which means to strengthen or support with evidence and authority. Corroboration implies a never ending process. As Clayton J. Thomas wrote [1989],

> Validation efforts, the many arduous endeavors to make our models better and more useful, are absolutely essential. . . We must realize, however, that our validation efforts will not result in absolutely validated models. . . Model use will remain conditional, dependent on the decisions to be made and the resources available for study. The user of a model will

continue to have the responsibility for its use.

Contrast With Ground Combat Models

The U. S. Army has been more assiduous than the Navy in attempting to validate its models of combat with historical battle data. There is a broad consensus among analysts that the Lanchester square law form does not validate well against data, either from exercises or historical battles. This need not deter the use of continuous fire or salvo fire models for the study of naval combat. The reason is that square law conditions rarely hold for ground combat but usually do for naval combat. In ground combat the defender secures a unit firepower advantage by employing terrain advantageously in prepared positions. Unless the defender advantage is taken into account in model validation, a square law model of ground combat will not conform with historical battle data. The problem vanishes at sea because there is no corresponding advantage of terrain or fortification. Secondly, the Lanchester model, which measures casualties produced by fire, inherently presumes that the only significant achievement of fire is casualties, and so victory must be expressed in casualties. But one of the most important effects of firepower in ground combat is suppression of enemy fire and movement, and a commonly observed cause of mission success is the domination or control of the enemy without severe attrition. There is no counterpart in naval combat (although Appendix A cites an example of suppression as an exception that proves the rule.) At sea, battles are won by putting enemy warships out of action and victory is measured by warships sunk. The first mathematical models of combat were developed by naval officers for the analysis of gunfire between opposing warships because the conditions in the model fit the conditions of combat. This is still the case today.

REFERENCES

Hughes, W. P., Jr., Fleet Tactics: Theory and Practice, 1986

Thomas, C. J., "Verification Revisited--1983," in Military Modeling, W. P. Hughes, Jr., editor; Second edition, 1989

IV. DEFINITIONS AND GENERAL ASSUMPTIONS

Definitions UNDERLINED are followed by mathematical symbols which appear in the models of combat processes represented in the study.

COMBAT UNIT

1. A ship or aircraft capable of delivering firepower.
2. An actual or notional warship/aircraft type comprising a homogeneous force.
3. A standard or benchmark unit in a heterogeneous force.

COMBAT FORCE

A group of COMBAT UNITS that operate and fight in concert.

FORCE STRENGTH A, B

1. The number of UNITS in a homogeneous COMBAT FORCE on sides **A** or **B**, designated A or B respectively.
2. The total value of a heterogeneous FORCE is a weighted sum of the individual UNIT values measured against a standard unit, e.g., an FFG-7. The FORCE STRENGTH is A or B measured in standard unit values.

SHOT

1. A single unit of ordnance, e. g., shell, torpedo, missile.
2. A notional unit of ordnance in a homogeneous force, e. g., an HC-type 8" shell, a Mark XII 21" torpedo, or a Harpoon missile.
3. A standard or benchmark unit of ordnance in a heterogeneous force.

HIT

Verb: To deliver an accurate SHOT to a UNIT

Noun: 1. A SHOT that inflicts damage proportionate to $1/a_1$ or $1/b_1$ of a target UNIT's STAYING POWER. 2. The fundamental unit of measurement of FIREPOWER, FIGHTING POWER, STRIKING POWER and COMBAT POWER.

SEEN TARGET

An enemy COMBAT UNIT that is detected, tracked and targetable. By targetable is meant a UNIT at which a SHOT may be fired with a utilitarian ACCURACY OF FIRE.

ACCURATE SHOT, OR GOOD SHOT

A SHOT that is so well aimed that it will HIT a SEEN TARGET, absent actions by the target to avoid it. ACCURATE, or GOOD, SHOT must in some cases be defined with respect to specific aspect and motion of the target.

ACCURACY OF FIRE π_A, π_B

The probability that a SHOT fired against a target will be an ACCURATE SHOT.

COMBAT KILL, OR MISSION KILL

Verb: To put out of action [OOA]; to render impotent for the duration of an engagement.

Noun: The state of a COMBAT UNIT that is OOA and rendered harmless, not necessarily sunk, but with no COMBAT POWER remaining. (COMBAT POWER is defined below.)

OFFENSIVE POWER

A casual expression of FIREPOWER, FIGHTING POWER, STRIKING POWER or COMBAT POWER as appropriate to the circumstances.

OVERKILL

A casual word for the excess or surplus of OFFENSIVE POWER in hits on targets that exceed the number necessary for COMBAT KILL. OVERKILL is a valuable, but not always desirable, margin to cover errors or miscalculations or chance. It is also a measure of damage beyond COMBAT KILL toward a sinking.

FIREPOWER Unit: a_2, b_2 Force: a_2A, b_2B

1. Of a COMBAT UNIT, the rate at which SHOTS are fired by the UNIT, whether ACCURATE or not,
   a. per unit of time for a stream of SHOTS, called continuous fire.
   b. per salvo for a pulse or bundle of SHOTS tightly spaced in time relative to the interval between pulses; called salvo fire.
2. Of a FORCE, the rate at which SHOTS are fired by all UNITS,
   a. per unit time for continuous fire.
   b. per tightly spaced pulse for salvo fire.

Firepower is frequently but unwisely used as a basis for comparison with other attributes.

FIGHTING, OR STRIKING, POWER Unit: α, β Force: αA, βB

1. Of a COMBAT UNIT, the number of ACCURATE SHOTS fired by it,
   a. per period of time for continuous fire
   b. per tightly spaced pulse for salvo fire.
2. Of a FORCE, the number of ACCURATE SHOTS fired by all UNITS,
   a. per period of time for continuous fire
   b. per tightly spaced pulse for salvo fire.

FIGHTING POWER is FIREPOWER diminished for ACCURACY OF FIRE. Also called STRIKING POWER for carrier air or missile strike.

STAYING POWER Unit: a_1, b_1 Force: a_1A, b_1B

The number of HITS that can be absorbed by a UNIT or FORCE before COMBAT POWER (equivalently, FIREPOWER and FIGHTING POWER) is reduced to zero for the remainder of the engagement. It is the converse of vulnerability.

FIGHTING STRENGTH

A composite value of FIREPOWER and SURVIVABILITY (defined below) that suitably represents deliverable firepower over the combat life of a UNIT or FORCE. For example, F. W. Lanchester showed that for continuous fire under square law conditions, what he called FIGHTING STRENGTH of a FORCE could be represented by αA^2 or βB^2; and for linear law conditions by αA or βB. It will be seen that the

FIGHTING STRENGTH of a <u>naval</u> FORCE using continuous fire with FIGHTING POWER α or β and STAYING POWER a_1 or b_1 can be represented by $a_1\alpha A^2$ or $b_1\beta B^2$.

The object of this study is to seek suitable measures of UNIT and FORCE FIGHTING STRENGTH for salvo fire so that various attributes of warships can be compared.

<u>SCOUTING EFFECTIVENESS</u> σ_A, σ_B

The degradation of FIGHTING POWER, measured in hits per salvo, lost due to imperfect detection or tracking of enemy targets. SCOUTING EFFECTIVENESS is a number between 0 and 1 that is the difference between the number of ACCURATE SHOTS delivered with optimal knowledge of enemy composition and location and the number of ACCURATE SHOTS delivered with the existing information.

SUSCEPTIBILITY

The degree to which a target is impotent to take action against ACCURATE SHOTS by the enemy. Total susceptibility results when the target can take no effective actions of maneuver, deception, or hard or softkill defense, so that the number of HITS equal the number of ACCURATE SHOTS.

VULNERABILITY

The ease with which a target may be FIREPOWER KILLED (put out of action) by enemy HITS. The converse of VULNERABILITY is STAYING POWER.

KILLABILITY

A composite of SUSCEPTIBILITY and VULNERABILITY.

SURVIVABILITY

The mathematical complement of KILLABILITY. A measure of all defensive actions, including design actions, that reduce SUSCEPTIBILITY and VULNERABILITY; i.e., that reduce damage and its effects.

COUNTERACTION or DEFENSIVE POWER

A composite of all defensive actions to reduce SUSCEPTIBILITY to HITS by the enemy. COUNTERACTIONS comprise COUNTERFIRE, SEDUCTION, EVASION, and DISTRACTION.

<u>COUNTERFIRE</u> or HARDKILL COUNTERACTION

Unit: a_3, b_3 Force: a_3A, b_3B

Weapon fire by a target to destroy enemy SHOTS. COUNTERFIRE is measured by the number of enemy SHOTS destroyed before they HIT.

SOFTKILL COUNTERACTIONS

<u>SEDUCTION</u> Force: a_4, b_4

The process of causing ACCURATE SHOTS to miss when COUNTERFIRE has failed, e.g., by seduction chaff. SEDUCTION is treated as equally effective against all GOOD SHOTS in a salvo. It is a multiplier applied to COMBAT POWER (defined below) taking values between 0 and 1. The value of a_4 or b_4 may or may not be proportional to the number of surviving targets exercising this mode of softkill defense, as appropriate.

