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Annapolis, Maryland: Naval Postgraduate School

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A STUDY OF THE RELATIONSHIP OF THE DISPLACEMENT OF CYCLONES TO VARIOUS ASPECTS OF THE UPPER AIR FLOW

BY

Theodore William Runk
Lieutenant Commander, United States Navy

Submitted in partial fulfillment
of the requirements
for the degree of
MASTER OF SCIENCE IN AEROLOGY

United States Naval Postgraduate School
Annapolis, Maryland
1948

This work is accepted as fulfilling
the thesis requirements for the degree of
MASTER OF SCIENCE IN AEROLOGY

from the
United States Naval Postgraduate School.

[redacted]

Chairman

Department of Aerology

Approved:

[redacted]

Academic Dean

7564

PREFACE

The initial purpose for this research was to examine quantitatively the effect of "thermal steering" as applied to constant pressure and differential analysis. This investigation was later expanded to a study of the correlation between the displacement of cyclones and (1) the thermal wind, (2) the geostrophic wind, and (3) the gradient wind. Since an integrated effect of the atmospheric winds was desired, the combined influence of the thermal wind for two layers and of the geostrophic and gradient wind for three levels was studied in order to add to the facts already known about the steering effects of individual layers or levels.

This work was conducted at the U. S. Naval Postgraduate School, Annapolis, Md. during the period December, 1947 to May, 1948.

I wish to express my appreciation to Assistant Professor George J. Haltiner for his helpful advice and guidance in the preparation of this paper.

T. W. Runk

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TABLE OF SYMBOLS

p	Pressure
p_o	Surface Pressure
g	Gravity
ρ	Density
V	Gradient Wind
V_7	Gradient Wind at the 700 mb. Level
V_5	Gradient Wind at the 500 mb. Level
V_3	Gradient Wind at the 300 mb. Level
V_m	Mean Gradient Wind
G	Geostrophic Wind
G_7	Geostrophic Wind at the 700 mb. Level
G_5	Geostrophic Wind at the 500 mb. Level
G_3	Geostrophic Wind at the 300 mb. Level
T	Isobaric Thermal Wind
T_7	1000-700 mb. Thermal Wind
T_5	700-500 mb. Thermal Wind
ψ	Cyclostrophic Term
C	Displacement of a Surface Center
C_T	Thermal Wind Component of Displacement
C_ψ	Cyclostrophic Component of Displacement
C_D	Divergence Component of Displacement
$\bar{X}$	Arithmetic Mean (easterly component)
$\bar{Y}$	Arithmetic Mean (northerly component)
σ	Unit of Standard Deviation (variance from the mean)
r	Correlation Coefficient
S	Standard Error of Estimate

INTRODUCTION

Numerous rules associating the movements of middle latitude cyclones[1] with various aspects of the upper winds have been promulgated in meteorological publications. All of these rules may be appropriately used to make a qualitative estimate of the movement of a surface low pressure system. Of these rules pertaining to the so-called "upper air steering," one which has received much emphasis concerns "thermal steering."[1,2] This rule suggests that a surface pressure system will move in the direction of the 1000-700 mb. mean isotherms with a speed which is proportional to the gradient of the mean isotherms. The application of the above rule would be augmented if some quantitative estimate of this proportionality were known. One analyst has, after repeated observations, suggested that a ratio (speed of surface system divided by the gradient of the mean isotherms) of 0.75 be used. The test of the validity of this suggestion and similar steering rules was the original purpose of this investigation.

It was initially intended to include the effect of the 700-500 mb. mean isotherms on the steering of surface lows, and also to establish the combined correlation between the movement of the surface lows and the isotherms for both the 1000-700 mb. and 700-500 mb. layers. The final step was to be the development of regression equations for the quantitative estimate of the movement of the surface system from the values of the thermal winds[2] over the system. In order to obtain a more complete picture of the upper air steering, the investigation was expanded to include a study of the role of geostrophic and gradient winds. To achieve the desired integrated effect for the whole atmosphere, insofar as data was available, the problem now became one of comparing the cyclone movement to the winds at several representative levels. It thus became a problem of multiple correlation between the movement of the pressure center and the winds at these levels. This task was divided into three parts; namely, the correlation of cyclone displacement to-

(1) 1000-700 mb. thermal wind, 700-500 mb. thermal wind, and the 500 mb.* geostrophic wind,

(2) geostrophic wind at the 700 mb., 500 mb., and 300 mb. levels,

(3) gradient wind at the 700 mb., 500 mb., and 300 mb. levels.

Since this is a statistical investigation, the relationships are presented in terms of simple and multiple correlation coefficients, four variate regression equations, and standard errors of estimate.

* The use of the 500 mb. geostrophic wind in this case is explained in Chapter III.

Time limitations restricted the test to one month's data. Because of the limited amount of data tested, the results are not conclusive; however, considerable information about the nature and magnitude of the correlations was obtained.

With these limitations in mind the following results of the investigation are advanced:

(1) That within certain limits of error, cyclones do move with a speed which is approximately 75% of the 1000-700 mb. thermal wind above the center, and in a direction which is approximately 10° to the left of the thermal wind.

(2) That the correlations obtained in the three cases of combined comparison are positive and large. The correlations between the movement of the cyclone center and the combined thermal winds were 0.56 for the easterly component of motion and 0.78 for the northerly component; their corresponding standard errors of estimate were 4.97 knots and 8.04 knots. For the combined geostrophic winds the correlations were o.63 and 0.75, and their respective standard errors of estimate were 6.21 knots and 7.95 knots.

Similarly, for the gradient wind comparison, the correlations were 0.69 and 0.77 with corresponding standard errors of estimate of 5.81 knots and 8.50 knots. These correlation coefficients indicate a pronounced relationship between the displacement of cyclones and the combined effect of the winds at representative levels in the atmosphere, but are not large enough to insure the reliability of regression coefficients computed from them. Furthermore, the standard errors of estimate are large in comparison with the mean value of the cyclone displacement, varying approximately from 20% to 50% of the displacement. It appears, therefore, that regression equations obtained here do not lend themselves to quantitative predictions but should be useful as a qualitative guide in forecasting.

CHAPTER I

EARLIER INVESTIGATIONS OF UPPER AIR STEERING

1. Concept of Steering.

German authors are credited by Petterssen(2) as having suggested the idea of steering. They have proposed that sea level pressure systems are steered by the circulation of upper troposphere. Petterssen has also referred to the principle of steering in a discussion of his displacement formulas. In his presentation he transformed the hydrostatic tendency equation into the form

$$(1) \qquad -\frac{\partial p_o}{\partial t} = \nabla p_o \cdot \vec{V}_m + p_o(\nabla \cdot \vec{V}_m)$$

in which $\vec{V}_m$ is the mean of the wind vectors as averaged over selected pressure intervals and is defined as

$$(2) \qquad \vec{V}_m = \frac{\sum_{i=1}^{n} a_i \vec{V}_i}{\sum_{i=1}^{n} a_i} .$$

The coefficient a_i is a weight factor assigned to each of the winds at the various levels. It would appear from equation (1) that the surface pressure change, and hence movement of surface pressure systems, is influenced by the upper winds. The quantity $\vec{V}_m$ is defined in terms of the geostrophic wind $\vec{G}$, the thermal wind $\vec{T}$, and the cyclostrophic deviation from the geostrophic wind (cyclostrophic component of the gradient wind) $\vec{\psi}$.[2,3]

$$(3) \qquad \vec{V}_m = \vec{G}_o + \frac{\sum_{i=1}^{n} a_i \vec{T}_i}{\sum_{i=1}^{n} a_i} + \frac{\sum_{i=1}^{n} b_i \vec{\psi}_i}{\sum_{i=1}^{n} b_i}$$

where $\vec{G}_o$ is the geostrophic wind at the surface, $\vec{T} = \vec{G}_{i+1} - \vec{G}_i$, and $\vec{\psi} = \vec{V}_i - \vec{G}_i$ ($\vec{V}$ is the gradient wind). Observe from equation (3) that the geostrophic, thermal, and cyclostrophic components may now be fitted into the sphere of influence of the upper winds on the variation of surface pressure. Further, Petterssen's extrapolation formulas[4] for the speed of a pressure center may be written[2]

$$(4) \qquad \begin{aligned} C_x &= V_{m,x} + \frac{P_o}{\frac{\partial^2 P_o}{\partial x^2}} \frac{\partial}{\partial x} (\nabla \cdot \vec{V}_m) \\ C_y &= V_{m,y} + \frac{P_o}{\frac{\partial^2 P_o}{\partial y^2}} \frac{\partial}{\partial y} (\nabla \cdot \vec{V}_m) \end{aligned}$$

where V_x and V_y stem from the transport term of the tendency equation. Equations (4) may now be written in vector form and expressed symbolically as

$$(5) \qquad \vec{C} = \vec{C}_T + \vec{C}_\psi + \vec{C}_D$$

where $\vec{C}$ is the movement of the pressure center, $\vec{C}_T$ is the component due to the weighted thermal wind, $\vec{C}_\psi$ is the cyclostrophic component, and $\vec{C}_D$ is the component involving divergence. This equation has had considerable qualitative application. It lends much weight to the concept of steering, indicating the role of each of the three functions of the wind in the steering process. The general aspects of the equation have been discussed comprehensively by Haltiner and Eaton.(5)

2. Empirical Investigation of Steering Levels.

Many investigators have sought to find a level or a layer in the atmosphere where the flow coincided consistently with the movements of surface pressure centers, such a level being referred to as a steering level. To date no such level has been discovered.

