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The negotiable alternatives identifier for group negotiation support

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Abstract

It has been observed that in single voting unanimity can rarely be reached. In many situations, however, members may not be aware that their preferences as expressed by their votes may contain room for compromise. This paper proposes a consensus-seeking methodology – the Negotiable Alternatives Identifier (NAI) – that searches for such a compromise. Starting with individual cardinal preferences on alternatives, NAI classifies alternatives into three classes of preferences: the most preferred, the less preferred and the least preferred. Within each class, relatively small differences in preferences among alternatives may make it reasonable for a decision maker to consider them interchangeable. As a result of this flexibility, a collective solution acceptable to all decision makers can be generated. In this paper we provide some theorems and their proofs to address the extreme conditions of the proposed heuristic. Also we provide an example to illustrate how to use the proposed heuristic to solve a real-world problem. © 1999 Elsevier Science Inc. All rights reserved.

Keywords: Social choices; Group decision making; Negotiation; Conflict resolution; Decision support systems

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1. Introduction

It is well known that modeling for group decision making is much more complex than that for individual decision making. As quoted from Hwang and Lin (1987):

> Moving from a single decision maker to a multiple decision maker setting introduces a great deal of complexity into the analysis. The problem is no longer the selection of the most preferred alternative among the nondominated solutions according to one individual's (single decision maker's) preference structure. The analysis must be extended to account for the conflicts among different interest groups who have different objectives, goals, criteria, and so on.

In effect, while the analysis of individual decision making benefits from a wide range of modeling tools ranging from descriptive statistics to sophisticated analytical algorithms, models that deal with group problem solving are often limited to the techniques of aggregation of preferences (e.g., Black, 1958; Borda, 1981; Cook and Seiford, 1978) and are faced with subtle interpersonal issues (Lax and Sebenius, 1986).

Thanks to their simple concepts and mechanisms, these techniques of aggregation of preferences have proven popular in many real life group decision-making situations (Hwang and Lin, 1987), for example, the selection of trajectories for the Mariner Jupiter/Saturn 1977 project. The decision rules were collective choice which were based on the sum of ordinal rankings and cardinal utility function values. However, such results in proposed compromise may or may not be acceptable to all the group members (Anscombe, 1976). Compromise has a role to play, of course. It should be embedded in a process of consensus seeking – information exchange, problem adaptation and restructuring seeking – if possible, to move to a single solution acceptable to all group members (Shakun, 1988 and 1991).

Recently, more research works in the development of new negotiation models or frameworks have been reported. For example, Holsapple et al. (1996) have summarized the negotiation theories and the parametric theory to support development of future negotiation support systems. Mumpower and Rohrbaugh (1996) proposed the Analytical Mediation Approach, which suggests hiring of an unbiased third party to assist the disputants to reach a mutually satisfactory agreement in resources allocation. Pinson and Moraitis (1996) applied the distributed artificial intelligence (DAI) and multi-agent theory to propose a general framework to support the development of intelligent distributed strategic decision making systems. Reeves and Bordetski (1995) have developed an interactive framework to support the identification of preferred range for the value of each objective of each decision maker. With such framework, decision makers are asked to specify their aspiration and reservation levels to establish the preferred ranges, group and individual solutions are, therefore, generated as a function of these ranges.

However, most of the above approaches cannot guarantee that a solution can be reached in one round of negotiation. For example, the approach proposed by Reeves and Bordetski (1995) requires the decision makers to specify their aspiration and reservation levels to develop the preferred ranges. However, decision makers always had difficulty in accurately specifying such values in the first trial. Therefore, multiple rounds of adjustment is inevitable. As consensus can rarely be reached in the first round of negotiation, this paper, therefore, proposes a consensus-seeking methodology called the Negotiable Alternatives Identifier (NAI) which entails multiple rounds of group consideration.

NAI is a formalized methodology based on an intuitive procedure observed in negotiations. This procedure typically is described as follows: "We group members failed to find a consensus. However, if some of us are willing to accept solutions other than our first choice, but which are not that far different preference-wise from the first choice, then we may find a common solution all of us can find acceptable".

The NAI algorithm, especially with its expansion/contraction/intersection concept, provides an aggregation approach which is less rigid yet intuitive enough for the decision makers to search for, and agree upon, a group solution. NAI first seeks to locate candidates for compromise, and next searches for alternatives which can be accepted by all the group members. The major difference between NAI and other approaches (e.g., Anscombe, 1976; Borda, 1981; Cook and Seiford, 1978) is that in most cases it requires only one round of negotiation to generate a solution.

Normally, a group's satisfaction decreases and the costs increase if longer meeting or more rounds of negotiations are required (Hackman and Kaplan, 1974). Also, the preferences that each member applied in the first round of negotiation reflected what he or she believed the most appropriate. Therefore, it is recommendable not to hold another round of negotiation or to adjust the preferences. But, based on the original preferences to identify an alternative which is not too far from every one's first choice.

