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Journal of Spacecraft and Rockets, Vol. 38, No. 4, September - October 2001
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JOURNAL OF SPACECRAFT AND ROCKETS
Vol. 38, No. 5, September–October 2001

Gravitational Effects of Earth in Optimizing ΔV for Deflecting Earth-Crossing Asteroids

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Analyses incorporating the gravitational effects of Earth to calculate optimal impulses for deflecting Earth-crossing asteroids are presented. The patched conic method is used to formulate the constrained optimization problem. Geocentric constraints are mapped to heliocentric variables by the use of the impact parameter. The result is a unified nonlinear programming problem in the sense that no distinctions are made for short or long warning times. Numerical solutions indicate that the ΔV requirements are considerably more than those of the previously published two-body analysis that excluded third-body effects. Generally speaking, the increments in the minimum ΔV due to the gravitational effects of the Earth are large (by as much as 60%) for near-Earth asteroids, and the errors diminish for orbits with large eccentricities ($e > 0.7$). Some interesting results for short warning times are also discussed.

Nomenclature

F, G	=	Lagrange coefficients for position
F_t, G_t	=	Lagrange coefficients for velocity
J	=	objective function for nonlinear programming problem
$\boldsymbol{R}$, R	=	radius vector from Earth to Earth-crossing asteroid (ECA), its magnitude
R_b	=	impact radius and its generalization
R_{critical}	=	proposed miss distance between Earth and ECA
R_{SOI}	=	radius of Earth's sphere of influence
$R_\oplus$	=	radius of Earth
$\boldsymbol{r}_a^0$	=	initial heliocentric position vector of ECA
t_b	=	time when $R(t) = R_b$
t_f	=	time of closest approach between Earth and asteroid after application of $\Delta\boldsymbol{V}$
$t_{\Delta V}$	=	time of application of $\Delta\boldsymbol{V}$
t_1	=	time when an ECA impacts Earth
$\boldsymbol{V}_a^0$	=	initial inertial velocity of ECA
$\boldsymbol{V}_\infty$	=	hyperbolic excess velocity
$\boldsymbol{V}_\oplus$, $\boldsymbol{V}_a$	=	inertial velocities of Earth and asteroid, respectively
$\Delta\boldsymbol{V}$	=	impulsive velocity increment imparted to an ECA
$\|\Delta\boldsymbol{V}\|$	=	magnitude of $\Delta\boldsymbol{V}$
$\mu_\oplus$	=	gravitational constant of Earth

Introduction

FOLLOWING the 1980 report of Alvarez et al.,[1] NASA and the Jet Propulsion Laboratory (JPL) cosponsored a 1981 workshop[2] with the goal of determining the probability and risk of impacts between near-Earth objects and Earth. Since then, there have been many workshops to study the fundamentals of the impact and impact mitigation problem.[3,4] Much of the concentration of the workshops have been related to the systems analysis of the impact problem, assessing the magnitude of the threat, impact effects and hazards to Earth, as well as the political implications of developing an impact mitigation capability. The survey in Ref. 5 shows that only a limited analysis has been performed on the astrodynamics of the impact mitigation problem. Much of the previous analyses[5–8] has been limited to analytical results obtained by approximations. One clear conclusion from these simplified analyses is that early detection gives longer reaction time, and asteroid interceptions far from Earth are much more desirable and easier than interceptions near Earth. This simply follows from the fact that small deflections far away will produce a greater miss distance at Earth. Recent numerical analyses show that there is much to be gained in analyzing the optimization problem.[9,10] This is primarily because of the coupling between warning times, available energy for deflection, and the sensitivity of the optimization problem.

A number of dynamics and control problems in hazard mitigation are defined in Ref. 5. One of these problems included an outline of an optimization problem as a nonlinear programming problem (NLP). This NLP formulation was furthered in Ref. 9, yielding some interesting results. For example, a slightly off-parallel impulse at perihelion will yield a higher miss distance than a parallel one. In this paper, we extend our previous heliocentric two-body analysis[9] to include the gravitational effects of Earth. This is achieved by simply mapping the geocentric constraints to heliocentric variables. The result is an NLP formed as an outgrowth of a patched-conic approximation. To distinguish this problem formulation from our previous two-body formulation,[9] we occasionally refer to it as the three-body model.

Our analysis centers on how optimal impulses applied to an asteroid at various points on the asteroid's orbit affect the outcome when there is a presumption of collision otherwise. The analysis tool presented may be utilized in determining a reasonably accurate estimate for optimizing the time and position of intercepting the asteroid for impact mitigation. Because our problem formulation does not really distinguish between short or long warning times, it represents a more unified approach to the deflection problem. We do, however, distinguish the results for short and long warning times because the differences between them are quite substantial.