Correlation studies between upper air flow and the movement of surface pressure centers were made by Austin(6) and Longley(7) working independently. Austin compared the surface movement of pressure centers to the following layers and levels:

a. mean of the 850 mb. and 700 mb. contours

b. mean of the 700 mb. and 500 mb. contours

c. 200 mb. contours

d. 850-700 mb. mean isotherms

e. 700-500 mb. mean isotherms.

Here the direction of movement of cyclones was compared to the direction of these isolines over the system; and the speed of cyclones was compared to the gradients of the isolines. For all cases investigated, Austin found that the average deviation angle of the direction of displacement from the wind direction at each level or layer mentioned above was within a range of values $26^{\circ} - 33^{\circ}$. For selected cases in which the speed of the cyclone exceeded 20 knots this limiting range was reduced to $21^{\circ} - 24^{\circ}$. By comparison of the displacement speed to the gradient of the isolines, he obtained correlations which were within the limits -0.22 to $+0.06$. For the same select cases the correlations were increased to values varying within the limits -0.13 to $+0.30$. These correlations for individual levels or layers are small. Austin concluded that comparison of directions was more favorable than the comparison of speeds. He found that although individual cases deviated in direction as high as $\pm 180^{\circ}$, roughly 60% of the cases deviated less than $\pm 15^{\circ}$. Regarding his results Austin states-

> This statistical study indicates that the forecasting principle of steering compares favorably with other prognostic procedures for determining the direction of motion of a cyclone. It appears, how-

ever, that the speed of a cyclone cannot be determined from the geostrophic wind at any particular level.

Longley's studies were along similar lines, but were confined to the flow at 700 mb. using the gradient wind at that level. The mean deviation angle between the direction of movement of a cyclone and the direction of the contours at 700 mb. was 31°. The dominant tendency was for the cyclone to be steered to the right of the 700 mb. flow (this tendency toward the right was also observed by Austin). Approximately 1/3 of the cases studied failed to move with the 700 mb. flow, and approximately 1/2 of the cases had a deviation angle greater than 20°. For selected cases the average deviation was 21°. The correlation of the speed of movement of cyclones to the gradient aloft was $0.45 \pm .07$. The ratio of the speed of a cyclone to the gradient of the contours varied from 0.26 to 2.38. It is to be noted, however, that this ratio was almost exactly 1.0 for 70% of the cases examined.

CHAPTER II

THEORETICAL BASIS FOR UPPER AIR STEERING

1. The Symbolic Steering Formula.

Petterssen's formulas(2) for the rate of displacement of a pressure center along the coordinate axes are

$$(6) \qquad C_x = -\frac{\dfrac{\partial^2 P_o}{\partial x \partial t}}{\dfrac{\partial^2 P_o}{\partial x^2}} \qquad\qquad C_y = -\frac{\dfrac{\partial^2 P_o}{\partial y \partial t}}{\dfrac{\partial^2 P_o}{\partial y^2}} \quad .$$

Equation (1) may be written as

$$(7) \qquad -\frac{\partial P_o}{\partial t} = V_{m,x}\frac{\partial P_o}{\partial x} + V_{m,y}\frac{\partial P_o}{\partial y} + P_o(\nabla \cdot \vec{V}_m) \, .$$

Differentiating partially with respect to x and y in (7) and assuming no acceleration, we obtain

$$(8) \qquad \begin{aligned} -\frac{\partial^2 P_o}{\partial x \partial t} &= V_{m,x}\frac{\partial^2 P_o}{\partial x^2} + P_o\frac{\partial}{\partial x}(\nabla \cdot \vec{V}_m) \\ -\frac{\partial^2 P_o}{\partial y \partial t} &= V_{m,y}\frac{\partial^2 P_o}{\partial y^2} + P_o\frac{\partial}{\partial y}(\nabla \cdot \vec{V}_m) \quad . \end{aligned}$$

The substitution of these values into (6) gives

$$C_x = V_{m,x} + \frac{P_o}{\frac{\partial^2 P_o}{\partial x^2}} \frac{\partial}{\partial x}(\nabla \cdot \vec{V}_m)$$

(9)

$$C_y = V_{m,y} + \frac{P_o}{\frac{\partial^2 P_o}{\partial y^2}} \frac{\partial}{\partial y}(\nabla \cdot \vec{V}_m) .$$

Petterssen has pointed out that although the gradient of $\nabla \cdot \vec{V}_m$ is relatively small, the terms $P_o / \frac{\partial^2 P_o}{\partial x^2}$ and $P_o / \frac{\partial^2 P_o}{\partial y^2}$ are extremely large and thus cannot in general be neglected. Combining C_x and C_y, we have for the displacement of a pressure center

$$(10) \quad \vec{C} = \vec{V}_m + \frac{P_o}{\frac{\partial^2 P_o}{\partial x^2}} (\nabla \cdot \vec{V}_m)\vec{i} + \frac{P_o}{\frac{\partial^2 P_o}{\partial y^2}} (\nabla \cdot \vec{V}_m)\vec{j} .$$

It appears then that the movement of a pressure center is governed by the wind over the center and by the gradient of the divergence.

If we assume now that the wind is gradient, then, since $\vec{G}_o = 0$ at the center of a cyclone, equation (10) by virtue of (3) is expressible in the form

$$(11) \quad \vec{C} = \frac{\sum_{i=1}^{n} a_i \vec{T}_i}{\sum_{i=1}^{n} a_i} + \frac{\sum_{i=1}^{n} a_i \vec{\psi}_i}{\sum_{i=1}^{n} a_i} + \frac{P_o}{\frac{\partial^2 P_o}{\partial x^2}} \frac{\partial}{\partial x}(\nabla \cdot \vec{V}_m)\vec{i} + \frac{P_o}{\frac{\partial^2 P_o}{\partial y^2}} \frac{\partial}{\partial y}(\nabla \cdot \vec{V}_m)\vec{j} .$$

Previously this result was written symbolically as

$$(5) \quad \vec{C} = \vec{C}_T + \vec{C}_\psi + \vec{C}_D \quad .$$

The three terms on the right side are terms involving the geostrophic wind shear (thermal wind), the cyclostrophic deviation from the geostrophic wind, and the gradient of the divergence of the wind field respectively. The movement of the pressure center is thus related to these quantities.

Alternatively, (2) may be introduced into equation (8). Resolving the right hand member into the geostrophic and cyclostrophic components of $\vec{V}_m$ into the form

$$(12) \quad \vec{V}_m = \frac{\sum_{i=1}^{n} a_i \vec{G}_i}{\sum_{i=1}^{n} a_i} + \frac{\sum_{i=1}^{n} a_i \vec{\psi}_i}{\sum_{i=1}^{n} a_i}$$

yields another symbolic expression analogous to (5)

$$(13) \quad \vec{C} = \vec{C}_G + \vec{C}_\psi + \vec{C}_D \quad .$$

This suggests that the movement of pressure systems is related to the gradient wind aloft.

2. Application of the Symbolic Steering Formula.

It is to the empirical and statistical treatment of equations (5) and (13) that this investigation is devoted.

Since the method of evaluating quantitatively the divergence of the wind field is complicated and lengthy[8], and therefore not readily adaptable to daily forecasting, the term involving divergence is henceforth neglected in the following treatment of the above equations. If one chooses to neglect the cyclostrophic term $\vec{C}_\psi$ also, then

$$\vec{C} \doteq \vec{C}_T \cdot$$

This approximate equality is the basis for correlation of cyclone displacement to the weighted thermal wind for one or more selected layers. (Comparisons mentioned in this section are described in detail in Chapter III). A second approach to the problem, neglecting again the cyclostrophic term, attempts to evaluate statistically the equation

$$\vec{C} \doteq \vec{C}_G$$

by introducing a weighted combination of the geostrophic winds for selected levels. Finally, to obtain a closer approximation of the integrated wind field, the cyclostrophic term will be introduced by evaluating the equation

$$\vec{C} \doteq \vec{C}_G + \vec{C}_\psi \qquad \text{or} \qquad \vec{C} \doteq \vec{C}_V ,$$

thus replacing the geostrophic wind by the gradient wind.

CHAPTER III

COMPARISONS OF CYCLONE MOVEMENT TO THE UPPER AIR WINDS

1. Organization of Data.

The month selected for the test was December, 1947. Originally, it was intended to evaluate the data for several months; however, time limitations restricted this project to one month's data. The charts used in this investigation were:

a. 0730* and 1930 surface (sea level),

b. 1100 and 2300 700 mb. (constant pressure),

c. 1100 and 2300 500 mb. (constant pressure),

d. 1100 and 2300 300 mb. (constant pressure).

The 300 mb. information was obtained from the chart file of the U. S. Weather Bureau Library, Washington, D. C. The 1000 mb. charts were constructed from the 0730 and 1930 surface charts. The 1000-700 mb. and 700-500 mb. differential analyses were constructed for 1100 and 2300.[5,9] It is realized that a short time interval exists between the surface charts and the upper air charts, however, no correction has been applied in the following work. All of the charts used were carefully examined by the author for accuracy of analysis.

The cyclones selected for the test were enclosed by at

* All times mentioned are Eastern Standard Time.

least one isobar and were sufficiently well defined to permit location of the center with reasonable accuracy. Only those areas on the charts were used where adequate reports guaranteed the certainty of analysis of both the surface and the upper air charts. Thirty-four distinct cases of cyclone movement were tested. The average speed and direction of movement for the periods 0730 - 1930 and 1930 - 0730 were recorded. Likewise, the speed and direction of the following upper winds at 1100 and 2300 were recorded:

a. 1000-700 mb. thermal wind,

b. 700-500 mb. thermal wind,

c. geostrophic wind at the 700 mb., 500 mb., and 300 mb. levels,

d. gradient wind at the 700 mb., 500 mb., and 300 mb. levels.

