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# P-Vector Method for Determining Arctic Ocean Circulation from the Joint US-Russian Hydrographic Data

Peter C Chu and C.W. Fan

Naval Ocean Analysis & Prediction Laboratory  
Naval Postgraduate School, Monterey California

Email: [pcchu@nps.edu](mailto:pcchu@nps.edu)

Website: [www.oc.nps.navy.mil/~chu](http://www.oc.nps.navy.mil/~chu)

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# Motivation

- Improving the weakness of diagnostic initialization
- Numerical model is usually integrated from (T, S) field (climatology or ...)  
 $u = v = w = 0$

Diagnostic Initialization

# Basic Equations for OGCM

$$\frac{\partial \mathbf{V}}{\partial t} = -\mathbf{V} \cdot \nabla \mathbf{V} - w \frac{\partial \mathbf{V}}{\partial z} - \mathbf{k} \times f \mathbf{V} - \frac{1}{\rho} \nabla p + \frac{\partial}{\partial z} \left( K_M \frac{\partial \mathbf{V}}{\partial z} \right) + \mathbf{H}_v$$

$$\frac{\partial T}{\partial t} = -\mathbf{V} \cdot \nabla T - w \frac{\partial T}{\partial z} + \frac{\partial}{\partial z} \left( K_H \frac{\partial T}{\partial z} \right) + H_T,$$

$$\frac{\partial S}{\partial t} = -\mathbf{V} \cdot \nabla S - w \frac{\partial S}{\partial z} + \frac{\partial}{\partial z} \left( K_H \frac{\partial S}{\partial z} \right) + H_S,$$

# Diagnostic Initialization

$$\frac{\partial T}{\partial t} = -\mathbf{V} \cdot \nabla T - w \frac{\partial T}{\partial z} + \frac{\partial}{\partial z} \left( K_H \frac{\partial T}{\partial z} \right) + H_T + F_T,$$

- Keep (T, S) constant

$$\frac{\partial S}{\partial t} = -\mathbf{V} \cdot \nabla S - w \frac{\partial S}{\partial z} + \frac{\partial}{\partial z} \left( K_H \frac{\partial S}{\partial z} \right) + H_S + F_S,$$

- Generate (u, v, w) fields

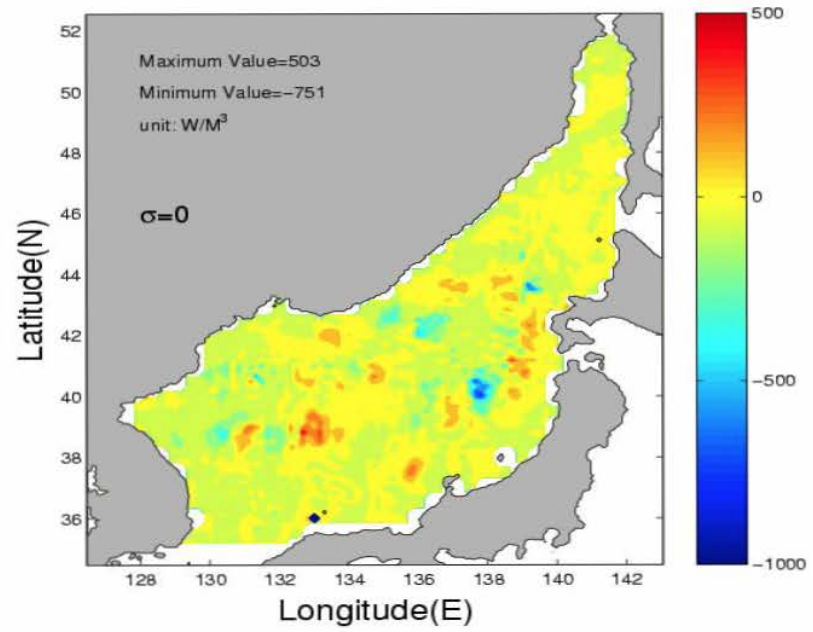
$$\frac{\partial T}{\partial t} = 0, \quad \frac{\partial S}{\partial t} = 0$$

# Extremely Strong Thermohaline Source/Sink Terms (Non-Physical)

$$F_T \equiv \mathbf{V} \cdot \nabla T + w \frac{\partial T}{\partial z} - \frac{\partial}{\partial z} \left( K_H \frac{\partial T}{\partial z} \right) - H_T,$$

$$F_S \equiv \mathbf{V} \cdot \nabla S + w \frac{\partial S}{\partial z} - \frac{\partial}{\partial z} \left( K_H \frac{\partial S}{\partial z} \right) - H_S.$$

