1997

On the Analytic Disassembly of Structural Matrices

Gordis, Joshua H.

On the Analytic Disassembly of Structural Matrices

Joshua H. Gordis
Naval Postgraduate School
Dept. of Mechanical Engineering Code ME/Go
Monterey, CA 9394315100
Voice: (408) 656.2866 FAX: (408) 656.2238
Email: gordis@me.nps.navy.mil

ABSTRACT The analytical dissasembly of global stiffness and mass matrices is examined. The conditions necessary for the unique disassembly of structural matrices are identified. The unique disassembly of global stiffness matrices is shown to be posssible for restricted structure topologies. The disassembly takes the form of a transformation which renders the global matrix block-diagonal, where each block represents an exact reduced element matrix. It is shown that it is not possible to uniquely dissassemble a global mass matrix. Graph-theoretic concepts are used to interpret the disassembly transformation matrices. The disassembly of structural matrices has applications in system identification and damage detection.

NOMENCLATURE

{f}	Force vector
[k],[K]	Element, global stiffness matrix
[m],[M]	Element, global mass matrix/Mapping matrix
[T]	Assembly transformation matrix
{x}	Response vector
e	Element quantity
Λ	Spectral matrix
Φ	Modal matrix
R	Equilibrium matrix
$\Re$	Range
u	Uncoupled
W	Work, strain energy

I. INTRODUCTION

The computer analysis of linear, self-adjoint structural systems generates square, symmetric matrices which represent the discretized distribution of mass, stiffness, and damping. The formation of these matrices is additive; that is, individual element matrices of mass, stiffness, and damping are added together using the appropriate connectivity to form the global matrices. In this paper, the disassembly of these global matrices is considered. The ability to analytically disassemble a global matrix would allow, for example, errors in calculated global static and/or dynamic response quantities to be related to individual element errors. Similarly, these errors could be interpreted as damage, and the location of the damage determined by relating the errors to particular elements.

The approach taken here involves the construction of a disassembly transformation, a transformation whose origin is found in frequency domain structural synthesis [1,2,3]. The transformation is constructed from knowledge of the various individual element comprising the structure. The ability to disassemble the structure is shown to be dependent on the algebraic independence of load paths, which will be discussed in a graph theoretic sense.

2 ASSEMBLY OF STRUCTURAL MATRICES: TRADITIONAL APPROACHES

Two common methods for assembling global matrices will be briefly reviewed.

Method I: Denoting the i'th element stiffness and mass matrices as k_i^e and m_i^e respectively, the assembly of "p" element mass and stiffness matrices into the global matrices of mass and stiffness can be denoted as

$$\mathbf{M} = \sum_{i=1}^{p} \overline{m}_i^e \qquad \mathbf{K} = \sum_{i=1}^{p} \overline{k}_i^e \qquad (1)$$

where the overbar indicates that the "p" individual element matrices are expanded to global structure sire and appropriately partitioned with zero entries according to the specified connectivity. There are various schemes for the computational implementation of this assembly process, such as the ID array [4].

Method 2: The second method for assembly involves the boolean transformation of uncoupled element matrices into the global matrix [5]. A block diagonal matrix of element matrices is assembled,

$$\mathbf{K}_u = \begin{bmatrix} [k_1^e] & [0] & \cdots & [0] \\ [0] & \ddots & & \vdots \\ \vdots & & \ddots & [0] \\ [0] & \cdots & [0] & [k_p^e] \end{bmatrix}$$

$$\mathbf{M}_u = \begin{bmatrix} [m_1^e] & [0] & \cdots & [0] \\ [0] & \ddots & & \vdots \\ \vdots & & \ddots & [0] \\ [0] & \cdots & [0] & [m_p^e] \end{bmatrix} \qquad (2)$$

and a non-square boolean matrix **T** relating the set of uncoupled element coordinates $\mathbf{x}_u$ to the set of coupled element coordinates $\mathbf{x}_c$ is constructed which reflects the element connectivity,

$$x_u = \mathbf{T}\mathbf{x}_c \tag{3}$$

and the global matrices are assembled by transformation,

$$K_c = T^T K_u T \qquad M_c = T^T M_u T \tag{4}$$

3. ASSEMBLY OF ELEMENT REDUCED MATRICES

We now describe an assembly procedure based on an alternative coordinate system, referred to as a reduced coordinate system. The purpose of describing this assembly procedure is to provide results needed in the analysis of the disassembly process. We will exclusively refer to the disassembly of stiffness matrices, due to the impossibility of disassembling mass matrices. to be shown in what follows.