These upper winds for selected layers and levels were considered to be representative of the wind field over the system at the beginning and end of the period of 12-hour surface motion.

It was desired to treat both components of a velocity vector alike. Since this is not possible for a vector expressed in speed and direction components, the velocity vectors of the recorded cyclone movement and upper winds were resolved into easterly and northerly directed components. This permitted similar treatments of both components in regard to descriptive effects and statistical correlation. Henceforth, the easterly components will be denoted by subscript x and the northerly components by subscript y.

2. Comparison A. Correlation of the Movement of Cyclones to the 1000-700 mb. Thermal Wind, the 700-500 mb. Thermal Wind, and the 500 mb. Geostrophic Wind.

The first comparison of cyclone motion to the upper winds was made with the 1000-700 mb. thermal wind $\vec{T}_7$, the 700-500 mb. thermal wind $\vec{T}_5$, and the 500 mb. geostrophic wind $\vec{G}_5$. These two thermal winds were selected to represent the lower half of the troposphere; the 500 mb. geostrophic wind was selected to be representative of the upper half of the troposphere. This is a somewhat heterogeneous combination of winds, but since differential analyses above the 500 mb. level were not available, the wind at the 500 mb. level was introduced. In order to compare the mean 12-hour upper air flow over the system, the like components of each of the upper winds at the beginning and the end of the 12-hour period were averaged arithmetically. This average was considered to be the mean flow over the surface pressure center during the 12-hour period.

Scatter diagrams indicating the relationship between the components of motion of the surface pressure centers and the like components of the winds at each level or for each layer are shown in figures 1 through 6. Simple statistical functions* -- simple correlation coefficient r of the movement of the cyclone to each of the winds over it, arithmetic averages $\bar{X}$ and $\bar{Y}$, units of standard deviation (variance from the mean) σ, and standard error of estimate S -- are listed in Table I.

	$\bar{X}$	σ_x	$r_{C_x \ldots}$	$S_{C_x \ldots}$
C_x	23.70	7.98	-------	-------
$T_{7,x}$	30.21	9.81	0.47	7.03
$T_{5,x}$	11.17	9.63	0.36	7.45
$G_{5,x}$	37.13	12.93	0.53	6.78
	$\bar{Y}$	σ_y	$r_{C_y \ldots}$	$S_{C_y \ldots}$
C_y	4.16	12.91	-------	-------
$T_{7,y}$	2.96	13.11	0.75	8.58
$T_{5,y}$	8.89	12.11	0.68	9.44
$G_{5,y}$	10.89	19.84	0.71	9.10

Table I. Simple Statistics of Comparison A.

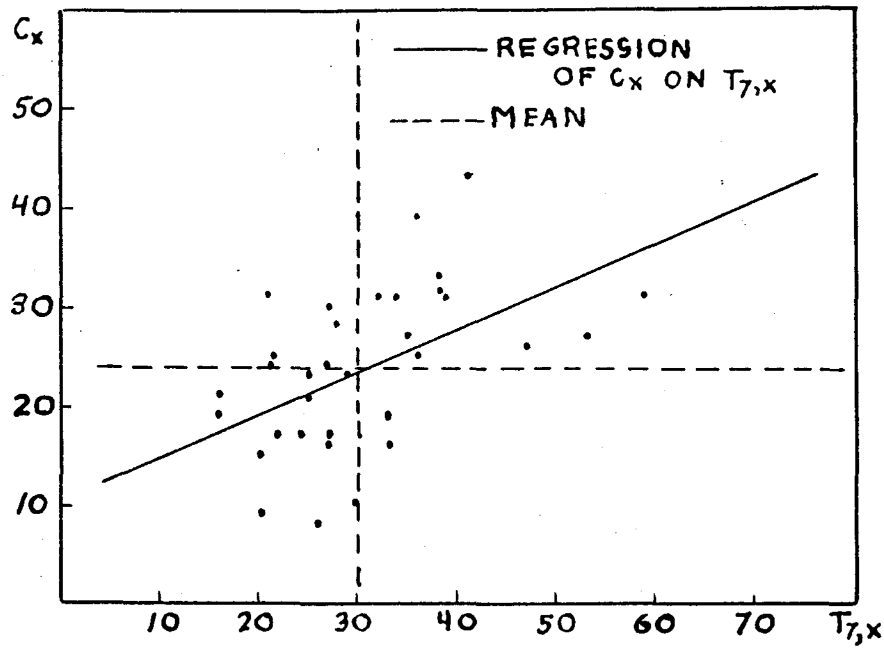
The relationship of the movement of cyclones to the combined winds over the system is expressed in terms of multiple statistical functions, i.e., the multiple correlation coefficients, the standard errors of estimate, and the multiple regression equations:

$$r_{C_x \cdot T_{7,x} T_{5,x} G_{5,x}} = 0.56$$

$$S_{C_x \cdot T_{7,x} T_{5,x} G_{5,x}} = 4.97$$

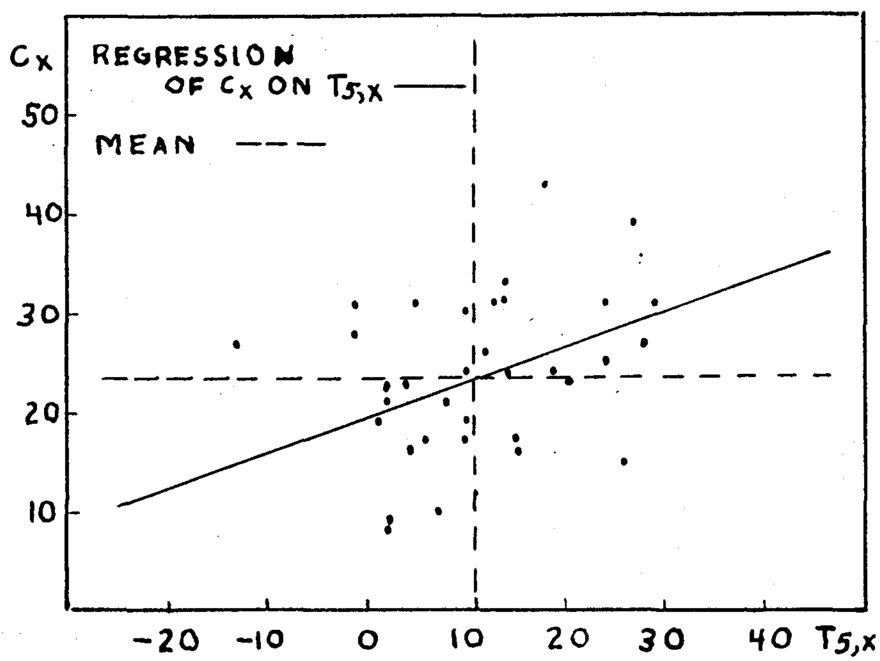
$$C_x = .29 T_{7,x} + .20 T_{5,x} + .07 G_{5,x} + 10.17$$

* See Appendix I for details of statistical analysis.



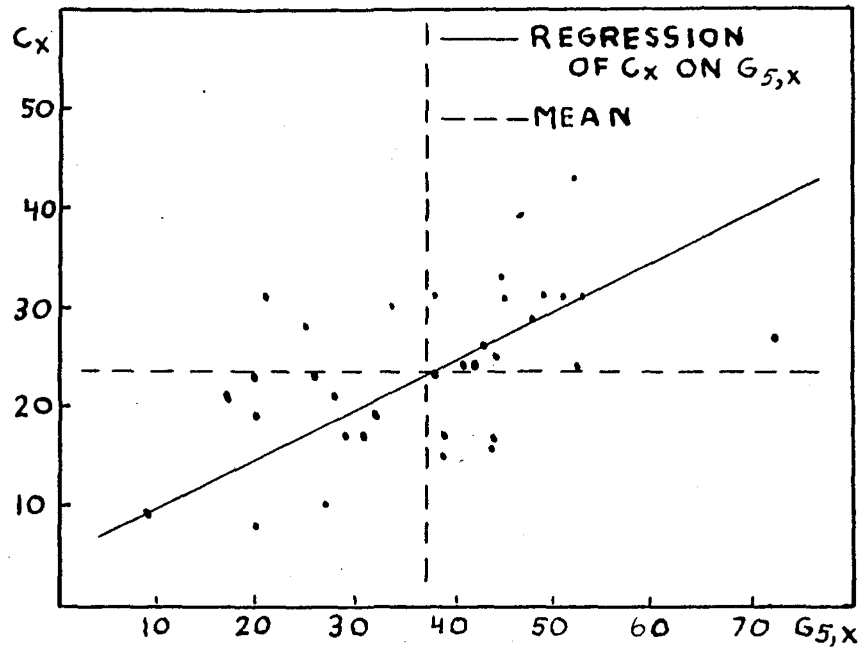
C_x
REGRESSION
OF C_x ON $T_{7,x}$
MEAN
50
40
30
20
10
10
20
30
40
50
60
70
$T_{7,x}$

FIG. (1)



