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Radar Bomb Scoring with Computer Controlled Bombing Systems

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THESIS

RADAR BOMB SCORING
WITH COMPUTER CONTROLLED
BOMBING SYSTEMS

by

Kurt Lee Keene

September 1974

Thesis Advisor: D.R. Barr

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Radar Bomb Scoring
With Computer Controlled
Bombing Systems

by

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I. INTRODUCTION

The measurement of bombing accuracy based on impact analysis has received and, undoubtedly, will continue to receive a great deal of attention. Such analysis is indispensable in the development, testing and evaluation of both ordnance and delivery systems. For an operational system, however, the military emphasis shifts to operator training and proficiency and tactical evaluations under varying conditions. The requirement to physically drop bombs, either training or tactical, in order to evaluate these exercises imposes severe limitations in terms of location, time and cost. The desirability of a method for predicting delivery accuracy from radar observed release conditions rather than impact data is then apparent.

The purpose of this thesis is to present the development of a proposed method for radar bomb scoring. Section II outlines the radar bomb scoring problem in general terms with emphasis on the requirements which must be met in order to obtain a reasonably accurate prediction model. Section III presents a discussion of a proposed radar bomb scoring model. The desire was to develop a system which would not require an extensive background in mathematics or statistics on the part of the bomb scoring personnel.

The impetus for this thesis was provided by some related CEP and bomb dispersion analysis for the AN/TPQ-27 radar bombing system which was conducted at the Naval Postgraduate School [Ref. 4]. The proposed model suggests the use of a computer assisted, radar controlled bombing system with the AN/TPQ-27 application serving as a prototype example.

The proposal envisions a computer software package for the AN/TPQ-27, or similar system, which would provide a direct readout of release condition errors or deviatiations from predetermined release conditions. Through ballistic considerations, these deviations are then translated into range and deflection aim errors on the ground. Computation of the estimated CEP is accomplished by means of a model based on the non-central chi-square distribution, where the non-centrality parameter is a function of the computed range and deflection aim errors.

II. GENERAL DISCUSSION OF RADAR BOMB SCORING

The concept of radar bomb scoring, although not new, is not yet fully developed. Reasons for this become more apparent upon examination of the many factors, some technical and some analytical, that must be considered.

A. THE PROBLEM OF PREDICTING BOMBING ACCURACY FROM RADAR DATA

Estimating bombing accuracy from samples of observed bomb impacts has occupied many analysts since the introduction of air-delivered weapons. Results of these efforts are reflected in many predictive models of Circular Error Probable (CEP) that have been proposed. Radar bomb scoring, on the other hand, compounds the analyst's problem by denying him the use of observed bomb impacts. The result is that the desired estimate of accuracy must be based on a prior estimate of where the bomb would have impacted had it been dropped. The many factors which influence this latter estimate are the subject of the remainder of this section.

1. Release Condition Dependence

For the purposes of this discussion, the delivery of a weapon from an aircraft may be considered to consist of two distinct phases. The first of these, which may be termed the positioning phase, includes the period of time from approach of the aircraft to the desired release point to that instant following release when the weapon is no longer influenced by the aircraft. The second, or free fall phase, begins upon termination of the positioning phase and ends on impact of the weapon in the ground plane.

The critical point, as far as radar bomb scoring is concerned, occurs at the juncture of these phases. It is at this time that the release conditions for the weapon are determined. The release conditions, in turn, become the initial conditions for the ballistic problem encountered in the free fall phase. The situation is analogous to that of computing rocket trajectories in which the powered and unpowered portions of the rocket trajectory correspond to the positioning and free fall phases of the bombing problem.

Obviously, there are a multitude of factors which determine what release conditions will be met for any given bomb drop. Prior to the instant of release, the bomb is subjected to the same aerodynamic forces and atmospheric perturbations that affect the flight of the delivery aircraft. As a result, one could expect the bomb to possess any one of an infinite set of velocity and acceleration components at the time of release. Compounding the problem is the air turbulence in the vicinity of the aircraft which exerts additional forces on the bomb even after physical separation of weapon and aircraft.

In addition to the dynamic state of the bomb at release, equal consideration must be given to the location of the release point relative to the target and the atmospheric conditions which prevail at the time.

All of these factors constitute the release conditions which, to a great extent, determine the eventual point of impact of the bomb. Not considered yet are the aerodynamic and gravitational forces present during the free fall phase which will further determine the point of impact. More will be said about these forces in subsequent discussion of the ballistic problem.

2. Measurement of Release Conditions

The intent of the preceding discussion of release conditions was to emphasize the magnitude of the aim point estimation problem and to hint at the direct relationship between the accuracy of the estimation and the accuracy with which the release conditions are measured.

With the possible exception of the transient effects of aircraft turbulence alluded to earlier, it is possible to measure quite accurately many forces acting on the bomb at the time of release. To do so, however, would require extensive instrumentation aboard the delivery aircraft and the means to transmit these measurements to the ground for analysis. The most attractive attributes of radar bomb scoring, i.e., low cost and flexibility, may be lost in an attempt to obtain the most accuraft measures possible.

An alternative is to base the trajectory estimation on data which can be obtained from a radar, or radars, tracking the delivery aircraft. This greatly simplifies the computational complexity of the estimation problem by limiting the parameters to be considered in any equations of motion. In general, it may be assumed that the radar could provide information on the coordinate location of the aircraft at time of release, as well as aircraft velocity and acceleration.

While this may seem like sketchy information on which to compute a bomb trajectory, it must be remembered that other input variables are available from sources outside the radar-aircraft system. Meterological data and the effects of earth curvature and rotation may be input as standards for the location of the bomb drop. In addition, bomb parameters such as drag curves may be available for the type bomb being dropped.

While many of the parameters mentioned have been measured precisely, the ultimate accuracy of the predicted impact point will depend on the accuracy with which the radar can measure the release conditions - location, velocity and direction of flight. The degree of accuracy obtainable is a function of the radar being used and will vary from one type to another.

An extensive discussion of radar errors and their determination is beyond the scope of this thesis. Reference 5 describes the problems associated with radar measurements and how these system errors may be quantified. This reference further points out the manner in which error magnitudes vary with the dynamics of the radar-aircraft system. These errors in measurement will, in general, be a function of the aircraft movement relative to the radar as well as the direction from the radar to the aircraft.

The effect of this on the development of an acceptable radar bomb scoring system is clear. The error functions for the radar employed should be well defined and generally applicable to all radars of that type. The use of multiple radars complicates the measurement problem due to the difficulties in calibration and collimation necessary to obtain an estimate of the "true" release conditions. It may be inferred that the introduction of a variety of types of radars into a bomb scoring system complicates the estimation problem for the same reasons.

3. Implications of the Bombing Mode

The problem of radar bomb scoring, in particular the determination of release conditions and estimation of the point of impact, changes considerably with the bombing mode employed. The intent here is to distinguish between bombing maneuvers which are pilot controlled (with or without the aid of on-board fire control equipment) and those which are computer controlled from a ground station.

a. Pilot Controlled

In this mode, the pilot, using pre-calculated release parameters, is free to attack the target from any point where these release parameters can be met. The only link between the delivery aircraft and the bomb scoring system is the radar tracking system and voice communications.

The problem of bomb scoring in this mode involves computing an estimated point of impact based solely on the release conditions as measured by the radar. The inaccuracies inherent in this system are due, in large part, to factors for which data cannot be provided by radar. Examples are deviations in dive angle, small variations in release velocity and direction and any last second violent maneuvers or gust perturbations which significantly affect the dynamics of the aircraft at release.

b. Computer Controlled

In this case, an integrated computer-radar-autopilot system attempts to control the aircraft to a predetermined point in space where release of the weapon occurs automatically. Calculation of the release point involves parameters which might significantly effect the weapon trajectory.

