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Hot Spot Policing: A Study of Place-Based Strategies to Crime Prevention*

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Abstract

Hot spot policing is a popular policing strategy that addresses crime by assigning limited police resources to areas where crimes are more highly concentrated. We evaluate this strategy using a game theoretic approach. The main argument against focusing police resources on hot spots is that it would simply displace criminal activity from one area to another. We provide new insights on the nature of the displacement effect with useful implications for the econometric analysis of crime-reduction effects of police reallocation. We also propose alternative place-based policies that display attractive properties in terms of geographic spillovers of crime reduction via optimal police reallocation.

JEL codes: D7, K4, R1,

Keywords: Hot spot policing, place-based strategies, crimes, displacement effect, social interactions.

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1 Introduction

Crime mapping is a powerful tool used by analysts in law enforcement agencies to visualize and study crime patterns. Such maps indicate that crimes are often not evenly distributed across geographic locations. Instead, clusters of crimes occur in specific areas, or hot spots. Hot spot policing is a place-based strategy that attempts to reduce crime by assigning limited police resources to places where crimes are more highly concentrated. This approach to crime prevention is relatively new and many crime experts argue it is one of the main reasons why New York City has achieved a dramatic decrease in crimes during the past two decades.¹ We evaluate this policy via a game theoretic approach and propose alternative strategies that display interesting features in terms of geographic spillovers of crime reduction. Our results offer useful hints to guide further empirical research.

The model we develop to study the effectiveness of hot spot policing incorporates various crime theories that capture different aspects of crime decisions. These theories have been so far studied in isolation and, by combining them in a single model, we are able to make predictions that are more consistent with observed patterns of crimes. Specifically, our approach is based on the rational choice model and uses game theory to incorporate strategic interactions among potential offenders into the analysis. We also borrow from the theory of environmental criminology, which highlights the role of spatial factors in the choice of crime location. More formally, we propose a two-stage game. We first divide the region under study into a finite number of areas that differ in terms of attractiveness for potential offenders. For instance, if the overall region represents NY City, then an area may correspond to one of its neighborhoods. We capture crime attractiveness via two attributes, namely, risk of apprehension and potential productivity. The riskiness of a place for a potential offender can be thought of as an index function that captures structural factors affecting the successful apprehension of offenders in that location, such as the presence of illumination or video cameras. The second attribute, potential productivity, relates to the expected gains from committing a crime in that place, such as the presence of a shopping mall or a bank. In the first period of the two-stage game, the enforcement agency decides how to allocate the limited police resources across alternative areas. In the second period, upon observing police allocation, people decide

¹See, e.g., Zimring (2011).

whether to commit a crime and, in case of doing so, where to perform the criminal act.

Using the standard backward induction principle we solve the game by first modeling people's choices for a given police assignment. Given the strategic interactions in crime decisions, this stage of the game is itself a game among potential offenders. **The first part of our work sheds light on one of the most controversial issues associated with hot spot policing, namely, the displacement effect.** That is, the possibility that redirecting police resources to hot spots would simply displace criminal activity from one area to another.² Empirical research has found some evidence against this argument (see, e.g., Braga (2008)). The model we provide features this empirical observation when the value of the outside option (not to commit a crime) does not depend on the number of people who opt not to become a criminal. Under this circumstance, the value of the outside option regulates people's utilities, and increasing resources in an area simply discourages people in that area from committing a crime. Alternatively, when the outside option displays congestion effects, increasing police resources in a given area pushes some criminals from this area to the others. Congestion effects in the outside option might occur if, for instance, an increment in the number of people searching for a legal job pushes salaries down or increases unemployment, thereby making this outside option less attractive. The last result raises a **simultaneity issue in the empirical studies** that address the effect of police levels on crime reduction by using cross-sectional data. Specifically, the crime rate in each area depends not only on the police resources allocated to that specific location, but on the whole vector of police allocation.

After we characterize the decisions of potential offenders, we go back to the first stage of the game and contrast the behavior of an enforcement agency that aims to reduce the overall crime rate with the one of a public authority that wants to reduce crime differences across areas. We interpret the latter as an extreme implementation of hot spot policing. We find that the optimal police allocation does not necessarily induce an even distribution of crimes across areas. In other words, though areas that are *a priori* more attractive to offenders (i.e., display a larger productivity-to-risk ratio) receive indeed more police attention, the extra efforts in these areas do not fully offset the impact of their initial structural differences. Thus, in our model, some **hot spots remain under the optimal allocation strategy**. This result is

²Repetto (1976) offers an early discussion on different types of displacement effects in the criminal activity. Ellen, Lacoë, and Sharygin (2013) studies displacement of crimes due to foreclosures.

robust to all the extensions we consider for our initial model. In particular, it remains valid independently of the displacement effects. Regarding the opportunity cost of the egalitarian policy, we find that it increases with the variability of the productivity-to-risk ratio across locations.

We then study an alternative place-based strategy. Specifically, we analyze the implications of introducing structural changes that aim to reduce the productivity-to-risk ratio of a certain area. This policy has been suggested by a number of crime theorists, including Braga and Wisburd (2010), who state that:

"The attributes of a place are viewed as key factors in explaining clusters of criminal events... To reduce and better manage problems at crime hot spots, the police need to change the underlying conditions, situations, and dynamics that make them attractive to criminals and disorderly persons."

We find that this policy reduces crime not only in the target area but also in all other locations. Positive (or negative) external effects of structural changes have been proposed earlier (see, e.g., Ehrlich (1973) and Glaeser and Gottlieb (2008)). An interesting aspect of our result is the mechanism that produces this outcome. The direct effect of the policy is to make the target area less attractive for potential offenders, thereby reducing its criminal activity. The indirect effect is due to subsequent police reallocation from the improved area to the other ones, where the criminal activity diminishes as well. In other words, **structural improvements in an area generate geographic spillovers of crime reduction in all other locations via optimal police reallocation.**

We finally consider various extensions to our initial model. The outside option people face (not to commit a crime) can be interpreted as the possibility to get a legal job. We introduce alternative specifications of the outside option and study the effects of improvements in the job market on the proportion of people who opt not to commit a crime, e.g., the labor supply of the economy.³ We also investigate the consequences of reversing the interaction effects among potential offenders. We show that, when interactions are positive—as in Freeman, Grogger, and Sonstelie (1996) and Sah (1991)—interventions become a delicate matter. The reason is

³Our theoretical results are consistent with empirical findings. See, for instance, Imrohroglu, Merlo, and Rupert (2004).

that these models often display multiple equilibria and policy interventions can easily affect equilibrium selection (see Blume (2006)). We explore this possibility with a simple example.

