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Empirical Relations for Moving Striations

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THE theory of moving striations in positive columns is proving difficult to develop accurately on a quantitative basis (Watanabe and Oleson 1955, Walsh 1957), because of the complexity of the phenomena and because the oscillations have large amplitude and require a non-linear theory. It is therefore of some interest to seek further guidance from data found experimentally, and to look for empirical relations between the quantities involved and attempt to account for these in the first instance on rather general grounds.

Although several papers on various experimental aspects of moving striations have appeared within the past few years (cf. Donahue and Dieke 1951, Pigg, Burton and Oleson 1957) the most extensive and useful data available appear still to be those of Pupp (1935). These have been summarized by Druyvesteyn and Penning (1940) and, more recently, by Francis (1956). Pupp's measurements were made in inert gases, although moving striations are not confined to these. The range of pressures covered was about 0·2–5 mm Hg, and currents up to 15 A were used.

Within a column sufficiently long for end effects to be negligible, the striations in a cylindrical tube of radius R are fairly well characterized by a spacing λ and a velocity v, related to their frequency f by the formula

$$f=v/\lambda. \qquad \ldots\ldots(1)$$

Actually v may vary with position in the tube and with time at a given position, but we shall neglect these complications, which are possibly connected with the non-linearity of the waves.

If p is the gas pressure (reduced to 0 °C, to allow for temperature changes, as it is largely the density which matters) and i is the current, Pupp found that

(*a*) $$\lambda/R=F(pR, i/R), \qquad \ldots\ldots(2)$$

(*b*) $$fR=\phi(pR), \qquad \ldots\ldots(3)$$

(*c*) f varied only slightly with i,

(*d*) if fRM is plotted against pR/V_1, where M is the atomic weight and V_1 is the ionization potential, the points for krypton, argon and neon lie nearly on one line, and those for helium a little above it. Using c.g.s. and volt-ampere units, F lies between about 9 and 3, and fR between 2×10^2 and 4×10^4, corresponding to striation velocities—towards the cathode—between 10^3 and 10^5 cm sec^{-1}.

We will now make an attempt to see whether equations (1), (2) and (3), and the results of (*d*), can be used to obtain further information, recognizing that this is likely to be at best only semi-quantitative.

Taking first equation (3), we notice that it would be similar to equation (1), if the striation spacing were a constant multiple of the tube radius or

$$\lambda=AR \qquad \ldots\ldots(4)$$

where A is a constant. In this case equations (1) and (3) give

$$v=A\phi(pR). \qquad \ldots\ldots(5)$$

Since p is inversely proportional to the atomic free path, equation (5) indicates that the velocity of the striations considered in equation (1) would depend on the ratio of tube radius to free path. Alternately, with the relation (4) assumed between R and λ, it would show that the velocity depends only on the number of free paths between neighbouring striations.

Equations (1), (2) and (3) can also be combined to give

$$v = \psi(pR, i/R) \qquad \ldots\ldots(6)$$

where ψ is a new function.

In order to explain at all the more complex relation (d) between fRM and pR/V_i, more detailed consideration of the processes taking place in the striations is necessary. The simplest assumptions seem to be that (a) the occurrence of M indicates that the mobility μ of the ions is involved; and (b) the appearance of V_i is connected with the mean longitudinal field X in the column.

If the relation (4) is valid, and the further likely assumption made that the potential difference between striations is V_i, or is proportional to or not much different from it, pR/V_i becomes proportional to the reciprocal of Townsend's parameter X/p. A simplified form of the empirical relation involving M and V_i can now be found if the striae were to have their speeds determined by the mobility motion of the ions in the mean longitudinal field, and the striation speed were equal to (cf. Druyvesteyn 1934) or proportional to the longitudinal drift velocity of the ions. Assuming for simplicity equality of the speeds and that the potential difference between striations is V_i, we have from equation (4)

$$v = X\mu = V_i\mu/(AR). \qquad \ldots\ldots(7)$$

For inert gas atomic ions in their parent gas (except He^+ in He for which the mobility is less) the value of μ at atmospheric pressure, μ_0 is given approximately by

