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Abstract

This article derives a dynamic model of the firm with endogenous investment and leverage ratio within the framework of the dividend discount model. Our valuation model incorporates both managerial flexibility and long-run growth. We derive closed-form solutions for the optimal policies and firm value, which allow us to compute the value of the real options as well as secular growth in a systematic way. A standard parameterization of the model suggests that the value of the real options can account for more than 8% of the market value of equity, while the present value of growth opportunities can represent more than 10% of share price. We also characterize the type of industries where traditional valuation models lead to considerable underpricing of securities.

JEL classification: G31, G32

Keywords: Dividend Discount Model; Gordon Growth Model; Real Options; Long-Run Growth; Dynamic Programming

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1 Introduction

The traditional dividend discount model (DDM) has been used as one of the main tools for firm valuation for decades. In its simplest formulation, this methodology consists in projecting the level of economic activity of the firm in the future years and then discounting the resulting cash flows to the present. One of the most attractive features of this model is that it dispenses with any utility specification that would capture the preferences of shareholders over alternative realizations of dividends.[1] Another attractive feature of the model is that it handles long-run growth (e.g., the Gordon Growth Model). However, the traditional DDM has been criticized because of its implicit assumption of precommitment to a deterministic plan of action. The fact that it does not include the value of the real options available to the firm reduces its usefulness as an asset pricing model in those cases where managerial flexibility plays an important role. We derive a dynamic model in the spirit of the DDM that, while preserving its two main features, incorporates the value of the real options.

It is implicitly assumed in the canonical DDM that projected cash flows derive from optimal firm behavior. We develop a dynamic model of the firm with endogenous choice of investment and leverage ratio that makes explicit the optimizing behavior that supposedly underlies firm decisions. In doing this, we provide some microeconomic foundations for the widely used DDM. By using dynamic programming techniques, we are able to introduce managerial flexibility into the model. We interpret the latter as the possibility of the firm to adapt itself to the future realization of uncertainty. The structural approach we follow allows us to find analytic formulas for firm decisions, market value of equity, the real options available to the firm, and long-term growth. We also ascertain the proportion of share price that is represented by the value of the real options, as well as by secular growth, and perform comparative statics analysis. We finally do a cross-sectional study of different industries that allows us to find those cases where managerial flexibility and long-run growth play a key role. These are the industries where our valuation method displays the largest advantage over existing ones. We next elaborate on these two distinctive components of our model and then describe our main findings.

[1] This important simplification is possible due to the Separation Principle. See, for example, Copeland, Weston, and Shastri (2005).

The firm in our dynamic model is subject to the influence of two different forces: secular growth and the business cycle. These forces are independent of each other and drive firm decisions. Long-run growth is a deterministic upward trend that induces the firm to increase its assets by a constant rate in each period. On the contrary, the business cycle evolves stochastically over time, consisting in short-term oscillations around the long-run growth trend. The corporate finance literature has usually modeled these short-term cycles as mean-reverting profit shocks. During these fluctuations, the CEO has the ability to adapt the firm in order to maximize shareholders' wealth; that is, the manager can take advantage of the real options available to the firm to capitalize on good fortune or to mitigate loss. When profits go up, the manager increases firm assets to take advantage of the incremental value (the expansion option). Because profits are persistent, as long as they continue to be high, the CEO will maintain the firm large (the extension option). After profits go down, the CEO responds by shrinking firm assets to reduce the impact of bad results (the contraction option). While profit shocks continue to be bad, the manager postpones new investments, keeping the firm small until better times come (the deferral option). Moreover, in the extreme case in which the firm experiences zero profits in a period, it has the possibility to completely stop its operations and wait until profits become positive again (the switching option). Thus, the real options emerge naturally in our model because the CEO has the possibility to adjust the firm over the business cycle in order to maximize share price.

Virtually all dynamic programming models of the firm in corporate finance have been developed without the possibility of long-run growth. In these models, relevant variables take on only bounded values as they fluctuate over time around a long-run constant. These oscillations can be interpreted as the firm making decisions in order to maximize its value over the business cycle with no possibility to engage in long-term growth. We contribute to this literature by introducing unbounded growth into the dynamic model. Many firms have experienced secular growth with an overlay of short-term cycles. We identify growth with the long-run increment in size of the firm and not with short-term increases in firm assets resulting from fluctuations over the business cycle. This type of long-run growth might be true for mature, stable firms, such as utilities.

It has been recognized long ago that the traditional DDM undervalues the price of corporate securities because it does not include the value of managerial flexibility. The relevant question is, then, by how much. We use the dynamic DDM to ascertain the proportion of share price that is represented by the value of the real options of the firm, as well as by secular growth. We parameterize the model with standard values used in the corporate finance literature and find that the value of the real options explains around 8% of the market value of equity. If we consider that in reality the firm also has other real options, such as the exit option, the conclusion is that the value of managerial flexibility can account for a substantial proportion of share price, which is not considered by the traditional DDM. The present value of growth opportunities is also an important component that represents another 10% of market equity. Furthermore, both proportions turn out to be counter-cyclical over the business cycle because the value of the real options and secular growth are less sensitive to profit shocks than is share price. This result suggests that the underpricing problem of the traditional DDM is more severe during economic recessions than expansions. We also find that these two features are complementary because the possibility to grow increases the value of the real options.

We finally perform a comparative statics analysis of managerial flexibility and secular growth, as well as a cross-sectional comparison of different industries. We find that the elasticity of the capital input in the production function as well as the volatility and persistence of profit shocks are the main determinants of the value of the real options. Not surprisingly, we also find that the market cost of equity and the growth rate are the two most important determinants of the present value of the growth opportunities. Because the downward bias in the stock price produced by standard valuation models depends on the specific firm under study, we characterize some industries where omitting managerial flexibility and/or long-run growth would lead to a considerable underpricing problem. For instance, we find that while for firms in the Oil and Gas Extraction industry not considering the real options would cause a large undervaluation of the stock price, for corporations in the Chemical industry not taking into account expected future growth would lead to an analogue problem. We believe that for these types of industries the model we propose is quite promising.

1.1 Literature Review

Discounted cash flow models in general, and the DDM in particular, can be traced back to Williams (1938). Copeland, Weston, and Shastri (2005) and Damodaran (2011), among many others, present a broad and updated review of the large body of literature about these traditional valuation models. We contribute to this literature by deriving a dynamic specification of that canonical model that takes into account the real options available to the firm.

Real options have been studied extensively in the economics and finance literature. Brennan and Schwartz (1985) use option pricing theory to describe the real options embedded in natural resource investment projects. McDonald and Siegel (1986) investigate the optimal timing of irreversible investments. Pindyck (1988) studies how randomness and irreversibility of investments impact the value of the real options held by the firm, as well as firm's capacity and market value. Dixit (1989) develops a model of investment decisions with "hysteresis" in a stochastic setting. Kulatilaka and Marcus (1992) show the systematic undervaluation of investments produced by the traditional discounted cash flow methodology in the presence of real options. Trigeorgis (1993) shows how to value the operating options of the firm, such as the option to defer, expand, and abandon investments. Quigg (1993) uses 2,700 land transactions in Seattle to study the value of the option to wait to develop the land and finds that the option premium represents, on average, 6% of the value of the land. Moel and Tufano (2002) study 285 decisions to open and close gold mines in North America and find that real options models provide better predictions of those decisions than other methods, such as the traditional discounted cash flow model. Our paper contributes to these strands of literature by providing a dynamic programming model of the firm based on the canonical DDM that allows us to calculate the value of managerial flexibility in a systematic manner.

Several papers in corporate finance use dynamic programming models of the firm to explain results observed by the empirical literature.[2] Moyen (2004) studies the sensitivity of firms' investments to changes in their internally generated funds. Hennessy and Whited (2005) explain some empirical findings apparently inconsistent with the trade-off theory, such as market timing. Hennessy and Whited (2007) estimate the costs of external finance. Moyen (2007) analyzes the

[2] See Strebulaev and Whited (2012) for a comprehensive review of this literature.

debt overhang problem in the sense of Myers (1977). Gamba and Triantis (2008) study the value and optimal management of financial flexibility. Tserlukevich (2008) develop a real options model to explain stylized facts about leverage behavior. Riddick and Whited (2009) explore the reasons why firms save cash. Gomes and Schmid (2010) use a real options model to analyze the relation between capital structure and stock returns. DeAngelo, DeAngelo, and Whited (2011) analyze the opportunity cost of borrowing today in terms of diminished future financial flexibility. We next relate our work to these papers.

Most of the papers we have just described assume shareholders are risk-neutral. This assumption allows them to use linear utility functions to capture shareholders' preferences. We develop the dynamic model of the firm within the framework of the DDM, which assumes the Separation Principle holds and, therefore, does not require any utility specification. According to this principle, in the context of a perfect capital market, the manager maximizes the lifetime expected utility of all current shareholders by maximizing the price of current shares (i.e., shareholders' wealth) independently of their individual subjective preferences. Thus, the CEO needs to know only the appropriate cost of capital of the firm to use as the discount rate. This property makes the present model a useful asset pricing tool.[3] In addition, our work differs from the extant literature on dynamic programming models of the firm in that we allow for long-run growth. As we show in our study, incorporating this feature into the model is important for firms where growth opportunities account for a large part of the stock price, such as companies in the Chemical industry. On the other hand, the previous papers contain other realistic features (i.e., diverse kinds of frictions, such as costly adjustment of capital, costly issuance of debt and/or equity, etc.) that we omit in this work. These features could be, nevertheless, easily introduced into the present model for asset pricing purposes. Even if the resulting model loses the closed-form, it could still be used to price securities properly and make meaningful quantitative (i.e., not only directional) predictions about firm decisions in the context of counter-factual policy analysis.

The paper is organized as follows. In Section 2, we obtain a dynamic version of the DDM that accounts for real options by applying dynamic programming. In Section 3, we separate the different components of market value of equity and study the economic importance of the real

[3]Dixit and Pindyck (1994) suggest this possibility for dynamic programming models of the firm.

options as well as long-run growth. The comparative statics analysis of the different components of share price and the cross-sectional study are in Section 4. Section 5 concludes. Appendix 1 contains propositions and proofs, while Appendix 2 describes the construction of model variables and calibration of model parameters.