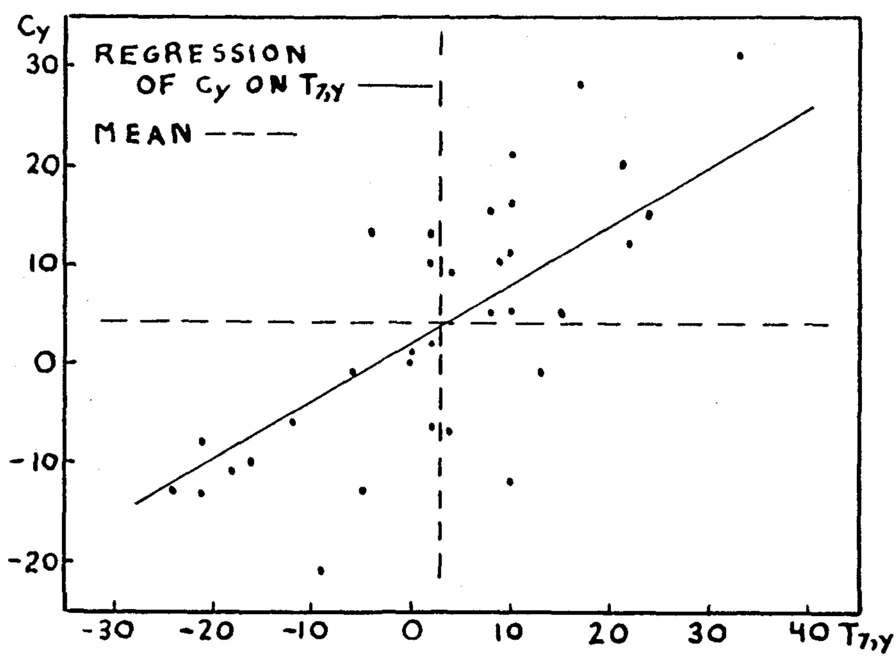
C_x
REGRESSION
OF C_x ON $T_{5,x}$
MEAN
50
40
30
20
10
-20
-10
0
10
20
30
40
$T_{5,x}$

FIG. (2)



C_x
REGRESSION OF C_x ON $G_{5,x}$
MEAN
50
40
30
20
10
10
20
30
40
50
60
70
$G_{5,x}$

FIG. (3)



C_y
REGRESSION OF C_y ON $T_{7,y}$
MEAN
30
20
10
0
-10
-20
-30
-20
-10
0
10
20
30
40
$T_{7,y}$

FIG. (4)

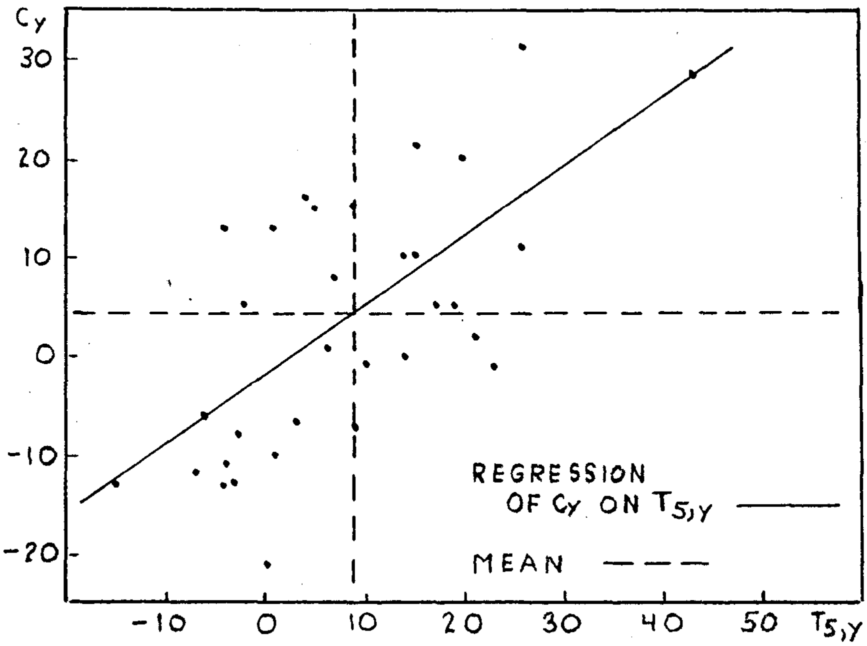

Cy
30
20
10
0
-10
-20
-10
0
10
20
30
40
50
$T_{5,Y}$
REGRESSION
OF Cy ON $T_{5,Y}$
MEAN

FIG. (5)

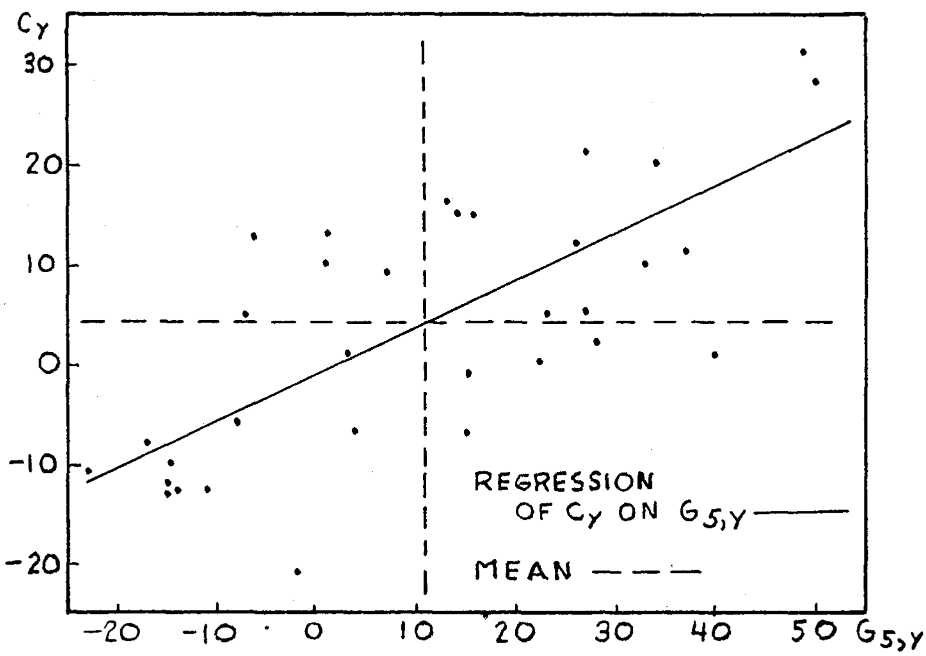

Cy
30
20
10
0
-10
-20
-20
-10
0
10
20
30
40
50
$G_{5,Y}$
REGRESSION
OF Cy ON $G_{5,Y}$
MEAN

FIG (6)

$$r_{C_y \cdot T_{7,y} T_{5,y} G_{5,y}} = 0.78$$

$$S_{C_y \cdot T_{7,y} T_{5,y} G_{5,y}} = 8.04$$

$$C_y = .50\,T_{7,y} + .22\,T_{5,y} + .06\,G_{5,y} + 0.09$$

3. Comparison B. Correlation of the Movement of Cyclones to the Geostrophic Wind at the 700 mb., 500 mb., and 300 mb. Levels.

Comparison B associated the displacement of cyclones with the geostrophic wind at the 700 mb., 500 mb., and 300 mb. levels. The surface centers tested in Comparison A were retested here and the same type of analysis was made. Scatter diagrams indicating the relation of surface motion to the geostrophic wind at each of these levels are shown in figures 7 through 12. Table II gives the simple statistical functions for Comparison B.

	$\bar{X}$	σ_x	$r_{C_x \ldots}$	$S_{C_x \ldots}$
C_x	23.70	7.98	--------	--------
$G_{7,x}$	29.22	9.15	0.41	7.34
$G_{5,x}$	37.13	12.93	0.53	6.78
$G_{3,x}$	73.46	26.41	0.59	6.43

	$\bar{Y}$	σ_y	$r_{C_y \ldots}$	$S_{C_y \ldots}$
C_y	4.16	12.91	--------	--------
$G_{7,y}$	3.65	11.84	0.66	9.73
$G_{5,y}$	10.89	19.84	0.71	9.11
$G_{3,y}$	28.33	35.13	0.71	9.12

Table II. Simple Statistics of Comparison B.

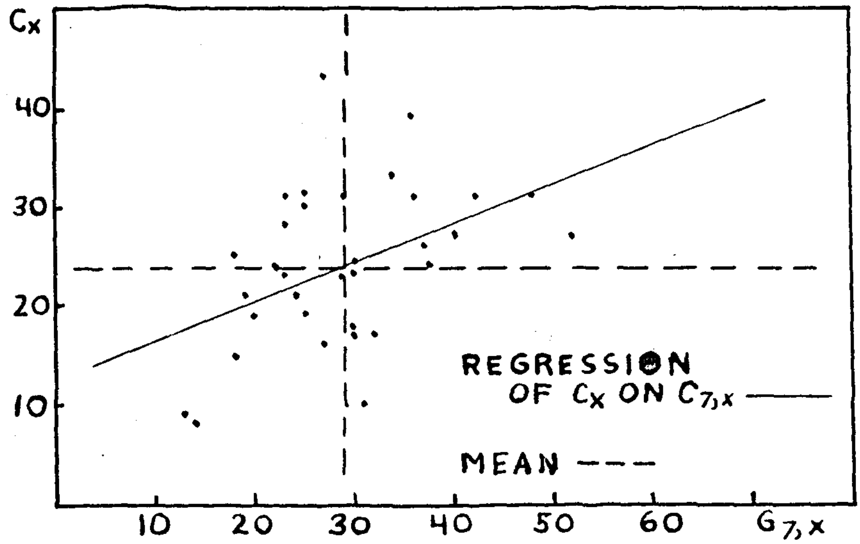

Cx
40
30
20
10
REGRESSION
OF Cx ON C7,x
MEAN
10
20
30
40
50
60
G7,x

FIG. (7)