It is our goal to transform the global stiffness matrix into a block-diagonal uncoupled matrix, where each block contains a matrix representing all the elastic mode information for a specific element, one block for each element. In order to construct such a tranformation, we recognize first that in order to preserve the clastic mode content of each individual element, the transformation matrix which operates on the global stiffness must be constructed from vectors which span the elastic mode subspace for each of the individual elements in the assembly. That is, we will construct the disassembly transformation based on the fact that the connectivity of an element is intrinsically related to its rigid body modes. Considering a specific element in an assembly, the surrounding elements of this specific element provide boundary conditions which suppress the element's rigid body modes in the response of the total structural assembly. In other words, a specific element's rigid body modes are superfluous to its contribution to the elastic behavior of the assembly, which is the transmission of elastic response across the element. The disassembly of a global stiffness matrix is a process which purges rigid body mode information from each element, and since the element's rigid body mode information is superfluous, the resulting reduced element matrices are "exact." However, with respect to a global mass matrix, the element's rigid body modes contribute to global kinetic energy, and hence the disassembly of a global mass matrix is not possible, as will be shown.

This assembly procedure is similar to that described by Eqs. (Z-4) in that a block diagonal (uncoupled) element matrix is transformed into the global matrix. However. as the transformation is constructed in an alternative coordinate system, it is necessary first to develop the form of the element stiffness matrix, referred to as the reduced matrix, consistent with the reduced coordinate system. This is one coordinate system in which it is possible, for restricted structural topologies, to disassemble global stiffness matrices. This reduced element matrix is not only purged of rigid body mode content, but is reduced in size as well. The feasibility of disassembly is. in part, related to the existence of an exact reduced element matrix, where "exact" means that the reduced element matrix contains all the elastic mode content of the clement. This exact matrix always exists.

We will first identify the reduced element stiffness matrix. This wil be followed by a description of the assembly process based on the reduced matrices. This process can be contrasted with the assembly process defined by Eqs. (4), and serves as the prerequisite for the description of the disassembly process which is the focus of this work.

3.1 Reduced Element Stiffness Matrices

We consider an arbitrary structural element, defined by a coordinate set $\{\mathbf{x}^e\}$ containing "n_e" physical displacements and rotations at the element nodes, and by the clement elastic and inertial relations,

$$\left\{f^e\right\} = \left[k^e\right]\left\{x^e\right\} \qquad \{f'\} = \left[m^e\right]\left\{\ddot{x}^e\right\} \tag{5a,b}$$

where $[\mathbf{k}^e]$ and [Id] are element stiffness and mass matrices of dimension n_e by n_e, and $\{\mathbf{f}^e\}$ is the vector of generalized forces consistent with the generalized nodal displacements $\{\mathbf{x}^e\}$ and accelerations $\{\ddot{\mathbf{x}}^e\}$. Note that all vectors are partitioned by nodal coordinates.

Considering first the statics of the element, the statement of equilibrium for the element can be written as

$$[R]\left\{f^e\right\} = \{0\} \tag{6}$$

where the matrix **[RI** is a matrix of dimension six by n_e and which contains the coefficients defined by the six equations of static equilibrium, and hence is of rank equal to six. At this point, we group the element nodes into two subsets, labelled "i" and "d" (independent and dependent), and partition the generalized forces in Eq. (6) according to this grouping

$$\begin{bmatrix} R_i & R_d \end{bmatrix}\begin{Bmatrix} f_i^e \\ f_d^e \end{Bmatrix} = 0 \tag{7}$$

Equation (7) can be r-arranged into the desired relation between the original physical coordinates of the element, and the new reduced coordinates,

$$\begin{Bmatrix} f_i^e \\ f_d^e \end{Bmatrix} = \begin{bmatrix} \mathbf{I} \\ -R_i^{-1}\mathbf{R}_d \end{bmatrix}\left\{f_i^e\right\} \quad \text{or} \quad \left\{f^e\right\} = [M]\left\{\tilde{f}^e\right\} \tag{8}$$

where the circumflex ($\tilde{\cdot}$) indicates a quantity associated with the new coordinate system.

