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Multifractal thermal characteristics of the southeastern GIN Sea upper layer

Chu, Peter C.

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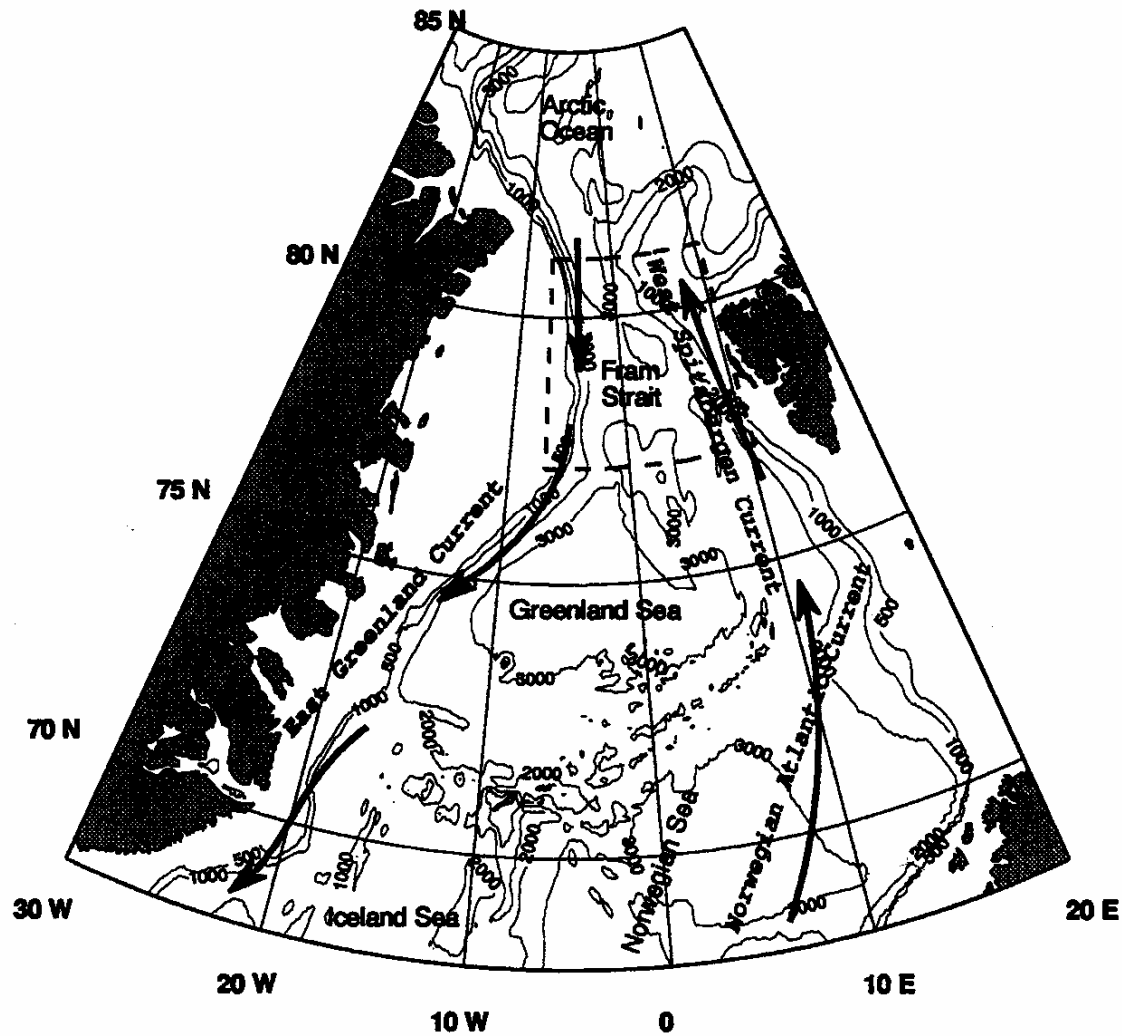
Multifractal Thermal Characteristics of the Southwestern GIN Sea Upper Layer