The advantage of this mode, insofar as radar bomb scoring is concerned, is the fact that there exists a continuous feedback of data between the aircraft and computer. The trajectory problem may be continuously solved to adjust for deviations in the desired release conditions. At release, many of the variables not obtainable in the pilot controlled mode are automatically input to the final computer solution of predicted impact.

B. ANALYSIS OF RADAR DATA AND ESTIMATION OF CIRCULAR ERROR PROBABLE

The process of radar bomb scoring has been described as an estimation problem involving the point of impact and the desired measure of accuracy, CEP. Each of these is dependent upon the radar measurement of release conditions and their accuracies. In this section, some of the problems encountered and methods that might be used in obtaining these estimates are discussed under the assumption that the release conditions are obtainable and known.

1. The Ballistic Problem

Most discussions of exterior ballistics for projectiles or bombs begin with developments of basic equations relating position with velocity components, time and the gravitational constant for trajectories that take place in a vacuum over a flat, non-rotating earth.

These equations provide a rough approximation of the distance a bomb will travel if released at a specified altitude and velocity in a specified direction. Unfortunately, this approximation does not provide the degree of accuracy necessary for a meaningful bomb scoring system unless corrected for more realistic conditions. Even more unfortunate is the fact that these basic equations exhaust the data available from radar measurements alone. It becomes obvious then that a radar bomb scoring system must have available much more information than is available from radar measurements.

The ballistic equations of motion in a useful form include a rather complete system of aerodynamic forces, variable winds, density and temperature variations, the effects of earth curvature and rotation and bomb parameters such as weight, diameter and configuration. The parameter values necessary to the solution of these more accurate equations may be available from sources outside the bomb scoring system. However, as is pointed out by Mc Shane, Kelley and Reno [Ref. 16], computer assistance is required for solution of such equations.

2. Estimating Point of Impact

The complexity of the equations of motion from ballistic considerations may make them too cumbersome for routine use. However, much of the work in computing trajectories from equations of this type has been accomplished and documented in the form of trajectory and bombing tables.

The use of these tables provides a quick and computationally simple means of computing the expected point of impact when the appropriate corrections for existing local conditions are applied to the tabled values. Furthermore, tables are available for all ordnance of interest to a bomb scoring system [Ref. 20].

The rationale for suggesting the use of pre-tabled data is quite simple. The use of these tables in precalculating desired bomb release conditions is accepted practice in a combat situation where the requirement for accuracy is critical. The requirement for greater accuracy for the radar bomb scoring function does not seem to be justified.

Estimation of the point of impact from existing trajectory and bombing tables appears appropriate for both the pilot controlled and the computer controlled modes of bombing. In the former case, the radar measurements of release conditions provide the points of entry into the appropriate tables. The tables provide range as a function of release altitude above the target and release velocity under assumed atmospheric conditions. To these tabled values, corrections due to non-standard conditions may be applied. With the direction of bomb release known, the estimated point of impact may be determined. Comparison of this point with the target location yields the desired estimates of range and deflection miss distances.

In the case of computer controlled bombing the procedure is somewhat different. Local wind conditions and atmospheric data, target and radar data and weapon ballistics are preset inputs to the computer. The desired release conditions calculated from these data provide the best available estimate of the release conditions which will place the expected point of impact on the target.

If it can be assumed that deviations from the desired conditions at release are relatively small and detectable, then only the magnitudes of these deviations need be considered in estimating range and deflection errors. The restriction that these deviations be small is necessary to insure that both the desired and achieved trajectories are subjected to very nearly the same conditions and forces. The trajectories will then be theoretically nearly identical in shape, so the release deviations may be translated through simple relationships to range and deflection aim errors in the target plane. It is then feasible to pre-calculate and tabularize range and deflection errors as a function of deviations in actual release conditions from desired release conditions.

A detailed discussion of how these deviations are translated and combined into total range and deflection error estimates is presented in Section II.

3. Estimating CEP

The most widely used measure of accuracy of air-to-surface weapons, CEP, is defined as the radius of a circle, centered at the target, which on the average contains fifty percent of the impact points of independently aimed weapons. Usually, the determination of this radius, or CEP, involves the assumption of some probability distribution of bomb impacts. Specifically, if x and y are the range and deflection components of the impact point and each is assumed to have some underlying probability distribution, then CEP is that value of R which satisfies

$$P[(x^2 + y^2) \leq R^2] = \frac{1}{2}. \qquad (1)$$

Jordan [Ref. 12], in a comprehensive survey of existing models for the estimation of CEP, discusses their salient features. The factor which distinguishes one from the other is, of course, the assumed distribution of impact points.

In developing a predictive model of CEP from a sample of observed data, one goal is to find some distribution which reasonably fits the observed data. The distribution parameters are often estimated from the sample. The controversy that arises over which distribution is most appropriate is a natural consequence of the factors which contribute to the distribution. These factors will certainly vary from one weapon delivery system to another. The result is that a model developed to estimate the accuracy of a given aircraft-weapon combination may or may not be acceptable for some other combinations, even though the release conditions are similar.

It may be possible, though tedious, to incorporate many distributions in an elaborate model, thus providing some selectivity according to the aircraft-weapon combination being scored. Conversely, the model could be designed on the basis of some general distribution type that is at least roughly descriptive of nearly all situations. Logistically, the latter approach is the more attractive option although obviously less accurate.

Having assumed a distribution for the points of impact, the determination of R from (1) may require estimation of the distribution parameters. We assume the parameters to be $\underline{\mu}$ and $\underline{\Sigma}$. The location of the mean of the distribution is determined by the estimate of the point of impact since each estimation problem must be based on a sample of size one. The remaining parameter of interest is the standard deviation, σ.

In the discussion thus far, it has been suggested that knowledge of release conditions permits the estimation of a mean point of impact through purely deterministic relationships. Under this assumption, the only allowable dispersion about the estimated mean point of impact is that due to ballistic dispersion. The standard deviations of ballistic dispersion in range and deflection then become the estimated parameter values for the impact distribution.

Values of ballistic dispersion for individual weapons are not available. However, the Joint Munitions Effectiveness Manual [Ref. 18] contains general expressions for ballistic dispersion as a function of range and bomb configuration. These expressions provide values of σ_D and σ_R which have been generally agreed upon by all the military services. It should be noted that values of these parameters are classified when applied to a specific weapon. For this reason, the values chosen for illustrative purposes in

Appendices B and C were selected for computational ease and are not intended to be representative of any specific weapon.

C. COMMENTS

From the complexity of the problem it should be apparent that successful radar bomb scoring cannot be performed using radar measurements alone, together with manual computations. The simplification required to make such a system manageable would lead to an unacceptable loss of accuracy. One alternative is an integrated radar-computer-autopilot system which, because of its capability to detect and compensate for additional delivery variables, could provide a much better estimate of bombing accuracy.

Tactical radar directed bombing systems incorporating the desired features of a radar bomb scoring system are currently in operation or under development. The modification of a tactical system of this type to perform the bomb scoring function has several distinct advantages over the development of a dedicated system.

- The system would be available in a much shorter period of time.
- Cost would be relatively small for the modifications required.
- Bomb scoring exercises would provide training for the bomb system crews as well as the air crews.

The modifications envisioned to perform this function should be relatively minor. The primary requirement is that the deviations of actual from desired release conditions, at the time of release, be output from the computer at the conclusion of each bomb run. Alternatively, the computer could be programmed to provide $\underline{\mu}$ directly. No hardware changes should be necessary, and the tactical functions of the system should be unaffected.

Under the assumption that the desired readout of release condition deviations can be made available, the following section describes a proposal for transforming these deviations into an estimate of achieved CEP.

