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An investigation of the relation of accelerations of surface lows to momentum changes at the 500 millibar level

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AN INVESTIGATION OF THE RELATION OF ACCELERATIONS

OF SURFACE LOWS TO MOMENTUM CHANGES

AT THE 500 MILLIBAR LEVEL

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AT THE 500 MILLIBAR LEVEL

by

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PREFACE

This work was conducted in the winter and spring of 1947-48 at the United States Naval Postgraduate School, Annapolis, Maryland. It was done to meet partial requirements for the degree of Master of Science in Aerology. The idea of the investigation was suggested by Dr. F. L. Martin of the staff of the Department of Aerological Engineering of the United States Naval Postgraduate School. His assistance and guidance in the preparation of this thesis is gratefully acknowledged.

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SYMBOLS AND ABBREVIATIONS

C	speed of upper air trough
U	mean west wind component
β	rate of change of Coriolis parometer with latitude
L	wave length of upper air trough
ρ	density
v	velocity
p	pressure
T	temperature in $^\circ K$.
R_a	gas constant per unit gram of air
A_E	eastward acceleration of surface low
X_{1E}	eastward component of momentum change in south section (no lag)
X_{2E}	eastward component of momentum change in east section (no lag)
X_{3E}	eastward component of momentum change in north section (no lag)
X_{4E}	eastward component of momentum change in west section (no lag)
X_{5E}	eastward component of momentum change in center section (no lag)
X_{6E}	eastward component of momentum change total all sections (no lag)
X_{7E}	eastward component of momentum change north + west + south + east sections (no lag)
X_{8E}	eastward component of momentum change north + east sections (no lag)
X_{9E}	eastward component of momentum change west + south sections (no lag)
X_{10E}	eastward component of momentum change north + south sections (no lag)

X_{11E} eastward component of momentum change west + east sections (no lag)

A_N northward acceleration of surface low

X_{1N} northward component of momentum change south section (no lag)

X_{2N} northward component of momentum change east section (no lag)

X_{3N} northward component of momentum change north section (no lag)

X_{4N} northward component of momentum change west section (no lag)

X_{5N} northward component of momentum change center section (no lag)

X_{6N} northward component of momentum change total all sections (no lag)

X_{7N} northward component of momentum change north + west + south + east sections (no lag)

X_{8N} northward component of momentum change north + east sections (no lag)

X_{9N} northward component of momentum change west + south sections (no lag)

X_{10N} northward component of momentum change north + south sections (no lag)

X_{11N} northward component of momentum change west + east sections (no lag)

Y_{1E} eastward component of momentum change south section (used for lag)

Y_{2E} eastward component of momentum change east section (used for lag)

Y_{3E} eastward component of momentum change north section (used for lag)

Y_{4E} eastward component of momentum change west section
(used for lag)

Y_{5E} eastward component of momentum change center section
(used for lag)

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Y_{7E} eastward component of momentum change north + west + south
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Y_{10N} northward component of momentum change north + south sections (used for lag)

Y_{11N} northward component of momentum change west + east sections (used for lag)

$\bar{x}$ arithmetic mean

σ standard deviation

x   any variate

N   number of variates considered

R   a determinant

S   standard error of estimate

$\sim$ correlation coefficient

$\sim(A_N \cdot X_{9N} Y_{8N})$ multiple correlation of northward acceleration of surface low on northward momentum changes of west + south sections (no lag) and north + east sections (lagged 1 day)

$\sim(A_N \cdot Y_{3N} Y_{8N})$ multiple correlation of northward acceleration of surface low on northward momentum changes of north section (lagged 1 day) and north + east section (lagged 1 day)

$\sim(A_N \cdot X_{9N} X_{5N})$ multiple correlation of northward acceleration of surface low on northward momentum changes of west + south sections (no lag) and center section (no lag)