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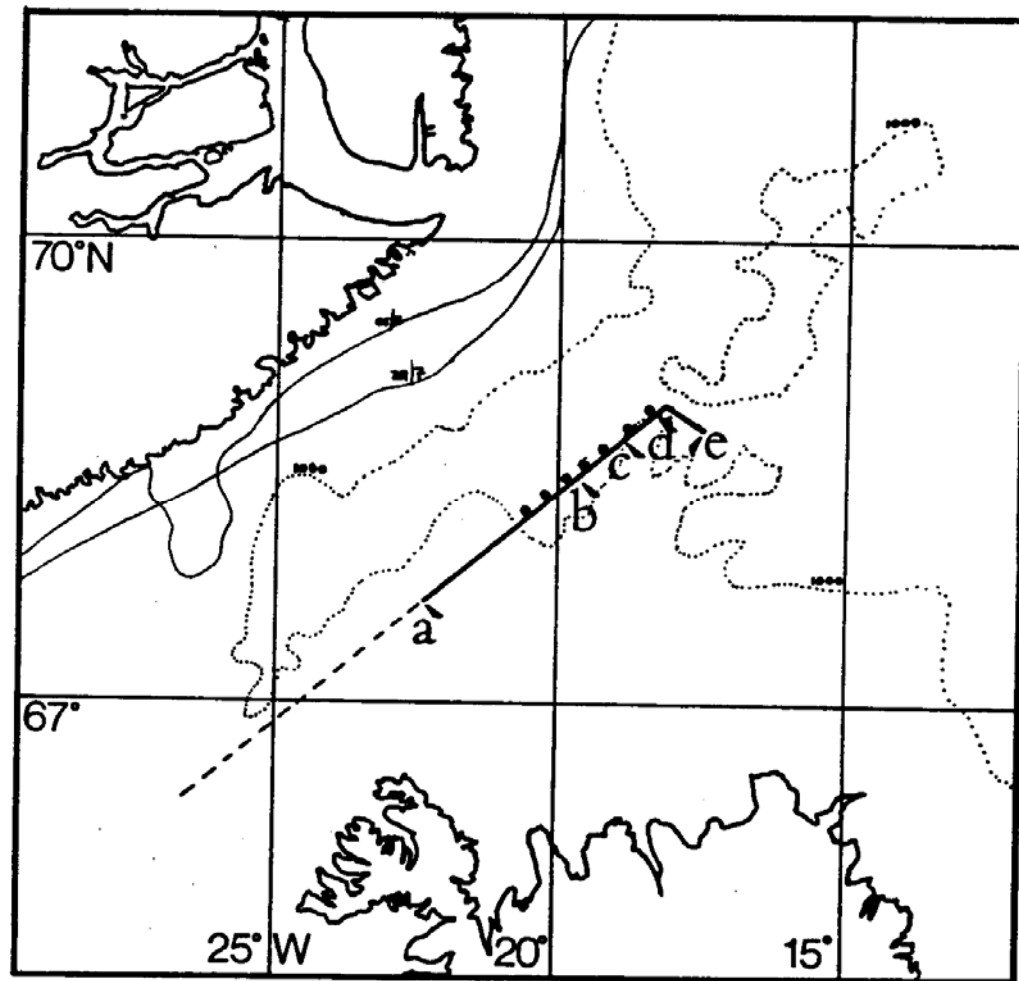
Reference

- Chu, P.C., 2004: Multifractal thermal characteristics of the southwestern GIN Sea upper layer, the journal "Chaos, Solitons and Fractals", 19 (2), 275-284.

