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# Military Cost-Benefit Analysis: A Multi-Attribute Three-Stage Procurement Model

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Acquisition Research Program:  
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## **Military Cost-Benefit Analysis: A Multi-Attribute Three-Stage Procurement Model**

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Defense Resources Management Institute  
Naval Postgraduate School

# Introduction

- Large & rising federal debt, shrinking discretionary budget
  - Budget uncertainty!
- Defense procurement typically requires vendors to submit bids which include
  - Price
  - Performance attributes
- Problem: Optimal vendor choice may change with changes in the budget!



# Budget Constraint

- Based on an “Economic Evaluation of Alternatives” (EEoA)\* approach:
  - The procurement agency buyer reveals desired attributes and the budget for the program
  - Vendor offers (bids) consist of product proposals to produce a set of performance attributes for a given budget authority
  - The procurement agency buyer selects a vendor according to the buyer’s (“secret”) weighting of the attributes (i.e. a multi-attribute value function)

\* See pp. 25-28 in Melese, F. “The Economic Evaluation of Alternatives,” Proceedings of the 6<sup>th</sup> Annual Acquisition Research Symposium: Defense Acquisition in Transition, Vol 1.



# Model Structure

Stage 1

Buyer specifies attribute set and budget information

Stage 2



Stage 3

Buyer selects winning bid according to its value function



# Model

- $n$  vendors
- Set of attributes  $A = (1, \dots, m)$
- Vendor  $i$ 's offer is  $A_i = [a_{i1}, \dots, a_{im}]$
- Buyer's "secret" value function (MOE) is  $V(A_i)$
- Budget level is  $B$
- Buyer makes selection decision according to:

$$\max_i V(A_i) = \sum_{j=1}^m w_j a_{ij}$$



# Vendor's Decision Problem

- Private information on production capabilities and costs:
  - Captured by cost functions  $c_{ij}(a_{ij})$
- Does not know  $V$ , but forms beliefs about the buyer's preferences
- “Best guess”  $\gamma_i = (\gamma_{i1}, \dots, \gamma_{im})$
- Results in a hypothetical value function to maximize:  $Q(A_i) = \sum_{j=1}^m \gamma_{ij} a_{ij}$



# Vendor's Decision Problem

- Vendor  $i$ 's problem can be expressed as:

$$\begin{aligned} \max_{a_{ij}} \quad & Q(A_i) = \sum_{j=1}^m \gamma_{ij} a_{ij} \\ \text{s.t.} \quad & TC_i = \sum_{j=1}^m c_{ij} (a_{ij}) \leq B \end{aligned}$$





# Simplified Approach

- For the sake of clarity, the remainder of the analysis will assume:

**Two attributes**

**Two vendors**



# Solution to Vendor's Problem

- A vendor's best offer (bid) will be a combination of attribute levels that uses the entire budget, and satisfies the condition:

$$\frac{\gamma_{i1}}{c'_{i1}(a_{i1})} = \frac{\gamma_{i2}}{c'_{i2}(a_{i2})}$$

- The buyer then chooses the vendor that maximizes its military effectiveness value,  $V$ , for the planned budget,  $B$



# Budget Uncertainty

- Now, instead of  $B$ , consider a range of possible budgets:  $B_1, \dots, B_k$
- Each vendor submits an offer (bid) for each of the  $k$  possible budgets
- This set of offers from a vendor constitutes an “expansion path”