$\sim(A_N \cdot X_{5N} Y_{3N})$ multiple correlation of northward acceleration of surface low on northward momentum changes of center section (no lag) and north section (lagged 1 day)

$\sim(A_N \cdot X_{9N} Y_{3N})$ multiple correlation of northward acceleration of surface low on northward momentum changes of west + south sections (no lag) and north section (lagged 1 day)

$\sim(A_E \cdot X_{2E} Y_{11E})$ multiple correlation of eastward acceleration of surface low on eastward momentum changes of east section (no lag) and west ⟵ east sections (lagged 1 day)

$\sim(A_E \cdot X_{5E} Y_{11E})$ multiple correlation of eastward acceleration of surface low on eastward momentum changes of center section (no lag) and west ⟵ east sections (lagged 1 day)

$\sim(A_E \cdot Y_{11E} Y_{2E})$ multiple correlation of eastward acceleration of surface low on eastward momentum changes of west ⟵ east sections (lagged 1 day) and east section (lagged 1 day)

$\sim(A_E \cdot X_{2E} Y_{2E})$ multiple correlation of eastward acceleration of surface low on eastward momentum changes of east section (no lag) and east section (lagged 1 day)

$\sim(A_E \cdot X_{6E} Y_{2E})$ multiple correlation of eastward acceleration of surface low on eastward momentum changes of center section (no lag) and east section . (lagged 1 day)

$\sim(A_E \cdot X_{5E} X_{2E})$ multiple correlation of eastward acceleration of surface low on eastward momentum changes of center section (no lag) and east section (no lag)

Lat.   Latitude

$\sum$ Summation symbol

CHAPTER I

INTRODUCTION

The problem of the movement of surface lows and the forecasting
of such movement has long been a paramount problem to the forecasting
meteorologist. To date no infallible technique has been developed to
accomplish this aim. No successful theoretical or statistical formula
has been set forth so that this complex physical process may be objec-
tively predicted. Success in most instances is the result of long
experience and intelligent and thorough analysis rather than from mas-
tery of "weather" by the tools of mathematics and physics.

The complexities of the processes involved in measuring meteor-
ological phenomena are well known. However investigations continue
in an attempt to solve the problem, or to shed whatever light possible.
The purpose of this paper is to investigate the relationship of momen-
tum changes at the 500 millibar level to the accelerations and decel-
erations of surface low pressure centers. The data for this investi-
gation has been made possible by the publication recently of Northern
Hemisphere synoptic 500 millibar and sea level charts (5).

The theory of the general circulation of the earth's atmosphere
as set forth by Rossby (3) states that surface lows are tertiary
phenomena imbedded in the west to east flow of the zonal westerlies.
Such cyclones are the ones considered in this paper. Rossby points
out that the westerlies in the middle latitudes form part of a reverse
meridional circulation cell which is driven by the direct cells to the
north and south. If the surface of the earth were homogeneous and the
energy received from the sun constant, the circulation of the earth's

atmosphere would be simple and we would have no turbulent eddies in the zonal westerlies. However the earth's surface is not homogeneous and the incoming solar radiation is not constant. These two factors, and possibly others (for example, friction) cause a break-down in the stability of the general circulation, and as a result the pattern of migratory cyclones, anticyclones, troughs, and wedges observed on the daily weather chart develops. As Haurwitz states (4) these turbulent eddies in the westerlies cause a meridional transport of heat and moisture. They also cause precipitation and outbreaks of cold air and so are of prime importance.

These outbreaks of cold and warm air at the upper levels may be seen in relation to the type of circulation taking place. At times the wind blows strongly from west to east with little north-south component. At such times outbreaks of cold and warm air are diminished. At other times there are pronounced wedges and troughs on the upper air chart. Then the north-south components of the flow are strong and the outbreaks of cold and warm air is pronounced.

The forecasting of the movement of these troughs and wedges is necessarily important to the forecaster. A formula for this purpose has been developed by Rossby (2).