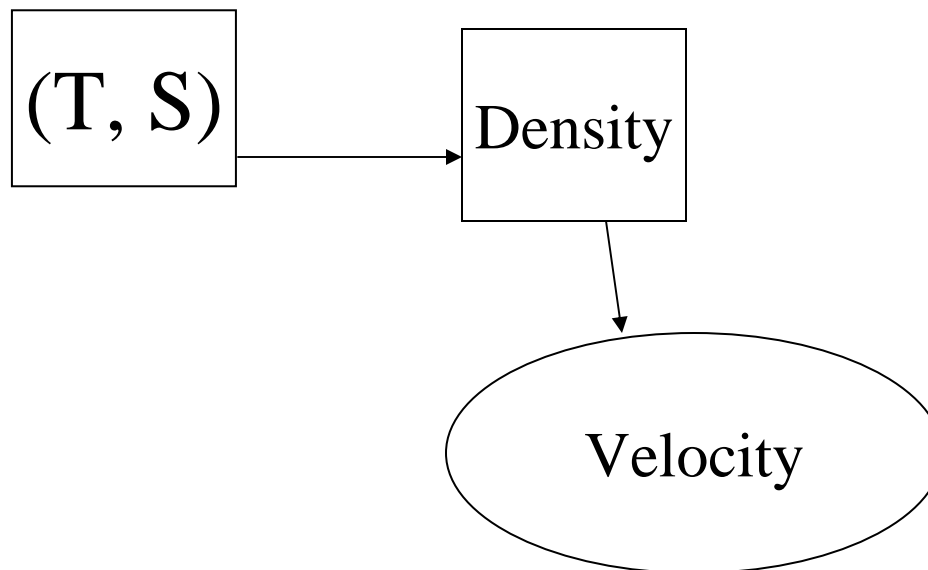
Chu, P.C., and J. Lan, 2003: Extremely strong thermohaline source/sinks generated by diagnostic initialization. *Geophysical Research Letters*, 30 (6), doi: 10.1029/2002GL016525





# Geostrophic Initialization

- Absolute geostrophic velocity computed from hydrographic data



# First Thought - Thermal Wind Relation

$$u = u_0 + \frac{g}{f\rho_0} \int_{z_0}^z \frac{\partial \rho}{\partial y} dz'$$

$$v = v_0 - \frac{g}{f\rho_0} \int_{z_0}^z \frac{\partial \rho}{\partial x} dz'$$

*How to determine  $(u_0, v_0)$  ?*

# Conservation of Mass and Potential Vorticity

$$\mathbf{V} \cdot \nabla \rho = 0$$

$$\vec{V} \cdot \nabla q = 0, \quad q \equiv f \frac{\partial \rho}{\partial z}$$

# Relationship Among Three Vectors

$$\vec{V} \perp \nabla \rho \qquad \vec{V} \perp \nabla q$$

$$\vec{V} \sim \nabla q \times \nabla \rho$$

# P-Vector

$$\vec{P} = \frac{\nabla q \times \nabla \rho}{|\nabla q \times \nabla \rho|}$$

$$\vec{P} \parallel \vec{V}$$

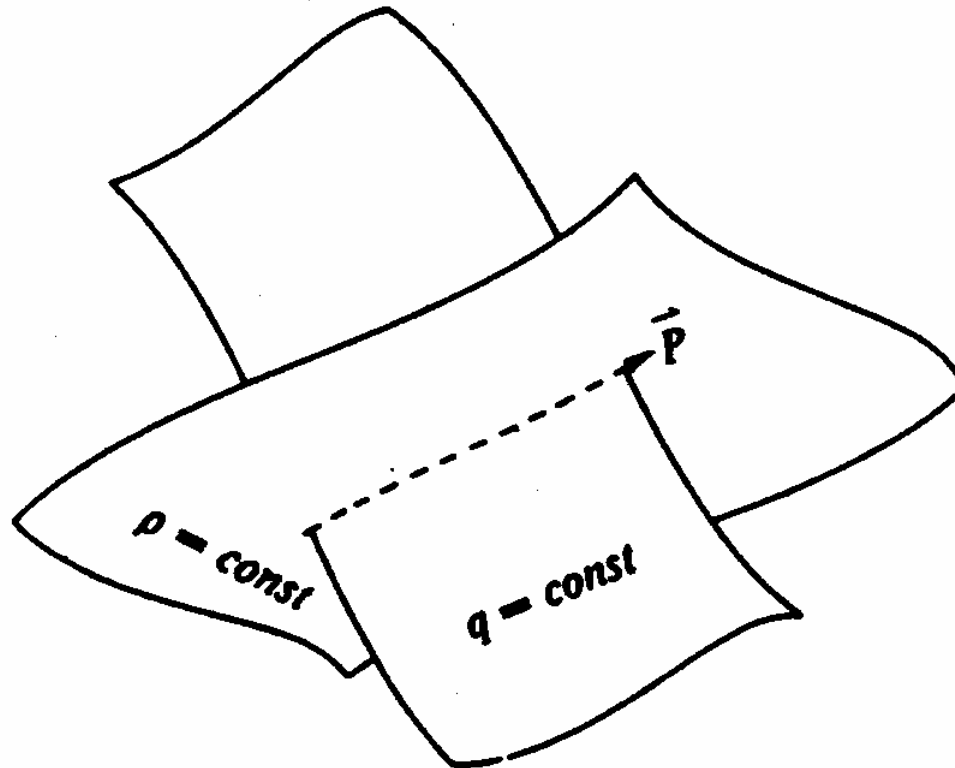
$$\vec{V} = r(\lambda, \phi, z) \vec{P}$$