# Examples

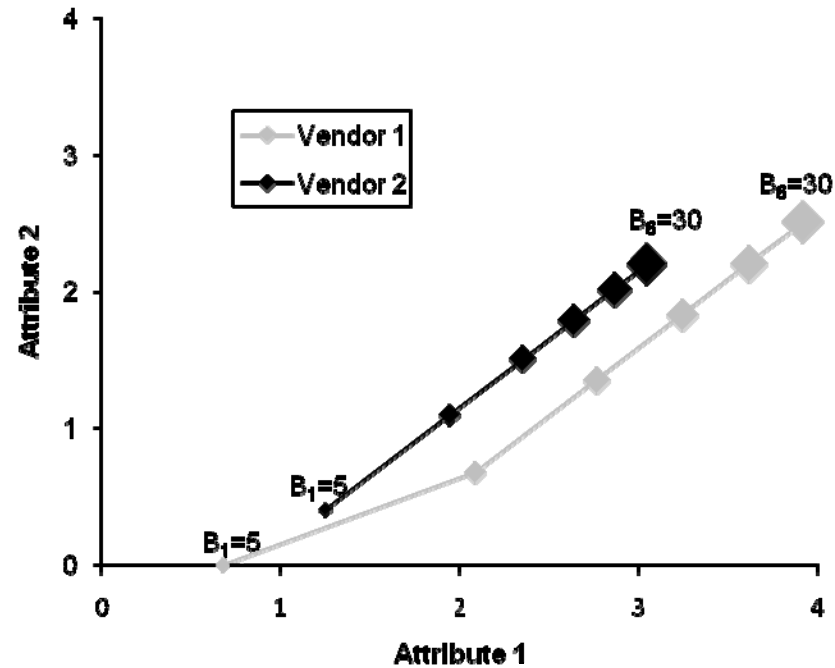
- Let the vendors have cost functions of the form:

$$c_{ij}(a_{ij}) = \alpha_{ij} e^{\beta_{ij} a_{ij}}, \text{ where } \alpha_{ij}, \beta_{ij} > 0$$

- $B_1=5, B_2=10, B_3=15, B_4=20, B_5=25, B_6=30$
- We will examine several cases where the vendors differ in their cost functions and/or beliefs about the weight the buyer places on the attributes



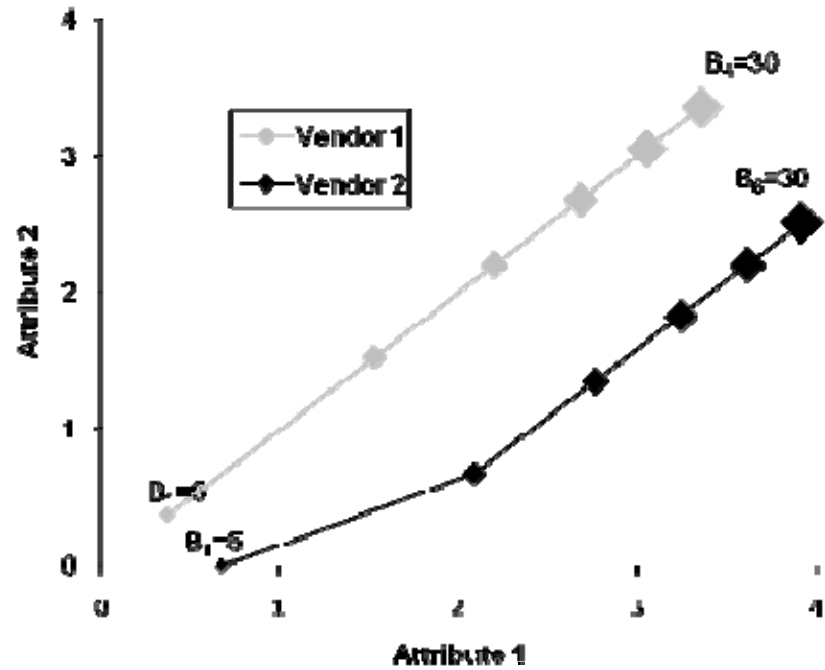
## Expansion Paths - Differing Cost Functions



$$\alpha_{11} = \alpha_{12} = 2.0, \beta_{11} = \beta_{12} = 0.6, \alpha_{21} = \alpha_{22} = 1.0, \beta_{21} = \beta_{22} = 1.0, \gamma_{11} = 0.7, \gamma_{21} = 0.7$$