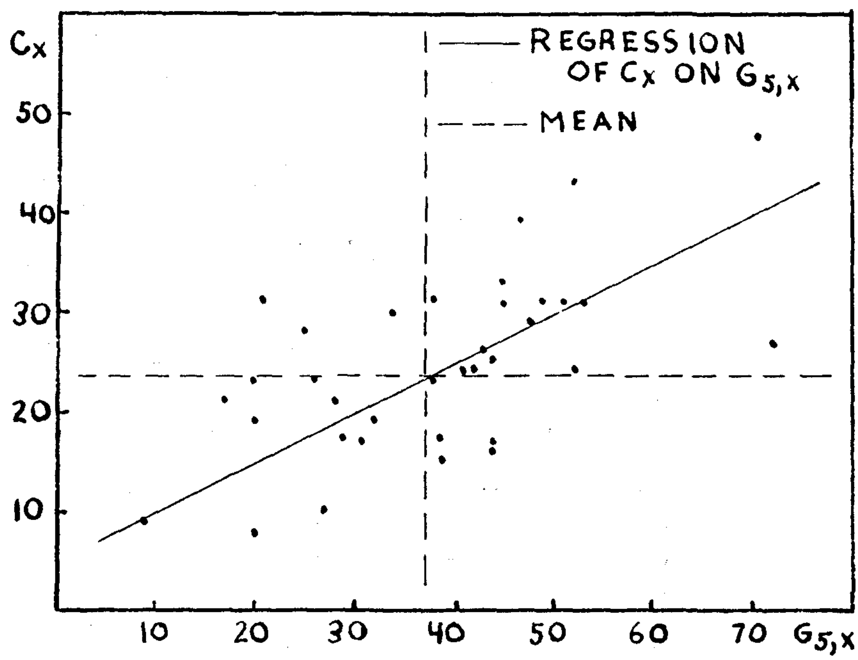
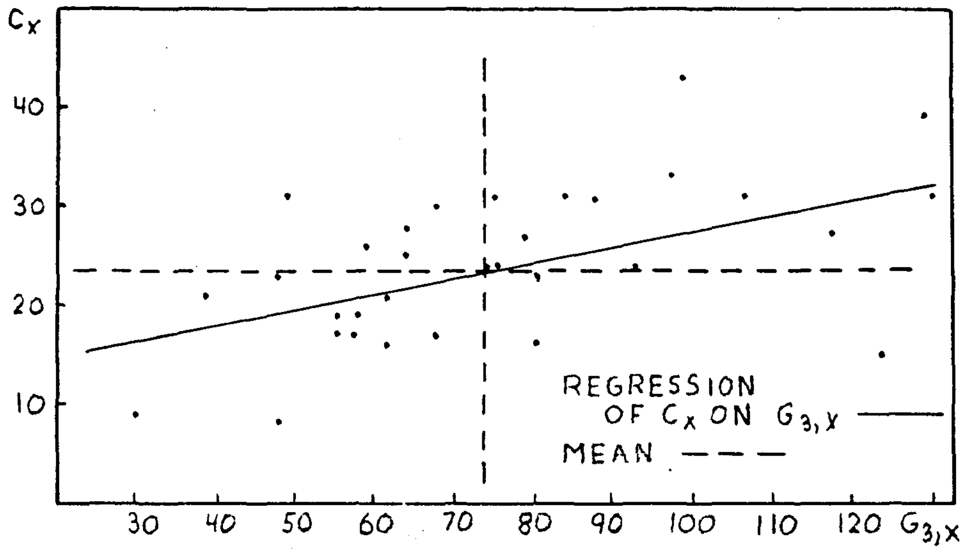
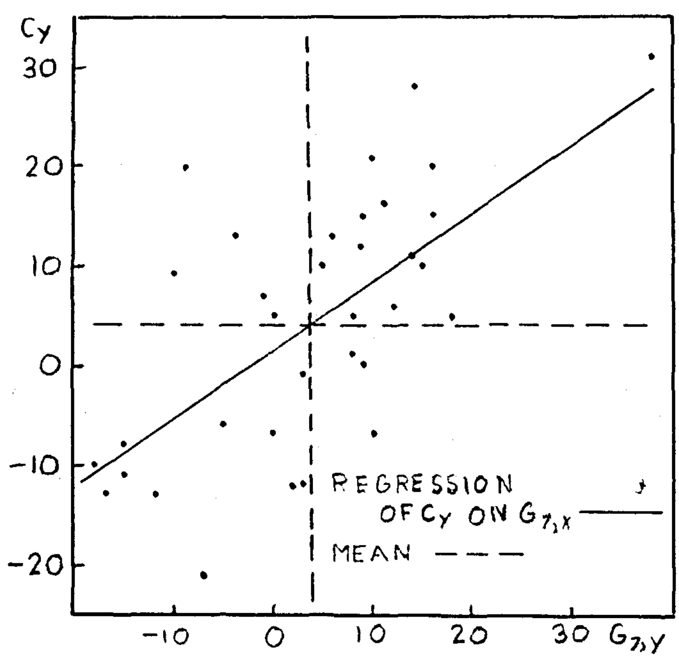

Cx
50
40
30
20
10
REGRESSION
OF Cx ON G5,x
MEAN
10
20
30
40
50
60
70
G5,x

FIG. (8)



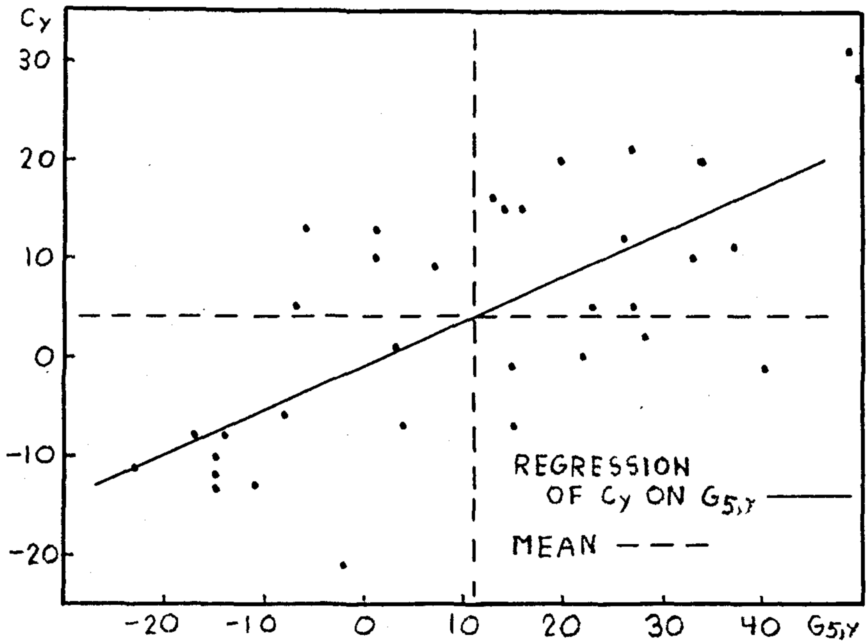
C_x
40
30
20
10
30
40
50
60
70
80
90
100
110
120
$G_{3,x}$
REGRESSION
OF C_x ON $G_{3,x}$
MEAN

FIG (9)



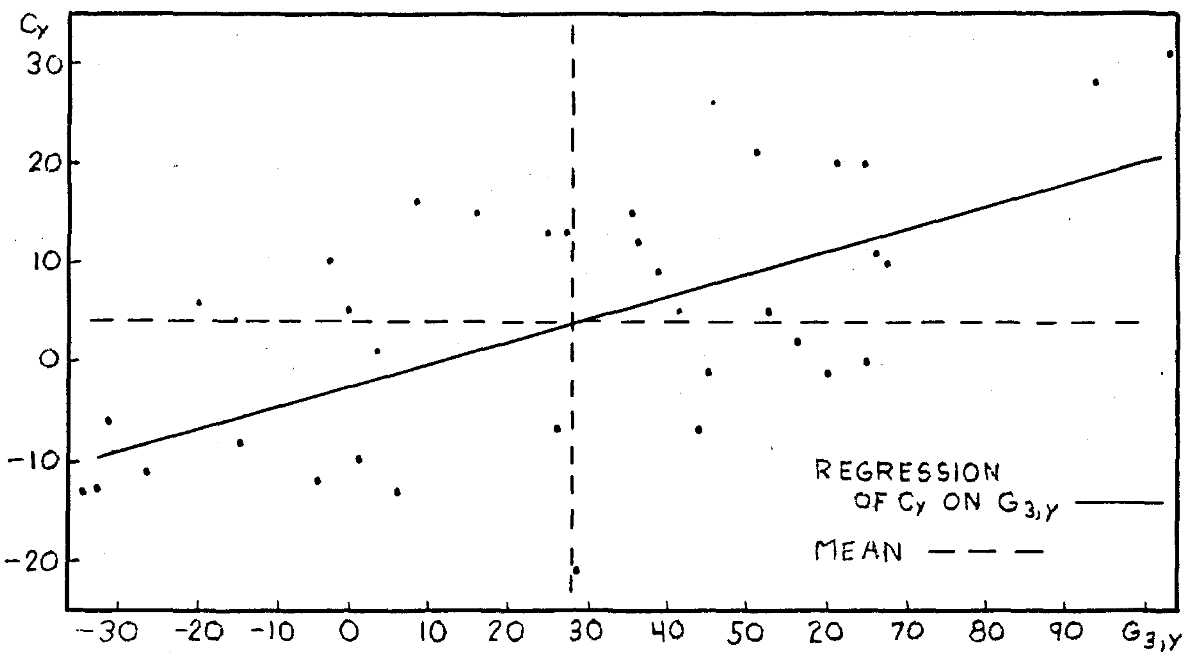
C_y
30
20
10
0
-10
-20
-10
0
10
20
30
$G_{7,y}$
REGRESSION
OF C_y ON $G_{7,x}$
MEAN

FIG (10)



Cy
30
20
10
0
-10
-20
-20
-10
0
10
20
30
40
G5,Y
REGRESSION
OF Cy ON G5,Y
MEAN

FIG. (11)



Cy
30
20
10
0
-10
-20
-30
-20
-10
0
10
20
30
40
50
20
70
80
90
G3,Y
REGRESSION
OF Cy ON G3,Y
MEAN

FIG. (12)

The relationship of the surface movement to the combined geostrophic winds in terms of multiple statistical functions was:

$$r_{c_x \cdot G_{7,x} G_{5,x} G_{3,x}} = 0.63$$

$$S_{c_x \cdot G_{7,x} G_{5,x} G_{3,x}} = 6.21$$

$$C_x = .15\, G_{7,x} + .07\, G_{5,x} + .14\, G_{3,x} + 4.63$$

$$r_{c_y \cdot G_{7,y} G_{5,y} G_{3,y}} = 0.75$$

$$S_{c_y \cdot G_{7,y} G_{5,y} G_{3,y}} = 7.95$$

$$C_y = .31\, G_{7,y} + .11\, G_{5,y} + .14\, G_{3,y} - 2.06$$

4. Comparison C. Correlation of the Movement of Cyclones to the Gradient Wind at the 700 mb., 500 mb., and 300 mb. Levels.

