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Analyzing the Vulnerability of Critical Infrastructure to Attack and Planning Defenses

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Abstract We describe new bilevel programming models to (1) help make the country's critical infrastructure more resilient to attacks by terrorists, (2) help governments and businesses plan those improvements, and (3) help influence related public policy on investment incentives, regulations, etc. An intelligent attacker (terrorists) and defender (us) are key features of all these models, along with information transparency: These are Stackelberg games, as opposed to two-person, zero-sum games. We illustrate these models with applications to electric power grids, subways, airports, and other critical infrastructure. For instance, one model identifies locations for a given set of electronic sensors that minimize the worst-case time to detection of a chemical, biological, or radiological contaminant introduced into the Washington, D.C. subway system. The paper concludes by reporting insights we have gained through forming "red teams," each of which gathers open-source data on a real-world system, develops an appropriate attacker-defender or defender-attacker model, and solves the model to identify vulnerabilities in the system or to plan an optimal defense.

Keywords critical infrastructure protection; bilevel program; mixed-integer program; homeland security

The Problem

What is critical infrastructure? The National Strategy for Homeland Security deems 13 infrastructure sectors critical to the United States; see Table 1 (DHS [18]). These include sectors such as "Government" and "Public Health," but a number, such as "Transportation" and "Information and Telecommunications," comprise physical systems that connect components of our economy: In essence, they enable the transfer and distribution of our economy's life forces. We focus on defending this type of infrastructure from attacks by terrorists, but we believe almost any type of critical infrastructure deserves analysis with the techniques we describe.

Any critical infrastructure system represents a huge investment of our nation's wealth, and minor disruptions to such a system's components—these disruptions can be random or deliberate—can severely degrade its performance as well as the performance of dependent systems. For instance, a massive power outage can result from the failure of just a few key lines and protective circuit breakers (U.S.-Canada Power System Outage Task Force [39]). The direct effect is to interrupt the energy supply to residential and industrial customers, but all other infrastructure systems listed in Table 1 will be affected if the power outage lasts long enough. So, how do we carry out a "vulnerability analysis" when terrorist attacks are the key concern? That is, how do we analyze the vulnerability of a critical infrastructure system to a terrorist attack, or set of coordinated attacks, and make informed proposals for reducing that vulnerability?

Most infrastructure systems are engineered to handle disruptions that result from accidents, or from random acts of nature, with little or no degradation in performance. Real-time reliability assessment of an electric power grid pronounces the system robust if no crippling "single point of failure" exists (e.g., Wood and Wollenberg [44]). Analysts of transportation

Table 1. Thirteen infrastructure sectors critical to the United States, as defined by the Department of Homeland Security [18].

Agriculture	Food
Water	Public Health
Emergency Services	Government
Defense Industrial Base	Information and Telecommunications
Energy	Transportation
Banking and Finance	Chemical Industry
Postal and Shipping	

systems, power plants, and other infrastructure often use fault trees to assess vulnerability (Roberts et al. [34]). Such an assessment helps identify minimal sets of events, or "cutsets," that are most likely to disrupt the system, and pronounce the system robust if their combined probability is sufficiently low. This assessment can suggest changes to the system to improve robustness, and the overall methodology can be used to evaluate alternative system configurations proposed by the analyst.

However, infrastructure that resists single points of random failure, or whose cutsets have low occurrence probabilities, may not survive a malicious, intelligent attack. For example, a lone attacker with a high-powered rifle could gravely damage an entire electric power grid by targeting highly reliable components at just a few key substations. (We reach this conclusion from our own analyses of electric power grids and from reports of gunfire disabling a substation; see Wallace [41].) Also, cutsets that are likely to occur due to random causes may not share any similarities to the cutsets that an attacker will likely find. An analyst might attempt a fault-tree assessment of a system subject to attack by guessing at the probability that each individual component might be attacked. In fact, such analysis is practiced (Garcia [22]), but the results must be classified as a guesses. We require a new paradigm for vulnerability analysis.

The new paradigm must account for an adversary's ability to collect information about an infrastructure system and use that information to identify weak spots in the system's architecture. A captured Al Qaeda training manual (Department of Justice [19]) advises: "Using public sources openly and without resorting to illegal means, it is possible to gather at least 80% of information about the enemy." We interpret that statement to mean: "It is possible to gather, from public sources, at least 80% of the information needed to plan a highly disruptive attack on an infrastructure system." Our experience indicates that one can often find *all* the information necessary to plan such an attack.

Our backgrounds compel us ask how a military analyst, faced with an intelligent enemy, would approach vulnerability analysis for military infrastructure. First, the analyst would assume that our infrastructure will be attacked and that we must take steps to protect it, i.e., harden the infrastructure or improve its active defenses. The budget for hardening or actively defending infrastructure will always be limited. So, typically, the analyst would be instructed to create a prioritized list of "defended assets" most in need of protection, along with a list of potential defensive measures, and deliver those lists to higher-level decision makers. The latter parties would make the final decisions after balancing costs, effectiveness, and intangibles, and after determining the true budget (which may be monetary or may be the number of aerial sorties, cruise missiles, tanks, etc., that can be spared for defensive purposes). Table 2 shows the doctrinal components that the U.S. Army uses to guide the prioritization of its defended assets (as well as its enemies').

Any person who has had a course in discrete optimization understands the fundamental flaw in the concept and use of a prioritized list. In addition to that shortcoming of the nominal military approach, we see that the civilian problem itself differs from the military one:

- almost every civilian U.S. asset is susceptible to surveillance or attack, and is thus vulnerable;

TABLE 2. Criteria for prioritizing defended assets (Department of the Army [20, 21]).

Criticality
How essential is the asset?

Vulnerability
How susceptible is the asset to surveillance or attack?

Reconstitutability
How hard will it be to recover from inflicted damage, considering time, special repair equipment, and manpower required to restore normal operation?

Threat
How probable is an attack on this asset?

- no matter how hard it is to recover from inflicted damage, we will, eventually, reconstitute and recover; and
- military planners have vast experience in determining the likelihood of alternative attacks; homeland-security planners do not. Thus, we must plan for what is possible, rather than what subjective assessments indicate is likely.

In fact, normally, we do not try to measure the importance, or value, of an asset directly. Rather, we model a complete infrastructure system, its value to society, and how losses of the system's components reduce that value, or how improvements in the system mitigate against lost value. The exact meaning of value will depend on the system under investigation: It may mean economic output, time to detection of a toxic substance, etc., and sometimes cost, the converse of value, will be a more convenient yardstick.

Al Qaeda teaches as its primary mission "overthrow of godless regimes (by) gathering information about the enemy, the land, the installations, and the neighbors, ... blasting and destroying the places of amusement, ... embassies, ... vital economic centers, ... bridges leading into and out of cities, ..." (Department of Justice [19]). Al Qaeda may not have a perfect model of a particular infrastructure system, but its operatives are instructed to gather (widely available) information about it. Clearly, that information is being used to plan the worst attacks Al Qaeda can devise. Consequently, prudence dictates the assumption that Al Qaeda, or any other terrorist organization, will use its limited offensive assets to maximize damage to the infrastructure system it decides to attack, and has all the data necessary to do this.

Our paradigm of an *attacker-defender model* does address criticality, vulnerability, reconstitutability, and threat, but in a very different way than military planners might. We incorporate reconstitutability by modeling how system components are repaired over time and how a repaired component contributes to improved system value (Salmerón et al. [36]). Unless strictly defended or hardened, every system component is assumed to be vulnerable. We address "threat" by positing different levels of offensive resources for the terrorists. At the end of our analysis, we can determine the criticality of a *group* of system components, i.e., the value of protecting them, hardening them, or the value of adding new components into the system for purposes of redundancy. Another paradigm, discussed later, directly identifies an optimal defense plan: This is the *defender-attacker model.*

To understand our approach, the reader must understand the basics of the next two sections. However, a reader not interested in the mathematics may feel free to skim those details.

Attacker-Defender Models

The core of an attacker-defender model is an optimization model of an infrastructure system whose objective function represents the system's value to society while it operates, or the cost to society when the system loses functionality. For instance, the maximum throughput of an oil pipeline system might measure that system's value, while power-generation costs, plus economic losses resulting from unmet demand, might measure the full cost of operating an electric power grid.

To set the mathematical context, we assume that the defender operates a system so as to minimize cost that can be represented by a linear function. The defender's problem is

$$\min_{\mathbf{y}\in Y} \mathbf{cy}, \tag{1}$$

where (i) $\mathbf{y}$ represents system operating decisions or activities, (ii) $\mathbf{c}$ defines the corresponding vector of costs (and/or penalties), and (iii) the set Y represents constraints on that operation and the requirements to be met, e.g., road capacities in a road network, the number of commuters wishing to travel between various points in that network, etc. Of course, by including auxiliary variables in $\mathbf{y}$, and auxiliary constraints in Y, we can also represent certain nonlinear cost functions.

