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Optimal probing control for wireless transmission when the payload is negligible

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SUMMARY

A mobile electronic device needs to periodically connect to a stationary receiver, but the information to transfer is minimal. One such example is the electronic bracelet used in house arrest, where the main purpose is to inform the receiver that the person is in the house. Because the mobile device does not know its current distance from the receiver, it has incentive to first send a low-strength signal to conserve its battery energy. If the low-strength signal fails to reach the receiver, the mobile device then gradually increases its signal strength until a successful connection occurs. By formulating the problem as a dynamic program, we characterize the structure of the optimal probing policy and develop an algorithm to compute it. We also consider a discrete approximation that can be easily implemented in practice. Numerical examples show promising improvement of the derived policy over naive heuristic policies, and that the derived policy is robust when there are small errors in estimating the distribution of the distance between the mobile device and the receiver. Copyright © 2010 John Wiley & Sons, Ltd.

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1. INTRODUCTION

In a low-power, wireless communication network, a mobile electronic device often draws energy from a battery when it periodically sends small data packets to other devices in the network. Examples of these mobile devices include *ad hoc* wireless sensor networks, electronic bracelets used in house arrest, and electronic tags used to track wild animals. Conservation of battery energy is important because, for many of these mobile devices, it is undesirable or uneconomical to frequently replace (or recharge) the battery. In addition, the battery usually accounts for a significant portion of the mobile device's weight and space; hence an energy-efficient protocol helps to reduce the battery size without compromising the device's performance. In some other cases, lengthening the battery life is crucial because the mobile device dies as soon as the battery runs out—such as transmitters used for wild animal tracking, geographical survey, and battlefield surveillance. For an introduction to low-power, wireless networks, see Akyildiz *et al.* [1] and Siva Ram Murthy and Manoj [2].

Because the distance between a mobile device and the nearest receiver changes from time to time, the energy required for each successful transmission is usually random. In order to establish a connection, typically the mobile device first goes through a probing process to determine the signal strength that is sufficient to reach the receiver. In the case of cellular phones, the energy

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consumed in the probing process is insignificant because much more energy is consumed *after* the connection is established. In the cases of electronic bracelets and electronic tags, however, the probing process accounts for a significant portion of the energy consumption, because there is minimal data to transfer. In these cases, it is important to design a probing policy that saves enough energy to lengthen the battery's life. The purpose of this paper is to determine optimal probing control when the payload is negligible.

Most work on transmission energy management for these wireless devices studies heuristic methods that would work well if the topology of the network is reasonably stable. Kubisch *et al.* [3] use the number of reachable neighbors in a wireless network to adjust the device transmission power. The idea is to increase the power level when the number of reachable neighbors is below a predetermined target, and to decrease the power level when the number is above that target. Agarwal *et al.* [4, 5], Ramanathan and Rosales-Hain [6], and Duncan and Malan [7] use the signal strength from a transmitter's neighbors to dynamically scale its transmission power. The transmission power increases when the signal strength drops below a predetermined threshold, and decreases when it is above that threshold. An experimental study of such an approach can be found in Son *et al.* [8]. Lin *et al.* [9] study a model-based approach, where the link quality between two neighboring nodes is continuously monitored in order to predict the required transmission power by linear regression. Jeong *et al.* [10] conduct an empirical study to compare different dynamic transmission power control algorithms. Their case study shows that those algorithms achieve more energy savings over the fixed transmission power control at lower data transmission rate. For a review of various transmission power control methods, refer to Khemapech *et al.* [11].

All these approaches share some deficiency for the following three reasons. First, they need to gather information from the environment and to adjust the transmitter's signal strength with an algorithm. As a result, they not only consume more processing energy, but also require a more powerful processor. Second, these methods are more suitable for wireless networks with relatively stable topology, but not for the cases when the topology changes continually. Third, none of the methods seek to optimize the probing power control; hence, when probing accounts for the majority of energy consumption in a wireless network, these methods will render little efficacy. Because of these three reasons, in those cases when the transmitters are carried by humans or animals, or mounted on mobile objects, none of the above methods are suitable.

In this paper, we seek to mathematically derive an optimal probing policy, where the distance between the mobile device and the receiver changes continually. For example, a criminal who wears an electronic bracelet used in house arrest can move freely within a certain distance from a stationary receiver. The electronic bracelet needs to send a signal to reach the receiver from time to time in order to acknowledge that the person under house arrest is still in the house. Because the electronic bracelet does not know its current distance from the receiver, it can potentially save battery energy by first sending a low-strength signal and hoping that the receiver is nearby. If the low-strength signal does not reach the receiver, the electronic bracelet then gradually increases its signal strength until a successful connection is made. In addition, because there is minimal data to transfer, the problem essentially ends as soon as the electronic bracelet connects to the receiver for the first time. Other applications include similar devices worn by people in home quarantine, by pets, and by wild animals to collect research data.

The remainder of this paper is organized as follows. Section 2 describes the problem and the mathematical model. Section 3 presents a dynamic-programming formulation and the structural properties of the optimal policy, and Section 4 provides an example. Section 5 uses numerical examples to show the improvement of the optimal policy over some naive heuristic policies, and to demonstrate the robustness of the optimal policy. Finally, Section 6 offers some concluding remarks.

2. MATHEMATICAL MODEL

Consider a mobile device that has to periodically send a wireless signal to reach a stationary receiver, but the information to transfer is minimal. Let 1 denote the normalized distance that the

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mobile device can reach with its maximum transmission power. Because we are concerned with energy conservation under the condition that the receiver is within range of the mobile device, throughout the paper we consider the case $P(X \leqslant 1) = 1$, where X represents the normalized distance between the mobile device and the receiver. The case $P(X > 1) > 0$ is beyond the scope of this work.

Let $F(x) \equiv P(X \leqslant x)$ denote the long-run percentage of time when the distance between the mobile device and the receiver is less than or equal to x, for $x \in [0, 1]$. We model X as a continuous random variable, and denote its tail distribution by $\bar{F}(x) \equiv 1 - F(x) = P(X > x)$. For convenience, we say that the *strength* of a wireless signal is x, if the signal can reach as far as x in distance, for $x \in [0, 1]$. The energy consumption of a signal with strength x is modeled by a cost function $c(x)$, where $c(x)$ is continuous and increases in x for $x \in [0, 1]$.