Literature Review

Our research contributes to work in both criminal studies and economics. In an early study, Becker (1968) examines individual decisions to commit crimes from an economic perspective.⁴ His cost-benefit analysis is consistent with the rational choice approach used by Cornish and Clarke (1986), which we follow as well.⁵ Our study also relates to subsequent work on the importance of social interactions in motivating criminal behavior (see, e.g., Ballester, Calvo-Armengol, and Zenou (2003, 2006), Chen and Shapiro (2007), Freeman, Grogger, and Sonstelie (1996), Glaeser, Sacerdote, and Scheinkman (1996), and Sah (1991)). Following the latter, we assume that each individual decision depends on other people’s criminal choices. In order to incorporate spatial factors into the analysis, we model people’s expected payoffs as in Hugie and Dill (1994), who study habitat selection by modeling the behavior of predators and prey (see also Helsley and Zenou (2013)).

Our work is also related to Espejo, Huillier, and Weber (2011), who provide an evaluation of hot spot policing by using a leader and follower model, as we do in this investigation. However, our aim and approach differ from theirs. Specifically, we want to characterize crime displacement in a simple (and testable) way in order to provide new insights to guide further empirical research. In this study, we assume that people observe the police allocation and make subsequent choices regarding criminal decisions and locations. Laezar (2006) shows that it may be optimal, under precise circumstances, to keep police allocation secret. Though our model is quite different from his—for instance, we introduce social interactions—it is important to remark that our results remain unchanged if we allow for randomized police allocations. The reason is that the effect of police enters people’s utilities in a linear fashion. It would be interesting to study the advantages of secret police allocation in a model similar to the one of Laezar (2006) but with the additional feature of social interactions. We leave

⁴See also Ehrlich (1973).

⁵Durlauf, Navarro, and Rivers (2010) provide a general description of criminal choices at the individual level to understand the implicit assumptions in aggregate crime regressions. They highlight the relevance of modeling the microfoundations of the empirical analysis of crimes.

this analysis for future research.

Braga and Wisburd (2010) offer a deep analysis of place-based policies to crime fighting.⁶ In addition to new interesting insights about hot spots and crime prevention, they provide a thorough and updated overview of the theoretical and empirical research regarding this topic. Our theoretical modeling assumptions are inspired by all their discussions and the literature therein, and the implications we derive are consistent with the empirical findings they describe. On the applied side of the literature, Fu and Wolpin (2013) perform a structural estimation of the effects of police reallocation on crime reduction. Their model does not contemplate the possibility of displacement of criminal activity, which is one of the main aspects of our analysis. Our theoretical results provide some justification to their modeling assumptions.

The rest of the paper is organized as follows. Section 2 describes our model. Section 3 presents our main findings. Section 4 evaluates the decision of an enforcement agency that aims to reduce crime by changing the attributes of a certain area. Section 5 discusses three extensions of our model. Section 6 concludes. We collect all proofs in the Appendix.

2 The Model

2.1 Main Variables

This subsection describes the main variables of our model, making a clear distinction between the features that we assume are exogenous to the incumbents (i.e., people and the enforcement agency) and those that are under their control. Sections 4 and 5 examine some extensions to this initial model structure.

Exogenous Variables We let N and M represent the size of the mass of people and police, respectively. There are K alternative areas where criminal activity can take place. With only a slight abuse of notation, K represents the set as well as the number of locations. These areas differ with respect to three attributes, namely, size of the area, risk of apprehension, and productivity of the criminal activity.

S_k refers to the geographic size of Area k (e.g., in square feet). Riskiness R_k is a probability

⁶See also Braga (2008), Eck et al. (2005), Felson and Clarke (1998), Sherman (1995), and Weisburd and Green (1995).

measure of the successful apprehension of offenders in Area k . Differences in riskiness across areas capture different characteristics of the areas that reflect the level of police search activity or the ability of police to capture offenders. For example, better lighting may increase the risk of apprehension as offenders are more likely to be seen by someone who might call the police. Conversely, the presence of nearby highways may reduce this risk, as it becomes easier for criminals to escape. We use f to indicate the fee an offender must pay if apprehended. The fee (f) could capture, for instance, the opportunity cost of time spent in prison.

Productivity A_k captures the richness of the area in terms of expected benefits to criminals. For example, a rich area may be a neighborhood that is populated by high-income people whose houses contain high-value items. It may also be a location with stores or banks available as potential targets.

Endogenous Variables The incumbents in the model are the people and the law enforcement agency. Specifically, people decide whether to commit a crime and, if they do so, where to perform the criminal act. In our model, p_k represents the fraction of people in N who decide to commit a crime in location k . We indicate the density of offenders in that location by $d_k = p_k N / S_k$.

The enforcement agency decides how to assign police resources to different areas. We let q_k denote the fraction of police resources in M that is assigned to location k ; consequently $e_k = q_k M / S_k$ is the corresponding police density. We assume $M / S_k \leq 1$, for all $k \in K$, so that the per capita apprehension rate (defined below) lies between 0 and 1.

2.2 Payoffs of People

We model encounters between police and offenders as a random process, such that the overall rate of apprehension in location k is given by

$$\mathcal{A}(k, p_k, q_k) = d_k e_k R_k S_k.$$

Furthermore, the per capita apprehension rate of an offender in location k is represented by

$$\mathcal{P}(k, p_k, q_k) = \mathcal{A}(k, p_k, q_k) / d_k S_k = R_k e_k.$$

It follows from the last two expressions that the expected penalty for a person who commits a crime is

$$\mathcal{P}(k, p_k, e_k) f.$$

Thus, the cost-side of each individual's analysis depends on both his perceived probability of being apprehended and the penalty he would have to pay in that case.⁷ (In Sub-section 5.2, we allow for congestion effects in the cost-side of the crime decision. These congestion effects can be motivated by the fact that a police officer cannot be at two places at the same time; thus, as the criminal activity in a certain area increases the probability of being apprehended in that area shifts down.)

On the other hand, the offender's expected benefit of committing a crime is

$$\mathcal{Y}(k, p_k, q_k) = A_k/d_k.$$

It follows that the expected payoff of the criminal act increases with the productivity of the area; by contrast, it decreases with the number of offenders in the area, as the total potential productivity has to be shared among more people.