# **Two-Step Inverse Method**

- (1) Density determines the P-vector.**
- (2) Thermal wind relation determines  $\gamma$ .**

# P-Vector

Intersection of density and potential  
vorticity surfaces



# Thermal Wind Relation Determines $\gamma$

$$r^{(k)} P_x^{(k)} - r^{(m)} P_x^{(m)} = \Delta u_{km}$$

$$r^{(k)} P_y^{(k)} - r^{(m)} P_y^{(m)} = \Delta v_{km}$$

$$\Delta u_{km} \equiv \frac{g}{f\rho_0} \int_{z_m}^{z_k} \frac{\partial \rho}{\partial y} dz'$$

$$\Delta v_{km} \equiv -\frac{g}{f\rho_0} \int_{z_m}^{z_k} \frac{\partial \rho}{\partial x} dz'$$



# Solution of $\gamma$

$$r^{(k)} = \frac{\begin{vmatrix} \Delta u_{km} & P_x^{(m)} \\ \Delta v_{km} & P_y^{(m)} \end{vmatrix}}{\sin(\alpha_{km})}$$

$$\begin{vmatrix} P_x^{(k)} & P_x^{(m)} \\ P_y^{(k)} & P_y^{(m)} \end{vmatrix} = \sin(\alpha_{km})$$

$$\alpha_{km} \neq 0$$

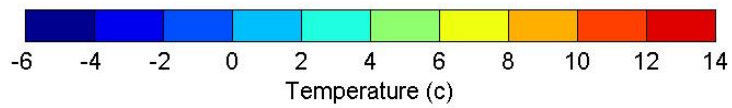
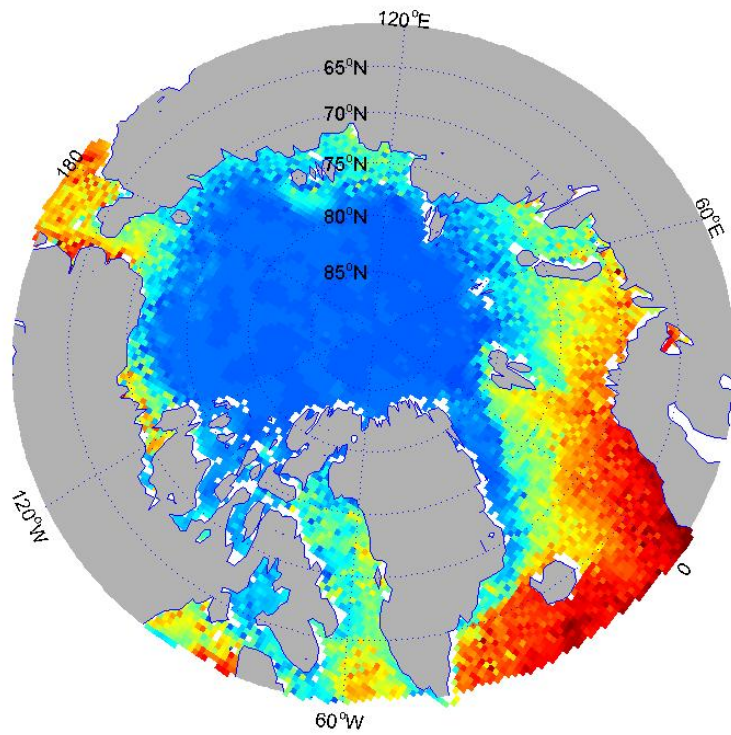
# US-Russian Arctic Hydrographic Data