When the mobile device needs to connect to the receiver, a feasible policy can be delineated by a probing sequence of signal strengths, denoted by $z_1, z_2, z_3, \ldots$, such that the mobile device sends signals along this sequence until a successful connection is made. After a successful connection, there is no need to find the minimal required signal strength, as the payload is negligible. In other words, the problem ends as soon as a signal reaches the receiver for the first time, and a feasible policy is completely defined by this probing sequence. Generally speaking, a probing sequence $\mathbf{z} = \{z_n\}_{n=1}^{\infty}$ is feasible as long as $z_n \in [0, 1]$, $n = 1, 2, \ldots$. However, we can rule out many suboptimal probing sequences by making three observations.

First, $z_1 = 0$ makes no sense, because a signal strength of 0 has no chance of reaching the receiver. Second, if the signal just sent did not reach the receiver, a weaker signal will only add extra cost without making progress; hence, $z_{n+1} > z_n$, for $n = 1, 2, \ldots$. Third, we need $\lim_{n\to\infty} z_n = 1$ to ensure that the mobile device will eventually reach the receiver, because otherwise the expected cost would be infinity. In the remainder of the paper, we will refer to a *feasible probing sequence* with the definition below.

Definition 2.1
We say a probing sequence $\mathbf{z} = \{z_n\}_{n=1}^{\infty}$ is feasible if it possesses the following three properties.

1. $z_1 > 0$.
2. $\mathbf{z} = \{z_n\}_{n=1}^{\infty}$ is an increasing sequence.
3. $\lim_{n\to\infty} z_n = 1$.

For a feasible probing sequence $\mathbf{z} = \{z_n\}_{n=1}^{\infty}$, the first signal (with strength z_1) will always be sent, but the signal with strength z_n, $n \geqslant 2$, will only be sent if $X > z_{n-1}$. Recall that $\bar{F}(x) = 1 - F(x) = P(X > x)$; the expected total energy consumed for a feasible probing sequence $\mathbf{z} = \{z_n\}_{n=1}^{\infty}$ can be expressed by

$$c(z_1) + \sum_{n=2}^{\infty} \bar{F}(z_{n-1}) c(z_n). \tag{1}$$

Our objective is to find the optimal probing sequence $\{z_n\}_{n=1}^{\infty}$ to minimize the preceding equation. Because $\bar{F}(1) = 0$, the minimized expected energy consumed is bounded by $c(1)$, which can be achieved by the probing sequence $z_1 = 1$.

3. OPTIMAL CONTROL POLICY

3.1. Dynamic-programming formulation

Because there are an infinite number of feasible probing sequences, it is impossible to enumerate all of them and compare their performances. To find the optimal policy that minimizes the objective function in Equation (1), we use dynamic programming. We say that the mobile device is in state y, $y \in [0, 1)$, if a signal strength y just failed to reach the receiver. In other words, in state y the mobile device learns that $X > y$, and needs to next select a signal strength $x \in (y, 1]$. By choosing signal strength x, a cost $c(x)$ is incurred immediately, and with probability $P(X \leqslant x | X > y)$ the

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problem ends with no extra cost. With probability $P(X>x|X>y)=\bar{F}(x)/\bar{F}(y)$, however, the signal strength x still cannot reach the receiver and the state becomes x. Denote by $V(y)$ the minimum expected additional energy consumed if the mobile device is in state y, which must satisfy the following optimality equation:

$$V(y)=\min_{x\in[y,1]} c(x)+\frac{\bar{F}(x)}{\bar{F}(y)}V(x). \tag{2}$$

Technically speaking, knowing $X>y$, the decision space should be $(y,1]$. However, in Equation (2) we use $[y,1]$; hence, we can properly define the minimum of a continuous function over a compact set (both $c(x)$ and $\bar{F}(x)$ are continuous by assumption, and the proof $V(x)$ is continuous is given in Section 3.2). Including y as a feasible action does not invalidate Equation (2) because y would never be the minimizer of the objective function.

Let $\pi(y)$ denote the optimal signal strength for the mobile device's next attempt if the mobile device is currently in state y, and define

$$\pi(y)\equiv\max\arg\min_{x\in[y,1]} c(x)+\frac{\bar{F}(x)}{\bar{F}(y)}V(x).$$

In other words, we choose the largest optimal signal strength in case there is a tie. The rationale of choosing the largest minimizer is that this policy will achieve the same minimum expected cost with fewer attempts. Note that $\pi(y)$ is a function that maps from the decision space $[0,1)$ to the action space $(0,1]$. In the remainder of this paper, we will refer $\pi(y)$ as the optimal policy in state y. The optimal probing sequence defined in Equation (1) can be expressed as $\{\pi(0),\pi(\pi(0)),\ldots\}$.

3.2. Characterization of the optimal policy

To characterize the structure of the optimal policy, we first want to show that $V(y)$ is continuous and nondecreasing in y, and that $\lim_{y\uparrow 1} V(y)=c(1)$. To see $V(y)$ is nondecreasing in y, consider $y'<y$ and denote the optimal probing sequence for state y by $z_1,z_2,\ldots$. Because $z_1,z_2,\ldots$ is a feasible probing sequence for state y', and the optimal expected cost is bounded by the expected cost of any feasible policy, it follows that

$$V(y')\leqslant c(z_1)+\sum_{n=2}^{\infty}\frac{\bar{F}(z_{n-1})}{\bar{F}(y')}c(z_n)\leqslant c(z_1)+\sum_{n=2}^{\infty}\frac{\bar{F}(z_{n-1})}{\bar{F}(y)}c(z_n)=V(y),$$

where the second inequality follows because $\bar{F}(y')\geqslant\bar{F}(y)$. Therefore, $V(y)$ is nondecreasing in y. To compute $\lim_{y\uparrow 1} V(y)$, consider the following inequality:

$$c(y)\leqslant V(y)\leqslant c(1),$$

where the left-hand side follows because in state y the mobile device has to next select a signal strength at least y, which incurs an immediate cost of at least $c(y)$. The right-hand side follows because selecting a full-strength signal (signal strength 1) is a feasible action, and the optimal expected cost $V(y)$ is bounded by the cost of this feasible policy. Consequently, we have $\lim_{y\uparrow 1} V(y)=c(1)$. To see that $V(y)$ is continuous in y, we need the following lemma, which will be also useful in proving Proposition 3.1.

Lemma 3.1
The function $\bar{F}(x)V(x)$ decreases in x.