2 A Dynamic Real Options Model of the Stock Price

In this section, we propose a dynamic valuation model that incorporates two key aspects of the firm, namely, managerial flexibility and secular growth. We view our model as a dynamic version of the standard DDM that could be used as a powerful valuation tool.

We use dynamic programming as the solution concept to derive the optimal policies as well as the market value of equity.[4] In our model, time is discrete and the firm makes decisions at the end of every period (e.g., quarter, year, etc.) The life horizon of the firm is infinite, which implies that shareholders believe it will run forever. The risk-free rate of interest in the economy is indicated by r_f.

The CEO makes investment and financing decisions such that the market value of equity is maximized. Variable $\widetilde{K}_t$ represents the book value of assets in period t.[5] The assets of the firm $\widetilde{K}_t$ will vary (i.e., increase or decrease) over time, reflecting the investment decisions. In each period, installed capital depreciates at constant rate $\delta > 0$. The debt of the firm in period t, $\widetilde{D}_t$, matures in one period and is rolled over at the end of every period. We assume the coupon rate c_B equals the market cost of debt r_B, which implies that book value of debt $\widetilde{D}_t$ equals market value of debt $\widetilde{B}_t$.[6] The amount of outstanding debt $\widetilde{B}_t$ will increase and decrease over time according to debt decisions. We assume the firm establishes leverage ratio $\ell_t \in [0, 1)$ as the maximum proportion of book assets that might be financed with debt such that it remains risk-free during all of the firm's life. Therefore, the firm can always repay its debt in full, and the market cost of debt r_B equals the risk-free interest rate r_f.

[4]Specifically, we use discrete-time, infinite-horizon, stochastic dynamic programming.

[5]The tilde on $\widetilde{X}$ indicates that variable X is growing over time.

[6]It is straightforward to generalize this component and assume a coupon rate c_B different from the market cost of debt r_B. Without any loss of generality and to simplify notation, we assume they are equal.

We introduce randomness to the model through the profit shock z_t. It is common in the corporate finance literature to assume that random shocks follow an AR(1) process in logs:

$$\ln(z_t) = \ln(c) + \rho \ln(z_{t-1}) + \varepsilon_t \tag{1}$$

where $\rho \in (0,1)$ is the autoregressive parameter that defines the persistence of profit shocks. In other words, a high ρ makes periods of high profit innovations (e.g., economics expansions) and low profit shocks (e.g., recessions) last more on average, and vice versa. The innovation term ε_t is assumed to be an *iid* normal random variable with mean 0 and variance σ^2. Constant $c > 0$ is a drift in logs that scales the moments of the distribution of z_t and has a direct impact on the expected profitability of the firm.[7] These assumptions result in lognormal conditional and unconditional distributions for profit shocks.[8]

Gross profits in period t are defined by the following function:

$$\widetilde{G}_t = (1+g)^{t(1-\alpha)} z_t \widetilde{K}_t^{\alpha} \tag{2}$$

where z_t is the profit shock in period t and parameter $\alpha \in (0,1)$ represents the elasticity of capital input. The level of technology in period t takes the form $(1+g)^{t(1-\alpha)}$, which implies the firm grows at rate $g \geq 0$ in each period. The exponent $(1-\alpha)$ makes the payoff function (dividends) homogeneous of degree-1 in growing variables, which is required to obtain constant long-run growth. Otherwise, the problem is ill-defined for that purpose. With this property, growing cash flows remain proportional to the market value of equity (the value function) and the firm becomes a scaled up replica of itself over time. This representation could be interpreted as technological change happening at a constant rate. Equation (2) says that gross profits depend on a profit innovation and a Cobb-Douglas production function with decreasing returns to scale in capital input. For existence of market value of equity, we impose the usual restriction that the secular growth rate must be lower than the market cost of equity (i.e., $g < r_S$). This market cost of equity is an exogenous rate that reflects the risk of the stock price. Subtracting operating costs $f\widetilde{K}_t$ (with $f > 0$) and depreciation $\delta\widetilde{K}_t$ from gross profits gives the firm's earnings before

[7] The usual assumption in the literature is $c = 1$, which makes term $ln(c)$ disappear from equation (1).

[8] Lazzati and Menichini (2013a) describe the steady-state of the model in closed-form.

interest and taxes in period t:

$$\widetilde{E}_t = (1+g)^{t(1-\alpha)} z_t \widetilde{K}_t^{\alpha} - f\widetilde{K}_t - \delta \widetilde{K}_t. \tag{3}$$

Corporate earnings are taxed at rate τ. Therefore, the firm's net profits in period t are

$$\widetilde{N}_t = \left(\widetilde{E}_t - r_B \widetilde{B}_t\right)(1-\tau). \tag{4}$$

With all the previous information, we can state the accounting cash flow equation or, equivalently, the cash flow that the firm pays to equity-holders in period t as

$$\widetilde{L}_t = \widetilde{N}_t + \widetilde{K}_t - \widetilde{B}_t - \widetilde{K}_{t+1} + \widetilde{B}_{t+1}. \tag{5}$$

Equation (5) is usually called the levered cash flow of the firm and implies that the dividend paid to shareholders in period t equals net profits minus the fraction of capital change that is financed with equity. Given the current state of the firm at $t = 0$, $\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right)$, the problem of the CEO is to choose an infinite sequence of functions $\left\{\widetilde{K}_{t+1}\left(\widetilde{K}_t, \widetilde{B}_t, z_t\right), \widetilde{B}_{t+1}\left(\widetilde{K}_t, \widetilde{B}_t, z_t\right)\right\}_{t=0}^{\infty}$, where z_t is the future profit shock in period t for all $t > 0$, such that the market value of equity is maximized. Accordingly, the stock price can be expressed as

$$\widetilde{S}_0\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right) = \max_{\left\{\widetilde{K}_{t+1}\left(\widetilde{K}_t, \widetilde{B}_t, z_t\right), \widetilde{B}_{t+1}\left(\widetilde{K}_t, \widetilde{B}_t, z_t\right)\right\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \frac{1}{(1+r_S)^t} \widetilde{L}_t \tag{6}$$

where E_0 is the expectation operator given information at $t = 0$ (i.e., z_0). Due to the introduction of long-run growth to the model, we need to normalize growing variables before proceeding with the maximization. Let vector $\widetilde{X}_t = \left\{\widetilde{K}_t, \widetilde{B}_t, \widetilde{G}_t, \widetilde{E}_t, \widetilde{N}_t, \widetilde{L}_t, \widetilde{S}_t\right\}$ contain the growing variables of the model. Then, we transform vector $\widetilde{X}_t$ in the following way: $X_t = \widetilde{X}_t/(1+g)^t$.

Finally, using the normalized variables and modifying the discount factor accordingly, the market value of equity can be expressed as

$$S_0(K_0, B_0, z_0) = \max_{\{K_{t+1}(K_t, B_t, z_t), B_{t+1}(K_t, B_t, z_t)\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \left(\frac{1+g}{1+r_S}\right)^t L_t. \tag{7}$$

Equation (7) provides some microfoundations for the canonical Gordon Growth Model in the stochastic setting. We obtain closed-form solutions for this problem by solving the Bellman equation associated to expression (7). We let variables with primes indicate values in the next period while normalized variables with no primes indicate current values (e.g., if we are at period zero, then next-period assets are $K' = K_1(K_0, B_0, z_0)$ and current assets are $K = K_0$). Then, the Bellman equation for the firm problem in equation (7) is given by

$$S(K, B, z) = \max_{K', B'} \left\{ L + \frac{(1+g)}{(1+r_S)} E\left[S\left(K', B', z'\right)|z\right] \right\}. \tag{8}$$

We solve equation (8) by using the usual backward induction argument, and the optimal policies for assets and debt are, respectively

$$K'^{*} = E\left[z'|z\right]^{\frac{1}{1-\alpha}} W^{*} \quad \text{and} \quad B'^{*} = \ell^{*} K'^{*} \tag{9}$$

where $E[z'|z] = cz^{\rho}e^{\frac{1}{2}\sigma^2}$ is the conditional expectation of the profit shock in the next period given current shock. In addition, $W^* = \left(\frac{\alpha}{\frac{r_S}{(1-\tau)}+f+\delta}\right)^{\frac{1}{1-\alpha}}$ is the time-invariant part of optimal capital and $\ell^* = \frac{1-(f+\delta)(1-\tau)}{1+r_B(1-\tau)}$ is the maximum (and optimal) book leverage ratio consistent with risk-free debt. This last ratio is constant over time and represents the target leverage of the firm. It shows the level of debt that maximizes the benefits of the tax shields while it keeps debt risk-free. Note that ℓ^* is (strictly) less than 1.

Equation (9) suggests that optimal capital increases with parameter α and the expected profit shock $E[z'|z]$. As α goes up, the concavity of the production function with respect to capital input diminishes and the marginal productivity (and profitability) of capital increases.[9] The expected profit shock increases with parameter c, current profit shock z, and volatility of innovations σ, which make the firm increase its optimal assets to take more advantage of their higher expected profitability. The effect of the persistence parameter ρ on expected profits depends on the value of current profit shock z. When $z > 1$, expected profits grow with ρ, while the opposite is true when $z < 1$.[10] A negative effect occurs with the four parameters in the denominator of W^*. A larger market cost of equity r_S decreases the present value of future periods' profits and the firm invests less. Higher income taxes τ, operating costs f, and depreciation δ reduce the profitability of capital and the firm diminishes optimal assets accordingly. The fact that financing decisions

[9] The positive effect of α on optimal capital is always true when $E[z'|z] > 1$, while it depends on W^* when $E[z'|z] < 1$.

[10] For standard values of the parameters, the invariant unconditional mean of profit shocks $E[z]$ is slightly above 1. Specifically, Table II shows that $E[z] = 1.05$. Thus, the event $z < 1$ is fairly frequent.

do not affect investing decisions and optimal capital is consistent with the Modigliani-Miller assumptions.