The transformation matrix **[Ml** defined by Eq. (8), referred to as a "mapping matrix," will be used to transform the element matrices into the reduced matrices. The contragradient transformation for displacements is found by requiring that the strain energy of the element be preserved. The work W done on the element by external forces. written in the original coordinate system is

$$W = \frac{1}{2}\left\{f^e\right\}^T\left\{x^e\right\} = \frac{1}{2}\left\{\tilde{f}^e\right\}^T[M]^T\left\{x^e\right\}, \tag{9}$$

and in the reduced system, denoting the as yet unknown transformation of displacements as $[\mathbf{T}_d]$,

$$\tilde{W} = \frac{1}{2}\left\{\tilde{f}^e\right\}^T\left\{\tilde{x}^e\right\} = \frac{1}{2}\left\{\tilde{f}^e\right\}^T\left[T_d\right]^T\left\{x^e\right\}. \tag{10}$$

Requiring that the work he equal, regardless of the coordinate system in which it is calculated, yields the transformation of displacements, $\mathbf{T_d}=\mathbf{M^T}$. The complete transformation is

$$\left\{f^e\right\}=[M]\left\{\tilde{f}^e\right\} \quad \left\{f^e\right\}=[M]\left\{\tilde{f}^e\right\}. \qquad (11a,b)$$

The reduced stiffness relation for an element is found using the transformations, Eqs. (11), and is

$$\{f"\} = [M]\left\{\tilde{f}^e\right\}, \qquad (12)$$

where $(\bullet)^\dagger$ indicates the pseudo-inverse, and the reduced stiffness matrix is

$$[k'] = [M]^\dagger\left[k^e\right]\left([M]^T\right)^\dagger. \qquad (13)$$

The development of the coordinate transformations. Eqs. (11) was based on the invariance of the strain energy of the element with respect to coordinate transformation. The disassembly process is exact only if the disassembly transformation applied to the global matrices preserves all elastic mode content of the elements. We now show that the generalized inverse transformation for the reduced element stiffness, Eq. (13), does indeed preserve all the elastic modes. We consider the stiffness relation for a single element. Comhining Eqs. (5a) and (6) one finds,

$$[R]\left[k^e\right]=[0] \qquad (14)$$

Considering Eq. (R) we see also that

$$[R][M]=[0] \qquad (15)$$

The element stiffness matrix $\mathbf{k^e}$ has a spectral decomposition,

$$\left[k^e\right]=\left[\Phi_{rb} \quad \Phi_{el}\right][\Lambda]\left[\Phi_{rb} \quad \Phi_{el}\right]^T=\left[\Phi_{el}\right]\left[\Lambda_{el}\right]\left[\Phi_{el}\right]^T \qquad (16)$$

where Λ is the spectral matrix of the element stiffness matrix $\mathbf{k^e}$, Φ_{rb} are the eigenvectors associated with the eigenvalues $\Lambda_i = 0$, $i=1,2,3...,n_r$ where n_r is the number of zero eigenvalues (rigid body modes) possessed by $\mathbf{k^e}$, and Φ_e arc the eigenvectors associated with the eigenvalues $\Lambda_i \neq 0$, $i=1,2,3...,n_e$ where n_e is the number of non-zero eigenvalues (elastic/dissipative modes) possessed by $\mathbf{k^e}$. Equation (16) makes clear that $\Re(k') = \mathrm{span}\left[\Phi_e\right]$, i.e. the elastic modes provide a basis for the range, denoted by $\Re(\bullet)$, of $\mathbf{k^e}$. We will also make use of the fact that $C" = \mathrm{span}\left\{\Phi_{rb}\right\} \oplus \mathrm{span}\left\{\Phi_{el}\right\}$, the rigid body modes Φ_r define a subspace C_r which is the orthogonal complement to the subspace C_e defined by the elastic modes Φ_e. Given these facts, we see that

