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Lanchester Models with Discontinuities: An Application to Networked Forces

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Lanchester models with Discontinuities: An application to Networked Forces

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Overview

- In this talk I plan to:
  - Provide a general overview of Lanchester Models
  - Discuss how they might be adapted to consider Network Effects
  - Show some details about how effectiveness is determined
  - Provide an historically motivated example.

Why do this?

- To provide a ‘bite sized’ analytic model that incorporates the effect of network loss with the kinetic battle. In short, it takes ‘kinetic-like’ and ‘cyber-like’ inputs and provides a ‘kinetic’ output.
- This may prove be useful for fast, transparent verification of a complex simulation model

Lanchester Models

Pit two sides, Blue and Red, against each other, and analyze the resulting combat as a deterministic model. In their most general form,

$$\frac{dR}{dt} = -\gamma_B$$

$$\frac{dB}{dt} = -\gamma_R,$$

where the gammas represent arbitrary functions. We explore specific choices, and their consequences subsequently

Common Lanchester Model ‘Flavors’

- For *Aimed fire*

$$\frac{dB}{dt} = -\beta_R R$$

$$\frac{dR}{dt} = -\beta_B B$$

- For *Area fire*

$$\frac{dB}{dt} = -\beta_R RB$$

$$\frac{dR}{dt} = -\beta_B RB$$

- For *Ambush situations*

$$\frac{dB}{dt} = -\beta_R R$$

$$\frac{dR}{dt} = -\beta_B BR$$

Solving the Lanchester Models

- Cumbersome, explicit time-dependent solutions exist,
- … but numerical integration via *Euler's Method* or others is fast, accurate, and easily implemented in a spreadsheet
- … While the explicit solutions are messy, the equations may be easily rewritten to find the time-independent state equation; for example, *Aimed fire* obeys the law:

$$\frac{B_0^2 - B_t^2}{R_0^2 - R_t^2} = \frac{\beta_R}{\beta_B}$$

Shock Action - modification

- Our contribution is to consider a model in which the dynamics of combat change suddenly and irrevocably at a deterministic time, t*.
- Our solutions to follow are implicit in the corresponding variables, which we call B* or R*

$$\frac{dR}{dt} = -\gamma_{BN} \quad t < t^*$$

$$\frac{dR}{dt} = -\gamma_{B} \quad t \geq t^*$$

$$\frac{dB}{dt} = -\gamma_{R} \quad \forall t$$

Aimed Fire → Aimed Fire
Model and results

- In this situation network loss causes us to go from highly effective aimed fire to less accurate aimed fire. The model is specified as:

$$\frac{dR}{dt} = - \beta_{BN} B \quad \mathrm{t} < \mathrm{t}^*$$

$$\frac{dR}{dt} = -\beta_B B \quad t \geq t^*$$

$$\frac{dB}{dt} = - \beta_R R \ \forall t$$

$$B_* = \sqrt{\frac{\beta_{BN} B_0^2 - \beta_B B_f^2 - \beta_R (R_0^2 - R_f^2)}{\beta_{BN} - \beta_B}}$$

Aimed Fire → Area Fire: Model and Result

- Conversely, in this situation, network reduction causes us to go from aimed fire to area fire

$$\frac{dR}{dt} = -\beta_{BN} B \quad \mathrm{t} < \mathrm{t}^{*}$$

$$\frac{dR}{dt} = -\beta_{B} BR \quad t \geq t^{*}$$

$$\frac{dB}{dt} = -\beta_{R} R \ \forall t$$

$$R_{*} = \frac{\beta_{BN}}{\beta_{B}} + \sqrt{\frac{\beta_{BN}^{2}}{\beta_{B}^{2}} + R_{0}^{2} - \frac{\beta BN}{\beta_{R}}[B_{0}^{2} - B_{f}^{2}] - 2\frac{\beta_{BN}}{\beta_{B}} R_{f}}$$