Thus, the overall expected utility of an offender in location k is given by

$$\mathcal{U}(k, p_k, q_k) = \mathcal{Y}(k, p_k, q_k) - \mathcal{P}(k, p_k, q_k) f = A_k/d_k - R_k e_k f.$$

Recall that our model allows people not to commit a crime. This outside option can be thought of as the possibility to work in a legal activity. Under this interpretation, the number of people who opt not to commit a crime comprises the labor supply in the economy. To simplify the exposition, we initially assume the expected payoff of this outside option is 0. We relax this restriction in Subsection 5.1 to evaluate the impact on crimes of public policies that affect the labor market in the economy. We refer to the outside option as $k = 0$, so that $\mathcal{U}(0) \equiv 0$ and the choice set of each person is $K_0 \equiv 0 \cup K$. The outcome of their decisions is a probability vector $\mathbf{p} \equiv (p_k)_{k \in K} \in \Delta^{K_0}$ where

$$\Delta^{K_0} \equiv \left\{ \mathbf{p} : p_k \geq 0 \text{ and } \sum_{k=0}^K p_k = 1 \right\}.$$

Thus, Np_0 represents the number of people who decide not to commit a crime.

⁷Durlauf and Nagin (2011) suggest that increasing the perceived risk of apprehension seems to have considerable deterrent effects on crimes.

2.3 Payoffs of Police Allocation Strategies

The public authority decides how to assign the police to different locations. Specifically, it chooses $\mathbf{q} \equiv (q_k)_{k \in K} \in \Delta^K$ where

$$\Delta^K \equiv \left\{ \mathbf{q} : q_k \geq 0 \text{ and } \sum_{k=1}^K q_k = 1 \right\}.$$

If the purpose of the enforcement agency is to minimize the overall level of criminal activity, then its payoff function is represented by

$$\mathcal{V}(\mathbf{p}) = p_0 \geq 0.$$

In our subsequent analysis, we contrast the behavior of a public authority interested in reducing the overall crime rate to that of a public authority aiming to minimize criminality while it keeps an even distribution of crimes across areas. The latter could be interpreted as an extreme version of implementing hot spot policing.

2.4 Structure of the Game

We model interactions between incumbents by using a leader and follower game, with the public authority as the leader and potential offenders as the followers.

In this game, the public authority first decides how to assign police to different locations with the objective of reducing the overall crime rate. This problem can be specified as follows

$$\max_{\mathbf{q}} \{ \mathcal{V}(\mathbf{p}) : \mathbf{q} \in \Delta^K \}.$$

Upon observing the distribution of police, each person, taking as given the decisions of the others, decides whether to commit a crime and, in that case, where to perform the criminal act. Thus, the problem faced by each person is

$$\max_k \{ \mathcal{U}(k, p_k, q_k) : k \in K_0 \}.$$

In the next section, we solve the game using the standard backward induction principle.

3 Equilibrium Analysis

3.1 People's Choices

The second stage of the game is itself a game among potential offenders. We use Nash equilibrium as our solution concept. Given a strategy profile \mathbf{p} , we let $b(\mathbf{p})$ indicate the best-response correspondence of an arbitrary person, that is,

$$b(\mathbf{p}) = \{k' \in K_0 : k' \in \arg \max_k \mathcal{U}(k, p_k, q_k)\}.$$

It follows that $\mathbf{p}(\mathbf{q}) \in \Delta^{K_0}$ is a Nash equilibrium if, for each $k' \in K_0$, we obtain

$$p_{k'}(\mathbf{q}) > 0 \text{ if } k' \in b(\mathbf{p}) \text{ and } p_{k'}(\mathbf{q}) = 0 \text{ otherwise.}$$

Given an initial police assignment and some beliefs regarding crime location, all people face the same choice set and expected payoffs. Thus, any action that is selected with a strictly positive probability will be among the options with the highest expected value. Since people are indifferent across these possibilities, we can interpret $\mathbf{p}(\mathbf{q})$ as either an asymmetric equilibrium in pure strategies or a symmetric mixed strategy equilibrium (see Hugie and Dill (1994)).

To simplify notation, we define $\theta_k \equiv (S_k)^2 A_k/R_k M N f$, for all $k \in K$. We can now describe, for each police assignment \mathbf{q} , the distribution of criminal activity across areas.

Proposition 1 *Fix some $\mathbf{q} \in \Delta^K$. The proportion of the population that decides to commit a crime in location k , for each $k \in K$, is given by*

$$p_k(\mathbf{q}) = S_k A_k / N [u(\mathbf{q}) + R_k q_k M f / S_k],$$

with $u(\mathbf{q}) \geq 0$. Moreover, $p_0(\mathbf{q}) > 0$ if and only if $\sum_{k \in K} \theta_k / q_k < 1$, in which case $u(\mathbf{q}) = 0$. The unique equilibrium is globally evolutionary stable.

Remark $u(\mathbf{q})$ is the utility level obtained by each person at the second-stage equilibrium when the police assignment is \mathbf{q} . In addition, requiring $\sum_{k \in K} \theta_k / q_k < 1$ is the same as assuming $M > \sum_{k \in K} (S_k)^2 A_k / R_k N f q_k$. Thus, in our model, some people will opt not to commit a crime ($p_0(\mathbf{q}) > 0$) only if the mass of police is large enough. This scenario is consistent with observed crime behavior.

Proposition 1 shows that the criminal activity in a certain location increases with the perceived productivity of the area and decreases with both its apprehension risk and the number of police officers in place. This proposition allows us to determine the patterns of displacement of criminal activity across locations as the public authority changes the initial police allocation. We elaborate next on this description.

Displacement Let $Q \equiv \{\mathbf{q} \in \Delta^K : \sum_{k \in K} \theta_k/q_k < 1\}$ and assume this set is non-empty.⁸ For all $\mathbf{q} \in Q$ and all $k \in K$, we get $\mathcal{U}(k, p_k(\mathbf{q}), q_k) = 0$ and $p_k(\mathbf{q}) = \theta_k/q_k$. That is, when the mass of police is large enough, the outside option (not to commit a crime) regulates the second-stage equilibrium payoffs of potential offenders, and the level of criminal activity in each area depends only on the amount of police assigned to that specific area (rather than on the whole vector of police allocation). This means that, if q_k increases, then location k becomes less attractive to potential offenders and its criminal activity decreases. Increasing q_k in and of itself does not induce any initial displacement of criminality from Area k to the other areas. However, we do observe an increase in crime in other areas due to the removal of police from the latter. In other words, the displacement effect occurs in our model because, in order to increase the police force in Area k , the law enforcement agency has to reduce it in other areas, which then experience an increase in crime rates. This displacement mechanism changes when the outside option displays congestion effects. We evaluate this possibility in Subsection 5.2 and state an important implication of this alternative specification for the econometric analysis of the effect of police on criminal activity. ■

As mentioned before, the literature on criminality defines a hot spot as an area with above-average level of crime relative to the entire space. Assuming $p_0(\mathbf{q}) > 0$, we get from Proposition 1 that Area k is a so-called hot spot if and only if

$$A_k/R_k e_k > (1/K) \sum_{k \in K} A_k/R_k e_k.$$

Thus, in our model, whether Area k is a hot spot depends on both the productivity-to-risk ratio of the area and its density of police. Sherman (1995) writes

"Drake Place was a "hot spot" of crime. It was so hot that the police said they stayed away from it as much as possible, unless they got a call."