III. RADAR BOMB SCORING: A PROPOSAL

In view of the preceding comments, it is suggested that a feasible approach to a radar bomb scoring function lies in the modification of a tactical radar bombing system. Using this approach, it is possible to arrive at an estimate of CEP by observing and recording only deviations from desired release conditions.

The procedures used in calculating these estimates are developed in the following sections. No claim of originality is made for this material. Rather, the intent has been to put together a number of simple relationships which can be easily applied.

In the comments about the proposed model, some thoughts on the accuracy of the model and areas for continued investigation are presented.

A. PREDICTION OF POINT OF IMPACT FROM RADAR OBSERVED RELEASE CONDITIONS

The basic contention that the point of impact may be estimated from observed release conditions requires some attention before specific relationships may be considered.

From the previous discussion of the ballistics problem, it was indicated that, in a vacuum over a flat, non-rotating earth, the range of a bomb could be calculated from the relationship

$$x = f_o(\dot{x},t) \quad . \tag{2}$$

The time of fall, t, is known to be a function of altitude, velocity and the acceleration due to gravity so that (2) may be rewritten as

$$x = f_1(\dot{x},\dot{z},z,g) \quad .$$

In order to use this relationship in a realistic situation, it is necessary to apply a correction factor so that

$$x = f_1(\dot{x},\dot{z},z,g) - C_f \quad , \tag{3}$$

where

$C_f = f(\dot{x},\dot{z},z,T,\rho,W,B,E,D)$
T = air temperature
ρ = air density
W = wind effects
B = bomb parameters
E = earth curvature and rotation effects
D = drag forces on the bomb.

The computer solution of (3) in a tactical bombing system results in a desired range, x_d, which will place the point of impact on the target. Any deviations in the release conditions will result in an achieved range, x_a, so that the expected range error may be found from

$$x_a - x_d = f_1(\dot{x}_a,\dot{z}_a,z_a,g) - f_2(\dot{x}_d,\dot{z}_d,z,g) - [C_{f_a} - C_{f_d}] \quad . \tag{4}$$

If the differences between the desired and achieved release conditions are small, then the third term of (4) is very small and may be neglected. The result is a simple relationship of the form

$$x_a - x_d = f_1(\dot{x}_a, \dot{z}_a, z_a, g) - f_2(\dot{x}_d, \dot{z}_d, z_d, g) \tag{5}$$

which calculates range error as a function of velocity and location errors at release. The functional form of C_f need not be known or considered in the estimation process. An analogous argument may be used in the development of an expression for deflection error.

1. The Coordinate System

The coordinate system and notation used in the remainder of this paper is shown in Figure 1. The system is centered on the target and oriented by the location of the desired release point and the target. The release angle, θ, is measured clockwise from any convenient reference. Mean deflection and mean range error are denoted by the quantities μ_D and μ_R respectively.

2. Translating Release Conditions to Errors in the Ground Plane

The assumptions pertinent to the development of the following error estimates have already been discussed in some detail but are repeated here for continuity and completeness:

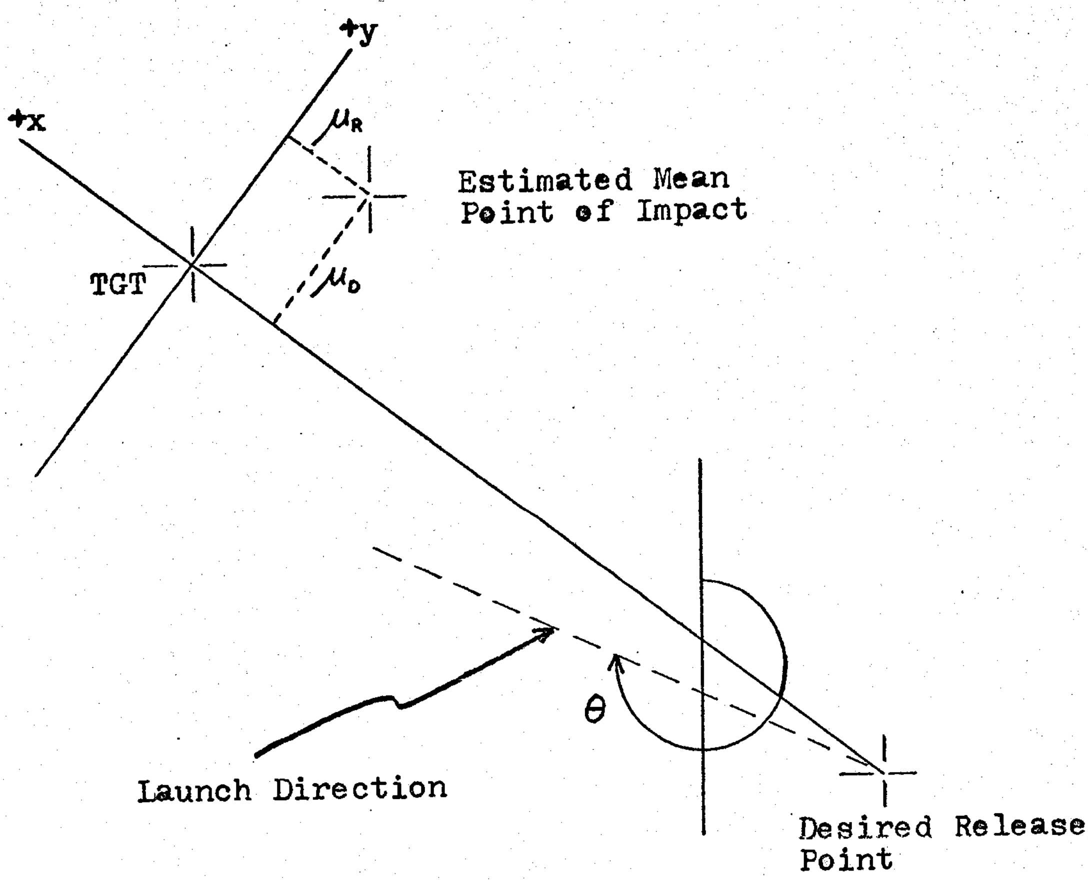

Fig. 1. Radar Bomb Scoring Coordinate System

- the desired release conditions are known from the computer solution of the ballistics problem.

- the desired conditions will place the point of impact on the target.

- deviations from the desired release conditions are detectable and may be output from the bombing system.

- deviations are small.

a. Velocity

Any deviation in velocity from the desired release velocity will be reflected in the range error due to velocity which will be denoted as e_{rv}. Using (5) the specified functions f_1 and f_2 may be replaced by well known physical relationships resulting in an expression of the form

$$e_{rv} = x_a - x_d = \dot{x}_a\sqrt{\frac{2z_a}{g}} - \dot{x}_d\sqrt{\frac{2z_d}{g}} \,, \qquad (6)$$

where

$$x = \dot{x}t$$

$$t = \sqrt{\frac{2z}{g}}$$

and $\dot{z}$ is assumed to be zero.

If the only observed deviation is in velocity, then (6) simplifies to

$$e_{rv} = \sqrt{\frac{2z}{g}}\,(\dot{x}_a - \dot{x}_d) \quad . \qquad (7)$$

b. Altitude

The form of (6) is a function of both velocity and altitude and is applicable to the range error due to altitude, e_{ra} . For the case in which only an altitude deviation exists, (6) becomes

$$e_{ra} = \dot{x}\sqrt{\frac{2}{g}}\,\left(\sqrt{z_a} - \sqrt{z_d}\,\right) \quad . \qquad (8)$$

c. Range

The effects of a small translation in range on the achieved trajectory is depicted in Figure 2.