EVASION Force: a_4, b_4

A process of maneuver to cause GOOD SHOTS to miss. Also design qualities of low-observability that cause GOOD SHOTS to miss. EVASION is treated as equally effective against all GOOD SHOTS in a salvo of, notably, non-homing torpedoes. Mathematically it affects results in the same way as SEDUCTION, and so the same symbol is applied over the same range of values.

DISTRACTION Force: ρ_A, ρ_B

A process of causing ACCURATE SHOTS to miss before COUNTERFIRE has its effect. DISTRACTION is treated as equally effective against all SHOTS (GOOD or not) in a salvo. It is a multiplier applied to σ_A or σ_B. The value of ρ may or may not be proportional to the number of surviving targets exercising this mode of softkill defense, as appropriate.

COMBAT POWER $P_A = \alpha A - b_3 B$, $P_B = \beta B - a_3 A$ in hits/salvo

The STRIKING POWER of a FORCE minus the total hits eliminated by COUNTERACTIONS of the target FORCE. COMBAT POWER cannot be defined or measured except against a specific enemy FORCE and the COUNTERACTIONS it takes to diminish the STRIKING POWER against it.

Note: The full effect of COMBAT POWER includes the suppression and demoralization of the enemy, but these are treated as of secondary importance in naval combat. For a discussion of the major effects in ground combat, see Hughes [1992].

DEFENDER ALERTNESS δ_A, δ_B

The extent to which a target UNIT fails to take defensive actions up to its designed combat potential, due to unreadiness or inattention caused by faulty EMCON or condition of readiness. It is normally a multiplier of a_3 or b_3 with values between 0 and 1.

SKILL, OR TRAINING EFFECTIVENESS τ_A, τ_B

The degree to which a firing or target UNIT does not reach its designed combat potential, due inadequate training, organization or motivation.

COMMAND AND CONTROL

Command and control is a function of command, a process that governs FORCES in a battle, and a system of people and material that perform the function by carrying out the process. It is the command-control (CC) process that is of interest in this study. Since CC governs very nearly all combat actions, its effect must be treated as a modifier of any value in the COMBAT POWER equation. If not readily apparent, it is easily shown that **A**'s CC can diminish his number of participating UNITS, A; his UNIT STRIKING POWER, a_2; his UNIT COUNTERFIRE, a_3; or his SOFTKILL COUNTERACTIONS, ρ_a or a_4; since STAYING POWER, a_1, is to a large extent inherent in ship design, a CC deficiency probably has least effect on a_1. Since the definitions are structured to represent performance with ideal CC, CC is always a factor deflator, never a force multiplier.

COMBAT WORK $\Delta B = P_A/b_1$, $\Delta A = P_B/a_1$, in ships out of action/salvo

The number of UNITS put OOA by a salvo or a period of continuous fire. WORK may also be the accumulated UNITS put OOA after a series of salvo exchanges.

COMBAT RESULT $B[T] = B[0] - \Delta B$, $A[T] = A[0] - \Delta A$

The conditions existing on both sides at a time T after the battle commenced (time 0). The COMBAT RESULT at time T is the FORCE STRENGTH remaining after subtracting enemy WORK done by that time.

COMBAT OUTCOME

A single-valued measure of the final conditions, or states, of both FORCES when the battle is over. An exchange ratio, $\Delta B/\Delta A$, is a common measure of OUTCOME, but in this study the FRACTIONAL EXCHANGE RATIO, defined below, is the preferred MOE for comparing the value of warship attributes.

FRACTIONAL EXCHANGE RATIO $FER = \frac{\Delta B/B}{\Delta A/A}$

COMBAT OUTCOME measured as the ratio of the fraction of each force remaining.

When the FER is greater than 1, then side **A** is winning; when the FER is less than 1, then **B** is winning, in that the winning side will have FORCES with COMBAT POWER remaining when the enemy is impotent. There are pathological exceptions in which both sides' COMBAT POWER is zero, suggesting a draw, but if one side suffers more overkill (computationally negative FORCES remaining) than the other, then the implication is that it has suffered more personnel casualties, greater damage beyond the point of COMBAT KILL, and more ships sunk or sinking.

* * * * * * * * * * * *

General Assumptions

1. A warship's staying power, a_1 or b_1, is the number of standard sized or notional hits required to put it out of action (OOA), not to sink it. Ships OOA, not ships sunk, measures work accomplished, ΔA or ΔB, by an enemy salvo.

2. A damaged unit's remaining staying power, a_1 or b_1, and firepower, a_2 or b_2, degrade linearly with (in direct proportion to) hits received, up to the point when the unit is OOA.

3. Counterfire (from area and point defense systems) is essentially a subtractive process, such that no shot will survive and hit up to the point that the defenses are saturated, after which all good shots will hit.

4. Countermeasures other than counterfire are equally effective against every good shot with a constant probability, a_4 or b_4, and the countermeasures take effect after counterfire is applied.

SUMMARY OF DEFINITIONS AND SYMBOLS

FORCE STRENGTH — A, B

FIREPOWER — Unit: a_2, b_2 — Force: a_2A, b_2B

ACCURACY OF FIRE — π_A, π_B

FIGHTING OR STRIKING POWER
Unit: α, β — Force: αA, βB

STAYING POWER — Unit: a_1, b_1 — Force: a_1A, b_1B

SCOUTING EFFECTIVENESS — Force: σ_A, σ_B

COUNTERFIRE, Hardkill Counteraction or Defensive Power
Unit: a_3, b_3 — Force: a_3A, b_3B

Softkill Defense Effectiveness:
- SEDUCTION — Force: a_4, b_4
- EVASION — Force: a_4, b_4
- DISTRACTION — Force: ρ_A, ρ_B

DEFENDER ALERTNESS — δ_A, δ_B

TRAINING EFFECTIVENESS — τ_A, τ_B

COMBAT POWER, $P_A = \alpha A - b_3B$, $P_B = \beta B - a_3A$ in hits/salvo

COMBAT WORK $\Delta B = P_A/b_1$, $\Delta A = P_B/a_1$ in ships put OOA per salvo

COMBAT RESULTS: $B[T] = B[0]-\Delta B$, $A[T] = A[0]-\Delta A$, in ships not OOA

FRACTIONAL EXCHANGE RATIO: $\dfrac{\Delta B/B}{\Delta A/A}$

FIGHTING STRENGTH: Varies with the combat mode; an index of the worth of a force in combat.

V. CONTINOUS FIRE EQUATIONS

CHASE EQUATION. Developed by Lieutenant (later Rear Admiral) J. V. Chase in 1902 and presented to Cdr. W. McCartey Little for calculating battle outcomes in Naval War College war games. His paper was classified Confidential and not declassified until 1972. See Fiske [1905, 1988]

The equations solved by Chase have been modified so that the terminology and symbology conform with this work. His original derivation appears in toto in Appendix C to Fiske [1988].

Assumptions

1. Fighting power is delivered in a continuous stream of shellfire.
2. All ships on the same side, A or B, have identical unit fighting power, α or β, and staying power, a_1 or b_1.
3. The value of unit fighting power is constant, i.e., it does not change because of a change in target range, target aspect, spotting effectiveness, or demoralization.
4. "Square law" conditions apply: every engaged ship is able to fire at every engaged enemy ship, and as soon as a ship is out of action the fact is known and fire is shifted to a ship with fighting power remaining.

Force-on-Force Differential Equations for the instantaneous rate at which units are being put out of action at any time t: Combat Work done by the enemy.

$$\frac{dB(t)}{dt} = \frac{\alpha A(t)}{b_1} \qquad \frac{dA(t)}{dt} = \frac{\beta B(t)}{a_1} \qquad [1]$$

State Equation For Results at Any Time, T

$$\alpha a_1[A(0)^2 - A(T)^2] = \beta b_1[B(0)^2 - B(T)^2] \qquad [2]$$

Fighting Strengths:

If $\alpha a_1 A^2$ > then $\beta b_1 B^2$, **A** will win a battle of annihilation.

[3]

If $\beta b_1 B^2$ > then $\alpha a_1 A^2$, **B** will win.

Model-based Conclusions

1. From Equation [3] it is seen that if there are twice as many units on one side as on the other, then for parity each unit of the force with the smaller number of units must be twice as strong in fighting power and twice as strong in staying power as the

numerically larger force.

2. From Equation [2] it is seen that equal increases in fighting power and staying power contribute equally to the fighting strength of a warship.

Discussion. With equation [1] we compute Combat Work done, i.e., losses to each side. Lanchester computed Combat Results, i.e., the forces remaining. Consistent with the salvo equations in this paper I have retained Chase's formulation of the pair of force-on-force equations. As they have no minus sign in front of the right-hand terms, they show not results but Work achieved. I have written the state equation solution in Lanchester fashion, which is the Result at any time T, rather than Work at time T.

The Chase formulation computes losses measured in ships and can be used to explore the value of staying power, as he wished to do. The Lanchester form can only evaluate the aggregate of total forces remaining.

Retrospectively in 1921 Chase wrote the following about his purpose in 1902.