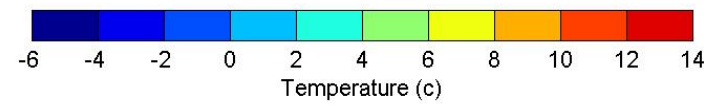
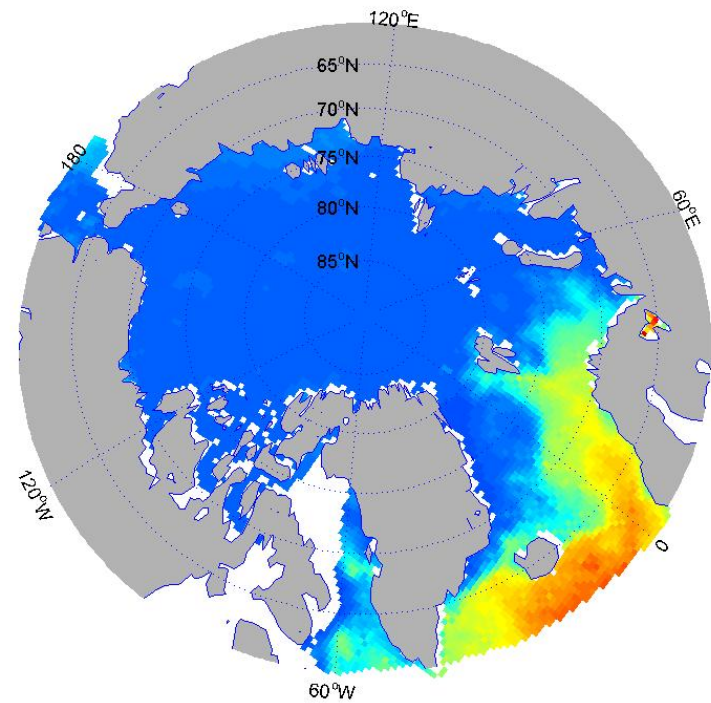
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# Temperature ( $z = 0$ )

Summer Surface Temperature

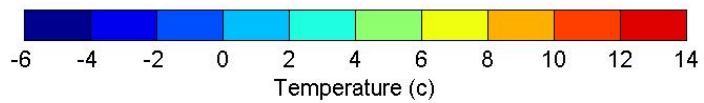
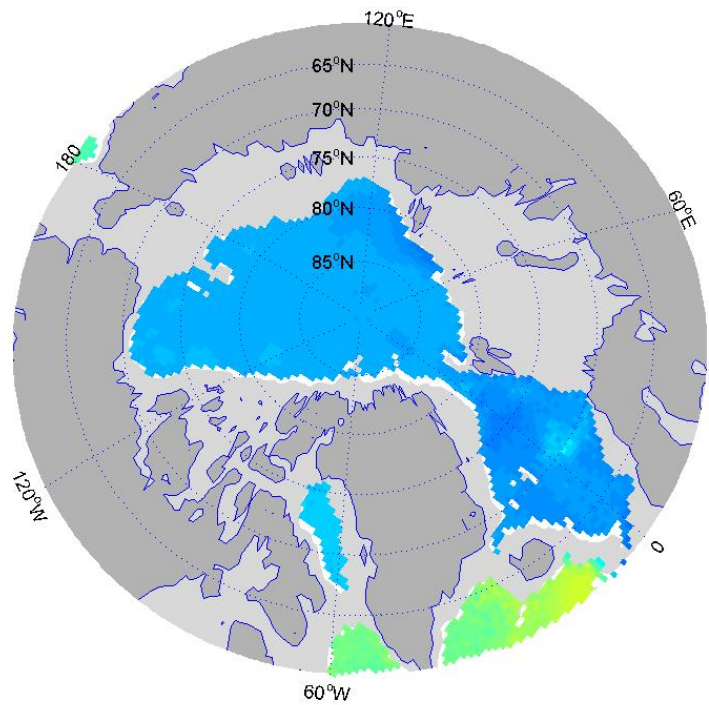