Formulation of the Optimization Problems

General Problem

Given an Earth-crossing asteroid (ECA) with an orbit that confirms an impending collision with Earth, the problem is to find the minimum ΔV to prevent a collision. Mathematically, the problem is posed as follows (see Fig. 1): minimize

$$J = \|\Delta\boldsymbol{V}\| \tag{1}$$

such that

$$\min_{t \geq t_{\Delta V}} R(\Delta\boldsymbol{V}, t) \geq R_{\text{critical}} \tag{2}$$

In Eq. (2), R is written as $R(\Delta\boldsymbol{V}, t)$ to emphasize its fundamental functional relationship as it pertains to the problem at hand. Thus, the requirement of an impending collision simply means that $R(\boldsymbol{0}, t_1) < R_{\text{critical}}$ for some t_1. In much of the literature, R_{critical} is the radius of Earth (~6378 km). For notational convenience and other

Received 21 January 2000; revision received 23 April 2001; accepted for publication 2 May 2001. This material is declared a work of the U.S. Government and is not subject to copyright protection in the United States.

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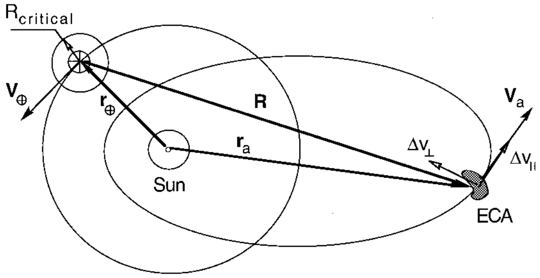

Fig. 1 Geometry of ECA (not to scale).

reasons, we simply prefer to keep it generic. The constraint [given by Eq. (2)] is itself a global minimization problem and, thus, represents a reformulation of another minimax problem, for example, maximizing the minimum of R. To convert the problem to a standard constrained minimization problem, we solve the minimization subproblem posed by Eq. (2) by noting that the minimum of R is required to be equal to R_{critical} and that R is continuous and differentiable for $t > t_{\Delta V}$. In other words,

$$R(\Delta \boldsymbol{V}, t_f) - R_{\text{critical}} = 0 \tag{3}$$

$$\dot{R}(\Delta \boldsymbol{V}, t_f) = 0 \tag{4}$$

$$\ddot{R}(\Delta \boldsymbol{V}, t_f) \geq 0 \tag{5}$$

is the solution to the subproblem of Eq. (2) for the minimum $\Delta \boldsymbol{V}$. In these equations, t_f denotes the time when these constraints are satisfied and is implicitly given by the equations themselves. Although Eqs. (3–5) are formal statements of a local minimum, we achieve global minimum by searching for the local minimum near the point (time, t_1) where $R(\boldsymbol{0}, t_1) = 0$. (A direct head-on collision was assumed in the numerical analysis.) Thus, the optimization problem is now reduced to an NLP of finding $[\Delta \boldsymbol{V}, t_f]$ that minimizes $J(\Delta \boldsymbol{V})$ subject to the constraints given by Eqs. (3–5). Note that our problem formulation is reasonably universal in that no approximations or assumptions on the gravity field have been made (yet). Obviously, these approximations come from a realization (propagation) of R and its time derivatives $\dot{R}$ and $\ddot{R}$.

A version of the preceding general formulation of the problem in given in Ref. 5 in addition to many other interesting problems associated with the mitigation problem. In the heliocentric two-body approximation,[9] Eqs. (3–5) are obtained by ignoring the gravitational effects of Earth. It is obvious that these ΔV underestimate the true value. To incorporate the effect of Earth and maintain the same problem structure outlined, three-body propagation techniques are required for R and its derivative. Perhaps a simpler method is to think of R_{critical} in terms of a dynamic variable $R_b(t)$, which has a property that if

$$R(\Delta \boldsymbol{V}, t_b) \leq R_b(t_b) \tag{6}$$

for some $t_b < t_f$, then,

$$\dot{R}(\Delta \boldsymbol{V}, t) < 0, \qquad \forall t \in [t_b, t_f] \tag{7}$$

$$\dot{R}(\Delta \boldsymbol{V}, t_f) = 0, \qquad R(\Delta \boldsymbol{V}, t_f) \leq R_{\text{critical}} \tag{8}$$