Proof
Let $x_1<x_2$. Consider a scenario where a mobile device in state x_1 is told whether $X>x_2$ and then follows the optimal policy. If $X>x_2$, the state becomes x_2; otherwise, the mobile device learns

that $X \in (x_1, x_2]$. Upon conditioning on whether $X > x_2$, we can write the optimal expected energy consumed in this scenario as

$$\frac{\bar{F}(x_2)}{\bar{F}(x_1)} V(x_2) + \left(1 - \frac{\bar{F}(x_2)}{\bar{F}(x_1)}\right) K,$$

where $K > 0$ represents the expected additional energy consumed if the mobile device learns $X \in (x_1, x_2]$. The preceding cost is a lower bound for $V(x_1)$ because having more information cannot hurt. Hence,

$$V(x_1) \geqslant \frac{\bar{F}(x_2)}{\bar{F}(x_1)} V(x_2) + \left(1 - \frac{\bar{F}(x_2)}{\bar{F}(x_1)}\right) K > \frac{\bar{F}(x_2)}{\bar{F}(x_1)} V(x_2).$$

Multiplying both sides by $\bar{F}(x_1)$ completes the proof. □

From Lemma 3.1, it follows that, if $y' < y$, then

$$V(y) - V(y') < \left(1 - \frac{\bar{F}(y)}{\bar{F}(y')}\right) V(y) \leqslant \left(1 - \frac{\bar{F}(y)}{\bar{F}(y')}\right) c(1),$$

which converges to 0 as $y' \to y$ because $\bar{F}(y)$ is continuous. Therefore, $V(y)$ is a continuous function. The proposition below states that the optimal policy $\pi(y)$ is nondecreasing in y for $y \in (0, 1]$.

Proposition 3.1
The optimal policy $\pi(y)$ is nondecreasing in y for $y \in (0, 1]$.

Proof
Suppose $y' > y$; we need to show that $\pi(y') \geqslant \pi(y)$ to complete the proof. Consider two cases:

1. $\pi(y) \leqslant y'$: Because $\pi(y) \leqslant y' < \pi(y')$, the proposition is trivially true.
2. $\pi(y) > y'$: We need to show that, for all $x < \pi(y)$,

$$c(\pi(y)) + \frac{\bar{F}(\pi(y))}{\bar{F}(y')} V(\pi(y)) \leqslant c(x) + \frac{\bar{F}(x)}{\bar{F}(y')} V(x), \tag{3}$$

so that any signal strength smaller than $\pi(y)$ cannot be optimal for state y'.

To do so, consider any $x < \pi(y)$ and rewrite the left-hand side of Equation (3) as

$$\begin{aligned}
&\left(c(\pi(y)) + \frac{\bar{F}(\pi(y))}{\bar{F}(y)} V(\pi(y))\right) - \frac{\bar{F}(\pi(y))}{\bar{F}(y)} V(\pi(y)) + \frac{\bar{F}(\pi(y))}{\bar{F}(y')} V(\pi(y)) \\
&\leqslant \left(c(x) + \frac{\bar{F}(x)}{\bar{F}(y)} V(x)\right) + \bar{F}(\pi(y)) V(\pi(y)) \left(\frac{1}{\bar{F}(y')} - \frac{1}{\bar{F}(y)}\right) \\
&\leqslant c(x) + \frac{\bar{F}(x)}{\bar{F}(y)} V(x) + \bar{F}(x) V(x) \left(\frac{1}{\bar{F}(y')} - \frac{1}{\bar{F}(y)}\right) \\
&= c(x) + \frac{\bar{F}(x)}{\bar{F}(y')} V(x),
\end{aligned}$$

where the first inequality follows from the definition of $\pi(y)$ and the second follows from Lemma 3.1.

Therefore, the proof is completed. □

Define $\Gamma_1 \subseteq [0, 1)$ as the set of states in which it is optimal to next send a full-strength signal; in other words,

$$\Gamma_1 \equiv \{y : \pi(y) = 1\}.$$

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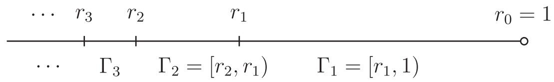

Figure 1. The structure of the optimal policy; $\Gamma_i=[r_i,r_{i-1})$, $i=1,2,\ldots$.

If Γ_1 is nonempty (as will be shown in Section 3.3), then from Proposition 3.1, Γ_1 must be an interval whose right endpoint is 1; see Figure 1.

For convenience, define the decision space $\Omega\equiv(0,1]$. If $\Omega-\Gamma_1$ is empty, then the optimal probing sequence for state 0 is trivial such that $\pi(y)=1$ for all $y\in\Omega$. On the other hand, if $\Omega-\Gamma_1$ is nonempty, then there must exist $\tilde{y}\in\Omega-\Gamma_1$ such that $\pi(\tilde{y})\in\Gamma_1$, because otherwise there is a positive probability $P(X\in\Gamma_1)$ that the probing process will go on indefinitely—resulting in an infinite expected total cost. Now consider any state $y>\tilde{y}$ and $y\notin\Gamma_1$. Because $\pi(y)<1$ $(y\notin\Gamma_1)$ and $\pi(y)>\pi(\tilde{y})$ (from Proposition 3.1), we can conclude that $\pi(y)$ is also in Γ_1. In other words, the set of states in which it is optimal to next select a signal strength in Γ_1, defined as

$$\Gamma_2\equiv\{y:\pi(y)\in\Gamma_1\},$$

is also an interval.

By following the same logic, we can define Γ_3, Γ_4, and so on. Consequently, we can describe the optimal policy by partitioning the decision space Ω into intervals $\Gamma_1,\Gamma_2,\ldots,$ such that $\pi(y)=1$ if $y\in\Gamma_1$ and $\pi(y)\in\Gamma_i$ for $y\in\Gamma_{i+1}$, $i=1,2,\ldots$.

The preceding argument of partitioning the decision space Ω into Γ_i, $i\geqslant1$, is based on the assumption that there exists some state in which it is optimal to next send a full-strength signal. In Section 3.3, we validate this assumption and, at the same time, show that each interval Γ_i is closed on the left-hand side and open on the right-hand side (see Figure 1). Therefore, we can write $\Gamma_i=[r_i,r_{i-1})$; in particular, $r_0=1$. We will also present a method to compute r_i, $i=1,2,\ldots$.

3.3. *Computation of the optimal policy*

In this subsection, we show how to compute the optimal policy. Specifically, we present an algorithm to recursively compute Γ_i, for $i=1,2,\ldots$.

3.3.1. *Compute* Γ_1.

To compute Γ_1, we first define $V_n(y)$ as the expected additional cost (energy consumed) in state y if the mobile device follows the optimal policy for the next n transmission attempts, but has to send a full-strength signal (signal strength 1) if all those n attempts fail. Denoting the optimal probing sequence for state y by $z_1,z_2,\ldots$, the difference between $V_n(y)$ and $V(y)$ can be expressed as

$$V_n(y)-V(y)=\frac{\bar{F}(z_n)}{\bar{F}(y)}(c(1)-V(z_n)).$$

Because for each feasible policy $\lim_{n\to\infty}z_n=1$ (Property 3 in Definition 2.1), the right-hand side of the preceding equation converges to 0 as $n\to\infty$, which implies $\lim_{n\to\infty}V_n(y)=V(y)$.