We note that "defender" is actually a misnomer in these models, because the models do not directly represent defensive actions; better terms might be "system user" or "system operator." However, our ultimate goal is to help identify defensive actions for the system user, so we feel justified in the slight abuse of terminology.

Now, our model posits that an attacker wishes to maximize the defender's optimal (minimal) operating cost, and will do so by restricting actions $\mathbf{y}$. Let $x_k = 0$ if the attacker *interdicts* the defender's kth asset, let $x_k = 0$ otherwise, and let $\mathbf{x}$ denote the vector of interdiction decisions. "Interdicting an asset" may be viewed as interdicting some component of the defender's infrastructure system. For simplicity in this paper, we assume that if $x_k = 1$, then $y_i = 0$ for any activity i that requires asset k. That is, interdiction of an asset stops the defender from carrying on activities that depend on that asset. We note that defender-attacker models often exhibit a one-to-one relationship between assets and activities; for example, interdiction of a pipeline segment between cities a and b stops the single activity that can occur on that segment, "flow from a to b."

Binary restrictions on $\mathbf{x}$, and some reasonable set of resource limitations on the attacker's resources, are represented by $\mathbf{x} \in X$. We represent the defender's set of feasible actions, restricted by interdictions $\mathbf{x}$, as $Y(\mathbf{x})$. Thus, the attacker solves this problem to guide his attacks:

$$\text{(MAX-MIN)} \quad \max_{\mathbf{x}\in X} \min_{\mathbf{y}\in Y(\mathbf{x})} \mathbf{cy}. \tag{2}$$

MAX-MIN is a type of bilevel program (e.g., Moore and Bard [29]), which is an instance of a Stackelberg game (von Stackelberg [40]). The attacker leads with an attack and the defender follows with a response, hence the standard phrases *leader* and *follower*, for attacker and defender, respectively. The key assumption here is that the attacker has a perfect model of how the defender will optimally operate his system, and the attacker will manipulate that system to his best advantage. That is a strong but prudent assumption for the defender: He can suffer no worse if the attacker plans his attacks using a less-than-perfect model of the defender's system. We find no difficulties in assuming that the defender will operate his system optimally, but a simple adjustment to the objective function can account for certain types of inefficiencies. (More general models of inefficiency seem unsupportable. For instance, one might be able to model a defender who always operates his system at a random point along, say, the "90%-efficiency frontier," but such a model would be hard to solve and, more importantly, hard to justify.)

One can devise many supportable generalizations of MAX-MIN including attacks that increase costs rather than limiting activities, or attacks that reduce the capacity of an asset less than 100%. We will cover some of these generalizations after establishing basic results.

Naturally, the defender may also lack perfect knowledge of the attacker's capabilities. That is, the defender may be guessing at the interdiction-resource constraints contained within $\mathbf{x} \in X$. However, the defender can solve the model over a range of posited interdiction resources and use those results to guide system improvements.

Solving an Attacker-Defender Model

For many situations, a linear program (LP) will provide an adequate model of the defender's system and its operations. For instance, the electric power industry commonly employs linearized optimal power-flow models for security analysis (Wood and Wollenberg [44]). Therefore, we express the optimal operation of the defender's system as

$$
\begin{aligned}
\text{(D0)} \qquad & \min_{\mathbf{y}} \ \mathbf{c}\mathbf{y} && (3)\\
& \text{s.t.} \quad A\mathbf{y} = \mathbf{b} && (4)\\
& \qquad F\mathbf{y} \le \mathbf{u} && (5)\\
& \qquad \mathbf{y} \ge 0. && (6)
\end{aligned}
$$

Constraints (4) correspond to general system-operations constraints (e.g., balance of flow in a transportation network), and constraints (5) represent capacity limitations for asset $k \in K$ (e.g., maximum flow across the kth network link, per unit of time). Assets might include power lines, pipelines, roads, ports, communications hubs, etc.

The attacker's interdictions might affect the system in any number of ways, but let us assume that only "assets" are in danger of being interdicted, and that interdiction of asset k causes the loss of all its capacity u_k. Thus, the full attacker-defender model is

$$
\begin{aligned}
\text{(AD0)} \qquad & \max_{\mathbf{x}\in X} \min_{\mathbf{y}} \ \mathbf{c}\mathbf{y}\\
& \text{s.t.} \quad A\mathbf{y} = \mathbf{b}\\
& \qquad F\mathbf{y} \le U(\mathbf{1} - \mathbf{x})\\
& \qquad \mathbf{y} \ge \mathbf{0}
\end{aligned}
$$

where $U = \text{diag}(\mathbf{u})$. We assume that the inner LP has been constructed to be feasible for any $\mathbf{x}$, because we expect the system to operate in some degraded fashion for any conceivable attack. This may require the use of auxiliary variables that are not susceptible to interdiction.

A natural approach to reformulating this problem fixes $\mathbf{x}$ temporarily, takes the dual of the inner linear program, and then releases $\mathbf{x}$. Unfortunately, an unappealing, nonlinear, mixed-integer program results. That model can be linearized in some instances (e.g., Wood [43], Salmerón et al. [35]), but an alternative model comes to mind: Change the paradigm of capacity interdiction to "cost interdiction," and then take the dual of the inner problem. (See Cormican et al. [16] for the mathematical details.) Specifically, let $-\mathbf{p}$ strictly bound the set of dual variables associated with $F\mathbf{y} \le U(\mathbf{1} - \mathbf{x})$, taken over all possible values of $\mathbf{x} \in X$. Thus, p_k bounds the value of a unit of asset k's capacity for the defender. Because we assume that AD0 is feasible even when asset k has been interdicted and has no capacity, it must be possible to set a cost on asset k's capacity that makes it too costly to use: p_k is just that cost. This is the standard approach to formulating an "elastic model"; see Brown et al. [8] for more discussion.

Thus, AD0 is equivalent to

$$
\begin{aligned}
\text{(AD1)} \qquad & \max_{\mathbf{x}\in X} \min_{\mathbf{y}} (\mathbf{c} + \mathbf{x}^T PF)\mathbf{y} \qquad && \text{[Dual vars. for fixed } \mathbf{x}]\\
& \text{s.t.} \quad A\mathbf{y} = \mathbf{b} && [\boldsymbol{\theta}]\\
& \qquad F\mathbf{y} \le \mathbf{u} && [\boldsymbol{\beta}]\\
& \qquad \mathbf{y} \ge \mathbf{0},
\end{aligned}
$$

where $P = \text{diag}(\mathbf{p})$. (Actually, nonstrict bounds $\mathbf{p}$ are also valid for identifying an optimal $\mathbf{x}$; see Cormican et al. [16].)

Now, when we take the dual of the inner minimization, a mixed-integer linear program (MILP) results:

$$
\text{(AD1-MILP)} \qquad \begin{aligned}
&\max_{\mathbf{x},\boldsymbol{\theta},\beta} \mathbf{b}^T\boldsymbol{\theta} + \mathbf{u}\boldsymbol{\beta} \\
&\text{s.t.} \quad A^T\boldsymbol{\theta} + F^T\boldsymbol{\beta} - F^T P\mathbf{x} \le \mathbf{c} \\
&\qquad\qquad\qquad\qquad\qquad\quad \mathbf{x} \in X \\
&\qquad\qquad\qquad\qquad\qquad\quad \boldsymbol{\beta} \le \mathbf{0}.
\end{aligned}
$$

We can solve this model directly, or by using Benders decomposition [4]. In fact, the standard Benders approach for integer $\mathbf{x}$ begins by taking the dual of AD1-MILP with $\mathbf{x}$ fixed, which obviously yields AD1. Thus, the max-min formulation of AD1 is a natural representation of the interdiction problem for application of Benders decomposition.