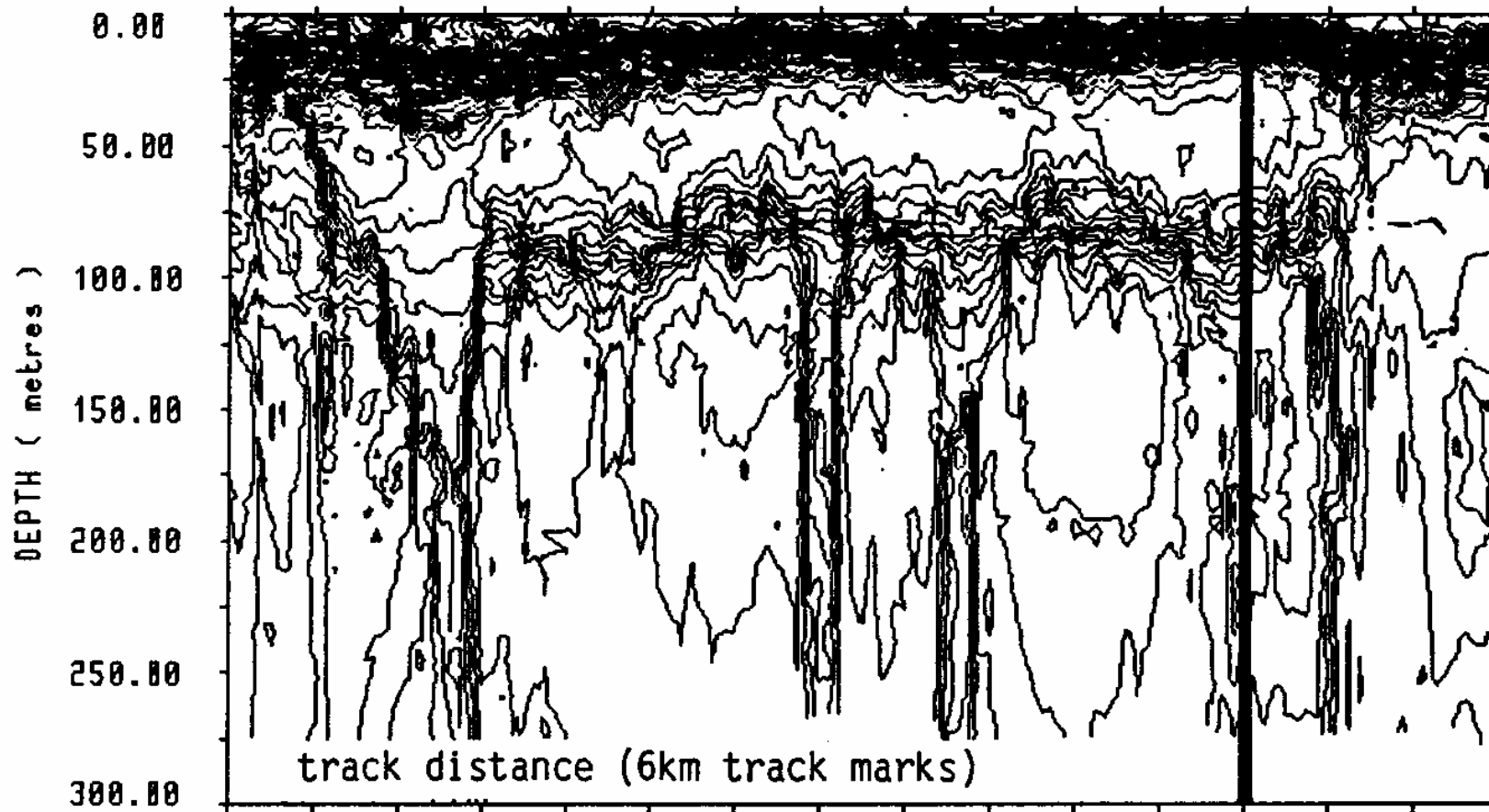
Greenland-Icelandic-Norwegian (GIN) Sea



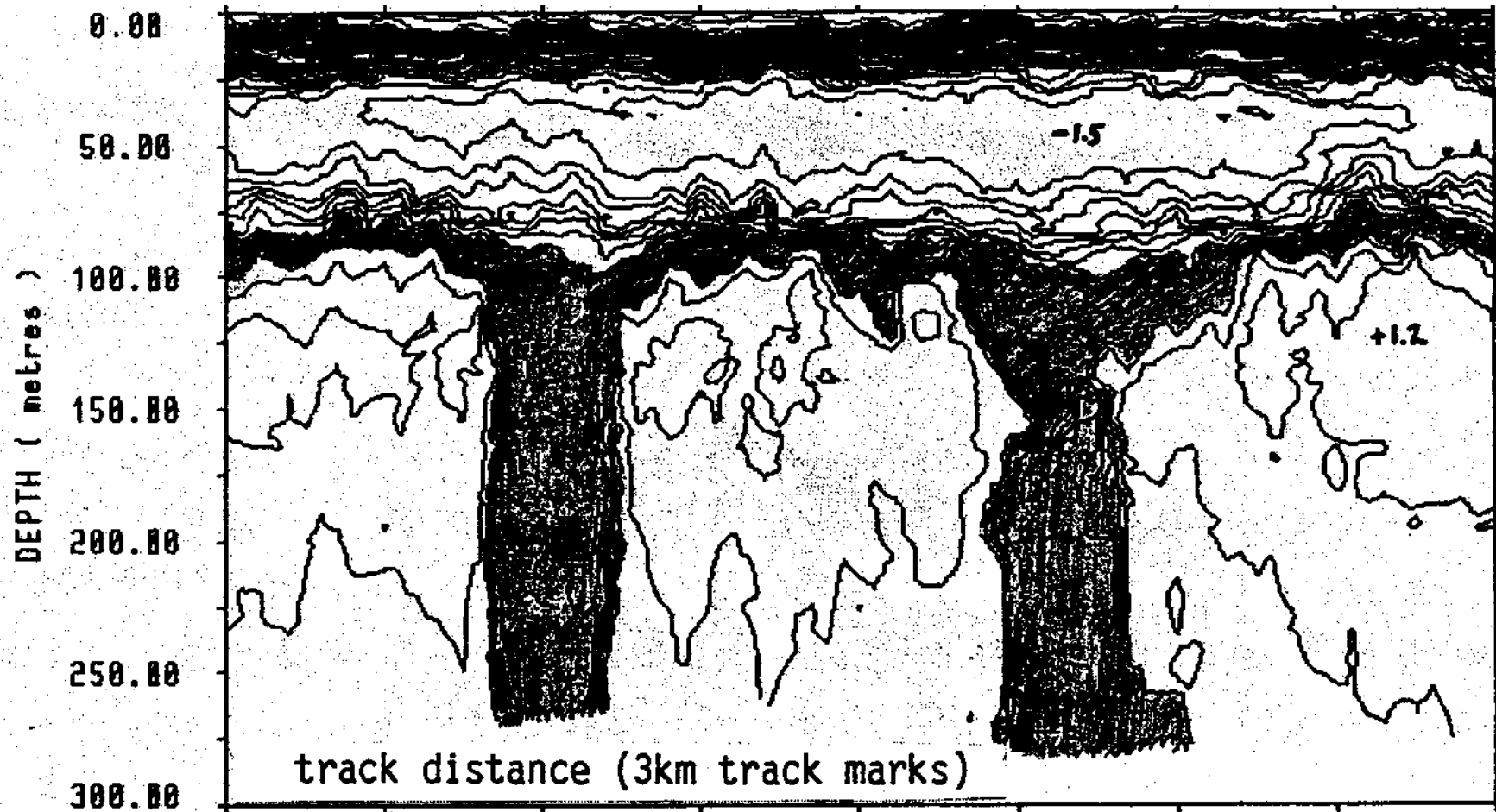
Thermister Chain



Temperature Field



Temperature Field (zoomed in)



Power Spectrum

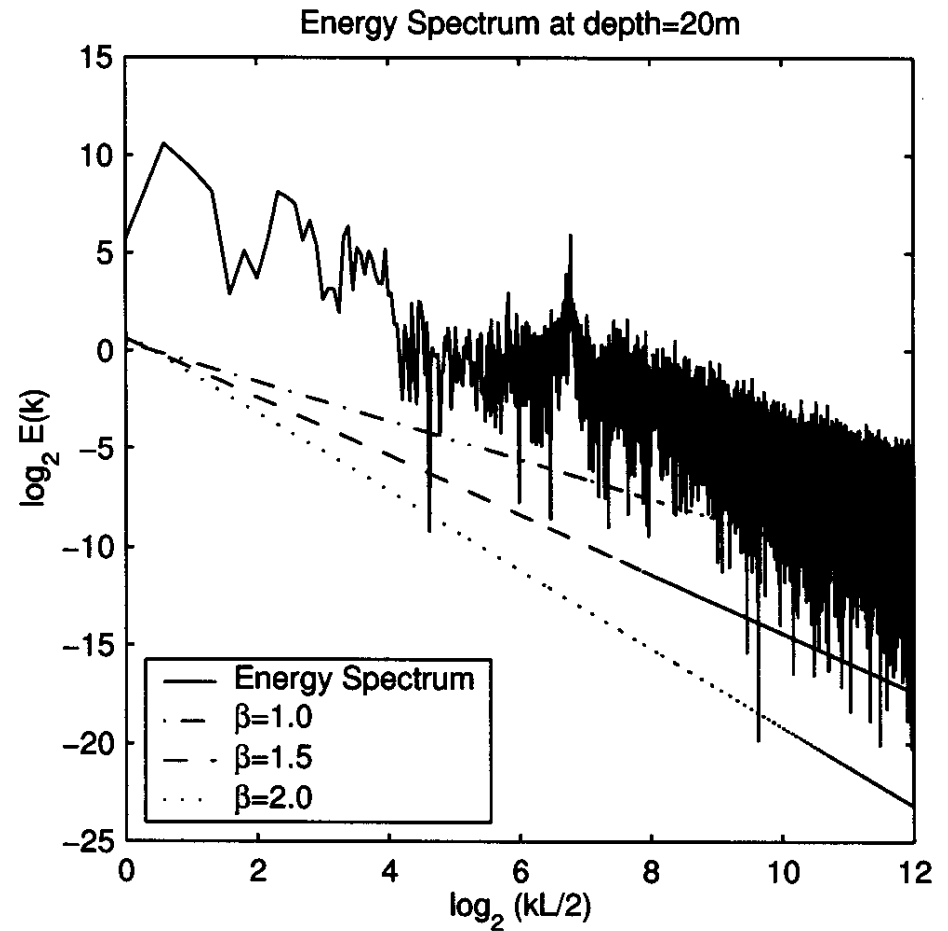
$$E(k) \propto k^{-\beta},$$

$$1 < \beta < 2,$$

Spike at

$$\text{Log}_2(kL/2) \approx 6.5,$$

**Corresponding to Chimney
Scale ~ 3 km**



Structure Function (1)

$$|\Delta T(r; \mathbf{x})| \equiv |T(x_{i+r}) - T(x_i)|, \quad i = 0, 1, \dots, \Lambda - r,$$