$$\Re(M)=\Re\left(\Phi^e\right) \qquad (17)$$

The transformation of the impedance relation begins by substituting $f = M\tilde{f}$ which yields

$$M\tilde{f}^e = k^e x^e \qquad (18)$$

We premultiply by $\mathbf{M^T}$,

$$M^T M\tilde{f}^e = M^T k^e x^e \qquad (19)$$

and solve for the reduced coupling forces.

$$\tilde{f} = \left(M^T M\right)^{-1} M^T k^e x^e \qquad (20)$$

We must solve the contragradient transformation for the displacements,

$$\tilde{x}^e = M^T x^e \qquad (21)$$

Requiring that the connection coordinate responses be a linear combination of the columns of $\mathbf{M}$ guarantees that we retain all elastic mode information, Eq. (17), then $\tilde{x} = M^T M\alpha$ and $\alpha = \left(M^T M\right)^{-1}\tilde{x}$. Therefore, the reduced stiffness matrix is

$$\tilde{f}^e=\left(M^T M\right)^{-1}M^T k^e M\left(M^T M\right)^{-1}\tilde{x}^e \qquad (22a)$$

or

$$\tilde{f}^e = \tilde{k}^e\tilde{x}^e \qquad (22b)$$

where

$$\tilde{k}^e = M^\dagger k^e\left(M^T\right)^\dagger \qquad (22c)$$

and $()^+$ indicates pseudo-inverse. This transformation, in effect, extracts the connectivity information from the original stiffness matrix, such that this information may be contained solely in the mapping matrix M. Therefore, the original stiffness matrix has the factorization.

$$k^e = M\tilde{k}^e M^T \qquad (22)$$

From this analysis, we see that::

(I) M is always full rank due to fact that it is constructed from the linearly independent equilibrium equations.
(2) The number of independent connection coordinates is determined by the connectivity to be established, and therefore so is the dimension of the reduced dimension coordinate system.
(3) If the number of independent coordinates equals the number of elastic modes of the element, the" $\Re(M)=\Re\left(\Phi^e\right)$, the reduced stiffness matrix will be full rank, and will provide an exact representation of the elastic mode content of the original stiffness matrix.
(4) The mapping transformation purges the stiffness matrix of rigid body mode information. Thk new coordinates here produced are differential response coordinates, which do not represent rigid body motion, hence the exclusion of mass terms in the impedance. It will he show" that the element's elastic modes can serve as a transformation, thereby guaranteeing the equality described in (3), above. However, a transformation constructed from equilibrium is independent of the elastic characteristics of the element, and avoids having to solve an eigenvalue problem for each element.
(5) The element mass matrix does not possess an exact reduced form because all eigenvalues are non-zero.
(6) The transformation matrix M is boolean only for lumped elements.

3.2 Modal Mapping Matrices

We can also construct the element mapping matrix from the elastic modes of the element, i.e.

$$M = \Phi_{el}$$

since this matrix obviously satisfies items (1), (3), and (4) above. However. with respect to the practical application of disassembly. constructing **M** from equilibrium avoids having to know the parameters describing the elasticity of the element, and also avoids solving the eigenvalue problem for each element. However, due to item (3) in the above list, the modal mapping matrix applies only to lumped elements and the two-noded beam element.

Examples of Mapping Matrices

We will start with the simplest structural element, the spring (see Fig. I), described by a single parameter, k.

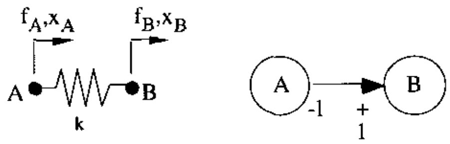


Figure 1. A spring element and the graph of its connectivity.