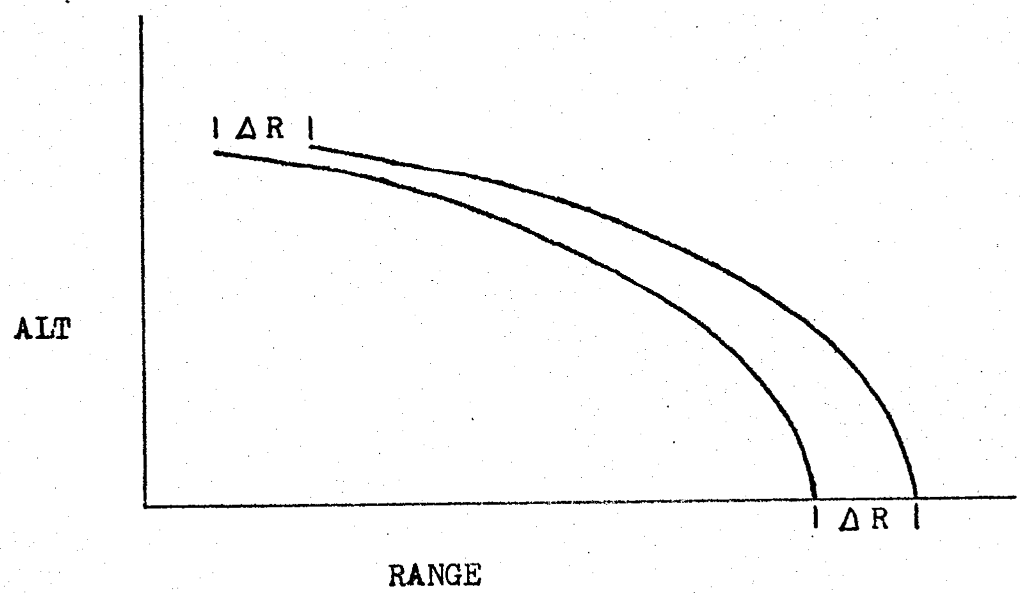

Fig. 2. The Effect of Small Range Errors on Ground Errors

Basically, Figure 2 implies that if the only observed deviation is either a premature or delayed release along the intended flight path resulting in a range error of ΔR, then the ground error is also ΔR. Using the notation of the previous work,

$$e_{rr} = \Delta R = x_d - R_a \quad ,$$

where R_a is the range from the actual release point to the target measured along the desired launch direction.

d. Direction

An error in launch direction will result in both range and deflection errors as shown in Figure 3.

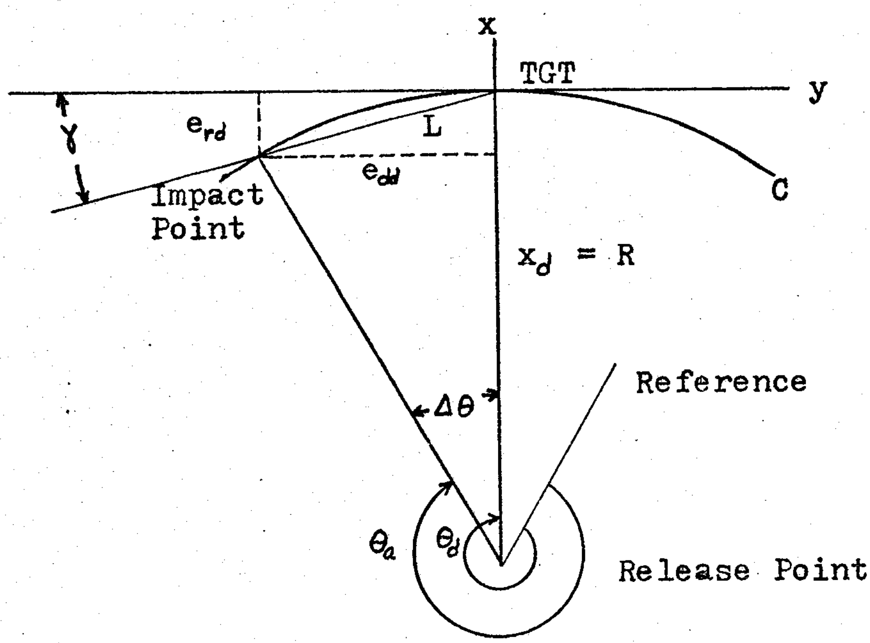

Fig. 3. The Effects of Launch Direction on Range and Deflection Errors

The error in launch direction, $\Delta\theta$, is shown to displace the point of impact along the arc C which passes through the target. The length of the chord, L, is

$$L = 2R \sin \frac{\Delta\theta}{2} \quad , \tag{10}$$

and

$$\gamma = \frac{\Delta\theta}{2} \quad .$$

It follows that

$$e_{dd} = L \cos \gamma$$

$$e_{rd} = -L \sin \gamma \quad .$$

Substituting and simplifying yields,

$$e_{dd} = R \sin \Delta\theta$$

$$e_{rd} = -R(1 - \cos \Delta\theta) \quad .$$

Converting to consistent notation results in

$$e_{dd} = x_d(\sin \Delta\theta) \quad , \tag{11}$$

where $\Delta\theta = \theta_a - \theta_d$ and e_{dd} takes the sign of $\Delta\theta$.

Also,

$$e_{rd} = -x_d(1 - \cos \Delta\theta) \quad . \tag{12}$$

e. Deflection

As in the case of small deviations in range, it may be shown that the translation of the release point in deflection, by the amount y_a, will result in a deflection error in the ground plane of the same magnitude. The same argument used in c. above applies. The result is an expression for deflection error, e_{df}, of the form,

$$e_{df} = y_a \quad . \tag{13}$$

f. Total Range and Deflection Errors

With the expressions developed thus far, it is possible to estimate the impact error due to certain individual errors in launch conditions. It is preferable however to consider the more general case where these release conditions may occur in any combination. To do so requires some concept of how these individual errors relate to expressions for total range and total deflection errors. Figure 4 illustrates a general case in which deviations are observed in velocity, altitude, range, direction and deflection. From the geometry of Figure 4 it can be seen that total range error, μ_R, may be expressed as

$$\mu_R = e_{rr} + e_r - (R_a + e_{rr} + e_r)(1 - \cos \Delta\theta) \quad ,$$

using the same trigonometric relationships used in the development of e_{rd}. Simplifying, the final expression becomes,

$$\mu_R = (x_d + \dot{x}_a\sqrt{\frac{2z_a}{g}} - \dot{x}_d\sqrt{\frac{2z_d}{g}}\ ) \cos \Delta\theta - R_a \ . \qquad (14)$$

Similarly, total deflection error, μ_D, is seen to be

$$\mu_D = e_{df} + (R_a + e_{rr} + e_r) \sin \Delta\theta$$

or

$$\mu_D = y_a + (x_d + \dot{x}_a\sqrt{\frac{2z_a}{g}} - \dot{x}_d\sqrt{\frac{2z_d}{g}}\ ) \sin \Delta\theta \ . \qquad (15)$$