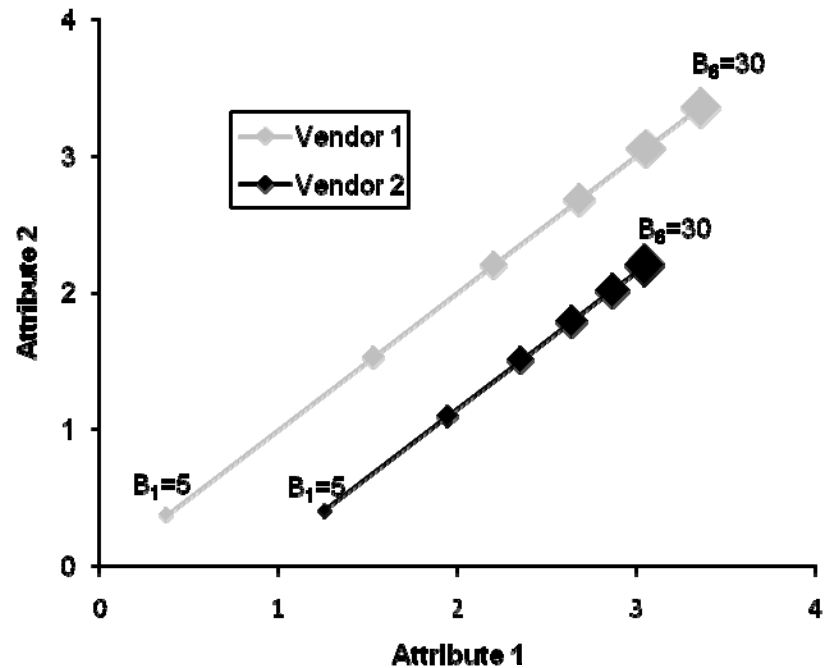
## Expansion Paths - Differing Beliefs ( $\gamma$ )



$$\alpha_{11} = \alpha_{12} = \alpha_{21} = \alpha_{22} = 2.0, \beta_{11} = \beta_{12} = \beta_{21} = \beta_{22} = 0.6, \gamma_{11} = 0.5, \gamma_{21} = 0.7$$



## Expansion Paths - Differing Beliefs and Cost Functions



$$\alpha_{11} = \alpha_{12} = 2.0, \beta_{11} = \beta_{12} = 0.6, \alpha_{21} = \alpha_{22} = 1.0, \beta_{21} = \beta_{22} = 1.0, \gamma_{11} = 0.5, \gamma_{21} = 0.7$$



# Switch to Budget-Value Space

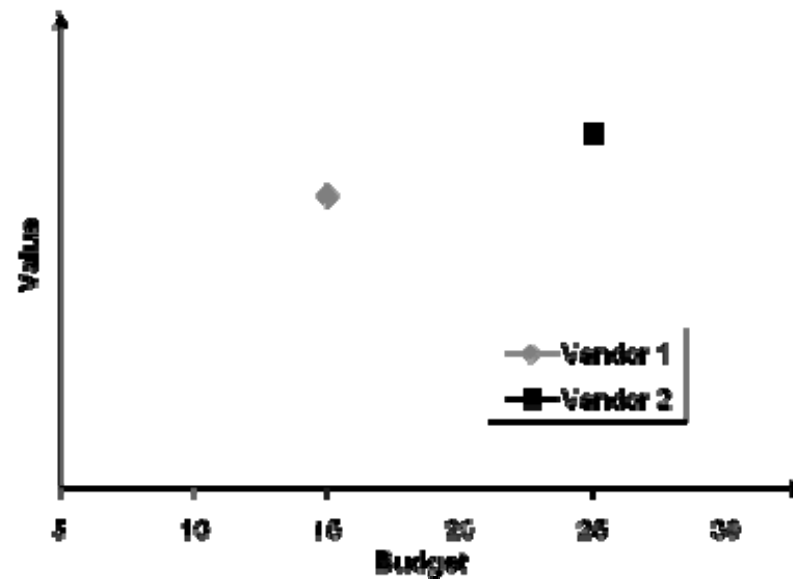
- What is the value to the buyer (procurement agency; warfighter) provided by each vendor for a specific budget authority?
- What is the value to the buyer provided by each vendor over all possible budget levels?
- Assume the two vendors have the properties from the last graph, and that the buyer places a weight of 0.7 on attribute 1