Returning to the previous notation with growing variables, where next-period assets are $\widetilde{K}_{n+1}\left(\widetilde{K}_n, \widetilde{B}_n, z_n\right)$ and current-period assets are $\widetilde{K}_n$, the solution of the Bellman equation leads to the following general formulation of the dynamic DDM:

Optimal firm decisions:

$$\widetilde{K}^*_{n+1}(z_n) = (1+g)^{n+1} E[z_{n+1}|z_n]^{\frac{1}{1-\alpha}} W^* \quad \text{and} \quad \widetilde{B}^*_{n+1}(z_n) = \ell^* \widetilde{K}^*_{n+1}(z_n). \tag{10}$$

Market value of equity:

$$\widetilde{S}_n\left(\widetilde{K}_n, \widetilde{B}_n, z_n\right) = \left(z_n \widetilde{K}^\alpha_n - f\widetilde{K}_n - \delta\widetilde{K}_n - r_B\widetilde{B}_n\right)(1-\tau) + \widetilde{K}_n - \widetilde{B}_n + (1+g)^n M_n P^* \tag{11}$$

where the first three terms on the right-hand side represent the (after-shock) book value of equity, while the last term is the going-concern value. Variable M_n is given by

$$M_n = e^{-\frac{1}{2}\sigma^2\frac{\alpha}{(1-\alpha)^2}}\left\{\left(\frac{1+g}{1+r_S}\right)E\left[z^{1/(1-\alpha)}_{n+1}|z_n\right] + \left(\frac{1+g}{1+r_S}\right)^2 E\left[z^{1/(1-\alpha)}_{n+2}|z_n\right] + ...\right\} \tag{12}$$

and represents the discounted sum of unconditional means of profit shocks. The general term turns out to be

$$E\left[z^{1/(1-\alpha)}_{n+t}|z_n\right] = \left(c^{\frac{1-\rho^t}{1-\rho}} z_n^{\rho^t} e^{\frac{1}{2}\sigma^2\frac{\left(1-\rho^{2t}\right)}{\left(1-\rho^2\right)}\frac{1}{(1-\alpha)}}\right)^{\frac{1}{1-\alpha}} \tag{13}$$

and indicates the expected profit shock t periods from today given current innovation z_n.[11] Variable P^* is given by

$$P^* = \left({W^*}^\alpha - fW^* - \delta W^* - r_B\ell^*W^*\right)(1-\tau) - r_S W^*(1-\ell^*) \tag{14}$$

[11] We obtain M_n in the following way. Let $A_0 = 0$ and, for $t = 1, 2, ...,$

$$A_t = A_{t-1} + \left(\frac{1+g}{1+r_S}\right)^t E\left[z^{1/(1-\alpha)}_{n+t}|z_n\right].$$

Then, we iterate the previous recursion until convergence (i.e., until $A_t = A_{t-1} = A$). Finally, we compute M_n as

$$M_n = Ae^{-\frac{1}{2}\sigma^2\frac{\alpha}{(1-\alpha)^2}}.$$

and denotes the (average) dollar return on equity minus the (average) dollar cost of equity at the optimum. Appendix 1 shows that P^* is (strictly) positive.

It is important to highlight that equation (11) deals with two independent aspects of growth, namely, *past* and *future* growth. The assumption that the firm is at an arbitrary current period $t = n$ implies that it has been running and growing for the past n periods. Thus, past growth refers to the n periods of growth that the firm has accumulated since the beginning of its life (at $t = 0$), which enters share price through $(1 + g)^n$. On the contrary, future growth refers to the growth opportunities that the firm expects to have in the future and enters the stock price through M_n. Specifically, future growth appears as $(1 + g)$ on the numerator of the discount factor in M_n. This distinction between past and future growth plays a fundamental role in the decomposition of share price that we perform in Section 3.

A nice property of dynamic programming models is that they treat firms as forward-looking agents and, thus, the market value of equity reflects not only current profit innovation but also the whole future sequence of expected profit shocks. This feature of the model is captured by variable M_n.

The market value of equity in equation (11) corresponds to the so-called Flow-to-Equity method of firm valuation. Lazzati and Menichini (2013b) describe those formulas in the context of other popular firm valuation methods, such as Weighted Average Cost of Capital and Adjusted Present Value.

In the next section, we decompose the stock price in its four primitive components and present the model predictions regarding managerial flexibility and secular growth.

3 The Value of Real Options and Long-Run Growth

As we mentioned before, one of the main disadvantages of the canonical DDM is that it does not consider managerial flexibility. It assumes that, at the beginning of the firm's life, the CEO projects the expected level of activity for all future periods and determines the required level of assets and debt for each of those periods in order to maximize shareholders' wealth. Then the firm is implicity assumed to follow that path of action passively, ignoring the fact that managers can react to future changes in the environment and make decisions to take advantage of good fortune or to alleviate loss. In this section, we investigate the economic importance of such undervaluation, as well as the relevance of the long-run growth opportunities. To this end, we first decompose the market value of equity into its primitive determinants, namely, the stock price with no managerial flexibility and no *future* growth opportunities, the value of the real options, the value of long-run growth, and the value of the interaction effect between the last two components. We then use parameter estimates that are standard in the literature to quantify each of these terms for a representative firm.

The aforementioned decomposition requires us to derive the stock price under different assumptions. First, we obtain the market value of equity for a firm that grows but ignores managerial flexibility. We do this by solving the problem of the firm in equation (6) with simultaneous optimization instead of dynamic programming techniques. That is, we assume the firm applies the expectation operator to all future cash flows first (conditioning on current profit shock z_0) and then maximizes the resulting objective function with respect to all decision variables at once. Because this optimization concept maximizes a deterministic stream of payoffs (i.e., the expected levered cash flows), it captures the assumption of precommitment to a deterministic plan of action that is embedded in the traditional DDM. This is equivalent to the CEO choosing an infinite sequence of functions $\left\{\widetilde{K}_{t+1}\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right), \widetilde{B}_{t+1}\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right)\right\}_{t=0}^{\infty}$, where z_0 is the realization of the current profit shock, such that the market value of equity is maximized. What in the previous section was an infinite sequence of stochastic functions that depended on future profit shock z_t, now is an infinite sequence of deterministic functions that depend on current (realized) profit shock z_0. **Saying it differently, the traditional DDM computes maximums of expectations while the real options approach calculates expectations of maximums.** With these assumptions, the problem of the firm in equation (6) becomes

$$\widetilde{S}_0\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right)_{WO} = \max_{\left\{\widetilde{K}_{t+1}\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right), \widetilde{B}_{t+1}\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right)\right\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \frac{1}{\left(1+r_S\right)^t} \widetilde{L}_t. \tag{15}$$

The subindex WO highlights that the solution corresponds to a firm without real options (but with growth). The solution to this problem is

$$\widetilde{S}_n\left(\widetilde{K}_n, \widetilde{B}_n, z_n\right)_{WO} = \left(z_n \widetilde{K}_n^{\alpha} - f\widetilde{K}_n - \delta\widetilde{K}_n - r_B\widetilde{B}_n\right)(1-\tau) + \widetilde{K}_n - \widetilde{B}_n + (1+g)^n \widehat{M}_n P^* \tag{16}$$

where

$$\widehat{M}_n = \left(\frac{1+g}{1+r_S}\right) E\left[z_{n+1}|z_n\right]^{\frac{1}{1-\alpha}} + \left(\frac{1+g}{1+r_S}\right)^2 E\left[z_{n+2}|z_n\right]^{\frac{1}{1-\alpha}} + ... \tag{17}$$

and the general term is

$$E\left[z_{n+t}|z_n\right] = c^{\frac{1-\rho^t}{1-\rho}} z_n^{\rho^t} e^{\frac{1}{2}\sigma^2 \frac{\left(1-\rho^{2t}\right)}{\left(1-\rho^2\right)}}. \tag{18}$$

The market value of equity in equation (11) differs from that of equation (16) via terms M_n and $\widehat{M}_n$. In particular, M_n captures the value of managerial flexibility, which is absent in $\widehat{M}_n$. Because the firm in the last model ignores the real options, the market value of equity in equation (11) is always greater than that of equation (16).

Second, we derive the stock price for a firm that has managerial flexibility but does not have the possibility to grow in the future. We obtain this value from equation (11) assuming $g = 0$ in the discount factor:

$$\widetilde{S}_n\left(\widetilde{K}_n, \widetilde{B}_n, z_n\right)_{WG} = \left(z_n\widetilde{K}_n^{\alpha} - f\widetilde{K}_n - \delta\widetilde{K}_n - r_B\widetilde{B}_n\right)(1-\tau) + \widetilde{K}_n - \widetilde{B}_n + (1+g)^n \overset{\triangle}{M}_n P^* \tag{19}$$

where

$$\overset{\triangle}{M}_n = e^{-\frac{1}{2}\sigma^2\frac{\alpha}{(1-\alpha)^2}} \left\{ \frac{1}{(1+r_S)} E\left[z_{n+1}^{1/(1-\alpha)}|z_n\right] + \frac{1}{(1+r_S)^2} E\left[z_{n+2}^{1/(1-\alpha)}|z_n\right] + ... \right\} \tag{20}$$

The subindex WG means that the solution corresponds to a firm without *future* growth opportunities (but with managerial flexibility). Note that while equation (19) does not include the present value of *future* growth opportunities, it does consider the impact of the n periods of *past* growth.