Comparison C associated the movement of cyclones to the gradient wind $\vec{V}$ at the 700 mb., 500 mb., and 300 mb. levels. This is an attempt to introduce into the correlation an evaluation of the cyclostrophic term $\vec{C}_{\psi}$ of equation (13). The geostrophic winds of Comparison B were corrected to gradient velocity by the addition of a graphically obtained correction involving the measurement of the radius of curvature of the contour lines.

Scatter diagrams indicating the degree of dependence of the surface motion on the gradient wind at each of these levels are shown in figures 13 through 18. The simple statistics are tabulated in Table III.

	$\bar{X}$	$\sigma_{\bar{x}}$	$r_{C_{x\ldots}}$	$S_{C_{x\ldots}}$
C_x	23.70	7.98	-------	-------
$V_{7,x}$	25.19	6.39	0.55	6.65
$V_{5,x}$	32.35	8.72	0.68	5.84
$V_{3,x}$	70.82	28.54	0.64	6.12

	$\bar{Y}$	$\sigma_{\bar{y}}$	$r_{C_{y\ldots}}$	$S_{C_{y\ldots}}$
C_y	4.16	12.91	-------	-------
$V_{7,y}$	3.10	10.41	0.73	8.79
$V_{5,y}$	10.81	16.55	0.71	9.07
$V_{3,y}$	22.24	34.66	0.57	10.67

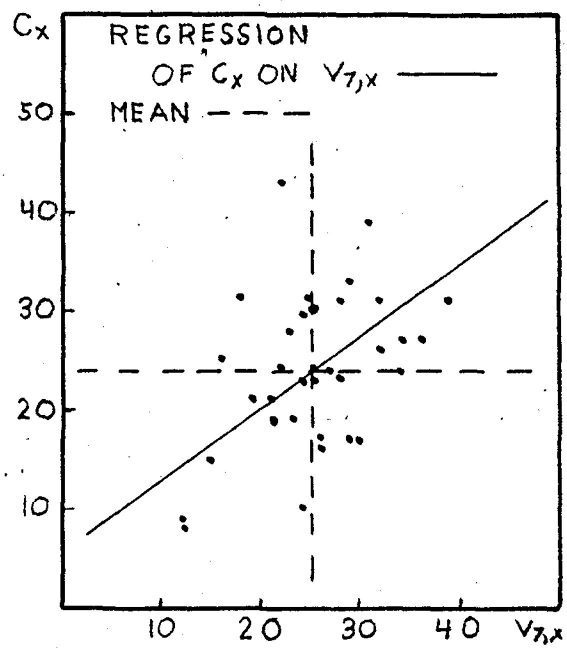
Table III. Simple Statistics of Comparison C.

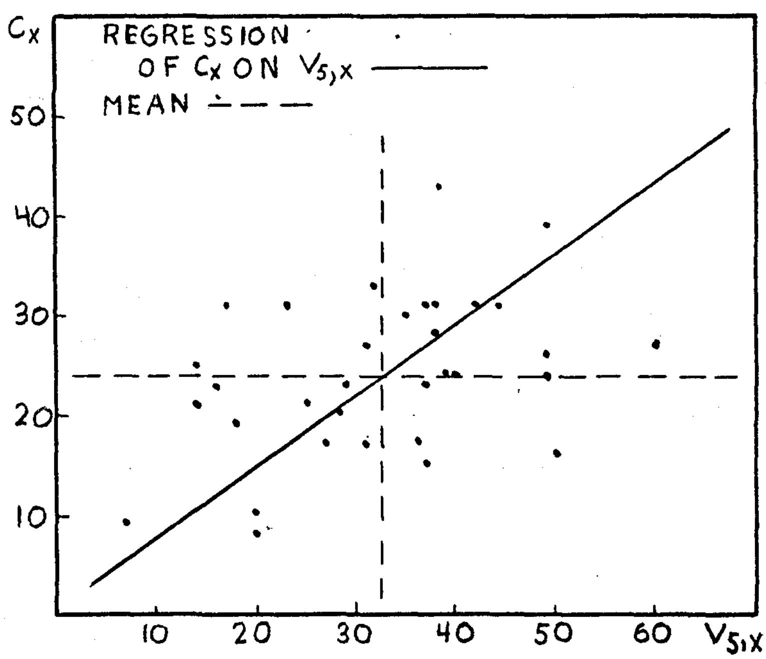
The relationship of cyclone movement to the combined gradient winds in terms of multiple statistics was:

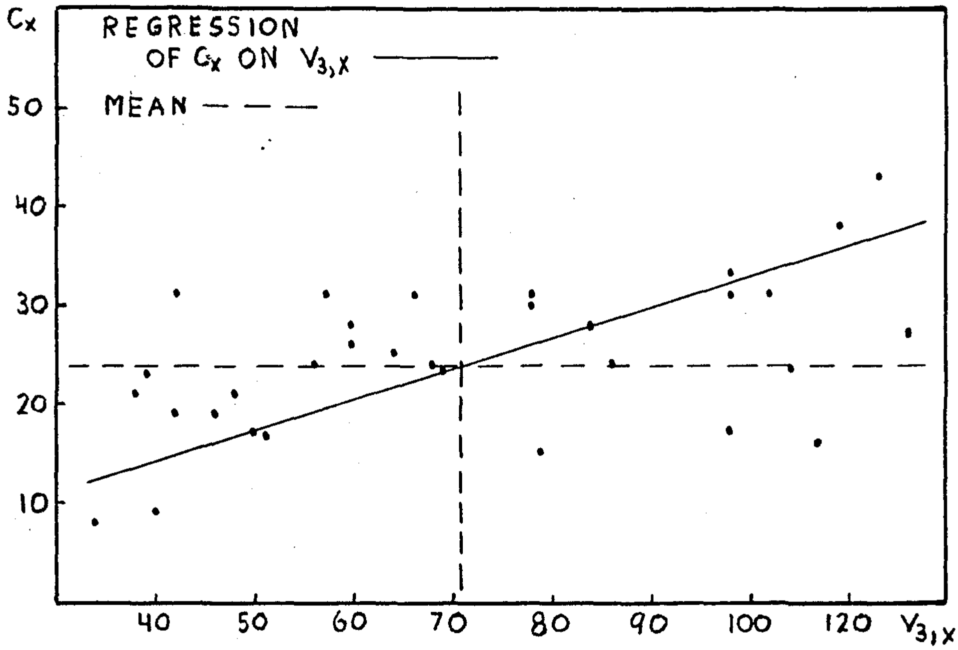
$$r_{c_x \cdot V_{7,x} V_{5,x} V_{3,x}} = 0.69$$

$$S_{c_x \cdot V_{7,x} V_{5,x} V_{3,x}} = 5.81$$

$$C_x = .11\, V_{7,x} + .37\, V_{5,x} + .06\, V_{3,x} + 4.94$$

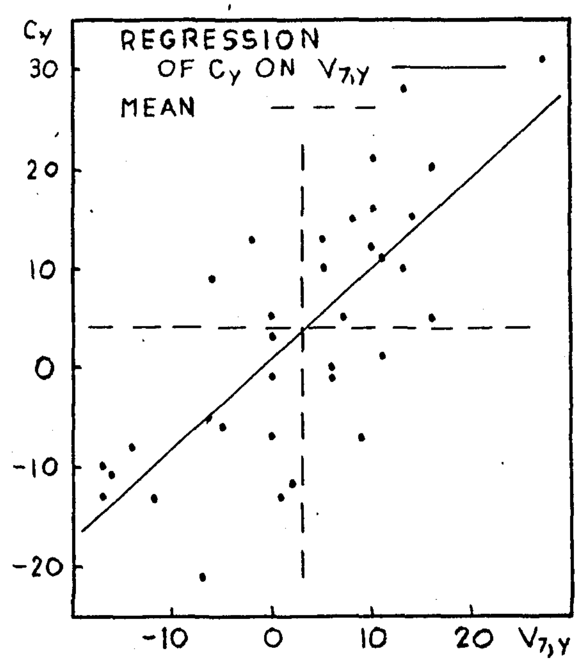

REGRESSION
OF Cx ON V7,x
MEAN
Cx
50
40
30
20
10
10
20
30
40
V7,x
FIG. (13)