> Some years ago when I was a member of the War College Staff, there was considerable discussion among the members of the staff as to the value of concentration of fire. Most of the statements pro and con were couched in vague general terms..."glittering generalities." [I] sought some more tangible expression of the advantages to be derived from concentration...the term "unit of destruction" is a quantity that does not admit of exact definition but it is readily seen that it serves as a measure of both the offensive and defensive qualities of a ship. By "unit of destruction delivered" by a ship is not meant the units leaving the muzzles of the battery of that ship but "unit delivered"...in this way the relative marksmanship of the contending forces may be taken into account.
>
> For example, let there be eight ships originally on each side and let one ship on one side be masked so the m = 8 and n = 7. Then the eight ships will destroy the seven ships and will have the equivalent of $\sqrt{15}$ (nearly four) intact ships with which to engage the remaining one intact ship...it will be seen that after destroying this one intact ship there will remain $\sqrt{14}$. In other words by blanking one ship [temporarily]...the eight ships have destroyed an exactly equal force and have remaining the equivalent of 3.74 ships...
>
> ...if there be twice as many units on one side as there are on the other, each unit of the force having the smaller number of units must be twice as strong offensively and twice as strong defensively as one of the hostile units. This has a bearing upon the question of large or small ships.

> Inasmuch as the displacement of a ship represents the total weights of the materials composing the ship and borne by her, the various materials could be segregated and transformed into separate masses of such material.
>
> Having certain definite quantities of the various materials the question of ship design [is], ...in the simplest form: "Shall we construct from these materials one ship or two ships?"...if we decide to build one ship instead of two, this single ship must be twice as strong offensively and twice as strong defensively as one of the two ships.
>
> It seems to me that while it may be possible to make a ship carry twice as many guns as one of half the displacement it is, at least, debatable if she can be made twice as strong defensively. The chances of hitting her certainly are much greater and she certainly is not twice as strong defensively against underwater attack.

Chase also noted at the time of his 1902 derivation that the equations applied only to gunfire, and was shrewd enough to observe "that sudden destruction arising from any cause whatsoever will [upset the analysis but] have least effect upon the accuracy of the results... if it take place near the end of the engagement. It would seem therefore that the force inferior in gunfire should use the ram or torpedo as early as possible." While he does not develop torpedoes further and reference to ramming bemuses us today, it is probably fair to credit him with an appreciation that a torpedo salvo had to be modeled separately as a pulse of destructiveness with a time delay to account for running time to the targets.

Chase foreshadows the Englishman F. W. Lanchester, and the Russian M. Osipov, who rediscover (!) in 1915 the aggregate, greater-than-linear advantage of numerical concentration when square law conditions obtain. Chase's equations are more powerful than Lanchester's in that not only force size and fighting power variations can be explored but also staying power.

REFERENCES

Fiske, Bradley A., "American Naval Policy," U. S. Naval Institute Proceedings, March 1905

Fiske, Bradley A., The Navy As a Fighting Machine, Annapolis, Md., Naval Institute Press, reprinted with Introduction and Appendices by Wayne P. Hughes, Jr., 1988. Originally published 1916.

VI. SALVO FIRE EQUATIONS

FISKE'S GUNNERY "EQUATIONS". Developed and first published in the U. S. Naval Institute's Prize Winning Essay of 1905 by Commander (later Rear Admiral) Bradley A. Fiske in the form of tables of results after 1,2,3...n salvoes. Fiske's salvo methodology approximates the results of using a continuous fire model.

In effect, Fiske employed the finite difference equations that appear below. He never recorded the equations that yield the tabled results, although as the U. S. Navy's leading electrical engineer he was fully equipped to do so. The relationships between the purely physical effects of superior numbers, fighting power, and staying power stand out more starkly by tabling the results salvo by salvo.

Assumptions

1. Fighting power of the force (called offensive power by Fiske) is delivered in discrete bundles, like salvoes.

2. In the force-on-force equations, the values of unit fighting power, α and β, are constant, as are the values of staying power, a_1 and b_1.

3. Fiske himself avoided such an assumption. Rather than assume, as Chase does, that all ships on the same side, A or B, comprise identical characteristics, Fiske finesses the issue by computing the value of fighting power of the whole force, relative to the staying power of the whole enemy force.

4. The attrition achieved by one side's fighting power in any time interval is small, typically 10% of the other's staying power. In effect, αA_n is about 10% of $b_1 B_n$, and vice versa. Not only is this expositionally convenient, but it is also about what was thought by naval leaders at the time to be the relationship between a battleship's fighting power and its staying power.

5. As with the Chase equations, "Square law" conditions apply: every engaged ship is able to fire at every engaged enemy ship, and fire is shifted efficiently as soon as a target is out of action.

Force-on-Force Equations for Combat Work achieved by a salvo delivered at any time step, n = 1,2,3...

$$\Delta(b_1 B_n) = \alpha A_n \qquad \Delta(a_1 A_n) = \beta B_n \qquad [4]$$

$$\text{and} \qquad b_1 B_{n+1} = b_1 B_n - \Delta(b_1 B_n) \qquad a_1 A_{n+1} = a_1 A_n - \Delta(a_1 A)$$

State Equation after exchange of n salvoes (adapted from J. Taylor [1988, Section 2.10])

$$a_1 \alpha \{A_n^2 - (1-\alpha\beta) A_0^2\} = b_1 \beta \{B_n^2 - (1-\alpha\beta) B_0^2\}$$

Fighting Strengths: Identical to continuous fire:

If $\alpha a_1 A_0^2 > \beta b_1 B_0^2$, then A will win a battle of annihilation.

If $\beta b_1 B_0^2 > \alpha a_1 A_0^2$, then B will win. [5]

Model-based Conclusion

Since the only difference in formulation between Fiske and Chase is the difference between a continuous function and a step function, for the small salvo effects that Fiske and his contemporaries conjectured the results are similar and conclusions are the same. For example, Fiske computes [1988; Table I, page 243] the results for a force with an initial fighting power of 1000 that fights a force with fighting power of 500, a 2:1 advantage. Each side's power per salvo is 10% of its remaining strength. The larger force will have 841 of 1000 units remaining when the weaker force is reduced to zero. Using continuous fire, the larger force would have 866 units remaining. The smaller the fighting power delivered in each salvo, the closer the result will approach the continuous fire outcome. The larger the salvo power, the greater the divergence from continuous fire. When we examine, next, World War II carrier battles, the same mathematical equations apply in 1942, but the single-salvo effectiveness is much greater, with detrimental consequences to the advantage of the larger force.

Discussion. Bradley Fiske's purpose was fully in the spirit of "exploratory modeling" as I have described it above. He wished to demonstrate the advantage of force concentration. He also discussed the value of warship attributes of the big gun era: number of units, unit fighting power and unit staying power.

Fiske uses care in defining terms. Offensive power having, say, a value of 1000 on one side and 500 on the other, is

> . . .of course, wholly arbitrary and some may say imaginary; but, as they are intended merely to show the comparative strength of the two forces, they are a logical measure, because numerical; there is always some numerical factor that expresses the comparative value of two contending forces, even though we never know what the factor is. [1988, p.240]

It may be, he says, 1,000 versus 500 men of equal average fighting value, commanded by officers of equal value. Or it may mean 10 warships opposed by 5 like ships, manned similarly. Thus, Fiske is acutely aware that he conjectures combat power which will perform combat work on the enemy. Nevertheless he conjectures actual combat power against a specified enemy; it is not designed firepower, nor numbers of units, that is given form for exploratory purposes.

He supposes that such a fighting force inflicts damage "in a given time that is proportional to the force" remaining. Fiske uses as his example salvo firepower that is 1/10th of the remaining offensive force, so that if side **A** has a value of 800 remaining, it will eliminate a value of 80 from side **B** in the next time period. Fiske's combat power destroys enemy combat power, not enemy ships. This is a subtle point. In our terminology, Fiske treats offensive power as αA in the aggregate. Although α is always associated with a period of time such that $\alpha = b_1/10$, Fiske would say that it is the enemy's entire bundle of remaining combat power, b_1B, that is being destroyed. Therefore nothing inherent in Fiske's formulation requires α and β to be constants. In effect he treats αA as a single variable. On the other hand, Fiske does not contemplate any change in unit fighting effectiveness caused by changes in range, visibility and such, but only change caused by enemy fire.

REFERENCE: Fiske, The Navy As a Fighting Machine, op. cit.

AIRCRAFT CARRIER STRIKE EQUATIONS. Developed and first published in Fleet Tactics: Theory and Practice to show quantitatively the value of attacking first with all air wings against an enemy carrier task force. See Hughes [1986; pp. 93-103]. The objective was expository and, much like Fiske, equations were avoided by simply tabling the results.

Assumptions

1. In Form 1, the value of the striking power of a carrier air wing is by convention the net value, after any and all degradations and counteractions have been taken into account. A single carrier's striking power, α or β, is the value in hits achieved by its whole air wing, conceived as a single, concentrated pulse of power.

2. In Form 2, the value of the combat power of a carrier air wing is by convention the value, in hits achievable, of that part of the air wing which strikes, before the effects of counteractions by the defender are subtracted. If some of the fighters in the wing are retained for defense (combat air patrol), their value appears as counterfire, a_3 or b_3.