The effect of the Network on Targeting

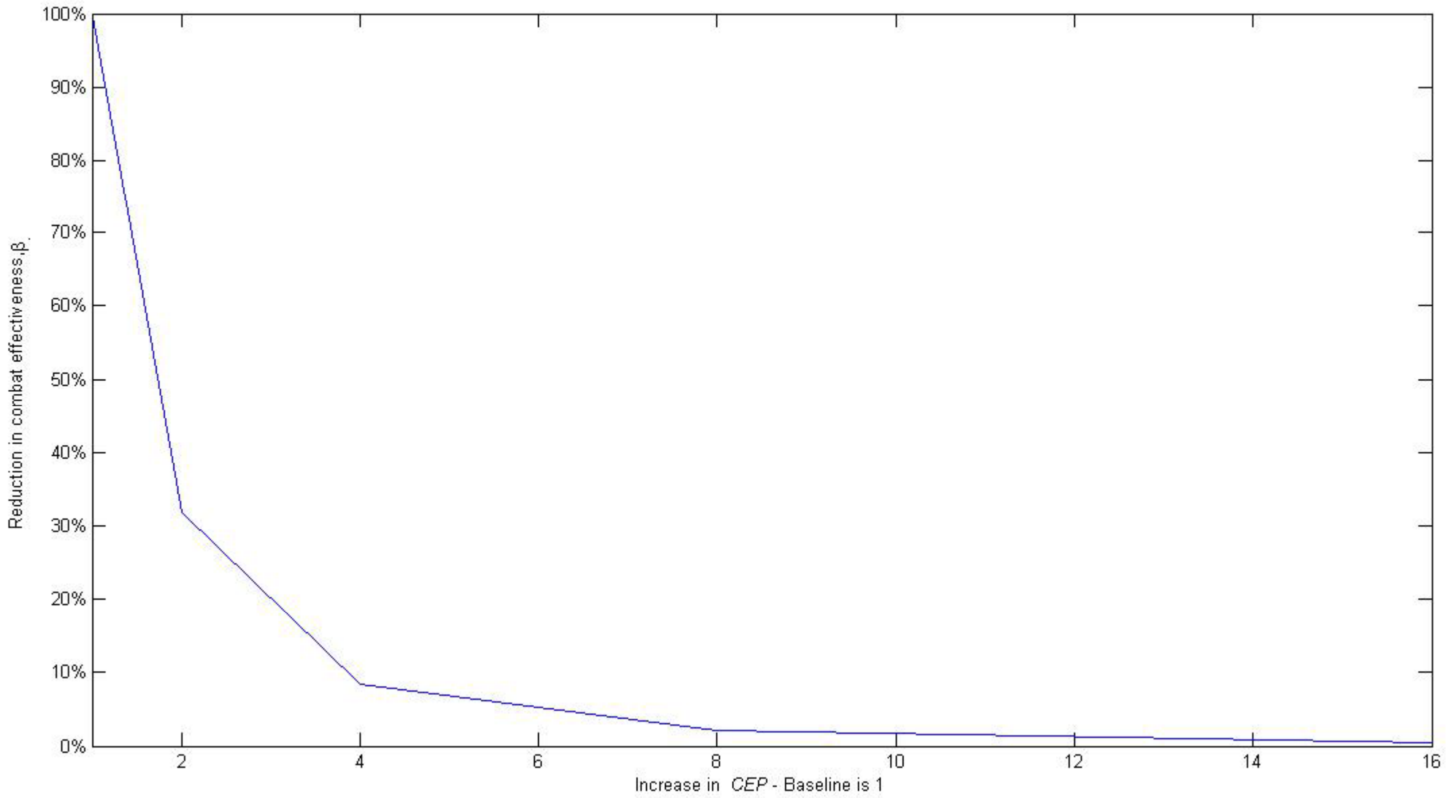
- If ordnance errors are equal and uncorrelated, we may say that they are *circularly distributed,* and

$$\Pr\{R < r\} = 1 - e^{-\frac{1}{2}\left(\frac{r}{\sigma}\right)^2}$$

Where the common unit of error is *Circular Error Probable* (The radius that encloses ½ of the rounds fired), which may be converted by:

$$\sigma = \frac{CEP}{\sqrt{\ln 4}} \approx \frac{CEP}{1.177}$$

Reduction in β as a function of CEP



When should we just switch from Aimed to Area fires?