It is

(1) $$C = U - \frac{\beta L^2}{4\pi}$$

where   C = speed of trough

        U = mean west wind component

2

β = rate of change of Cariolis parometer with latitude

L = wave length of trough

This formula has been used with some success (1) by the U. S. Weather Bureau long range unit.

A period of strong west-east flow is known as high index. A relatively weak west-east flow is known as low index, in such a case it is generally considered that the north-south flow will be relatively strong. However this definition is not comprehensive enough to adequately describe all flow patterns. For example let us take a situation with weak west-east flow and little or no north-south component. This is a low index situation but very different from the normal concept of low index. Another case is that in which there is very strong northwest-southeast and southwest-northeast flow. This by index measurements is high index, but it has a large north-south component.

A new, more complete idea of index has been put forth by Willett (6). He not only considers east-west (zonal) but also north-south (meridional) index (6). With the use of these two measures of index the flow pattern can better be objectively described. This investigation uses the concepts of both zonal and meridional index, and attempts to compare the daily changes of each with the daily accelerations of surface lows.

Actually the day to day changes in zonal index are generally used subjectively as a guide to accelerations and or decelerations of pressure systems in view of (1). However since zonal index is calculated from the belt 35-55° N. latitude all around the globe, it could be, and very often is, a relatively coarse index when applied to any individual migratory

low. Thus the basic idea of this thesis is to develop more sensitive indices than either zonal or meridional, but analogous to these. To this end it was decided to examine the 24-hour changes of specific momentum at the 500 millibar level immediately above the sea level low and at systematically determined points nearby in the 500 millibar windstream. The relationship of these points to the sea level low will be shown in Figure 1.

Certain similarities in concept may be noted between this approach and that of "steering". Numerous investigators including Austin (7), Longley (8), and Runk (9) have found varying degrees of correlation of the movement of the sea level low with the actual wind velocities at several upper levels including the 500 millibar level. However, it is to be empphasized that the present approach may be thought of as an attempted refinement of such investigations inasmuch as accelerations of the sea level low are to be correlated with changes in specific momentum. It is to be noted that both of the last two variables are essentially differences of variables discussed by the above mentioned authors.

THEORETICAL CONSIDERATIONS

On the 500 millibar chart we will consider this distribution of specific momentum in the vicinity of the sea level low. Specific momentum is defined as ρv where v is the vector velocity of the wind field at any point, and ρ is the density at this point. From the equation of state, we have

$$\rho = \frac{p}{R_a T}$$

where   p = pressure in millibars

T = temperature in $^{\circ}$Kelvin

Ra = gas constant per unit gram of air.

Since we are dealing with a constant pressure chart we find that

$$\rho v = K \frac{v}{T} \quad , \qquad K = \frac{p}{R_a}$$

the value of $K = \frac{p}{R_a}$ being a constant for the 500 millibar level. The value of the specific momentum is then effectively determined, apart from the constant K, by the simple relation

$$\rho v = \frac{v}{T}$$

Five values of the specific momentum on the 500 millibar chart are considered for every position of a sea level low as indicated in Figure 1. The five values are taken at south, east, north, west, and

center sections.