We finally find the share price of a firm that ignores managerial flexibility and does not have possibilities of *future* growth. This value is obtained from equation (16) assuming $g = 0$ in the discount factor:

$$\widetilde{S}_n\left(\widetilde{K}_n, \widetilde{B}_n, z_n\right)_{AP} = \left(z_n\widetilde{K}_n^{\alpha} - f\widetilde{K}_n - \delta\widetilde{K}_n - r_B\widetilde{B}_n\right)(1-\tau) + \widetilde{K}_n - \widetilde{B}_n + (1+g)^n \overline{M}_n P^* \tag{21}$$

where:

$$\overline{M}_n = \frac{1}{(1+r_S)} E\left[z_{n+1}|z_n\right]^{\frac{1}{1-\alpha}} + \frac{1}{(1+r_S)^2} E\left[z_{n+2}|z_n\right]^{\frac{1}{1-\alpha}} + ... \tag{22}$$

Equation (21) represents the part of share price that depends purely on the expected profitability of the firm's assets. The subindex AP refers to the fact that this part of the stock price is explained by the value of assets in place. As with equation (19), while $\widetilde{S}_n\left(\widetilde{K}_n, \widetilde{B}_n, z_n\right)_{AP}$ does not include the present value of *future* growth opportunities, it does include the n accumulated periods of *past* growth.

Next, we derive the different components of share price using the previous formulas. The value of the real options is given by

$$\widetilde{O}_n(z_n) = \widetilde{S}_n\left(\widetilde{K}_n, \widetilde{B}_n, z_n\right)_{WG} - \widetilde{S}_n\left(\widetilde{K}_n, \widetilde{B}_n, z_n\right)_{AP} = (1+g)^n\left(\overset{\triangle}{M}_n - \overline{M}_n\right)P^*. \tag{23}$$

That is, the present value of managerial flexibility equals the market value of equity of a firm that takes advantage of the real options but does not expect to grow in the future minus the stock price of a firm with no managerial flexibility and no future growth.

The value of long-run growth results from

$$\widetilde{G}_n(z_n) = \widetilde{S}_n\left(\widetilde{K}_n, \widetilde{B}_n, z_n\right)_{WO} - \widetilde{S}_n\left(\widetilde{K}_n, \widetilde{B}_n, z_n\right)_{AP} = (1+g)^n\left(\widehat{M}_n - \overline{M}_n\right)P^*. \tag{24}$$

Equation (24) means that the present value of secular growth equals the market value of equity of a firm with no managerial flexibility but with the possibility to grow in the future minus the stock price of a firm with no real options and no future growth.

We finally compute the value of the interaction effect between managerial flexibility and growth as a residual in the following way:

$$\begin{aligned}\widetilde{I}_n(z_n) &= \widetilde{S}_n\left(\widetilde{K}_n, \widetilde{B}_n, z_n\right) - \left[\widetilde{S}_n\left(\widetilde{K}_n, \widetilde{B}_n, z_n\right)_{AP} + \widetilde{O}_n(z_n) + \widetilde{G}_n(z_n)\right] \\ &= (1+g)^n\left[\left(M_n - \overset{\triangle}{M}_n\right) - \left(\widehat{M}_n - \overline{M}_n\right)\right]P^*.\end{aligned} \tag{25}$$

Term $\widetilde{I}_n(z_n)$ can be interpreted as the impact of growth on the value of the real options.[12] The firm increases its assets over time as a consequence of growth, and the value of its future real options goes up accordingly. Moreover, in Section 4 we do a comparative statics analysis and

[12] Equation (25) implies that the interaction effect depends on $\left(M_n - \overset{\triangle}{M}_n\right) - \left(\widehat{M}_n - \overline{M}_n\right)$, which is an infinite summation with general term:

$$\left[\left(\frac{1+g}{1+r_S}\right)^t - \frac{1}{(1+r_S)^t}\right]\left(e^{-\frac{1}{2}\sigma^2\frac{\alpha}{(1-\alpha)^2}}E\left[z_{n+t}^{1/(1-\alpha)}|z_n\right] - E\left[z_{n+t}|z_n\right]^{\frac{1}{1-\alpha}}\right).$$

This expression shows why $\widetilde{I}_n(z_n)$ represents an interaction effect between the two components: the first factor reflects the pure impact of growth while the second factor denotes the sole influence of the real options.

show that a higher growth rate augments the value of the real options. Appendix 1 generalizes this result showing that the interaction term is (weakly) positive.

In summary, the previous formulas suggest the market value of equity with dynamic programming can be decomposed as follows.

Primitive decomposition of the stock price:

$$\widetilde{S}_n\left(\widetilde{K}_n, \widetilde{B}_n, z_n\right) = \widetilde{S}_n\left(\widetilde{K}_n, \widetilde{B}_n, z_n\right)_{AP} + \widetilde{O}_n\left(z_n\right) + \widetilde{G}_n\left(z_n\right) + \widetilde{I}_n\left(z_n\right). \tag{26}$$

We next analyze the relevance of these components of share price for a representative firm. Table I contains the values we use for each of the model parameters. These values are standard in the corporate finance literature. We let parameter c be 1, which is the norm in the literature. The autoregressive parameter (ρ) is set at 0.75 while the standard deviation of the innovation term (σ) equals 0.20. These parameter values are close to the estimates of DeAngelo, DeAngelo, and Whited (2011). We choose the curvature of the production function (α) to be 0.65, which is close to the parameter estimate of Hennessy and Whited (2007). Both the operating costs (f) and the depreciation rate of capital (δ) are set equal to 0.10. Finally, we fix the corporate income tax rate (τ) at 0.35, the market cost of debt (r_B) at 0.02 (which equals the risk-free interest rate r_f), the market cost of equity (r_S) at 0.08, and the long-run growth rate (g) at 0.01. This parameterization is consistent with a yearly period.

We assume that the firm is at the outset of its life (i.e., $t = 0$) and the current state $\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right)$ is at the mean of the stationary unconditional distribution of profit shocks:

$$z_0 = E\left[z\right], \quad \widetilde{K}_0 = E\left[z\right]^{\frac{1}{1-\alpha}} W^*, \quad \text{and} \quad \widetilde{B}_0 = \ell^* \widetilde{K}_0 \tag{27}$$

where

$$E\left[z\right] = c^{\frac{1}{1-\rho}} e^{\frac{1}{2}\sigma^2 \frac{1}{(1-\rho^2)}}. \tag{28}$$

Table II exhibits the values of the current state $\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right)$ according to equation (27). These variables are constructed with the parameter values described in Table I and used in the parameterization of the different dynamic models we described above. Table II also shows that optimal book leverage ratio (ℓ^*) is 0.86. As we explained in Section 2, this ratio is the target leverage of the firm and is a constant that depends only on primitive parameters.[13]

Next we present the main quantitative results of this section. Table III shows that the market value of equity, $\widetilde{S}_0\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right)$, is 26.24. The value of the real options, $\widetilde{O}_0(z_0)$, is 2.23 and represents 8.48% of share price. Considering that in reality the firm has more real options than the ones included in $\widetilde{S}_0\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right)$, such as the exit option, the conclusion is that the value of managerial flexibility explains an important fraction of the stock price. Firm value, $\widetilde{B}_0 + \widetilde{S}_0\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right)$, turns out to be 33.46 and the real options represent 6.65% of that value. This proportion is close to the 6% reported by Quigg (1993) as the value of the land explained by the option to wait to develop. The value of long-run growth, $\widetilde{G}_0(z_0)$, is 2.77 and explains 10.57% of the market value of equity as well as 8.29% of firm value. The interaction term between managerial flexibility and growth has a value of 0.40 and represents 1.51% and 1.18% of share price and firm value, respectively.

Fact 1: Overall, managerial flexibility plus secular growth represent around 20% of share price and 16% of the value of the firm.

One of the main methodological contributions of the present article is to introduce the possibility to engage in long-term growth. Figure 1 displays the stochastic evolution over time of the market value of equity, $\widetilde{S}_n\left(\widetilde{K}_n, \widetilde{B}_n, z_n\right)$, for a firm that grows ($g = 0.01$). The model is simulated over 100 periods (i.e., years) starting at moment $t = 0$. The parameterization of the model uses the values in Table I and the initial state $\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right)$ is as described in equation (27), with the values shown in Table II. The solid dotted line shows the stochastic path of market equity for the growing firm. This dotted line oscillates around an exponential solid line that represents the long-run growth trend. As benchmark, we added a dashed line that represents the share price of the same firm when it does not grow (i.e., $g = 0$). The horizontal solid line around which this dashed line fluctuates is the mean of the stationary distribution of share price. Because both simulations are created for the same resolution of uncertainty, the dotted and dashed lines exhibit the same peaks and troughs over time. The only difference between the two firms is the

[13] Lazzati and Menichini (2013c) study the capital structure implications of this type of models.

assumption about growth. All four lines are normalized by the first value in the simulation of market equity for the no-growth firm.

Figure 2 shows the evolution over time of the stock price (solid line with dots), the value of managerial flexibility (solid line), the present value of growth opportunities (dashed line), and the value of the interaction effect between the real options and growth (dotted line). The simulation in this figure uses the same sequence of profit shocks used to construct market value of equity in Figure 1 and the four lines are normalized by the first value of the corresponding time series. We next highlight the main feature of Figure 2.

Fact 2: The stock price is considerably more sensitive to profit shocks than are the real options, secular growth, and the interaction term.

The reason for this different behavior is that the stock price, $\widetilde{S}_n\left(\widetilde{K}_n, \widetilde{B}_n, z_n\right)$, receives the full impact of z_n through M_n. On the contrary, z_n enters $\widetilde{O}_n(z_n)$, $\widetilde{G}_n(z_n)$, and $\widetilde{I}_n(z_n)$ through $\overset{\triangle}{M}_n - \overline{M}_n$, $\widehat{M}_n - \overline{M}_n$, and $\left(M_n - \overset{\triangle}{M}_n\right) - \left(\widehat{M}_n - \overline{M}_n\right)$, respectively, and these subtractions reduce the influence of profit shocks.

In Figure 3, we can observe how the proportion of share price explained by managerial flexibility (solid line), long-run growth (dashed line), and the interaction effect (dotted line) evolve over time. This simulation is also based on the same realization of uncertainty of Figure 1. We next summarize the main result in Figure 3.

Fact 3: The proportions of share price explained by managerial flexibility, long-run growth, and the interaction effect are counter-cyclical. That is, when profit shocks are high, the fraction of market equity explained by each of those terms falls and vice versa.