By definition, $V_0(y)=c(1)$ for $y\in\Omega$. Letting $V_n(1)\equiv0$ for $n\geqslant0$, we can properly write the following recursive equation for $V_n(y)$, $n\geqslant1$:

$$\begin{aligned}V_n(y)&=\min_{x\in[y,1]}c(x)+\frac{\bar{F}(x)}{\bar{F}(y)}V_{n-1}(x)\\&=\min\left\{c(1),\min_{x\in[y,1]}c(x)+\frac{\bar{F}(x)}{\bar{F}(y)}V_{n-1}(x)\right\}.\end{aligned}\tag{4}$$

In Equation (4), we write the objective function as the minimum between two terms so that the problem can be interpreted as an *optimal stopping problem*. In this optimal stopping problem, after each unsuccessful attempt, the mobile device can decide whether to *stop* or *continue*. If the mobile

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device decides to stop, it then sends a full-strength signal to complete the transmission, in which case the total additional cost is $c(1)$. If the mobile device decides to continue, it then attempts a signal strength strictly less than 1 and continues the probing process. Because the mobile device needs to decide the best signal strength to send once it decides to continue, the cost function for continuing is itself an optimization problem.

Technically speaking, once the mobile device decides to continue, the decision space should be an open interval $(y,1)$ rather than a closed interval $[y,1]$, as seen in Equation (4). We use a closed interval so that the minimum can be properly defined on a compact set. In addition, using the closed interval $[y,1]$ as the decision space does not invalidate Equation (4) because $x=y$ will not be the minimizer for the continuing decision, while $x=1$ will just yield $c(1)$—the cost if the mobile device decides to stop. In other words, with a closed decision space, the optimal decision for this n-stage problem is to stop if and only if

$$c(1)\leqslant \min_{x\in[y,1]} c(x)+\frac{\bar{F}(x)}{\bar{F}(y)}V_{n-1}(x).$$

Let $B_1\subseteq[0,1)$ denote the set of states in which it is better to stop, rather than to continue (send a signal strength less than 1) and then stop if that first transmission attempt fails. In other words,

$$B_1\equiv\left\{y:c(1)\leqslant \min_{x\in[y,1]} c(x)+\frac{\bar{F}(x)}{\bar{F}(y)}c(1)\right\}. \tag{5}$$

When y increases, the tail distribution function $\bar{F}(y)$ decreases, and therefore

$$\min_{x\in[y,1]} c(x)+\frac{\bar{F}(x)}{\bar{F}(y)}c(1)$$

increases. Consequently, if there exists $\bar{y}\in B_1$, then $y\in B_1$ for all $y>\bar{y}$. We say B_1 is *absorbing* because once the mobile device selects a signal strength in B_1, all future signal strengths must be in B_1.

Because B_1 is absorbing, the *one-stage look-ahead policy*—stop for the first time when the mobile device enters a state in B_1—is optimal for this optimal stopping problem; for example, see Ross [12] or Bertsekas [13] for more discussions on optimal stopping problems. We present this result in the next proposition.

Proposition 3.2
The sets B_1 and Γ_1 are identical.

Proof
We first show that $\Gamma_1\subseteq B_1$. Suppose $y\in\Gamma_1$; then by definition

$$c(1)\leqslant \min_{x\in[y,1]} c(x)+\frac{\bar{F}(x)}{\bar{F}(y)}V(x)\leqslant \min_{x\in[y,1]} c(x)+\frac{\bar{F}(x)}{\bar{F}(y)}c(1),$$

where the second inequality follows because $V(x)\leqslant c(1)$ for all $x\in\Omega$. In other words, $y\in B_1$, and therefore $\Gamma_1\subseteq B_1$.

To prove that $B_1\subseteq\Gamma_1$, we first use mathematical induction to show that for $y\in B_1$, $V_n(y)=c(1)$, $n\geqslant 0$. The statement is clearly true for $n=0$. Supposing that the statement is also true for $n-1$ so that $V_{n-1}(y)=c(1)$ for $y\in B_1$, then using Equation (4) we have

$$V_n(y)=\min\left\{c(1),\ \min_{x\in[y,1]} c(x)+\frac{\bar{F}(x)}{\bar{F}(y)}c(1)\right\}=c(1),$$

where the first equality follows from the induction hypothesis, and the second equality follows from the definition of B_1. Consequently, $V(y)=\lim_{n\to\infty}V_n(y)=c(1)$ for $y\in B_1$, which implies that in state $y\in B_1$ it is optimal to stop—to send a full-strength signal. Hence, $B_1\subseteq\Gamma_1$, and the proof is completed. □

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Because both the tail distribution function $\bar{F}(x)$ and the cost function $c(x)$ are continuous, it follows that B_1 is closed, and therefore we can define

$$r_1 \equiv \max\{0, \min\{y : y \in B_1\}\}.$$

We then arrive at the conclusion that $\Gamma_1 = [r_1, 1)$. If $r_1 = 0$, then in state 0 it is optimal to immediately send a full-strength signal; otherwise, $\Gamma_1^c = [0, r_1)$ is nonempty and we need to compute Γ_2.

3.3.2. Compute Γ_2. Suppose $r_1 > 0$ so that $\Gamma_1^c = [0, r_1)$ is nonempty. In this subsection, we show how to compute Γ_2, the set of states in which it is optimal to next send a signal strength in Γ_1.

For $y < r_1$, define $J_n(y)$ as the expected additional cost in state y if the mobile device follows the optimal policy for the next n transmission attempts, but has to select a signal strength in $[r_1, 1]$ if all those n attempts fail. By definition, for $y < r_1$, we have that

$$J_0(y) = \min_{x \in [r_1,1]} c(x) + \frac{\bar{F}(x)}{\bar{F}(y)} V(x) = \min_{x \in [r_1,1]} c(x) + \frac{\bar{F}(x)}{\bar{F}(y)} c(1), \tag{6}$$

where the second equality follows because $V(x) = c(1)$ for $x \geqslant r_1$.

Denoting the optimal probing sequence for state y by $z_1, z_2, \ldots$, the difference between $J_n(y)$ and $V(y)$ can be expressed as

$$J_n(y) - V(y) = \begin{cases} \dfrac{\bar{F}(z_n)}{\bar{F}(y)}(J_0(z_n) - V(z_n)) \leqslant \dfrac{\bar{F}(z_n)}{\bar{F}(y)}(c(1) - V(z_n)) & \text{if } z_n < r_1; \\ 0 & \text{if } z_n \geqslant r_1. \end{cases}$$

Because for each feasible policy $\lim_{n\to\infty} z_n = 1$ (Property 3 in Definition 2.1), the preceding equation shows that $\lim_{n\to\infty} J_n(y) = V(y)$.

