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Let $\vec{\Phi}_0$ have components $\phi_i > 0$. Then, $\vec{S}_1 \equiv \nu_1(1 + \beta)\vec{\Phi}_1 - \beta\eta_0\vec{\Phi}_0$ has components:

$$S_i = \nu_1(1 + \beta)\psi_i - \beta\eta_0\phi_i.$$

For $\beta \geq 0$, S_i is positive $\langle = \rangle (1 + \beta)/\beta > (1/\nu_1)(\eta_0\phi_i/\psi_i)$, $\psi_i > 0$ since the matrix T has nonnegative elements. But $\eta_0\phi_i/\psi_i \leq \bar{\eta}_1$, so that S_i is positive for all j if $(1 + \beta)/\beta > \bar{\eta}_1/\nu_1$. This proves the first part. The second part follows from definition. *QED.*

The above iteration sequence $\{\vec{\Phi}_i^*\}_{i=0}^\infty$ can be thought of as an extrapolation and renormalization performed on the sequence $\{\vec{\Phi}_i\}_{i=0}^\infty$. The norm-preserving feature of the definition of $\eta_i\vec{\Phi}_i^*$ is important when the last used fast group flux $\vec{\psi}_0$ is used as an *initial* approximation in the matrix equation:

$$A_2\vec{\psi} = B_2(\eta_i\vec{\Phi}_i^*). \quad (36)$$

Thus, the norm of the vector of the right-hand side of the equation above is kept constant in the course of the iterations. It is fully expected that using the extrapolation and renormalization scheme with the proper β 's will result in a reduction of effort in solving discrete matrix problems, the reductions being as much as 50 per cent.

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High-Energy Particle Nomograph*

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Summary—A nomograph is presented which has proved to be useful in calculations related to the design of high-energy particle accelerators. Rest mass is given in the range from 0.0001 to 100 atomic mass units with the energy equivalent of this mass in electron volts. The final velocity and mass of the particle are given for a range of energies from 5000 to 10^{15} electron volts. The equation for particle velocity is given in the form

$$v/c = [1 - 1/(1 + E/E_0)^2]^{1/2},$$

where E is the particle energy and E_0 is the energy equivalent of its rest mass. The frequency required for operation of a proton synchrotron is determined to illustrate the use of the nomograph.

THE solution of many problems concerning high-energy particles requires a knowledge of the relationships between their energy, velocity, and mass. The design of particle accelerators and the measurement of particle energy are typical problems involving these relationships. Since the solution of the exact equations is

somewhat laborious, a nomograph (Fig. 1) opposite, has been constructed to facilitate the calculations where two-place accuracy is sufficient.

There are three axes on the nomograph. The first gives the rest mass of the particle, m_0 , in atomic mass units (amu) as well as the energy equivalent of this mass, E_0 , in electron volts. The second axis gives the kinetic energy of the particle, E , in electron volts. The third axis gives the velocity of the particle, plotted as $(1 - v/c)$, and the ratio of its actual to rest mass, m/m_0 . Any straight line intersecting the three axes shows the relationship between these quantities. Approximate equations are given in the Appendix for velocities lower and higher than those covered by the nomograph.

The frequency range of the accelerating voltage in a proton synchrotron will be determined to illustrate the use of the nomograph. Assume that the protons are injected with an initial energy of 4 mev and accelerated to an energy of 2.3 bev in an accelerating tube having a

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Fig. 1—Nomograph of mass and velocity of high-energy particles.

circumference of 68 meters. Reference to Fig. 1 shows that the final velocity of the protons is 0.958 cycles. The frequency at which the particles finally rotate is, therefore, $(0.958 \times 3 \times 10^8 \text{ meters}) / (68 \text{ meters}) = 4.2 \times 10^6 \text{ cps}$. The energy of the injected particles is too low for their

velocity to be given by the nomograph. From the approximate relationship given in the Appendix for low-energy particles, $(v/c) \approx (2E/E_0)^{1/2} = (2 \times 4/938)^{1/2} = 0.0924$. The corresponding rotational frequency is $(0.0924 \times 3 \times 10^8 \text{ meters}) / (68 \text{ meters}) = 0.41 \times 10^6 \text{ cps}$.

An electron synchrotron is planned by M.I.T. and Harvard to accelerate electrons to an energy of 6 bev. Reference to the nomograph shows that the electrons will emerge from this machine with a mass 13,000 times their rest mass, and at a velocity within 4×10^{-7} per cent of the velocity of light.

APPENDIX

Equations for Mass and Velocity

$$\frac{m}{m_0} = 1 + \frac{E}{E_0}$$

$$\frac{v}{c} = \sqrt{1 - \frac{1}{(1 + E/E_0)^2}}$$

For $E/E_0 \ll 1$ (Error < 2 per cent if $E/E_0 < 0.025$, $v/c < 0.22$)

$$\frac{v}{c} \approx \sqrt{\frac{2E}{E_0}}$$

For $E/E_0 \gg 1$ (Error < 2 per cent if $E/E_0 > 3$, $(1 - v/c) < 0.03$)

$$\left(1 - \frac{v}{c}\right) \approx \frac{1}{2(1 + E/E_0)^2}$$

Energy Equivalents of Certain Particles

Particle	E_0 (mev)
Electron	0.51098
Proton	938.23
Neutron	939.52
Deuteron	1875.5
Alpha	3727.2

A Portable Scintillation Alpha Survey Instrument*

W. G. SPEAR†

Summary—A scintillation alpha counter utilizing a zinc sulfide fluor, a multiplier phototube, and a two-stage vacuum tube amplifier has been developed. A neon-bulb oscillator high-voltage supply is used to supply 900 volts to the multiplier phototube, and headphones are used to obtain an aural indication of counting.

The average geometry or counting efficiency over the 7.43-inch² probe area is approximately 13 per cent with variations from a maximum counting efficiency at probe center of 18 to 22 per cent

down to approximately 7 to 8 per cent at the probe edges. The background counting rate is less than one count per minute, and the battery life is approximately 150 hours of continuous operation for the 3-1/2 pound instrument.

INTRODUCTION

SCINTILLATION alpha counters are preferable, in general, to air-proportional counters because they are inherently nonmicrophonic, little affected by humidity and temperature variations, and can be made to

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