This pattern is due to the different sensitivities to profit shocks described in Figure 2 and suggests that both managerial flexibility and growth opportunities are more important during economic recessions than expansions. In other words, the undervaluation problem of the traditional DDM is exacerbated during periods of low profits shocks. Figure 4 exhibits the same three proportions but with respect to firm value, and the conclusions are the same.

In Section 4, we elaborate on the key determinants of the primitive components of the stock price and perform a cross-sectional comparison of different industries.

4 Comparative Statics and Cross-Sectional Analysis

In the previous section, we showed that the value of the real options is sensitive to the fluctuations of the business cycle. Now we show that this value is also sensitive to the primitive characteristics of the firm. For instance, Quigg (1993) finds that the mean value of the option to wait to invest is 6% of the value of the land, ranging from 1% to 30% according to land characteristics. Accordingly, in Subsection 4.1, we do a comparative statics analysis of relevant model variables, while in Subsection 4.2, we perform a cross-sectional analysis for different SIC industries.

4.1 Comparative Statics

We study how market value of equity, $\widetilde{S}_0\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right)$, and the proportion of share price explained by the real options, $\widetilde{O}_0\left(z_0\right)/\widetilde{S}_0\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right)$, and by long-run growth, $\widetilde{G}_0\left(z_0\right)/\widetilde{S}_0\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right)$, vary when we change the base case parameter values by up to $\pm$ 20%. The analytic solutions make it easy to understand the relative impact of the different model parameters on these terms. Briefly, we find that while the stock price and the value of the real options are very sensitive to the curvature of the production function (α), the present value of growth opportunities is very sensitive to both the market cost of equity (r_S) and (naturally) the growth rate (g). The persistence of profit shocks (ρ) is also an important determinant of the value of managerial flexibility. We next elaborate on these results.

We assume again that the firm is at the beginning of its life (i.e., $t = 0$). Table IV displays how market equity value changes with different parameterizations of the model. **One of the most important parameters is the curvature of the production function** (α). The higher this parameter, the higher the marginal productivity of capital, which allows the firm to increase optimal assets to take more advantage of profit shocks. A 20% increment in the value of α (i.e., from 0.65 to 0.78) increases market value of equity by a factor of 6 from 26.24 to 170.25. Another important parameter is the drift in logs (c), which scales the moments of the distribution of profit shocks. The importance of this parameter stems from the fact that it has a direct impact on the mean profitability of the firm. When c increases from 1 to 1.2, share price goes up from 26.24 to 150.69, an almost 6 times increment. The other parameters affect market equity to a lesser extent, with persistence of profit shocks (ρ) and market cost of equity (r_S) at the top of this second group.

Table V exhibits the comparative statics analysis of the fraction of market equity explained by managerial flexibility. It is clear that **the elasticity of capital input** (α) **has a significant influence on the value of the real options**, specially on the expansion and contraction options. The possibility of the CEO to take advantage of high profits and to protect shareholders from bad earnings by adapting firm size is more valuable when the curvature of the production function is lower. For the base case parameter value of $\alpha = 0.65$, managerial flexibility represents 8.48% of market equity, percentage that increases to 25.24% when α goes up to 0.78. For the same reasons, the volatility of profit shocks (σ) also has a considerable influence on the value of the real options. The higher the variability of profit shocks, the more the firm can benefit from having the possibility to adapt itself to changes in the environment. When σ goes up from the base case value of 0.2 to 0.24, the fraction of share price represented by real options increases from 8.48% to 12.08%.

The persistence of profit shocks (ρ) **also plays a major role in the valuation of real options**. Economic expansions and recessions are longer on average when profit shocks are more persistent, and the value of managerial flexibility increases, specially the value of the extension and deferral options. The reason for this result is that the CEO is able to mantain the firm large during periods of high earnings and keep it small during periods of low profit shocks. A 20% increase of ρ from 0.75 to 0.9 augments the proportion of real options from 8.48% to 22.42%. The other model parameters have a smaller effect on this proportion.

Finally, the impact of different parameter values on the proportion of market equity explained by long-run growth appears in Table VI. As expected, **the market cost of equity** (r_S) **and the growth rate** (g) **are among the most important parameters**. A 20% reduction in the value of r_S (i.e., from 0.08 to 0.064) increases the proportion of secular growth from 10.57% to 13.38% while a 20% increase in the value of g (i.e., from 0.01 to 0.012) augments the fraction of long-run growth from 10.57% to 12.69%. Similar in importance is the drift in logs (c), which strongly affects the expected profitability of the firm. When c increases from 1 to 1.2, the proportion of secular growth goes up from 10.57% to 13.84%. The other model parameters play a smaller role regarding this proportion.

In the next subsection, we compare the importance of real options and long-run growth across industries that sharply differ regarding key parameter values, such as the elasticity of capital input.

4.2 Cross-Sectional Analysis

In this subsection, we extend our previous results by comparing the relevance of the real options and growth opportunities across different SIC industries. In particular, we focus on three industries that display considerably different capital elasticity according to Jones (2003). We choose Oil and Gas Extraction (OGE) as an industry with high capital elasticity ($\alpha = 0.75$), Printing and Publishing (PP) as an industry with low capital elasticity ($\alpha = 0.25$), and Chemicals (C) as an industry between those two extremes ($\alpha = 0.50$). We compute the other parameters for each industry using Compustat data and show their values in Table VII. Appendix 2 describes the data items used to construct model variables as well as the procedure employed to calibrate the parameters for each industry.

Table VIII exhibits our main findings. OGE firms have the greatest proportion of share price explained by managerial flexibility, 10.15%, while C firms are in second place with 4.69% and PP firms are third with 1.28%. Although these industries exhibit considerable variation in the values of all other parameters, the curvature of the production function (α) remains as the main driver of the value of the real options, as it defines how much the CEO can take advantage of the future business cycles. For instance, PP firms have a substantially higher persistence of profit shocks (ρ) than OGE firms do, but the proportion of share price explained by managerial flexibility is roughly 8 times less as a consequence of their lower elasticity of capital. These findings are consistent with the results in the previous subsection.

Finally, it is noticeable the impact of the growth rate (g) on the present value of growth opportunities. OGE firms have a modest growth rate of 0.43% per year, which explains 4.93% of the stock price. On the contrary, C firms, with a growth rate of 3.47% per year, have 35.46% of their stock price explained by future growth opportunities. Between these two extremes are PP firms with a growth rate of 2.51% per year, which explains 26.74% of share price.

In summary, for industries such as OGE, which display low curvature of the production function, valuation methods that omit managerial flexibility (e.g., the traditional DDM) substantially underprice share price. Similarly, for industries such as C, which experience considerable growth over time, models of stock valuation that do not incorporate the possibility of secular growth (e.g., most of the existing dynamic programming models of the firm) produce an important downward bias in the estimates. **We conclude that for these cases the valuation model we derive in this article displays the largest benefits.**

5 Conclusion

We derive a dynamic model of the firm within the framework of the dividend discount model (DDM). The model we propose includes two important features: real options and secular growth. The introduction of the real options is an improvement with respect to the traditional DDM, which assumes precommitment to a deterministic decision plan and, therefore, does not consider managerial flexibility. The introduction of secular growth enhances the existing dynamic programming models of the firm in corporate finance, which so far have been developed assuming the firm does not have long-run growth opportunities. Furthermore, being based on the Separation Principle, our model does not require any assumption about shareholders' preferences, as long as market discount rates are available. We believe all these feaures make our dynamic model a useful asset pricing tool.

The introduction of real options and secular growth into the valuation model is essential to value the firm correctly. By decomposing the stock price in its primitive components, we are able to quantify for different industries the magnitude of the underpricing problem created by neglecting those features. For instance, we find that for firms in the Oil and Gas Extraction industry the proportion of share price explained by managerial flexibility can easily exceed 10%. Thus, valuation models such as the traditional DDM would lead to a large underpricing of this type of firms. Analogously, we find that for firms in the Chemical industry more than 30% of their stock price can be explained by future growth opportunities. Then, employing valuation models that do not handle long-run growth would produce severely undervalued estimates. Finally, we find that the proportions of share price represented by both managerial flexibility and secular growth are counter-cyclical, which suggests that the aforementioned undervaluation problems become aggravated during economic recessions.

There are several ways to extend the present dynamic DDM, the most obvious one being the addition of other real life features as described in the comprehensive review of Strebulaev and Whited (2012). Another promising research direction is to find assumptions that allow us to separate the value of the different real options (e.g., expansion, extension, etc.) contained in this model.

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6 Appendix 1: Propositions and Proofs

Proposition 1 *At the optimum, the (average) dollar return on equity minus the (average) dollar cost of equity is (strictly) positive. That is*

$$P^* > 0. \tag{29}$$

Proof We need to show that the following inequality is true:

$$\left(W^{*^{\alpha}} - fW^* - \delta W^* - r_B \ell^* W^*\right)(1-\tau) - r_S W^*(1-\ell^*) > 0 \tag{30}$$

which can be reformulated as

$$\alpha < \frac{\frac{r_S}{1-\tau} + f + \delta}{\frac{r_S}{1-\tau} + f + \delta - \ell^*\left(\frac{r_S}{1-\tau} - r_B\right)}. \tag{31}$$

Given that $\alpha \in (0,1)$ and the right-hand side is greater than or equal to 1, the last inequality is true, which completes the proof.