That is, $R_b(t)$ may be viewed as a dynamic impact radius. If such a variable can be found, then Eq. (2) can be replaced by

$$\min_{t_{\Delta V} \leq t \leq t_b} \tilde{R}(\Delta \boldsymbol{V}, t) \geq 0 \tag{9}$$

where

$$\tilde{R} \equiv R(\Delta \boldsymbol{V}, t) - R_b(t) \tag{10}$$

and Eqs. (3–5) are modified to

$$\tilde{R}(\Delta \boldsymbol{V}, t_b) = 0 \tag{11}$$

$$\dot{\tilde{R}}(\Delta \boldsymbol{V}, t_b) = 0 \tag{12}$$

$$\ddot{\tilde{R}}(\Delta \boldsymbol{V}, t_b) \geq 0 \tag{13}$$

Because $\dot{\boldsymbol{R}} + \boldsymbol{V}_\oplus = \boldsymbol{V}_a$, we can write

$$\dot{R}(\Delta \boldsymbol{V}, t) = [\boldsymbol{V}_a(\Delta \boldsymbol{V}, t) - \boldsymbol{V}_\oplus(t)] \cdot (\boldsymbol{R}/R) \tag{14}$$

from which it is apparent that B-plane parameters[11] provide an excellent choice for approximating $R_b(t)$. We propose two slightly different approaches for this approximation.

Patched-Conic Formulation

In the patched-conic approximation, we choose the impact radius $R_b(t)$ to be equal to the time-varying impact parameter:

$$R_b(t) = R_{\text{critical}} \sqrt{1 + \frac{2\mu_\oplus}{R_\oplus V_\infty^2(t)}} \tag{15}$$

where $\mu_\oplus$ and $R_\oplus$ are the gravitational constant and the radius of Earth, respectively, and $V_\infty(t)$ is the magnitude of $\boldsymbol{V}_\infty(t;\, \Delta \boldsymbol{V})$ for a given $\Delta \boldsymbol{V}$, defined by

$$\boldsymbol{V}_\infty(t;\, \Delta \boldsymbol{V}) = \boldsymbol{V}_a(\Delta \boldsymbol{V}, t) - \boldsymbol{V}_\oplus(t) \tag{16}$$

When $V_\infty(t)$ given by Eq. (16) is computed at the edge of the Earth's sphere of influence (SOI), it conforms to the strict definition of the patched-conic approximation. Deferring a discussion of this for the moment, we use a further simplification typically used in interplanetary orbital transfers: the method of matched asymptotes or zero-SOI patched-conic approximation. We compute the hyperbolic excess velocity of the asteroid from Eq. (16) at the point of (heliocentric) intersection between the Keplerian orbits of the asteroid and Earth. Also, at this same point, the impact radius is equal to the impact parameter typically used in patched-conic approximations. From the notion of the impact parameter, it is apparent that if at some point t_b, $R(\Delta \boldsymbol{V}, t_b) \leq R_b(t_b)$, then the conditions given by Eqs. (7) and (8) will hold. Hence, from Eq. (14), we can write

$$\dot{R}(\Delta \boldsymbol{V}, t_b) = \boldsymbol{V}_\infty \cdot (-\boldsymbol{V}_\infty / V_\infty) = -V_\infty(t_b) < 0 \tag{17}$$

for the hyperbolic section of the trajectory relative to Earth. Because $V_\infty(t_b)$ is known from the geometry of the ECA's orbit, Eqs. (17) and (14) provide a patched-conic approach for computing the left-hand side of Eq. (12).

The global optimization problem may now be formalized using Lagrange coefficients (see Ref. 12) as follows. We begin with a presumption of an asteroid on a head-on collision course with Earth. This implies that we have established a time t_1 (or alternatively, a time interval $t_1 - t_0$) such that

$$\boldsymbol{R}(\boldsymbol{0}, t_1) \equiv \boldsymbol{r}_a^0 F(t_1) + \boldsymbol{V}_a^0 G(t_1) - \boldsymbol{r}_\oplus(t_1) = \boldsymbol{0} \tag{18}$$

where $\boldsymbol{r}_a^0$ and $\boldsymbol{V}_a^0$ denote initial conditions for the heliocentric radius and velocity vectors for the asteroid (see Fig. 1) and $\boldsymbol{r}_\oplus(t_1)$ is the heliocentric radius vector of the Earth's orbit. The quantities F and G are the usual Lagrange coefficients (see Ref. 12) that denote the components of $\boldsymbol{r}_a$ in $(\boldsymbol{r}_a^0, \boldsymbol{V}_a^0)$ basis. The NLP reduces to finding $\Delta \boldsymbol{V}$ and a t_b nearest t_1 that minimizes $J(\Delta \boldsymbol{V})$ [see Eq.(1)] subject to the constraint given by Eqs. (11) and (12) [Eq. (13) is only used to validate the result]. Although Eq. (12) may be used to constrain t_b, a somewhat simpler choice for the constraint is to use the approximation that all of the parameters of interest occur at the intersection points of the two orbits, that is,