REGRESSION
OF Cx ON V5,x
MEAN
Cx
50
40
30
20
10
10
20
30
40
50
60
V5,x
FIG. (14)



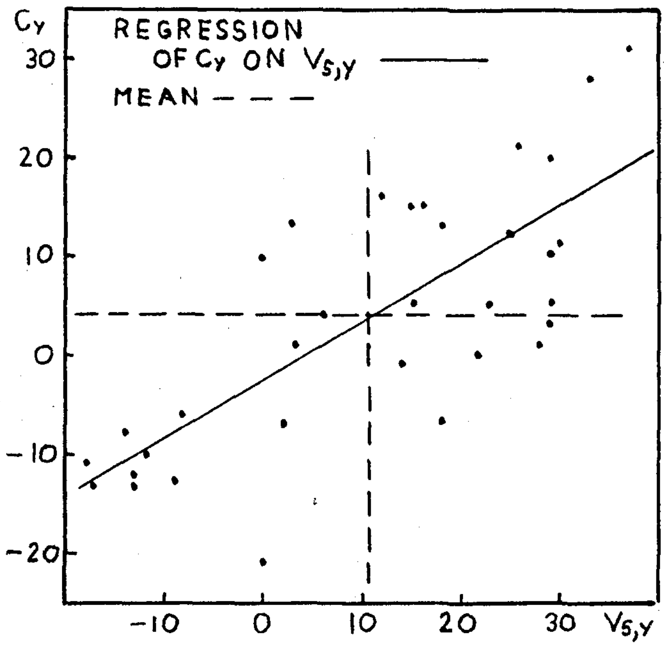
REGRESSION OF C_x ON $V_{3,x}$
MEAN
C_x
50
40
30
20
10
40
50
60
70
80
90
100
120
$V_{3,x}$

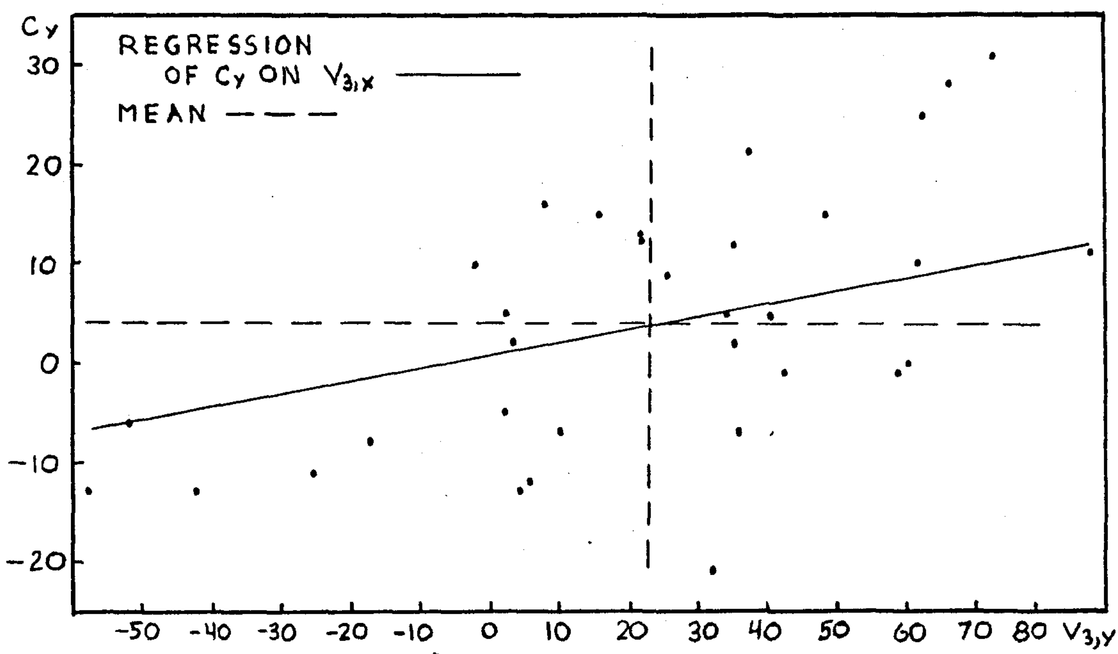
FIG. (15)



REGRESSION OF C_y ON $V_{7,y}$
MEAN
C_y
30
20
10
0
-10
-20
-10
0
10
20
$V_{7,y}$

FIG. (16)


Cy
REGRESSION
OF Cy ON $V_{5,y}$
MEAN
30
20
10
0
-10
-20
-10
0
10
20
30
$V_{5,y}$
FIG. (17)


Cy
REGRESSION
OF Cy ON $V_{3,x}$
MEAN
30
20
10
0
-10
-20
-50
-40
-30
-20
-10
0
10
20
30
40
50
60
70
80
$V_{3,y}$
FIG. (18)

$$r_{c_y \cdot v_{7,y} v_{5,y} v_{3,y}} = 0.77$$

$$S_{c_y \cdot v_{7,y} v_{5,y} v_{3,y}} = 8.50$$

$$C_y = .59\, V_{7,y} + .41\, V_{5,y} + .09\, V_{3,y} - 0.24$$

5. Comparison D. Correlation of the Movement of Cyclones to the Thermal Wind for the 1000-700 mb. Layer.

The final comparison was made between the speed and direction of the cyclones and the speed and direction of the 1000-700 mb. thermal wind. The thermal wind speeds and directions at the beginning and end of the 12-hour period of cyclone motion were averaged to represent the mean thermal wind speed and direction over the cyclone during the period. Additional data for the month of November, 1947 was included in this test increasing the number of test cases to 54.

Comparison of the direction of motion to the direction of the thermal wind yielded a wide range of deviation angles of the cyclone's movement from the thermal wind, varying from -100° to $+48^\circ$* with a maximum grouping at -10°. The frequency distribution is listed in Table IV.

* Minus angle denotes movement of the cyclone to the left of the thermal wind, plus angle denotes movement to the right.

Dev. Angle	-55°	-50°	-40°	-30°	-20°	-10°
Frequency	3	3	2	4	9	10
Dev. Angle	0°	10°	20°	30°	40°	45°
Frequency	9	4	3	3	2	2

Table IV. Frequency Distribution of Cyclone Deviation From the Thermal Wind.

Such a frequency distribution approximates a normal curve, as is shown in figure 19.

Comparison of the speed of cyclone movement to the thermal wind was accomplished by computing the ratio (C/T_7) of the former to the latter. This computation likewise yielded a wide range of ratios, varying from 0.30 to 1.85 with a maximum frequency occurring at 0.75 (See Table V for detailed frequency distribution). As before, this frequency distribution can be approximated by a normal curve (See figure 20).

C/T_7	.30	.35	.45	.55	.65	.75	.85
Frequency	1	2	2	7	8	10	9
C/T_7	.95	1.05	1.15	1.25		1.35	1.40
Frequency	7	2	1	2		1	2

Table V. Frequency Distribution of the Ratio of Cyclone Speed to the Thermal Wind Speed.

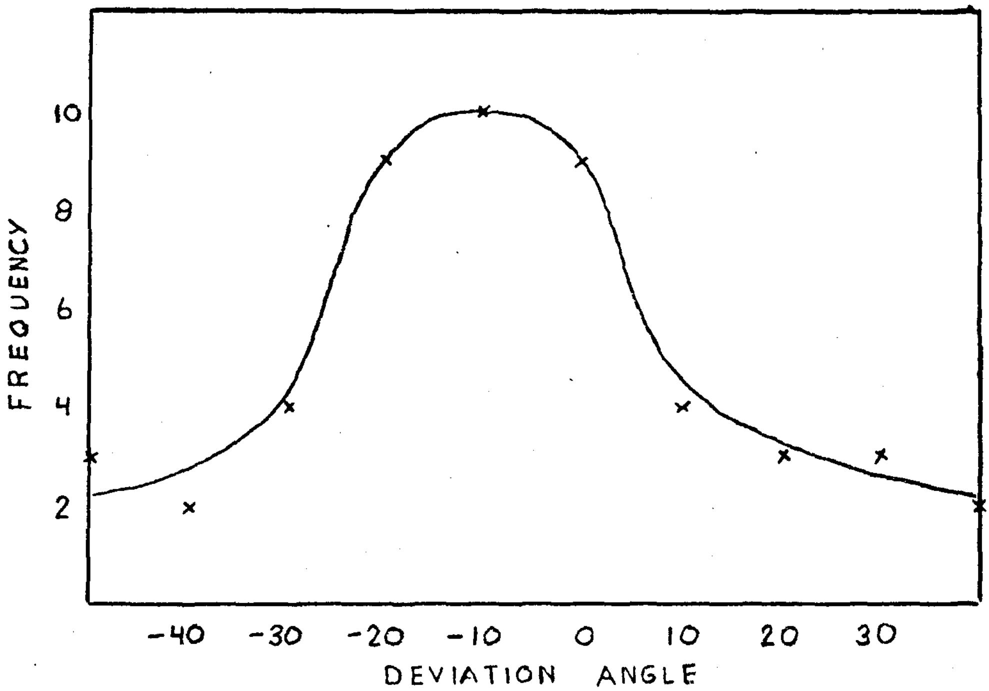

FREQUENCY
10
8
6
4
2
-40
-30
-20
-10
0
10
20
30
DEVIATION ANGLE

Fig. (19)