Proposition 2 *The interaction effect between managerial flexibility and growth is (weakly) positive. That is*

$$\widetilde{I}_n(z_n) \geq 0. \tag{32}$$

Proof Because proposition (1) shows that $P^* > 0$, inequality (32) is equivalent to saying that $\left(M_n - \overset{\triangle}{M}_n\right) - \left(\widehat{M}_n - \overline{M}_n\right) \geq 0$ is true. Then, the proof consists in showing that all terms in that infinite summation are nonnegative. That is, we need to show that, for all t, the following inequality is true:

$$\left[\left(\frac{1+g}{1+r_S}\right)^t - \frac{1}{(1+r_S)^t}\right]\left(e^{-\frac{1}{2}\sigma^2\frac{\alpha}{(1-\alpha)^2}} E\left[z_{n+t}^{1/(1-\alpha)}|z_n\right] - E\left[z_{n+t}|z_n\right]^{\frac{1}{1-\alpha}}\right) \geq 0 \tag{33}$$

The first factor can be rearranged as $\frac{(1+g)^t - 1}{(1+r_S)^t}$ and, because $0 \leq g < r_S$, it is (weakly) positive. The second factor is also nonnegative and, in order to prove this, we need to show that, for all t, the following inequality is true:

$$e^{-\frac{1}{2}\sigma^2\frac{\alpha}{(1-\alpha)^2}} E\left[z_{n+t}^{1/(1-\alpha)}|z_n\right] \geq E\left[z_{n+t}|z_n\right]^{\frac{1}{1-\alpha}} \tag{34}$$

which can be expanded as

$$e^{-\frac{1}{2}\sigma^2\frac{\alpha}{(1-\alpha)^2}}\left(c^{\frac{1-\rho^t}{1-\rho}}z_n^{\rho^t}e^{\frac{1}{2}\sigma^2\frac{\left(1-\rho^{2t}\right)}{\left(1-\rho^2\right)}\frac{1}{(1-\alpha)}}\right)^{\frac{1}{1-\alpha}} \geq \left(c^{\frac{1-\rho^t}{1-\rho}}z_n^{\rho^t}e^{\frac{1}{2}\sigma^2\frac{\left(1-\rho^{2t}\right)}{\left(1-\rho^2\right)}}\right)^{\frac{1}{1-\alpha}}. \tag{35}$$

After working inequality (35) algebraically, it reduces to

$$\frac{1}{2}\sigma^2\frac{\alpha}{(1-\alpha)}\frac{\left(\rho^2-\rho^{2t}\right)}{\left(1-\rho^2\right)} \geq 0. \tag{36}$$

Given that $\sigma > 0$, $\alpha \in (0,1)$, and $\rho \in (0,1)$, the last inequality is true, which completes the proof.

7 Appendix 2: Model Variable Construction and Parameter Calibration

We need to find parameter values for $c, \rho, \sigma, f, \delta, \tau, r_B, r_S$, and g for each of the three industries (parameter α for each group of firms was defined above). We calibrate the model using Compustat annual data for all firms in each of the three SIC codes (i.e., Oil and Gas Extraction (OGE) is SIC 13, Printing, Publishing, and Allied Products (PP) is SIC 27, and Chemicals and Allied Products (C) is SIC 28). The sample includes 9,476 firm-years for the OGE industry, 1,859 firm-years for the PP industry, 11,162 firm-years for the C industry, and covers the period 1990-2013.

In order to obtain parameter f, we average the ratio Selling, General, and Administrative Expense (XSGA)/Assets - Total (AT) for all firm-years in each industry. We follow the same procedure to get δ as the ratio of Depreciation and Amortization (DP) over Assets - Total (AT), and τ as the fraction Income Taxes - Total (TXT)/Pretax Income (PI). We obtain parameters c, ρ, and σ for each industry using the firm's autoregressive profit shock process of equation (1) and the gross profits equation (2). The data we use with these equations are Gross Profit (GP) and Assets - Total (AT). We keep the assumption that the risk-free interest rate ($r_f = r_B$) is 0.02. We derive r_S using CAPM with the corresponding industry betas estimated by Fama and French (1997) and assuming an expected market return (r_M) of 0.08. Finally, we obtain g for each industry from Jorgenson and Stiroh (2000).

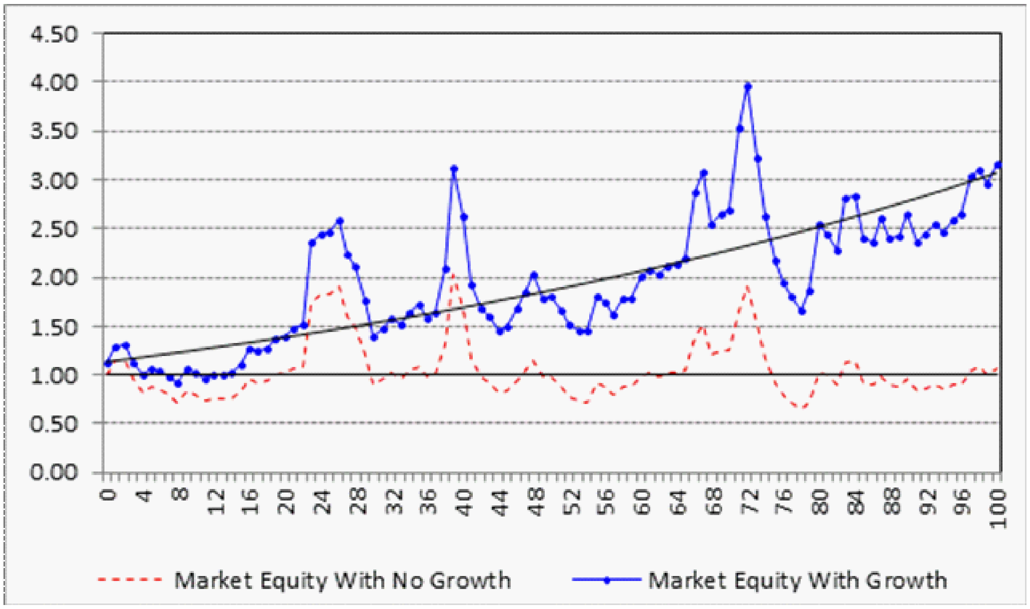

Figure 1. Simulation of market value of equity. The model is simulated over 100 periods (years) with the parameterization described in Section 3. The figure exhibits the evolution over time of the market value of equity of a firm that grows in the long-run (solid line with dots). The figure also shows the random path of the stock price for the same firm when it does not grow (dashed line). In both cases, overlaying market equity is the secular growth trend (solid lines). All values are normalized by the first observation of the no-growth share price.

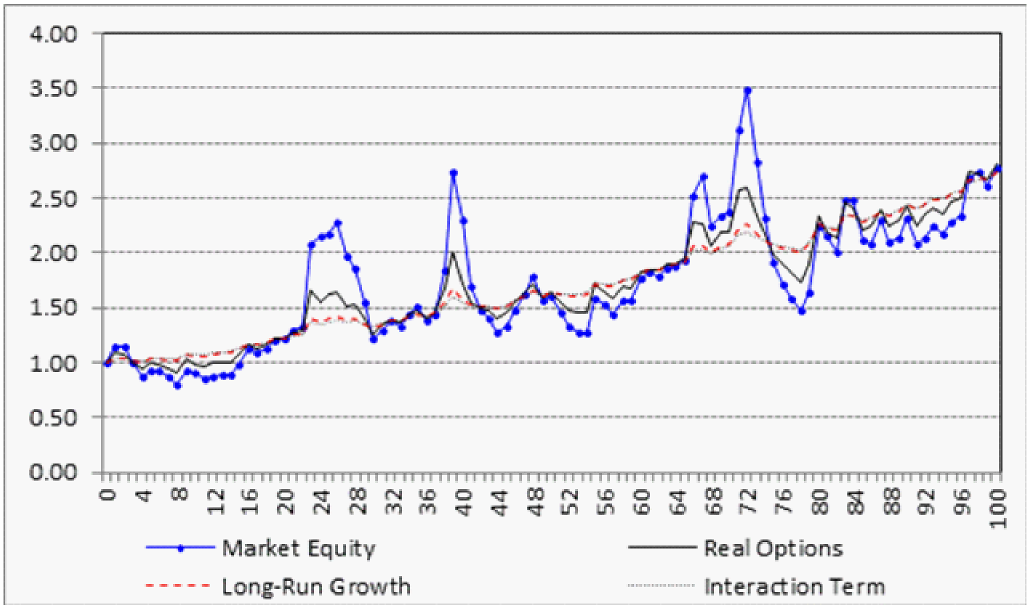

Figure 2. Simulation of market value of equity, real options, and long-run growth. The model is simulated over 100 periods (years) with the parameterization described in Section 3. The figure exhibits the evolution over time of market value of equity (solid line with dots), managerial flexibility (solid line), secular growth (dashed line), and the interaction term (dotted line). The four lines are normalized by the first observation of the corresponding time series.

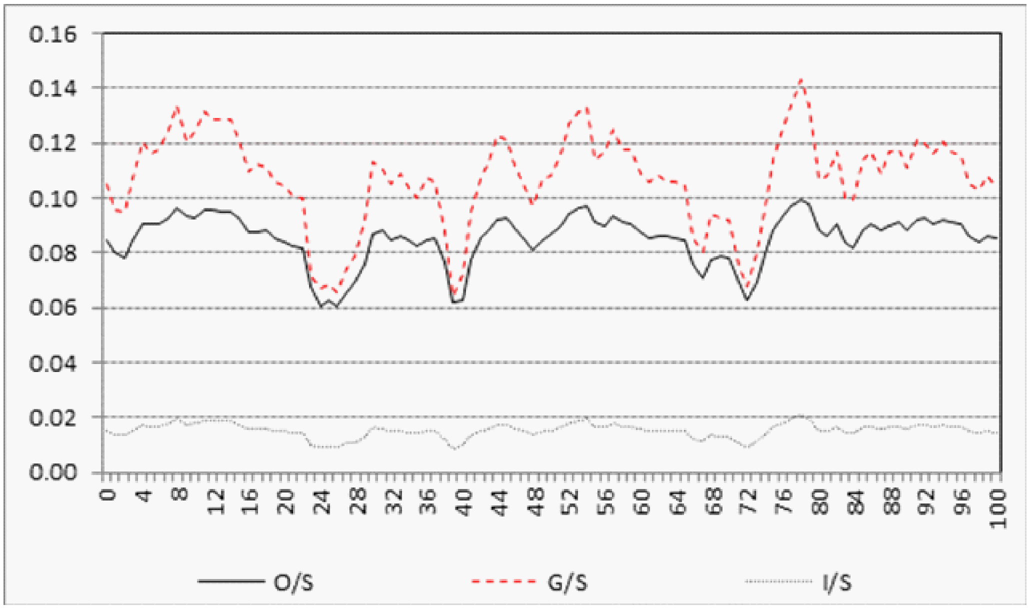

Figure 3. Simulation of the proportion of real options and long-run growth. The model is simulated over 100 periods (years) with the parameterization described in Section 3. The figure exhibits the evolution over time of the proportion of market value of equity explained by managerial flexibility (solid line), secular growth (dashed line), and the interaction term (dotted line).