The spring is a uniaxial element of zero length with a force f_i and displacement x_i at each end, and with a stiffness relation,

$$\begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} \tag{23}$$

We will develop the transformation between the original element coordinate system and the reduced coordinate system. We begin by writing equilibrium for the element,

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} = [R]\{f\} = 0 \tag{24}$$

where the f_i are the forces applied at each end of the spring. The mapping matrix for the spring is either of the following.

$$M = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \qquad M = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \tag{25}$$

and the reduced stiffness matrix for the spring element is

$$\tilde{k}^e = M^{\dagger} k^e \left(M^T\right)^{\dagger} = [k] \tag{26}$$

Turning now to a simple beam element of length L, as shown in Fig. 2, the mapping matrix is constructed from equilibrium and is

$$\{f\} = \begin{Bmatrix} f_A \\ m_A \\ f_B \\ m_B \end{Bmatrix} = \begin{Bmatrix} \tilde{f}_A \\ \tilde{f}_B \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ -L & -1 \end{bmatrix} \begin{Bmatrix} f_A \\ m_A \end{Bmatrix} = [M]\{\tilde{f}\} \tag{27}$$

The element stiffness matrix for the beam [5] is

$$\left[k^e\right] = \frac{EI}{L^3} \left[\begin{array}{cc|cc} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ \hline -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{array}\right] = \begin{bmatrix} A & B \\ B^T & C \end{bmatrix} \tag{28}$$

and using Eq. (22c) the reduced element stiffness matrix is

$$\left[\tilde{k}^e\right] = [A] = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L \\ 6L & 4L^2 \end{bmatrix} \tag{29}$$

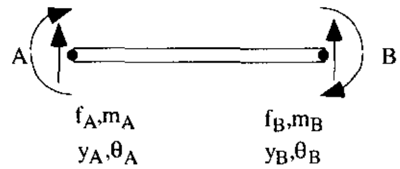


Figure 2. A simple beam element.

As can he seen from Eq. (27), the mapping matrix for the beam element is non-boolean.

3.3 Assembly of Reduced Element Stiffness Matrices

The global stiffness matrix can be assembled from the reduced stiffness matrices and the associated mapping matrices M, in a manner analogous to Eq. (4). The individual element mapping matrices arc assembled into a global mapping matrix $\mathbf{M}_g$ which reflects the connectivity to be established. Noting that the partitioned rows of M correspond to the independent and dependent coordinates, $\mathbf{M}_g$ relates the uncoupled reduced element forces $\tilde{\mathbf{f}}$ to the assembled system forces **f**, and the displacements are related via the transpose, i.e.

$$f = M_g \tilde{f} \qquad \tilde{x} = M_g^T x \tag{30a,b}$$

These transformations operate on a block diagonal matrix of un assembled element reduced stiffness matrices.

$$\tilde{K}^e = \begin{vmatrix} \tilde{k}_1^e & & & & \\ & \ddots & & 0 & \\ & & \tilde{k}_i^e & & \\ & 0 & & \ddots & \\ & & & & \tilde{k}_p^e \end{vmatrix} \tag{31}$$

producing the assembled global stiffness matrix,

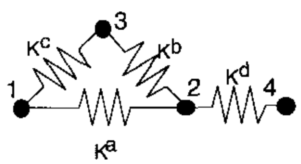
$$K = M_g \tilde{K}^e M_g^{\ T} \tag{32}$$

4. DISASSEMBLY OF GLOBAL STIFFNESS MATRICES

4.1 Rank Of M_g Graph Theoretic Considerations

The columns of $\mathbf{M}_g$ are constructed from the mapping matrices of the individual elements. Each element mapping matrix $\mathbf{M}$ is constructed from the imposition of static equilibrium on the element. We will show here that an alternative interpretation of $\mathbf{M}$, and hence $\mathbf{M}_g$, can be made based on the connectivity established by the element, schematically represented by its graph.