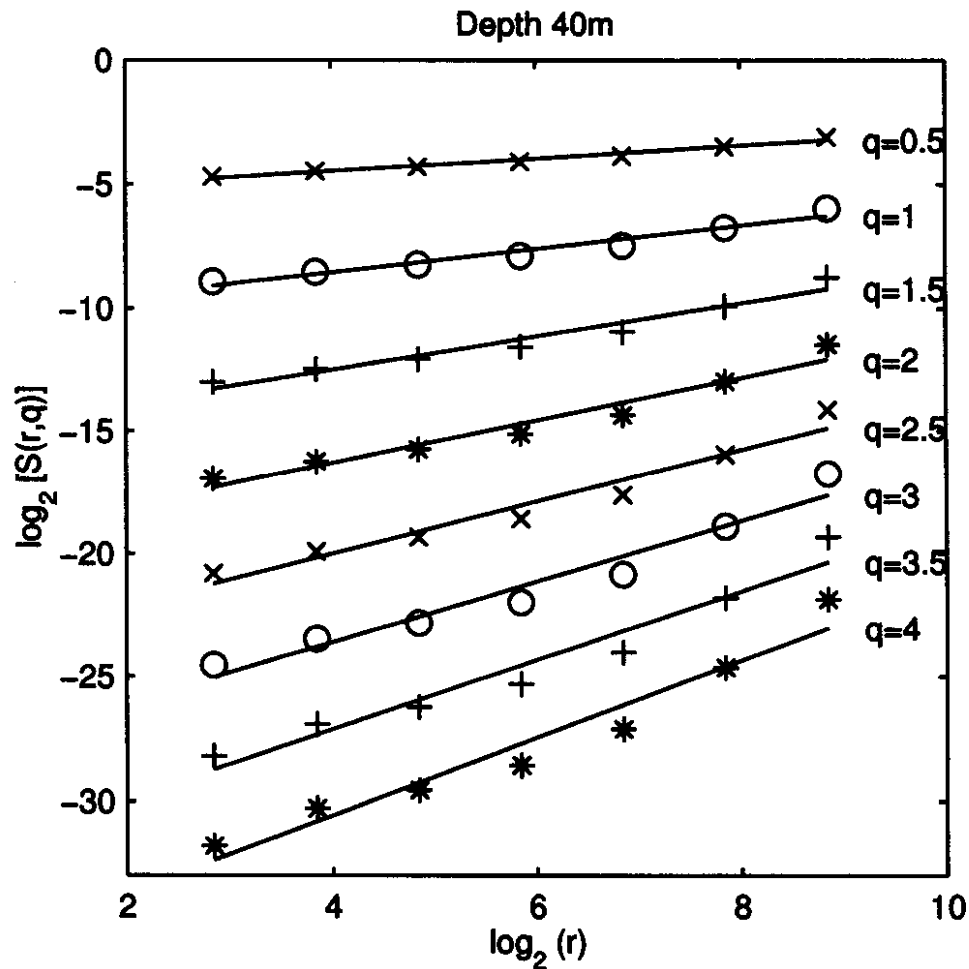
$$S(r, q) \equiv \langle |\Delta T(r; \mathbf{x})|^q \rangle = \frac{1}{\Lambda - r} \sum_{i=0}^{\Lambda-r} |\Delta T(r; \mathbf{x})|^q.$$

Here, r is the lag, q is the order of the structure function

$$S(1, 1) = \frac{1}{\Lambda - 1} \sum_{i=0}^{\Lambda-1} |T(x_{i+1}) - T(x_i)|$$

$S(1, 1)$ is the mean gradient.