Although $J_n(y)$, $n \geqslant 0$, is only defined for $y < r_1$, we let $J_n(r_1) \equiv c(1)$ for $n \geqslant 0$; hence we can conveniently write the recursive equation for J_n as follows:

$$J_n(y) = \min\left\{ J_0(y), \min_{x \in [y, r_1]} c(x) + \frac{\bar{F}(x)}{\bar{F}(y)} J_{n-1}(x) \right\}. \tag{7}$$

We can again interpret this problem as an optimal stopping problem. In this case, a mobile device in state $y \in [0, r_1)$ can either *stop* by selecting a signal strength in $[r_1, 1]$, in which case the cost is $J_0(y)$, or *continue* by selecting a signal strength in $[y, r_1)$, in which case the mobile device is allowed at most $n-1$ additional transmission attempts in $[y, r_1)$. Note that in Equation (7) we use the closed interval $[y, r_1]$ as the decision space if the mobile device decides to continue so that the minimum can be properly defined on a closed interval. However, using a closed decision space $[y, r_1]$—as opposed to $[y, r_1)$—makes no mathematical difference in Equation (7) because the optimal decision is still to stop if the stopping cost is smaller than or equal to the continuing cost.

Let B_2 denote the set of states—as a subset of $[0, r_1]$—in which it is better to stop, rather than to continue (send a signal strength less than r_1) and then stop if that first transmission attempt fails. In other words,

$$B_2 \equiv \left\{ y : J_0(y) \leqslant \min_{x \in [y, r_1]} c(x) + \frac{\bar{F}(x)}{\bar{F}(y)} J_0(x) \right\}. \tag{8}$$

Unfortunately, from this equation it is rather difficult to determine whether B_2 is absorbing; otherwise, we can use a similar argument to that in the previous subsection to show that $B_2 = \Gamma_2$. We can, however, see from this equation that B_2 is closed because all functions involved are continuous and the minimum is properly defined. Technically speaking, however, we cannot rule out the possibility that B_2 may contain multiple disjoint closed intervals; see Figure 2 for an example.

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Figure 2. The definition of r_2 in the case B_2 contains three closed intervals.

First note that $r_1 \in B_2$ trivially. Define r_2 as the left endpoint of the interval that contains r_1. That is (see Figure 2 for an example),

$$r_2 \equiv \max\{0, \min\{y : x \in B_2 \text{ for all } x \in [y, r_1]\}\}.$$

Recall from Section 3.2 that $\Gamma_2 = \{y : \pi(y) \in \Gamma_1\}$—the set of states in which it is optimal to next select a signal strength in Γ_1—is an interval whose right endpoint is r_1. The next proposition shows that $\Gamma_2 = [r_2, r_1)$.

Proposition 3.3
The set $\Gamma_2 = [r_2, r_1)$.

Proof
We first show that $\Gamma_2 \subseteq [r_2, r_1)$. Suppose $y \in \Gamma_2$; then by definition in state y it is optimal to next select a signal strength in Γ_1. Therefore, for $y \in \Gamma_2$,

$$J_0(y) \leqslant \min_{x \in [y, r_1]} c(x) + \frac{\bar{F}(x)}{\bar{F}(y)} V(x) = \min_{x \in [y, r_1]} c(x) + \frac{\bar{F}(x)}{\bar{F}(y)} J_0(x),$$

where the equality follows because if $y \in \Gamma_2$, then according to Proposition 3.1 we have that $x \in \Gamma_2$ for all $x \in [y, r_1)$, and therefore $V(x) = J_0(x)$ for all $x \in [y, r_1)$. This equation shows that $y \in B_2$, so that $\Gamma_2 \subseteq B_2$. In addition, because Γ_2 is an interval whose right endpoint is r_1, it follows that $\Gamma_2 \subseteq [r_2, r_1)$.

To prove that $[r_2, r_1) \subseteq \Gamma_2$, we first use mathematical induction to show that for $y \in [r_2, r_1)$, $J_n(y) = J_0(y)$, $n \geqslant 0$. The statement is trivially true for $n = 0$. Supposing that the statement is also true for $n-1$ so that $J_{n-1}(y) = J_0(y)$ for $y \in [r_2, r_1)$, then starting from Equation (7) we have

$$J_n(y) = \min\left\{ J_0(y), \min_{x \in [y, r_1]} c(x) + \frac{\bar{F}(x)}{\bar{F}(y)} J_0(x) \right\} = J_0(y),$$

where the first equality follows from the induction hypothesis, and the second equality follows because $[r_2, r_1) \subseteq B_2$. Consequently, $V(y) = \lim_{n\to\infty} J_n(y) = J_0(y)$ for $y \in [r_2, r_1)$, which implies that in state $y \in [r_2, r_1)$ it is optimal to next select a signal strength in Γ_1. Hence, $[r_2, r_1) \subseteq \Gamma_2$, and the proof is completed. □

If $r_2 = 0$, then in state 0 it is optimal to first select a signal strength in Γ_1, and then to send a full-strength signal if that first transmission attempt fails. In other words, the optimal probing sequence for state 0 is $z_1 = \pi(0)$ and $z_2 = 1$, where $\pi(0)$ can be solved from Equation (6) by letting $y = 0$. On the other hand, if $r_2 > 0$, then we can use the same method to compute r_3 to find $\Gamma_3 = [r_3, r_2)$, and examine whether $r_3 = 0$. Consequently, we can repeat this process to compute $r_1, r_2, \ldots$ until $r_k = 0$ for some k. We can completely calculate the optimal policy function $\pi(y)$ for $y \in \Omega$. The optimal probing sequence for state 0 can be described by $\pi(0), \pi(\pi(0)), \ldots, 1$.

Remark 3.1
One interesting observation, out of mathematical curiosity, is that an optimal probing policy does not always exist. To construct such an example, suppose that the distance X follows a uniform distribution. The expected total cost for a probing sequence $z_1, z_2, \ldots, z_n$ can be written as

$$c(z_1) + c(z_2)(1 - z_1) + c(z_3)(1 - z_2) + \cdots + c(z_n)(1 - z_{n-1}). \tag{9}$$

DOI: 10.1002/oca

Consider another probing sequence by inserting $y < z_1$ at the very beginning to obtain $y, z_1, z_2, \ldots, z_n$. The expected total cost for this new probing sequence is

$$c(y)+c(z_1)(1-y)+c(z_2)(1-z_1)+c(z_3)(1-z_2)+\cdots+c(z_n)(1-z_{n-1}). \tag{10}$$

By taking the difference between Equations (9) and (10), this new probing sequence $y, z_1, z_2, \ldots, z_n$ is better if

$$c(y)+c(z_1)(1-y)-c(z_1)=c(y)-yc(z_1)<0,$$

or equivalently, if $c(y)/y < c(z_1)$. If the cost function $c(x)$ has the property that $\lim_{x\to 0} c(x)/x = 0$, then for any sequence $z_1, z_2, \ldots, z_n$ it is possible to find another sequence that is strictly better; hence, no sequence is optimal. In practice, however, this property of the cost function implies that the internal circuits do not consume any energy (the case $b=0$ in Section 4), which is most likely not the case.