3. Damage to aircraft carriers is the only work or result from an attack. Damage to escorts is ignored as of secondary importance. Defensive effectiveness is not inconsequential but is imbedded in other terms defined above.

4. Scouting effectiveness is implicit and is either 0 or 1. If it is zero for side **A** and one for **B** then side **B** alone attacks, and vice versa. If scouting effectiveness is one for both **A** and **B**, then an exchange of air wing "salvoes" transpires.

Force-on-Force Equations for combat work achieved by a salvo (air wing attack) at any time step:

Form 1

$$\Delta B = \frac{\alpha A}{b_1} \qquad \Delta A = \frac{\beta B}{a_1} \qquad [6]$$

Form 2

$$\Delta B = \frac{\alpha A - b_3 B}{b_1} \qquad \Delta A = \frac{\beta B - a_3 A}{a_1} \qquad [7]$$

Fighting Strengths

The fighting strength equation in Form 1 is similar to Fiske's Equations [5].
The fighting strength equation in Form 2 anticipates that of Modern Missile Combat (Equation [9], Section VII below), and is:

If $a_1\alpha A^2 - a_1 A b_3 B > b_1\beta B^2 - b_1 B a_3 A$, then **A** wins a salvo exchange

If $b_1\beta B^2 - b_1Ba_3A > a_1\alpha A^2 - a_1Ab_3B$, then **B** wins a salvo exchange

We now have to be careful not to overstate the value of superior striking power, because the superior side may have overkill: more than enough striking power to combat kill the inferior.

Model-based Conclusions

Form 1

1. In theory, the strength of an airwing's pulse of striking power is an open question. In practice it is shown in Fleet Tactics, Chapter 4, that Form 1 can be calibrated against the four Carrier Battles in 1942 such that $\alpha = b_1$ and $\beta = a_1$, i.e., a Japanese or American carrier air wing in action was able to put one enemy carrier out of action, all things considered.

2. When salvoes (air wing attacks) are exchanged, the superior side loses the square-law property of cumulative advantage. While the larger force wins, it suffers more. When calibrated for 1942, the results are the same as would hold for the linear law, which is much less advantageous than the square law advantage of the superior side.

3. The problem of representing salvo overkill arises for the first time. When the superior side has more than enough striking power to put the entire enemy force out of action with one salvo, the mathematical result is that the weaker side has a negative number of ships remaining. While this can be easily corrected formally by disallowing combat work that is greater than the whole enemy force, it is worth emphasizing that a negative value in a simple, expected value model indicates an operationally advantageous margin for error, which is in effect a surplus of offensive power to hedge against tactical blunder or chance. In addition, "all ships out of action" does not mean all ships sunk, so overkill is an indicator of the amount of damage inflicted beyond firepower kill.

4. The presence of "overkill" circumstances is the beginning of an unstable situation, which, as we will see under Modern Missile Combat equations below, leads to problems both mathematical and operational when missile salvoes are introduced.

5. For now, the most significant conclusion is that one wins handsomely only if he succeeds in attacking effectively first, and not by exchanging salvoes. This underscores the advantage of superior scouting.

Form 2

1. To match battle results in 1944, Form 2 must be adopted. Defensive power must be taken into account explicitly to obtain a respectable fit with the data in the only carrier battle that year, the Battle of the Marianas in June. In particular, U. S. fighter

defenses and AA gunnery had become too formidable for Form 1 to be useful.

2. Since counterfire is best represented as a subtractive process, it also exhibits the possibility of producing a negative value of enemy combat power: a defense so strong that mathematically it would compute a negative number of hits. Again, the amount of defensive overkill is worth knowing and within limits is worth having, for it represents a hedge against unforeseeable conditions.

3. When the numerator of Form 2, say ($\alpha A - b_1 B$), is taken all together, then it is a bundle of combat power that behaves the same as the numerator (the striking power) in Form 1. But there is an essential complication, in that the first term, αA, comes from the attacker's attributes and the second term, $b_1 B$, comes from the defender's attributes.

Discussion

The central question addressed in Chapter 4 of Fleet Tactics is, "How effective was an air wing attack in World War II? An air wing attack turned out to be less effective than naval air proponents anticipated. The pre-war expectation that a single carrier's air wing would destroy more than one carrier in a concerted attack had considerable influence on tactics, especially those of the IJN. It threatened to create the aforementioned unstable situation, which made scouting and first attack vital to success. See Fleet Tactics, pp. 103-106. But one CV knocked out per air wing was devastating enough. Using the MOE of this study, CVs out of action rather than CVs sunk, in 1942 of 23 CVs engaged in the four big carrier battles of 1942, 13 were put out of action. By November of that year each side had been reduced to one operational CV.

The trend seen even by late 1942, however, was toward improved survivability, of which the CAP was the single most important ingredient. By 1944, the defense was so strong that it has to be accounted for explicitly, and Form 1 is no longer adequate. The value of counterfire--the effectiveness of CAP and AA batteries--has to be incorporated, resulting in Form 2. With Form 2 the pros and cons of allocating air wing effort between striking and defending become apparent. There was only one carrier battle, off the Marianas in June 1944, but it is suffient to show how far the U. S. had come in reducing IJN air wing effectiveness and U. S. CV susceptibility to damage. Subsequent air battles in the Pacific were essentially U. S. sea based air versus Japanese land based air.

Observe that the carrier is the combat unit of interest, and aircraft are "expendable shots." The perspective herein offers no means of computing how many aircraft should survive an attack to exploit their advantage of reusability. In a refined model this would be a required feature. But no refinement was necessary at all to study the Pacific campaign, because aircraft losses were so atrocious. In each the five big carrier battles, the U. S. lost an average of 40% of our aircraft and the Japanese lost 60%. Notably, a large proportion of aircraft lost were on board a carrier, or because the aircraft had no carrier on which to land when they returned, or because they ran out of fuel before they could land.

The effect of attacking effectively before the enemy can launch his own strike, even though not explicit in the equations, can be demonstrated with them. Consider three carriers facing two, so that A = 3 and B = 2. If **A** attacks first, **A** puts both of **B** out of action with overkill. If **B** attacks first, **B** reduces **A** to a single carrier. After **B**'s first attack, if **A** then counterattacks, he cannot win by either sequential attack or simultaneous exchange. There is a four-carrier difference in the results of first attack due to superior scouting.

What of an exchange of attacks? While **A** destroys **B**, he does so at the cost of two carriers. If continuous fire is applied, **A** will only lose a fraction--.76--of a single carrier. **B** is more effective in salvo warfare because he delivers a massive pulse of combat power before he is destroyed.

The consequences of an unanswered attack are enormous, not to say decisive. Unlike in the big gun era when unanswered gunfire was anomalous, in carrier warfare the overriding ambition of every tactical commander was to deliver an unanswered, decisive strike, after which the enemy would be too weak to respond effectively.

With Equation [7] we have built a model that will, with the addition of terms for scouting effectiveness and defensive readiness, be able to address the essential aspects of modern missile combat for surface-to-surface and air-to-surface warfare.

REFERENCE

Hughes, Wayne P. Jr. Fleet Tactics: Theory and Practice, op. cit.

VII. MODERN MISSILE COMBAT EQUATIONS

THE BASIC SALVO EQUATION. Developed in Fleet Tactics: Theory and Practice to show the tactical consequences if a warship had the combat power to destroy more than one similar warship with a single salvo. The mathematical structure is identical with Equation [7] of the Aircraft Carrier Strike Equations. But the new possibility now exists that the striking power of a single unit may be strong enough to take out several similar enemy units, with profound tactical effects.

The salient result of the many-for-one assumption seems to be the creation of operational and mathematical instability. As to actual operations, Fleet Tactics, Chapter 10, concludes that all classical concepts of force concentration are nullified when a many-for-one situation obtains. Victory through superior scouting is promoted in importance, and new tactics of dispersal and sequential engagement become attractive. But Chapter 10 also concludes that defensive power can reduce enemy striking power, in which case concentration for defense makes sense tactically. As to the mathematical model, exploratory computations seem to be very parameter dependent, i.e., case-specific. In fact, case-by-case results of the computations appear to be patternless and chaotic. This strongly suggests that detailed modeling and computations will be misleading until a clearer pattern of both the analytical and operational behavior of the various attributes is grasped first.

Assumptions

1. The striking power of the attacker is the number of accurate (good) shots launched.

2. Good shots are spread equally over all targets. A uniform distribution is not necessarily the best distribution. If each target's defense extracts an equal number of accurate shots, the whole strike may be defeated, whereas an uneven distribution concentrated against only some targets would put at least those targets out of action. (It is easy to compute the correct distribution when everything is known and control of fire is perfect; but knowledge and control were never sufficient in the past when targets were in plain view, and it is less likely that optimal distribution of fire will be achieved in the future.)

3. Counterfire by the target force eliminates with no "leakage" all good shots until the force defenses are saturated, after which all good shots are hits. Mathematically a subtractive process best describes the effect of counterfire.