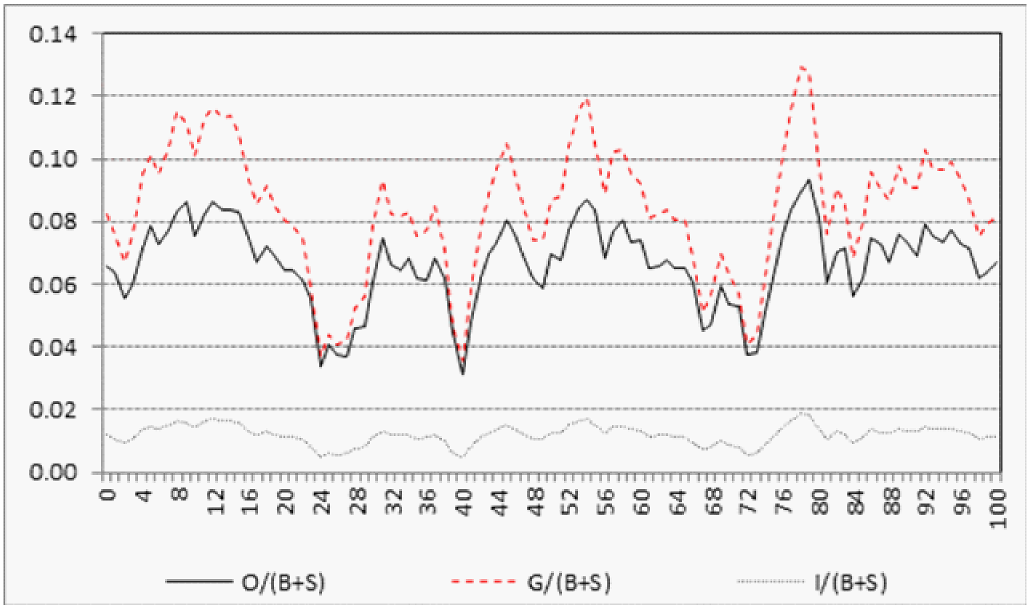

Figure 4. Simulation of the proportion of real options and long-run growth. The model is simulated over 100 periods (years) with the parameterization described in Section 3. The figure exhibits the evolution over time of the proportion of firm value explained by managerial flexibility (solid line), secular growth (dashed line), and the interaction term (dotted line).

Table I

Base Case Parameter Values

The table presents the values used to parameterize the base case of the dynamic dividend discount model. The parameters are the drift in logs (c), the persistence of profit shocks (ρ), the standard deviation of the innovation term (σ), the concavity of the production function (α), the operating costs (f), the capital depreciation rate (δ), the corporate income tax rate (τ), the market cost of debt (r_B), the market cost of equity (r_S), and the growth rate (g).

Parameter	*Value*
c	1.00
ρ	0.75
σ	0.20
α	0.65
f	0.10
δ	0.10
τ	0.35
r_B	0.02
r_S	0.08
g	0.01

Table II

Parameterization of the Current State of the Dynamic Dividend Discount Model

The table shows the base case parameterization of the current state of the dynamic dividend discount model. The variables are the mean of the invariant unconditional distribution of profit shocks ($E[z]$), the part of optimal assets that is constant with respect to the passage of time (W^*), optimal book leverage ratio (ℓ^*), and the current state of capital $\left(\widetilde{K}_0\right)$, debt $\left(\widetilde{B}_0\right)$, and profit shocks ($z_0$).

Variable	*Value*
$E[z]$	1.05
$W*$	7.37
$l*$	0.86
$\widetilde{K}_0$	8.40
$\widetilde{B}_0$	7.21
z_0	1.05

Table III

Value of Market Equity, Real Options, and Long-Run Growth

The table exhibits the base case results for the dynamic dividend discount model. The variables are the market value of equity with no real options and no future growth, $\widetilde{S}_0\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right)_{AP}$; the value of the real options, $\widetilde{O}_0(z_0)$; the value of long-run growth, $\widetilde{G}_0(z_0)$; the interaction term between managerial flexibility and growth, $\widetilde{I}_0(z_0)$; the market value of equity with managerial flexibility and long-run growth, $\widetilde{S}_0\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right)$; optimal next-period assets, $\widetilde{K}_1^*(z_0)$; optimal next-period debt, $\widetilde{B}_1^*(z_0)$; current debt, $\widetilde{B}_0$; and the market value of the firm, $\widetilde{B}_0 + \widetilde{S}_0\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right)$.

Variable	*Value*	*% of* $\tilde{S}_0$	*% of* $\tilde{B}_0 + \tilde{S}_0$
$\tilde{S}_{0,AP}$	20.85	79.44	62.31
$\tilde{O}_0$	2.23	8.48	6.65
$\tilde{G}_0$	2.77	10.57	8.29
$\tilde{I}_0$	0.40	1.51	1.18
$\tilde{S}_0$	26.24	100.00	78.44
$\tilde{K}_1^*$	8.61	32.79	25.72
$\tilde{B}_1^*$	7.39	28.16	22.09
$\tilde{B}_0$	7.21	27.48	21.56
$\tilde{B}_0 + \tilde{S}_0$	33.46	127.48	100.00

Table IV

Sensitivity Analysis of Market Equity Value

The table shows share price, $\widetilde{S}_0\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right)$, for different values of model parameters. The column labeled Base Case contains the base case parameter values described in Table I while the other columns contain proportional changes of those initial values. The parameters are the drift in logs (c), the persistence of profit shocks (ρ), the standard deviation of the innovation term (σ), the concavity of the production function (α), the operating costs (f), the capital depreciation rate (δ), the corporate income tax rate (τ), the market cost of debt (r_B), the market cost of equity (r_S), and the growth rate (g).

	BC-20%	BC-16%	BC-12%	BC-8%	BC-4%	Base Case (BC)	BC+4%	BC+8%	BC+12%	BC+16%	BC+20%
c	0.800	0.840	0.880	0.920	0.960	1.000	1.040	1.080	1.120	1.160	1.200
$\tilde{S}$	6.06	7.64	10.01	13.51	18.68	26.24	37.20	52.93	75.28	106.77	150.69
ρ	0.600	0.630	0.660	0.690	0.720	0.750	0.780	0.810	0.840	0.870	0.900
$\tilde{S}$	23.83	24.14	24.52	24.98	25.54	26.24	27.15	28.36	30.06	32.59	36.79
σ	0.160	0.168	0.176	0.184	0.192	0.200	0.208	0.216	0.224	0.232	0.240
$\tilde{S}$	24.30	24.64	25.01	25.39	25.81	26.24	26.71	27.20	27.73	28.28	28.87
α	0.520	0.546	0.572	0.598	0.624	0.650	0.676	0.702	0.728	0.754	0.780
$\tilde{S}$	13.46	14.76	16.46	18.72	21.82	26.24	32.86	43.36	61.38	95.63	170.25
f	0.080	0.084	0.088	0.092	0.096	0.100	0.104	0.108	0.112	0.116	0.120
$\tilde{S}$	29.94	29.14	28.37	27.63	26.92	26.24	25.59	24.97	24.37	23.80	23.25
δ	0.080	0.084	0.088	0.092	0.096	0.100	0.104	0.108	0.112	0.116	0.120
$\tilde{S}$	29.94	29.14	28.37	27.63	26.92	26.24	25.59	24.97	24.37	23.80	23.25
τ	0.280	0.294	0.308	0.322	0.336	0.350	0.364	0.378	0.392	0.406	0.420
$\tilde{S}$	30.04	29.28	28.52	27.76	27.00	26.24	25.49	24.73	23.98	23.22	22.47
r_B	0.016	0.017	0.018	0.018	0.019	0.020	0.021	0.022	0.022	0.023	0.024
$\tilde{S}$	26.57	26.51	26.44	26.38	26.31	26.24	26.18	26.11	26.05	25.98	25.92
r_S	0.064	0.067	0.070	0.074	0.077	0.080	0.083	0.086	0.090	0.093	0.096
$\tilde{S}$	36.01	33.61	31.46	29.54	27.81	26.24	24.82	23.52	22.34	21.25	20.25
g	0.0080	0.0084	0.0088	0.0092	0.0096	0.0100	0.0104	0.0108	0.0112	0.0116	0.0120
$\tilde{S}$	25.54	25.68	25.82	25.96	26.10	26.24	26.39	26.54	26.69	26.84	26.99

Table V

Sensitivity Analysis of Real Options Value

The table shows the proportion of share price, $\widetilde{S}_0\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right)$, that is explained by the value of the real options, $\widetilde{O}_0\left(z_0\right)$, for different values of model parameters. The column labeled Base Case contains the base case parameter values described in Table I while the other columns contain proportional changes of those initial values. The parameters are the drift in logs (c), the persistence of profit shocks (ρ), the standard deviation of the innovation term (σ), the concavity of the production function (α), the operating costs (f), the capital depreciation rate (δ), the corporate income tax rate (τ), the market cost of debt (r_B), the market cost of equity (r_S), and the growth rate (g).