⁸It can be easily shown that Q is a convex set.

The structural characterization of hot spots we offer revises the causality of this expression.

The next section evaluates efficient and egalitarian police allocations.

3.2 Efficient and Egalitarian Police Assignments

The last subsection described the behavior of potential offenders given different police assignments. Using this result, we now characterize the optimal distribution of police ($\mathbf{q}^* \equiv (q_k^*)_{k \in K}$) for a public authority whose goal is to reduce the overall rate of criminal activity. We then compare the optimal policy with one that targets hot spots until they disappear.

Proposition 2 (Efficient Allocation) *Let $M > \underline{M}$. Then, for each $k \in K$,*

$$q_k^* = \sqrt{\theta_k} / \sum_{k \in K} \sqrt{\theta_k} \quad \text{and} \quad p_k^* = \sqrt{\theta_k} \sum_{k \in K} \sqrt{\theta_k}.$$

Remark $\underline{M} \equiv \left(\sum_{k \in K} S_k \sqrt{A_k/R_k} \right)^2 / Nf$ is the minimum mass of police such that $p_0^* > 0$. We assume $M > \underline{M}$ as, otherwise, the problem of the enforcement agency is trivial.

Proposition 2 indicates that the optimal amount of police in each area depends on both the productivity-to-risk ratio and the size of the area. This proposition also implies that, at equilibrium, the ratio of criminal densities of Area k over Area l is given by

$$d_k^* / d_l^* = \sqrt{A_k/R_k} / \sqrt{A_l/R_l}.$$

That is, though the public authority makes a greater effort in areas that are a priori more attractive to offenders, this extra effort is not enough to eliminate the effects of their initial attribute differences. Therefore, areas that are a priori more attractive remain so in an efficient police allocation. In other words, our model contains hot spots as an equilibrium outcome.

Hot Spots Let $M > \underline{M}$. Area k is a hot spot at the efficient equilibrium if and only if

$$\sqrt{A_k/R_k} > (1/K) \sum_{k \in K} \sqrt{A_k/R_k}.$$

We next examine the case in which the goal of the enforcement agency is to obtain an even distribution of criminal activity across all areas (i.e., $d_k^{**} = d_l^{**}$ for all $k, l \in K$). To this end, we assume that the available mass of police is large enough to induce some people to opt for the outside option at the egalitarian allocation. This result holds for all $M > \underline{M}' \equiv$

$\left(\sum_{k \in K} S_k \sqrt{A_k/R_k}\right)^2 / Nf$. Under this assumption, we obtain the following specification, for each $k \in K$,

$$q_k^{**} = (\theta_k/S_k) / \sum_{k \in K} (\theta_k/S_k).$$

Comparing this egalitarian policy with the efficient allocation strategy, we obtain

$$e_k^{**}/e_l^{**} = (A_k/R_k) / (A_l/R_l) > \sqrt{(A_k/R_k)} / \sqrt{(A_l/R_l)} = e_k^*/e_l^*,$$

whenever $(A_k/R_k) > (A_l/R_l)$. That is, the egalitarian approach targets areas that are *a priori* more attractive to offenders more intensively than does a public authority who aims to reduce overall crime levels. This leads to our next proposition.

Proposition 3 (Opportunity Cost of Egalitarian Policy) *Let $M > \underline{M}'$. The opportunity cost of equity in terms of overall crime levels is given by*

$$p_0^* - p_0^{**} = (1/MNf) \sum_{k < l} S_k S_l \left(\sqrt{A_k/R_k} - \sqrt{A_l/R_l} \right)^2.$$

Proposition 3 indicates that the opportunity cost (in terms of criminal activity) of implementing an extreme hot spot policing strategy increases with the variability of the productivity-to-risk ratio across locations.

The next example illustrates our results so far and anticipates the analysis in the next section.

Example 1: Let $K = \{1, 2\}$, $A_1 = 8$, $A_2 = 4$, $R_1 = 1$, $R_2 = 2$, $S_1 = S_2 = 1$, $f = 1$ and $NM > 20$. Thus, Area 1 is both more productive and less risky than Area 2. Furthermore,

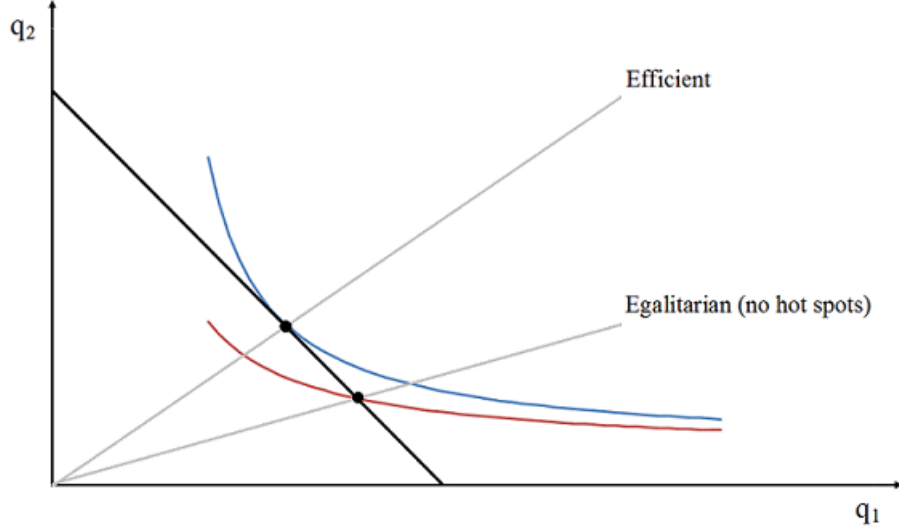
$$\mathcal{U}(1, p_1, q_1) = 8/Np_1 - Mq_1 \text{ and } \mathcal{U}(2, p_2, q_2) = 4/Np_2 - 2Mq_2.$$

Given that $MN > 20$, by Proposition 1, the efficient policy solves

$$\max_{q_1, q_2} \{1 - (\theta_1/q_1 + \theta_2/q_2) : 0 \leq q_1 \leq 1, 0 \leq q_2 \leq 1, q_1 + q_2 = 1\} \quad (1)$$

with $\theta_1 = 8/MN$ and $\theta_2 = 2/MN$. Figure 1 exhibits a graphical representation of this result. In this figure, the constraint set is denoted by the bold line. The two curves can be thought of as different indifference curves: each displays combinations of q_1 and q_2 that induce the same level of criminal activity and higher indifference curves are associated with lower crime levels.