$$\mu_0 = 4M_{Ne}/M \qquad \ldots\ldots(8)$$

where M_{Ne} is the atomic mass of neon, and the units the conventional v cm sec ones. At pressure p mm

$$\mu = 760\mu_0/p = 3040M_{Ne}/(Mp). \qquad \ldots\ldots(9)$$

The velocity (equation (7)) becomes

$$v = 3040V_iM_{Ne}/(ARMp). \qquad \ldots\ldots(10)$$

Eliminating v between equations (1) and (10), using equation (4) to eliminate λ, and rearranging, with $M_{Ne} = 20{\cdot}2$,

$$fRM = 6{\cdot}14\ 10^4A^{-2}/(pR/V_i). \qquad \ldots\ldots(11)$$

According to this, the relation between fRM and pR/V_i should be hyperbolic. The graph connecting them is roughly of this form (Druyvesteyn and Penning 1940). The product is not however accurately constant, being for example for the Ne–A–Kr line about 8×10^3 and 6×10^3 for pR/V_i equal to 0·2 and 0·02 respectively; this could be due to changes in A, which are known to occur for smaller currents than those in most of Pupp's discharges. For He the products are also about 30% larger; this might be due to the ratio of molecular ions to atomic ions tending to be greater than for the other gases, resulting in a larger value of effective mobility (Hornbeck and Molnar 1951). For Ne–A–Kr, the constant of proportionality in equation (11) would have the mean experimental value of 7×10^3, if the value of A were approximately 3. A value of 8 can be deduced from data given by Pupp (1935, figure 8). Close agreement cannot be looked for in view of the approximations and assumptions made, but the grouping of p, R, M, p and V_i in equation (11) is definitely in agreement with Pupp's empirical relation and the form of the latter reproduced rather closely. The use of what is essentially uniform column theory for a longitudinally periodic column may be tentatively justified by the fact that the flux of ions at any point is maintained jointly by the local concentration and potential gradients, and that the mobility is proportional to the diffusion coefficient.

For smaller currents where equation (4) is not satisfactory and the relation (c) not valid, similar, but less simple, relations are also likely to exist. It must however be remembered that the purity of the gases may have been different in high current and low current experiments done hitherto; if, therefore, the properties of the striations depend to any considerable extent, for example, on cumulative ionization via metastable or other excited states, the behaviour of the high current and low current discharges may not be strictly comparable.

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REFERENCES

DONAHUE, T., and DIEKE, G. H., 1951, *Phys. Rev.*, **81**, 248.
DRUYVESTEYN, M. J., 1934, *Physica*, **1**, 1003.
DRUYVESTEYN, M. J., and PENNING, F. M., 1940, *Rev. Mod. Phys.*, **12**, 87.
FRANCIS, G., 1956, *Handb. d. Physik*, **XII**, II (Berlin: Springer), p. 140.
HORNBECK, J. H., and MOLNAR, J. P., 1951, *Phys. Rev.*, **84**, 621.
PIGG, M. K., BURTON, J. B., and OLESON, N. L., 1957, *Proc. Third International Conference on Ionization Phenomena in Gases* (Italian Physical Society, Venice), p. 833.
PUPP, W., 1935, *Phys. Z.*, **36**, 61, and a series of earlier papers.
WALSH, P. J., 1957, *Tenth Annual Gaseous Electronic Conference*, Cambridge, Mass.
WATANABE, S., and OLESON, N. L., 1955, *Phys. Rev.*, **99**, 1701.

Unitary Phase Shift Scattering Approximation

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§ 1. INTRODUCTION

PARK showed in a recent paper (Park 1957) how to improve the first Born approximation using, essentially, the unitarity relation of the S-matrix. He applies his theory to S-wave square-well potential scattering and finds that his method was better for angles less than 20°.

It is the purpose of this work to generalize such an approach to all orders in the interaction potential. We start from the well-known S-matrix theory (Lippmann and Schwinger 1950, Klein 1956, Garrido 1959) and introduce the unitarity relation in the expression for the phase shifts. By expanding in powers of the interaction we obtain an approximation that, with certain