The concepts underlying the NAI methodology are discussed in Section 2. Section 3 describes the NAI methodology and its mathematical models. In Section 4, we further investigate the search procedures by providing some theorems and their proofs to discuss the extreme conditions of NAI heuristics. An example adapted from a real life decision situation illustrating the proposed methodology is provided in Section 5. Section 6 presents concluding remarks.

2. Consensus-seeking: Problem definition and basic concepts

2.1. Definition of the problem

(1) All participants of the group problem solving share the same set of exhaustive and mutually exclusive alternatives, where n is the number of alternatives, and d the number of decision makers participating in the group problem-solving.

(2) Prior to the group decision making process, each decision maker or group member i has performed his/her own assessment of preferences. For example, the decision maker can use an additive utility method (Fishburn, 1974a, b) or the Analytic Hierarchy Process (Saaty, 1980) to obtain utilities. The output of such analysis is a vector of normalized cardinal preferences on n alternatives $\mathbf{r}_i = [r_{ij}]$, where $r_{ij} \geqslant 0$ for $i = 1, \ldots, d$, $j = 1, \ldots, n$, and $\sum_{j=1}^{n} r_{ij} = 1$.

(3) Furthermore, the vector of ranking, $\mathbf{r}_i$, is sorted according to an order of decreasing importance. This notion of preference corresponds to a complete asymmetry preorder. In other words, $r_{i,1}$ represents the relative preference of the most preferred alternative and $r_{i,n}$ the relative preference of the least preferred alternative.

(4) Given the vector $\mathbf{r}_i$, $R_{i,s}$ can be defined as the cumulative preference that a decision maker gives to the first s alternatives:

$$R_{i,s} = \sum_{j=1}^{s} r_{ij}. \tag{1}$$

2.2. Expansion/contraction/intersection concept

Starting with individual and cardinal rankings of alternatives, the proposed methodology is motivated by the following observations. First, the possibility of reaching consensus can be improved if the decision makers exhibit some flexibility regarding their individual assessment of preferences. Second, they should be able to identify exchangeable or negotiable alternatives.

Bui and Shakun (1989) propose a methodology that attempts to help the decision makers who exhibit flexibility in the assessment of preferences. It is based on the observations that the determination of the cardinal ranking of a set of alternatives is influenced by two factors:

1. The total number of alternatives being evaluated affects the intensity of preferences. Often, the greater the number of alternatives, the weaker the relative importance of the alternatives. That is, it is more difficult to tell which one is more important than the other.

2. The distribution of marginal difference among the alternatives is rarely uniform. For example, some alternatives share close evaluation (e.g., A and B

with respective scores of 0.33 and 0.32); while others score significant marginal difference (e.g., C and D with 0.25 and 0.11, respectively).

NAI is characterized by a triplet of operations: expansion, contraction, and interaction. The objective of the first operation is to assess individual preferences by locating possible areas of compromise. In effect, when a decision-maker ranks his preferences, the order is constantly subject to re-evaluation. He logically chooses the alternative that is ranked first; however, he may consider the others depending on their relative distances from the first.

The methodology proposed in this paper groups ranked alternatives into three classes of preferences: the most preferred, the less preferred, and the least preferred sets. Within each class, negligible differences in preferences among alternatives would increase the confidence of the decision makers not to discriminate among them. As a consequence, it would make it easier for the decision makers to trade them interchangeably. In other words, grouping alternatives that share close evaluation corresponds to expanding the preference space(s) of the decision maker from one best alternative to a set of more or less equally preferred alternatives.

The contraction operation constitutes the second phase of the NAI algorithm. Given a subset of comparatively satisfactory alternatives obtained from the expansion mapping, the second operation attempts to identify those that might exhibit a stronger preferential distribution than others. Thus, if among the preferred alternatives, there still remains an unequal distribution of preferences, the NAI algorithm provides an indicator that distinguishes the most preferred alternatives from the preferred ones.

The third and the last step is the intersection operation. It derives a collective solution(s) that is (are), in principle, acceptable to all group members. Consensus is reached when there is at least one alternative that appears in every group member's subset of the most preferred alternatives. As a result, a collective solution is one that is essentially acceptable to all can be suggested.

Conversely, if the intersection operation fails to identify a collective solution, this could be seen as an indicator suggesting that some other form of consensus seeking or compromise should be tried (Shakun, 1991; Zarhman, 1994).

3. Heuristics for consensus seeking

Although comprehensive, Bui and Shakun's (Bui and Shakun, 1989) work lacks a complete formalism that allows exploration of all properties of the NAI concept.

