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Modeling a Severe Supply Chain Disruption and Post-Disaster Decision Making with Application to the Japanese Earthquake and Tsunami

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Modern supply chains are increasingly vulnerable to disruptions, and a disruption in one part of the world can cause supply difficulties for companies around the globe. This paper develops a model of severe supply chain disruptions in which several suppliers suffer from disabled production facilities and firms that purchase goods from those suppliers may consequently suffer a supply shortage. Suppliers and firms can choose disruption management strategies to maintain operations. A supplier with a disabled facility may choose to move production to an alternate facility, and a firm encountering a supply shortage may be able to use inventory or buy supplies from an alternate supplier. The supplier's and firm's optimal decisions are expressed in terms of model parameters such as the cost of each strategy, the chances of losing business, and the probability of facilities reopening. The model is applied to a simulation based on the 2011 Japanese earthquake and tsunami, which closed several facilities of key suppliers in the automobile industry and caused supply difficulties for both Japanese and U.S. automakers.

Key words: supply chain risk; disruption management; Japanese earthquake and tsunami; simulation; automobile industry

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1. Introduction

The earthquake and tsunami that struck Japan in March 2011 directly impacted over 27,000 businesses whose production, warehousing, and retail facilities were destroyed or disabled by the natural disaster. One year after the disaster, 22% of those businesses had not yet resumed operations (Daily Yomiuri 2012). Several months after the Japanese earthquake and tsunami, floods in Thailand forced the closure of almost 1,000 manufacturing factories (The Nation 2011). Japanese and Thai businesses deliver parts and supplies to firms and consumers throughout the globe, and
these natural disasters disrupted global supply chains, especially in the automotive and electronics industries. As a result, production was temporarily halted at some facilities around the world, customer orders were delayed, and inventory in the pipeline fell dramatically (Nakata 2011).

The supply chain disruptions caused by the Japanese earthquake and tsunami and the Thai flood exemplify the increasing vulnerability of modern supply chains. The globalization of supply chains and the emphasis on lean manufacturing and efficiency in logistics have increased the risk of disruption in those same supply chains (Christopher 2005). A firm in one country may receive parts, supplies, and raw materials from suppliers in multiple countries. If a disruptive event strikes one of those countries and hinders a company’s ability to produce or deliver supplies, supply shortages may suddenly occur. Efficient supply chains often mean less inventory and reliance on a single supplier for production inputs, which can make it more difficult for a firm if its supplier defaults on its obligations.

A severe supply chain disruption is defined in this paper as a disruptive event that causes production difficulties for multiple suppliers and in which at least two of these suppliers deliver non-identical goods or services to at least two competing firms. When severe disruptions occur, companies must make decisions about maintaining operations, repairing facilities in the case of a physical disruption, reducing production, obtaining supplies from different sources, and recovering from the disruption. The severity of a supply chain disruption can be measured by the number of entities that encounter difficulties in receiving or delivering materials or goods due to an unplanned event (Craighead et al. 2007). The interdependent and multiplying effects of at least two suppliers and firms being impacted will complicate the decisions of each entity.

This paper proposes a model for severe disruptions in which a disruption simultaneously impacts several suppliers and the suppliers’ customers, referred to here as “firms,” may face supply shortages as a result. The model incorporates decisions made by both suppliers and firms in the midst of this disruption, including whether or not to move production to an alternate facility, using inventory to continue production and meet customer demand, and acquiring supplies from other suppliers that are not impacted.

Unlike much of the supply chain risk literature (reviewed in the next section) which focuses either on decisions taken by companies prior to a disruption or on the equilibrium strategies resulting from the interaction of suppliers and firms, this paper focuses on decisions made by suppliers and firms during and after the disruption. If the disruption is severe and impacts several suppliers or multiple nodes in the supply chain, an entity may need to adjust its scheduling, production, and delivery of products and services beyond that foreseen by its preparedness activities. By focusing
on post-disruption decision making, this paper explores what suppliers and firms could do even if they had not adequately planned for a particular disruption. The optimal decisions for suppliers and firms are solved as functions of input parameters, such as cost, revenue, and customer loyalty. Threshold parameters are derived to serve as decision rules for whether and when a supplier should move production from a disabled facility to an alternate facility. A firm’s decision about how much to produce during the disruption trades off between two conflicting objectives: maximizing profit and satisfying demand. Changing the values of input parameters provides clarity for a decision maker about the benefits of different risk management strategies, such as maintaining inventory, buying from an alternate supplier, and helping a primary supplier recover more quickly.

This model quantifies the response of suppliers and firms to a disruption in terms of the level of demand that each entity is able to satisfy. Section 2 reviews the literature in supply chain risk and disruption management, the latter of which models optimal strategies for recovering from production disruptions. Section 3 presents the model and optimal decisions for suppliers and firms in the midst of production and supply difficulties. Section 4 applies this model to an example based on the supply chain disruption that occurred in the automobile sector as a result of the Japanese earthquake and tsunami.

2. Literature Review

Previous work in supply chain risk includes both qualitative and quantitative treatments. Qualitative studies (Chopra and Sodhi 2004, Sheffi 2005, Tang 2006) generally classify supply chain risk into several categories and review or recommend best practices for companies to prevent and prepare for potential disruptions. The causes of risk in the supply chain include supply-side risk, demand-side risk, operational risk, and security or catastrophic risk (Manuj et al. 2007). This paper models catastrophic risk, or severe disruptions, which can lead to supply-side, demand-side, and operational disruptions.

Quantitative models of supply chain risk broadly follow one of two methodologies: (i) adapting traditional inventory or sourcing models to account for the possibility of supply shortages or (ii) developing game theoretic models to explore the interdependent decision making between suppliers and firms. Traditional supply chain and inventory models (see Hopp and Spearman 2008) have been modified to incorporate the possibility of a firm’s supplier failing to satisfy the firm’s demand. A modified economic order quantity inventory policy is optimal if supply is available and then unavailable for a random amount of time (Parlar and Berkin 1991). If disruptive events vary the supply leadtime, a discrete-time Markov process can determine the optimal inventory level (Song
Tomlin (2006) builds on this discrete-time Markov model to determine the optimal inventory level where a firm can also purchase raw materials from an alternate supplier if its primary supplier suffers a disruption. Further extensions include the impact on a firm’s decision when it obtains more information about the reliability of a supplier (Tomlin 2009), incorporating multiple supply sources each with variable leadtimes (Song and Zipkin 2009), and designing a supply chain with dynamic sourcing that is subject to disruptions (Mak and Shen 2012).

Because suppliers and firms are both making decisions to mitigate risks in the supply chain, game theory provides a useful modeling construct. Babich et al. (2007) explore whether a firm should source from one or two suppliers when each supplier may suffer from a disruption. The answer depends, in part, on whether the probabilities of failure for the two suppliers are positively correlated or not. Given that contracts usually govern supplier-firm relationships, understanding the contracting process when suppliers may default on their obligation provides insight into managing supply chain risks (Swinney and Netessine 2009, Xia et al. 2011). Other approaches include a network-based model where several suppliers, manufacturers, and distributors are simultaneously maximizing their profit and minimizing their risk (Nagurney 2006).

In contrast to the above models, which primarily focus on decisions made prior to a disruption, disruption management models decision making during and after a disruption (Yu and Qi 2004). Despite preventative measures, disruptions can occur in the production and distribution of goods and services. When disruptions occur, suppliers and firms must adjust their plans, and disruption management explores the optimal way to manage disruptions with the understanding that a firm’s objective function may change as a result of the disruption (Xia et al. 2004). Supply chain disruption studies include production difficulties or operational risks (Xia et al. 2004), sudden drops in demand (Xiao et al. 2005), supply shortages (Xiao and Yu 2006), and cost fluctuations that impact wholesale prices (Xiao and Qi 2008). Rescheduling production (Bean et al. 1991, Adhyitya et al. 2007), moving production to other machines (Lee et al. 2006), transporting goods by alternate modes if one mode fails (MacKenzie et al. 2012a), and purchasing supplies from a backup supplier (Hopp and Spearman 2008) are examples of disruption management strategies.

The model proposed in this paper follows most closely with the approach of the disruption management literature. The model analyzes what suppliers and firms should do during a severe supply chain disruption under certain conditions. Although we assume that suppliers and firms behave optimally, a firm’s objective function during the disruption changes from what it was before the disruption. Quantitative models from the supply chain risk literature motivate several aspects of the modeling paradigm provided here. As in Tomlin (2006), a firm that suffers a supply shortage
can buy from a more expensive alternate supplier or produce less, and its decision depends on its inventory. Following the game theoretic approach, our model examines both firms and suppliers that are making decisions based on the decisions of the other entities. As time progresses, their optimal decisions can change. Finally, Nagurney (2006) has inspired the multitude of suppliers and firms, each of which is solving an optimization problem to maximize its objective.