NORTH SECTION

SEA LEVEL LOW

CENTER SECTION

WEST SECTION

EAST SECTION

SOUTH SECTION

600 MILES
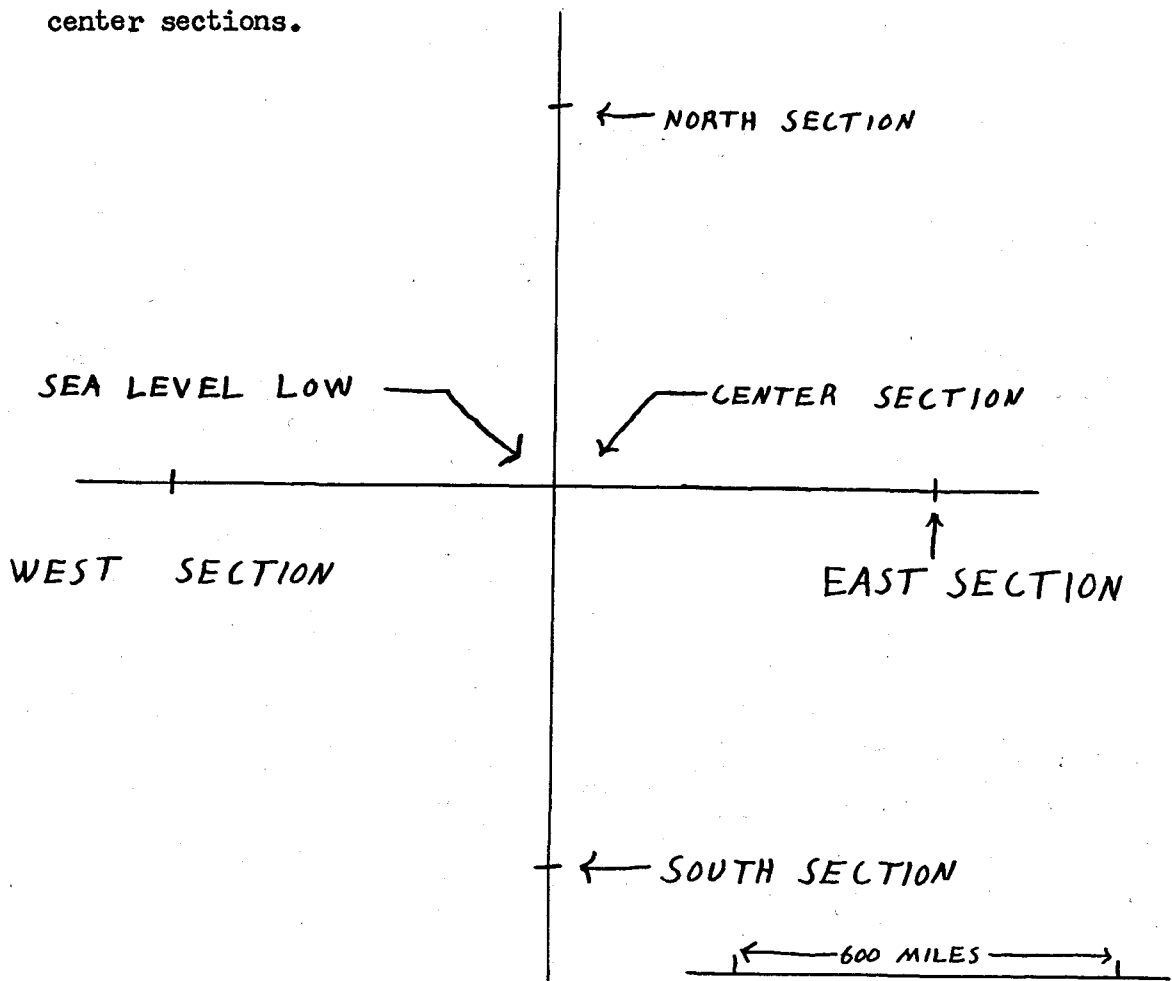
FIGURE 1

Diagram of the position of sections on the 500 millibar chart.

We will investigate the effect of changes of specific momentum in these sections taken singly and in various combinations upon the acceleration of the surface low pressure system.

The data was taken from the historical weather map series published by the Army Air Forces (5) using the 500 millibar and sea level charts for the same day. Unfortunately there is a time discrepancy of $8\frac{1}{2}$ hours between the map times of the two charts, the map time of the surface chart being 1230 GMT and that of the 500 millibar chart 0400 GMT.

Observations were taken only in the Northern Hemisphere, but otherwise there were no restrictions as to the location of the surface lows considered if 500 millibar data were available. Each individual low was followed until a central position could no longer be defined on the map. The observations were taken for the months of October and November 1945.