Referring to Fig. 1, the spring element establishes a connection between point "A," with the single coordinate x_A and point "B," with the single coordinate x_B. The graph is a means of pictorially representing connection information, and the graph for the spring element is shown in Fig. 1, where an arrow is drawn from point "A" to "B," (the "vertices") and the tail and head of the arrow (the "edge") are labelled with a "-1" and a "+1". Since either mapping matrix of Eq. (25) is valid, it is immaterial which end of the arrow is positive. From this development, it is seen that the mapping matrix for a system of springs can be constructed from the graph of the system. Referring to Fig. 3, a network of four springs is shown along with the global mapping matrix which can be assembled by inspection.



Node-to-Node Connections

$$\mathbf{M}_g = \begin{array}{c} \begin{array}{cccc} \text{1-2} & \text{2-3} & \text{1-3} & \text{2-4} \end{array} \\ \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right] \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \text{ Nodes}$$

Figure 3. A network of springs and its mapping matrix

For the disassembly of global stiffness matrices, we will make use of the results found for the mapping transformation given by Eq. (22). The pseudo-inverse transformation provides an exact reduced stiffness matrix because the transformation purges only rigid body mode information, which is associated exclusively with element connectivity. The only additional requirement is that the global mapping matrix $\mathbf{M}_g$ must be full rank. If this is so, than the exact disassembled global matrix is given by

$$\tilde{K}^e = M_g^{\dagger} K \left(M_g^T\right)^{\dagger} \tag{33}$$

The global mapping matrix $\mathbf{M}_g$ is constructed from the individual element mapping matrices, and therefore guarantees that the subspace spanned by the elastic modes of all element is spanned. We now consider the issue of the rank of $\mathbf{M}_g$.

By visual inspection of $\mathbf{M}_g$ as shown in Fig. 3, the matrix is clearly rank deficient. Column three is dependent on columns one and two, and as each column corresponds to a load path in the structure, the conclusion is that the load paths joining nodes 1.2. and 3 are dependent. For example, spring k^b provides the same load path as provided by springs k "and k^c. Spring k^d provides a unique load path. Because of the redundancy of load paths in the structure, and the resulting rank deficiency of the mapping matrix, it is not possible to disassemble the stiffness matrix.

We now consider the disassembly of two simple beam structures (see Fig. 4), one a truss topology, and the other a star topology. As will be seen, disassembly is only possible for one of these structures, and from this result we will draw general conclusions about the feasibility of stiffness disassembly. For this development, we will consider all beams for both structures to be colinear (all angles between beams being 0 or 180 degrees), with no loss of generality. We assemble the global mapping matrices for each structure using the element mapping matrix shown in Eq. (27), as follows

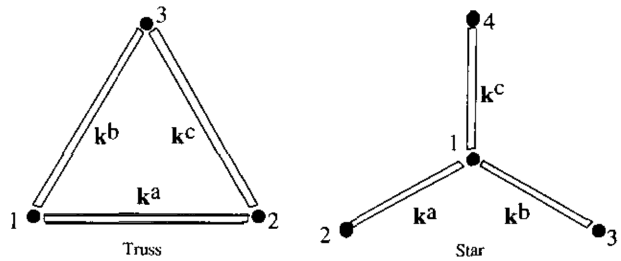


Figure 4. Beam topologies; truss and star

$$M_{truss} = \left[\begin{array}{cccc|cc} 0 & 0 & 0 & & 1 & 0 \\ & 1 & 0 & 0 & 0 & 1 \\ -1 & 0 & 1 & & & 0 \\ -L_a & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & & 0 \\ 0 & 0 & L_b & -1 & -L_c & -1 \end{array}\right]$$

$$\mathbf{M}_{star} = \left[\begin{array}{cccc|cc} 1 & & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ -1 & 0 & & 0 & 0 & 0 \\ -L_a & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -L_b & 0 & 0 & 0 \\ 0 & 0 & -0 & 1 & 0 & 0 \\ 0 & & & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -L_c & -1 \end{array}\right] \tag{34}$$