3. Supply Chain Disruption Model

The supply chain contains $N$ suppliers and $M$ firms. Supplier $n$ ($n = 1, 2, \ldots, N$) delivers product to $M_n \leq M$ firms, and firm $m$ ($m = 1, 2, \ldots, M$) has $N_m \leq N$ suppliers. A firm receives a different product from each supplier, and each product is required for production. The firms compete with each other to sell their products to final consumers. Prior to a disruption, the system is in equilibrium, and each firm has constant demand.

Figure 1 outlines the decision-making framework for the model relationships between suppliers and firms. In order to make the graphic interpretable, Figure 1 depicts one supplier and one firm, but the model includes several suppliers each of which delivers product to several firms.

The disruption begins when an event directly impacts the $N$ suppliers and temporarily closes each supplier’s facility but does not impact the firms’ production facilities. Each supplier may choose to move production to an alternate facility. Moving production to alternate facilities has proved to be an important disruption management strategy for several businesses, including Texas Instruments, Renesas, and the Fujistu Group after the Japanese earthquake and tsunami (Godinez 2011, Greimel 2011, McGrath 2011), Sony after the floods in Thailand (Sony Corporation 2012), and energy service provider Enterprise Products after a fire closed one of its liquid natural gas facilities (Passwaters 2011). Intel replicated the manufacturing process at multiple facilities in order to quickly shift production if one of its facilities is damaged (Sheffi 2005).

If a supplier chooses not to move production, each firm that usually receives product from that supplier must deal with the lack of supplies. A firm uses inventory of raw materials, if available, to continue production. If the firm has not maintained raw material inventory—the model assumes the supplier does not maintain inventory—the firm determines its optimal production as described below by purchasing from more costly alternate suppliers. The firm decides how much to produce and buy from alternate suppliers based on two objectives: maximizing its profit and satisfying customer demand. For example, automobile manufacturers attempted to find other sources for a special pigment used in automobile paint after the Japanese earthquake and tsunami disabled the main facility (Walsh 2011). When Hurricane Mitch destroyed 80% of Honduras’ banana crop,
Supplier's facility is closed

Move production to alternate facility? yes no

Supplier inventory? no yes

Supply shortage for firm
Buy from alternate supplier?

Customer demand satisfied yes no

Firm receives required supplies

Demand not satisfied or customers buy from other firms yes no

Finished goods inventory?

Stop yes no

Supplier's facility reopens?

Chiquita relied on other Central American nations for its banana supply (Sheffi 2005). These alternate sources of supply may have been more expensive than the firms’ primary suppliers, but the firms’ desire not to lose demand motivated them to seek these alternate suppliers.

If the firm’s optimal production is less than its demand, the firm can make up the shortfall by using any finished goods inventory that it has maintained. If the sum of firm \( m \)'s production and finished goods inventory is less than demand, the firm’s customers will buy from its competitors with probability \( \tau_m \). Any customers who do not purchase from the firm’s competitors become backorders for the firm in the next period. At the end of the period, supplier \( n \)'s production facility reopens with a constant probability, \( p_n \). The parameters \( \tau_m \) and \( p_n \) describe the uncertainty in customer loyalty and the time when a supplier’s facility reopens, respectively. The disruption ends when all the suppliers’ facilities have reopened.

The main decisions in the model are each supplier’s decision about moving production to an alternate facility and each firm’s decision about buying from alternate suppliers. The optimal decision for a supplier and a firm in this model is examined in separate subsections.

3.1. Supplier

Before the disruption, a supplier produces \( z \) units in each period at a per-unit cost \( c \) and receives \( r \) revenue per unit produced. In period \( k = 0 \), a disruption closes the supplier’s facility, and the supplier’s facility will reopen at the beginning of the next period with a constant probability \( p \).
(Although each parameter differs for each supplier, the subscript \( n \) that represents a supplier is dropped for simplicity.) The decision for the supplier is whether to move its production to an alternate facility or wait for its primary facility to reopen. Moving production from one facility to another could require standardization across assembly lines before the disruption occurs. In case of a closure, a supplier would need to alter the flow of inbound material and reroute transportation of its goods to customers (Sheffi 2005). To represent this complexity in the model, a supplier who moves production to an alternate facility incurs a one-time fixed cost \( C \) and encounters a new per-unit cost of production \( c^+ \), where \( c^+ \geq c \). (The parameter \( C \) can be set to an extremely large amount in the model to represent a situation in which a supplier has no alternate facility.)

We assume that the supplier is able to produce in the same period in which it decides to move production. If the supplier does not move production, the supplier cannot produce in that period, and its customers (i.e., firms) may buy from other suppliers.

A fear of losing business to competitors would almost certainly be one of potentially several factors that would motivate a supplier to move production to an alternate facility. For example, Boston Scientific builds additional production lines at multiple facilities for some of its highly technical medical devices. Without backup facilities, the time it would take to repair and recertify a disrupted facility would mean lost business for Boston Scientific (Sheffi 2005). In the model, we assume that a supplier cannot predict whether firms will purchase from alternate suppliers, but it assesses that firms will buy from other suppliers with a probability \( \theta \). As will be described in Section 3.3, a supplier increases \( \theta \) if firms buy from alternate suppliers and decreases \( \theta \) if firms do not purchase from alternate suppliers.

The supplier decides to move production in the current period or a future period in order to maximize its expected profit over the entire time of the disruption. Equation (1) describes the supplier’s expected profit \( E[\pi(k)] \) if it decides to move production \( k \) periods after the current period where \( k = 0, 1, 2, \ldots \). Each period in the model represents the length of time during which supply and production decisions remain fixed. The application in Section 4 assigns one week for a period, and each supplier or firm’s decision determines production for a one-week period. The expected profit is the sum of \( E[\pi_{\leq k}(k)] \), the expected profit if the primary production facility opens within \( k \) periods, and \( E[\pi_{> k}(k)] \), the expected profit if the primary facility opens after \( k \) periods.

\[
E[\pi(k)] = E[\pi_{\leq k}(k)] + E[\pi_{> k}(k)]
\]  

(1)

We assume that once the supplier moves production to an alternate facility it will continue to produce \( z \) units per period at the alternate facility until its primary facility reopens. Thus, the
supplier’s only decision is to select \( k \) although it can reexamine its preferred alternative in each period as long as it has not already moved production.

The supplier’s expected profit if the facility reopens within \( k \) periods is given in Equation (2) where \( Z \) represents backordered demand. Because the supplier does not move production until period \( k \), the expected profit is the supplier’s profit generated by the primary facility reopening before or on the \( k \)th period. This expectation is the product of: the per-unit profit \( r - c \); any demand that was not purchased via alternate suppliers where each period has demand \( z \) and firms do not buy from alternate suppliers with probability \( 1 - \theta \) in every period; and the probability the facility reopens in a given period where \( p \) is the probability the facility reopens in the next period and \( 1 - p \) is the probability it remains closed. After the facility reopens, the supplier moves production back to the original facility and produces at a per-unit cost of \( c \).

\[
E\left[\pi_{\leq k}(k)\right] = \begin{cases} p(1 - \theta)(Z + z)(r - c) & \text{facility reopens in next period} \\ + (1 - p)p \left[ (1 - \theta)^2 (Z + z) + (1 - \theta) z \right] (r - c) & \text{reopens in second period} \\ + (1 - p)^2 p \left[ (1 - \theta)^3 (Z + z) + (1 - \theta)^2 z + (1 - \theta) z \right] + (r - c) + \ldots \\ + p(1 - p)^{k-1} \left[ (1 - \theta)^k (Z + z) + \sum_{l=0}^{k-1} (1 - \theta)^l z \right] (r - c) & \text{reopens in } k \text{th period} \end{cases}
\]

\[
= p \left( 1 - \frac{[(1 - p)(1 - \theta)]^k}{\theta + p(1 - \theta)} \right) \left[ \frac{1 - \theta}{\theta} \left[ \theta Z + (1 - \theta) z \right] + \frac{1 - (1 - p)^k}{p\theta} z \right] (r - c)
\]

(2)

The supplier’s expected profit if the facility reopens after the supplier moves production is given in Equation (3). The expected profit is the summation of three terms: the first term represents the expected product that would still be demanded of the supplier even though it did not produce in periods 0 through \( k - 1 \); the second term represents the expected profit of producing at the original facility after it reopens; and the third term represents the expected profit of producing at the alternate facility.

\[
E[\pi_{>k}(k)] = (1 - p)^k \left[ \left( 1 - \theta \right)^k Z + \sum_{i=0}^{k-1} (1 - \theta)^i z \right] (r - c) + \sum_{i=1}^{\infty} (1 - p)^i z (r - c^+) - C
\]

\[
= (1 - p)^k \left[ \left( 1 - \theta \right)^k Z + \frac{1 - (1 - \theta)^{k+1}}{\theta} z \right] (r - c^+) + z (r - c)
\]

(3)
Equations (2) and (3) can be combined into a single profit function. If the supplier wants to maximize its expected profit over the course of the disruption, it should choose \( k \) such that Equation (4) is maximized.

\[
A[(1-p)(1-\theta)]^k + B(1-p)^k
\]  

(4)

where

\[
A = \frac{[\theta Z - (1-\theta)z][\theta (r-c^+) - p(1-\theta)(c^+-c)]}{\theta + p(1-\theta)\theta}
\]

\[
B = \frac{z[\theta (r-c^+) - p(1-\theta)(c^+-c)]}{p\theta} - C
\]

This objective function serves as the basis for describing the conditions under which a supplier will choose to move production in a future period.