3.1. NAI heuristic

As discussed earlier, the distribution of preferences among alternatives reflects the extent to which alternatives are related to each other. With alternatives ranked by cardinal preferences, the *Structural Index of Preferences* of the Subset consisting of the first j alternatives, SI_{ij} can be defined as follows. For a decision maker i,

$$\mathrm{SI}_{ij} = \left(\frac{1}{j}\right) M_i^{(j)}, \tag{2}$$

where

$$M_i^{(j)} = \left(\frac{1}{j-1}\right) \sum_{k=1}^{j-1} M_{ijk}, \tag{3}$$

and

$$M_{ijk} = \frac{\left(\frac{R_{ik}}{k}\right)}{\left(\frac{R_{ij}-R_{ik}}{j-k}\right)}, \tag{4}$$

where $j = 2, \ldots, n$; and $k = 1, \ldots, j-1$ is a summation index. Here we assume that the denominator differs from zero. Otherwise we define $M_{ijk} = \infty$.

In other words, M_{ijk} is the ratio between the cumulative preference per alternative assigned to better alternatives and that of the residual alternatives. $M_i^{(j)}$ is an average value of M_{ijk}. The structural index SI_{ij} puts this average $M_i^{(j)}$ on a per alternative basis. The value of SI_{ij} is a function of the number of alternatives j, as well as the distribution of the decision maker's preferences r_{ij}. Theoretically, SI_{ij} varies between $1/j$ (i.e., situation in which the decision maker is completely indifferent with regard to alternatives) and ∞ (i.e., the maximum of "disequilibrium" among distribution of preferences). A detailed proof is given in Section 4. Furthermore, one might argue that the closer the value of SI_{ij} to $1/j$ the easier for the decision maker to negotiate with other members of the group. Conversely, the higher the value of SI_{ij}, the smaller the degree of flexibility in negotiation.

Phase 1: The expansion operation. Given a set of n ranked alternatives, the subset of preferred alternatives can be defined as the one comprising the top alternatives, say p_i^*, that are clearly more preferred than the other alternatives, i.e. $n - p_i^*$. The identification of the number of preferred alternatives p_i^* as well as the rationale of the approach are described below:

(1) Define $n-1$ subsets of alternatives: The first subset is composed of the first two alternatives ($j = 2$). The second is composed of the first three alternatives ($j = 3$), etc. And the $(n-1)$th subset is the entire set itself ($j = n$).

(2) For each subset of j alternatives, compute its structural index of preferences, SI_{ij}, where $j = 2, \ldots, n$.

(3) The subset containing the preferred alternatives is the one that has the lowest SI_{ij}

$$\mathrm{SI}_{i,p_i^*} = \min\{\mathrm{SI}_{ij}\}, \tag{5}$$

where p_i^* represents the first p_i^* alternatives that form the subset of the preferred alternatives $(2 \leqslant p_i^* \leqslant n)$. In the case of multiple optimum, the largest among the optimal p^* values is selected.

The rationale of using the smallest SI_{ij} as the cut-off point can be intuitively justified by the observation that the lower the value of SI_{ij} the more uniform the distribution of preferences among alternatives. Thus, by choosing p_i^* that has the minimum value of SI_{ij} as the cut-off point is that the decision maker i has more or less evenly distributed his preferences among the p_i^* alternatives. In other words, numerical differences between p_i^* alternatives are not significant enough to assert that none of the alternatives is clearly worse than another to the extent that it should be discarded.

From a group problem solving point of view, a higher SI_{i,p_i^*} value indicates that the decision maker i has a strong and clear choice. As a consequence, there may be little room left for concession. On the other hand, a SI_{i,p_i^*} with a small value suggests that the decision maker i would exhibit some indifference to the alternatives, and as a consequence, any of these could be acceptable.

Phase 2: The contraction operation. The idea here is to find out which subset of the preferred set constitutes the most preferred subset. Given p_i^* preferred alternatives the identification of a second cut-off can be done by applying the following steps:

(1) Define $p_i^* - 1$ subsets of alternatives in a bottom-up manner: The first bottom subset is composed by the p_i^* minus the top one. The second subset is composed by p_i^* alternatives minus the first two top alternatives; etc. And the $(p_i^* - 1)$th bottom subset contains only one alternative, the one just above the cut-off point for the preferred set.

(2) Compute the arithmetic mean r_{i,m^*}^- of the cardinal preferences of each subset m^*, where $m^* = 1, \ldots, p_i^* - 1$ corresponds respectively to the first to the $(p_i^* - 1)$th bottom subset as defined in step (1).

(3) For $m^* = 1, \ldots, p_i^* - 1$, compute the preference ratio index, C_{i,m^*} as follows:

$$C_{i,m^*} = \frac{r_{i,m^*}}{r_{i,m^*}^-}, \tag{6}$$

where r_{i,m^*} is the cardinal preference for alternative m^*, the last top alternative defining a cut-off point separating the bottom subset from the alternatives above. Note that the cardinal preferences of the alternatives in the preferred set, p_i^*, are renormalized so that their sum equals one.