Winter Surface Temperature

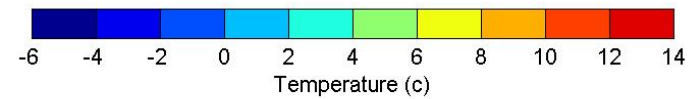
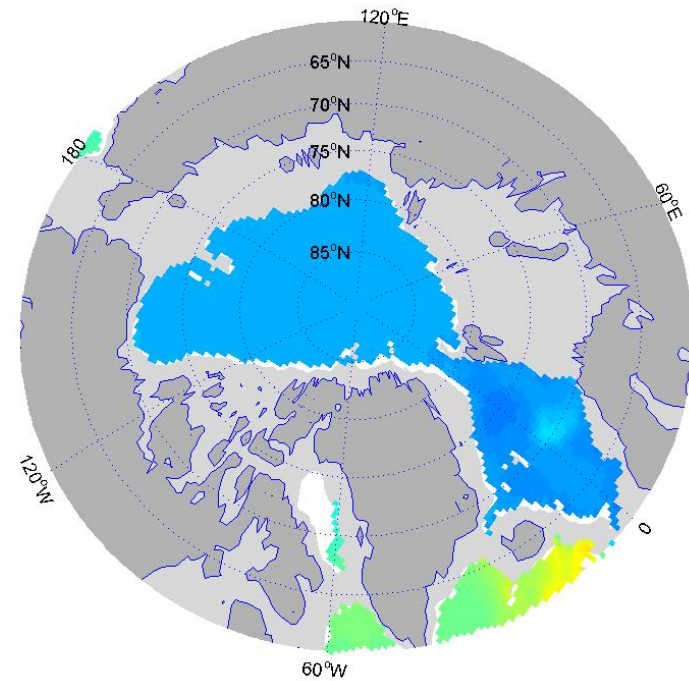


# Temperature ( $z = -1000$ m )

Summer Temperature: Water Depth: 1000(m)

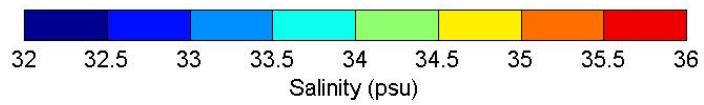
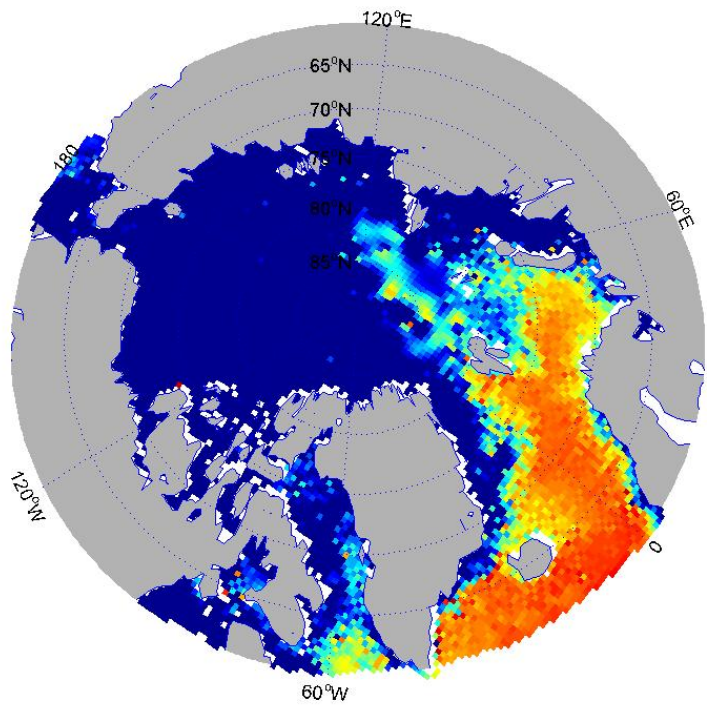


Winter Temperature: Water Depth: 1000(m)