**Proposition 1.** A supplier that wants to maximize Equation (4) will move production in a future period \( k^{*} > 0 \) as given in Equation (5) if and only if \( A < 0 \), \( B > 0 \), and \( -BG/A < 1 \) where

\[
G = \log((1-p)/(1-\theta))
\]

\[
k^{*} = \frac{\log(-BG/A)}{\log(1-\theta)}
\]  

(5)

**Proof** The solution \( k^{*} \) is obtained by setting the first derivative in Equation (4) to 0 and solving for \( k \). The solution \( k^{*} > 0 \) if and only if \( 0 < -BG/A < 1 \). This is true because \( \log(1-\theta) < 0 \). Because \( G \) is positive, \( A \) and \( B \) must have opposite signs in order that \( -BG/A > 0 \).

Equation (4) has at most one critical point, so the maximum profit either occurs at \( k = 0 \), \( k = k^{*} \), or \( k \to \infty \). Thus, to prove that \( k^{*} \) is a unique maximum if and only if \( A < 0 \) and \( B > 0 \), it suffices to show that the first derivative of Equation (4) is greater than 0 for all \( k < k^{*} \) and is less than 0 for all \( k > k^{*} \) if and only if \( A < 0 \) and \( B > 0 \).

The condition \( k > k^{*} \) is examined first. If \( A < 0 \), then \( B > 0 \). Under this assumption, if \( k > \log(-BG/A)/\log(1-\theta) \), then

\[
k\log(1-\theta) < \log\left(\frac{-BG}{A}\right)
\]

\[
\Rightarrow A\log\left[(1-p)(1-\theta)\right][\left(1-p\right)(1-\theta)]^k + B\log(1-\theta) < 0
\]  

(6)

where the left-hand side of the expression is the first derivative of Equation (4). Thus, if \( A < 0 \) and \( B > 0 \), the first derivative is less than 0. If \( A > 0 \) and \( B < 0 \), Equation (6) results in the first derivative being greater than 0. Thus, if the first derivative is less than 0 for \( k > k^{*} \), then \( A < 0 \) and \( B > 0 \).
Likewise, for \( k < k^* \), if \( A < 0 \) and \( B > 0 \),

\[
k < \frac{\log \left( \frac{-B G}{A} \right)}{\log (1 - \theta)} \quad (7)
\]

and the first derivative is greater than 0. Thus if \( A < 0 \) and \( B > 0 \), the first derivative is greater than 0 when \( k < k^* \). If \( A > 0 \) and \( B < 0 \), Equation (7) results in the first derivative being less than 0. Thus, if the first derivative is greater than 0 for \( k < k^* \), then \( A < 0 \) and \( B > 0 \). □

The solution \( k^* \) is generally not an integer. Choosing the optimal period requires comparing the objective function of the two integers nearest to \( k^* \) to determine the optimal period in which the supplier will move production to an alternate facility.

If the conditions of Proposition 1 are not satisfied, no interior maximum exists, and the supplier will compare the alternatives of moving production immediately, \( k = 0 \), and never moving production.

**Proposition 2.** If the conditions of Proposition 1 are not satisfied, a supplier that wants to maximize Equation (4) will choose to move production immediately if and only if the fixed cost \( C \) is less than \( \bar{C} \) as given in Equation (8).

\[
\bar{C} = \frac{(pZ + z) \left[ (r - c^+) - p (1 - \theta) (c^+ - c) \right]}{p (\theta + p (1 - \theta))} \quad (8)
\]

*Proof.* The expected profit evaluated at \( k = 0 \) is greater than expected profit as \( k \to \infty \) if and only if \( A + B > 0 \), which is obvious by setting \( k = 0 \) and letting \( k \to \infty \) in Equation (4).

\[
A + B = \frac{\left[ \theta Z - (1 - \theta) z \right] \left[ \theta (r - c^+) - p (1 - \theta) (c^+ - c) \right]}{p (\theta + p (1 - \theta))} + \frac{z \left[ \theta (r - c^+) - p (1 - \theta) (c^+ - c) \right]}{p (\theta + p (1 - \theta))} - C
\]

\[
= \frac{(pZ + z) \left[ (r - c^+) - p (1 - \theta) (c^+ - c) \right]}{p (\theta + p (1 - \theta))} - C \quad (9)
\]

Clearly, \( A + B > 0 \) if and only if \( C < \bar{C} \) as stated in Equation (8). □

The final element to explore in the supplier’s decision is the impact of \( p \), the probability that the original production facility will reopen in the next period.

**Proposition 3.** A supplier that wants to maximize Equation (4) will never move production if the probability that its facility reopens in the next period is greater than \( \bar{p} \) where \( \bar{p} \) is given in Equation (10).

\[
\bar{p} = \frac{\theta (r - c^+)}{(1 - \theta) (c^+ - c)} \quad (10)
\]
Table 1  Supplier’s decision of when to move production

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Optimal decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>If $p &gt; \bar{p}$</td>
<td>Never move</td>
</tr>
<tr>
<td>If $A &lt; 0$, $B &gt; 0$, and $\frac{-BG}{A} &lt; 1$</td>
<td>Move in $k^*$ periods</td>
</tr>
<tr>
<td>If $A \geq 0$, $B \leq 0$, or $\frac{-BG}{A} \geq 1$ and ...</td>
<td>Move immediately</td>
</tr>
<tr>
<td>... $C &lt; \bar{C}$</td>
<td></td>
</tr>
<tr>
<td>... $C &gt; \bar{C}$</td>
<td>Never move</td>
</tr>
</tbody>
</table>

Figure 2  Examples of selecting $k$ to maximize expected profit: (i) interior optimal point, (ii) move immediately, (iii) never move, and (iv) never move

Proof The proof follows from Propositions 1 and 2. If $p \geq \bar{p}$, then $B \leq 0$, which means no interior optimal point exists, and $\bar{C} \leq 0$. Thus, letting $k \to \infty$ maximizes the expected profit in Equation (4). □

Table 1 summarizes the supplier’s optimal decision of when, if ever, to move production to an alternate facility, and Figure 2 graphically depicts those conditions. If the chances that the primary facility will reopen in the next period are high (i.e., if $p > \bar{p}$), it is optimal for the supplier to wait and choose not to move to the alternate facility.

A supplier will plan to move production to an alternate facility in the future if $A < 0$, $B > 0$, and $BG < -A$. As can be seen via Equations (4) and (10), a necessary condition for $B > 0$ is that $p < \bar{p}$. If the probability that the primary facility will reopen in the next period is small and the per-period demand for the supplier as given by $z$ is large, $B$ is likely to be greater than 0, as long as the fixed cost $C$ is not very large. If $p < \bar{p}$, according to Equation (4), $A < 0$ if and only if $\theta Z - (1 - \theta) z < 0$. 


The backordered demand $Z$ multiplied by $\theta$—the assessed probability that firms will buy from alternate suppliers—must be smaller than $(1-\theta)z$. Thus, an interior maximum is more likely to exist in situations where demand for the supplier is large, the probability that the primary facility will reopen is small, and backordered demand is 0 or close to 0. Backordered demand will be small at the beginning of the disruption because the model assumes $Z = 0$ when the disruption first occurs. No interior maximum exists, however, if $B$ is very large such that $BG > -A$, in which case the supplier will immediately move production to an alternate facility. This situation is most likely to occur when $C$ is close to 0.

Because the backordered demand $Z$ can change from period to period, the supplier’s optimal decision may change during the course of the disruption. For example, if the supplier has no backordered demand when the disruption initially closes its facility, the supplier may determine that never moving production is optimal because $C > \bar{C}$. The actual fraction of the supplier’s customers that buy from other suppliers may be less than the probability $\theta$. If $Z$ is larger than what the supplier expected, the supplier may choose to move production to an alternate facility if $\bar{C}$ increases and $C < \bar{C}$.

3.2. Firm

Without loss of generality, it is assumed that one unit of product produced by firm $m$ requires one unit from each of its $N_m$ suppliers. Each one of these $N_m$ supplies is also required for firm $m$ to produce one unit.

Before the disruption, the per-unit selling price for a firm is $\rho$ and the per-unit cost of production is $\bar{w}$ where $\rho \geq \bar{w}$. The cost of production includes both the cost of inputs and the cost of other factors of production such as labor and capital. The firm produces $\bar{x}$ amount of product in each period.

After the disruption occurs, if at least one of the firm’s suppliers is unable or chooses not to move production to an alternate facility, the firm must decide how to resolve this supply shortage. It may have inventory of supplies or raw material inventory. The model assumes that the firm will use this inventory before seeking to buy supplies from an alternate supplier.