The disassembly of the global stiffness matrix is not possible for the truss structure due to the fact that $\mathbf{M}_{truss}$ is rank deficient. The disassembly of the star topology is possible because $\mathbf{M}_{star}$ is full column rank. The stiffness matrix for the star structure, where $EI/L^3=1$ for each beam, is

$$K_{star} = \left[\begin{array}{cccccccc} 36 & 18 & -12 & 6 & -12 & 6 & -12 & 6 \\ 18 & 12 & -6 & 2 & -6 & 2 & -6 & 2 \\ -12 & -6 & 12 & -6 & 0 & 0 & 0 & 0 \\ 6 & 2 & -6 & 4 & 0 & 0 & 0 & 0 \\ -12 & -6 & 0 & 0 & 12 & -6 & 0 & 0 \\ 6 & 2 & 0 & 0 & -6 & 4 & 0 & 0 \\ -12 & -6 & 0 & 0 & 0 & 0 & 12 & -6 \\ 6 & 2 & 0 & 0 & 0 & 0 & -6 & 4 \end{array}\right]$$

and by Eq. (37), the disassembled global stiffness matrix is

$$
\tilde{\mathbf{K}}^{e}_{star} = \begin{bmatrix} 12 & 6 & 0 & 0 & 0 & 0 \\ 6 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 12 & 6 & 0 & 0 \\ 0 & 0 & 6 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 12 & 6 \\ 0 & 0 & 0 & 0 & 6 & 4 \end{bmatrix}
$$

The rank deficiency of the global mapping matrices of Eqs. (34) can be understood from a graph theoretic viewpoint. Each pair of columns in $\mathbf{M}_{truss}$ and $\mathbf{M}_{star}$ represent a connection, or load path. In the truss, there are redundant load paths from node 1 to node 3, i.e. beam "b" as well as beams "a" and "c." This redundancy in load paths is manifest in the rank deficiency of $\mathbf{M}_{truss}$. In the star structure, each beam provides a unique load path, and hence $\mathbf{M}_{star}$ is full rank. Graph theory provides a theorem, directed at boolean mapping matrices however, which says that if a graph is a "tree," then its mapping matrix is full rank [6]. A tree is a graph of n nodes connected by n-l edges. and therefore cannot contain any loops. The result presented here follows heuristically from this theorem.

5. SUMMARY AND CONCLUSIONS

The conditions for which the analytic disassembly of structural matrices is possible are presented. It is shown that a unique analytic disassembly is possible only for stiffness matrices comprised of beam elements, and for topologies containing no "loops," such as the standard truss. Hence, disassembly of a global stiffness matrix is only possible in very restricted cases. The disassembly is made possible by a transformation which exploits the orthogonality of the subspaces spanned by the elastic modes and rigid body modes of an element stiffness matrix, and the fact that element connectivity is exclusively associated with the element's rigid body modes. The disassembly of a global mass matrix is not possible, due to the fact that the element's rigid body modes contribute to the global mass matrix.

The disassembly transformation was interpreted in a graph-theoretic sense, and the inability to disassemble a global stiffness matrix was shown to be due to the algebraic redundancy of load paths, such as that found in a simple truss. This conclusion is analagous to a result from graph theory relating the rank of an arc-incidence matrix to the presence of loops in the graph. The extension of these results to general structural assemblages is straightforward, and a determination of the ability to disassemble can be made simply by determining the rank of the global mapping matrix, $\mathbf{M}_g$.

6. REFERENCES

[II Gordis, J. H. Bielawa, R. L. Flannelly, W. G. "A General Theory for Frequency Domain Structural Synthesis" *Journal of Sound and Vibration* 150(1), 1991.

[2] Gordis, 1. H. and Flannelly, W. G. "Analysis of Stress due to Fastener Tolerance in Assembled Components." *AIAA Journal*, Vol. 32, No. 12, 1992.

[3] Gordis, J. H. "Structural Synthesis in the Frequency Domain: A General Formulation." ***Shock and Vibration, Volume*** 1, Issue 5, 1994.

[4] Cook, R. D., Concepts and Applications of Finite Element Analysis John Wiley&Sons, Inc.. 1974.

[5] Gallagher, R. H., Finite Element Analysis Fundamentals. Prentice-Hall, Inc., 1975.

[6] Brualdi, R. and Ryser, H., Combinatorial Matrix Theory. Cambridge University Press. Cambridge, England, 1991.