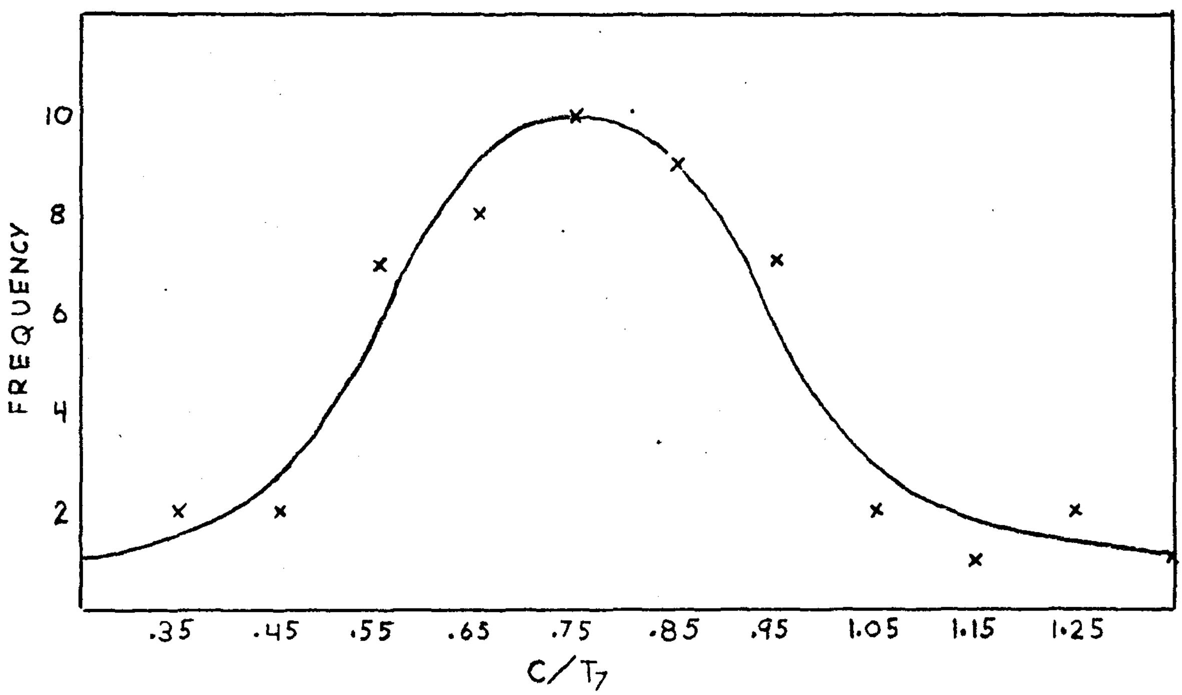

FREQUENCY
10
8
6
4
2
.35
.45
.55
.65
.75
.85
.95
1.05
1.15
1.25
C/T7

Fig. (20)

CHAPTER IV

SUMMARY OF RESULTS

1. Evaluation of Comparisons A, B, and C.

One may observe from the scatter diagrams that a positive relation exists between the variates; however, the scatter in all cases is large and the pattern is not well defined. The simple correlation coefficients are thus positive and are relatively large for meteorological data. The fact that the correlation is positive and large in every case indicates that when the winds over a system increase (decrease) the speed of movement of the surface low pressure system will also increase (decrease) and that the direction of the movement will in general follow the orientation of the isolines over the system. The magnitudes of simple correlation of the x-components vary from 0.36 to 0.68, of the y-components from 0.57 to 0.75. It would appear that a better correlation to north-south motion exists than to east-west motion. One may observe that the units of standard deviation and the standard errors of estimate are both large as compared to the mean of their respective components.

The ultimate objective of the investigation was the computation of multiple correlation coefficients, and thence the development of regression equations from which the easterly and northerly components of cyclone movement could be estimated. The multiple correlation coefficients in the three comparisons were not significantly better than the simple coefficients. Although large for meteorological data, these multiple correlations are somewhat small to be useful in objective forecasting. As was expected, the introduction of the gradient wind for the geostrophic wind produced a small improvement in correlation. Some slight similarity exists in the regression coefficients, the y coefficients being generally larger than the x coefficients. Also, in Comparison A greater weight appears to be given to the lower layers, this being in agreement with Petterssen's results.[2] Apart from these similarities, the regression coefficients are more or less heterogeneous in nature. The standard errors of estimate, though reduced somewhat from those of the simple correlations, are still large in comparison with the magnitude of average surface movement.

2. Evaluation of Comparison D.

Observe from Comparison A that the correlation of the easterly component of surface motion with the easterly component of the 1000-700 mb. thermal wind was 0.47. Likewise, for northerly components it was 0.74. For the 700-500 mb. thermal wind the correlations were 0.36 and 0.68 for the respective easterly and northerly components. If both the 1000-700 mb. and 700-500 mb. layers are introduced into the correlation, the coefficients are increased to 0.58 and 0.75

respectively. The regression equations derived from these coefficients are:

$$C_x = .39\,T_{7,x} + 11.09$$

$$C_y = .48\,T_{7,y} + 2.78 \quad .$$

Because of the small magnitude of the above coefficients and of the large value of the standard errors of estimate involved, these equations cannot in general be used quantitatively.

To visualize better the qualitative value of the thermal wind, especially in a more practical qualitative form, consider the results of Comparison D. It was observed that the deviation angle whose frequency was a maximum was -10°; that is , the surface low pressure center tends to be steered most frequently about 10° to the left* of the 1000-700 mb. thermal wind. However, the curve of frequency distribution, figure 19, is relatively flat; thus, this deviation of -10° occurs only 19% of the time. A deviation from 0° to -20° occurs 50% of the time (See Table IV). The comparison of the speed of cyclones to the 1000-700 mb. thermal wind speed

* This result does not agree with that of Austin whose data indicated that the major deviation was to the right of the thermal wind.

indicates a maximum frequency of occurrances of the ratio C/T_7 of 0.75. Again, the flatness of the curve (See figure 20) is such that this ratio occurs only 19% of the time. 76% of the cases have ratios ranging from 0.55 to 0.95. We must conclude then that the modal values, because of the spread of the frequency distribution, do not lend themselves to quantitative forecasting. We may conclude that they will be of assistance in making qualitative estimates of the movement of cyclones, and should be a helpful supplement to other forecasting tools.

Author's Comment.

The author regrets that time limitations did not permit the assembly and test of more data; also, that time was not available to expand the investigation to a statistical examination of the influence of the upper winds on the movement of anticyclones. It is believed that, though inclusion of additional data may alter somewhat the regression coefficients and perhaps may indicate a more striking similarity between all of the regression equations, the magnitude of the correlations obtained will not be substantially changed.

APPENDIX I

METHOD OF CORRELATION[10]

The calculation of correlation in this paper is based on the assumption that the relationship of two co-variates is linear. Positive correlation signifies an increase of one variate as the other increases; likewise, a decrease of one when the other decreases. A simple correlation coefficient is defined as

$$r_{x_1, x_2} = \frac{\sum (x_1 - \bar{x}_1)(x_2 - \bar{x}_2)}{\sigma_{x_1} \; \sigma_{x_2}} ,$$

where x_1 and x_2 are the co-variates. $\bar{x}_1$ and $\bar{x}_2$ are the arithmetic means of x_1 and x_2 respectively. σ_{x_1} and σ_{x_2} are the standard deviations of x_1 and x_2 from the mean, and are defined by the formula

$$\sigma_x = \left[\frac{1}{N} \sum (x - \bar{x})^2 \right]^{\frac{1}{2}} .$$

N is the number of co-variates considered.

A linear regression line of two variates is written in the form

$$(x_1 - \bar{x}_1) = r_{x_1 x_2} \frac{\sigma_{x_1}}{\sigma_{x_2}} (x_2 - \bar{x}_2) \quad .$$

Regression of x_1 on x_2 simply says that the above equation of the form $x_1 = ax_2 + b$ may be used to estimate x_1 when x_2 is known. The development of a regression of x_1 on x_2, as represented in the above equation, permits the estimation of x_1 only when x_2 is known. This equation may not be used to estimate x_2.

A standard error of estimate S is a mean of the differences between an observed x_1 and an estimated x_1. It is , therefore, a measure of the mean error involved when x_2 is used to estimate x_1; and thus, is a standard for judging the reliability of the derived regression equation. S is by definition

$$S_{x_1 x_2} = \sigma_{x_1} (1 - r^2_{x_1 x_2})^{1/2} \quad .$$

The calculation of multiple correlations of a four variate relationship between x_1, x_2, x_3, and x_4, such as presented in this paper, is also based on the assumption of linear regression. A multiple correlation coefficient is determined from the relation

$$r_{1\cdot 234} = \left(1 - \frac{R}{R_{11}}\right)^{1/2} ,$$

where R is the determinant

$$R = \begin{vmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ r_{21} & r_{22} & r_{23} & r_{24} \\ r_{31} & r_{32} & r_{33} & r_{34} \\ r_{41} & r_{42} & r_{43} & r_{44} \end{vmatrix}$$

and R_{11} is the minor of r_{11}

$$R_{11} = \begin{vmatrix} r_{22} & r_{23} & r_{24} \\ r_{32} & r_{33} & r_{34} \\ r_{42} & r_{43} & r_{44} \end{vmatrix} .$$

The standard error of estimate of the regression of x_1 on x_2, x_3, and x_4 is defined as

$$S_{1\cdot 234} = \sigma_1 \left(\frac{R}{R_{11}}\right)^{1/2} .$$

The regression equation of x_1 on x_2, x_3, and x_4 is given by the formula

$$(x_1 - \bar{x}_1)\frac{R_{11}}{\sigma_{x_1}} + (x_2 - \bar{x}_2)\frac{R_{12}}{\sigma_{x_2}} + (x_3 - \bar{x}_3)\frac{R_{13}}{\sigma_{x_3}} + (x_4 - \bar{x}_4)\frac{R_{14}}{\sigma_{x_4}} = 0 .$$

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