The unit of velocity used is degrees latitude per day ($^{\circ}$Lat./day) which is numerically equal to miles per hour multiplied by 0.338.

CHAPTER III

STATISTICAL RELATIONSHIPS DEDUCED

FROM THE DATA

The methods of collecting the data and computing the statistical variates will be described in some detail to lend clarity to the method of attacking the problem and the meaning of the results obtained.

In general lows that maintained their identity for 4-5 days or longer were chosen for observation. When a surface low was first observed on the chart, its position and the specific momentum values at the 5 sections (see Figure 1) were recorded. One day later the position of the low was again recorded and its northward and eastward components of motion noted. The 500 millibar specific momentum components were recorded section by section and for each section the 24 hour differences in these components were recorded. These differences in specific momentum components were also recorded for each subsequent day that the low was observed.

On the third day, two successive displacements of the surface center have been recorded. By taking the difference in these displacements (eastward and northward components) we obtain essentially the eastward and northward acceleration components of the surface low. Acceleration components are also computed on all subsequent days until the position of the low is no longer clearly defined.

It is readily seen that for the calculated acceleration components which exist on any particular day, there are available values of specific momentum changes both for that day and the day before. This enables us not only to compare concurrent specific momentum changes and acceler-

ations of the surface low, but also to compare the acceleration of the surface low to the specific momentum changes which occurred 1 day previously. The specific momentum changes concurrent with the acceleration of the surface low are referred to as "no lag" or "not used for lag". The specific momentum changes which precede the surface low accelerations are referred to as "used for lag" or "lagged 1 day".

The temperatures observed ranged from 268° K. to 228° K. or 40° K. Thus the values of specific momentum could differ by as much as 17.5% from values obtained if temperatures were not considered. Thus it is felt that the difference in temperature is a significant factor.

As stated in Chapter II all velocities are converted into $^\circ$Lat./ day which is a convenient measure of the velocities of the surface low. Since the magnitudes of the differences in specific momentum are of the order of 10^{-2} in the units of $\frac{v}{T}$ employed, their values are multiplied by 100 to make them of the same order of magnitude as the accelerations of the surface low.

Such data were recorded and computed for a number of low pressure systems appearing on the October and November 1945 weather maps. The arithmetic means and standard diviations were calculated from the standard formulas shown below and are tabulated in Tables II and III.

$$(2) \qquad \bar{X} = \frac{1}{N} \sum_{i=1}^{N} X_i \qquad\qquad \sigma = \frac{1}{N} \sum_{i=1}^{N} (X_i - \bar{X})^2$$

where $\bar{X}$ – arithmetic mean

N – number of variates

X – values of variate

$\sum\limits_{i=1}^{N}$ – summation of 1 to N

σ – standard deviation from the mean

TABLE II

ARITHMETIC MEANS AND STANDARD DEVIATIONS
(Not Used For Lag)

East Components			North Components		
Variable	Mean	Standard Deviation	Variable	Mean	Standard Deviation
A_E	-1.03	4.45	A_N	-0.02	5.20
X_{1E}	-0.124	3.18	X_{1N}	-0.42	4.36
X_{2E}	-0.25	2.93	X_{2N}	+1.38	4.23
X_{3E}	-1.14	3.62	X_{3N}	-2.30	4.19
X_{4E}	-0.12	3.32	X_{4N}	-0.14	4.25
X_{5E}	-0.70	3.82	X_{5N}	-0.25	4.06
X_{6E}	-2.45	8.42	X_{6N}	-0.01	7.99
X_{7E}	-1.27	6.75	X_{7N}	+0.31	7.23
X_{8E}	-1.35	4.51	X_{8N}	+1.24	5.54
X_{9E}	0.00	4.81	X_{9N}	-0.42	5.93
X_{10E}	-1.19	4.29	X_{10N}	-0.72	5.43
X_{11E}	-0.06	4.42	X_{11N}	+1.18	5.23