Structure Function (Power Law)



$$S(r, q) \propto r^{\zeta(q)},$$

$$\zeta(q) = H(q)q,$$

Dimension of GIN Sea Temperature Field

$$D_{g(T)} = 2 - H_1.$$

Stationary

$$S(r, 1) = \text{const} ,$$

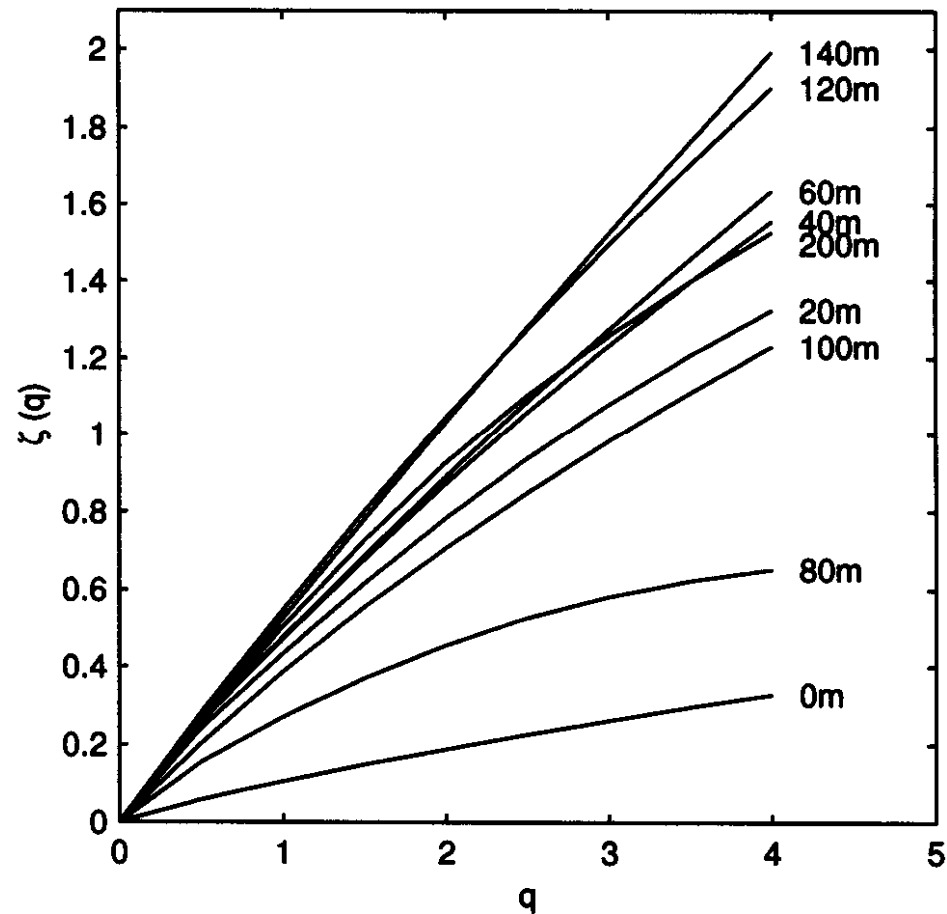
$$\zeta(1) = H(1) = 0 .$$

$$D_{g(T)} = 2 ,$$

Power of the structure function $S(r, 1)$ and the dimension of GIN Sea temperature field

Depth (m)	0	20	40	60	80	100	120	140	200
$\zeta(1) = H_1$	0.11	0.43	0.47	0.48	0.28	0.40	0.56	0.52	0.50
$D_{g(T)}$	1.89	1.57	1.53	1.52	1.78	1.60	1.44	1.48	1.50

Dependence of structure function's power $\zeta(q)$ on q and depth



Normalized Small Scale Absolute Gradient and Running Mean

$$\langle |\Delta T(1; x_i)| \rangle = \frac{1}{\Lambda} \sum_{i=0}^{\Lambda-1} |\Delta T(1; x_i)|,$$

$$\varepsilon(1; x_i) = \frac{|\Delta T(1; x_i)|}{\langle |\Delta T(1; x_i)| \rangle},$$

$$\varepsilon(r; x_i) = \frac{1}{r} \sum_{j=i}^{i+r-1} \varepsilon(1; x_j), \quad i = 0, 1, \dots, \Lambda - r.$$