(4) Choose the second cut-off point m_i^* by maximizing the C_{i,m^*} preference ratio, i.e., Max $\{C_{i,m^*}\}$ for $m^* = 1, \ldots, p_i^* - 1$. The rationale for this is as follows. If C_{i,m_i^*} is large, then there is a big relative drop between the preference value r_{i,m_i^*} of the alternative just above the cut-off point compared to the average preference $r_{i,m_i^*}^-$ of the alternatives in the subset below. Thus, Max $\{C_{i,m^*}\}$ is a criterion for the subset of most preferred alternatives at the top of the

preferred set. In the case of multiple optimum, the largest optimal m^* value is selected.

In other words, the alternatives that are situated above this second cut-off point are considered most preferred. It is assumed that the decision maker would be reluctant to drop these alternatives. In a situation of complete indifference, all $C_{i,m_i^*} = 1$ are maximum and we would set $m_i^* p_i^*$.

Phase 3: The intersection operation. Given all individual subsets of m_i^* (most preferred) alternatives, an intersection operation can be performed to identify possible consensus solution(s). Similarly, an intersection operation can be performed on individual subsets of p_i^* (less preferred) alternatives.

3.2. Breaking the intersection impasse

The above has proved to be useful, since it partially captured the human behavior in group decision making (Bui, 1987). If each decision maker selects his own m_i^* and deletes all the others, then there might be no common alternative in every decision maker's most preferred set m_i^*. That is,

$$m_1^* \cap m_2^* \ldots m_{d-1}^* \cap m_d^* = \emptyset, \tag{7}$$

where d is the number of decision makers. In other words, the most preferred alternatives have an empty intersection. To overcome this difficulty, we propose the following two procedures.

Procedure 1. Procedure 1 is an iterative process. In every iteration, it removes the alternatives that have the least possibilities to be accepted as candidates. That is, if no common alternative is identified in every decision maker's most preferred set, then the alternatives that fall in the least preferred set, p_i^*, will be removed and the preferences distribution will be reallocated to the left-over alternatives in the more preferred set p_i^*.

In every iteration, the procedure recalculates r_{ij}, SI_{ij} and C_{i,m^*} to find the new cut-off points to divide the n into p_i^* and $n - p_i^*$ and to divide the p_i^* into m_i^* and $p^* - m_i^*$. This procedure continues until a common alternative(s) is identified in every decision maker's most preferred set m_i^*. This procedure assumes that the preference distribution is indifferent to the removal of the least preferred alternatives. There is no general guarantee that this procedure will lead to a consensus.

Procedure 2. Procedure 2 differs from procedure 1 in that it expands the size of the most preferred set downward until an alternative is identified in every decision maker's most preferred set. The length of the procedure depends on the distribution of preferences. If the allocations do not deviate widely, it takes only two steps to reach the solution. Otherwise, it takes the whole procedure (four steps discussed below).

Step 1. For each decision maker i, find the following two bounds:

$$C_{i,\min} = \min_{m_i^* < j \leqslant p_i^*} \{C_{ij}\},$$
$$C_{i,\max} = \max_{m_i^* < j \leqslant p_i^*} \{C_{ij}\}, \quad (8)$$

which are the upper and lower limits of C_{ij} for $j = m_i^* + 1, \ldots, p_i^*$.

Step 2. For each decision maker i and value $t \in [0, 1]$, identify $m_i(t)$, which is the largest value of j, such that

$$C_{ij} \geqslant C_{i,\max} - t \cdot (C_{i,\max} - C_{i,\min}). \quad (9)$$

These steps represent the fact that because the decision makers cannot find an alternative that appears in everyone's most preferred set, they have to further expand the size of the most preferred set by including part of the alternatives in the less preferred set until alternative p_i^*. Then the smallest value of t is selected such that the decision makers have at least one common alternative of all expanded most preferred sets, and anyone of the common alternatives can be accepted as the solution.

The application of the above two steps will always give a solution unless there is no common alternative in the preferred sets of the decision makers. In such case the preferred sets should be expanded again before applying the above procedure. The expansion can be done in the following way:

Step 3. For each decision maker i, find the following two bounds:

$$\mathrm{SI}_{i,\min} = \min_{p_i^* < j \leqslant n},$$
$$\mathrm{SI}_{i,\max} = \max_{p_i^* < j \leqslant n} \{\mathrm{SI}_{ij}\}. \quad (10)$$

Step 4. For each decision maker i and value $t \in [0, 1]$, calculate $p_i(t)$, which is the largest value of j, such that

$$\mathrm{SI}_{ij} \leqslant \mathrm{SI}_{i,\min} + t \cdot (\mathrm{SI}_{i,\max} - \mathrm{SI}_{i,\min}). \quad (11)$$

The smallest value of t is selected such that the expanded preferred alternative sets have at least one common element. Steps 1 and 2 are then used with the expanded preferred alternative sets.

4. Theorems and proofs

In this section, we provide theorems to address the extreme conditions of the heuristics we proposed.