If the firm is able to purchase product from alternate suppliers at a higher cost, the firm must decide how much to produce in each period until all of its primary suppliers are producing again. Ultimately, the firm desires to maximize its long-run profit, which is the sum of its profit in each period and requires a dynamic programming model. However, while the firm is managing a disruption in which events are rapidly changing, calculating the long-run profit may not be possible. A more realistic objective is proposed, which seeks to mirror how a firm will actually respond
during a disruption (Yu and Qi 2004). In the model, the firm has two objectives: (i) maximize its profit in the current period and (ii) satisfy the current demand \(d\) for its product. If the cost of supplies increases due to the disruption, a firm that solely maximizes its profit in the current period may choose to produce less or even not to produce at all. However, a firm that just focuses on maximizing profit in the short term may hurt its long-run profit because its customers will buy from the firm’s competitors. Therefore, a firm will also seek to meet customer demand over the course of the disruption.

Alternate suppliers sell supplies to the firm at a higher cost than the primary suppliers. The firm’s cost function during the disruption \(w(x)\) has a non-decreasing marginal cost to represent that each additional scarce supply costs more during a disruption. As given in Equation (11), the function \(f(x)\) combines these two objectives into a single objective, and the firm seeks to maximize \(f(x)\). By maximizing \(f(x)\), a firm simultaneously attempts to maximize its profit and minimize the deviation from the current demand \(d\). The parameter \(\alpha \geq 0\) allows the firm to trade off between these two competing objectives.

\[
\begin{align*}
\text{maximize } f(x) &= \rho x - w(x) - \alpha (x - d)^2 \\
\text{subject to } &0 \leq x \leq d
\end{align*}
\] (11)

The demand for the firm may fluctuate in each period because \(d\) includes not only the constant customer demand but also customers from previous periods who chose not to buy from the firm’s competitors. We assume that a firm knows this demand before deciding how much to produce in a period although it does not know how many customers will want to purchase from it if a competitor does not satisfy demand. Demand from competitors may be satisfied by the firm’s finished goods inventory. Assuming that demand is known before determining production is a frequent assumption in supply chain risk and disruption models (e.g., Xia et al. 2004, Tomlin 2006). Such an assumption may be reasonable as the disruption may not change what is demanded of the firm. Further extensions of the model could explore how production decisions would change if demand were uncertain.

The firm’s desire to satisfy current demand is assumed to follow a quadratic function, where \(x = d\) is the ideal production amount. Similar to quadratic cost functions (Holt et al. 1955), this functional form implies that a firm really wants to avoid producing substantially less than demand. Conversely, a firm may be willing to accept producing slightly less than demand, especially in the
midst of a disruption when supplies may be difficult to acquire. A linear functional relationship does not allow us to effectively represent the firm’s strong preference for meeting demand.

The trade-off parameter $\alpha$ is in units of dollars per unit of production squared, and it represents the firm’s willingness to sacrifice profit in order to satisfy current demand. As will be discussed in Proposition 5, a firm will produce $x$ such that $\rho - w'(x) - 2\alpha(x - d) = 0$ where $w'(x) = \frac{dw}{dx}$ is the firm’s marginal cost. The trade-off parameter can be assessed by asking a firm to choose a production level $\hat{x}$ where $\hat{x} < d$. Then, $\alpha = \frac{[\rho - w'(\hat{x})]}{[2(\hat{x} - d)]}$.

The cost function for a firm in our model, as expressed in Equation (12), is a piecewise linear function, where $x_1 < x_2 \ldots < x_{L-1}$ are different production quantities for the firm and $\bar{w} < w_1 < w_2 \ldots < w_L$ are different per-unit costs of production corresponding to the different production quantities. For example, a firm that produces $x$ where $x_1 < x < x_2$ has a cost of $w_1x_1$ for the first $x_1$ units of production because it can buy $x_1$ units from one alternate supplier and a cost of $w_2(x - x_1)$ for the remaining units of production because it buys those from a different alternate supplier.

$$w(x) = \begin{cases} 
  w_1x & \text{if } x \leq x_1 \\
  w_1x_1 + w_2(x - x_1) & \text{if } x_1 < x \leq x_2 \\
  \vdots & \text{ } \\
  \sum_{i=1}^{L-2} w_i(x_i - x_{i-1}) + w_{L-1}(x - x_{L-2}) & \text{if } x_{L-2} < x \leq x_{L-1} \\
  \sum_{i=1}^{L-1} w_i(x_i - x_{i-1}) + w_L(x - x_{L-1}) & \text{if } x_{L-1} < x 
\end{cases}$$

Because the firm’s cost function is unbounded, assigning $x_0 = 0$ and $x_L = \infty$ provides a useful way to bound the cost function.

The values of $w_i$ and $x_i$ depend on the number of suppliers that move production to alternate facilities and the number of alternate suppliers used by a firm. A simple example demonstrates how the firm’s cost function may increase as more suppliers do not produce and alternate suppliers may not be able to produce as much as the firm’s primary suppliers. Assume that a firm produces $\bar{x} = 3$ before the disruption, the firm has two suppliers X and Y, and the per-unit cost of supplies from X and Y are 1 and 2, respectively. If the firm’s cost of production is comprised entirely from purchasing supplies, $\bar{w} = 3$. If Y moves production to an alternate facility but X does not, and an alternate supplier to X (called X1) can sell no more than 3 units at a per-unit cost of 2 to the firm, then $w_1 = 2 + 2 = 4$ and $x_1 = 3$. If X1 can sell at most 2 units per period but another supplier X2 can sell no more than 1 unit at a per-unit cost of 3, then $w_1 = 2 + 2 = 4$, $x_1 = 2$ (the maximum amount produced by X1), $w_2 = 3 + 2 = 5$, and $x_2 = 3$. Finally, assume that supplier Y does not move production either, the first alternate supplier Y1 can sell at most 1 unit at a per-unit cost of 3, and the second alternate supplier Y2 can sell no more than 2 units at a per-unit cost of 4.
If the constraints remain the same for X1 and X2, \( w_1 = 2 + 3 = 5, \) \( x_1 = 1 \) (suppliers X1 and Y1); \( w_2 = 2 + 4 = 6, \) \( x_2 = 2 \) (suppliers X1 and Y2); and \( w_3 = 3 + 4 = 7, \) \( x_3 = 3 \) (suppliers X2 and Y2).

The firm’s cost function in Equation (12) can be expressed more simply as a maximum function in Equation (13).

\[
    w(x) = \max_{1 \leq l \leq L} \left\{ l - \sum_{i=1}^{l-1} w_i (x_i - x_{i-1}) + w_l (x - x_{l-1}) \right\}
\]  

(13)

We explore the firm’s optimal production for two different cases: (i) where the firm’s sole objective is to maximize profit in the current period, or \( \alpha = 0 \), and (ii) where the firm sacrifices profit to satisfy current demand, or \( \alpha > 0 \).

**Proposition 4.** If \( \alpha = 0 \), the firm will produce \( \min (x^*, d) \) where \( x^* \) is given in Equation (14) and \( i^* = \arg\min_{1 \leq i \leq L} \{ \rho - w_i \} \) subject to \( \rho - w_i \geq 0 \).

\[
    x^* = \begin{cases} 
    0 & \text{if } \rho < w_1 \\
    \text{anywhere in the interval } [x_{i^*-1}, x_{i^*}] & \text{if } \rho = w_{i^*} \\
    x_{i^*} & \text{if } \rho > w_{i^*}
    \end{cases}
\]  

(14)

**Proof** From classical microeconomic theory, a firm produces at a level where marginal revenue equals marginal cost, and this proof demonstrates that \( x^* \) is the quantity at which marginal revenue equals marginal cost. The firm’s marginal cost as given in Equations (12) and (13) is non-decreasing. If \( \rho < w_1 \), \( i^* \) is undefined because no feasible solution exists. The firm is unable to make a profit, and it will choose to produce \( x^* = 0 \).

If \( \rho = w_{i^*} \) for some \( i^* = 1, 2, \ldots, L \), then the firm’s profit increases for all \( x < x_{i^*-1} \) and decreases for all \( x > x_{i^*} \). The firm’s profit remains constant for \( x_{i^*-1} \leq x \leq x_{i^*} \), and the firm is indifferent to producing any amount within this interval. The firm’s profit is positive if \( i^* > 1 \) because \( \rho > w_i \) for all \( i < i^* \).

If \( \rho > w_{i^*} \), no point exists where the firm’s marginal cost equals its marginal revenue. Given the firm’s non-decreasing marginal cost, the firm’s profit decreases for all \( x > x_{i^*} \). A firm that maximizes its profit will choose \( x^* = x_{i^*} \).