TABLE III

ARITHMETIC MEANS AND STANDARD DEVIATIONS
(Used For Lag)

East Components			North Components		
Variable	Mean	Standard Deviation	Variable	Mean	Standard Deviation
A_E	-1.03	4.45	A_N	-0.02	5.20
Y_{1E}	+0.47	3.16	Y_{1N}	+0.22	3.79
Y_{2E}	+0.07	3.08	Y_{2N}	+0.61	4.21
Y_{3E}	-1.33	3.43	Y_{3N}	+0.20	3.57
Y_{4E}	-0.76	3.47	Y_{4N}	-0.75	4.24
Y_{5E}	-0.05	3.54	Y_{5N}	-0.28	3.90
Y_{6E}	-1.55	7.90	Y_{6N}	+0.57	7.68
Y_{7E}	-1.39	6.26	Y_{7N}	-0.24	7.33
Y_{8E}	-1.27	4.76	Y_{8N}	+0.28	5.50
Y_{9E}	-0.18	4.88	Y_{9N}	-0.55	5.74
Y_{10E}	-0.83	3.81	Y_{10N}	-0.23	5.25
Y_{11E}	-0.59	4.67	Y_{11N}	-0.15	5.27

TABLE IV

TABLE OF SIMPLE CORRELATION COEFFICIENTS OF
EASTWARD SPECIFIC MOMENTUM CHANGES AT 500
MILLIBAR LEVEL WITH EASTWARD ACCELERATION
OF SURFACE LOW

Section	Simple Correlation Coefficients	
	No Lag	Surface Low Lagging 1 Day
South	-.059	+.082
East	+.203	+.210
North	-.021	+.038
West	-.013	+.152
Center	+.206	+.122
Total All Sections	+.144	+.208
Total – North,West,South,East	+.094	+.193
North + East	+.190	+.114
West + South	-.148	+.209
North + South	+.041	+.008
West + East	+.033	+.301

TABLE V

TABLE OF SIMPLE CORRELATION COEFFICIENTS OF
NORTHWARD SPECIFIC MOMENTUM CHANGES AT 500
MILLIBAR LEVEL WITH NORTHWARD ACCELERATION
OF SURFACE LOW

Section	Simple Correlation Coefficients	
	No Lag	Surface Low Lagging 1 Day
South	+.189	-.120
East	-.141	+.265
North	-.129	+.293
West	+.133	+.018
Center	+.250	-.081
Total All Sections	+.192	+.201
Total – North,West,South,East	-.002	+.224
North + East	-.204	+.378
West + South	+.329	-.077
North + South	+.109	+.086
West + East	-.028	+.225

Simple correlation coeficients were then computed, using the accelerations of the surface low to the specific momentum changes at the 500 millibar level. In all 65 values of each variate were used. The correlation coeficients were computed from the formula

(3)

$$r = \frac{\frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x})(y_i - \bar{y})}{\sigma_x \; \sigma_y}$$

where r = correlation coeficient

x and y   = variates correlated

$\bar{x}$ and $\bar{y}$ = arithmentic means of variates correlated

σ_x and $\bar{y}$ = standard deviations of the variates correlated.

From (2) and (3) it can be seen that in order for the existence of a high correlation coefficient the respective signs of $(x - \bar{x}$) and $(y - \bar{y}$) must not only be consistent but their magnitudes must also be very nearly the same. The simple correlation coefficients are presented in Tables IV and V. Table IV gives correlations of eastward components and Table V northward components.

Next certain of the variates which yielded the highest values of simple correlations were selected to obtain multiple correlations. The multiple correlation coefficient is written $r(1.23)$ and read the multiple correlation of variate 1 on variates 2 and 3. The formula for the multiple correlation coefficient is

(4) $$r(1.23) = \left(1 - \frac{R}{R_{11}}\right)^{\frac{1}{2}}$$

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where

$$R = \begin{vmatrix} \nu(11) & \nu(12) & \nu(13) \\ \nu(21) & \nu(22) & \nu(23) \\ \nu(31) & \nu(32) & \nu(33) \end{vmatrix}$$

and R_{11} is the minor of $\nu(11)$. In (4), the νs are simple correlations between the variates. In general in order to get high multiple correlation the simple correlations must be high.