Theorem 1. *For all j, $\mathrm{SI}_{ij} \geqslant 1/j$, and equality holds if and only if $r_{i1} = r_{i2} = \cdots = r_{ij}$.*

Proof. Eqs. (3) and (4) imply that

$$\begin{aligned}\mathrm{SI}_{ij} &= \frac{1}{j}\frac{1}{j-1}\sum_{k=1}^{j-1}\frac{R_{ik}(j-k)}{k(R_{ij}-R_{ik})}\\ &= \frac{1}{j}\frac{1}{j-1}\left(\frac{R_{i1}(j-1)}{1(R_{ij}-R_{i1})}+\frac{R_{i2}(j-2)}{2(R_{ij}-R_{i2})}+\cdots+\frac{R_{ij-1}}{(j-1)(R_{ij}-R_{ij-1})}\right).\end{aligned}$$

Notice for the general term $k = 1, 2, \ldots, j-1$,

$$\frac{R_{ik}(j-k)}{k(R_{ij}-R_{ik})} = \frac{\sum_{l=1}^{k} r_{il}(j-k)}{k\sum_{l=k+1}^{j} r_{il}} \geqslant \frac{kr_{ik}(j-k)}{k(j-k)r_{ik}} = 1,$$

and equality holds if and only if $r_{i1} = r_{i2} = \cdots = r_{ij}$. The statement follows immediately from this observation. □

Remark. The case $r_{i1} = r_{i2} = \cdots = r_{ij}$ is called the "most equilibrium" condition, when the decision maker cannot distinguish among the alternatives.

In our next theorem we show that there is no general upper bound for the values of SI_{ij}.

Theorem 2. *Assume that for $k = 1, 2, \ldots, j-1$, $r_{ik}/r_{ik+1} \geqslant \alpha_k$. Then*

$$M_i^{(j)} \geqslant \frac{1}{j-1}\sum_{k=1}^{j-1}\alpha_k \quad \text{and} \quad \mathrm{SI}_{ij} \geqslant \frac{1}{j(j-1)}\sum_{k=1}^{j-1}\alpha_k. \tag{12}$$

Proof. Notice that for $j > k$,

$$M_{ijk} = \frac{\sum_{l=1}^{k} r_{il}(j-k)}{k\sum_{l=k+1}^{j} r_{il}} \geqslant \frac{kr_{ik}(j-k)}{k(j-k)r_{ik+1}} = \frac{r_{ik}}{r_{ik+1}} \geqslant \alpha_k,$$

therefore Eqs. (3) and (2) imply that

$$M_i^{(j)} \geqslant \frac{1}{j-1}\sum_{k=1}^{j-1}\alpha_k,$$

$$\mathrm{SI}_{ij} \geqslant \frac{1}{j}\frac{1}{j-1}\sum_{k=1}^{j-1}\alpha_k. \qquad \square$$

Corollary 1. *If the α_k values are large enough that the SI_{ij} value can be arbitrarily large. That is, they are not bounded in general.*

Corollary 2. *Since the alternatives are ordered in decreasing ranking, $r_{ik} \geqslant r_{ik+1}$. Therefore $\alpha_k = 1$ is always an appropriate selection, and so $M_i^{(j)} \geqslant 1$ and $\mathrm{SI}_{ij} \geqslant (1/(j(j-1)))(j-1) = 1/j$. Hence Theorem 2 is a straightforward generalization of Theorem 1.*

To search for another cut-off point to divide the preferred set into the most preferred set and the less preferred set, we have to compute the preference ratio index, C_{i,m^*}:

$$C_{i,m^*} = \frac{r_{i,m^*}}{r^-_{i,m^*}}.$$

The cut-off point is selected where C_{i,m_i^*} is the maximum; that is, $\max\{C_{i,m^*}\}$ for $m^* = 1, 2, \ldots, p_i^* - 1$.

This step does not lead to less preferred solutions, if the C_{ij} values are the same for $j = 1, 2, \ldots, p_i^* - 1$. This is the case if and only if

$$\frac{r_{i1}}{\frac{1}{p_i^*-1}\sum_{k=2}^{p_i^*} r_{ik}} = \frac{r_{i2}}{\frac{1}{p_i^*-2}\sum_{k=3}^{p_i^*} r_{ik}} = \cdots = \frac{r_{i,p_i^*-1}}{r_{i,p_i^*}}.$$

It is easy to see that these equations hold if and only if for all $j = 1, 2, \ldots, p_i^* - 2$,

$$r_{ij} = r_{ij+1}\frac{\frac{1}{p_i^*-j}\sum_{k=j+1}^{p_i^*} r_{ik}}{\frac{1}{p_i^*-j-1}\sum_{k=j+2}^{p_i^*} r_{ik}}. \tag{13}$$

The values of $r_{ip_i^*}$ and $r_{ip_i^*-1}$ can be arbitrary, but Eq. (13) implies that all other values $r_{i1}, r_{i2}, \ldots, r_{ip_i^*-2}$ are uniquely determined. If $r_{ip_i^*-1} = r_{ip_i^*}$ then it is easy to see that $r_{i1} = r_{i2} = \cdots = r_{ip_i^*}$.