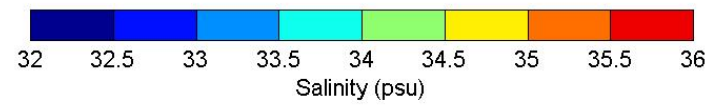
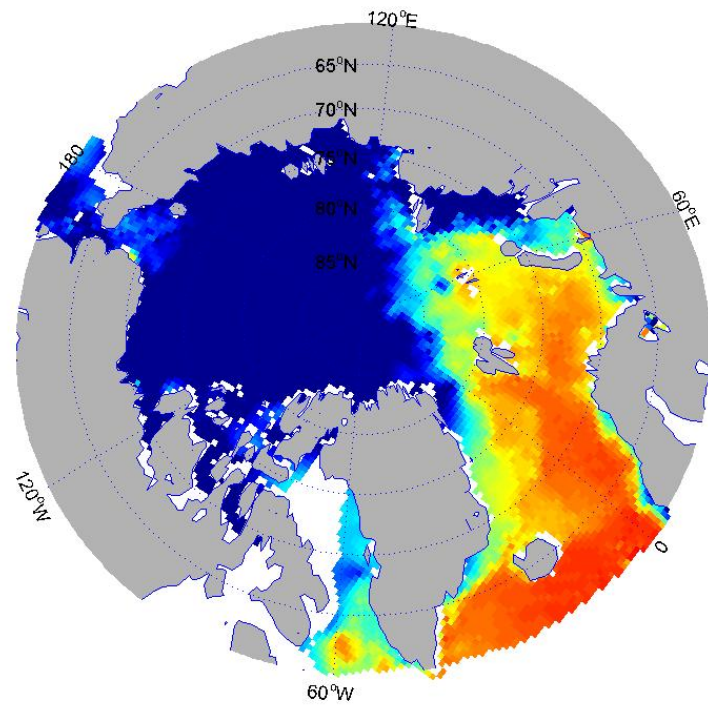


# Salinity ( $z = 0$ )

Summer Surface Salinity

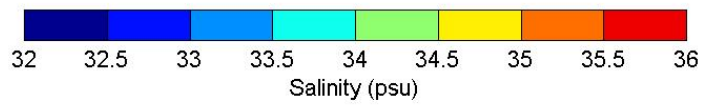
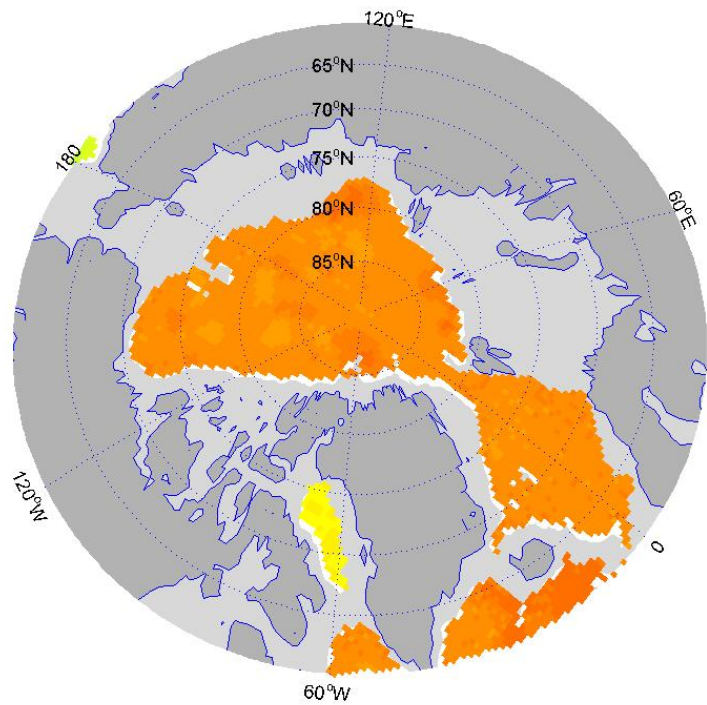


Winter Surface Salinity

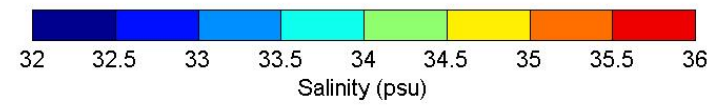
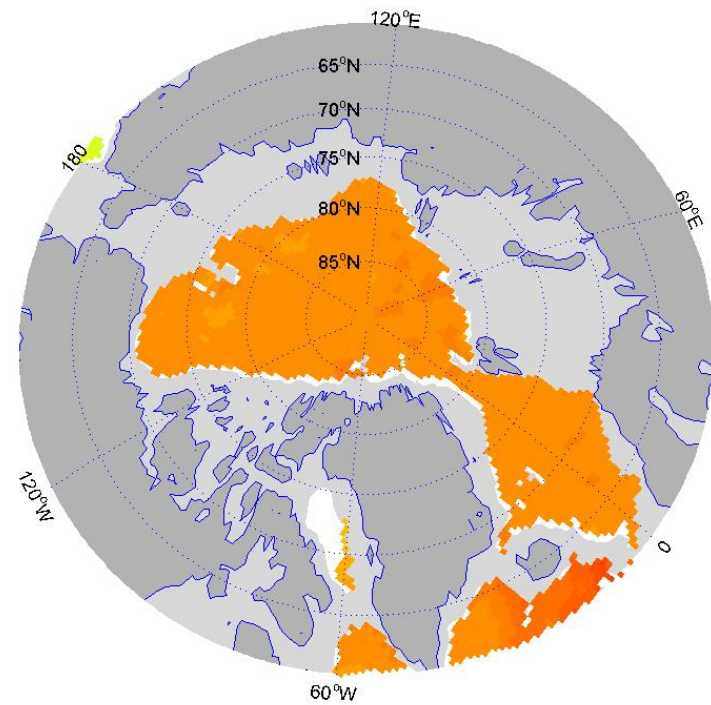