$$\boldsymbol{r}_a^0 F(t_b) + \left(\boldsymbol{V}_a^0 + \Delta \boldsymbol{V}\right) G(t_b) = \boldsymbol{r}_\oplus \tag{19a}$$

$$\left\| \boldsymbol{r}_a^0 F(t_b) + \left(\boldsymbol{V}_a^0 + \Delta \boldsymbol{V}\right) G(t_b) \right\| = \|\boldsymbol{r}_\oplus\| = r_\oplus \tag{19b}$$

We prefer Eq. (19b) to Eq. (19a) because it is a scalar, and we choose the time t_b that satisfies Eq. (19b) and is nearest to t_1. For a more accurate solution, Eq. (12) may be used along with Eqs. (14) and (17). To summarize, the NLP is defined as the problem of finding $\Delta \boldsymbol{V}$ and a t_b nearest the point t_1 [given by Eq. (18)] that minimizes

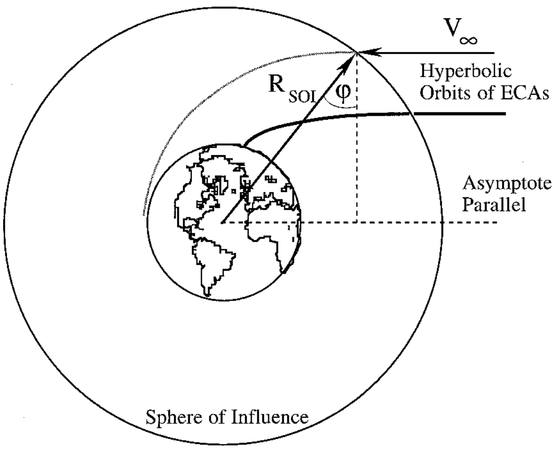

Fig. 2 Definition of angle φ at SOI (not to scale).

$J = \|\Delta V\|$ subject to the constraints given by Eqs. (11) and (12) [or, alternatively, Eq. (19)]. In Eq. (11), $\|\boldsymbol{R}(\Delta \boldsymbol{V}, t_b)\|$ is computed from

$$\boldsymbol{R}(\Delta \boldsymbol{V}, t_b) \equiv \boldsymbol{r}_a^0 F(t_b) + \left(\boldsymbol{V}_a^0 + \Delta \boldsymbol{V}\right) G(t_b) - \boldsymbol{r}_\oplus(t_b) \tag{20}$$

and $R_b(t_b)$ is determined from Eqs. (15) and (16), where $\boldsymbol{V}_a(\Delta \boldsymbol{V}, t)$ is determined from

$$\boldsymbol{V}_a(\Delta \boldsymbol{V}, t_b) = \boldsymbol{r}_a^0 F_t(t_b) + \left(\boldsymbol{V}_a^0 + \Delta \boldsymbol{V}\right) G_t(t_b) \tag{21}$$

where F_t and G_t are the well-known Lagrange coefficients (see Ref. 12) that correspond to the components of velocity at time t_b in $(\boldsymbol{r}_a^0, \boldsymbol{V}_a^0)$ basis.

SOI-Based Patched-Conic Formulation

An alternative to the preceding method is to continue to use a static critical radius and to patch the conics at Earth's SOI. In principle, this is a more accurate method. In this method,[5] the problem is subject to the heliocentric Keplerian equations outside the Earth's SOI and geocentric two-body equations inside the SOI. Figure 2 shows the geometry of this approach for the two-dimensional case. Under this framework, the constraints for the optimization problem can be described in terms of the terminal boundary conditions at the time t_{SOI} when the asteroid intersects the Earth's SOI. These constraints are given by

$$R - R_{\text{SOI}} = 0 \tag{22}$$

$$R_{\text{SOI}} \cos\varphi - R_b(t_{\text{SOI}}) = 0 \tag{23}$$

$$\dot{R} < 0 \tag{24}$$

where φ is given by (see Fig. 2)

$$\cos(\varphi + 90\,\text{deg}) = \frac{V_\infty R_{\text{SOI}}}{\boldsymbol{V}_\infty \cdot \boldsymbol{R}_{\text{SOI}}} \tag{25}$$

Here, V_∞ is determined from Eq. (16), but at the point t_{SOI}. $\dot{R}$ is computed from the usual two-body dynamic equations.[9] Because the radius of the Earth's SOI is about 9.0×10^5 km, or, equivalently, about 0.0060 astronomical units (AU), it is apparent that this patched conic method (patching the conic at $R = 0.0060$) should produce nearly the same result as the one described earlier (patching the conic at $R = 0$).