If the chosen \( x^* \) exceeds the current demand \( d \), the firm chooses to produce \( d \). By producing \( d \), the firm’s marginal revenue exceeds its marginal cost, but it is unable to sell more than demand. □

Although the firm desires to maximize its profit during a disruption, it may be willing to sacrifice some profit in order to satisfy the current demand. The firm’s optimal production is detailed for the case where \( \alpha > 0 \).
The right-hand derivative production during a supply shortage. If \( \alpha \) which occurs at any production level from 1 to 2. For large \( \alpha \) but wait to see whether its primary facility will reopen before moving. It might also be optimal for a supplier to plan to move production in the future cost of waiting for the primary facility to reopen, especially if the primary facility is expected to reopen quickly. This model of a severe supply chain disruption reveals several observations about supplier and firm behavior. A supplier may be able to produce at an alternate facility but choose not to move production to that facility immediately. The cost of moving production may exceed the expected cost of waiting for the primary facility to reopen, especially if the primary facility is expected to reopen quickly. It might also be optimal for a supplier to plan to move production in the future but wait to see whether its primary facility will reopen before moving.

3.3. Model insights

This model of a severe supply chain disruption reveals several observations about supplier and firm behavior. A supplier may be able to produce at an alternate facility but choose not to move production to that facility immediately. The cost of moving production may exceed the expected cost of waiting for the primary facility to reopen, especially if the primary facility is expected to reopen quickly. It might also be optimal for a supplier to plan to move production in the future but wait to see whether its primary facility will reopen before moving.

Proof The objective function \( f(x) \) given in Equation (11) is concave because it is the summation of three concave functions: \( \rho x \) is linear and \(-w(x) \) and \(-\alpha (x - d)^2 \) are both concave. The cost function \( w(x) \) is convex because it is the maximum of piecewise affine functions and the maximum of multiple convex functions is also convex (Boyd and Vandenberghe 2004).

Because the objective function is concave, a unique maximum exists if the first derivative, when it exists, equals 0. The function is differentiable except at the points where the cost function changes, i.e., \( x = x_1, x_2, \ldots, x_{L-1} \), and \( f'(x) = \rho - w_i - 2\alpha (x - d) \) for all other \( x \).

If \( \rho - w_i - 2\alpha (x_i - d) \leq 0 \), a point \( x^* = (\rho - w_i)/(2\alpha) + d \) exists within the interval \([x_{i-1}, x_i] \) where \( f'(x^*) = 0 \). This is true because \( \rho - w_i - 2\alpha (x_{i-1} - d) \geq 0 \geq \rho - w_i - 2\alpha (x_i - d) \).

If \( \rho - w_i - 2\alpha (x_i - d) > 0 \), no point exists where the first derivative equals 0. This is true because \( \rho - w_{i+1} - 2\alpha (x_i - d) < 0 \). If \( \rho - w_{i+1} - 2\alpha (x_i - d) \) were not less than 0, then \( \rho - w_{i+1} - 2\alpha (x_{i+1} - d) < \rho - w_i - 2\alpha (x_i - d) \) because \( w_{i+1} > w_i \) and \( x_{i+1} > x_i \). This would contradict the definition that \( i^* = \arg\min_{1 \leq i \leq L} \{\rho - w_i - 2\alpha (x_i - d)\} \). Because of the concavity of the objective function, \( x_i^* \) maximizes the firm’s objective function if \( f(x_i^*) > f(x_i^* - \delta) \) and \( f(x_i^*) > f(x_i^* + \delta) \) for an arbitrary small \( \delta > 0 \). The objective function \( f(x) \) is not differentiable at \( x = x_i^* \), but the left-hand derivative \( f'_{\text{left}}(x_i^*) = \rho - w_i - 2\alpha (x_i - d) > 0 \), and \( f(x_i^*) > f(x_i^*) - \delta f'_{\text{left}}(x_i^*) > f(x_i^* - \delta) \). The right-hand derivative \( f'_{\text{right}}(x_i^*) = \rho - w_{i+1} - 2\alpha (x_i - d) < 0 \), and \( f(x_i^*) > f(x_i^*) + \delta f'_{\text{right}}(x_i^*) > f(x_i^* + \delta) \). □

As illustrated in Figure 3, the parameter \( \alpha \) can have a large impact on the firm’s level of production during a supply shortage. If \( \alpha = 0 \) (chart i), the firm is solely maximizing its profit which occurs at any production level from 1 to 2. For large \( \alpha \) (chart iv) the firm is willing to produce without any profit in order to satisfy demand, \( d = 3 \).
Although the model assumes that a supplier cannot predict what firms will do if the former does not move production to an alternate facility, the supplier observes the past actions of firms and alters its original assessment. The supplier changes its probability that firms will buy from alternate suppliers, the parameter $\theta$, based on its observations. The model assumes that a supplier calculates $\theta$ in the current period so that it equals the amount of supplies that the supplier’s customers purchased from alternate suppliers divided by the total number of supplies required in the previous period. We assume that each supplier has complete information about the firms’ and other suppliers’ actions in that previous period. As will be discussed in Section 4.3, $\theta$ can suddenly change from one period to the next (if the firms’ buying behavior changes). Incorporating a smoothing parameter so that the new value of $\theta$ integrates information from the most recent period with the previous value of $\theta$ could be another approach to model the supplier’s beliefs. A more careful study of suppliers’ actions during a disruption would be necessary to know if a supplier’s beliefs change suddenly (as is currently assumed) or more gradually (as a smoothing parameter would suggest).

As $\theta$ increases, the fixed cost threshold $\bar{C}$ increases (see Proposition 2). A larger value of $\bar{C}$ may influence the supplier to move production immediately as opposed to waiting. Thus, the supplier’s optimal decision changes during the course of the disruption as a result of the firms’ behavior. If no alternate suppliers are available for the firms, which could be modeled by setting each firm’s cost function $w(x)$ to an extremely high value, a supplier will not lose any business and $\theta = 0$. 

Figure 3  Examples of impact on $\alpha$ on firm’s production when the current demand $d = 3$
Consequently, a supplier will not move production to an alternate facility even if one is available because the supplier is not losing profit while waiting for its primary facility to reopen.

A firm may also react to its suppliers’ decisions. If a firm has several non-producing suppliers, the firm may not produce because purchasing from so many alternate suppliers is too costly. If a few of those suppliers move production to alternate facilities or if their primary facilities reopen, the firm’s cost of production decreases and it will not need to buy from as many alternate suppliers. Consequently, the firm may decide to produce, depending on its willingness to meet demand. If the firm does produce, the suppliers who have not been producing will observe that they are losing business, which may incentivize them to move production. In this manner, a supplier’s individual decision to move production can create a ripple effect whereby firms begin to produce more, and more suppliers move production. These interactions between suppliers and firms are numerically explored in Section 4.3.

The model explains why a firm may continue to produce at higher costs during a disruption, even if those costs exceed its revenue. The firm’s desire to trade off maximizing profit with satisfying current demand seeks to reflect a situation where calculating profit over the long term poses a challenge. A key determinate of the value of $\alpha$ is the cost of an additional alternate supplier. If the cost of the alternate supplier is much larger than the selling price, then the firm requires a large value of $\alpha$ in order to produce close to the current demand. If the cost of the alternate supplier exceeds the selling price by a small margin, a small value of $\alpha$ can enable the firm to satisfy most of the current demand.

4. Application to Automobile Sector Disruption

The Japanese earthquake and tsunami impacted the automobile sector most heavily. The production of the Japanese motor vehicle industry dropped by almost 50% in March and April 2011 compared to the industry’s production in the same months in 2010 (Japan 2011). Companies such as Renesas, which manufactures electronics for the automobile industry, and Merck, which produces a chemical agent used in automobile paint, had facilities that were closed for months. Renesas moved some production to other facilities in Japan and Singapore and raced to reestablish its supply chain in order not to lose customers to its competitors (Greimel 2011, Okada 2011). Toyota and Honda did not return to normal production until about six months after the tsunami (Bunkley 2011c). Nissan was positioned slightly better before the disaster because it had been surging its inventory to prepare for a production increase. It was able to resume full production a couple of months before Toyota and Honda (Bunkley 2011b, Woodyard 2011). Automobile production difficulties in Japan impacted automobile manufacturers around the globe, leading to temporary halts.
in some production lines, longer waits for certain vehicles, and extremely low inventory levels.

An application of the supply chain disruption model is based on this disruption in the automobile sector, and we use simulation to arrive at numerical results for the model. This application is designed to simulate automobile production in North America in the months following the Japanese earthquake and tsunami. Although the results mirror what actually occurred, the model simplifies the complexities of modern automobile supply chains and requires several data assumptions. This section numerically explores several model features, including: three different scenarios; the interactions between suppliers and firms; incorporating non-constant probabilities into the model; and sensitivity analysis for a supplier and a firm. This application demonstrates how this model can provide insight into the impacts of a severe supply chain disruption and numerically quantify the benefit of different risk management strategies.

4.1. Input data

The application has $N = 4$ suppliers and $M = 3$ firms. Firm 1 receives supplies from Suppliers 1 and 2; Firm 2 receives supplies from Suppliers 1, 2, and 3; and Firm 3 receives supplies from Suppliers 1, 2, and 4. The disruption closes the four suppliers’ production facilities, and Table 2 depicts the input parameters for the suppliers.