4. QUADRATIC COST FUNCTION AND UNIFORM DISTANCE DISTRIBUTION

In this section, we demonstrate the algorithm discussed in Section 3 by considering a quadratic cost function and a uniform distribution for the distance between the mobile device and the receiver.

To model the energy consumption, we choose a simple quadratic cost function such that $c(x) = b+(1-b)x^2$, where $b \in [0, 1]$ is a constant. The first term in the cost function, b, represents the energy consumption incurred for each attempt and is independent of the signal strength. This portion includes the energy consumed by the electronic circuits and the mobile device's communication module in the receiving mode, while awaiting the receiver's acknowledgment. The second term in the cost function, $(1-b)x^2$, represents the transmission energy that is proportional to the square of the signal strength x. This quadratic cost term is motivated by the assumption that the mobile device has an isotropic antenna in a free space; that is, it emits radio energy homogeneously in all directions in an unobstructed space. In such a case, the signal strength at a certain distance is determined by the radio energy density, which is the total energy emitted divided by the spherical surface area at the distance. Because the spherical surface area is proportional to the square of the distance, the energy needed to provide sufficient signal strength to establish communication is also proportional to the square of the distance.

To choose a distribution function for X—the normalized distance between the mobile device and the stationary receiver—we consider the application of house arrest, in which the person under house arrest wears an electronic bracelet that contains a transmitter and a stationary receiver is typically placed in the center of the house. We assume that the percentage of time the person spends in one square foot is proportional to the reciprocal of the location's distance from the center of the house. The density function for X evaluated at x is proportional to $2\pi x \cdot (1/x) = 2\pi$—a constant. Therefore, X follows a uniform distribution. Below we demonstrate the algorithm developed in Section 3 by letting $c(x) = b+(1-b)x^2$ and $F(x) = x$, for $x \in [0, 1]$.

4.1. *Compute* Γ_1

In this subsection, we find Γ_1—the set of states in which it is optimal for the mobile device to next send a full-strength signal. From Proposition 3.2 and the definition of B_1 in Equation (5), $y \in \Gamma_1$ if and only if

$$1 \leqslant \min_{x\in[y,1]} (1-b)x^2+b+\frac{1-x}{1-y}.$$

Note that the objective function in the right-hand side of the preceding is a quadratic function with a positive leading coefficient, and its value is equal to 1 when $x = 1$. Therefore, the inequality will

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hold if and only if the first derivative of the quadratic function evaluated at $x=1$ is less than or equal to 0. In other words, $y\in\Gamma_1$ if and only if

$$\left(2(1-b)x-\frac{1}{1-y}\right)\bigg|_{x=1}\leqslant 0,$$

or equivalently

$$y\geqslant\frac{1-2b}{2-2b}. \tag{11}$$

If $b\geqslant 0.5$—the fixed energy consumption accounts for more than 50% of the energy consumption of a full-strength signal—then $r_1=\max\{0,(1-2b)/(2-2b)\}=0$, in which case $\Gamma_1=[0,1)$ and the optimal policy in state 0 is $\pi(0)=1$. In other words, the mobile device should simply send a full-strength signal each time it attempts to connect to the receiver. On the other hand, if $b<0.5$, then $r_1>0$ and Γ_1^c is nonempty, which we discuss in Section 4.2.

4.2. *Compute* Γ_2

In this subsection, we consider the case $b<0.5$, so that in state 0 it is not optimal for the mobile device to send a full-strength signal. In particular, we want to find Γ_2—the set of states in which it is optimal to next select a signal strength in $\Gamma_1=[(1-2b)/(2-2b),1)$.

For $y\in\Gamma_1^c$, using Equation (6) we have that

$$J_0(y)=\min_{x\in[(1-2b)/(2-2b),1]}(1-b)x^2+b+\frac{1-x}{1-y}. \tag{12}$$

The minimizer for the quadratic function in Equation (12) is

$$x=\frac{1}{2(1-b)(1-y)}, \tag{13}$$

which is indeed in Γ_1 for $y\in\Gamma_1^c$. Substituting the preceding into Equation (12) yields

$$J_0(y)=-\frac{1}{4(1-b)(1-y)^2}+\frac{1}{1-y}+b\quad\text{for } y<\frac{1-2b}{2-2b}. \tag{14}$$

According to Equation (8), for $y\in[0,(1-2b)/(2-2b))$, $y\in B_2$ if and only if

$$-\frac{1}{4a(1-y)^2}+\frac{1}{1-y}+b\leqslant\min_{x\in[y,(1-2b)/(2-2b)]}(1-b)x^2+b+\frac{1-x}{1-y}\left(-\frac{1}{4(1-b)(1-x)^2}+\frac{1}{1-x}+b\right),$$

which, after consolidating similar terms on both sides, is equivalent to

$$\min_{x\in[y,(1-2b)/(2-2b)]}g(x,y)\geqslant 0, \tag{15}$$

where

$$g(x,y)\equiv(1-b)x^2+\frac{1-x}{1-y}b+\frac{1}{4(1-b)}\left(\frac{1}{(1-y)^2}-\frac{1}{(1-y)(1-x)}\right).$$

It is rather difficult to find a closed-form solution for B_2 because Equation (15) involves a minimization problem of a complicated function, where the argument y appears in both the constraint and the objective function. However, we can at least get an idea what the set B_2 defined by Equation (15) looks like. Take the partial derivative of $g(x,y)$ with respect to y to obtain

$$\begin{aligned}\frac{\partial g(x,y)}{\partial y}&=\frac{(1-x)b}{(1-y)^2}+\frac{1}{4(1-b)}\left(\frac{2}{(1-y)^3}-\frac{1}{(1-y)^2(1-x)}\right)\\&=\frac{(1-x)b}{(1-y)^2}+\frac{1}{4(1-b)}\frac{1}{(1-y)^3(1-x)}(1+y-2x),\end{aligned}$$

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which is positive for $x \in [0, (1-2b)/(2-2b)]$. In other words, for a fixed x, $g(x, y)$ increases in y, and therefore

$$\min_{x \in [y, (1-2b)/(2-2b)]} g(x, y)$$

also increases in y because the feasible region for x in the minimization problem becomes a smaller set as y increases. Consequently, we can use a simple search algorithm to numerically solve for $\bar{y}$ in

$$\min_{x \in [\bar{y}, (1-2b)/(2-2b)]} g(x, \bar{y}) = 0.$$

If $\bar{y} > 0$, then $\Gamma_2 = [\bar{y}, (1-2b)/(2-2b))$; otherwise $\Gamma_2 = [0, (1-2b)/(2-2b))$, in which case the optimal probing sequence for state 0 consists of two transmission attempts.