4. Hits on a target force will diminish its whole fighting strength linearly and proportionate to the remaining hits the target force can take before it is completely out of action.

5. Weapon range is "sufficient" on both sides. In other words, neither side has a weapon range and scouting advantage such that it

can detect, track and target the other while standing safely outside the range of the enemy's weapons.

Force-on-Force Equations for combat work achieved by a single salvo at any time step:

$$\Delta B = \frac{\alpha A - b_3 B}{b_1} \qquad \Delta A = \frac{\beta B - a_3 A}{a_1} \qquad [8]$$

The combat power, P_A or P_B, of a salvo is measured in hits that damage the target force, and is the numerator of the left and right hand equations, respectively. Combat power achieves combat work in hits. When divided by the number of hits a target can take before it is out of action, work on the enemy is measured in ships OOA.

Fighting Strengths

If $a_1\alpha A^2 - a_1 A b_3 B > b_1\beta B^2 - b_1 B a_3 A$, then A wins a salvo exchange.

[9]

If $b_1\beta B^2 - b_1 B a_3 A > a_1\alpha A^2 - a_1 A b_3 B$, then B wins an exhange.

These equations hold when the first term on both sides of the inequality sign is larger than the second term. When the second term is larger than the first, the defense is too strong, no damage is done by the attacker, and a 0 (not a negative) loss results.

Model-based Conclusions

1. "Excess" offensive and defensive power in the form of overkill now have a significant effect on results.

2. Missile combat is force-on-force, so that we need to examine the fraction of each force that can be put OOA by a salvo:

$$\frac{\Delta B}{B} = \frac{\alpha A - b_3 B}{b_1 B} \qquad \frac{\Delta A}{A} = \frac{\beta B - a_3 A}{a_1 A} \qquad [10]$$

Comparative effectiveness of the two sides can be seen by dividing one equation by the other to obtain a Fractional Exchange Ratio (FER):

$$\text{FER} = \frac{\Delta B/B}{\Delta A/A} = \frac{(\alpha A - b_3 B)(a_1 A)}{(\beta B - a_3 A)(b_1 B)} \qquad [11]$$

When FER > 1 then **A** will have forces remaining when **B** is out of action, and when FER < 1 then **B** will have forces remaining.

3. The fractional exchange ratio is unreliable when overkill exists, i.e., when the combat power of either side results in more combat work than there are enemy units to accept it, or when negative combat power results because the defense of either side is strong enough to eliminate more than the number of good shots in the enemy strike.

4. From Equation [11] it may be deduced that for **B** to achieve parity in FER when A is twice as numerous as B, then each B unit must have twice the striking power, twice the defensive power, and twice the staying power of each A unit. This advantage of numerical superiority relative to the other attributes seems to hold over many if not all situations.

5. Experimentation with all other parameters shows no consistent preference in favor of striking power, counterfire, or staying power. Preference seems to depend on some kind of unstable relationship between the values.

6. In general, instability is great when force combat power (numerator in Equation [10]) is large in comparison with force staying power (denominator in Equation [10]). If unit staying power, a_1 or b_1, cannot easily and affordably be added, then force staying power can only be increased and stability restored by increasing the quantity of A or B with units that have affordable attributes.

Discussion

Although further refinements to the salvo equations are possible and in some cases desirable, Equations [8], [10] and [11] are the basic form for exploratory analysis.

The unstable circumstance of very strong combat power on both sides relative to their staying power argues under all circumstances in favor of delivering unanswered strikes. First effective attack is achieved by outscouting the enemy. Since scouting plays a crucial role, we will build it into the next and final model.

The apparent instability and chaotic behavior of the simple salvo model imply the limited value of studies using specific scenarios and ship characteristics in any detail until the general nature of warship attributes and their interrelationships is understood. Studies tend to concentrate on the numerator, specifically increases in single unit striking power or single unit counterfire, without sufficient regard for the denominator, specifically unit staying power and numbers of units. To avoid putting too much capability in a single package of combat value, studies should carefully consider the relative value of greater numbers and staying power vis-a-vis offensive and defensive firepower, and do so in a force-on-force context.

REFERENCE

Hughes, W. P., Jr., Fleet Tactics: Theory and Practice, op. cit., Chapter 10.

VIII. SALVO EQUATION EMBELLISHMENT

ADDITIONAL TERMS FOR SCOUTING, DEFENSIVE READINESS, SOFTKILL AND SKILL OR TRAINING. The basic salvo equations, [8] and [10] above, treat attacks and defenses as full up or zero. By the introduction of multipliers, σ_A and δ_A, on each side, values of partial offensive and defensive effectiveness may be explored. Other terms such as a_4 for seduction, softkill and evasion are also incorporated.

These terms enrich the analytical potential and flexibility of the model. They also complicate and confuse our basic understanding of the interrelationships, for we increase the number of parametric attributes from 8 to over 14.

Assumptions

1. Scouting effectiveness, σ_A or σ_B, takes values between 0 and 1 that measure the extent to which striking power is diminished due to less than perfect targeting and distribution of fire against the target force.

2. Similarly, defender alertness, or readiness, δ_A or δ_B, takes values between 0 and 1 that measure the extent to which counterfire is diminished due to less than perfect readiness or fire control designation to destroy the missiles of an enemy attack.

3. Seduction chaff causes otherwise accurate shots to miss after counterfire has failed. We assume that it draws off all such good enemy shots with the same probability, a_4 or b_4, against each. The model also assumes that the probability does not change as the number of defenders employing it are reduced. Evasion, by low observability or avoidance of a weapon such as a torpedo, is treated mathematically in the same way as seduction chaff.

4. Distraction chaff draws off shots before counterfire, thereby reducing the number of accurate shots that must be destroyed by counterfire. It is given a fixed probability of distracting each enemy shot. Designated ρ_A or ρ_B, it is a multiplier between 0 and 1 applied to βB and αA respectively.

5. A force will fail to reach its full combat potential in part due to inadequate training, organization or motivation. The degree to which a firing unit or target unit thereby fails to achieve its potential is a skillfulness, or "training," multiplier, τ_A or τ_B that takes values between 0 and 1 and is applied where appropriate. E. Hatzopoulis [1990] is the first naval officer to think of this straightforward way to reflect human factors in naval force-on-force equations.

Force-on-Force Equations

Let $\alpha' = \sigma_A \tau_A \rho_B \alpha$ be the fighting power in hits of an attacking unit of side **A** modified for scouting and training deficiencies and the effect of defender **B**'s distraction chaff.

Let $\beta' = \sigma_B \tau_B \rho_A \beta$ be the fighting power in hits of an attacking unit of side **B** modified for scouting and training deficiencies and the effect of defender **A**'s distraction chaff.

Let $b_3' = \delta_B \tau_B b_3$ be the hits denied to **A** by defender counterfire of **B**, degraded for defender alertness and training deficiencies.

Let $a_3' = \delta_A \tau_A a_3$ be the hits denied to **B** by defender counterfire of **A**, degraded for defender alertness and training deficiencies.

Then the embellished force-on-force equations, including seduction and evasion terms, are:

$$\Delta B = \frac{(\alpha' A - b_3' B) b_4}{b_1} \qquad \Delta A = \frac{(\beta' B - a_3' A) a_4}{a_1} \qquad [12]$$

Model-based Conclusions

1. Both striking power, α' or β', and defensive power, a_3' or b_3', depend on good scouting. Concentration of forces (the number of participating units, A or B) depends on effective leadership and tactics. But force staying power, $(a_1 A)$, is in the main a design attribute that is independent of the degree of success in scouting and tactical concentration.

2. For exploratory analysis, the whole of scouting's effect on a combat outcome can be reduced to four multipliers, σ_A, σ_B, δ_A, and δ_B, that take values between 0 and 1 to diminish each side's striking power and counterfire.

3. Similarly, the effects of any and all "training" deficiencies are simply inserted. This seems useful to know, however constrained we are in quantifying τ for ourselves and the enemy.

Discussion

As to explicit representations of own and enemy scouting, they are very difficult to express quantitatively in the absence of combat specifics. The mathematics of own scouting effectiveness is well developed and robust; the principal failure is in marrying the scouting and shooting process into a single system of evaluation. In sharp contrast, the mathematics--indeed, the entire art--of diminishing enemy scouting capability is weak and undeveloped.

Nevertheless the equations tell us how and where to introduce scouting's effect. It seems particularly important to include it as a factor that degrades defensive power. Many things, including too-strict EMCON, enemy deception and stealth, and confusion over whether all enemy units have been detected and targeted, can cause a surprise attack, which in salvo warfare is likely to be fatal. We don't know in most exploratory analyses what numbers to assign to σ and δ. The vital point, which is that success in modern salvo warfare centers on superior scouting, has already been made. It is also worth reaffirming that the best way to soften the consequences of scouting weakness is to increase the value of the salvo equations' denominator with either greater numbers of units or greater staying power per unit, or both.