	BC-20%	BC-16%	BC-12%	BC-8%	BC-4%	Base Case (BC)	BC+4%	BC+8%	BC+12%	BC+16%	BC+20%
c	0.800	0.840	0.880	0.920	0.960	1.000	1.040	1.080	1.120	1.160	1.200
$\tilde{O}/\tilde{S}$	4.11%	5.17%	6.20%	7.11%	7.88%	8.48%	8.95%	9.29%	9.55%	9.74%	9.87%
ρ	0.600	0.630	0.660	0.690	0.720	0.750	0.780	0.810	0.840	0.870	0.900
$\tilde{O}/\tilde{S}$	3.96%	4.59%	5.33%	6.20%	7.23%	8.48%	10.02%	11.94%	14.43%	17.75%	22.42%
σ	0.160	0.168	0.176	0.184	0.192	0.200	0.208	0.216	0.224	0.232	0.240
$\tilde{O}/\tilde{S}$	5.47%	6.02%	6.60%	7.20%	7.83%	8.48%	9.16%	9.85%	10.57%	11.32%	12.08%
α	0.520	0.546	0.572	0.598	0.624	0.650	0.676	0.702	0.728	0.754	0.780
$\tilde{O}/\tilde{S}$	3.52%	4.14%	4.90%	5.84%	7.01%	8.48%	10.36%	12.77%	15.88%	19.94%	25.24%
f	0.080	0.084	0.088	0.092	0.096	0.100	0.104	0.108	0.112	0.116	0.120
$\tilde{O}/\tilde{S}$	8.60%	8.58%	8.56%	8.53%	8.51%	8.48%	8.46%	8.43%	8.41%	8.38%	8.36%
δ	0.080	0.084	0.088	0.092	0.096	0.100	0.104	0.108	0.112	0.116	0.120
$\tilde{O}/\tilde{S}$	8.60%	8.58%	8.56%	8.53%	8.51%	8.48%	8.46%	8.43%	8.41%	8.38%	8.36%
τ	0.280	0.294	0.308	0.322	0.336	0.350	0.364	0.378	0.392	0.406	0.420
$\tilde{O}/\tilde{S}$	8.51%	8.51%	8.50%	8.50%	8.49%	8.48%	8.47%	8.47%	8.46%	8.45%	8.44%
r_B	0.016	0.017	0.018	0.018	0.019	0.020	0.021	0.022	0.022	0.023	0.024
$\tilde{O}/\tilde{S}$	8.49%	8.49%	8.49%	8.49%	8.48%	8.48%	8.48%	8.48%	8.47%	8.47%	8.47%
r_S	0.064	0.067	0.070	0.074	0.077	0.080	0.083	0.086	0.090	0.093	0.096
$\tilde{O}/\tilde{S}$	8.68%	8.66%	8.62%	8.58%	8.53%	8.48%	8.42%	8.36%	8.30%	8.23%	8.16%
g	0.0080	0.0084	0.0088	0.0092	0.0096	0.0100	0.0104	0.0108	0.0112	0.0116	0.0120
$\tilde{O}/\tilde{S}$	8.72%	8.67%	8.62%	8.58%	8.53%	8.48%	8.44%	8.39%	8.34%	8.29%	8.25%

Table VI

Sensitivity Analysis of Secular Growth Value

The table shows the proportion of share price, $\widetilde{S}_0\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right)$, that is explained by the value of long-run growth, $\widetilde{G}_0(z_0)$, for different values of model parameters. The column labeled Base Case contains the base case parameter values described in Table I while the other columns contain proportional changes of those initial values. The parameters are the drift in logs (c), the persistence of profit shocks (ρ), the standard deviation of the innovation term (σ), the concavity of the production function (α), the operating costs (f), the capital depreciation rate (δ), the corporate income tax rate (τ), the market cost of debt (r_B), the market cost of equity (r_S), and the growth rate (g).

	BC-20%	BC-16%	BC-12%	BC-8%	BC-4%	Base Case (BC)	BC+4%	BC+8%	BC+12%	BC+16%	BC+20%
c	0.800	0.840	0.880	0.920	0.960	1.000	1.040	1.080	1.120	1.160	1.200
$\tilde{G}/\tilde{S}$	4.16%	5.51%	6.92%	8.27%	9.50%	10.57%	11.47%	12.23%	12.87%	13.40%	13.84%
ρ	0.600	0.630	0.660	0.690	0.720	0.750	0.780	0.810	0.840	0.870	0.900
$\tilde{G}/\tilde{S}$	11.15%	11.06%	10.97%	10.86%	10.72%	10.57%	10.38%	10.15%	9.85%	9.46%	8.92%
σ	0.160	0.168	0.176	0.184	0.192	0.200	0.208	0.216	0.224	0.232	0.240
$\tilde{G}/\tilde{S}$	10.89%	10.84%	10.77%	10.71%	10.64%	10.57%	10.49%	10.42%	10.34%	10.25%	10.17%
α	0.520	0.546	0.572	0.598	0.624	0.650	0.676	0.702	0.728	0.754	0.780
$\tilde{G}/\tilde{S}$	10.71%	10.68%	10.66%	10.64%	10.61%	10.57%	10.48%	10.33%	10.07%	9.65%	9.00%
f	0.080	0.084	0.088	0.092	0.096	0.100	0.104	0.108	0.112	0.116	0.120
$\tilde{G}/\tilde{S}$	10.72%	10.69%	10.66%	10.63%	10.60%	10.57%	10.54%	10.51%	10.47%	10.44%	10.41%
δ	0.080	0.084	0.088	0.092	0.096	0.100	0.104	0.108	0.112	0.116	0.120
$\tilde{G}/\tilde{S}$	10.72%	10.69%	10.66%	10.63%	10.60%	10.57%	10.54%	10.51%	10.47%	10.44%	10.41%
τ	0.280	0.294	0.308	0.322	0.336	0.350	0.364	0.378	0.392	0.406	0.420
$\tilde{G}/\tilde{S}$	10.61%	10.60%	10.59%	10.58%	10.58%	10.57%	10.56%	10.55%	10.54%	10.53%	10.52%
r_B	0.016	0.017	0.018	0.018	0.019	0.020	0.021	0.022	0.022	0.023	0.024
$\tilde{G}/\tilde{S}$	10.58%	10.58%	10.58%	10.57%	10.57%	10.57%	10.56%	10.56%	10.56%	10.56%	10.55%
r_S	0.064	0.067	0.070	0.074	0.077	0.080	0.083	0.086	0.090	0.093	0.096
$\tilde{G}/\tilde{S}$	13.38%	12.72%	12.11%	11.55%	11.04%	10.57%	10.13%	9.72%	9.34%	8.99%	8.66%
g	0.0080	0.0084	0.0088	0.0092	0.0096	0.0100	0.0104	0.0108	0.0112	0.0116	0.0120
$\tilde{G}/\tilde{S}$	8.45%	8.87%	9.29%	9.72%	10.14%	10.57%	10.99%	11.42%	11.84%	12.27%	12.69%

Table VII

Cross-Sectional Parameter Values

The table presents the values used to parameterize the dynamic dividend discount model for three different SIC industries: Oil and Gas Extraction (OGE), Chemicals (C), and Printing and Publishing (PP). The parameters are the drift in logs (c), the persistence of profit shocks (ρ), the standard deviation of the innovation term (σ), the concavity of the production function (α), the operating costs (f), the capital depreciation rate (δ), the corporate income tax rate (τ), the market cost of debt (r_B), the market cost of equity (r_S), and the growth rate (g).

Parameter	*Value*		
	OGE	*C*	*PP*
c	1.0000	1.0000	1.0000
ρ	0.4865	0.8152	0.8476
σ	0.2785	0.2414	0.2145
α	0.7500	0.5000	0.2500
f	0.0845	0.2964	0.3024
δ	0.0855	0.0421	0.0544
τ	0.1609	0.1296	0.2413
r_B	0.0200	0.0200	0.0200
r_S	0.0710	0.0854	0.0902
g	0.0043	0.0347	0.0251

Table VIII

Cross-Sectional Value of Market Equity, Real Options, and Long-Run Growth

The table exhibits the results of the dynamic dividend discount model for three different SIC industries. The variables are the market value of equity with no real options and no future growth, $\widetilde{S}_0\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right)_{AP}$; the value of the real options, $\widetilde{O}_0\left(z_0\right)$; the value of long-run growth, $\widetilde{G}_0\left(z_0\right)$; the interaction term between managerial flexibility and growth, $\widetilde{I}_0\left(z_0\right)$; the market value of equity with managerial flexibility and long-run growth, $\widetilde{S}_0\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right)$; optimal next-period assets, $\widetilde{K}_1^*\left(z_0\right)$; optimal next-period debt, $\widetilde{B}_1^*\left(z_0\right)$; current debt, $\widetilde{B}_0$; and the market value of the firm, $\widetilde{B}_0+\widetilde{S}_0\left(\widetilde{K}_0, \widetilde{B}_0, z_0\right)$.

Variable	*Oil And Gas Extraction*			*Chemicals*			*Printing And Publishing*		
	Value	*% of $\tilde{S}_0$*	*% of $\tilde{B}_0+\tilde{S}_0$*	*Value*	*% of $\tilde{S}_0$*	*% of $\tilde{B}_0+\tilde{S}_0$*	*Value*	*% of $\tilde{S}_0$*	*% of $\tilde{B}_0+\tilde{S}_0$*
$\tilde{S}_{0,AP}$	179.57	84.16	61.66	9.16	55.64	52.21	6.73	71.30	68.83
$\tilde{O}_0$	21.67	10.15	7.44	0.77	4.69	4.40	0.12	1.28	1.23
$\tilde{G}_0$	10.51	4.93	3.61	5.84	35.46	33.27	2.53	26.74	25.81
$\tilde{I}_0$	1.62	0.76	0.56	0.70	4.22	3.96	0.06	0.67	0.65
$\tilde{S}_0$	213.37	100.00	73.27	16.47	100.00	93.83	9.44	100.00	96.53
$\tilde{K}_1^*$	97.14	45.53	33.36	1.60	9.73	9.13	0.48	5.08	4.90
$\tilde{B}_1^*$	81.91	38.39	28.13	1.11	6.75	6.33	0.34	3.65	3.52
$\tilde{B}_0$	77.86	36.49	26.73	1.08	6.57	6.17	0.34	3.60	3.47
$\tilde{B}_0+\tilde{S}_0$	291.23	136.49	100.00	17.55	106.57	100.00	9.78	103.60	100.00