To illustrate with a concrete, albeit simplified, example, consider the following model of a crude-oil pipeline network:

Data

A node-arc incidence matrix for the pipeline network

$\mathbf{b}$ vector of supplies and demands: $b_i > 0$ defines a supply of b_i million barrels per day (mmbbl/day) at node i, $b_i < 0$ defines a demand of b_i mmbbl/day at i, and $b_i = 0$ implies i is a transshipment node (pumping station)

$\mathbf{c}_1$ vector of shipping costs by pipeline segment, i.e., arc (\$/mmbbl/day)

$\mathbf{c}_2$ vector of penalties for not taking available supply ("take-or-pay penalties") (\$/mmbbl/day)

$\mathbf{c}_3$ vector of penalties for unmet demand (e.g., spot-market cost) (\$/mmbbl/day)

$\hat{I}_2$ incomplete diagonal matrix with a 1 for each supply node, but 0 elsewhere

$\hat{I}_3$ incomplete diagonal matrix with a 1 for each demand node, but 0 elsewhere

Variables

$\mathbf{y}_1$ flows on pipelines (mmbbl/day)

$\mathbf{y}_2$ unused supply (mmbbl/day)

$\mathbf{y}_3$ unmet demand (mmbbl/day)

Formulation

$$
(\mathrm{D_P0}) \qquad \min_{\mathbf{y}} \; \mathbf{c}_1\mathbf{y}_1 + \mathbf{c}_2\mathbf{y}_2 + \mathbf{c}_3\mathbf{y}_3
$$

$$
\text{s.t.} \quad A\mathbf{y}_1 - \hat{I}_2\mathbf{y}_2 + \hat{I}_3\mathbf{y}_3 = \mathbf{b} \tag{7}
$$

$$
I\mathbf{y}_1 \le \mathbf{u} \tag{8}
$$

$$
\text{all variables} \ge 0.
$$

Constraints (7) are elastic flow-balance constraints, and constraints (8) represent pipeline capacities. For simplicity, we

1. have ignored the oil's purchase price,
2. will assume $\mathbf{c}_2 = \mathbf{0}$ and $\mathbf{c}_1 > \mathbf{0}$,
3. set all unmet demand penalties equal, i.e., $\mathbf{c}_3 = (c_3 \; c_3 \ldots c_3)$, and
4. assume that only pipeline segments can be interdicted (not, say, pumping stations).

Now we proceed directly to create a cost-interdiction model in the form of AD1. Let $x_k = 1$ if the attacker interdicts asset k, let $x_i = 0$ otherwise, and let $\mathbf{x} \in X$ denote the binary restrictions on $\mathbf{x}$ along with some plausible resource constraints. For example, intelligence indicates that the attacker can form at most T squads to carry out simultaneous attacks, so $X = \left\{x_i \in \{0,1\} \; \forall i \in I \,\middle|\, \sum_{i \in I} x_i \le T\right\}$. We further note that $p = c_3$ exceeds the penalty incurred by not supplying one mmbbl/day (because $\mathbf{c}_1 > \mathbf{0}$). Thus, letting $\mathbf{p} = (c_3\, c_3 \cdots c_3)$ and $P = \text{diag}(\mathbf{p})$, the max-min interdiction model is

$$\begin{aligned} (\text{AD}_\text{P}1) \qquad & \max_{\mathbf{x}\in X} \min_{\mathbf{y}} (\mathbf{c}_1 + \mathbf{x}^T P)\mathbf{y}_1 + \mathbf{c}_2\mathbf{y}_2 + \mathbf{c}_3\mathbf{y}_3 \\ & \text{s.t.} \quad A\mathbf{y}_1 - \hat{I}_2\mathbf{y}_2 + \hat{I}_3\mathbf{y}_3 = \mathbf{b} \\ & \qquad\qquad\qquad\qquad I\mathbf{y}_1 \le \mathbf{u} \\ & \qquad\qquad\quad \text{all variables} \ge 0. \end{aligned}$$

We leave it to the reader to take the dual of the inner minimization to create ADP1-MILP, but a caveat is in order: The quality of the LP relaxation of that MILP will depend directly on how small the penalties p_k are, and the modeler may need to expend some effort in identifying small, valid values. For instance, each p_i in ADP1 can be validly reduced to $p_k - c_{1,\min} + \varepsilon$ where $c_{1,\min}$ is the smallest shipping cost a demand might incur while being satisfied, and where ε is some small, positive constant.

In some instances, a cost-interdiction model like AD1 can actually be a more natural paradigm than AD0. In such cases, the analyst can avoid the AD0-to-AD1 transition and will not have to worry about bounds on dual variables. For instance, suppose D0, with constraints (5) eliminated, corresponds to a shortest-path problem on a road network. In some situations, we may replace the capacity constraints by modeling the interdiction of a link k in the network as a delay d_k on the nominal length c_k (transit time). Thus, this model becomes:

$$\begin{aligned} (\text{AD}_\text{R}1) \qquad & \max_{\mathbf{x}\in X} \min_{\mathbf{y}} (\mathbf{c} + \mathbf{x}^T D)\mathbf{y} \\ & \text{s.t.} \quad A\mathbf{y} = \mathbf{b} \\ & \qquad\quad \mathbf{y} \ge \mathbf{0}, \end{aligned}$$

where $D = \text{diag}(\mathbf{d})$, with $\mathbf{d}$ being the vector of delays d_k. See Israeli and Wood [25] for details on this model and solution techniques for it. We note that AD_R1 also fits into the framework of defender-attacker models, described next; § 4.4 provides an example.

Defender-Attacker Models

By solving an attacker-defender model, we identify a set of most critical components for an infrastructure system. This leads to some obvious heuristics for solving an "optimal defense problem," i.e., identifying the best possible defense plan given a limited defense budget. We prefer truly optimal solutions, however.

In theory, one merely embeds the bilevel attacker-defender model in a trilevel defender-attacker-defender model such as

$$\min_{\mathbf{z}\in Z} \max_{\mathbf{x}\in X(\mathbf{z})} \min_{\mathbf{y}\in Y(\mathbf{x})} \mathbf{cy}. \tag{9}$$

Here, $\mathbf{z}$ denotes a binary vector of defense decisions ($z_k = 1$ if asset k is hardened and made invulnerable, say, and $z_k = 0$, otherwise); $\mathbf{z} \in Z$ denotes the binary restrictions on $\mathbf{z}$ together with budgetary (and possibly other) constraints; and the inner max-min problem simply represents an attacker-defender model with a restricted set of attack strategies, $X(\mathbf{z})$. Thus, the defender seeks to identify a defense plan $\mathbf{z}^*$ so that when the attacker solves

$$\max_{\mathbf{x}\in X(\mathbf{z}^*)} \min_{\mathbf{y}\in Y(\mathbf{x})} \mathbf{cy}, \tag{10}$$

the benefit the attacker sees, i.e., the damage the attacker can guarantee to inflict, is minimized.

Unfortunately, these trilevel problems solve only with extreme difficulty, and no conversion to an MILP appears possible, in general. (See Israeli and Wood [25] for more details, and for the description of one special-case solution technology.)

Fortunately, certain optimal-defense problems lend themselves to easier bilevel, defender-attacker models. The defender becomes the leader in this new Stackelberg game, so we essentially reverse the meanings of $\mathbf{x}$ and $\mathbf{y}$, and make the following definitions:

Indices

k asset the defender might want to defend, and the attacker might want to attack (this simple defender-attacker model assumes a one-to-one relationship between potentially attacked and potentially defended assets)

Data

c_k value to the attacker of attacking undefended asset k (vector form $\mathbf{c}$)

p_k reduction in value of attacking the defender's kth asset if that asset is defended, i.e., the attacker receives benefit $c_k + p_k$, $p_k \le 0$, by attacking defended asset k (vector form $\mathbf{p}$)

Variables

$$x_k = \begin{cases} 1 & \text{if the defender defends his } k\text{th asset} \\ 0 & \text{otherwise} \end{cases}$$

$$y_k = \begin{cases} 1 & \text{if the attacker attacks the defender's } k\text{th asset} \\ 0 & \text{otherwise} \end{cases}$$

$\mathbf{x}, \mathbf{y}$ vector forms of y_k and x_k, respectively

Constraints

$\mathbf{x} \in X$ resource constraints and binary restrictions on the defender's defense plan, e.g., $X = \{\mathbf{x} \in \{0,1\}^n \mid G\mathbf{x} \le \mathbf{f}\}$

$\mathbf{y} \in Y$ resource constraints and binary restrictions on the attacker's attack plan, e.g., $Y = \{\mathbf{y} \in \{0,1\}^n \mid A\mathbf{y} = \mathbf{b}\}$

Formulation

$$\text{(DA1)} \qquad \min_{\mathbf{x} \in X} \max_{\mathbf{y}} (\mathbf{c} + \mathbf{x}^T P)\mathbf{y} \quad \text{s.t. } \mathbf{y} \in Y.$$

A simplified example illustrates. Suppose intelligence reports indicate that a terrorist organization, "the attacker," intends to send out b teams to attack b different subway stations in a city encompassing $M > b$ total stations. Municipal authorities, "the defender," have m teams, $m < M$, with which to defend stations; a defended station becomes invulnerable to attack. The value to the defender of station k is $c_k > 0$, and we assume the attacker assigns the same values. (If not, the defender's optimal defense plan may perform better than predicted.) Let $p_k = -c_k$; thus, if station k is defended, the attacker will gain no benefit by attacking it. This "subway-defense problem" may be formulated as

$$\text{(DA1}_{\text{SUB}}) \qquad \min_{\mathbf{x} \in X} \max_{y} \sum_{k=1}^{M} (c_k + x_k p_k) y_k \tag{11}$$

$$\text{s.t.} \quad \sum_{k=1}^{M} y_k = b \tag{12}$$

$$y_k \in \{0,1\} \quad \forall k \tag{13}$$

where $X = \{\mathbf{x} \in \{0,1\}^M \mid \sum_{k=1}^{M} x_k = m\}$.