Supplier 1 resembles Renesas, whose production facility was closed for 12 weeks (Okada 2011). With a geometric probability distribution, the expected number of periods that Supplier 1’s facility will be closed is 12 if the probability of reopening in each period is 1/12. Supplier 2 represents Merck, whose production facility was closed for 8 weeks (Agence France Press 2011). Supplier 3 represents Toyota and Honda combined, and Supplier 4 represents Nissan. Because Nissan’s production resumed more quickly than that of Toyota and Honda, the probability of Supplier 4’s facility reopening is twice as large as that of Supplier 3.

Table 3 depicts the input parameters for the three firms. Firm 1 represents Ford, General Motors, and Chrysler (the “Detroit 3”) combined; Firm 2 represents Toyota and Honda in North America; and Firm 3 represents Nissan in North America. Because the Detroit 3 are less dependent on Japanese suppliers than Japanese automakers, Firm 1 only receives supplies from two of the

<table>
<thead>
<tr>
<th>$n$</th>
<th>$r$</th>
<th>$c$</th>
<th>$c^*$</th>
<th>$z$</th>
<th>$p$</th>
</tr>
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<tr>
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<td>1</td>
<td>2</td>
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<td>1/12</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>84</td>
<td>1/8</td>
</tr>
<tr>
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<td>3</td>
<td>1</td>
<td>2</td>
<td>21</td>
<td>1/26</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>8</td>
<td>1/13</td>
</tr>
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</table>
Table 3  Inputs for firms

<table>
<thead>
<tr>
<th>m</th>
<th>ρ</th>
<th>ϵ</th>
<th>w</th>
<th>x</th>
<th>w</th>
<th>Raw materials inventory</th>
<th>Finished goods inventory</th>
<th>τ</th>
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</thead>
<tbody>
<tr>
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<td>3</td>
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<td></td>
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<tr>
<td>2</td>
<td>4</td>
<td>21</td>
<td>3</td>
<td>42</td>
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<td>0.39</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>8</td>
<td>3</td>
<td>24</td>
<td>48</td>
<td>0.46</td>
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Table 4  Cost curves for firms when suppliers are not producing

<table>
<thead>
<tr>
<th>Number of suppliers with inoperable facilities and who have not moved production</th>
<th>w₁</th>
<th>w₂</th>
<th>w₃</th>
<th>w₄</th>
<th>x₁</th>
<th>x₂</th>
<th>x₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm 1</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>28</td>
<td>55</td>
</tr>
<tr>
<td>Firm 2</td>
<td>1</td>
<td>4</td>
<td>5</td>
<td>8</td>
<td>11</td>
<td>21</td>
<td>31</td>
</tr>
<tr>
<td>Firm 3</td>
<td>1</td>
<td>4</td>
<td>5</td>
<td>8</td>
<td>11</td>
<td>21</td>
<td>31</td>
</tr>
</tbody>
</table>

Suppliers. As shown in Table 3, each firm’s pre-disruption production, \( \bar{x} \), reflects the percentage of total production in North America in 2010: the Detroit 3 produced 55% of vehicles in North America, Toyota and Honda produced 21%, and Nissan produced 8% (Ward’s 2011). The customer loyalty parameter \( \tau \) (or how likely customers will buy from a competitor) is derived from the 2012 Customer Retention Study published by J. D. Power and Associates (2010). Generally, the automotive pipeline has about eight weeks worth of inventory (Snyder 2011), and Firms 1 and 2 have two weeks of raw materials inventory and six weeks of finished goods inventory at the start of the simulation. Because Nissan had begun an inventory surge before the disruption, Firm 3 begins with three weeks of raw materials inventory.

Simplifying assumptions are necessary for the cost and revenue parameters for both suppliers and firms. For a supplier, the marginal cost of producing at an alternate facility is twice as much as the marginal cost of producing at the primary facility. Before the disruption, firms make a profit of one for each unit produced. The firms’ cost curves during the disruption (Table 4) reflect the assumptions that the firms’ costs rise as more suppliers are unable to produce and the marginal cost increases as a firm produces more.

We explore this disruption for three different scenarios: (1) suppliers have no alternate facilities and firms concentrate solely on maximizing profit; (2) suppliers have no alternate facilities and firms are willing to sacrifice profit to meet demand; and (3) suppliers have alternate facilities and firms
4.2. Results

Setting the fixed cost $C$ of moving production to an extremely high value deters suppliers from moving to alternate facilities and represents a situation where alternate facilities are not available. If $\alpha = 0$, firms are solely concerned with maximizing their profit. Such a case represents a situation with little disruption management as firms rely on inventory and only buy from alternate suppliers if this action is profitable. Suppliers have no decisions to make in this specific case, and each waits until its primary facility reopens before producing. The firms follow the decision rule detailed in Proposition 4. As can be deduced from Table 4, none of the firms produce when all the suppliers are inoperable because $w_1 > \rho$ for all the firms. After Supplier 1 or 2’s primary facility reopens, Firm 1 produces $x_1 = 28$ because $\rho = w_1 = 3$. Firms 2 and 3 do not produce until two of their suppliers’ primary facilities reopen (i.e., one non-producing supplier) because $\rho = w_1 = 4$ for both firms with one non-producing supplier but $\rho < w_1 = 5$ with two non-producing suppliers. When two of their suppliers are producing, Firm 2 produces $x_1 = 11$ and Firm 3 produces $x_1 = 4$.

Tables 5 and 6 depict the average results from 10,000 simulations of the above example. Because suppliers are passive actors in this case, each supplier’s performance, as measured by the percentage of demand it fulfills during the total length of the disruption, depends on the length of time each supplier’s primary facility is closed. The supply shortage hinders the firms’ abilities to meet customer demand, and Firm 2 satisfies 69% of its demand on average. Because it relies on only two suppliers, Firm 1 performs the best, satisfying 89% of its customer demand on average.
Table 7  Average percentage of demand for firms when $\alpha > 0$ and suppliers have no alternate facilities

<table>
<thead>
<tr>
<th>Firm</th>
<th>Satisfied by firm</th>
<th>Taken by another firm</th>
<th>Not satisfied by any firm</th>
<th>Taken from another firm</th>
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<tbody>
<tr>
<td>Firm 1</td>
<td>96.2</td>
<td>0.0</td>
<td>3.8</td>
<td>2.5</td>
</tr>
<tr>
<td>Firm 2</td>
<td>86.6</td>
<td>5.9</td>
<td>7.5</td>
<td>0.1</td>
</tr>
<tr>
<td>Firm 3</td>
<td>91.4</td>
<td>3.0</td>
<td>5.6</td>
<td>1.3</td>
</tr>
</tbody>
</table>

Firms impacted by the supply shortage from Japan apparently sacrificed some short-term profit in order to maintain production levels. Choosing $\alpha > 0$ reflects this desire to meet demand, and each firm follows the rule established in Proposition 5 in order to determine how much it will produce in each period. If Suppliers 1 and 2 are inoperable and $\alpha = 0.005$ and demand $d = 55$ for Firm 1, the firm will not produce anything because $\rho - w_1 - 2\alpha(x_{i-1} - d) < 0$ for all $i \geq 1$. If the demand accumulates so that $d = 110$ in the next period, Firm 1 will produce $(\rho - w_{i^*})/(2\alpha) + d = 10$ where $i^* = 1$ (Table 4). Because Firm 1’s demand exceeds that of Firm 2 and Firm 3, the values of $\alpha$ for these latter firms are multiplied by the ratio of Firm 1’s demand to each firm’s demand, i.e., $55/21$ and $55/8$, in order that Firms 2 and 3 behave similarly.

We perform 10,000 simulations with $\alpha$ values chosen at 0.005, 0.0131, and 0.0344 for Firms 1, 2, and 3, respectively. These values of $\alpha$ are chosen so that each firm is incentivized to buy from alternate suppliers if demand for its product accumulates beyond one period. Alternate facilities are not available for suppliers, and the results for suppliers are almost the same as the previous results. Firms satisfy much more of their customer demand (Table 7), and Firm 1 satisfies 96% of its demand on average. Firm 2 loses almost 6% of its demand on average to Firms 1 and 3. This corresponds with Toyota and Honda’s actual production, where their share of production in North America declined by about 5% following the Japanese earthquake and tsunami (Ward’s 2011). The Detroit 3’s share of production increased by about 3.5%, compared to an average of 2.5% for Firm 1 from the simulation. On average, Firm 3 loses 3.0% of its customers but gains 1.3%, which reflects reality where Nissan’s share of production remained relatively constant.

Setting $C$ equal to 475, 295, 270, and 49 for the four suppliers provides an opportunity for them to move production to an alternate facility. As will be discussed in the next subsection, these values of $C$ generally incentivize suppliers to move production to alternate facilities after the disruption has lasted for several periods. We initialize $\theta = 0.5$ for all the suppliers to represent ignorance on the part of the supplier about what firms will do. If $Z = 0$ and the other inputs are those given in Table 2, the conditions of Proposition 1 are satisfied because $A < 0$ and $B > 0$ as calculated by Equation (4). Each supplier initially plans to move production in the second period. During the
Table 8  Average percentage of demand for firms when $\alpha > 0$ and suppliers have alternate facilities

<table>
<thead>
<tr>
<th></th>
<th>Satisfied by firm</th>
<th>Taken by another firm</th>
<th>Not satisfied by any firm</th>
<th>Taken from another firm</th>
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<tr>
<td>Firm 1</td>
<td>98.1</td>
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<td>0.9</td>
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<tr>
<td>Firm 2</td>
<td>93.4</td>
<td>2.0</td>
<td>4.1</td>
<td>0.0</td>
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<tr>
<td>Firm 3</td>
<td>95.6</td>
<td>1.2</td>
<td>3.0</td>
<td>0.5</td>
</tr>
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</table>

simulation, the firms use their raw materials and finished goods inventory and do not immediately purchase from alternate suppliers. Consequently, each supplier reassesses its value of $\theta$ and delays moving production to alternate facilities to later periods.