4.3. Find conditions for $0 \in \Gamma_2$

When designing the optimal probing sequence for a given value of b, it is helpful to find the condition for $0 \in \Gamma_2$, in which case the optimal probing sequence consists of two transmission attempts. To answer this question, note that $0 \in \Gamma_2$ if and only if $y = 0$ satisfies Equation (15), or equivalently

$$\min_{x \in [0, (1-2b)/(2-2b)]} (1-b)x^2 + (1-x)b - \frac{1}{4(1-b)} \frac{x}{(1-x)} \geqslant 0.$$

After multiplying $4(1-b)(1-x)$, the preceding equation becomes

$$\min_{x \in [0, (1-2b)/(2-2b)]} -4(1-b)^2 x^3 + 4(1-b)x^2 - (1+8b-8b^2)x + 4b(1-b) \geqslant 0. \tag{16}$$

If the preceding equation holds for some $b \in [0, 0.5)$, then in state 0 it is optimal to next select a signal strength in Γ_1, which implies that the optimal probing sequence consists of two attempts. We are interested in finding the values for $b \in [0, 0.5)$ such that Equation (16) holds.

Note that

$$h(x, b) \equiv -4(1-b)^2 x^3 + 4(1-b)x^2 - (1+8b-8b^2)x + 4b(1-b) \tag{17}$$

is a cubic polynomial in x with a negative leading coefficient. We consider the following two cases:

1. The function $h(x, b)$ is nonincreasing in x:
   Taking derivative with respect to x yields

   $$\frac{\partial h(x, b)}{\partial x} = -12(1-b)^2 x^2 + 8(1-b)x - (1+8b-8b^2). \tag{18}$$

   This quadratic polynomial in x—with a negative leading coefficient—is always less than or equal to 0 if and only if its discriminant is less than or equal to 0; that is,

   $$64(1-b)^2 - 4(12)(1-b)^2(1+8b-8b^2) \leqslant 0,$$

   which is equivalent to

   $$24b^2 - 24b + 1 \leqslant 0,$$

   implying

   $$\frac{6-\sqrt{30}}{12} \leqslant b \leqslant \frac{6+\sqrt{30}}{12}.$$

DOI: 10.1002/oca

Therefore, for $(6-\sqrt{30})/12 \leqslant b < 0.5$, the left-hand side of Equation (16) is minimized when $x = (1-2b)/(2-2b)$, and the minimized value is equal to

$$\frac{2b^2}{1-b} \geqslant 0.$$

Consequently, Equation (16) holds for $(6-\sqrt{30})/12 \leqslant b < 0.5$.

2. The function $h(x, b)$ has one local minimum:
If $0 < b < (6-\sqrt{30})/12 \approx 0.04356$, then by setting Equation (18) equal to 0 we can see that $h(x, b)$ has a local minimum at

$$x^* = \frac{2-\sqrt{24b^2-24b+1}}{6(1-b)}.$$

Therefore, Equation (16) holds if and only if $h(x^*, b) \geqslant 0$, which—after substituting x^* into Equation (17) and some algebra—is equivalent to

$$10b^3 - 38b^2 + 30b - 1 \geqslant 0.$$

By plotting the function $10b^3 - 38b^2 + 30b - 1$, it follows that the preceding holds if and only if b is greater than or equal to the smallest root to $10b^3 - 38b^2 + 30b - 1$, which is approximately equal to 0.034858.

From these two cases, we can conclude that if $b > 0.034858$, then $0 \in \Gamma_2$ and using Equation (13) we have

$$\pi(0) = \frac{1}{2(1-b)(1-0)} = \frac{1}{2(1-b)},$$

and $\pi(\pi(0)) = 1$. The minimized expected total cost is equal to

$$\frac{3-4b^2}{4(1-b)}$$

by substituting $y = 0$ into Equation (14). Finally, we summarize our findings in a corollary.

Corollary 4.1
Let $c(x) = b + (1-b)x^2$ and X follow a uniform distribution. If $b \geqslant 0.5$, then the optimal probing sequence consists of one attempt, namely 1 (maximum transmission power). If $0.034858 \leqslant b < 0.5$, then the optimal probing sequence consists of two attempts: $\{1/2(1-b), 1\}$. If $b < 0.034858$, then the optimal probing sequence consists of at least three transmission attempts.

5. NUMERICAL EXAMPLES

As seen in Section 4, it is rather complicated to compute the optimal policy. In practice, we can use a discrete model to approximate the optimal policy. Specifically, divide $[0, 1]$ into n equal-length subintervals such that the distance a signal can reach must be a multiple of $1/n$. Define the following notations from their counterparts in the continuous model:

$$c_i \equiv c\left(\frac{i}{n}\right), \quad \bar{F}_i \equiv \bar{F}\left(\frac{i}{n}\right), \quad i = 1, \ldots, n.$$

When a signal of strength i/n, $i = 0, \ldots, n$, fails to reach the receiver, let V_i denote the optimal additional expected cost and π_i (an integer between $i+1$ and n) the optimal signal strength to send next. The recursive equation can be written as

$$V_i = \min_{i+1 \leqslant j \leqslant n} c_j + \frac{\bar{F}_j}{\bar{F}_i} V_j, \quad i = 0, \ldots, n-1,$$

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Table I. Performance of six naive heuristic policies, reported as a ratio to the optimal cost. The distance X follows a beta distribution with parameters (α, β), and $c(x)=b+(1-b)x^2$.