In general, the model DA1 and instances like $\mathrm{DA1}_{\mathrm{SUB}}$ are difficult to solve because the inner minimization is not an LP. Thus, no general transformation exists to convert DA1 into an MILP as we converted AD1 into AD1-MILP. This situation can be resolved in several ways:

1. We decide that continuous attack effort represents a reasonable approximation of reality and convert Y to $Y_{CONT} = \{\mathbf{y} \in R_+^n \mid A\mathbf{y} = \mathbf{b}\}$ (Golden [23]).

2. The LP relaxation of Y, $Y_{LP} = \{\mathbf{y} \in R_+^n \mid A\mathbf{y} = \mathbf{b}\}$, yields intrinsically binary solutions, so a conversion from DA1 into "DA1-MILP" is, in fact, possible. This is the situation with $\mathrm{DA}_{\mathrm{SUB}}1$, and we invite the reader to work out the details. See Brown et al. [9] for an example involving theater ballistic-missile defense.

3. Or, neither of the cases above pertains, and we really must include restriction $\mathbf{y} \in \{0,1\}^n$ in the definition of Y.

Case 3 requires special techniques to solve, but solution methods better than brute-force enumeration do exist (e.g., Israeli and Wood [25], Brown et al. [10]). This paper focuses on the simpler case, Case 2.

What We Have Done

A terrorist organization can learn just about everything it needs to know to plan a perfect attack on our critical infrastructure. This key insight leads us to apply attacker-defender and defender-attacker models to problems of protecting this infrastructure. This section describes a number of these models (the first embedded in a complete decision-support system), along with applications.

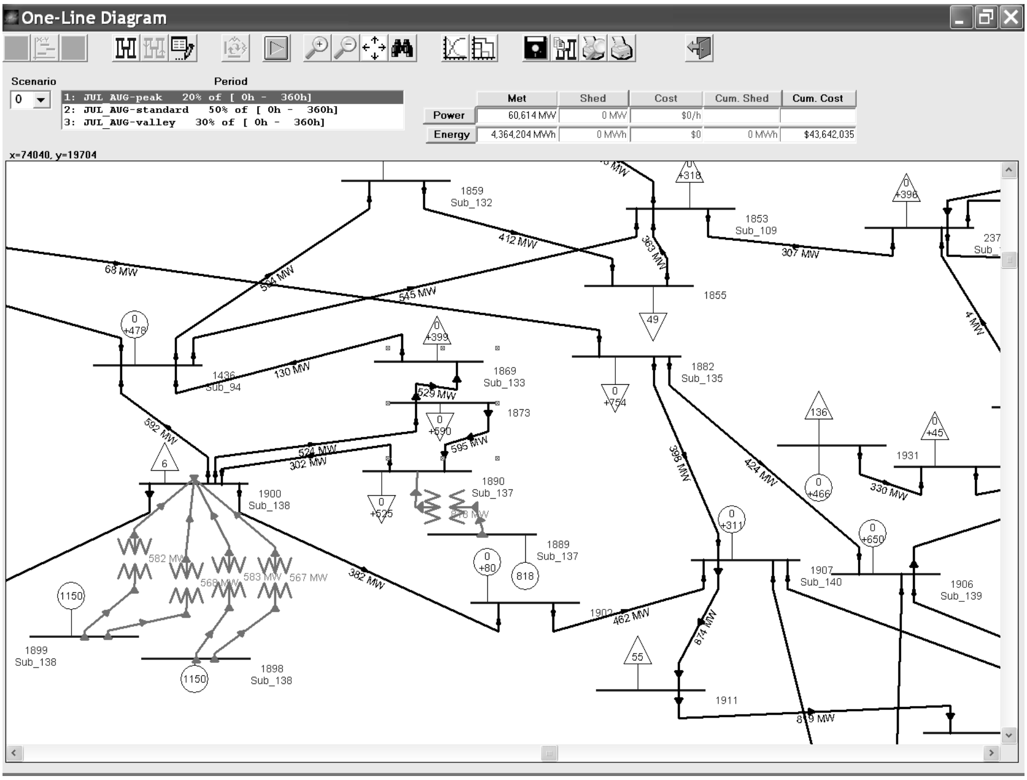
These models reflect our experience as military planners who have been asked to help target enemy infrastructure and defend our own infrastructure, such as road, communication, electric power, and pipeline networks. Most of the models have been derived in the course of our research and/or our students'. We have been fortunate to be able to test many of these models by (i) defining a hypothetical but real-world scenario; (ii) assembling a "red team" of well-trained, military officer-students to gather scenario data from strictly public sources; (iii) guiding the team in building, instantiating, and running an appropriate model, and then analyzing the results.

The results are always interesting, and usually lead to valuable insights. We find cases in which a given set of attackers can do more damage than we would have predicted, or less; and sometimes the attacks do not target the "obvious" components revealed in single-point-of-failure analyses. An anecdote illustrates this last point. Suppose that a terrorist organization wants to attack and close down the operations of a specific airline, at a single airport, for the purpose of disrupting the airline's finances. Based on passenger-revenue data obtained from the Internet, a red-team analysis indicates that "City A" is the most damaging airport to strike for one large U.S. airline. If the terrorists can afford two strikes, Cities B and C would be best (Brown et al. [7]).

Electric Power Grids: An Attacker-Defender Model

We have produced a decision-support system called the Vulnerability of Electric Grids Analyzer (VEGA) (Salmerón et al. [37]), which identifies an optimal or near-optimal attack (i.e., a set of coordinated attacks) on an electric power grid. VEGA also animates the system operator's optimal response to that attack. Given a scenario extracted from an electric grid database and an assessment of the level of effort needed for an attacker to target each component, VEGA determines, and illustrates graphically, which equipment-loss patterns lead to maximal damage measured in terms of load (demand for power) that must be shed (dropped). Figure 1 depicts one of VEGA's many interface screens. We note that VEGA has been built with the intention of analyzing regional, bulk-power transmission systems as opposed to local distribution systems, but it could certainly be used in the latter case.

FIGURE 1. A screen shot of the "one-line diagram" view in VEGA.



One-Line Diagram

Scenario: 0

Period:

```
1: JUL_AUG-peak      20% of [ 0h -   360h]
2: JUL_AUG-standard  50% of [ 0h -   360h]
3: JUL_AUG-valley    30% of [ 0h -   360h]
```

	Met	Shed	Cost	Cum. Shed	Cum. Cost
Power	60,614 MW	0 MW	$0/h		
Energy	4,364,204 MWh	0 MWh	$0	0 MWh	$43,642,035

x=74040, y=19704

Note. The icons depict generators, transformers, buses, transmission lines, and customer demands. For any given level of attacker capability, VEGA finds an optimal or near-optimal target set of vulnerable components (Salmerón et al. [36].)

In VEGA, an "optimal DC power-flow model" comprises D0, the inner, minimizing LP. This model incorporates elastic current-balance (flow-balance) constraints along with linearized admittance constraints for AC lines. This power-flow model approximates the "true" active power flows and disregards reactive power flows, but the electric power industry normally deems this approximation adequate for analyzing system security.

When an electric grid possesses sufficient generating and transmission capacity to meet all demand, the power-flow model reflects how a system operator would set generating levels to minimize cost. When capacity is insufficient, as after an attack, the model reflects how the operator will react to minimize the amount of load shed, while using generation cost as a secondary criterion.

Given a fixed attack plan, VEGA must solve a sequence of power-flow models. This is true because we normally model long-term unmet demand for energy (amount of load shed, integrated over time), taking into account (i) differing repair times for components, as well as (ii) daily demand variations ("load duration curves"), and (iii) seasonal demand variations. Modeling restoration is crucial because damaged transmission lines might be repaired in a few days, other components might be repaired in a week or two, but a damaged transformer might take many months to replace. Transformers pose special difficulties because they are big, heavy, and expensive; few spares exist; and a replacement might have to be ordered from, built by, and shipped from an overseas manufacturer.

An attacker-defender model can be embedded in a formal trilevel model to optimize the upgrading or hardening of a system against terrorist attack (Israeli and Wood [25]); see also §3 in this chapter). Such models exist for electric grids, but real-world instances are impossible to solve at this time (Salmerón et al. [35]). Consequently, we use heuristic procedures as illustrated here. We consider a small section of the U.S. grid containing roughly 5,000 buses, 500 generators, 3,000 loads, 5,000 lines, 1,000 transformers, 500 substations, a total reference load of 60 gigawatts (GW), and a total generating capacity of 70 GW.