Figure 1: Efficient versus Egalitarian Police Allocation Strategies



We define an efficient allocation as one that occurs whenever the marginal efficacy of police resources is the same across areas. Upon a simple calculation, we obtain that this holds whenever

$$q_2 = \sqrt{\theta_1/\theta_2}q_1.$$

Using the fact that $q_1^* + q_2^* = 1$, we get $q_1^* = 2/3$ and $q_2^* = 1/3$. This police assignment corresponds to the upper-left intersection in Figure 1. Under this allocation, $p_1^* = 12/NM > 6/NM = p_2^*$, meaning that Area 1 is a hot spot at equilibrium.

In contrast to the efficient allocation strategy, the egalitarian policy satisfies the following condition:

$$q_2 = (\theta_1/\theta_2)q_1.$$

Using the constraint, we obtain $q_1^{**} = 4/5$ and $q_2^{**} = 1/5$. This police assignment corresponds to the lower-right intersection in Figure 1. Note that these two intersections coincide if only if $\theta_1 = \theta_2$. Under the egalitarian allocation, $p_1^{**} = p_2^{**} = 10/NM$. Thus, by construction, there are no remaining hot spots. However, as Figure 1 shows, the egalitarian allocation is on a lower indifference curve. The opportunity cost of this policy in terms of crime level is $2/NM$.

Note that, if we increase either the penalty in case of being caught (f) or the amount of police (M), then both θ_1 and θ_2 decrease by the same percentage. Thus, while in these two scenarios the police allocation remains the same, the indifference curves get re-leveled with the

induced criminal activity curves shifting downward. Alternatively, if we reduce θ_1 by reducing productivity (A_1) and/or increasing riskiness (R_1), then the criminal activity shifts downward by $1/q_1^*$ (this follows by applying the envelope theorem to expression (1)). This change flattens the indifference curves, so that now both the efficient and the egalitarian allocations entail a lower q_1 and a higher q_2 . This means that structural changes in Area 1 have beneficial spillover effects on crime levels in Area 2 via subsequent police reallocation. The discussion in the next section elaborates on this argument. ■

4 Modifying the Attributes of the Areas

This section extends our model to consider an enforcement agency that aims to change the characteristics of places that give rise to criminal opportunities while sustaining an optimal police allocation. Specifically, we are interested in two questions: (i) What is the effect of changing the attributes of an area on the overall rate of criminal activity? and (ii) What is the impact on the criminal level of the areas not directly benefited by such a policy?

The question of how to best fight crime has received both academic and practical consideration. For example, Braga and Wisburd (2010) state that the aim of place-based policy strategies should go beyond hot spot policing. In their own words

"We should solve the conditions and situations that give rise to the criminal opportunities that sustain high-activity crime places."

Similarly, public authorities have instrumented a number of area changes to increase apprehension risk or decrease productivity potential. These measures include, for instance, improving the lighting in dark areas or inking store merchandise.

To illustrate such initiatives, notice that by Proposition 1 (assuming $M > \underline{M}$) the problem of the enforcement agency regarding the allocation of police resources is as follows

$$\max_{\mathbf{q}} \left\{ p_0 = 1 - \sum_{k \in K} \theta_k / q_k : \mathbf{q} \in \Delta^K \right\}. \quad (2)$$

Note that lowering the productivity-to-risk ratio in Area k is similar to decreasing θ_k . By applying the envelope theorem on (2), we then obtain

$$\partial p_0^* / \partial (-\theta_k) = 1/q_k^*.$$

Thus, slightly reducing θ_k increases the number of people who opt not to commit a crime by $1/q_k^*$. We next elaborate on the mechanism by which this change happens.

Specifically, there are two forces behind the last result that reinforce each other. First, the target area becomes less attractive to potential offenders and, therefore, its criminal activity naturally diminishes. Second, by using Proposition 2 we get that, for each $l \neq k$,

$$\partial q_l^* / \partial(-\theta_k) = (1/2)(q_l^*)^2 / \sqrt{\theta_l \theta_k} > 0 \quad \text{and} \quad \partial p_l^* / \partial(-\theta_k) = -(1/2)p_l^* / p_k^* < 0.$$

This means that the police force is optimally reallocated from Area k to the others, thereby reducing the criminal activity in these areas as well. **We conclude by saying that structural changes in a certain area have beneficial spillover effects on all other locations via subsequent optimal police reallocations.**

5 Extensions of the Model

In this section, we consider three natural extensions of our initial framework. While the first two modify the way in which we model the outside option, the last one changes the nature of the interaction effects among potential offenders.

5.1 Outside Option and the Labor Market

In our model, the expected payoff of the outside option (not to commit a crime) is assumed to be 0. This restriction simplifies our exposition without changing the two main implications, namely, the nature of the displacement of criminal activity and the characterization of hot spots. Nevertheless, it impacts both the effectiveness of the public authority in reducing the overall crime rate and the optimal police allocation. To formalize this effect, we extend Proposition 2 with $\mathcal{U}(0) \equiv c$, so that c measures the opportunity cost of committing a crime. This specification leads to the next result.

Proposition 4 *Let $M > \underline{M} / (1 + c \sum_{k \in K} \gamma_k)$ with $\gamma_k = S_k / R_k M f$. Then, for each $k \in K$,*

$$q_k^{***} = q_k^* (1 + c \sum_{k \in K} \gamma_k) - c \gamma_k \quad \text{and} \quad p_k^{***} = p_k^* / (1 + c \sum_{k \in K} \gamma_k)$$

where \underline{M} , q_k^* and p_k^* are defined as in Proposition 2.

Proposition 4 states that a higher opportunity cost of committing a crime facilitates the condition under which $p_0(\mathbf{q}) > 0$ and reduces the level of criminality in all locations.

As we mentioned above, the outside option could be thought of as the possibility to work in a legal activity. Under this interpretation, the number of people who opt not to commit a crime comprises the labor supply in the economy. Thus, an increase in c could correspond, for instance, to a decrease in the unemployment rate or an increase in the minimum wage. Consistent with the empirical evidence, we find that the criminal activity decreases when labor market conditions improve. Alternatively, improvements in the attributes of areas would also induce more potential offenders to choose the outside option, therefore increasing the labor supply of the economy.