The standard error of estimate for each multiple correlation was computed. The formula is,

(5)
$$S_{(1.23)} = \sigma_1 \left(\frac{R}{R_{11}} \right)^{\frac{1}{2}} .$$

It is called the standard error of estimating X_1 from X_2 and X_3. Examining (4) and (5), it is seen that high values of multiple correlation will give low values of standard error of estimate. The value of the standard error of estimate is also directly proportional to the standard deviation of the dependent variate correlated. It is seen that the smaller the standard error of estimate the better the chance of predicting the dependent variate. The multiple correlations considered and the corresponding standard errors of estimate are presented in Table VI.

Although the correlations are not high as correlation coefficients are generally rated in most fields, they do show a rather interesting pattern.

TABLE VI

TABLE OF MULTIPLE CORRELATION COEFFICIENTS

AND STANDARD ERRORS OF ESTIMATE

Variates Correlated	Correlation Coefficient	Standard Error of Estimate
$\sim (A_E \cdot X_{2E} \, Y_{11E})$	.393	4.09
$\sim (A_E \cdot X_{5E} \, Y_{11E}$	.373	4.15
$\sim (A_E \cdot Y_{11E} \, Y_{2E})$	.373	4.12
$\sim (A_E \cdot X_{5E} \, X_{2E})$	.279	4.27
$\sim (A_E \cdot X_{5E} \, Y_{2E})$	.281	4.27
$\sim (A_E \cdot X_{2E} \, Y_{2E})$	.314	4.22
$\sim (A_N \cdot X_{9N} \, Y_{8N})$	.460	4.62
$\sim (A_N \cdot Y_{3N} \, Y_{8N})$	.438	4.67
$\sim (A_N \cdot X_{9N} \, X_{5N})$	.425	4.71
$\sim (A_N \cdot X_{5N} \, Y_{3N})$	.398	4.77
$\sim (A_N \cdot X_{9N} \, Y_{3N})$	.388	4.79

The north sector when correlated concurrently with the northward acceleration of the surface low shows a negative correlation of -.129, but when the specific momentum changes are lagged by 1 day, the correlation coefficient becomes a + .293. Thus an increase of northward momentum in the north sector in the latter case could be interpreted as a surge of mass northward away from the position of the surface center. This surge of mass northward leaves deficit of mass to the north of the surface center and the effect is reflected the following day by an increase in the northward movement of the surface low.

Consider now the west + south section's values. The correlation of the northward component of this combination of sections to the northward acceleration of the surface low concurrently gives a value of + .329, but when the south + west sections are lagged by one day the correlation coefficient becomes - .077. Thus a northward surge in the rear sectors causes an excess of mass to the south of the low and the effect is concurrently noted by a northward acceleration of the surface center.

The array of correlations for the eastward components indicates that such an explanation as given above does not suffice for eastward accelerations of the surface low. The best simple correlations are obtained by the combination west + east sectors lagged by 1 day and the east sector lagged by 1 day, the correlation coefficients being respectively + .301 and + .210. Although the combination of west + east gives the greater correlation, its standard deviation is 4.67 as compared to 3.08 for the east section alone,which reduces its apparent advantage.

The center sector which is analogous to the 500 millibar wind

shows significant correlation, only with no lag. The correlation
coefficients obtained are smaller than those obtained by other in-
vestigators correlating upper wind to movement of surface lows, but
it must be remenbered that the correlations presented here are corre-
lations of accelerations rather than velocities and hence it was ex-
pected that they would be smaller.