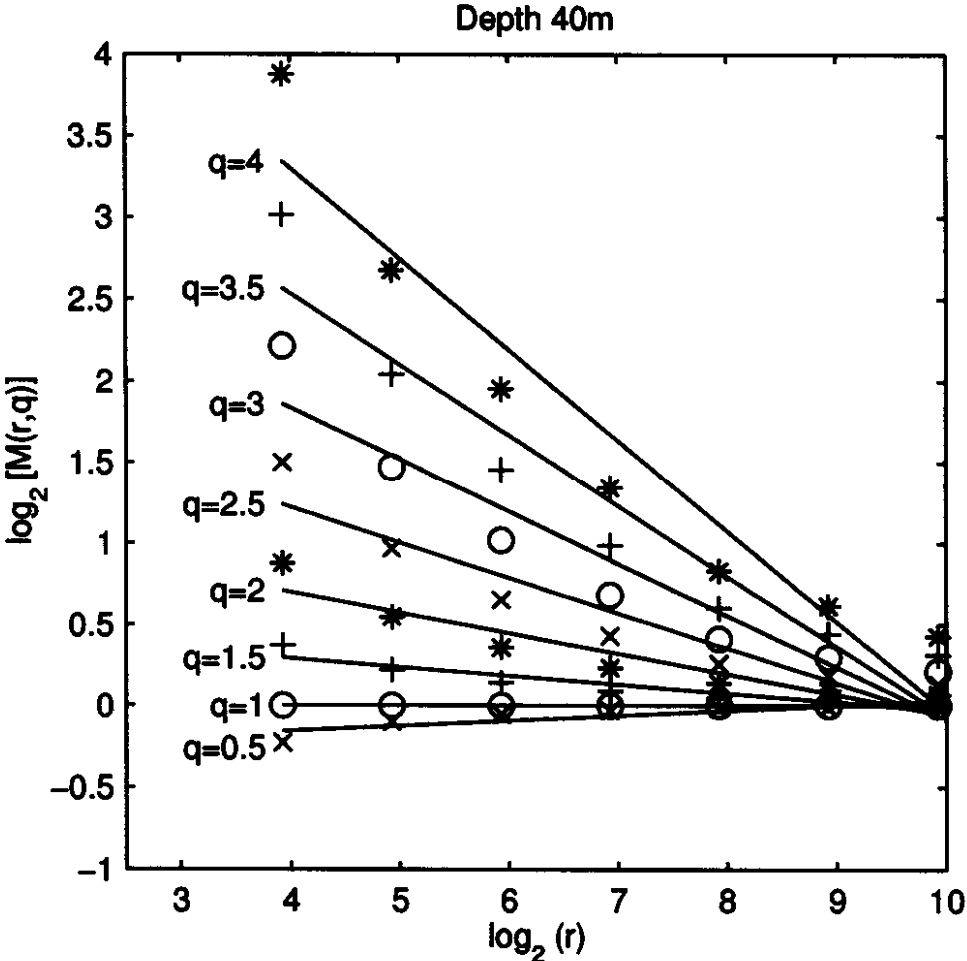
Singular Measure

$$M(r, q) \equiv \langle \varepsilon(r; x_i)^q \rangle = \frac{1}{\Lambda - r} \sum_{i=0}^{\Lambda-r} [\varepsilon(r; x_i)]^q ,$$

$$M(r, 1) \equiv \langle \varepsilon(r; x_i) \rangle = \frac{1}{\Lambda - r} \sum_{i=0}^{\Lambda-r} [\varepsilon(r; x_i)]$$

$$= \frac{1}{\Lambda - r} \sum_{i=0}^{\Lambda-r} \left[\frac{1}{r} \sum_{j=i}^{i+r-1} \varepsilon(1; x_j) \right] = 1.$$

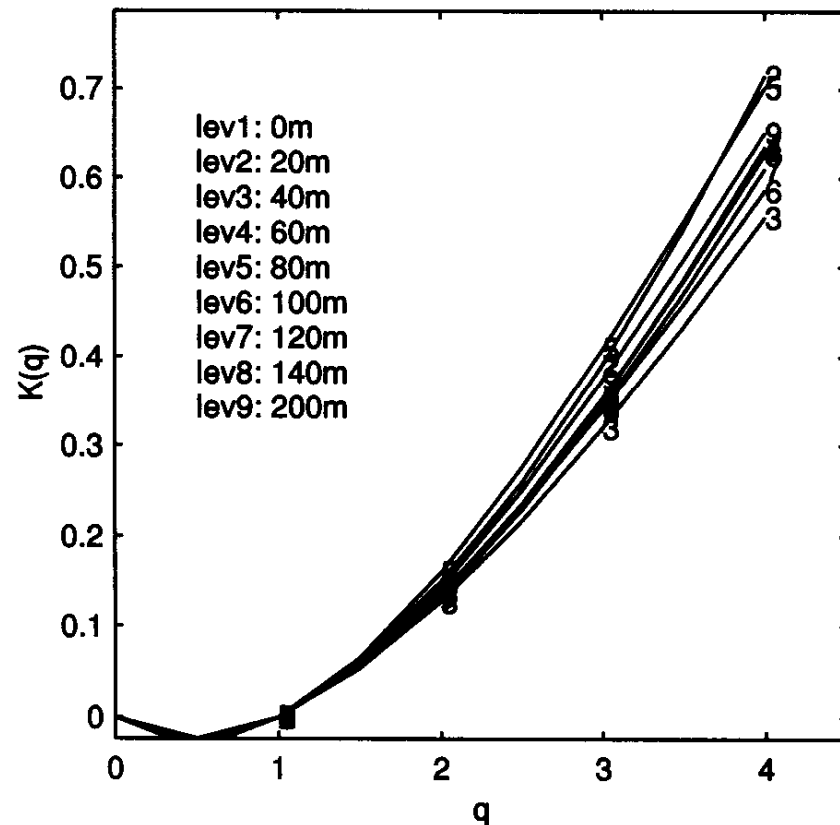
Singular Measure (Power Law)