# Salinity ( $z = -1000$ m)

Summer Salinity: Water Depth: 1000(m)



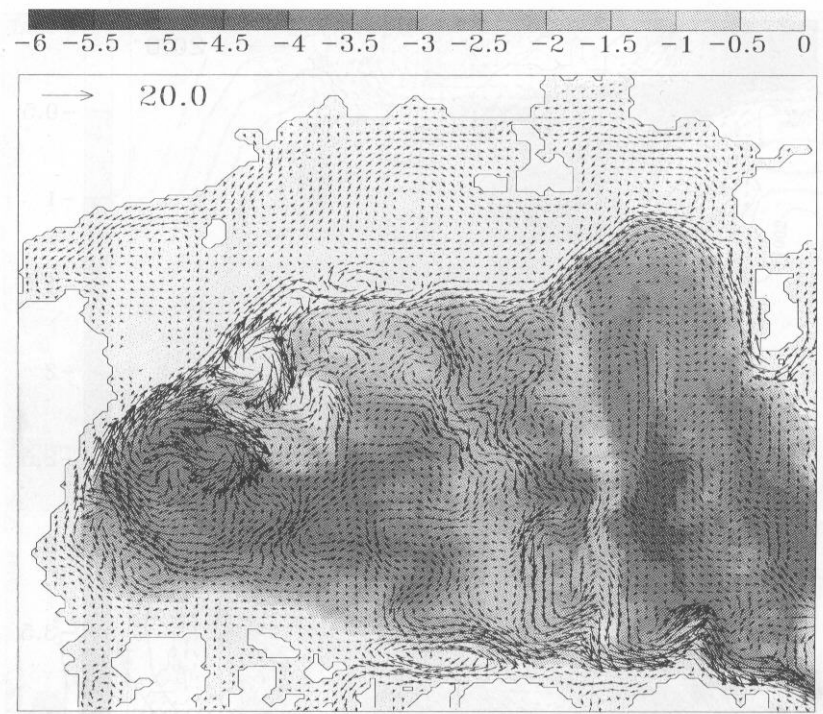
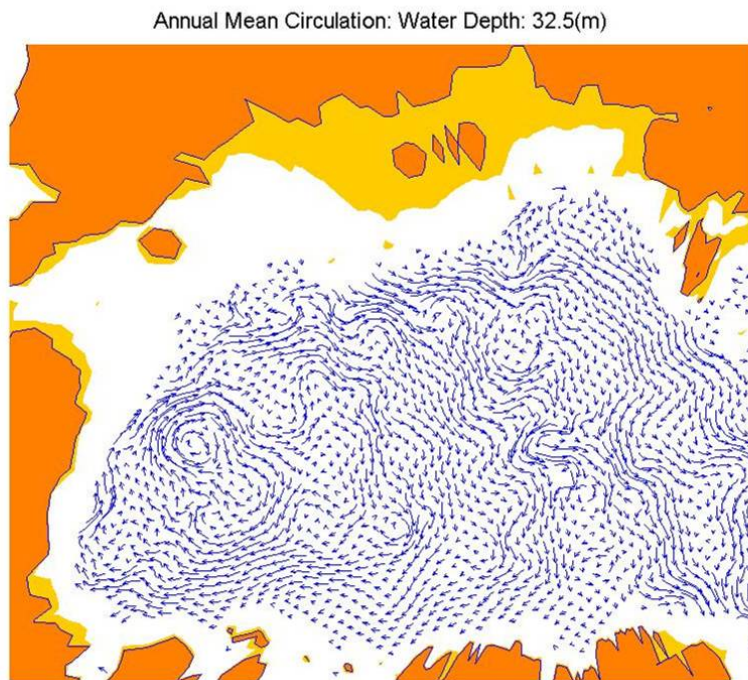
Winter Salinity: Water Depth: 1000(m)



# Mean Circulations ( $z = -32$ m)

P-vector

7 years mean of POPS model results (Zhang, Maslowski, Semtner, 1999, JGR)