5.2 Congestion Effects in the Outside Option

Our previous analysis rules out the possibility of congestion effects in the outside option. However, we can imagine a simple mechanism by which the opposite is true. For instance, when the number of people searching for a legal job increases, salaries may be pushed down or unemployment increased, thereby making this outside option less attractive. This possibility reduces the effectiveness of the police force to fight crimes.

In this section, we incorporate congestion effects in the outside option by assuming $\mathcal{U}(0, p_0) \equiv A_0/d_0$. Under this specification, the model does not have a closed form solution neither for people's choices conditional on police assignments nor for the optimal police allocation. Nevertheless, it still delivers relevant information regarding both the displacement mechanism and the characterization of hot spots at the optimal allocation of police resources. We start by describing the implication of congestion effects on the displacement mechanism.

Proposition 5 *Let $\mathcal{U}(0, p_0) \equiv A_0/d_0$ and $Q \equiv \{\mathbf{q} \in \Delta^K : q_k > 0, k \in K\}$. For all $\mathbf{q} \in Q$, all $k \in K$ and all $m \in K$ with $m \neq k$, we get $\partial p_k(\mathbf{q})/\partial q_k \leq 0$ and $\partial p_m(\mathbf{q})/\partial q_k \geq 0$.*

Proposition 5 states that increasing police resources in Area k reduces its criminal activity, but it also increments the criminal level in all other locations. The reason is as follows: When q_k increases, location k becomes less attractive to potential offenders, and this pushes some criminals to the outside option. When the value of this outside option is independent of the number of people who decide not to commit a crime, there are no further consequences.

However, when the outside option displays congestion effects, the value of not to commit a crime decreases, incentivizing people to commit crimes in other locations. This has the undesired effect of shifting $p_m(\mathbf{q})$ up in all other areas. We next describe an econometric challenge raised by Proposition 5.

Estimates of Crime-Reducing Effect of Police Academics have long studied the relationship between the scale of policing and the level of criminal activity by using panel data. The first few studies on this issue did not find evidence of a strong causal effect of police on crimes. As Levitt and Miles (2007) explain, one of the reasons behind such disappointing result is that early studies did not take into account an endogeneity bias. Namely, jurisdictions with higher crime rates react by hiring more police, and this response induces a positive cross-sectional correlation between police and crimes. Marvell and Moody (1996) and Levitt (1997) address this difficulty by using an approach based on Granger causality, and Lazzati (2013) proposes a partial identification approach that relies on the use of police resources as monotone instrumental variables.⁹

Proposition 5 poses a new identification challenge. Under congestion effects in the outside option, the crime rate in each area depends not only on the police resources assigned to that location but also on the whole vector of police allocation. **That is, if congestion effects prevail, then any study that uses cross-sectional data to evaluate the effect of police on crimes should implement a simultaneous equations approach.** ■

The next result shows that some hot spots still remain at the optimal police allocation in the presence of congestion effects in the outside option. It also states that whether an area is a hot spot depends on its productivity-to-risk ratio in the same way as when $\mathcal{U}(0) \equiv 0$.

Proposition 6 *Let $\mathcal{U}(0, p_0) \equiv A_0/d_0$. Area k is a hot spot at the efficient equilibrium if and only if*

$$\sqrt{A_k/R_k} > (1/K) \sum_{k \in K} \sqrt{A_k/R_k}.$$

This proposition corroborates that equilibrium hot spots are a robust feature of our model.

⁹See also MacCray and Chalfin (2012).

5.3 Complementarities in Criminal Activity

In the previous analysis, the game induced in the second stage displays negative interaction effects among potential offenders. We next evaluate the consequences of an alternative specification.

The overall expected utility of an offender in location k is given by

$$\mathcal{U}(k, p_k, q_k) = \mathcal{Y}(k, p_k, q_k) - \mathcal{P}(k, p_k, q_k) f.$$

Differentiating this expression with respect to p_k , we obtain

$$\partial \mathcal{U}(k, p_k, q_k) / \partial p_k = \partial \mathcal{Y}(k, p_k, q_k) / \partial p_k - (\partial \mathcal{P}(k, p_k, q_k) / \partial p_k) f. \quad (3)$$

We expect both derivatives on the right-hand-side of (3) to be (weakly) negative. Specifically, congestion effects in the rewards are expected as the higher the number of criminals in an area, the lower the piece of the pie for each offender. Congestion effects in costs are also expected since one police officer cannot be at two different places at the same time. Therefore, the higher the number of criminals in a given area, the lower the probability that any one of them is apprehended (Freeman, Grogger, and Sonstelie (1996) and Sah (1991)). The sign of the total effect will, thereby, depend on the relative size of these two forces. That is, for each $k \in K$,

$$\partial \mathcal{U}(k, p_k, q_k) / \partial p_k \geq (\leq) 0 \text{ if } |\partial \mathcal{Y}(k, p_k, q_k) / \partial p_k| \leq (\geq) |\partial \mathcal{P}(k, p_k, q_k) / \partial p_k| f.$$

In our previous analysis, the second term dominates the first one, thereby inducing a congestion game among potential offenders. When the opposite holds, the second-stage game is a game of strategic complements. Such games often display multiple equilibria and involve coordination problems. In our case, all people may coordinate in the same option and police allocation choices can easily affect the selected one. The possibility that policy interventions may affect equilibrium selection is well-described by Blume (2006) for a discrimination model. The next example applies this phenomenon to our model of crimes.

Example 2: Let $K = \{1, 2\}$, $N = 1$, and $M = 1$. We further assume that $\mathcal{U}(0) \equiv 0$. In addition,

$$\mathcal{U}(1, p_1, q_1) = 1/p_1 - (q_1 + 1/2) / (p_1)^2 \text{ and } \mathcal{U}(2, p_2, q_2) = 1/p_2 - 4(q_2 + 1/2) / (p_2)^2.$$

In this example, Area 2 shows greater apprehension risk than Area 1 and the second stage of the game displays strategic complementarities.

Specifically, when $q_1 = q_2 = 1/2$, the second-stage game has two Nash equilibria: $\mathbf{p}(1/2, 1/2) \in \{(1, 0, 0), (0, 1, 0)\}$. While these two equilibria imply the same level of utility for people (zero), the last one is much riskier for potential offenders. The reason is that the first equilibrium guarantees each person a payoff of zero independently of what other people choose. Alternatively, the second equilibrium gives each offender a payoff of zero if and only if all other people follow the equilibrium strategy and select to commit a crime in Area 1. Otherwise, the payoff is negative. Thus, choosing not to commit a crime is a dominant strategy and it is, therefore, reasonable to predict that everyone will choose this option.