Power of the Singular Measure

$$M(r, q) \propto r^{-K(q)}, \quad q \geq 0,$$

$$K(0) = K(1) = 0.$$



Characteristics of the Power of the Singular Measure

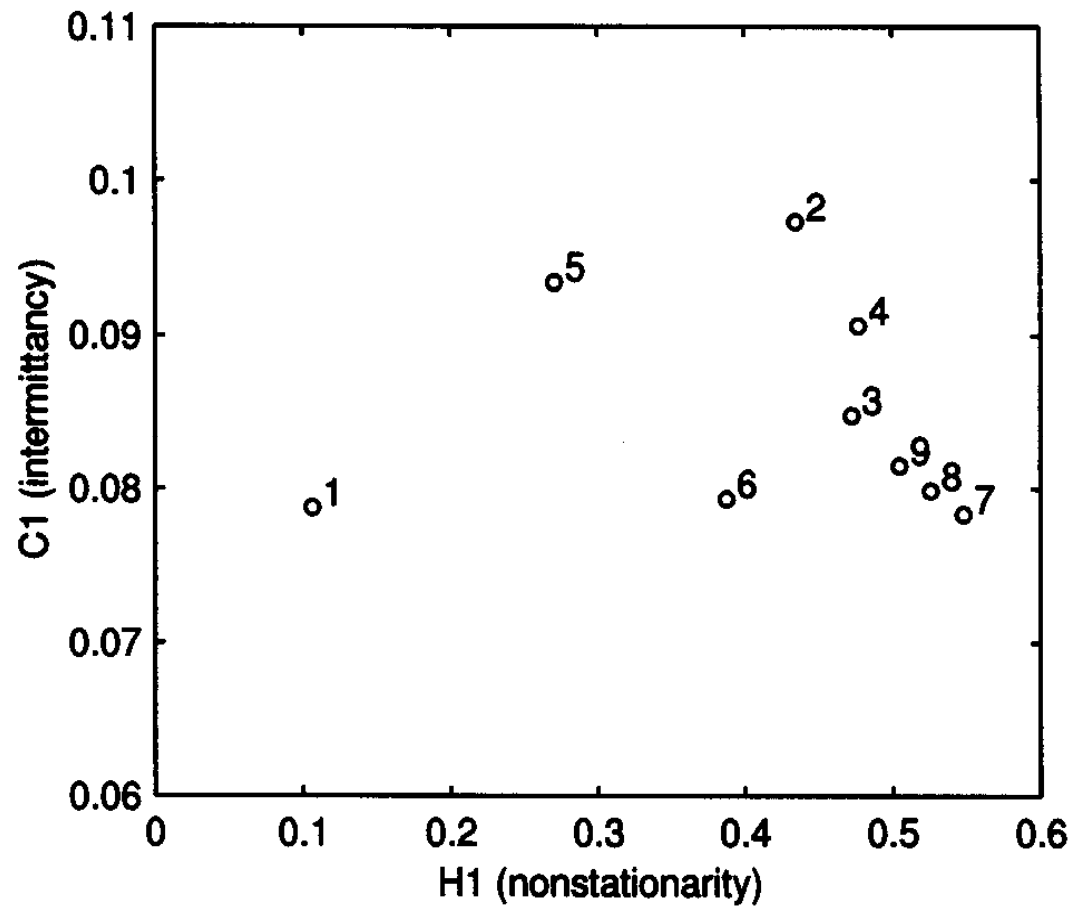
$$\frac{d^2K(q)}{dq^2} > 0,$$

$$K(q) < 0 \quad \text{only if} \quad 0 < q < 1$$

$$C(q) = \frac{K(q)}{q-1}.$$

$$C_1 \equiv C(1) = K'(1) \geq 0,$$

Mean Multifractal Plane (number
representing level: 1 ~ 0 m, ..., 9 ~ 200m)



Conclusions

- (1) Chimney width is around 3 km.
- (2) GIN Sea sublayer is nonstationary.
- (3) The structure function has multifractal characteristics.