Numerical Analysis

Because of the enormous number of possibilities between the various parameters, we limit the scope of the numerical analysis to coplanar orbits. As expected, the two NLPs posed in the preceding section yield nearly the same results. The MATLAB® optimization toolbox[13] was used to solve the NLPs. For well-documented reasons and numerical scaling, heliocentric canonical units were used. In this system, the distance units (DU), time units (TU), and speed units (SU) are 1 DU = 1 AU = 1.49596E8 km, 1 TU = $1/2\pi$ year = 58.17 days, and 1 SU = 29.80 km/s. In the discussions to follow, we will use the term impulse time to mean specifically either $(t_1 - t_{\Delta V})$ or its absolute value; the exact usage will be obvious from the context. Without loss in generality, we choose $t_1 = 0$. This initialization has the advantage of interpreting $t_{\Delta V}$ as the time interval before impact if no action, that is, ΔV maneuver, is undertaken. Also, because the warning time (i.e., the time interval between detection and collision) must be greater than the impulse time, the latter provides a necessary lower bound for the former.

Note the sensitivity of the optimization algorithm to initial guesses in the numerical solution of the optimization problem. The sensitivity reduction procedure described in Ref. 9 for the two-body problem works with little modification for the current problem formulation as well. To conform to some of the analysis established in the literature, we distinguish the results between long and short warning times. In addition, we choose $R_{\text{critical}} = 1$ Earth radius (4.263E−5 DU).

Analysis for Long Warning Times

Long warning times loosely correspond to $t_{\Delta V}$ greater than one orbital period of the asteroid. Consider a fictitious asteroid whose orbital elements are given by $a = 1.5$ AU (period of about 1.85 years) and $e = 0.5$. In Figs. 3 and 4, the magnitudes and angles of the optimal impulses are compared with those from Ref. 9 that excluded the gravitational effects of Earth. The impulse time is normalized to the period of the unperturbed asteroid for ease of interpretation. Not surprisingly, the minimum required ΔV increases dramatically as t approaches 0. As expected, the minimum ΔV is larger when the gravitational effects of the Earth are included. However, what is noteworthy is that the three-body requirements (i.e., the ones obtained by the patched-conic approximations) are significantly higher than those obtained from two-body approximations, that is, ignoring the gravity of Earth. For example, at the second perihelion point (which is at $t = -2.0746$ asteroid's orbital periods), the two-body ΔV requirement is 0.8694 cm/s, whereas

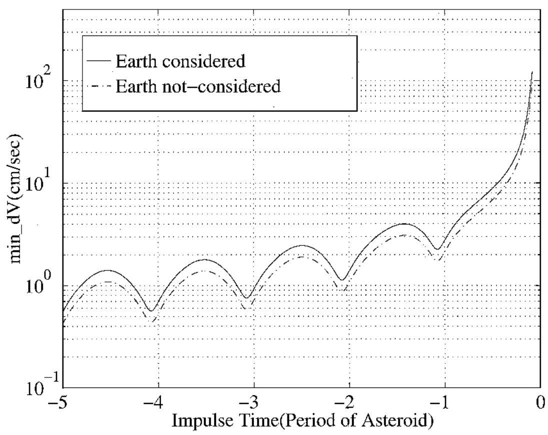

Fig. 3 Minimum ΔV vs impulse time.

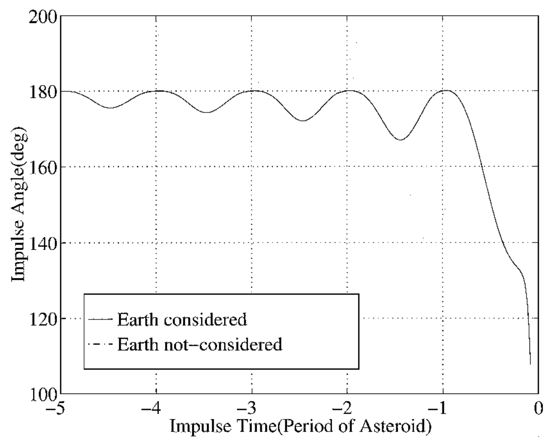

Fig. 4 Optimal impulse angle vs impulse time.