We posit a group of 10 terrorists: A single terrorist can destroy a line, which takes 48 hours to repair; two terrorists can destroy a transformer or a bus, which takes 168 hours to repair; and three terrorists can destroy a substation, which takes 360 hours to repair. (These repair times are likely to be optimistic and serve for purposes of illustration only.) Three hundred and sixty hours also represents the study's time horizon because the system can be fully repaired in that time.

We employ a load-duration curve (a staircase function) that states: The actual load is 100% of the reference load 20% of the time ("peak load"), 70% of the reference 50% of the time ("standard load"), and 45% of the reference 30% of the time ("valley load"). This load-duration curve implies a total demand for energy, over the course of the study, of about 15,000 gigawatt-hours (GWh). For simplicity, we set all generation costs to $10 per megawatt-hour (MWh) and set the cost of any unmet demand to $1,000 per MWh.

VEGA identifies a near-optimal interdiction plan for the terrorists in about 30 minutes on a 3 gigahertz personal computer. The plan interdicts three substations and one line, which results in 356 GWh of energy being shed over the study period, and a peak unmet load of 2.8 GW. These values are small percentagewise, but 2.8 GW represents the requirements of nearly three million residential customers. The economic effects of this attack would be substantial.

From these results, it is clear that protecting substations must be a priority. Therefore, we assume utility companies will spend enough money on increased security at the three hypothetically attacked substations to make them invulnerable to such attacks. We rerun VEGA with this information and find that total unmet demand reduces to less than 160 GWh and peak unmet load decreases to 1.4 GW. Once again, the terrorists attack three substations and one line.

We have reduced the disruption that the 10 terrorists can cause by about 50%, but suppose the defense budget enables us to harden the three substations attacked in the second round, plus one more: We choose one that seems to be important in a model variant that allows 15 terrorists. In the ensuing third round of attacks, the 10 terrorists attack three substations and one line, but this attack results in total unmet demand for energy of only 90 GWh and a peak unmet load of less than 600 GW. Thus, we can substantially reduce the vulnerability of this power grid by improving security at only seven substations, from a total of roughly 500. This may be deemed cost effective by utility planners.

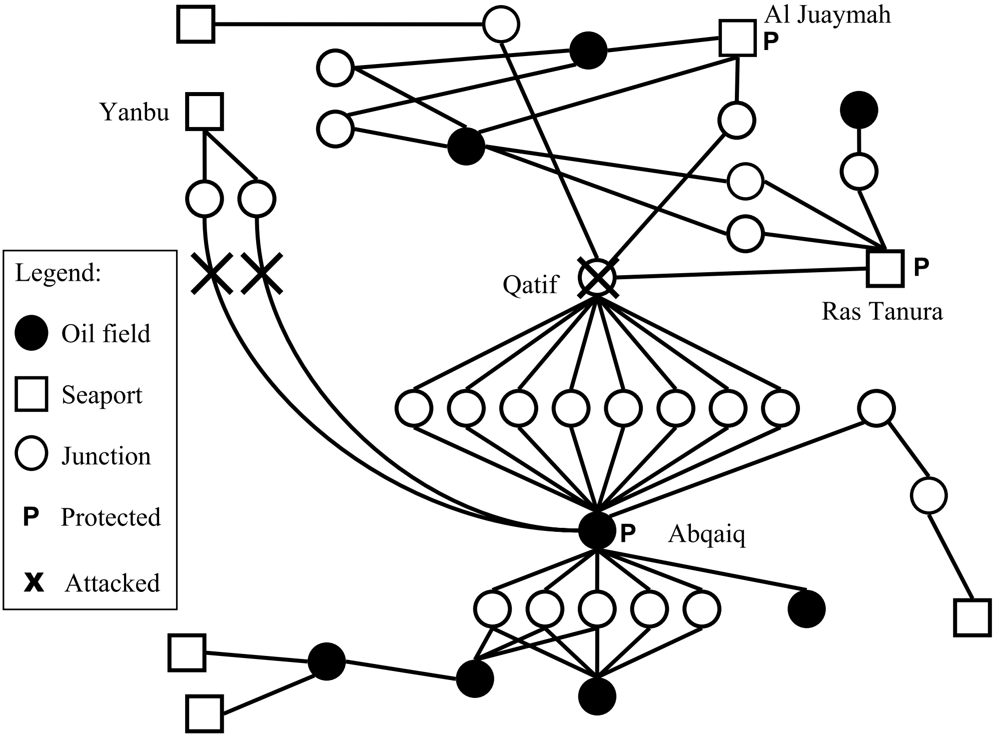
VEGA has been funded, in part, by the U.S. Department of Homeland Security, Office of Domestic Preparedness, and by the Department of Energy. It uses an Intel-based computer, a Microsoft operating system, and modeling software, all of which costs about five thousand dollars per seat.

Oil Pipelines: An Attacker-Defender Model

Pipeline systems for crude oil and refined petroleum products (and natural gas) are sparsely connected because of the enormous expense required to acquire rights of way, lay pipe, build pumping stations and maintain the system once it is complete. For instance, consider Figure 2, which is a schematic of the crude-oil pipeline network in Saudi Arabia (found, with all capacity data, through a simple Internet search). This network is clearly sparse, although our experience indicates that it is more densely connected than the typical gas or oil pipeline in the United States (e.g., Avery et al. [2]). In fact, the Saudi network may have substantial redundant capacity (Bremmer [6]) and, consequently, may be more resilient to attack than pipeline networks elsewhere.

An enormous security force guards the Saudi pipeline network (Sparshott [38]), but the network covers a huge area that cannot be patrolled completely. Where should the Saudi

Figure 2. Three attacks on the Saudi Arabian crude-oil pipeline system reduce capacity by 3.7 mmbbl/day.



Note. The Saudi Arabian oil pipeline network has some heavily protected, invulnerable components, indicated by "P," but most of the network is hard to defend and vulnerable to attack. Assuming insurgents have only enough resources to attack three different facilities, the three attacks shown maximally reduce Saudi capacity, even after the pipeline operator optimally redirects flows to use reserve capacity. The reduced output here exceeds a breakpoint estimated to cause a worldwide economic recession (Andrews et al. [1]).

government concentrate its security efforts? A full answer to that question would require a more detailed study than we can undertake. However, one of our red teams has found an "Achilles cutset," so to speak, whose components ought to be considered closely for potential protection.

For purposes of analysis, we play the part of a terrorist organization. First, what is our goal? Well, analysts at Morgan Stanley (Chaney and Berner [14]) report that a reduction in Saudi crude-oil output to 4 mmbbl/day (million barrels per day), from a current 8 to 9 mmbbl/day, would cause worldwide economic distress. The loss would only amount to about 5% of world demand, but Chaney and Berner estimate the price of oil would jump to $80/bbl from a 2004 price of $40/bbl. Furthermore, this jump could lead to a global recession if damaged facilities could not be repaired in a few months. So, taking the lead from Morgan Stanley, we set a goal of reducing Saudi oil output to 4 mmbbl/day or less.

Naturally, we would like to implement a coordinated strategy that requires as few individual attacks as possible. What is the minimum number necessary to reach our goal? We assume that the largest oil field at Abqaiq is well protected, i.e., invulnerable to attack, as are the two seaports on the Persian Gulf, Ras Tanamura and Al Juaymah. However, all other system components, pipeline segments, and junctions are potential targets.

We can solve this problem via the max-flow interdiction model of Wood [43], which minimizes maximum flow given a fixed amount of interdiction resource. (Thus, we must solve a min-max attacker-defender model rather than paradigmatic max-min model, AD0.) Each seaport in Figure 2 is connected to a supersink, with the arc's capacity equaling the port's capacity. Similarly, each oil field is connected by an arc to a supersource, with the arc's capacity equaling the production capacity of the field. Pipeline arcs are assigned their known capacities, and junctions are split into arcs, as required, to represent limited pumping capacity.

The best single attack targets the junction at Qatif. Worldwide oil prices spike on the news, but moderate quickly when it is learned that maximum output has only been reduced to 8.7 mmbbl/day, in a system whose current total capacity is about 10 mmbbl/day, with current output around 9 mmbbl/day. (Exact values for these numbers would depend on when the hypothetical attack occurs. These values are close to current numbers, but Saudi Arabia may add capacity in the near future, and demand could increase or decrease.)

The best attack on two targets adds one of the pipelines connecting Abqaiq and Yanbu, and reduces maximum output to 5.8 mmbbl/day. The world gets really worried. The best attack on three targets adds the second Abqaiq-Yanbu pipeline, Saudi output drops to 3.7 mmbbl/day, our goal has been reached, and worldwide oil prices shoot skyward.