Tables 5 and 8 depict the average results from 10,000 simulations with this fixed cost of using alternate production facilities. Supplier 2 benefits the least from moving production because its primary facility has the largest probability of reopening, but demand met by Supplier 3 increases from 27% to 66% with the alternate facility. With supply shortages occurring less frequently, the firms are able to satisfy almost all of their customers’ demand by relying on inventory and purchasing from alternate suppliers until the primary suppliers resume production.

Firms and suppliers can use simulation results such as those presented in these three scenarios to analyze and quantify the marginal benefit of different disruption management strategies. Comparing demand satisfied by firms in the first two examples provides insight into the impact of firms sacrificing profit in order to satisfy customer demand. A firm could use this type of comparison to measure how partially operating at a loss in the short term can help it retain more customers during the disruption. Comparing the second and third examples shows the additional benefit if suppliers move production to alternate facilities. This model quantifies the benefit of these decisions in terms of both the increased demand satisfied by suppliers and the ability of firms to meet their own customer demand.

4.3. Supplier-firm interaction

The supply chain disruption model incorporates different interactions among the suppliers and firms, and each player’s decision can influence the decisions of the other players. Exploring an individual sample path of the simulation demonstrates this interaction. This sample path uses the same parameters as the third scenario (suppliers have alternate facilities and $\alpha > 0$ for the firms).

Figure 4(a) depicts each supplier’s decision in each period as the number of periods from the current period until it plans to move production to an alternate facility (i.e., parameter $k$ from Equation (4)). Figure 4(b) depicts the percent of required supplies that the firms purchase from alternate suppliers in each period. The suppliers begin with $\theta = 0.5$ in the first period, and each
supplier plans to move production in the next period. The firms use inventory in the initial periods and do not buy from alternate suppliers. Consequently, the suppliers reevaluate $\theta$ equal to 0 in the second period and decide to wait to move production through period 7. At the end of period 7, Supplier 2’s primary facility reopens, and Firm 1 only needs one alternate supplier. Firm 1 begins to purchase from an alternate supplier to Supplier 1 in period 8. Supplier 1 observes that it is losing market share to the alternate supplier (between 30 and 40% of its demand during periods 8 through 10) and updates $\theta$. In period 9, the supplier decides to move production two periods later, in period 11. Mathematically, Supplier 1 has an interior maximum defined by Proposition 1, and $k^* = 2$.

In period 10, Firm 2 begins to purchase about 10% of its supplies from alternate suppliers to Suppliers 1 and 3, and Supplier 3 reacts by deciding in period 11 to move production 5 periods later. After Supplier 1 produces at an alternate facility beginning in period 11, Firm 2 only needs one alternate supplier. Firm 2 produces 21 units in period 11, which means that Supplier 3 loses all of its demand in that period, as depicted in Figure 4(b). Consequently, Supplier 3 adjusts its plans and moves production to an alternate facility in period 12. Similarly, Firm 3 begins to purchase some supplies from an alternate supplier of Supplier 4 in period 11. Supplier 3, who until this point had not planned to move to an alternate facility, reacts by moving production in period 12. By period 12, Suppliers 1, 3, and 4 are operating at alternate facilities and delivering supplies even
though their primary facilities do not reopen until several periods later. Because the firms initially rely on inventory and do not purchase from alternate suppliers, none of the suppliers intend to move production from periods 3 through 7. After Supplier 2’s primary facility reopens (which decreases all of the firms’ production costs), and no inventory remains, the firms begin purchasing from alternate suppliers. This incentivizes the remaining three suppliers to move production within a few periods.

Perhaps more interesting than the specific results of this sample path is the manner in which suppliers and firms influence each other. First, suppliers take their cue from the firms. If firms are not purchasing from alternate suppliers, suppliers are not incentivized to move production to another facility. If a firm suddenly purchases all of its supplies from alternate suppliers, the primary supplier will quickly try to move production to another facility, if feasible. Second, firms may not purchase as much from alternate suppliers if several suppliers are not producing because it is too difficult or too costly to replace all of those primary suppliers. However, if only one or two suppliers are disabled, it may be easier for a firm to replace a couple of suppliers.

4.4. Non-constant probabilities

An important assumption of the model is that $p$—the probability that a supplier’s primary facility reopens in a period—remains constant. In reality, the probability that the facility reopens likely changes, and presumably, the facility would be more likely to reopen as time progresses. Incorporating non-constant probabilities into the model complicates a supplier’s profit function, and the three propositions describing the optimal decision for a supplier are no longer valid. The overarching model and simulation can still be used if a supplier knows the probability of reopening in each period. For example, $p$ as a function of the time period $k$ could increase at an exponential rate: $p(k) = 1 - (1 - p(0)) \exp(-\beta k)$ where $p(0)$ is the probability the facility reopens at the end of the current period and $\beta$ describes the rate at which the probability increases (Figure 5(a)).

We use Supplier 1 to demonstrate the effect of non-constant probabilities on a supplier’s initial decision to move production to an alternate facility. Supplier 1, which represents Renesas, supplies to all three firms. The primary facility’s probability of reopening increases according to the functional form given in the preceding paragraph. At the beginning of the disruption, Supplier 1 chooses the period in which it plans to move production based on maximizing its expected profit. The expected profit incorporates $p(k)$, which increases as $k$ increases. Figure 5(b) depicts Supplier 1’s initial decision for several values of $\beta$ ranging from 0 to 0.2 and for six different fixed costs, $C$. Setting $\beta = 0$ implies a constant probability, $p(k) = p(0) = 1/12$ as before.
Immediately

\( \beta = 0.2 \)

\( \beta = 0.1 \)

\( \beta = 0.05 \)

\( \beta = 0.02 \)

\( \beta = 0.05 \)

\( \beta = 0.2 \)

Figure 5  Effect of non-constant probabilities on Supplier 1’s decision

The supplier is less likely to move production to an alternate facility for larger values of \( \beta \). The probability that the primary facility reopens increases at such a quick rate that the supplier should wait for the facility to reopen even if \( C = 0 \). When \( C \geq 475 \) and \( \beta = 0.02 \) (which means that \( p(k) \) increases at a relatively slow rate), the supplier will plan to never move to an alternate facility. When \( C = 0 \), the supplier should wait one period before moving production if \( \beta > 0.03 \). This numerical exploration of non-constant probabilities suggests that the rate at which the probability changes can have a large impact on the supplier’s decision. The model is flexible enough to incorporate this additional complexity.

4.5. Sensitivity on parameters for suppliers

Sensitivity analysis on the key parameters for a supplier generates important insights about the model and provides details of how managers may want to prepare for a disruption. Supplier 1, which represents Renesas, is chosen because it supplies all three firms, and the insights derived from sensitivity analysis for this supplier can be applied to the other suppliers. The parameter \( p \) is varied for different fixed costs, \( C \) (Figure 6). As the fixed cost increases for a given value of \( p \), Supplier 1 meets less demand because larger fixed costs make moving production to an alternate facility less appealing. For example, if \( p = 0.1 \), \( \theta = 0.5 \), and \( Z = 0 \) (the initial values of \( \theta \) and \( Z \) for each supplier), the conditions of Proposition 1 are satisfied when \( 235 < C < 756 \). If \( 235 < C \leq 378 \), \( k^* < 1 \) as calculated by Equation (5), and moving production immediately generates a higher
expected profit than waiting a single period. Because $\bar{C} = 687$, the conditions of Proposition 2, but not Proposition 1, are satisfied if $C \leq 235$. Thus, Supplier 1 moves production immediately if $C \leq 378$, which explains why Supplier 1 satisfies 100% of its demand when $p = 0.1$ and $C = 32, 64, 128,$ or $256$ in Figure 6. When $C = 512$ and $p = 0.1$, the supplier waits at least one period to move production and only satisfies 69% of its demand on average.

If a supplier does not move production, increasing $p$ helps a supplier meet more demand because its primary facility reopens more quickly. This can be seen in Figure 6 when $C = 2048$, Supplier 1’s ability to meet demand always increases as $p$ increases. As can be seen from Equation (8), $\bar{C}$ decreases as $p$ increases, which makes it less likely that a supplier will move production. For example, if $C = 64$, Supplier 1 meets 100% of its demand if $p = 0.33$ but only meets 87% of its demand on average when $p = 0.4$. This leads to an interesting phenomenon, in which the larger probability that a facility reopens may lead to a more severe supply shortage because a supplier anticipates that its facility will reopen quickly. The supplier has less incentive to increase capacity at other facilities.