Deficiencies in combat skill--what we have called "training"--are also placed in combat context with the salvo equation. There is a continuing undercurrent of emotional appeals for the enhancement of organizational, doctrinal, motivational and other human factors without specifying where and how the factors affect combat results. The placement of τ in the equations is apparently the necessary and sufficient way to do this. It is not difficult in the framework of the salvo equation to compare the value of money spent on men vs. the value of money spent on their machines. Most importantly, the equations declare that there is no such thing as a training bonus or synergism. Our studies almost always assume the skillful employment of sensors and weapons when we measure their designed combat potential or estimate their combat power. When human operators enter combat, they seldom achieve the full potential of their machines. The most we can hope for is that they come close to doing so.

REFERENCE

Hatzopoulos, Epaminondas, "A Modern Naval Combat Model," 1990

IX. SUMMARIES OF PERTINENT STUDIES

This is a catalogue and summary of fourteen studies or analyses of simple force-on-force models of naval combat. It will be made clear whether the study is for exposition, validation, application, or the accumulation of historical battle data. Studies are reported chronologically.

It is of some significance that all but one of the authors are naval officers. Each of them had a keen interest in tactics and all but one (Brian Galvin) had a proclivity toward science. Chase was a lieutenant and Fiske a commander when they conceived their mathematical models.

Rear Admiral Bradley A. Fiske, **The Navy as a Fighting Machine**, 1916, Reissued 1988. Develops and applies the salvo equations for the gunnery age, incidentally showing why the square law (called the N-square law) was applicable at sea in the big gun era [pp. 241-245; 313-319]. Appendix C to the 1988 edition gives Rear Admiral J. V. Chase's original development of continuous fire equations for naval combat in 1902 [pp. 375-382].

Captain Wayne P. Hughes, Jr., **Fleet Tactics: Theory and Practice**, 1986. First book in which this study's concepts were explained and employed. With continuous and salvo fire equations, it illustrates the value and limitations of various attributes of warships and naval forces. Contrasts how the phenomena--and so the mathematics--of concentrated combat power changed over time, in the age of fighting sail, age of the big gun, age of the aircraft carrier, and finally in modern air-surface warfare with missiles. Show that naval officers saw empirically the square law phenomenon at work in combat, so that it was naval officers who first worked out the mathematics of concentrated firepower and staying power, anticipating the work of Lanchester and Osipov.

Lieutenant T. J. McKearney, **The Solomons Naval Campaign: A Paradigm for Surface Warships in Maritime Strategy**, September 1985, NPS Masters Thesis. Strategic and tactical aspects of the night surface battles between the U. S. and Imperial Japanese navies in 1943-44. Gathers information about the gun and torpedo firepower and staying power of opposing forces, and timelines of the events in 11 engagements. Shows how U. S. prewar war games and fleet exercises, concentrating on daytime engagements, served us badly by emphasis on long range gunnery [pp. 105-138]. Shows quantitatively how IJN prewar preparations for night actions using salvoes of torpedoes as the decisive weapon paid off in the early engagements, until the USN learned to combine a scouting (radar) advantage with torpedo salvoes in 1943 [pp. 139-141; 149-159]. Appendix A is a convenient compendium of data from the 11 battles.

Midshipmen 1/c Keith W. Brzozowsky and Robert M. Memmesheimer, **The Application of the Sochard Ship Damage Model to World War II Ship**

Damage, June 1988, NSWC unpublished monograph. Develops simple, clear relationships between number of hits by bombs and torpedoes and the damage inflicted on warships from 1,000 to 50,000 tons displacement in World War II. Compiles data for 78 sinkings and 98 mission kills. [See ENDNOTE]

Richard Humphrey, **Warship Damage Rules for Naval Wargaming**, May 1990, ORSA/TIMS Presentation. Draws from Brzozowsky and Memmesheimer, Korotkin, and Chesneau to develop probabilities of sinking or firepower kill as a function of hits by torpedoes and bombs of different calibers. Three noteworthy conclusions from the data are (1) the wide disparity in the damage resulting from a given amount of ordnance (2) compared with small ships, big ships were sunk or put out of action by a quantity of ordnance that was in strikingly less than linear proportion to their displacement, and (3) the imprecision evident in even hard historical data from combat.

Lieutenant Thomas R. Beall, **The Development of a Naval Battle Model and its Validation Using Historical Data**, March 1990 NPS Masters Thesis. Describes the validation of this study's models using 14 battles from World Wars I and II. Battles are fought by continuous fire (guns only), salvoes (air attacks), or a combination of continuous fire and salvoes (guns and torpedoes). Beall uses survivability data from Brzozowsky and Memmesheimer to derive new curves and adds an ordnance-damage relationship for all calibers of guns. A valuable data source of past warship firepower, fighting power and staying power. Concludes that the combat models are "a fair representation of reality" observed in each battle. An unstated inference is that without knowing the particulars of a battle it cannot be reproduced, nor would a more detailed model serve better. Beall is primarily interested in the combat models as tactical planning aids, and believes (as I do) that their limited power to foresee a battle's details in no way nullifies their value for tactical planning. The model shows the weaknesses of USN tactics at Savo Island, shows that even without surprise the IJN had the tactical advantage against our defective plan, and suggests (by the model's very nature) better USN tactics. The model also could have shown USN tacticians why their cruiser gunnery tactics were doomed against superior IJN torpedo tactics in the night battles of the Solomons of 1942.

Lieutenant Jeffrey R. Cares, **The Fundamentals of Salvo Warfare**, March 1990, NPS Master's Thesis. Since Beall's validation effort ended with World War II, Cares' objective was to examine the model of modern missile warfare with a controlled experiment. For his "real world" he uses the well shaken down NAVTAG wargame in a simulation mode (no man in the loop). The test vehicle is a single warship class in various tactical configurations and numbers ranging from one-on-one to three-on-two. He "fought" 275 battles with 1900 missiles exchanged between 700 ships. NAVTAG results, both mean and variance, were compared with salvo model predictions. The major departures were attributable to two things. One was the friction of units in combat (called "entropy" by Cares) manifest as duplicated targeting, and a "sump effect," (the maldistribution of the effect of defender **B**'s distraction chaff.

Let $\beta' = \sigma_B \tau_B \rho_A \beta$ be the fighting power in hits of an attacking unit of side **B** modified for scouting and training deficiencies and the effect of defender **A**'s distraction chaff.

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Nevertheless the equations tell us how and where to introduce scouting's effect. It seems particularly important to include it as a factor that degrades defensive power. Many things, including too-strict EMCON, enemy deception and stealth, and confusion over whether all enemy units have been detected and targeted, can cause counter-measures do not necessarily reduce the bombers' damage on carrier and escorts if they continue to close the range, but bomber losses increase severely. Thus, the final choice lies with the bombers, and the analysis suggests that the effect of countermeasures will not, in all circumstances, reduce warship losses to the first bomber attack but will very much reduce the number of bombers that reattack.

Lieutenant Brian R. Galvin, **"Punching Combat's Equations,"** in Naval Institute **Proceedings**, July 1991. An operations-oriented paper, espousing the use of salvo equations to organize tactical thought aboard ship. Galvin is the first to explore the correct way to introduce soft-kill counteractions into the equations, and does so with his equation 3. As the Proceedings' lead says, "the simple tactical models of old still seem to work."

Lieutenant Aristomenis P. Lalis, **Sensitivity Analysis of the Modern Naval Combat Model**, September 1991, NPS Masters Thesis. An attempt to use mathematical manipulation and parametric values to derive general lessons about the relative value of different warship attributes. One method is a "battle trace" developed by D. Barr, M. Weir and J. Hoffman [1991], which is the sequential use of the FER. The second method is to take partial derivatives and examine rates of change in a sensitivity analysis. Perhaps the most striking result is the absence of a clear pattern when two parameters of one side's force, such as σ_A and δ_A, are varied and compared with each other. Lalis constructed tables of results for many circumstances. Hoped-for patterns of comparative advantage did not emerge. The relative advantage of, say, superior scouting or defender alertness seems to be very case specific, frequently depending on many or all of the circumstances modeled in the equations. To date at least, a consistent advantage of one attribute (such as staying power) over any other is not generalizable. Navy friends say why is this a surprise? Nevertheless, it is a disappointment. A principal aim of follow-on research must be to seek patterns of advantage.

[The principal exceptions, already evident and so not examined by Lalis, are (1) the pervasive and general advantage of greater numbers and (2) the often crucial need to attack decisively before the enemy can launch his attack in an exchange.]

Richard Humphrey, **Damage and Losses of Warships in Modern Warfare**, November 1991, ORSA/TIMS Presentation. Although ship damage and loss data since 1945 is less extensive than that available for World War II studies, recent data agree rather well with Humphrey's previous Warship Damage Rules against which they are compared. Gulf War results "were of particular interest for the demonstrated difficulty in sinking modern patrol boats with air to surface weapons. . ."

ENDNOTE For design study, ordnance-damage data should of course be taken from the U. S. Navy's most authoritative source. None of the

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X. CONCLUSIONS

1. Terminology. For clear communication, unambiguous definitions of naval combat terms should be agreed within the Navy and adopted. In particular, "combat power" and "survivability" take on a variety of meanings.