This situation might not last for long—pipelines can usually be repaired fairly quickly—but at the very least, a painful spike in oil prices would result. The three targeted pipeline-system components need security measures reviewed at the very least. At first glance, it seems that a reasonable strategy to mitigate such attacks would add a third Abqaiq-Yanbu pipeline, parallel but not collocated for obvious reasons. However, this pipeline would extend 1,200 kilometers and, estimating from other pipeline construction projects around the world, might cost one billion dollars (Pipeline & Gas Journal [32]). Clearly, other options require exploration.

The D.C.-Metro System: A Defender-Attacker Model

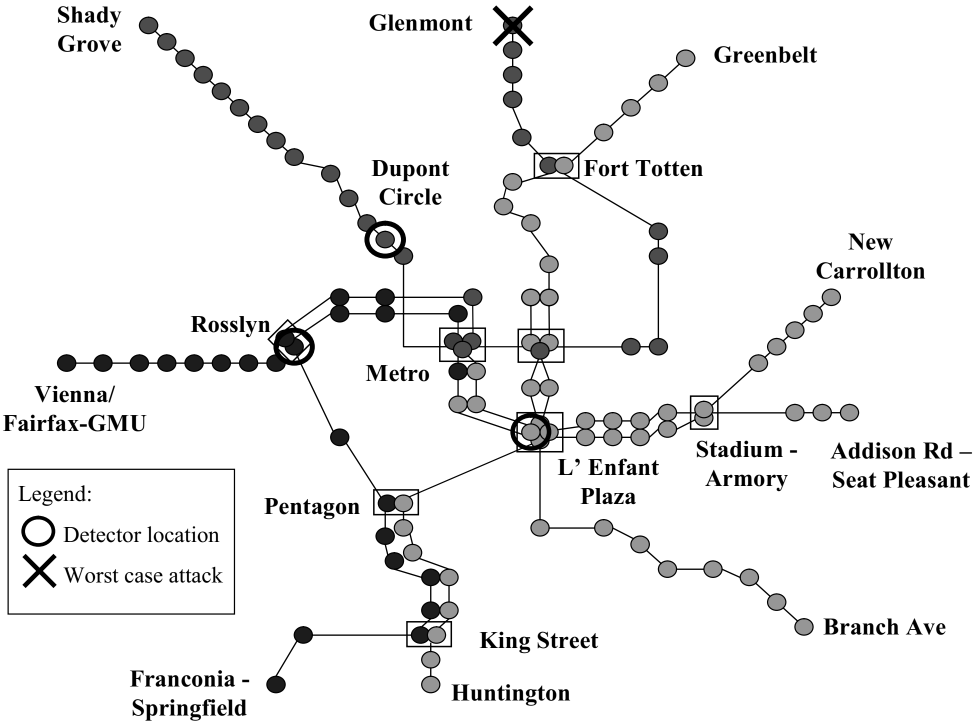
Terrorists have certainly considered the possibility of attacking the United States with nuclear, biological, or chemical (NBC) agents. In likely scenarios, terrorists contaminate a civilian population with a chemical or biological agent, or with radioactive debris from a "dirty bomb." Subway systems in metropolitan areas seem to be attractive targets for this purpose, because their efficiency in moving large numbers of people, quickly, over large distances, would also spread a contaminant among large numbers of people, quickly, over large distances. Consequently, authorities have already begun to install NBC sensors in the Washington, D.C., subway system ("D.C. Metro") and in other transportation facilities around the country (Chang [15]). NBC sensors are expensive, so given limited budgets, how should these detectors be deployed? Figure 3 displays a diagram of the D.C. Metro, and depicts optimal locations given a supply of three sensors. "Optimal" implies that the locations minimize the worst-case time to detection (i.e., no matter where a terrorist might strike). By minimizing detection time, trains could be stopped as quickly as possible after an attack and hazardous-material response teams called in to help reduce casualties. The detection-time objective function only takes transit times and interplatform transfer times into account, but it could certainly account for passenger volumes, if desired.

We will not provide details of this min-max defender-attacker model, but we note that related models have been studied for detecting malevolent contamination of a municipal water system (e.g., Berry et al. [5]).

Figure 4 shows the value of the optimal solution for varying numbers of detectors. This diagram leads to the key insight for policymakers: casualties versus dollars.

Before leaving this topic, we must add a caveat, lest the reader be lulled into a false sense of security. At this stage in the development of NBC detectors, especially biological detectors, noxious substances cannot be quickly and reliably identified. Such detectors may be able to identify a "suspicious" substance instantaneously, but verification may take many hours; sensitivity must be increased and false positives decreased if such technology is to prove useful. A Defense Science Board report states "...in fact, a technological breakthrough is needed" (Defense Science Board [17]).

FIGURE 3. Locations of NBC detectors in the D.C. Metro (subway) system to minimize maximum time to detect an attack.



Note. Using public Metro maps and schedules, we model the circulation of an NBC agent throughout the network. The solution installs detectors at Dupont Circle, L'Enfant Plaza, and Rosslyn. Observing this, an optimizing attacker would choose Glenmont to maximize the time to first detection: 31 minutes (Avital et al. [3]).

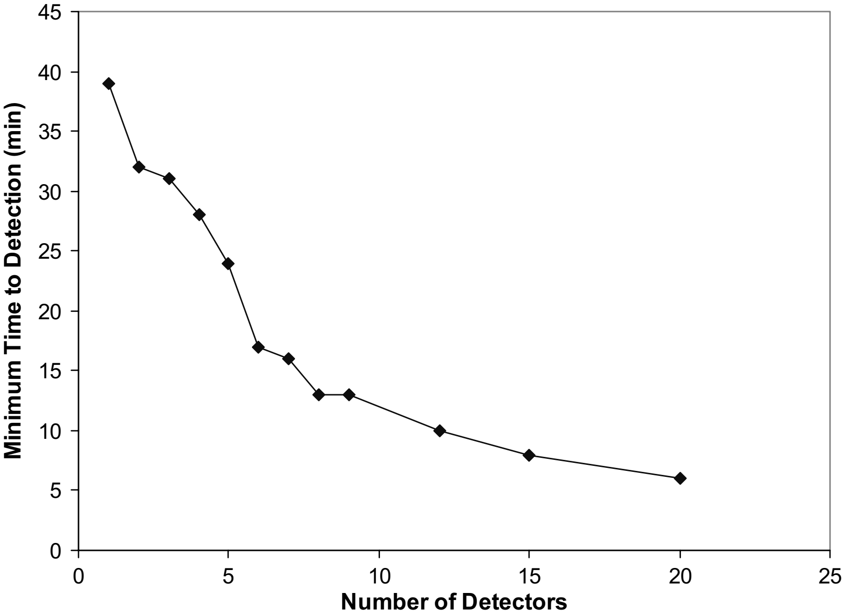
Improving Airport Security: A Defender-Attacker Model

Airport security has received much attention in recent years, mostly regarding the effectiveness, or ineffectiveness, of personnel and equipment at security checkpoints (Miller [28]). However, the system aspects of airport security deserve the attention of OR analysts. Here, we investigate techniques to improve the probability of detecting a terrorist who is trying to: infiltrate Terminal 1 at the Los Angeles International Airport (LAX); reach an airline gate; and hijack or sabotage an airplane. For simplicity, we consider only a single terrorist, or "infiltrator," who moves along the standard paths that legitimate passengers use.

Figure 5 shows a map of Terminal 1, along with a skeleton of the "infiltration network" that describes the paths that an infiltrator could take from "curbside" into the terminal, through a check-in procedure, through one or more security checkpoints, and finally out to the airline gates. (The full network contains too many arcs to depict.) We shall represent the airport's administration: Our goal is to spend a limited "defense budget" on screening devices and procedures that increase detection probabilities on individual arcs, with the purpose of maximizing the overall detection probability. The options for changing procedures include, for instance, simply closing off certain ingress routes, or performing a physical search of, say, every third passenger, rather than every tenth. In addition to improving standard screening equipment, the red team analyzing this scenario (Landon et al. [26]) also includes the potential installation of advanced imaging devices now undergoing field tests (Levine [27]).

Probability of nondetection proves to be a more convenient concept with which to describe a defender-attacker model for this problem. For simplicity, we assume every arc k in the

FIGURE 4. Minimum detection time of an NBC attack on the D.C. Metro as a function of the number of detectors.