- Let η be the firing rate. For Aimed fire:

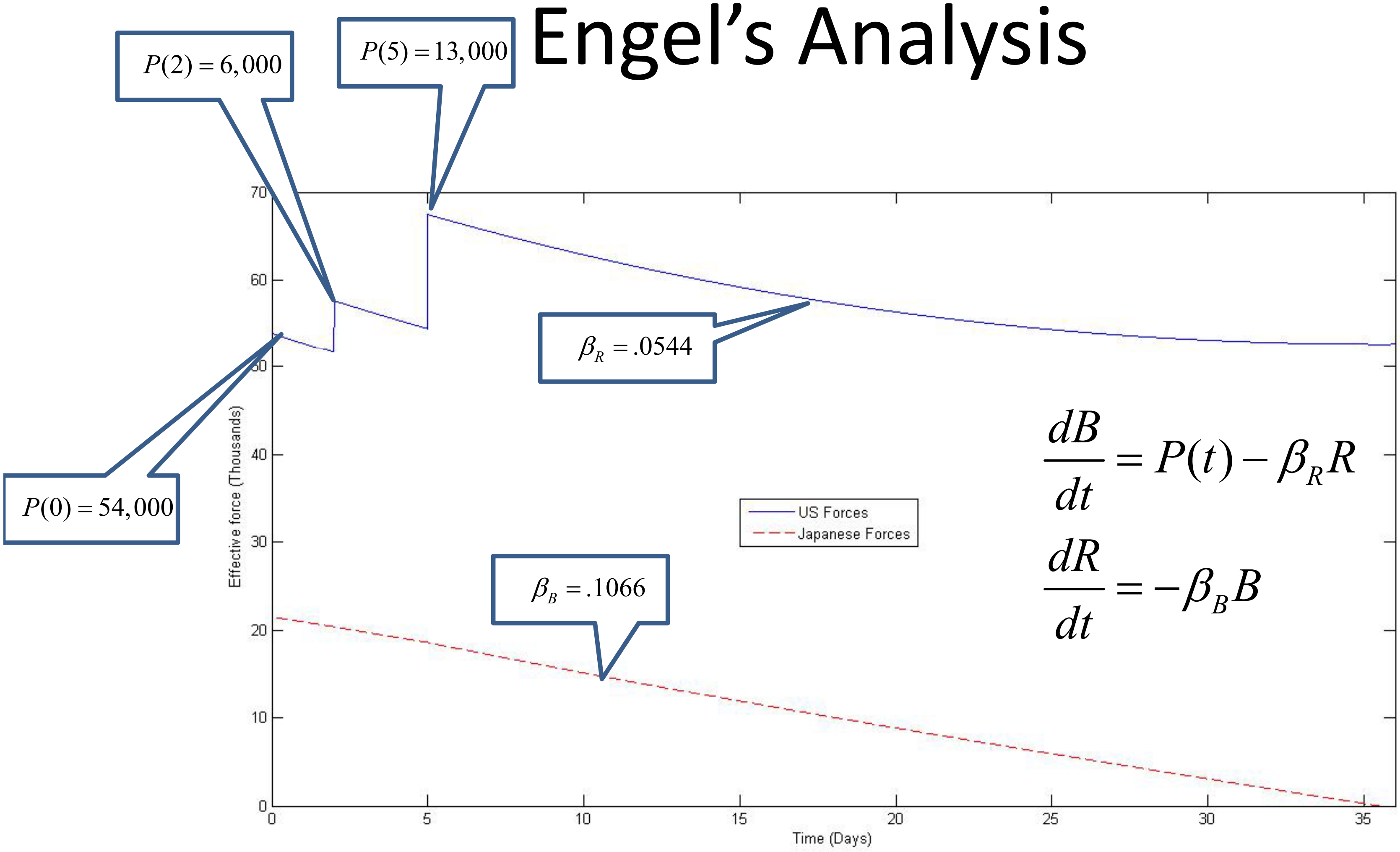
$$E[k(h)] = \eta \, p_{kill|hit} \, p_{hit} Bh$$