4.6. Risk management insights for firms

Sensitivity analysis for a firm can provide some indication of risk management strategies for the firm. Several parameters are varied for Firm 2 (the most impacted firm in the application), alternating among low, base, and high values for each parameter as given in Table 9. The base values for each parameter are identical to those given in Tables 3 and 4.

Five thousand simulations are run with each parameter moved to its low or high level and the other parameters remaining at their base levels. The tornado diagram in Figure 7 depicts the
Table 9  Sensitivity on parameters for Firm 2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Low</th>
<th>Base</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective trade-off</td>
<td>$\alpha$</td>
<td>0</td>
<td>0.0131</td>
<td>10</td>
</tr>
<tr>
<td>Finished goods inventory</td>
<td></td>
<td>0</td>
<td>126</td>
<td>252</td>
</tr>
<tr>
<td>Cost of alternate supplier</td>
<td>$w_1, w_2, w_3, w_4$</td>
<td>3, 3, 4, 4</td>
<td>6, 9, 12, 15</td>
<td>9, 15, 21, 27</td>
</tr>
<tr>
<td>Selling price</td>
<td>$\rho$</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Probability that Supplier 3’s primary facility reopens</td>
<td>$p$</td>
<td>1/36</td>
<td>1/26</td>
<td>1/3</td>
</tr>
<tr>
<td>Raw materials inventory</td>
<td></td>
<td>0</td>
<td>42</td>
<td>84</td>
</tr>
<tr>
<td>Probability that customers buy from competitors</td>
<td>$\tau$</td>
<td>0.01</td>
<td>0.39</td>
<td>0.99</td>
</tr>
</tbody>
</table>

average demand satisfied by Firm 2 at each value. Whether Firm 2 satisfies demand depends most on the value of $\alpha$, which determines the relative importance of meeting demand to maximizing profit. The firm can satisfy all of its demand if it is willing to sacrifice a lot of its profit. Assuming that a firm will always be able to find enough supplies if it is willing to pay a lot may not always be valid, and some supplies may be constrained in reality.

The results are also very sensitive to finished goods inventory because going from 6 weeks of inventory (the base case) to no inventory really hurts the firm’s ability to satisfy demand. On the other end of the spectrum, the loyalty of the firm’s customers has relatively little impact on the firm’s ability to satisfy demand. This is somewhat surprising, but the firm’s competitors (Firms 1 and 3 in the model) also suffer from a supply shortage and are not easily able to produce more to take customers away from Firm 2. If the model were altered to include a firm who is not impacted by the supply chain disruption and who could easily produce more, the impacted firm may lose more of its demand if customers are truly willing to buy from the firm’s competitors.

Sensitivity analysis usually reveals whether conclusions from a model are robust to changes in input parameters, but Figure 7 also provides managerial insights into potential risk management strategies for Firm 2, which may be the true benefit of sensitivity analysis for this application. Many of the parameter inputs are choices that a firm can make before or potentially during the disruption, and a firm could use this information to quantify the benefits of a particular action. Removing the 6 weeks’ worth of finished goods inventory reduces satisfied demand from 87% to 75%, but doubling the inventory from 6 to 12 weeks only increases satisfied demand to 93%. Eliminating raw materials inventory reduces satisfied demand to 83%, but increasing raw materials inventory from 2 to 4 weeks only increases satisfied demand to 91%. This decreasing marginal return in holding inventory occurs because inventory will be depleted during a long disruption. When supplies are difficult to acquire for a span of several months, having enough inventory to cover such a long disruption will be difficult without resorting to other risk management strategies.
Finding a low-cost alternate supplier can really help a firm manage the risk of the disruption. If an alternate supplier is available that is willing to sell its supplies at roughly the same price as the primary supplier, the firm can rely on that alternate supplier to satisfy its demand. Once a disruption occurs and supplies become scarce, it might become more difficult for the firm to find an alternate supplier at the same cost. A better strategy may be to source from multiple suppliers or arrange with another supplier before a disruption in order that the firm can purchase supplies from them if a disruption occurs. However, sourcing from multiple suppliers may increase the firm’s operational costs and the risk of a disruption may not outweigh the efficiency and cost benefits of sourcing from a single supplier. A discussion of risk management strategies should also account for the possibility that alternate suppliers will not be available during a disruption. Although not included in the model, developing close and deep relationships with suppliers can benefit a firm during a supply chain disruption, as exemplified by Toyota’s relationship with the valve manufacturer Aisin-Seiki (Sheffi 2005, Whitney et al. 2012).

Another strategy that could provide benefit for the firm is if it can increase the probability that its supplier’s primary facility reopens. If \( p \) for Supplier 3 increases from 1/26 to 1/3 (or, equivalently, if the expected number of periods that Supplier 3 is closed reduces from 26 to 3 periods), Firm 2 can satisfy 94% of its demand. Firm 2 could help Supplier 3 recover more quickly by working with the supplier to find alternate modes of production for the supplier or spending money to rebuild the supplier’s facility. The model does not specifically include this alternative in the firm’s decision making, but the feasibility, costs, and benefits of such a strategy can be examined in future iterations of the model.
This type of model and simulation and the resulting sensitivity analysis can provide insight into the potential benefit of different risk management strategies and allow a firm to compare among a set of strategies to determine what may be optimal. This analysis examines each input independently, however, and any correlation among parameters could alter the results of this sensitivity analysis. Additionally, this approach does not attempt to include the cost of different strategies. Before determining which strategy is the best, the cost of each strategy must also be incorporated into the decision making.

5. Conclusion

This paper has proposed a model of severe supply chain disruptions in which several suppliers’ facilities are suddenly closed. Insights from the model reveal that a supplier may not move production to an alternate facility if it expects the primary facility to reopen quickly, the cost of moving production or producing at the alternate facility is significant, and/or firms are not purchasing from alternate suppliers. Threshold parameters for these factors generate decision rules for the supplier. The firm’s bi-objective function enables the firm to trade off maximizing profit with satisfying current demand. This formulation seeks to mirror how a firm will make production decisions during a severe disruption in which calculating profit over the long term may not be realistic. The firm’s optimal production decision is based on the difference between the selling price and the cost of an alternate supplier as well as its desire to trade off between the two objectives.

Based on the supply disruption caused by the Japanese earthquake and tsunami, a simulation of this model includes four suppliers and three firms. The firms represent the Detroit 3 automakers, Toyota, Honda, and Nissan operating in North America. Three different cases of this simulation are presented, and the results of Case 2 where the suppliers have no alternate facility and the firms have some desire to meet current demand closely reflect automobile production in the summer of 2011. Toyota and Honda lost market share, but the Detroit 3 gained market share in the United States. Nissan suffered from the natural disaster but to a lesser extent than Toyota and Honda. Other important factors that contributed to this shift in market share but are not included in the model were Toyota’s problems with its braking system early in 2011 and the restructuring of the U.S. automakers.

The application serves as an example of how this model can be used to analyze and quantify the impacts of different disruption management strategies. The simulation measures the amount of demand satisfied by each firm and supplier, and a user can understand the effect of suppliers moving production to alternate facilities at different times during the disruption, firms holding
raw materials inventory or finished goods inventory, and firms purchasing from alternate suppliers. Changing the input parameters for a firm can represent different risk management strategies. The model can serve as a planning tool for a firm exploring different scenarios and disruptions in order to select the optimal mix of strategies.

An enhanced model of the disruption in the automobile industry would include several firms, maybe as many as 15 to account for the other automobile manufacturers, and at least two to three times as many suppliers as firms. Some suppliers to the automobile manufactures may also be purchasers from other suppliers. The model and simulation could incorporate this additional complexity if parameters could be estimated for the 50 to 70 entities that would be included in such a simulation. This additional complexity should not substantially increase the simulation’s runtime, however.

The supply chain disruption model is a multi-period repetitive decision framework in which firms and suppliers influence each other’s actions through probabilities and cost parameters. Alternatively, a game theoretic model in which entities follow Nash equilibrium strategies could provide other insights, and comparing the two modeling approaches may lead to a richer understanding of the behavior of suppliers and firms during severe supply chain disruptions. A firm may incentivize a supplier to move production to an alternate facility immediately by purchasing from alternate suppliers, regardless of the increased cost. Such post-disruption management strategies in which firms motivate suppliers to act more quickly could be an interesting component of a game theoretic model.

Further extensions of this model can include the broader economic impacts of supply shortages and industry mitigation strategies. Some research has already been pursued in this direction, including the demand-driven input-output model (see also MacKenzie et al. 2012b) and the Inventory Inoperability Input-Output Model (Barker and Santos 2010a,b).

The model in this paper provides new insights into managing supply chain disruptions. Understanding the optimal disruption management strategy can help suppliers and firms determine appropriate actions during a disruption. From a broader perspective, anticipating how businesses may react following a disruption can serve to quantify the business interruption losses from a natural disaster and supply chain disruption.

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