Note. This is a display for policymakers: The horizontal axis converts to investment of millions of dollars, and the vertical axis converts to the number of people exposed to NBC contaminants, i.e., potential casualties. An analysis like this frequently reveals sharp break points (say, at six detectors here) that may affect policy making (Avital et al. [3]).

network possesses some nominal probability of nondetection, $q_k > 0$. This is the probability that the infiltrator will not be detected if he traverses arc k. If we spend exactly c_k dollars at arc k, a new device will be installed, or a new procedure implemented, and the nondetection probability becomes $\bar{q}_k > 0$, with $\bar{q}_k < q_k$. (Notes: (i) The model extends easily to handle multiple options for reducing nondetection probability on an arc, (ii) completely closing off a route can be handled by setting $\bar{q}_k$ arbitrarily close to 0, and (iii) every artificial arc k connecting t in $\mathcal{G}$ has $q_k = \bar{q}_k = 1$.) Our overall task is to expend a total budget of c' dollars so as to maximize the minimum probability of nondetection along any path the infiltrator might take. Assuming independence of detection events, this model can be formulated as follows (see the related stochastic-programming model in Pan et al. [31]):

Indices and Structural Data

$i \in \mathcal{N}$ nodes of the infiltration network

$k \in \mathcal{A}$ directed arcs of the infiltration network

$\mathcal{G} = (\mathcal{N}, \mathcal{A})$ infiltration network

Variables

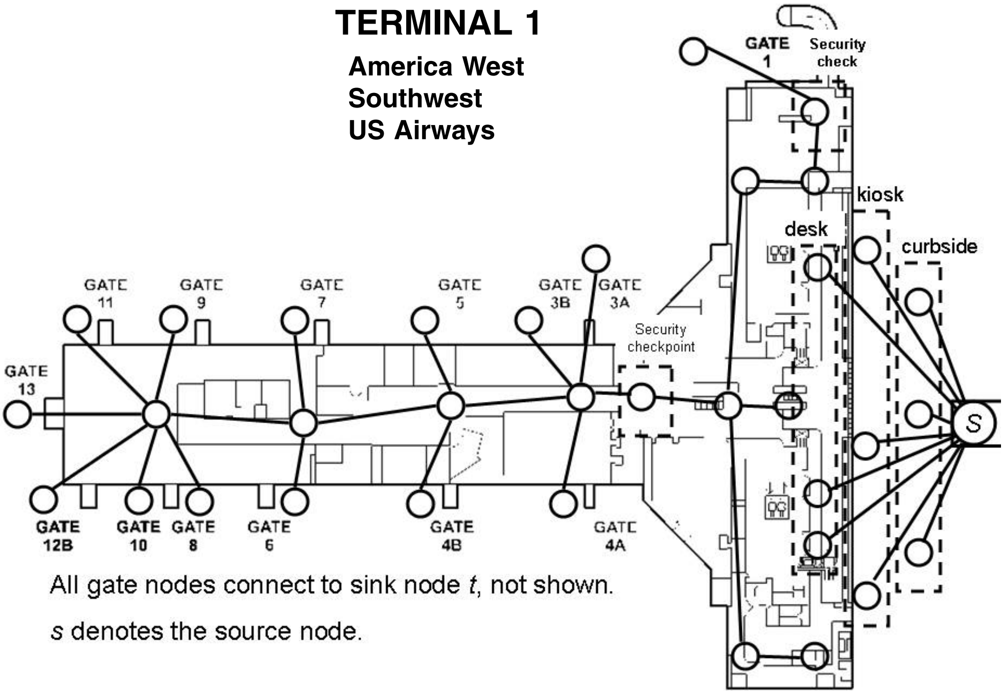
$$x_k = \begin{cases} 1 & \text{if the defender upgrades security on arc } k \\ 0 & \text{otherwise} \end{cases}$$

$$y_k = \begin{cases} 1 & \text{attacker traverses arc } k \text{ when } x_k = 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\bar{y}_k = \begin{cases} 1 & \text{attacker traverses arc } k \text{ when } x_k = 1 \\ 0 & \text{otherwise} \end{cases}$$

$\mathbf{x}, \mathbf{y}, \bar{\mathbf{y}}$ vector forms of x_k, y_k, and $\bar{y}_k$, respectively

Figure 5. A limited security budget can be optimally allocated to protect Los Angeles International Airport (LAX) Terminal 1.



Note. This figure displays a map of the terminal along with a skeleton of the "infiltration network" that represents infiltration routes for terrorists (and the routes the legitimate passengers use). Arcs not shown represent movements from check-in desks or automated check-in kiosks to screening stations, through screening stations, through physical-search stations, and also artificial arcs connecting each gate node to a single sink node t (Landon et al. [26].)

Data

A node-arc incidence matrix corresponding to $\mathcal{G}$
$\mathbf{b}$ node-length vector with $b_s = 1$, $b_t = -1$ and $b_i = 0$ for all $i \in \mathcal{N} - s - t$
q_k nominal probability of nondetection on arc k when $x_k = 0$ ($q_k > 0$, vector form $\mathbf{q}$)
$\bar{q}_k$ probability of nondetection on arc k when $x_k = 1$ ($q_k > \bar{q}_k > 0$, vector form $\bar{\mathbf{q}}$)
d_k $\ln q_k$ (natural log of q_k) (vector form $\mathbf{d}$, matrix form $D = \operatorname{diag}(\mathbf{d})$)
$\bar{d}_k$ $\ln \bar{q}_k$ (vector form $\bar{\mathbf{d}}$, matrix form $D = \operatorname{diag}(\bar{\mathbf{d}})$)
c_k cost, in dollars, to upgrade security on arc k (vector form $\mathbf{c}$)
c' total budget, in dollars, for upgrading security

Formulation

$$(\mathrm{DA}_{\mathrm{LAX}}1) \quad \min_{\mathbf{x} \in X} \max_{\mathbf{y}, \bar{\mathbf{y}}} \prod_{k \in A} q_k^{(1-x_k)y_k} \bar{q}_k^{x_k \bar{y}_k} \tag{14}$$

$$\text{s.t.} \quad A\mathbf{y} + A\bar{\mathbf{y}} = \mathbf{b} \tag{15}$$

$$\mathbf{y}, \bar{\mathbf{y}} \in \{0,1\}^{|\mathcal{A}|} \tag{16}$$

where $X = \{\mathbf{x} \in \{0,1\}^{|\mathcal{A}|} \mid \mathbf{cx} \le c'\}$.

Constraints (15) and (16) ensure that one unit of "unsplittable flow," representing the infiltrator, moves from s to t. Constraints (15) are standard flow-balance constraints, just like those one could use to model a shortest-path problem in $\mathcal{G}' = (\mathcal{N}, \mathcal{A} \cup \mathcal{A})$, which is simply $\mathcal{G}$ with each arc duplicated.

The standard reformulation technique for this model takes a logarithm of the objective function, say the natural logarithm. This leads to the essentially equivalent model, $\mathrm{D2}_{\mathrm{LAX}}$, below. It is clear then that we can replace constraints (16) with simple nonnegativity restrictions, because the constraint matrix (15) is totally unimodular, and for fixed $\mathbf{x}$, the model defines a straightforward shortest-path problem on $\mathcal{G}'$ (if we multiply the nonpositive objective function by -1, and switch the inner maximization to a minimization). Note also that the infiltrator's objective can only worsen by putting flow around a cycle, so no difficulty analogous to negative-length cycles in a shortest-path problem can arise here.

$$(\mathrm{DA2}_{\mathrm{LAX}}) \qquad \begin{aligned} &\min_{\mathbf{x}\in X}\max_{\mathbf{y},\bar{\mathbf{y}}} (\mathbf{1}+\mathbf{x})^T D\mathbf{y} + \mathbf{x}^T \bar{D}\bar{\mathbf{y}} \\ &\text{s.t.} \quad A\mathbf{y} + A\bar{\mathbf{y}} = \mathbf{b} \\ &\qquad\qquad \mathbf{y},\bar{\mathbf{y}} \geq \mathbf{0} \end{aligned}$$

Clearly, this model converts easily into an MILP.

Before reporting computational results, we note that modifying security equipment and procedures can both increase delays for legitimate passengers, or decrease them. For instance, increasing the percentage of people receiving physical searches on an arc will certainly increase the detection probability for an infiltrator traversing that arc, but it will also raise the average passenger's delay there. On the other hand, adding parallel metal detectors, parallel imaging devices, and parallel personnel to oversee this equipment will reduce average delays. $\mathrm{DA2}_{\mathrm{LAX}}$ can be modified to incorporate constraints that limit, at least approximately, the average delay for a legitimate passenger. However, for simplicity, we simply report the changes in delay that result from changes in security procedures and equipment, under the pessimistic assumption that passengers do not adjust their routes to reduce delay for themselves.