2. Own Attributes. The key attributes that bear heavily on success in modern surface and air-surface naval combat are:

- o Striking power
- o Staying power
- o Counterfire (defensive firepower)
- o Scouting (detection and targeting) effectiveness
- o Softkill counteractions
- o Defensive readiness
- o Training, organization, doctrine and motivation (resulting in skill and referred to collectively herein as "training.")

No attribute may be neglected in warship design, peacetime drills, or combat operations. While this conclusion is derived from an examination of the phenomenon of modern missile combat rather than from manipulation of the salvo equations, the explicit relationships between the attributes may be grasped from the equations, permitting the study of any element's contribution alone or in combination.

3. Enemy Attributes and the FER. It is a fact of combat that the attributes of the enemy in battle are coequal determinants of its outcome. That his numbers and quality are a vital ingredient in ship design which cannot be known during warship configuration has, to say the least, complicated the design problem. The Fractional Exchange Ratio (FER) is advanced as a robust way to compare warship attributes in the absence of knowledge about the circumstances in which a warship will fight, including the attributes of the enemy.

4. C^2. Command and control do not appear in the equation because they affect many of the terms on both sides:

- o Scouting effectiveness
- o Striking power generated
- o Counteractions generated against enemy striking power
- o State of training
- o Even numbers of engaged units, A or B, are affected, because it is a primary command responsibility to bring all forces to bear in a battle while at the same time inhibiting the enemy's ability to do so.

5. Staying Power Robustness. Ship staying power is uniquely the ship design element least affected by the particulars of a battle, including poor tactics. Staying power's inherent robustness suggests that it should be treated with great respect; and specifically with greater respect in U. S. Navy ship designs, which have little staying power relative to other attributes such as striking power and defensive hard and soft kill power.

hits on targets). The other was overkill: an excess of missile firepower which works to the disadvantage of the larger force in an exchange. While the departures of model predictions from NAVTAG results are not startling, neither is NAVTAG the real world, which would be considerably more chaotic. Cares also develops some thoughtful theory of modern combat and proposes three "Laws of Salvo Warfare" which together are a theoretical foundation of the basic salvo equation. This appears in Appendix A.

Wayne P. Hughes, Jr., **"Survivability of Warships: the Historical Data,"** June 1990, MORS Presentation. An encapsulation of the simple ordnance-damage relationships developed by Humphreys and Beall, the presentation is notable for a warm reception among the MORS working group attendees, and the infrequency of such war-data-based presentations in current naval operations research forums.

Lieutenant Timothy T. Smith, **Combat Modeling Low Intensity Conflict Anti-Surface Warfare for Engagement Analysis**, March 1991, SECRET NPS Masters Thesis. The first published practical application of the salvo model. Smith drew on his personal experience on board a Navy FFG during Operation EARNEST WILL escort of tankers in the Persian Gulf. The analysis pits an FFG escort against differing numerical and tactical combinations of potential enemy Fast Attack Craft (FAC). While the quantitative results are classified, it should come as no surprise that single DD- or FFG-sized escorts are in trouble against small FACs in modest numbers. Perhaps the principal conclusion is that to win the FFG must outscout the enemy with helicopters or other reliable search systems and then attack very effectively first.

Lieutenant Epaminondas Hatzopoulos, **A Modern Naval Combat Model**, September 1990, NPS Masters Thesis. Explores how to deal with human factors that affect the outcome of a naval battle: scouting and alertness, leadership, morale, and training. Concludes that while the value of human factors will always be difficult to quantify, the manner in which they affect salvo warfare outcomes is easy to see and represent. Since a large body of opinion believes that the subject of human factors is too intricate to model, the simple, concise solution in the thesis is of more than passing interest. Hatzopoulos compares and contrasts his approach for sea combat with that of T. N. Dupuy for ground combat. But he does not deal with the phenomena of command-control, which has an influence pervasive throughout the model and very case specific, for reasons that are summarized where Command and Control is defined in Section IV. Also shows that the form of the salvo equation for hard-kill point defense is different from area defense (SAMs and CAP).

Lieutenant Ray L. Snell, **Countertargeting in Modern Naval Combat**, March, 1991, NPS Masters Thesis. Shows that the salvo model is suitable for study of air launched missile attacks against surface warships. With a combination of salvo equations and computer simulation, Snell explores the effects of jamming and deception, ρ_B, to reduce bomber scouting effectiveness, σ_B. He finds that CVBG

10. Warship Design Goal. Maximum fighting strength (i.e., hits achieved during a combat lifetime) is the proper warship design goal. Experimentation with all parameters except numbers of forces shows no consistent preference in favor of striking power, counterfire or staying power to achieve the goal. Preference seems to depend on some kind of varying relationships between the values on both sides in a sea battle. The instability and seemingly chaotic behavior that is seen under many circumstances imply a limited value of studies that use specific scenarios and detailed ship characteristics, until the general nature of warship attributes and their interrelationships is better understood. Procurement and design studies tend to concentrate only on fleet combat power, or even more narrowly on single unit striking power and single unit counterfire, without sufficient regard for other attributes. In particular, this study indicates the need for attention to whole-force staying power, which is the product of unit staying power multiplied by the number of units engaged. To achieve balance and avoid building an unsustainable amount of offensive and defensive firepower into individual warships, detailed studies must be done in a force-on-force context in which the relative worth of offensive and defense power is compared with the worth of greater numbers and staying power.

11. Further About Staying Power.

a. The salvo equations do not embrace undersea warfare. It is important to realize that design features suggested by the salvo equations will understate the value of staying power, which when present contributes uniquely to survival against torpedoes and mines as well as shells and missiles.

b. In littoral operations the effectiveness of defensive systems, a_3 and a_4, will be curtailed because of short response time, in which case survival and the ability to fulfill a mission will depend more heavily on staying power.

XI. RECOMMENDATIONS

1. Evaluate warship attributes in the context of force-on-force, in which the quantity and vulnerability of the force are evaluated in competition with unit offensive and defensive firepower.

2. Measure the quantity of accurately delivered ordnance over the combat life of a warship or force. This is a force's true fighting strength. Use this MOE to measure the contribution of an equal cost increment, whether of additional ships or of an attribute within a force of ships.

3. Use the salvo equations to solve the fundamental and confounding problem in warship design, which is that one needs a vehicle for comparison of warship attributes without knowing what battles the warship will fight. Battle variables are not simply those related to a scenario. Variability of outcomes is also related to tactics and the adept handling of forces in the battle, and also to chance, for historical results show wide swings that cannot be attributed to the relative fighting strengths of the opposing sides. Salvo equations are recommended as the best vehicle for exploratory, front end analysis of surface warship attributes for strike, AAW, ASUW, and some aspects of search, patrol, inshore warfare and amphibious operations. ASW and mine warfare are noteworthy exceptions.

4. Use the Fractional Exchange Ratio (FER) with the salvo equations. The FER is as a strong and well-tested, but abstract, measure of the relative worth of warship attributes which can be used with any appropriate form of salvo equation or a continuous fire equation. The limitations of analysis with such an exploratory model are well set forth in this study, but the salvo equations in conjunction with the FER are a superior tool of exploratory analyis.

5. Conduct exploratory analysis first, and use scenario-specific, detailed analysis next for corroboration. Complicated warfare simulations used prematurely for exploratory analysis will inhibit a basic grasp of the desirable design goals, because data for such simulations is intrinsically unknowable. Use of the most complex simulations also tends to restrict the analysis to that of a single ship in a small number of tactical and parameter variations. Engineering design studies with detailed models should be used iteratively, to test feasibility of a design concept.

6. It is also recommended that the salvo model be used to explore complementary mixes of warships comprising task groups. Methods of force mix analysis have not been introduced in this paper. Complex models and simulations have been strikingly unsuccessful at deriving a mix of several classes of fighting ships in a force. For various reasons, admittedly not all analytical in nature, the tendency has been to derive one attractive class of all purpose surface warship, disregarding the possibility that two or more

cooperative classes are preferable. This is seen in the past balance of battleships, cruisers, and destroyers, and in fact is seen in the <u>imbalance</u> of past U. S. peacetime shipbuilding programs which tended to concentrate on capital ships and neglect cruisers, destroyers, amphibious ships and minecraft. The salvo model cannot of itself determine a preferable mix, but it is thought to have promise to <u>explore</u> many aspects of a mix, once one is hypothesized. For example, lack of force staying power creates an unstable situation which is very undesirable for a superior force when it confronts a smaller one in an exchange of salvoes. The two ways to reduce the danger of an unstable combat situation by warship design are (1) to increase unit staying power, and (2) to increase the number of units engaged. The first way is attained by building less vulnerable ships with the same combat power. The second way is attained by building more numerous ships, some of which could be lost without having critical consequences to the task force as a whole, and which could operate in the most exposed positions or dangerous waters. The first category has the character of 1930s-era capital ships (battleships, battlecruisers, and aircraft carriers) that were seen as the backbone of the fleet. The second category has the character of 1930s-era destroyers that were seen as expendable. Historical design experience suggests that an as yet untried mix of two classes might be superior to a single surface warship class of present characteristics.