Consider next the special case when the utility values are assigned so that

$$r_{i1} > r_{i2} > \cdots > r_{ic} > r_{ic+1} = r_{ic+2} = \cdots = r_{in} = 0.$$

This case can be considered as a "maximum disequilibrium", when the decision maker assigns all his preferences to some alternatives, and totally ignores the others. In this case $\mathrm{SI}_{ij} = \infty$ for $j \geqslant c + 1$, since the denominator in Eq. (4) becomes zero for $c \leqslant k < j$.

In many situations, a decision maker would like to divide the alternatives into two groups: preferred and less preferred, and assign the same preference to all the alternatives in the same group. That is, if there are c alternatives that are preferred more by the decision maker and each receives a higher preference: $r_{i1} = r_{i2} = \cdots = r_{ic} = r_{ih}$, then there are $n - c$ of less preferred alternatives and each receives a lower preference: $r_{ic+1} = r_{ic+2} = \cdots = r_{in} = r_{il}$, where $r_{ih} > r_{il}$. The continuity of the SI_{ij} values and Theorem 1 imply that if the difference $r_{ih} - r_{il}$ is small enough, then

$$\mathrm{SI}_{i1} > \mathrm{SI}_{i2} > \cdots > \mathrm{SI}_{in},$$

that is, the expansion operation does not reduce the alternative set. If the difference $r_{ih} - r_{il}$ is large enough, then $p_i^* = c$. That is, the preferred set coincides with the set of alternatives with single utility values. To prove the assertion, consider first the case of $r_{il} = 0$. If $j \leqslant c$, then

$$\mathrm{SI}_{ij} = \frac{1}{j(j-1)} \sum_{k=1}^{j-1} \frac{R_{ik}(j-k)}{k(R_{ij}-R_{ik})} = \frac{1}{j(j-1)}(j-1)\frac{r_{ih}}{r_{ih}} = \frac{1}{j}.$$

If $j \geqslant c+1$, then

$$\mathrm{SI}_{ij} = \infty,$$

since at least one denominator in Eq. (4) equals zero. Hence the minimal value occurs at $j = c$. The continuity of SI_{ij} implies that the same j value is optimal for small enough values of r_{il}.

There is a special situation in which a decision maker allocates his preferences according to an algebraic sequence:

$$r_{ij} = r_{ij-1} - \delta, \text{ or } r_{ij} = (n-j+1)\delta,$$

where $j = 1, 2, \ldots, n$. In such condition $r_{i1} = n\delta$, $r_{i2} = (n-1)\delta$, ..., $r_{in} = \delta$, and $((n(n+1)/2)\delta = 1.0$. That is, all the preferences are allocated like a -45-degree line pointing downward. The cumulative preferences can be written as follows:

$$R_{ij} = \left(nj - \frac{j(j-1)}{2}\right)\delta,$$

and therefore

$$\begin{aligned}
\mathrm{SI}_{ij} &= \frac{1}{j(j-1)} \sum_{k=1}^{j-1} \frac{R_{ik}(j-k)}{k(R_{ij}-R_{ik})} \\
&= \frac{1}{j(j-1)} \sum_{k=1}^{j-1} \frac{\left(kn - \frac{k(k-1)}{2}\right)\delta(j-k)}{k\left(\left(jn - \frac{j(j-1)}{2}\right) - \left(kn - \frac{k(k-1)}{2}\right)\right)\delta} \\
&= \frac{1}{j(j-1)} \sum_{k=1}^{j-1} \frac{2n-(j-1)-k+j}{2n-(j-1)-k} \\
&= \frac{1}{j(j-1)} \sum_{k=1}^{j-1} \left(1 + \frac{j}{2n-(j-1)-k}\right) \\
&= \frac{1}{j(j-1)} \left((j-1) + \sum_{k=1}^{j-1} \frac{j}{2n-(j-1)-k}\right) \\
&= \frac{1}{j} + \frac{1}{j-1} \sum_{k=1}^{j-1} \frac{1}{2n-j-k+1}.
\end{aligned}$$

We are unable to solve the optimal j in closed form, so we used simple enumeration. The results are shown in Table 1. The results show that for $j = \frac{7}{10}n$ the SI_{ij} is always the minimum. Therefore, $j = 0.7n$ is a good cut-off point to divide n into the preferred set and the rest. For $n > 10$, the differences among SI_{i,p_i^*-1}, SI_{i,p_i^*} and SI_{i,p_i^*+1} become very small, for example, at $n = 20$. Therefore,