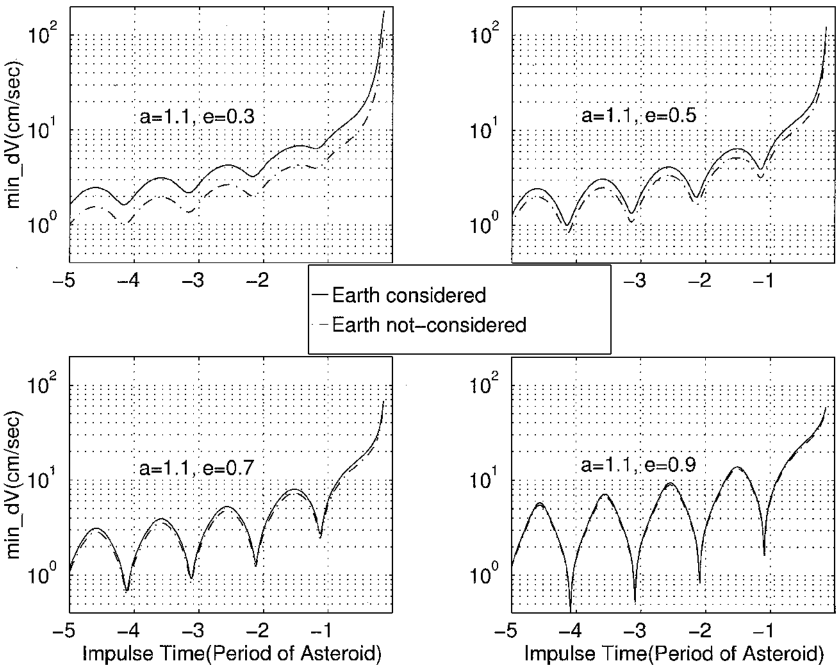

Fig. 5 Minimum ΔV with Earth's gravitational effects for various eccentricities.

the three-body requirement jumps up by 29.7% to 1.1275 cm/s. Figure 4 shows a corresponding plot of the optimal impulse angle with respect to impulse time. The impulse angle is defined as the angle from the ECA's original velocity vector to the impulse vector through the sunward direction. It is apparent from Fig. 4 that the optimal angles for both the two-body and three-body cases are identical. In other words, the effect of Earth's gravity simply increases the magnitude of the minimum ΔV (leaving the angle unchanged). This can be attributed to that, because the ΔV requirements are quite small, it has a linear relationship to the miss distance. Because Earth's gravity has the effect of essentially enlarging the miss distance, the optimal impulse angle should really be unchanged. Thus, Fig. 4 confirms this observation and, in some way, validates our methods.

The effects of Earth's gravity can have quite a dramatic effect on near-Earth asteroids on a collision course with Earth. Figure 5 shows the histories of the minimum ΔV for various fictitious asteroids whose orbital elements are given by $a = 1.1$ AU, $e = 0.3$, 0.5, 0.7, and 0.9. The plots clearly show that the two-body assumptions do not hold for near-Earth ECAs, particularly for those that have lower eccentricities. For example, the minimum ΔV obtained by adding the effects of the third body (Earth) is 59.6% larger than the two-body solution at the second perihelion for the case of $a = 1.1$ AU and $e = 0.3$ (first subfigure in Fig. 5). The large differences are a direct consequence of the increasing value of the impact parameter for slower and, hence, nearer-Earth asteroids.

To confirm the results from the optimization algorithm, a simple two-body propagation code was developed for geocentric hyperbolic trajectories. The idea is to use the converged results from the optimization code as initial conditions for propagating the position of the asteroid inside the SOI. Thus, $\boldsymbol{R}_{\mathrm{SOI}}$, $\boldsymbol{V}_{\infty}$, and t_{SOI} form the initial conditions for the propagation. The propagation was performed by the method of Lagrangian coefficients for hyperbolic orbits (see Ref. 12). The results of this propagation for the case corresponding to Fig. 1, that is, $a = 1.5$ and $e = 0.5$, are displayed in Figs. 6 and 7. The unperturbed trajectory is on a rectilinear collision course with Earth. The asteroid perturbed by two-body assumptions still collides with Earth, but with a reduced impact angle. The trajectory obtained from the patched-conic concept (or three-body approximation) provides sufficient ΔV for the asteroid to miss Earth (by the specified amount of one Earth radius).