APPENDIX A. THE AGGREGATION OF TERMS

A thoughtful reader of the assumptions that accompany each model in Sections V through IX will appreciate their effects on computation, among which are:

o The linear degradation of unit capabilities after hits.

o The uniform spread of hits from a salvo over all targets.

o The presumption that force striking power is the product of unit striking power times the number of units; and defensive firepower is similarly multiplicative.

The salvo equation expresses a dynamic process in a specific, rigid mathematical construction. One way, perhaps the only way, to escape its strictures and assumptions is to aggregate terms in an even more abstract and primitive formulation. The value of the aggregate terms can be whatever is experimentally "true," or best fits the data. For instance, if we observe that the number of hits, $\boldsymbol{\alpha}[2]$, achieved by salvoes from two identical warships is only 1.7 of the value of α for one ship, then the striking power of the two is not $\alpha A = \alpha \times 2$ but a non-linear aggregate value, $\boldsymbol{\alpha}[2] = 1.7 \times \alpha[1]$. If one has experimental evidence for $\boldsymbol{\alpha}[A]$, A = 1,2,3..., then he can and should use it in the manner described below.

The theoretical structure that follows is adapted from J. Cares [1990]. It is expressed as three "laws of salvo warfare." When the laws describe the process, the general form applies. If and when the particular conditions of linearity also hold (and in general they would not for real combat) then the model forms above would produce "true" results. If one wishes to deny the assumptions of additivity, then he should retreat into the still more basic form appearing below, using the laws of salvo warfare as guide. He will have to have numerical values for the synthesized terms that the laws imply. Finally, if any of the three laws themselves appear not to hold and must be repudiated, then even this most primitive form is in default.

<u>The First Law of Salvo Warfare</u>: Salvoes are interactions of pulses of combat power with their targets and therefore are event-stepped phenomena (not continuous processes) of attrition, in which damage is propertional to the ratio of combat power to staying power. Therefore we say the effect of a salvo by **A** against **B** is:

Losses to A = [Combat power of B] / [Staying power of A]

<u>The Second Law of Salvo Warfare</u>: Combat power is the attacker's pulse of lethal energy minus the defender's actions to attenuate the energy. Therefore we say:

[Combat power of B] = [Striking power of B] - [Defensive power (or Counteractions) of A]

The Third Law of Salvo Warfare: Combat power may be measured in units of hits, staying power in units of hits per ship, and combat potential and damage in units of ships.

Therefore, when the law holds, we need not be concerned with maneuver or advantageous tactical position, or with the effects a salvo might have to demoralize or suppress the enemy's actions. Contrariwise, when the Third Law does not hold, the model is defective. As an example, at the climax of the Battle of Jutland, Admiral Scheer, the German tactical commander, in desperation ordered a torpedo attack by his Third Torpedo-boat Flotilla on the British battle line. The salvo of torpedoes achieved no hits and damage "in units of [battle]ships" was zero. But the combat power visible in the wakes of the torpedoes caused the British commander to turn his battle line away, letting the German battleships escape their awkward posture. The torpedo salvo's effect on Jutland's outcome by briefly suppressing British firepower was very great, even decisive in the view of some historians. When a combat outcome hinges on effects of fighting power other than hits and losses (as it often does in a land battle), then the Third Law is broken and the model is deficient.

A form of the salvo model with aggregated terms may be written that is consistent with the three Laws. Both Beall [1990] and Cares [1990] conclude from their research (summarized in Section IX) that the following representation of Equation [12] is the most generally satisfying:

$$\Delta B = \frac{\boldsymbol{\sigma}_A \boldsymbol{\alpha}[A] - \mathbf{b}_3[B]}{\mathbf{b}_1[B]}$$

The **bold** symbols represent the not necessarily linear aggregate of whole **Force** scouting power, striking power, counteractions, and staying power for the number of units, [A] or [B], attacking or defending. This "four-element model" (Cares' expression; p.25]) is in force-on-force analysis really an eight-element model. Eight elements with the flexibility to aggregate attributes in non-linear fashion, are superior for *a posteriori* corroboration against the experimental battle results.

However, for exploratory analysis to study individual ship design variants, in the absence of battle data one has little choice but to begin with the assumptions that accompany each of the combat model forms.

REFERENCES

Beall, T. R., "The Development of a Naval Battle Model and its Validation Using Historical Data," 1990

Cares, J. R., "The Fundamentals of Salvo Warfare," 1990

BIBLIOGRAPHY

Ancker, C. J., Jr., One-on-One Stochastic Duels, ORSA MAS, Washington, DC, 1982

Ball, R. E., The Fundamentals of Aircraft Combat Survivability Analysis and Design, AIAA, Inc., New York, 1985

Ball, R. E., and C. N. Calvano, The Fundamentals of Surface Ship Combat Survivability Analysis and Design, USNPS, Monterey, CA, 1992 Draft

Bankes, S. C., "Exploratory Modeling and the Use of Simulation for Policy Analysis," RAND Note N-3093-A, Rand Corporation, Santa Monica, CA, 1992

Barr, D., M. Weir, and J. Hoffman, "Evaluation of Combat," Naval Postgraduate School, Monterey, CA, 1991

Beall, T. R., "The Development of a Naval Battle Model and its Validation Using Historical Data," NPS Masters Thesis, Monterey, CA, 1990

Brzozowsky, K. W. and R. M. Memmesheimer, "The Application of the Sochard Ship Damage Model to World War II Ship Damage," unpublished NSWC monograph, White Oak, MD, 1988

Cares, J. R., "The Fundamentals of Salvo Warfare," NPS Masters Thesis, Monterey, CA, 1990

Chase, J. V. "A Mathematical Investigation of the Effect of Superiority of Force in Combats Upon the Sea," unpublished CONFIDENTIAL paper, 1902, printed in Appendix C to Fiske, The Navy As a Fighting Machine cited below.

Epstein, J. M., "The Calculus of Conventional War: Dynamic Analysis Without Lanchester Theory," The Brookings Institution, Washington, DC, 1985

Fiske, B. A., "American Naval Policy," U. S. Naval Institute Proceedings, March, 1905

Fiske, B. A., The Navy As a Fighting Machine, originally published 1916, reissued in Classics of Sea Power Series, Naval Institute Press, Annapolis, MD, 1988

Gafarian, A. V. and Ancker, C. J., Jr., "The Two-on-One Stochastic Duel," Naval Research Logistics, June 1984

Galvin, B. R., "Punching Combat's Equations," U. S. Naval Institute Proceedings, July, 1991

Hatzopoulis, E., "A Modern Naval Combat Model," NPS Masters Thesis, Monterey, CA, 1990

Horrigan, T. J., "The Configural Theory of Weapons Effectiveness: A Means to Enhance the Impact of Technology on Combat Effectiveness," Horrigan Analytics, 1460 N. Sandburg Terrace, Chicago, IL, January 1988

Hughes, W. P., Jr., "Combat Science: An Organizing Study," Draft Research Paper, NPS, Monterey, CA, 1992

Hughes, W. P., Jr., Fleet Tactics: Theory and Practice, Naval Institute Press, Annapolis, MD, 1986

Humphrey, R. L., "Warship Damage Rules for Naval Wargaming," ORSA/TIMS Joint Meeting, Las Vegas, NV, May 1990

Humphrey, R. L., "Damage and Losses of Warships in Modern Combat," ORSA/TIMS Joint Meeting, Anaheim, CA, November 1991

Lalis, A. P., "Sensitivity Analysis of the Modern Naval Combat Model," NPS Masters Thesis, Monterey, CA, 1991

Mahan, A. T., The Lessons of the War With Spain, Little Brown, Boston, MA, 1898

Mahan, A. T., Naval Strategy, Compared and Contrasted With the Principles and Practices of Military Operations on Land, Little Brown, Boston, MA, 1911

McKearney, T. J., "The Solomons Naval Campaign: A Paradigm for Surface Warships in Maritime Strategy," NPS Masters Thesis, Monterey, CA, 1985

Osipov, M., "The Influence of the Numerical Strength of Engaged Forces on Their Casualties," originally in Russian, Military Collection, 1915; translated by R. L. Helmold and A. S. Rehm and republished as U. S. Army CAA Research Paper CAA-RP-91-2, Bethesda, MD, 1991

Smith, T. T., "Combat Modeling Low Intensity Conflict Anti-Surface Warfare for Engagement Analysis," SECRET NPS Masters Thesis, Monterey, CA, 1991

Snell, R. L., "Countertargeting in Modern Naval Combat," NPS Masters Thesis, Monterey, CA, 1991

Taylor, J. G., Lanchester Models of Warfare, ORSA MAS, Washington, DC, 2 volumes, 1983

Taylor, T. C., "A Salvo Exchange Model for Modern Tactical Conditions," unpublished article, 16546 Chalet Terrace, Pacific Palisades, CA, 90272, Revision 5, July 1990

Thomas, C. J., Military Operations Research Course: Selected Topics of Military Operations, WORC-ORSA, 1966

Thomas, C. J., "Verification Revisited," in Military Modeling, MORS, Alexandria, VA, 2nd Edition, 1989

U. S. Army DARCOM, Engineering Design Handbook, DARCOM-P 706-101, Alexandria, VA, 1977

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