Table 1
Relationship between n and p_i^*

n	p^*	SI_{i,p^*-1}	SI_{i,p^*}	SI_{i,p^*+1}
3	3	0.75	0.75	
4	3	0.666	0.558	0.611
5	4	0.488	0.455	0.521
6	5	0.394	0.389	0.457
7	5	0.362	0.336	0.343
8	6	0.307	0.296	0.309
9	7	0.268	0.266	0.281
10	7	0.251	0.241	0.242
15	11	0.164	0.163	0.165
20	14	0.125	0.123	0.124
30	21	0.0833	0.0829	0.0830
40	28	0.0626	0.0624	0.0625
50	35	0.0502	0.0501	0.0502
100	70	0.0198	0.0197	0.0198

for this particular distribution of r_{ij} the best usage of the heuristic is recommended to be $n \leqslant 15$.

To further divide the preferred set into the most preferred set and the less preferred set, if we use C_{i,m_i^*}, for this particular case, $m_i^* = p_i^* - 1$, and therefore the most preferred alternative set contains $p_i^* - 1$ alternatives, and the less preferred set contains only one alternative. Therefore we have to use SI_{ij} again. However, the preferences r_{ij} for the most preferred alternatives need to be renormalized as discussed in Section 3. That is, now the total preference, 1.0, of decision maker i must be redistributed to p_i^* alternatives instead of n. This step is needed, since part of the original preference has been allocated to the other alternatives, so the total preference that can be allocated to the less preferred set is less than 1.0.

Table 2 provides the values of p_i^* and m_i^* based on different n values. From Table 2 we know that if $n = 10$, $p_i^* = 7$, $m_i^* = 5$, And the less preferred set consists of only two alternatives. The rest belongs to the least preferred set.

5. NAI as a tool for group decision and negotiation support

In this section, we provide an example of using the NAI heuristic in solving a real-world problem. The example presented here is a simplified description of a real-life application of the NAI heuristic (a more detailed discussion of the case is given by Jacquet-Lagreze and Shakun, 1988). It demonstrates the ability of the heuristic to enlarge the decision set and consequently, reduce the conflict caused by individual differences.

A group of four decision makers: three department heads and the company president. They worked together to design a new product for the ultralight

Table 2
Selections of p_i^* and m_i^*

n	p^*	m^*	SI_{i,m^*-1}	SI_{i,m^*}
3	3	2	1.0	1.0
4	3	2	1.0	1.0
5	4	3	0.75	0.75
6	5	4	0.666	0.558
7	5	4	0.666	0.558
8	6	5	0.488	0.455
9	7	5	0.488	0.455
10	7	5	0.488	0.455
15	11	8	0.307	0.296
20	14	10	0.177	0.174
30	21	15	0.118	0.117
40	28	20	0.0889	0.0886
50	35	25	0.0713	0.0714
100	70	50	0.0359	0.0358

airplane market. Player P1 is Marketing, Player P2 is Engineering, Player P3 is Finance, and Player P4 is the company president.

They have identified six alternatives: A, B, C, D, E and F. The players used two computer programs to perform their group decision making. A multi-attribute utility model (Jacquet-Lagreze, 1985) that is based on a piece-wise linear additive utility method was used to derive individual utilities (see Table 3). Second, we developed a simple program to calculate the r_{ij}, SI_{ij} and C_{i,m^*} to determine the locations of the cut-off points.

Table 4 shows the individual normalized utilities as r_{ij}, where $\sum_{j=1}^{n} r_{ij} = 1$, SI_{ij} and C_{i,m^*} for each player. The cut-off point defining the preferred set of alternatives for a given player is based on $\min(\mathrm{SI}_{ij})$, as discussed in Section 3, and is indicated by an asterisk (*) in the SI_{ij} for that player. Similarly, an asterisk (*) in the C_{i,m_i^*} for each player indicates the most preferred set which is based on $\max(C_{i,m^*})$.

Table 3
Utilities of players for six alternatives

Player 1		Player 2		Player 3		Player 4	
Alt.	Util.	Alt.	Util.	Alt.	Util.	Alt.	Util.
E	0.77	F	0.67	D	0.71	E	0.77
F	0.69	E	0.58	B	0.57	F	0.71
C	0.53	A	0.43	E	0.57	B	0.39
D	0.47	D	0.42	F	0.55	C	0.34
B	0.44	B	0.35	C	0.39	D	0.27
A	0.02	C	0.29	A	0.29	A	0.13