# Mean Circulations ( $z = -32$ m)

P-vector

7 years mean of POPS model results (Zhang, Maslowski, Semtner, 1999, JGR)

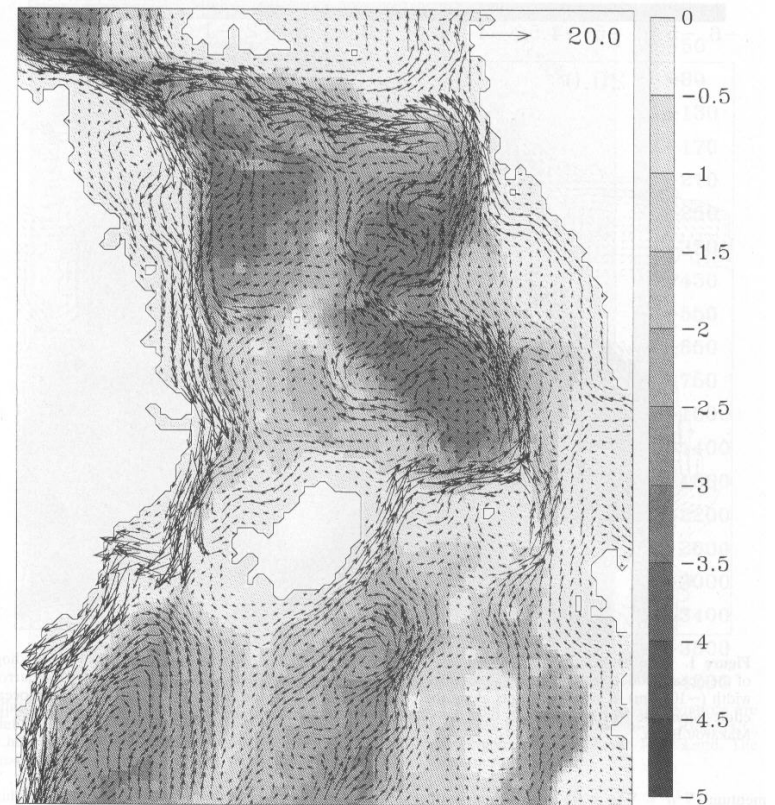
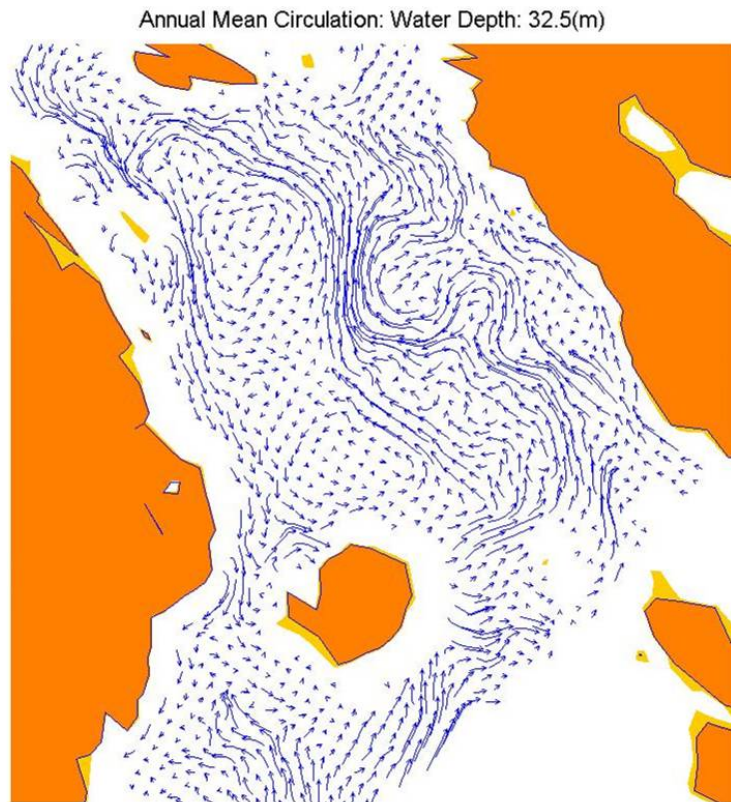


Figure 2. The 7-year mean of POPS model results for the North Atlantic circulation at 32.5 m depth.



# Conclusions

- (1) P-Vector method is effective to calculate the absolute velocity from hydrographic data.
- (2) The initial velocity field can be calculated using the P-vector method (Geostrophic Initialization).