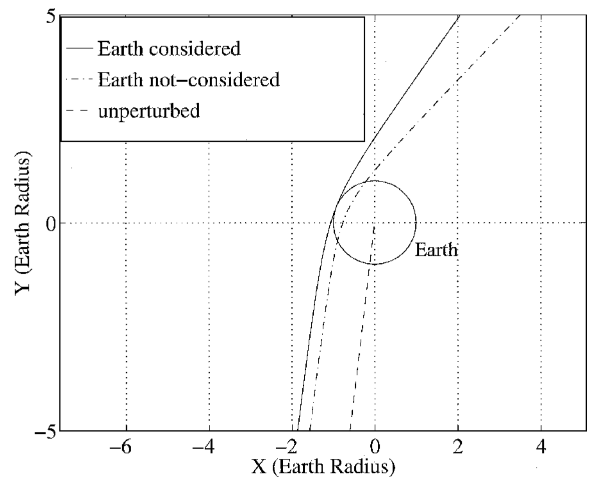

Fig. 6 Hyperbolic trajectories with respect to Earth.

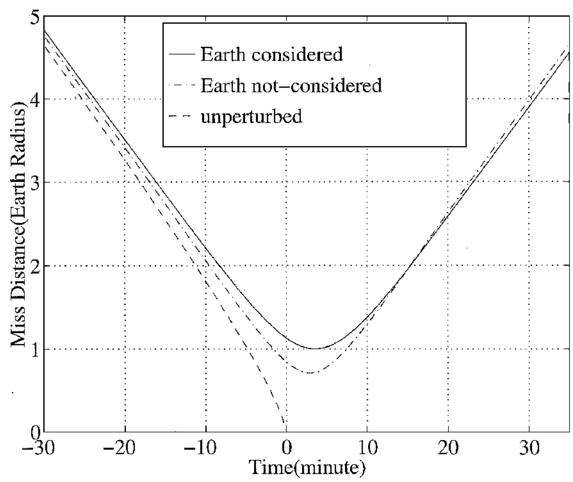

Fig. 7 Miss distances from Earth.

Analysis for Short Warning Times

Short warning times loosely correspond to $t_{\Delta V}$ less than one orbital period of the asteroid. In previous studies[6,14,15] the Earth's gravitational effects were ignored, and a rectilinear motion was assumed. The following simple computation underscores the effect of this assumption. If a ΔV is imparted to an asteroid for missing Earth by R_{critical}, then the trajectory is bent toward Earth by the perigee of a hyperbolic orbit,

$$r_p = -(\mu_\oplus/V_\infty) + \sqrt{\left(\mu_\oplus/V_\infty^2\right) + R_{\text{critical}}^2} \tag{26}$$

Using R_{critical} of one Earth-radius and a typical impact velocity of 20 km/s, we get $r_p = 0.856$ of Earth radius. Thus, preliminary approximations of energy required to deflect a threat may result in an error of 14% in miss distance.

Consider once again the fictitious asteroids characterized by a semimajor axis of $a = 1.1$ (orbital period ≈ 1.15 years), and eccentricities given by $e = 0.3$, 0.5, 0.7, and 0.9. Figures 8 and 9 display the optimal impulse angles and minimum ΔV required for impulse times of up to about 15 days before impact. It is apparent from Figs. 8 and 9 that the larger the orbit eccentricity, the greater the fluctuations in both the minimum ΔV and the optimal impulse angle. What is more interesting is that the optimal impulse angles are not perpendicular to the velocity but are instead about 40 deg for the specific examples considered here. Note, however, that the optimal angles are measured from the ECA's original heliocentric velocity and not from the ECA's velocity with respect to the Earth. Another interesting feature of the results is that the optimal ΔV appear to target a specific impact parameter. For a given orbit of the asteroid, the optimal trajectories yielded nearly the same position vector, velocity vector, and impact parameter for all impulse times (of less than about 15 days). Table 1 shows the converged values for the cases considered. To confirm these results, the same propagation technique adopted for the long warning times were used. Figure 10 shows the propagated hyperbolic trajectories for the position and velocity vectors given in Table 1. The asteroid misses the Earth by the specified amount of one Earth radius in all of the cases, thus validating (to some degree) the optimality of the results.

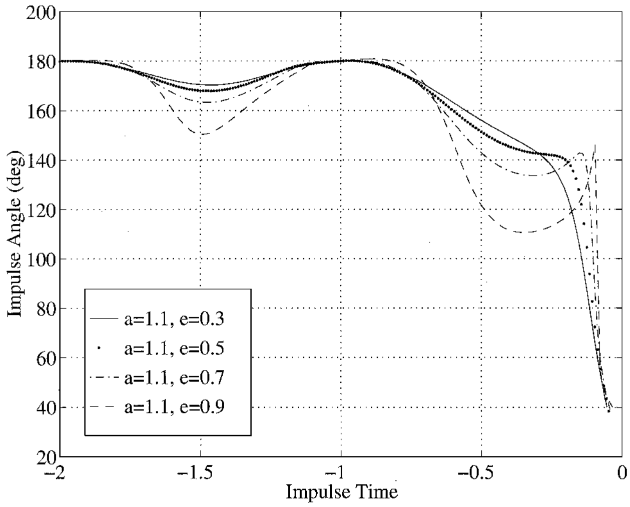

Fig. 8 Histories of optimal impulse angles for short warning times.