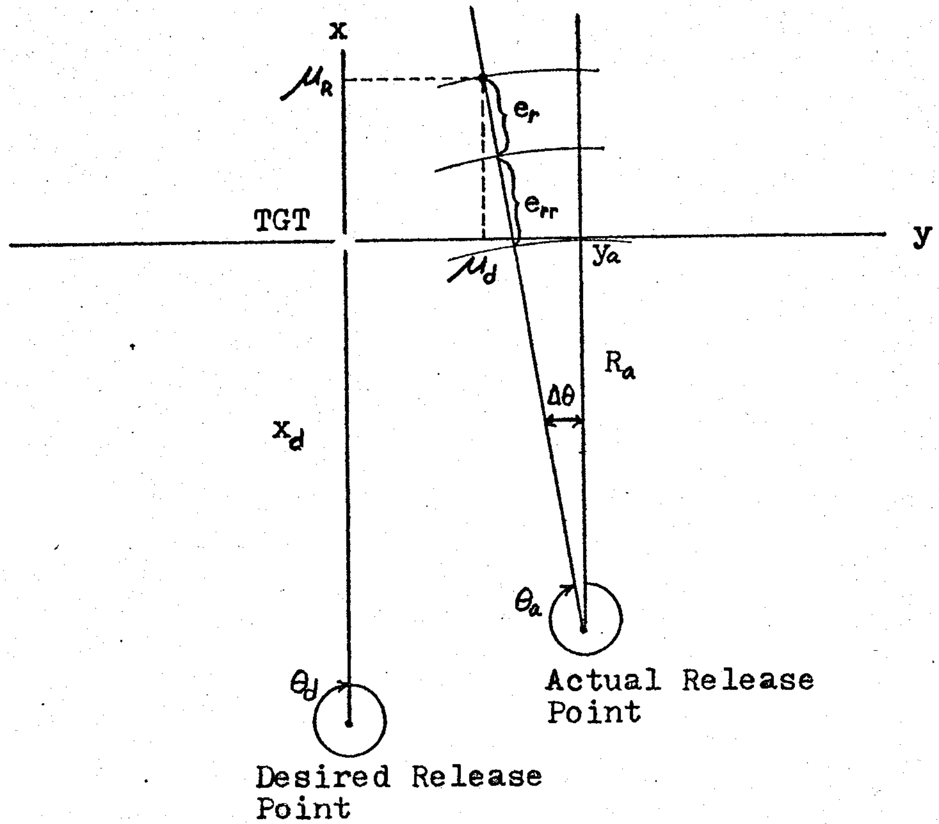

Fig. 4. The Effect of Error Combinations

Note that the last term of (15) should take the same sign as $\Delta\theta$. Expressions (14) and (15) give the total estimates of range and deflection errors, respectively.

B. ESTIMATION OF CIRCULAR ERROR PROBABLE

It is necessary to assume some distribution of bomb impacts about the target in order to estimate CEP. It is also assumed that the range and deflection components of the point of impact are independently distributed with the mean of each located at the target and with variances due to ballistic dispersion, σ_R^2 and σ_D^2.

The joint distribution of range and deflection is then assumed to be bivariate normal with density function,

$$f(x,y) = \frac{1}{2\pi\sigma_R\sigma_D} \exp\left\{-\frac{1}{2}\left[\left(\frac{x}{\sigma_R}\right)^2 + \left(\frac{y}{\sigma_D}\right)^2\right]\right\} .$$

The use of this function in subsequent estimation of CEP is acceptable in only those instances where the aim point or mean point of impact is coincident with the target. In general this will not be the case. Instead, errors in bomb release have the effect of offsetting the aim point in deflection and range by the amounts μ_D and μ_R respectively. The problem then is finding the probability that an impact will occur within a circle of radius R, centered at the target, when the aim point has been offset.

Grubbs [Ref. 6] approaches this problem through an interesting application of the non-central chi-square distribution. This approach has been adopted for use in this proposed model for estimating CEP. There are two principle reasons for this selection. The first of these is computational ease. Secondly, there is intuitive appeal and ease of interpretation in the use of a distribution whose parameter is directly related to the offset in the aim point.

1. <u>The Non-Central Chi-Square Distribution</u>

For the specific problem of estimating CEP, using the assumptions of the previous section,

$$x \sim N(\mu_R, \sigma_R{}^2)$$

$$y \sim N(\mu_D, \sigma_D{}^2)$$

since the effect of μ_D and μ_R is to center the impact distributions about the estimated point of impact rather than the target. Letting

$$u_R = \frac{x - \mu_R}{\sigma_R}$$

and

$$a_R = \frac{\mu_R}{\sigma_R}$$

then $u_R \sim N(0,1)$ and $(u_R + a_R)^2$ is distributed as a non-central chi-square random variable with probability density function

$$f_{\chi'^2}(x) = \frac{\exp\{-\frac{1}{2}(x+\lambda)\}}{2^{\nu/2}} \sum_{i=0}^{\infty} \frac{(x)^{\frac{\nu}{2}+i-1}}{\Gamma(\frac{\nu}{2}+i)\, 2^{2i}} \frac{\lambda^i}{i!} \quad ,$$

where $\lambda = a_R{}^2$ is the non-centrality parameter and $\nu = 1$ represents the degrees of freedom.

From the above,

$$\left(\frac{x - \mu_R + \mu_R}{\sigma_R}\right)^2 = \left(\frac{x}{\sigma_R}\right)^2 \sim \chi'^2_{(1)} \quad ,$$

and, similarly,

$$\left(\frac{y}{\sigma_D}\right)^2 \sim \chi'^2_{(1)} \quad .$$

The expression

$$\frac{x^2}{\sigma_R{}^2} + \frac{y^2}{\sigma_D{}^2} \quad , \tag{16}$$

is therefore distributed as the sum of two non-central chi-square random variables. From the reproductive property of this distribution, the sum of non-central chi-squares is also non-central chi-square with $\lambda = \sum_i \lambda_i$ and $\nu = \sum_i \nu_i$. Therefore,

$$\frac{x^2}{\sigma_R{}^2} + \frac{y^2}{\sigma_D{}^2} \sim \chi'^2_{(2)}$$

$$\lambda = \frac{\mu_D{}^2}{\sigma_D{}^2} + \frac{\mu_R{}^2}{\sigma_R{}^2} \quad .$$

2. Solutions

In this form, it is not obvious how one would proceed to determine the desired probability that $x^2 + y^2 \leq R^2$. However, by letting

$$\sigma^2 = \sigma_R{}^2 + \sigma_D{}^2 \quad ,$$

$$v_1 = \frac{\sigma_R{}^2}{\sigma^2} \quad ,$$

and

$$v_2 = \frac{\sigma_D{}^2}{\sigma^2} \quad ,$$

then, from (16)

$$x^2 + y^2 = \sigma^2[v_1(\frac{x}{\sigma_R})^2 + v_2(\frac{y}{\sigma_D})^2] \quad . \tag{17}$$

The bracketed term of (17) is seen to be the weighted sum of two non-central chi-square random variables.

Setting

$$\psi^2 = [v_1(\frac{x}{\sigma_R})^2 + v_2(\frac{y}{\sigma_D})^2] \quad ,$$

the expression

$$x^2 + y^2 = \sigma^2\psi^2 \tag{18}$$

is obtained. Finally, from (18)

$$\frac{x^2 + y^2}{\sigma^2} = \psi^2 \quad ,$$

where

$$\psi^2 \sim \chi'^2 \quad ,$$

so that the desired probability may now be written as,

$$P[\frac{x^2 + y^2}{\sigma^2} \leq \frac{R^2}{\sigma^2}] \quad . \tag{18a}$$

Solutions are possible through the application of one of several available approximating methods to the non-central chi-square. Johnson and Kotz [Ref. 11] discuss in some detail many of the approximations which have been suggested. One of the most tractable and easily computed of these involves the transformation to an approximate chi-square and then using a normal approximation to this function. The details of this procedure are reported in reference 6 and, for continuity, are described here.