When the specific momentum changes at the 500 millibar lavel are
lagged by the accelerations of the surface low the situation lends it-
self to a forecasting use. Two regression equations are presented
below. The regression equation is an implement for predicting the
value of the dependent variate when the values of the independent
variates are put into the equation. The general form of the regression
equation for 3 variates is

(6)

$$R_{11} \frac{X_1 - \bar{X}_1}{\sigma_1} + R_{12} \frac{X_2 - \bar{X}_2}{\sigma_2} + R_{13} \frac{X_3 - \bar{X}_3}{\sigma_3} = 0$$

The symbols in equation (6) have been described in the preceeding pages.
Regression equations for predicting both eastward and northward accel-
eration of the surface low have been computed. The equations are

(7)

$$A_E = .25 \, Y_{11E} + 1.049 \, Y_{2E} - 1.712 \quad , \quad S(A_E) = \pm 4.12$$

$$A_N = 1.31 \, Y_{3N} + .302 \, Y_{8N} + .155 \quad , \quad S(A_N) = \pm 4.67.$$

In (7) the independent variates are in units of $\dfrac{°Lat./day}{T°K.} \times 10^2$.

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Their corresponding multiple correlations of + .373 and + .438 respectively are not high enough to give good quantitative results, although the qualitative indications should be significant.

For qualitative prediction of northward acceleration of surface centers the north component of specific momentum change in the north section lagged one day was observed to be, with few exceptions, of the same sign (compared to respective means) as the northward acceleration of the surface low.

CHAPTER IV

CONCLUSIONS

It has been shown by this investigation that there is correlation between accelerations of surface lows and specific momentum changes at the 500 millibar level. This, in effect, is also a correlation between day-to-day zonal and meridional index changes and movements of surface lows. It shows the effect of index on a micro-scale.

Although the correlation coefficients obtained were comparatively low, this was not too surprising. There are several factors contributing to this:

1. Only one level of the whole atmosphere is considered. If the investigation were to be conducted for numerous levels the results should be more significant.

2. The correlations of accelerations rather than velocities leads to smaller numerical coefficients, especially if the day to day velocities of the surface low are fairly uniform.

3. The distance of 600 nautical miles between the center and north, south, east, and west sections was arbitrarily chosen, and that it is the best distance to use is open to question. In several instances it was observed that specific momentum values were obtained from outside the limits of the 500 millibar trough reflected from the surface low. In cases where the east or west sections from one day to the next passes from the confines of the trough across the ridge line the changes in the north component of the specific momentum would be quite large and not necessarily reflected in accelerations of the surface low. It is surmised that using a shorter distance from center to outlying sections would give better results.

4. Geostrophic winds were used throughout. In regions of marked curvature the errors are quite significant.

5. The scale of the maps (5) was quite small. Even though dividers were used, for wind velocities greater than 75 miles per hour errors of less than 10% are difficult to avoid.

Although the correlation coefficients obtained in this investigation are smaller than those obtained in other investigations which correlated velocities, it does not mean that they are less useful. For when predicting from velocity correlations one must predict the total displacement, but in predicting from acceleration correlations only the expected change from the previous days displacement is predicted. In cases where the differences in day-to-day displacements of the surface low are small compared to the displacements themselves the value of predicting from acceleration correlations is enhanced. Thus in such cases the regression equations based on multiple correlation of acceleration and analogous specific momentum changes present a refinement over regression equations based on velocity correlations.

This investigation has scarcely made a start in the possible lines of investigation. The best distances of center to outer section is not determined. The investigation could be carried out at other levels if data is available. The time of lag that gives best results is undetermined. The lag enforced by the data in this investigation was $32\frac{1}{2}$ hours. It is most probable that a different time lag would give better results.

In conclusion, it is felt that the results obtained are significant and that micro scale changes in meridional and zonal indices are re-

flected in the day to day accelerations of surface lows. However, much more extensive work needs to be done to complete the problem.

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