For obvious reasons, our red team can only make educated guesses about the cost of, and improved detection probabilities for, these devices. The team must also make similar guesses regarding the delay that new imaging devices will cause passengers. Therefore, the absolute statistics reported by the team cannot be taken literally. However, the relative results are believable, and the methodology can accept any system-describing parameters, which field testers and manufacturers should eventually be able to provide. We summarize the red team's computation results below. Note that "Risk" reflects probability of detection only as a relative value, and the expenditures are probably optimistic and the delay values are probably pessimistic:

1. Baseline, Scenario 1, no security improvements: Budget = \$0, "Risk" (to the infiltrator) = 10, Delay (incremental) = 0 hours, Actions = {}.
2. Scenario 2: Budget = \$100,000, Risk = 126, Average Delay = 1.5 hours, Actions = {Add two imaging devices, screen 1 in 10 at two locations, close three check-in kiosks}. (Note: Closing an automated kiosk increases the reliability of identification checks.)
3. Scenario 3: Budget = \$250,000, Risk = 249, Average Delay = 2.5 hours, Actions = {Add 15 imaging devices, screen 1 in 3 at all locations security checkpoints, close three check-in kiosks}.

Supply Chains

Supply chains, i.e., physical distribution systems, are a key infrastructure of private-sector companies that manufacture and/or distribute goods. "Supply chains" do not appear on the list of critical infrastructure systems shown in Table 1, but they are certainly critical to our nation's well-being.

Strategic supply-chain design has a long and successful record in the United States, reducing costs and increasing service levels. Unfortunately, efficient supply chains are fragile. In fact, after scrupulously investing exactly the right amount of money in a supply chain, on exactly the right bottlenecks, the resulting product-flow patterns resemble one or more spanning trees. However, as any OR analyst knows, a spanning tree is maximally fragile: Breaking any link disconnects the network.

To address supply-chain vulnerability, we have teamed with Prof. Terry Harrison of Pennsylvania State University and Dr. Jeffrey Karrenbauer, President of INSIGHT, Inc., a company devoted to supply-chain optimization for over 25 years (INSIGHT [24]). Together, we have analyzed detailed corporate supply-chain data for many companies, including the majority of the FORTUNE 50. Also, we have developed new features for INSIGHT's supply-chain optimization tools to evaluate and mitigate supply-chain vulnerability. Many key results have already been presented by Brown et al. [11, 12, 13], so we provide only an overview here.

The first key "result" is an observation: We still encounter considerable confusion in the private sector between random acts of nature—these have been studied by insurance actuaries for centuries—and belligerent acts of intelligent terrorists who observe defensive preparations and act to maximize damage. We strongly suggest remedying this confusion before proceeding with any analysis.

On occasion, one can reduce vulnerability substantially with simple planning and with only a modest investment in new physical infrastructure. Sometimes, just strategically relocating surge capacity can provide benefit at virtually no cost. This contrasts with the high cost of adding redundant capacity, or hardening components, in other types of infrastructure such as pipelines and electric power grids.

We have learned that labor unions and competitors can be just as clever and determined as terrorists, and have similar goals: maximize damage inflicted (to market share, profit, reputation, etc.). The denial of access to West Coast ports in the United States in 2002 due to a labor dispute was no less damaging than the anthrax attacks of 2001 that closed postal and shipping services on the East Coast.

We have presented our findings to numerous companies, with enthusiastic responses to even simple discoveries. American companies now have senior executives focused on "preserving corporate continuity." These positions were originally motivated by threats to information systems, and thus back-up computer facilities and doubly backed-up data have become ubiquitous. Now, these same companies are coming to realize that they must also back up their physical operations to handle attacks on their own infrastructure (e.g., equipment, warehouses) as well as on public infrastructure they use (e.g., roads, communications networks).

Other Systems

Our work on critical infrastructure protection represents just one aspect of a research program that has also led to new military and diplomatic planning models; two have already been incorporated into comprehensive decision-support systems. One system helps plan theater ballistic-missile defense (Brown et al. [9]). The embedded defender-attacker model optimally locates antimissile platforms (ships or ground-based units supplied with antimissile missiles), while assuming the attacker can see some or all of our defensive preparations. The other system identifies optimal actions (e.g., embargoes of key materials, economic sanctions, military strikes) to delay a covert nuclear weapons program (Brown et al. [10]). This is an attacker-defender model where we, for a change, are the attacker. As with the missile-defense model, analysis can be carried out under different assumptions regarding the adversary's (defender's) ability to observe our actions. This model applies to any complex industrial project that can be delayed by a competitor.

A key insight from these military and diplomatic exercises is that deception and secrecy can make huge contributions to successful defense of our critical assets, or to successful attacks on an adversary's critical assets. (The techniques of two-person game theory can also be useful here; for example, see Owen [33].) Secrecy is already becoming an important (and debated) issue in the general area of homeland security.

Although this work is all relatively new, there is already an emerging body of unclassified publications including about fifty case studies, over twenty graduate theses, open-literature

Table 3. Case studies of these topics have evaluated roles of both attacker and defender.

Electric grids
Road networks
Strategic rail networks
Domestic water systems
Sea lanes, canals, restricted straits
Multicommodity supply chains
Petroleum distribution networks for
- U.S. Southwest
- Northern California
- Defense Fuel Supply System, Japan

Theater ballistic missile attacks
Weapons of mass destruction (WMDs) at
- Reliant Stadium, Houston, Texas
- Washington, D.C., Metro
- Melbourne, Australia
- Changi Naval Base, Singapore
- Manhattan
- Norfolk, Virginia

Insurgent incursions
Economic warfare
WMD development project

publications, and a number of prototypic decision-support tools. Table 3 shows some of the topics studied. We are working with certain institutions that address these threats and welcome inquiries from others. We also provide classified products to planners when the need arises.

What We Have Learned

We have discovered much through our own mathematical modeling of critical-infrastructure protection, and from applications and red-team studies. We have also learned from reading the literature, attending conferences, and speaking with colleagues, clients, and students. This section summarizes the lessons we have gleaned from all these sources.

The attacker has the advantage. This is the reverse of classical military theory and accrues from the asymmetric nature of this conflict: The defender must protect a huge, dispersed target set, while the attacker need only focus on a small set of targets chosen to maximize damage.

Some systems are naturally robust, while others are not. It turns out that our road systems are remarkably robust, fuel-distribution systems are highly fragile, and most other systems lie somewhere in between.

Hardening an infrastructure system from attack can be expensive. However, if you understand what the most damaging attacks must look like, you can better improve the system's robustness against attack for a given budget.

Critical infrastructure has been built to be "cost-effective" with little concern for coordinated, belligerent attacks. Consequently, these systems are fragile with respect to such attacks, and even four years after September 11th, private owners of infrastructure have few economic incentives to spend large sums of money to reduce this fragility. This calls for (i) government subsidies, changes to tax codes, and regulatory reform, and/or (ii) proving the secondary economic benefit of these expenditures, if such exist (for example, spare electric transmission capacity could provide new, profitable trading opportunities).

The data are out there, and if we can get them, anybody can. "Sunshine laws" in the United States require that our governments, federal to local, conduct their affairs with transparency to the public. As a result government agencies have produced lots of excellent websites with lots of useful information for terrorists based anywhere in the world. Many websites have been redesigned in recent years to reduce access to potentially dangerous information, but we find stunning exceptions. We advise owners of public websites associated with infrastructure to appoint an independent "red team" to analyze the website with intent to cause harm to the owners or to the users of the infrastructure.

The answers are not always obvious. The most damaging coordinated attacks, or the most effective defenses, can be nonintuitive. Key U.S. infrastructure systems are huge, and analysis at large scale deserves rigorous, purpose-built, optimizing decision-support tools to formalize the notion of a transparent, two-sided conflict. Heuristics have their place here but, preferably, not for identifying worse-case attacks. What is an "approximately worse-case attack"?

Malicious, coordinated attacks can be much more damaging than random acts of nature. Our audiences usually arrive with the opposite point of view. However a, skillful, small-scale attack can inflict more damage than a major hurricane or earthquake.

Reliability is not the answer. We must protect the most *critical* components in our infrastructure systems, rather than backing up the least reliable components. Many infrastructure owners still think that a "reliable system," i.e., a system that fails rarely due to random events, will be a "robust system" in the face of malicious, coordinated attacks. However, common sense (for a terrorist) dictates: Destroy the most reliable components. After all, they have been made most reliable because they are most critical for system operations.

The right redundancy may be the answer. For any given level of investment, there is usually a dominant set of incremental changes to infrastructure that returns maximal immediate benefit. For some types of infrastructure—e.g., supply chains—benefit can be achieved at relatively modest cost by adding a few alternate shipment paths or by installing excess capacity at just the right locations, etc.

Secrecy and deception may be valuable. Our military applications of attacker-defender and defender-attacker models have shown that much can be gained from secrecy and/or deception. For instance, hiding the location of a defensive asset can cause the attacker to strike a target that is essentially invulnerable. Clearly, in the world of suicide terrorists and physical infrastructure, such an outcome could be desirable.

Worst-case analysis using optimization is key to a credible assessment of infrastructure vulnerability, and to reducing that vulnerability. We cannot depend on standard reliability analyses to protect us adequately because we cannot assume that attacks occur randomly. We face a determined, intelligent enemy who seeks to do us maximal harm.

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