If

$$\sigma^2\psi^2 = x^2 + y^2 \quad ,$$

it is possible to determine some function of ψ^2 that is approximately distributed as a central chi-square. Observing that

$$m = E(\psi^2) = 1 + \frac{\mu_D{}^2 + \mu_R{}^2}{\sigma^2} \quad , \tag{19}$$

and

$$v = \mathrm{Var}(\psi^2) = \frac{2(\sigma_R{}^4 + \sigma_D{}^4) + 4(\sigma_R{}^2\mu_R{}^2 + \sigma_D{}^2\mu_D{}^2)}{\sigma^4} \quad , \tag{20}$$

then,

$$E[\frac{2m\psi^2}{v}] = \frac{2m^2}{v} \quad ,$$

and

$$\mathrm{Var}[\frac{2m\psi^2}{v}] = \frac{4m^2}{v} \quad ,$$

which implies that

$$\frac{2m\psi^2}{v} \doteq \chi^2_{\frac{2m^2}{v}} \quad .$$

The Wilson-Hilferty normal approximation to the central chi-square, reported by Grubbs, states

$$P[\chi_f^{\,2} \leq x] \doteq \Phi(\{(\tfrac{x}{f})^{1/3} - 1 + \tfrac{2}{9f}\}\sqrt{\tfrac{9f}{2}}) \quad . \tag{21}$$

Substituting into the right hand side of (21), the result is

$$\frac{\sqrt[3]{\frac{\psi^2}{m}} - 1 + \frac{v}{9m^2}}{\sqrt{\frac{v}{9m^2}}} \sim N(0,1) \quad . \tag{22}$$

The solution for the desired estimate of CEP is found by equating (22) to zero and solving for ψ. The result is,

$$\widehat{CEP} = \sqrt{\sigma^2 m(1 - \frac{v}{9m^2})^3} \quad . \tag{23}$$

The simple form of (23) lends itself to the development of tables of estimated CEP as a function of the estimated aiming errors, μ_D and μ_R. Appendix C provides an example of how such tables might be organized.

C. COMMENTS ON THE PROPOSED MODEL

The estimation of CEP through the use of the proposed model is unique in that the estimation is based upon a single observation of release conditions. Further, it should be noted that the approximations used to arrive at an estimate of CEP are included primarily as an aid to computation. If exact values of the non-central chi-square distribution are available, then equation (18a) may be used directly to compute the estimated CEP.

The limitations of the model are not known since it has not been tested. It is possible, however, to say something about the expected accuracies of the aiming errors and CEP estimations. A check of random entries of trajectory tables in Reference 20 showed that equations (14) and (15) provided results that agreed quite closely with values obtained by direct interpolation in the tables. The maximum difference found in this random check was approximately six percent. This was considered to be acceptable in view of the fact that many of the interpolations were made over 5000 feet intervals in altitude and 100 knots in velocity. The use of more refined tables should show better agreement between the two methods.

The accuracies of the approximating methods used to estimate CEP are better known. Reference 11 reports a difference of approximately 0.33 between the exact value of the non-central chi-square and the approximate value, at the upper five percent point of the distribution. The difference was obtained for $\nu = 2$ and $\lambda = 25$. In terms of CEP, this difference translates into an error of less than one-half of one percent.

Further work on the model, in addition to testing, could profitably include the analysis of radar system errors and how these errors should be integrated into the model. In its present form, only aiming errors and ballistic dispersion are considered.

In addition, the model could be strengthened by providing for situations other than level bombing ($\dot{z} \neq 0$) and zero accelerations at release. The former could be easily accomodated by resolving the aircraft velocity into horizontal and vertical components and revising the ballistic equations accordingly. The latter is less easily incorporated due to the increased complexities of the ballistic equations and the technical difficulties associated with obtaining acceleration data.

The final comment to be made involves the application of the model to other than the computer controlled mode of bombing. Although the bomb scoring function envisions the use of a tactical radar bombing system, the system could be employed to score the results of a pilot controlled bombing mission. In this case, the pilot would be instructed to achieve a set of pre-calculated release conditions and the radar would observe the deviations from these conditions. The estimation of achieved CEP is then identical to the computer controlled mode.

APPENDIX A

SAMPLE CALCULATIONS OF RANGE AND DEFLECTION ERRORS

For the purpose of illustrating the calculation of range and deflection errors, the following conditions are assumed:

Desired launch conditions:

$\dot{x}_d$ = velocity = 300 kts = 506.7 ft/sec

z_d = altitude = 5200 ft

x_d = range = 8850 ft

θ_d = launch direction = 337°

Conditions at release:

$\dot{x}_a$ = 308 kts = 520.2 ft/sec

z_a = 5270 ft

R_a = 8730 ft

θ_a = 340°

y_a = deflection offset = 0 ft

From (15)

$$\mu_D = y_a + \left(x_d + \dot{x}_a\sqrt{\frac{2z_a}{g}} - \dot{x}_d\sqrt{\frac{2z_d}{g}}\right) \sin \Delta\theta$$

where

$$\Delta\theta = \theta_a - \theta_d = 340 - 337 = +3^\circ$$

Direction substitution yields,

$$\mu_D = 0 + (8850 + 520.2\sqrt{\frac{10540}{32.2}} - 506.7\sqrt{\frac{10400}{32.2}}) \sin 3°$$

$$\mu_D = 479.2 \text{ feet}$$

Similarly, from (14),

$$\mu_R = (8850 + 9412 - 9106)(0.9986) - 8730$$

$$\mu_R = 413.2 \text{ feet}$$

The estimated point of impact is then located approximately 413 feet over and 479 feet to the right of the intended target.

APPENDIX B

SAMPLE CALCULATIONS OF CEP

Deflection and range components of ballistic dispersion are assumed to possess the following values of standard deviation:

$\sigma_D = 60$ feet $\qquad \sigma_D^2 = 3600$ feet^2

$\sigma_R = 80$ feet $\qquad \sigma_R^2 = 6400$ feet^2

Using the Normal approximation to the assumed distribution, recall that

$$\frac{\sqrt[3]{\frac{\psi^2}{m}} - \left(1 - \frac{v}{9m^2}\right)}{\sqrt{\frac{v}{9m^2}}} \sim N(0,1)$$

where

$$m = 1 + \frac{\mu_1^2 + \mu_2^2}{\sigma^2}$$

$$v = \frac{2(\sigma_D^4 + \sigma_R^4) + 4(\sigma_D^2\mu_D^2 + \sigma_R^2\mu_R^2)}{\sigma^4}$$

$$\sigma^2 = \sigma_D^2 + \sigma_R^2$$

Assuming

$$\mu_D = 300 \text{ feet}$$

$$\mu_R = 420 \text{ feet}$$

the estimated value of CEP may be found by setting (22) equal to zero and rearranging terms. The result is

$$\widehat{CEP} = \sqrt{\sigma^2 m(1 - \frac{v}{9m^2})^3}$$

Substituting the assumed values,

$$m = 27.6$$

$$v = 59.2$$

and

$$\widehat{CEP} = \sqrt{10000(27.6)(1 - \frac{59.2}{9(761.8)})^3}$$

$$\widehat{CEP} = \sqrt{268940} \quad \text{ft}$$

$$\widehat{CEP} = 519 \text{ ft}$$

APPENDIX C

TABLES OF ESTIMATED CEP

The sample tables of estimated CEP included in this appendix were computed using the normal approximation to the noncentral chi-square distribution. The standard deviations for deflection and range dispersion were arbitrarily selected to be 80 and 90 feet respectively and are not intended to represent the actual dispersion parameters for a particular weapon. The accepted parameters for a specified weapon may be found in Reference 20.

Entry to the tables is made using μ_D, the deflection error, and μ_R, the range error computed from the release conditions. The tabled values were computed from equation (23).