Let us now assume the public authority assigns all police force to the riskier area, so that $q_1 = 0$ and $q_2 = 1$. Though the equilibrium set does not change, the two predictions differ regarding expected payoffs. While the payoff of coordinating not to commit a crime is zero, the payoff of coordinating to commit a crime in Area 1 is $1/2$ for each offender. Thus, it may now be reasonable to predict that people will coordinate in the second equilibrium. ■

Whether crime decisions are substitutes or complements is ultimately an empirical question with relevant policy implications. De Paula and Tang (2012) and Aradillas-Lopez and Gandhi (2012) provide theoretical results on identification of signs of interaction effects in games. Their work could be very useful in addressing whether crime decisions are complements or substitutes. The answer to this fundamental question would provide relevant insights for developing further theoretical and empirical research in the area of crimes. An alternative approach would consist in developing policies that are robust to the alternative possibilities.

6 Final Discussion

Crime rates fell sharply in the U.S. during the 1990's, including both violent and property crimes. In NY City the fall was so strong that the media often refers to this phenomenon as the New York "miracle." This drop in crimes generated deep debates among crime experts and hot spot policing appears as one of the most cited explanations.¹⁰ The main argument

¹⁰Levitt (2004) evaluates frequently cited reasons for the crime decline in articles in major newspapers over the 1990's. He presents a list of six factors, which has innovative policing strategies at the top and increased

against focusing police resources on hot spots is that it would simply displace criminal activity from one area to another. We evaluate hot spot policing via a game theoretic approach with a special emphasis on the displacement mechanism. Our characterization of the displacement mechanism offers new insights for the empirical analysis of the deterrent effect of police on crimes. We find that, while areas that are initially more attractive for potential offenders should indeed receive more police resources, some hot spots still remain at the optimal allocation. Thus, further hot spot policing strategies should be carefully studied in terms of ultimate objectives. We finally study alternative place-based policies that display attractive properties in terms of geographic spillovers of crime reduction. The mechanism by which the spillovers take place is particularly interesting: By making a target area less attractive for potential offenders, the public authority directly reduces its criminal activity. The spillover effect is due to the subsequent optimal police reallocation from the improved area to the other ones, where the criminal activity diminishes as well.

number of police as the least cited factor among the six. While he finds innovative policing strategies do not appear to have played an important role in the drop in crime, he suggests increased number of police may have been an important determinant.

7 Appendix: Proofs

Proof of Proposition 1 By Sandholm (2001), $\mathbf{p}(\mathbf{q})$ is a Nash equilibrium in the second stage of the game if it satisfies the Kuhn-Tucker conditions for the Lagrangian

$$\mathcal{L}(\mathbf{p}, \mathbf{q}) = \int_0^{p_0} \mathcal{U}(0) dt + \sum_{k \in K} \int_{\varepsilon}^{p_k} \mathcal{U}(k, t, q_k) dt + \sum_{k \in K_0} \varphi_k p_k + \lambda (1 - \sum_{k \in K_0} p_k).$$

That is, if $(\mathbf{p}(\mathbf{q}), \boldsymbol{\varphi}(\mathbf{q}), \lambda(\mathbf{q}))$ satisfies the following conditions:

$$A_k S_k / N p_k(\mathbf{q}) - R_k M f q_k / S_k = -\varphi_k(\mathbf{q}) + \lambda(\mathbf{q}), \text{ for all } k \in K,$$

$$0 = -\varphi_0(\mathbf{q}) + \lambda(\mathbf{q})$$

$$\varphi_k(\mathbf{q}) \geq 0, p_k(\mathbf{q}) \geq 0 \text{ and } \varphi_k(\mathbf{q}) p_k(\mathbf{q}) = 0, \text{ for all } k \in K_0,$$

$$\lambda(\mathbf{q}) \geq 0 \text{ and } (1 - \sum_{k \in K_0} p_k(\mathbf{q})) = 0.$$

It is readily verified that the non-negativity constraints are non-binding, i.e., $\varphi_k^* = 0$, for all $k \in K$. Thus, the previous conditions reduce to

$$A_k S_k / N p_k(\mathbf{q}) - R_k M f q_k / S_k = \lambda(\mathbf{q}) \text{ for all } k \in K$$

$$\varphi_0(\mathbf{q}) = \lambda(\mathbf{q})$$

$$\varphi_0(\mathbf{q}) \geq 0, p_0(\mathbf{q}) \geq 0 \text{ and } \varphi_0(\mathbf{q}) p_0(\mathbf{q}) = 0$$

$$p_k(\mathbf{q}) \geq 0 \text{ for all } k \in K$$

$$\lambda(\mathbf{q}) \geq 0 \text{ and } \sum_{k \in K_0} p_k(\mathbf{q}) = 1.$$

As a consequence, we need to consider only two cases, namely, $\varphi_0(\mathbf{q}) > 0$ and $\varphi_0(\mathbf{q}) = 0$.

We first suppose $\varphi_0(\mathbf{q}) > 0$. Then $p_0(\mathbf{q}) = 0$ and $p_k(\mathbf{q}) = S_k A_k / N (\lambda(\mathbf{q}) + R_k q_k M f / S_k)$, for all $k \in K$. However, this is possible if and only if there exists $\lambda(\mathbf{q}) > 0$, such that $\sum_{k \in K} p_k(\mathbf{q}) = 1$. Note that $\sum_{k \in K} p_k(\mathbf{q})$ is decreasing in $\lambda(\mathbf{q})$ and $\sum_{k \in K} p_k(\mathbf{q}) \rightarrow 0$ as $\lambda(\mathbf{q}) \rightarrow \infty$. Thus, by the intermediate value theorem, this condition holds if and only if $\sum_{k \in K} \theta_k / q_k \geq 1$.

We now suppose $\varphi_0(\mathbf{q}) = 0$. This yields the following equation:

$$p_k(\mathbf{q}) = (S_k)^2 A_k / N R_k q_k M f = \theta_k / q_k,$$

for all $k \in K$. Since $p_0(\mathbf{q}) \geq 0$, then $\sum_{k \in K} \theta_k / q_k \leq 1$ with strict inequality if $p_0(\mathbf{q}) > 0$.