Table 4
The first NAI trial

Player	Alt.	r_{ij}	SI_{ij}	C_{i,m^*}
1	E	0.26	–	1.45*
	F	0.24	0.56	1.44
	C	0.18	0.44	1.16
	D	0.16	0.35	1.07
	B	0.15	0.29*	–
	A	0.01	–	1.25
2	F	0.24	–	1.62*
	E	0.21	0.58	1.56
	A	0.16	0.46	1.22
	D	0.15	0.35	1.31
	B	0.13	0.30	1.21
	C	0.11	0.27*	–
3	D	0.23	–	1.50
	B	0.19	0.62	1.27
	E	0.19	0.39	1.39
	F	0.18	0.29	1.62*
	C	0.13	0.27*	1.34
	A	0.09	0.27*	–
4	E	0.30	–	1.80
	F	0.27	0.54	2.13*
	B	0.15	0.55	1.28
	C	0.13	0.46	1.26
	D	0.10	0.41*	–
	A	0.05	0.46	–

Given the individual player preferred sets we find, in the first trial, by intersection, the NAI heuristic has identified B, C, D, E and F as preferred negotiable alternatives for the group. Alternative A is a negligible alternative which can be dropped from further consideration. However, since no alternative appears in every one's most preferred set, we have to use the procedures as discussed at the end of Section 3 to search for a solution. We first discuss Procedure 1.

Procedure 1. Procedure 1 suggests that we drop alternative A and make another trial. We obtain Table 5, which shows alternative E as the most preferred while alternatives B, C, D and F are less preferred. Evidently, airplane E is a good candidate for the group's new product. Also from earlier literature (Jacquet-Lagreze and Shakun, 1988), we note that airplane E is also the max/min solution to the group decision problem.

Procedure 2. The first step in Procedure 2 uses Eq. (8) to find the values of $C_{i,\min}$ and $C_{i,\max}$ of the less preferred set. The second step uses Eq. (9) to expand the

Table 5
The second NAI trial

Player	Alt.	r_{ij}	SI_{ij}	C_{i,m^*}
1	E	0.27	–	1.45*
	F	0.24	0.56	1.44
	C	0.18	0.44	1.16
	D	0.16	0.35	1.07
	B	0.15	0.29*	–
2	F	0.29	–	1.63
	E	0.25	0.58	1.64*
	D	0.18	0.47	1.31
	B	0.15	0.39	1.21
	B	0.13	0.34*	–
3	D	0.25	–	1.37
	B	0.20	0.62	1.13
	E	0.20	0.39	1.21
	F	0.20	0.29	1.41*
	C	0.14	0.27*	–
4	E	0.31	–	1.80
	F	0.29	0.54	2.13*
	B	0.16	0.55	1.28
	C	0.14	0.46	1.26
	D	0.11	0.41*	–

size of the most preferred set by taking some of the alternatives from the less preferred set. We present the procedure and the results in Table 6.

It only took one step to obtain the solution–Alternative E. When some decision makers are willing to give up their choices in the most preferred set and to consider alternatives in the less preferred set, solution is much easier to obtain. The solution was identified when each decision maker moved the first alternative in the less preferred set into the most preferred set. That is, at $t = 0$,

Table 6
The alternative search by procedure 2

t	Player	$C_{i,\min}$	$C_{i,\max}$	$C_{i,j}$	m^*
	Player 1				E
	Player 2				F
	Player 3				DBEF
	Player 4				EF
0.0	Player 1	1.07	1.44	1.44	EF
	Player 2	1.21	1.56	1.56	FE
	Player 3	1.34	1.34	1.34	DBEFC
	Player 4	1.26	1.28	1.28	EFB

the first alternative was moved. This example is a special case in that it took only one step to obtain the solution. Solution E is the same as the one obtained in Procedure 1.

6. Summary

Recent studies in social choice continue to challenge the efficiency of voting systems (e.g., Benoit and Kornhauser, 1994). Extending Arrow's pioneering work (Arrow, 1951), many authors argue that searching for a common solution based on the maximal element of an ordering simply violates the concept of justice (Anscombe, 1976; Knight and Johnson, 1994; Kolm, 1994). Instead, we argue that there is a need for deliberative procedures that help (i) search for social consensus, and when they fail, (ii) identify disconcerting, perhaps insurmountable, difficulties as a basis for problem re-definition.

We have proposed a formalized heuristic for consensus seeking in a single voting context, the Negotiable Alternatives Identifier (NAI). NAI first seeks to locate candidates for compromise, and next searches for a collective solution. This algorithm contributes to the literature of social choice and reasoning in two areas. First, the reasoning in this paper is a further evidence of the importance of using a complete asymmetry preorder for expressing preferences over a set of alternatives. Second, the NAI algorithm, especially with its expansion/contraction/intersection concept, provides an aggregation approach which is less rigid yet intuitive enough for the decision makers to search for, and agree upon, a group solution. A limitation of the NAI heuristic resides in the fact that it operates on a given set of cardinal distribution. One can argue, however, that if impasse is shown to be unavoidable, then voters are at least "confirmed" by NAI, and encouraged to search for more creative settlement.

The NAI consensus-seeking heuristics has been integrated into Co-op, a decision support system for group multi-criteria decision making on a network of personal computers. From its use in real world cases, NAI has proven to be helpful in supporting group consensus seeking.

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