μ_R (feet) \ μ_D (feet)	0	5	10	15	20	25	30	35	40	45
0	100	100	101	101	102	103	104	105	106	108
5	100	100	101	101	102	103	104	105	106	108
10	100	101	101	101	102	103	104	105	107	108
15	101	101	101	102	102	103	104	106	107	109
20	101	102	102	102	103	104	105	106	108	109
25	102	102	103	103	104	105	106	107	108	110
30	103	103	103	104	105	105	107	108	109	111
35	104	104	105	105	106	107	108	109	110	112
40	105	105	106	106	107	108	109	110	111	113
45	107	107	107	108	108	109	110	111	113	114
50	108	108	109	109	110	111	112	113	114	116
55	110	110	111	111	112	113	114	115	116	118
60	112	112	112	113	114	114	116	117	118	120
65	114	114	115	115	116	117	118	119	120	122
70	116	117	117	117	118	119	120	121	122	124
75	119	119	119	120	120	121	122	123	125	126
80	122	122	122	122	123	124	125	126	127	129
85	124	124	125	125	126	126	127	129	130	131
90	127	127	127	128	129	129	130	131	133	134
95	130	130	130	131	132	132	133	134	136	137
100	133	133	134	134	135	135	136	137	139	140
105	137	137	137	137	138	139	140	141	142	143
110	140	140	140	141	141	142	143	144	145	147
115	143	143	144	144	145	145	146	147	149	150
120	147	147	147	148	148	149	150	151	152	153
125	151	151	151	151	152	153	153	155	156	157
130	154	154	155	155	156	156	157	158	159	161
135	158	158	159	159	159	160	161	162	163	164
140	162	162	162	163	163	164	165	166	167	168
145	166	166	166	167	167	168	169	170	171	172
150	170	170	171	171	171	172	173	174	175	176

μ_R (feet)	μ_D (feet) 0	5	10	15	20	25	30	35	40	45
155	174	174	175	175	175	176	177	178	179	180
160	179	179	179	179	180	180	181	182	183	184
165	183	183	183	183	184	184	185	186	187	188
170	187	187	187	188	188	189	189	190	191	192
175	191	191	192	192	192	193	194	194	195	196
180	196	196	196	196	197	197	198	199	200	201
185	200	200	200	201	201	202	202	203	204	205
190	205	205	205	205	205	206	207	207	208	209
195	209	209	209	210	210	210	211	212	213	214
200	213	214	214	214	214	215	216	216	217	218
205	218	218	218	219	219	219	220	221	222	223
210	223	223	223	223	223	224	225	225	226	227
215	227	227	227	228	228	228	229	230	231	232
220	232	232	232	232	233	233	234	234	235	236
225	236	236	237	237	237	238	238	239	240	241
230	241	241	241	241	242	242	243	244	244	245
235	246	246	246	246	246	247	247	248	249	250
240	250	250	251	251	251	252	252	253	254	254
245	255	255	255	255	256	256	257	257	258	259
250	260	260	260	260	260	261	261	262	263	264
255	264	264	265	265	265	266	266	267	267	268
260	269	269	269	270	270	270	271	271	272	273
265	274	274	274	274	275	275	276	276	277	278
270	279	279	279	279	279	280	280	281	282	282
275	283	283	284	284	284	285	285	286	286	287
280	288	288	288	289	289	289	290	290	291	292
285	293	293	293	293	294	294	295	295	296	296
290	298	298	298	298	299	299	299	300	301	301
295	303	303	303	303	303	304	304	305	305	306
300	307	307	308	308	308	308	309	309	310	311

μ_R (feet)	μ_D (feet)									
	0	5	10	15	20	25	30	35	40	45
305	312	312	312	313	313	313	314	314	315	316
310	317	317	317	317	318	318	319	319	320	320
315	322	322	322	322	323	323	323	324	324	325
320	327	327	327	327	327	328	328	329	329	330
325	332	332	332	332	332	333	333	333	334	335
330	336	337	337	337	337	337	338	338	339	340
335	341	341	342	342	342	342	343	343	344	344
340	346	346	346	347	347	347	348	348	349	349
345	351	351	351	351	352	352	352	353	353	354
350	356	356	356	356	357	357	357	358	358	359
355	361	361	361	361	361	362	362	363	363	364
360	366	366	366	366	366	367	367	367	368	369
365	371	371	371	371	371	371	372	372	373	373
370	376	376	376	376	376	376	377	377	378	378
375	380	380	381	381	381	381	382	382	383	383
380	385	385	385	386	386	386	386	387	387	388
385	390	390	390	391	391	391	391	392	392	393
390	395	395	395	395	396	396	396	397	397	398
395	400	400	400	400	401	401	401	402	402	403
400	405	405	405	405	405	406	406	406	407	407
405	410	410	410	410	410	411	411	411	412	412
410	415	415	415	415	415	416	416	416	417	417
415	420	420	420	420	420	420	421	421	422	422
420	425	425	425	425	425	425	426	426	427	427
425	430	430	430	430	430	430	431	431	431	432
430	434	435	435	435	435	435	436	436	436	437
435	439	439	440	440	440	440	440	441	441	442
440	444	444	444	445	445	445	445	446	446	447
445	449	449	449	450	450	450	450	451	451	452
450	454	454	454	454	455	455	455	456	456	456

μ_R (feet) \ μ_D (feet)	50	55	60	65	70	75	80	85	90	95
0	110	112	114	114	118	121	124	127	130	133
5	110	112	114	114	119	121	124	127	130	133
10	110	112	114	114	119	121	124	127	130	133
15	110	112	114	114	119	122	124	127	130	134
20	111	113	115	115	120	122	125	128	131	134
25	112	114	116	116	120	123	126	129	132	135
30	113	114	117	117	121	124	127	129	132	136
35	114	116	118	118	122	125	128	130	133	137
40	115	117	119	119	124	126	129	132	135	138
45	116	118	120	120	125	127	130	133	136	139
50	118	120	122	122	126	129	132	134	137	140
55	120	121	124	124	128	131	133	136	139	142
60	121	123	125	125	130	132	135	138	141	144
65	123	125	127	127	132	134	137	140	143	146
70	126	128	130	130	134	136	139	142	145	148
75	128	130	132	132	136	139	141	144	147	150
80	130	132	134	134	139	141	144	146	149	152
85	133	135	137	137	141	144	146	149	151	154
90	136	138	140	140	144	146	149	151	154	157
95	139	141	142	142	147	149	151	154	157	159
100	142	143	145	145	150	152	154	157	159	162
105	145	147	148	148	153	155	157	160	162	165
110	148	150	152	152	156	158	160	163	165	168
115	151	153	155	155	159	161	163	166	168	171
120	155	157	158	158	162	164	167	169	171	174
125	158	160	162	162	166	168	170	172	175	177
130	162	164	165	165	169	171	173	176	178	181
135	166	167	169	169	173	175	177	179	181	184
140	170	171	173	173	176	178	180	183	185	187
145	173	175	176	176	180	182	184	186	189	191
150	177	179	180	180	184	186	188	190	192	195

μ_D
(feet)

μ_R (feet)	50	55	60	65	70	75	80	85	90	95
155	181	183	184	184	188	190	192	194	196	198
160	185	187	188	188	192	193	195	197	200	202
165	189	191	192	192	196	197	199	201	203	206
170	194	195	196	196	200	201	203	205	207	210
175	198	199	200	200	204	205	207	209	211	213
180	202	203	205	205	208	209	211	213	215	217
185	206	207	209	209	212	214	215	217	219	221
190	211	212	213	213	216	218	220	221	223	225
195	215	216	217	217	220	222	224	226	228	230
200	219	220	222	222	225	226	228	230	232	234
205	224	225	226	226	229	231	232	234	236	238
210	228	229	230	230	233	235	237	238	240	242
215	233	234	235	235	238	239	241	243	244	246
220	237	238	239	239	242	244	245	247	249	250
225	242	243	244	244	247	248	250	251	253	255
230	246	247	248	248	251	252	254	256	257	259
235	251	252	253	253	255	257	258	260	262	263
240	255	256	257	257	260	261	263	264	266	268
245	260	261	262	262	264	266	267	269	271	272
250	265	266	267	267	269	270	272	273	275	277
255	269	270	271	271	274	275	276	278	279	281
260	274	275	276	276	278	279	281	282	284	286
265	278	279	280	280	283	284	285	287	288	290
270	283	284	285	285	287	289	290	291	293	295
275	288	289	290	290	292	293	295	296	297	299
280	293	293	294	294	297	298	299	301	302	304
285	297	298	299	299	301	303	304	305	307	308
290	302	303	304	304	306	307	308	310	311	313
295	307	308	309	309	311	312	313	314	316	317
300	312	312	313	313	315	317	318	319	320	322