				Expected cost of six heuristic policies using the following probing sequences* (reported as a ratio to the optimal cost)					
b	α	β	Optimal cost	Mean of X	Mode of X	Median of X	Quartiles of X	0.5	0.25, 0.5, 0.75
0.01	2	8	0.1471	3.30	4.85	3.68	2.34	1.88	1.09
	4	6	0.4006	1.63	1.73	1.66	1.49	1.28	1.10
	6	4	0.7093	1.25	1.20	1.23	1.47	1.41	1.29
	8	2	0.9693	1.25	1.12	1.21	1.74	1.28	1.63
0.03	2	8	0.1737	2.91	4.22	3.23	2.23	1.68	1.06
	4	6	0.4191	1.59	1.70	1.62	1.51	1.26	1.13
	6	4	0.7185	1.25	1.20	1.23	1.49	1.42	1.33
	8	2	0.9706	1.25	1.12	1.22	1.76	1.29	1.68
0.1	2	8	0.2544	2.25	3.15	2.47	2.12	1.35	1.04
	4	6	0.4792	1.52	1.61	1.54	1.60	1.21	1.23
	6	4	0.7493	1.26	1.21	1.24	1.57	1.43	1.46
	8	2	0.9750	1.27	1.13	1.24	1.81	1.34	1.82

*For each probing sequence consisting of n signal strengths, we list only the partial sequence $z_1, \ldots, z_{n-1}$, with the understanding that $z_n = 1$. For instance, the first sequence consists of $z_1 =$ mean of X, $z_2 = 1$; the fourth sequence consists of $z_i = i$th quartile of X, $i = 1, 2, 3$, and $z_4 = 1$; the last sequence consists of $z_i = i/4$, $i = 1, 2, 3, 4$.

with the boundary condition $V_n = 0$, and the optimal policy is

$$\pi_i = \max \arg \min_{i+1 \leqslant j \leqslant n} c_j + \frac{\bar{F}_j}{\bar{F}_i} V_j, \quad i = 0, \ldots, n-1.$$

We can use V_i to approximate $V(i/n)$ and π_i to approximate $\pi(i/n)$.

In the rest of this section, we present two numerical examples by using this discrete approximation and by letting $n = 10^4$. In the first numerical example, we compare the optimal policy with a few naive heuristic policies. In the second numerical example, we study the robustness of the optimal policy.

In the first numerical example, we let X follow the beta distribution with parameters (α, β) such that $\alpha + \beta = 10$, and let $c(x) = b + (1-b)x^2$. The beta distribution with the chosen parameters exhibits a unimodal density function with the expected value equal to $\alpha/(\alpha+\beta) = \alpha/10$. By varying α and b, we compare the optimal cost and the expected cost of six heuristic policies in Table I. The fourth column in Table I gives the expected cost with the optimal policy, whereas the costs of the six heuristic policies are reported as the ratio to the optimal cost. In the first four heuristic policies, we consider probing sequences that are based on the distribution of X. For each probing sequence consisting of n signal strengths, we list only the partial sequence $z_1, \ldots, z_{n-1}$, with the understanding that $z_n = 1$. For instance, the first sequence consists of $z_1 =$ mean of X, $z_2 = 1$; the fourth sequence consists of $z_i = i$th quartile of X, $i = 1, 2, 3$, and $z_4 = 1$. In the last two heuristic policies, the probing sequence is independent of the distribution of X. For instance, the last sequence consists of $z_i = i/4$, $i = 1, 2, 3, 4$.

As seen in Table I, it is not surprising that none of these heuristic policies perform uniformly well in all cases. The best heuristic policy in the group is probably the last one. This sequence performs better when α is small—when the mobile and the receiver tend to be closer—but the performance deteriorates quickly as α increases. These examples demonstrate that a naive heuristic policy is likely to perform poorly compared with the optimal policy.

In the second numerical example, we study the robustness of the optimal policy when the estimated distribution of X—the distance between the mobile device and the receiver—is different from the true distribution. Again, the distance X follows a beta distribution with parameters (α, β),

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Table II. Performance when using the policy that would have been optimal for another distance distribution, reported as a ratio to the optimal cost. The distance X follows a beta distribution with parameters (α, β), and $c(x)=b+(1-b)x^2$.

				Expected cost as ratio to the optimal cost*			
b	α	β	Optimal cost	(α^-, β^+)	(α^+, β^-)	(α^-, β^-)	(α^+, β^+)
0.01	2	8	0.1471	1.020	1.031	1.010	1.011
	4	6	0.4006	1.020	1.026	1.002	1.001
	6	4	0.7093	1.023	1.026	1.002	1.002
	8	2	0.9693	1.035	1.023	1.019	1.017
0.03	2	8	0.1737	1.023	1.030	1.012	1.010
	4	6	0.4191	1.020	1.025	1.002	1.001
	6	4	0.7185	1.023	1.025	1.003	1.002
	8	2	0.9706	1.034	1.022	1.018	1.017
0.1	2	8	0.2544	1.026	1.026	1.011	1.008
	4	6	0.4792	1.020	1.022	1.001	1.001
	6	4	0.7493	1.022	1.023	1.003	1.002
	8	2	0.9750	1.032	1.019	1.016	1.016

*Use the policy that would have been optimal for another beta distribution, where $\alpha^-=\alpha-0.5$, $\beta^-=\beta-0.5$, $\alpha^+=\alpha+0.5$, and $\beta^+=\beta+0.5$.

and $c(x)=b+(1-b)x^2$. In Table II, the first three columns give the parameters and the fourth column gives the optimal cost if the parameters (α, β) are known. In the fifth column, the mobile device estimates the distribution of X to follow a beta distribution with parameters $(\alpha^-, \beta^+)=(\alpha-0.5, \beta+0.5)$ and uses the policy that would have been optimal for parameters (α^-, β^+). Of course, this policy is suboptimal for parameters (α, β), and its cost is reported as a ratio to the optimal cost. The last three columns compare the costs of another three suboptimal policies.

It is encouraging to see that the derived policy performed quite well, as all the ratios in Table II are very close to 1. This observation suggests that the derived policy is robust, when there are small errors in estimating the distance distribution. It is also intuitive that the two cases (α^-, β^+) and (α^+, β^-) are worse than the other two cases, because their deviations of α and β change $E[X]=\alpha/(\alpha+\beta)$ in the same direction.

6. CONCLUDING REMARKS

In this paper, we consider optimal probing control for a mobile device whose main purpose is to connect to the receiver with minimal data to transfer. We use dynamic programming to formulate the problem and show how to compute the optimal policy. In practice, one can implement the optimal probing policy via a discrete approximation.

When designing a probing policy, we assume that the distance between the mobile device and the nearest receiver is available by a probability distribution. In practice, such a distribution can be obtained through a site survey prior to deploying the wireless system. If a site survey is not feasible, one possible approach is to allow the mobile device to estimate this distribution in real time and to adjust its policy accordingly. Another possibility is to develop a robust policy that would work reasonably well for a range of common distributions. If the nearest receiver is often out of reach, even with the maximum transmission power, then that possibility needs to be taken into account when designing the probing sequence. These observations motivate a few possible future research directions.

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