Uniqueness follows as the potential function is strictly concave in \mathbf{p} on Δ^{K_0} . Sandholm (2010) shows that global evolutionary stability follows by the same condition. ■

Proof of Proposition 2 For M large enough, any efficient police allocation satisfies the following condition:

$$\mathcal{U}(k, p_k(\mathbf{q}), q_k) = 0.$$

Thus, for all $k \in K$,

$$p_k(\mathbf{q}) = \theta_k/q_k.$$

The problem of the public authority can then be posed as

$$\max_{\mathbf{q}} \left\{ 1 - \sum_{k \in K} \theta_k/q_k : \mathbf{q} \in \Delta^K \right\}. \quad (4)$$

In equation (4), the objective function is differentiable and strictly concave for all \mathbf{q} in the interior of Δ^K . Thus, the solution to (4) exists and is unique. Moreover, \mathbf{q}^* is an equilibrium if it satisfies the Kuhn-Tucker conditions for the Lagrangian:

$$\mathcal{L}(\mathbf{q}) = 1 - \sum_{k \in K} \theta_k/q_k - \sum_{k \in K} \varphi_k q_k - \lambda (\sum_{k \in K} q_k - 1).$$

In this case, $(\mathbf{q}^*, \varphi^*, \lambda^*)$ satisfies the following conditions:

$$\begin{aligned} \theta_k/(q_k^*)^2 - \varphi_k^* &= \lambda^*, \text{ for all } k \in K, \\ \varphi_k^* &\geq 0, q_k^* \geq 0 \text{ and } \varphi_k^* q_k^* = 0, \text{ for all } k \in K, \\ \lambda^* &\geq 0 \text{ and } (1 - \sum_{k \in K} q_k^*) = 0. \end{aligned}$$

It is readily verified that the non-negativity constraints are non-binding, i.e., $\varphi_k^* = 0$ for all $k \in K$. The characterization of \mathbf{q}^* follows through a simple calculation.

To find \underline{M} , note that $\sum_{k \in K} \theta_k/q_k^* < 1$ needs to hold for $p_0^* > 0$ to be true. That is,

$$\left(\sum_{k \in K} S_k \sqrt{A_k/R_k} \right)^2 / N M f < 1.$$

It follows that $\underline{M} = \left(\sum_{k \in K} S_k \sqrt{A_k/R_k} \right)^2 / N f$. ■

Proof of Proposition 3 From the previous analysis we know that

$$p_0^* = 1 - \left(\sum_{k \in K} \sqrt{\theta_k} \right)^2 \quad \text{and} \quad p_0^{**} = 1 - \sum_{k \in K} S_k \sum_{k \in K} \theta_k/S_k.$$

Then,

$$p_0^{**} - p_0^* = \left(\sum_{k \in K} \sqrt{\theta_k} \right)^2 - \sum_{k \in K} S_k \sum_{k \in K} \theta_k / S_k.$$

By applying the Multinomial Theorem to the first term in the right hand side and expanding the second term, the last expression takes the form of

$$\begin{aligned} p_0^{**} - p_0^* &= \sum_{k \in K} \theta_k + \sum_{k, l \in K, k \neq l} \sqrt{\theta_k} \sqrt{\theta_l} - \sum_{k \in K} \theta_k - \sum_{k, l \in K, k \neq l} (S_l / S_k) \theta_k \\ &= \sum_{k, l \in K, k \neq l} \sqrt{\theta_k} \sqrt{\theta_l} - \sum_{k, l \in K, k \neq l} (S_l / S_k) \theta_k. \end{aligned}$$

Since $\theta_k \equiv (S_k)^2 A_k / R_k M N f$, then

$$\begin{aligned} p_0^{**} - p_0^* &= (1/MNf) \left[\sum_{k, l \in K, k \neq l} S_k S_l \sqrt{A_k / R_k} \sqrt{A_l / R_l} - \sum_{k, l \in K, k \neq l} S_k S_l (A_k / R_k) \right] \\ &= (1/MNf) \sum_{k, l \in K, k < l} S_k S_l \left(\sqrt{A_k / R_k} - \sqrt{A_l / R_l} \right)^2 \end{aligned}$$

which completes the proof. ■

Proof of Proposition 4 The proof of this result is very similar to the proofs of Propositions 1 and 2, thus we omit it. ■

Proof of Proposition 5 Under this specification, for all $\mathbf{q} \in Q$, people will re-distribute across options till the utility obtained in each of them is the same. Therefore, $p_0 = S_0 A_0 / N u(\mathbf{q})$ and, for each $k \in K$, we have

$$p_k(\mathbf{q}) = S_k A_k / N [u(\mathbf{q}) + R_k q_k f M / S_k] \quad (5)$$

where $u(\mathbf{q})$ is the constant that solves $\sum_{k \in K_0} p_k(\mathbf{q}) = 1$. Differentiating (5) $p_k(\mathbf{q})$ and $p_m(\mathbf{q})$ with respect to q_k we get

$$\begin{aligned} \partial p_k(\mathbf{q}) / \partial q_k &= - \left\{ S_k A_k / N [u(\mathbf{q}) + R_k q_k f M / S_k]^2 \right\} [\partial u(\mathbf{q}) / \partial q_k + R_k f M / S_k] \\ \partial p_m(\mathbf{q}) / \partial q_k &= - \left\{ S_k A_k / N [u(\mathbf{q}) + R_k q_k f M / S_k]^2 \right\} \partial u(\mathbf{q}) / \partial q_k. \end{aligned}$$

By the Implicit Function Theorem applied to $\sum_{k \in K_0} p_k(\mathbf{q}) = 1$ we get

$$\partial u(\mathbf{q}) / \partial q_k = - \frac{\left\{ S_k A_k / N [u(\mathbf{q}) + R_k q_k f M / S_k]^2 \right\} R_k f M / S_k}{\sum_{k \in K} \left\{ S_k A_k / N [u(\mathbf{q}) + R_k q_k f M / S_k]^2 \right\} + S_0 A_0 / N u(\mathbf{q})^2} \leq 0.$$

Substituting the last expression in the previous two, we obtain that $\partial p_k(\mathbf{q}) / \partial q_k \leq 0$ and $\partial p_m(\mathbf{q}) / \partial q_k \geq 0$. ■

Proof of Proposition 6 At the second stage equilibrium, $p_0 = S_0 A_0 / Nu(\mathbf{q})$. Thus, maximizing p_0 is the same as selecting the vector \mathbf{q} that minimizes $u(\mathbf{q})$. It follows that any optimal \mathbf{q}^* must satisfy, for all $k, m \in K$,

$$\partial u(\mathbf{q}) / \partial q_k = \partial u(\mathbf{q}) / \partial q_m.$$

Using intermediate results from the proof of Proposition 5 we get that, for all $k, m \in K$,

$$\sqrt{A_k R_k} / [u(\mathbf{q}) + R_k q_k f M / S_k] = \sqrt{A_m R_m} / [u(\mathbf{q}) + R_m q_m f M / S_m] = H.$$

Thus, for each $k \in K$, we have

$$N p_k(\mathbf{q}) / S_k = d_k = \sqrt{A_m / R_m} H \tag{6}$$

and the result follows immediately. ■

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