μ_R (feet) \ μ_D (feet)	50	55	60	65	70	75	80	85	90	95
305	316	317	318	318	320	321	322	324	325	327
310	321	322	323	323	325	326	327	328	330	331
315	326	327	328	328	330	331	332	333	334	336
320	331	331	332	332	334	335	337	338	339	340
325	335	336	337	337	339	340	341	342	344	345
330	340	341	342	342	344	345	346	347	348	350
335	345	346	347	347	349	350	351	352	353	354
340	350	351	351	351	353	354	355	357	358	359
345	355	355	356	356	358	359	360	361	363	364
350	359	360	361	361	363	364	365	366	367	369
355	364	365	366	366	368	369	370	371	372	373
360	369	370	371	371	372	373	374	376	377	378
365	374	375	375	375	377	378	379	380	381	383
370	379	380	380	380	382	383	384	385	386	387
375	384	384	385	385	387	388	389	390	391	392
380	389	389	390	390	392	393	394	395	396	397
385	393	394	395	395	396	397	398	399	401	402
390	398	399	400	400	401	402	403	404	405	406
395	403	404	405	405	406	407	408	409	410	411
400	408	409	409	409	411	412	413	414	415	416
405	413	414	414	414	416	417	418	419	420	421
410	418	418	419	419	421	422	422	423	425	426
415	423	423	424	424	426	426	427	428	429	430
420	428	428	429	429	430	431	432	433	434	435
425	432	433	434	434	435	436	437	438	439	440
430	437	438	439	439	440	441	442	443	444	445
435	442	443	444	444	445	446	447	448	449	450
440	447	448	448	448	450	451	452	452	453	454
445	452	453	453	453	455	456	456	457	458	459
450	457	458	458	458	460	460	461	462	463	464

μ_R (feet)	μ_D (feet) 100	105	110	115	120	125	130	135	140	145
0	136	139	143	146	150	154	158	162	166	170
5	136	140	143	147	150	154	158	162	166	170
10	136	140	143	147	150	154	158	162	166	170
15	137	140	144	147	151	155	158	162	166	170
20	137	141	144	148	151	155	159	163	167	171
25	138	141	145	148	152	156	159	163	167	171
30	139	142	146	149	153	156	160	164	168	172
35	140	143	147	150	154	157	161	165	169	173
40	141	144	148	151	155	158	162	166	170	174
45	142	145	149	152	156	160	163	167	171	175
50	144	147	150	154	157	161	165	168	172	176
55	145	148	152	155	159	162	166	170	174	177
60	147	150	153	157	160	164	168	171	175	179
65	149	152	155	159	162	166	169	173	177	180
70	151	154	157	160	164	167	171	175	178	182
75	153	156	159	162	166	169	173	177	180	184
80	155	158	161	165	168	171	175	179	182	186
85	157	160	164	167	170	174	177	181	184	188
90	160	163	166	169	172	176	179	183	186	190
95	162	165	168	172	175	178	182	185	189	192
100	165	168	171	174	177	181	184	188	191	195
105	168	171	174	177	180	183	187	190	194	197
110	171	174	177	180	183	186	189	193	196	200
115	174	177	179	182	186	189	192	195	199	202
120	177	180	182	185	188	192	195	198	202	205
125	180	183	186	188	192	195	198	201	204	208
130	183	186	189	192	195	198	201	204	207	211
135	187	189	192	195	198	201	204	207	210	214
140	190	193	195	198	201	204	207	210	213	217
145	193	196	199	201	204	207	210	213	217	220
150	197	200	202	205	208	211	214	217	220	223

μ_D (feet)

μ_R (feet)	100	105	110	115	120	125	130	135	140	145
155	201	203	206	208	211	214	217	220	223	226
160	204	207	209	212	215	218	220	223	226	230
165	208	210	213	216	218	221	224	227	230	233
170	212	214	217	219	222	225	227	230	233	236
175	216	218	221	223	226	228	231	234	237	240
180	220	222	224	227	229	232	235	238	240	243
185	224	226	228	231	233	236	238	241	244	247
190	228	230	232	235	237	240	242	245	248	251
195	232	234	236	238	241	243	246	249	251	254
200	236	238	240	242	245	247	250	253	255	258
205	240	242	244	246	249	251	254	256	259	262
210	244	246	248	251	253	255	258	260	263	266
215	248	250	252	255	257	259	262	264	267	270
220	252	254	257	259	261	263	266	268	271	273
225	257	259	261	263	265	267	270	272	275	277
230	261	263	265	267	269	272	274	276	279	281
235	265	267	269	271	273	276	278	280	283	285
240	270	272	274	276	278	280	282	285	287	289
245	274	276	278	280	282	284	286	289	291	294
250	278	280	282	284	286	288	291	293	295	298
255	283	285	287	288	291	293	295	297	299	302
260	287	289	291	293	295	297	299	301	304	306
265	292	293	295	297	299	301	303	306	308	310
270	296	298	300	302	304	306	308	310	312	314
275	301	302	304	306	308	310	312	314	316	319
280	305	307	309	310	312	314	316	318	321	323
285	310	311	313	315	317	319	321	323	325	327
290	314	316	318	319	321	323	325	327	329	331
295	319	320	322	324	326	328	330	332	334	336
300	323	325	327	328	330	332	334	336	338	340

μ_R (feet) \ μ_D (feet)	100	105	110	115	120	125	130	135	140	145
305	328	330	331	333	335	337	338	340	342	345
310	333	334	336	337	339	341	343	345	347	349
315	337	339	340	342	344	346	347	349	351	353
320	342	343	345	347	348	350	352	354	356	358
325	346	348	350	351	353	355	356	358	360	362
330	351	353	354	356	357	359	361	363	365	367
335	356	357	359	360	362	364	365	367	369	371
340	360	362	363	365	367	368	370	372	374	376
345	365	367	368	370	371	373	375	376	378	380
350	370	371	373	374	376	377	379	381	383	385
355	375	376	377	379	380	382	384	385	387	389
360	379	381	382	384	385	387	388	390	392	394
365	384	385	387	388	390	391	393	395	396	398
370	389	390	391	393	394	396	398	399	401	403
375	393	395	396	398	399	401	402	404	406	407
380	398	399	401	402	404	405	407	408	410	412
385	403	404	406	407	408	410	411	413	415	416
390	408	409	410	412	413	415	416	418	419	421
395	412	414	415	416	418	419	421	422	424	426
400	417	418	420	421	422	424	425	427	429	430
405	422	423	424	426	427	429	430	432	433	435
410	427	428	429	431	432	433	435	436	438	440
415	432	433	434	435	437	438	440	441	443	444
420	436	437	439	440	441	443	444	446	447	449
425	441	442	444	445	446	447	449	450	452	454
430	446	447	448	450	451	452	454	455	457	458
435	451	452	453	454	456	457	458	460	461	463
440	456	457	458	459	460	462	463	465	466	468
445	460	461	463	464	465	466	468	469	471	472
450	465	466	467	469	470	471	473	474	475	477

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