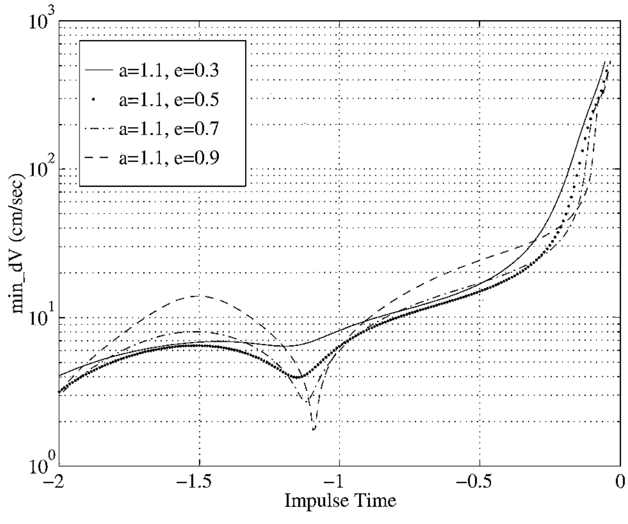

Fig. 9 Histories of minimum ΔV for short warning times.

Table 1 Position and velocity of asteroids (a = 1.1) at Earth's SOI

Parameter	$e = 0.3$	$e = 0.5$	$e = 0.7$	$e = 0.9$
Impact parameter b_i	1.601 $R_\oplus$	1.230 $R_\oplus$	1.112 $R_\oplus$	1.058 $R_\oplus$
X, km	−11,700	175,000	308,200	479,300
Y, km	−928,900	−912,300	−876,300	−795,700
V_X, km/s	0.01	−2.82	−7.78	−16.87
V_Y, km/s	8.94	15.35	21.59	27.56

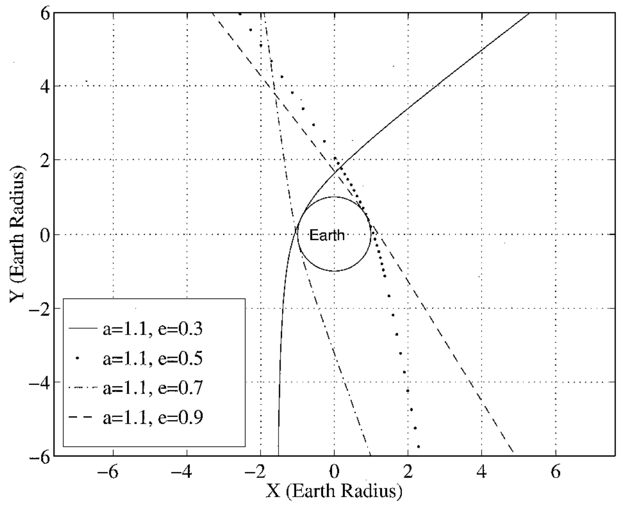

Fig. 10 Trajectories of asteroids at near Earth for short warning times.

Conclusions

The minimum ΔV required to deflect an ECA is substantially greater than previous estimates established elsewhere, particularly for nearer-Earth asteroids, that is, those asteroids whose semimajor axes are near 1 AU. This is a direct consequence of the impact radius having an inverse relationship to the hyperbolic excess velocity: The nearer and slower asteroids are more influenced by the Earth's gravity than the farther and faster ones. The increase in the ΔV due to the gravitational effects of Earth may be quickly estimated from a rule of thumb: The ΔV obtained from the Keplerian two-body model underestimates the true ΔV by approximately the impact parameter normalized by the miss distance $\{= \sqrt{[1 + 2\mu_\oplus/R_\oplus V_\infty^2(t)]}\}$.

Acknowledgments

Funding for this research was provided, in part, by the Air Force Space Command and the Naval Postgraduate School (NPS). We would like to thank Brian Neuenfeldt for sponsoring this research and Paul Penzo of the Jet Propulsion Laboratory, California Institute of Technology, for suggesting the use of the impact parameter to modify our original analysis. This work was performed while the second author was a National Research Council Postdoctoral Associate at NPS. A version of this paper (AAS 98-186) was presented at the AAS/AIAA Spaceflight Mechanics Meeting, Monterey, CA, 9–11 February 1998.

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Associate Editor