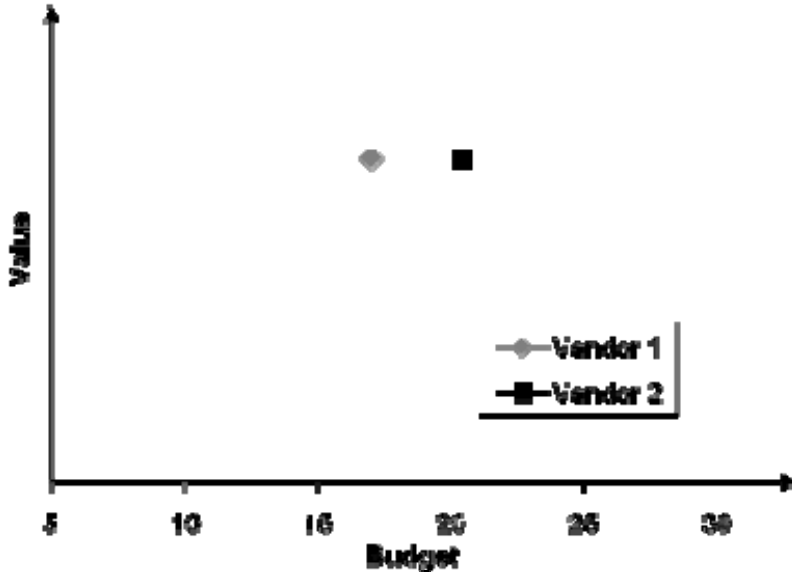
# Traditional Price & Performance Bid

## Value by Budget Level

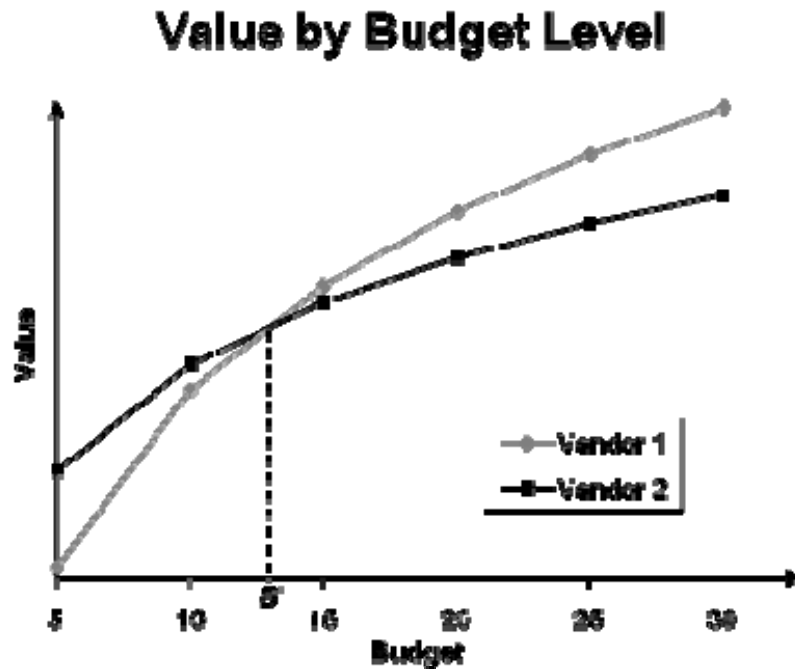


# Air Tanker Costs for Given Level of Effectiveness (Boeing vs. EADS?)

Value by Budget Level



# Vendor Bids: Performance Offers over a Range of Budgets



# Next Steps

- Model the budget uncertainty with a probability distribution, and determine the expected utility provided by each vendor
- Include uncertainty in vendor performance (quantity, quality, schedule) promises
  - May be framed as either cost uncertainty or performance uncertainty or both (depends on the particular contract structure)