- For Area fire: $E[k(h)] = \eta \, p_{kill|hit} \frac{A_L}{A_T} BRh$

- We should prefer aimed fire iff:

$$\frac{A_L}{A_T} R > p_{hit}$$

Case Study: The battle of Iwo Jima
Engel's Analysis



$P(2) = 6,000$

$P(5) = 13,000$

$\beta_R = .0544$

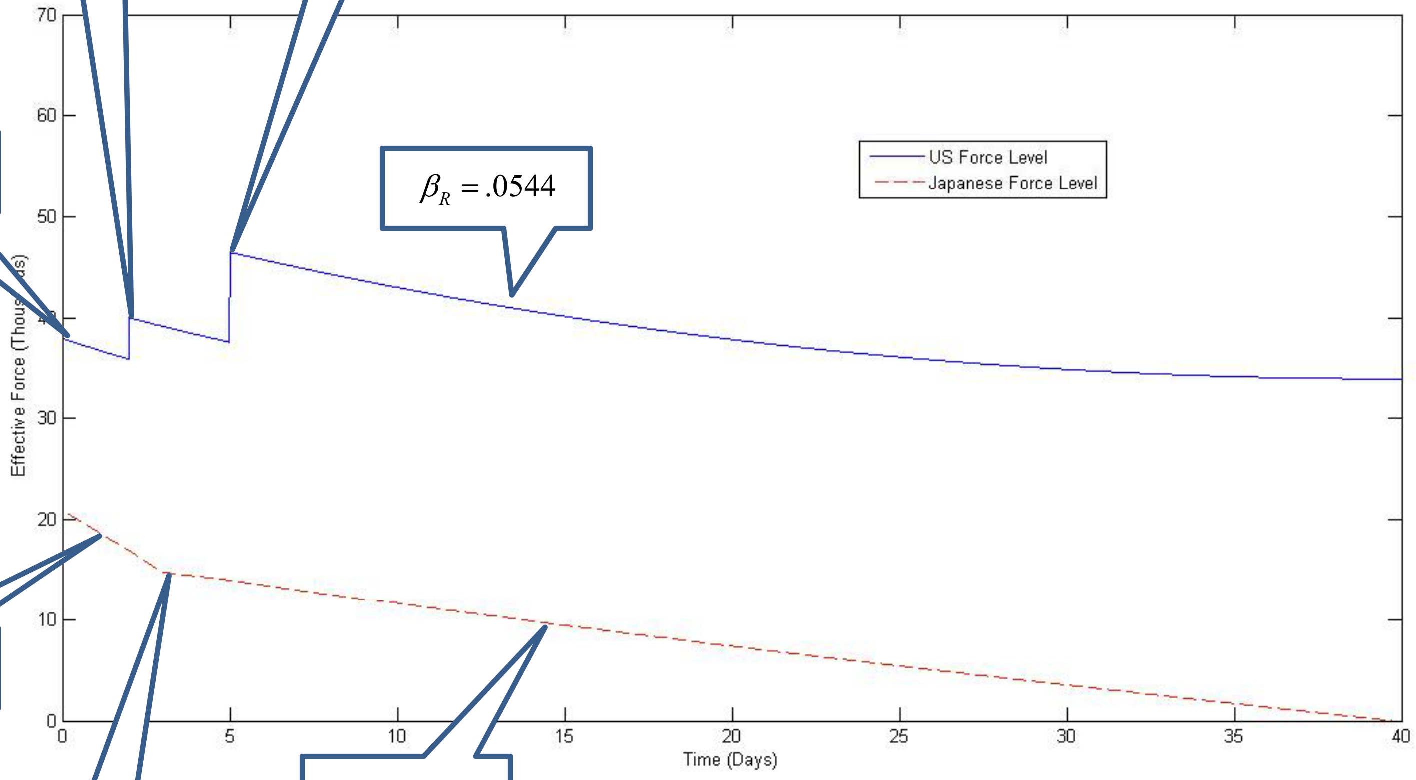
$P(0) = 54,000$

$\beta_B = .1066$

$$\frac{dB}{dt} = P(t) - \beta_R R$$

$$\frac{dR}{dt} = -\beta_B B$$

Case Study II: Networked Battle of Iwo Jima



$P(2) = 5{,}000$

$P(5) = 11{,}000$

$\beta_R = .0544$

$\beta_R = .0544$

$\beta_{BN} = .0544$

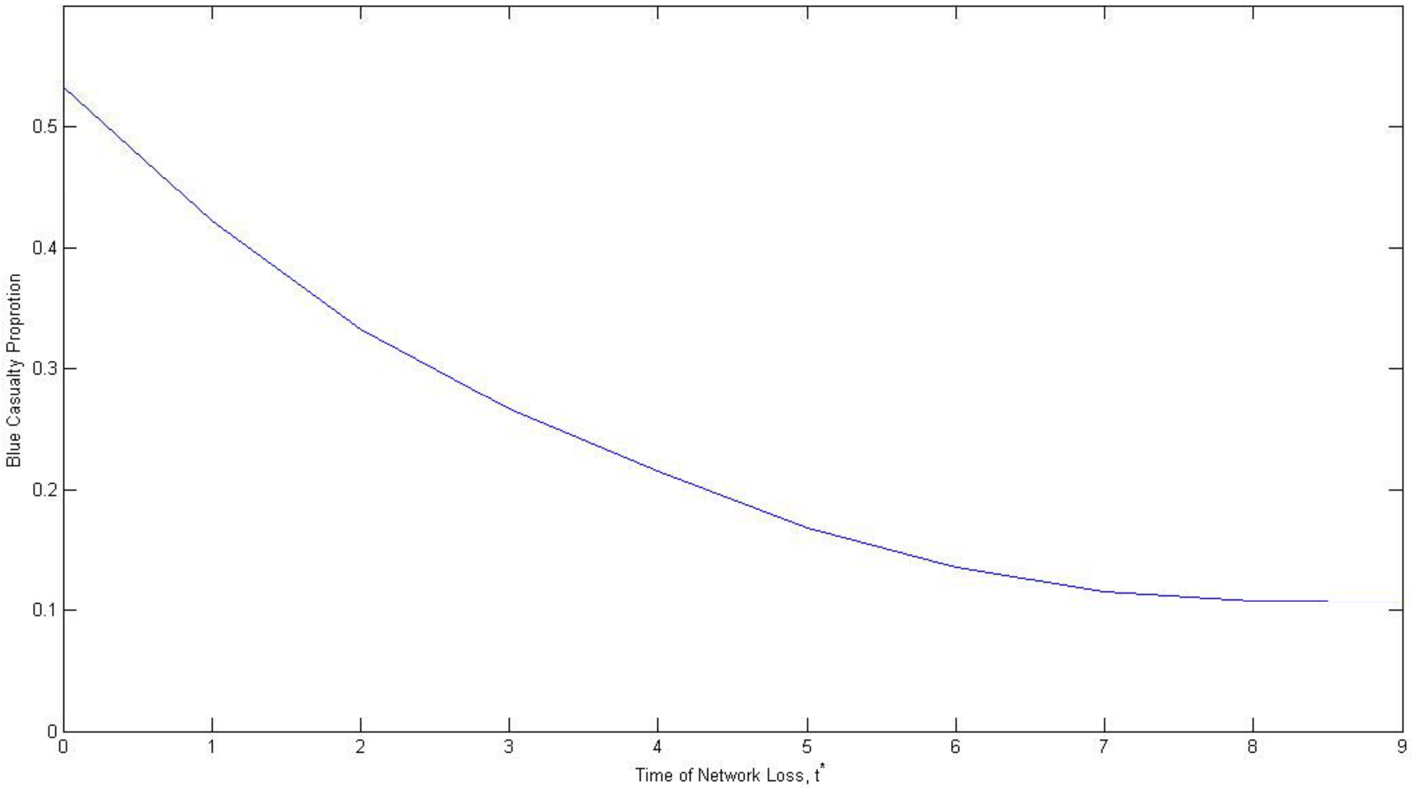
$\beta_B = .0106$

$t^* = 3$

We may ask…

- Suppose that Blue has a vulnerable network, but plans like his network was invulnerable, uses Lanchester for his planning and plans for a 10% casualty rate.
- Suppose further that the quality of his network gives him parity with the advantage for being ‘dug in’
- We may ask: What’s the impact of having a his network fail?

The Impact of Network Failure



Conclusion

- Our aim was to:
  - Demonstrate an analytical approach to incorporate the effects of network loss
  - Show that planning for combat with a networked force, and then losing said network, my lead to negative results
  - Demonstrated how a Service may compare the value of network reliability to capability.