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Revisiting the Hot Hand Theory with Free Throw Data in a Multivariate Framework

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Abstract

Despite the conventional wisdom among players and fans of the existence of the "hot hand" in basketball, studies have found only no evidence or weak evidence for the hot hand in game situations, although stronger evidence in controlled settings. These studies have considered both free throws and field goals. Given the heterogeneous nature of field goals and several potential sources that could cause a positive or negative correlation between consecutive shots (such as having a weak defender), free throws may provide for a more controlled setting to test for the hot hand. Almost all studies have tested for the hot hand at the individual level in univariate frameworks, and as some have pointed out, the studies may not have had enough power to detect the hot hand. In this study, I use a sample based on every free throw attempted during the 2005-06 NBA season. I used a multivariate framework with individual player fixed effects. I find that hitting the first free throw is associated with a 2- to 3-percentage-points higher probability of hitting the second free throw. Furthermore, the infrequent foul-shooters have more streakiness in their shooting than frequent foul-shooters.

1. Introduction

A common perception among basketball fans, players, and commentators is the existence of the hot hand or streak shooting, where a player has a higher probability of making the next shot had he or she made a few shots in a row. Many people who play basketball believe in the hot hand, as sometimes they just have a better feel for the ball, are more confident in their ability to make the shot, and have better concentration. And teams will often find the player who has made a few shots in a row to take the next shot, under the assumption that they have a higher probability than anyone else of making the next shot (Burns, 2004).

Despite this conventional wisdom that the hot hand in basketball is a real phenomenon, there has been little evidence supporting the theory. Researchers have used "runs" tests, binomial tests, and conditional probability tests, and generally found that the streaks of consecutive made shots are statistically natural. That is, such runs of consecutive made shots occur about as frequently as they would be expected given the natural variation in the probability of making a shot.

One potential problem with these studies was insufficient power. Many test players individually rather than in a larger framework. In this study, I test for conditional probabilities within a pooled, multivariate framework. I examine free throws from the 2005-06 season. With tens of thousands of observations in my sample, I find statistically significant evidence for the hot hand, as hitting a prior free throw leads to a 2- to 3-percentage-point higher probability of hitting a given free throw.

2. Literature Review

Bar-Eli et al. (2006) provide a nice review of the literature on the "hot hand" in all sports. For basketball, the studies examining the hot hand in basketball have used three types of shots: field goals, free throw, and shots in a controlled experiment. Testing for the hot hand with field goals could be problematic because of several possible sources of correlation between the probability of success for consecutive shots. The direction of this correlation, however, is ambiguous. If a player has a weak defender on him who allows him to drive easily to the basket for a lay-up or to take open shots, then there would be a positive correlation between the success of consecutive shots. On the other hand, several factors could contribute to a negative correlation between the success of consecuvitve field goal attempts. Players may naturally mix up their shot selection so that they will drive for a lay-up (perhaps an easier shot) on one possession and then be more likely to shoot from outside (a more difficult shot) on the next possession. Similarly, if a player misses a few shots from the outside and loses confidence, he may be more likely to wait for an easy shot to get his confidence back. In addition, one could argue that if a player hits one shot from outside, then the defender will play him tighter on the next opportunity so that he may not get as easy a shot. These factors could contribute to the findings of no evidence for the hot hand, when field goal data are used, as discussed below.

Free throw data does not have these problems because each shot is the same, without any defenders. Consecutive shots could, however, be correlated due to a few factors. First, the pressure of a free throw could be positively correlated, as it would depend on the game situation . Second, a player may be fatigued, and this fatigue may only slightly ease between the first and second free throws. Thus, free throws are not perfect for testing for the "hot hand," but it may be optimal over using field goals.

Shots from a controlled experiment, therefore, may allow for the cleanest test of the hot hand. However, shooting uncontested in a gym is a far different situation from shooting in a game situation. Thus, the applicability of such results to a game situation is uncertain.

In the seminal paper on the hot hand, Gilovich et al. (1985) used all three types of data: free throw shooting among the Philadelphia 76ers, Boston Celtics, and Cornell University basketball teams, field goals among several field goals among players from the New York Knicks and New Jersey Nets, and a controlled shooting experiment among the Cornell University men's and women's teams. Analyzing conditional probability, run counts, and serial correlations, they find no evidence for the hot hand.

In the only other study on free throws in game situations, Wardrop (1995) finds no statistically significant evidence across individuals members of the Boston Celtics (over the 1980-81 and 1981-82 seasons) that players were more likely to make their second free throw had they made their first free throw. Pooling all players, however, the data do suggest that making the first free throw is correlated with making the second free throw, as players hit 78.9% of their free throws after a make, but only 74.3% after a miss. However, pooling data like this, without any control for the players, naturally produces such a result because the good free throw shooters will have many observations in which he hits both the first and second free throws, while the poor shooters will have many observations in which he misses both. If the best and worst free throw shooters (Larry Bird and Rick Robey) are taken out of the data, then the 4.6% difference is reduced to a 3.4% difference. [[IS THIS A VALID CRITIQUE??]]

Other studies testing for differences in conditional probabilities have also failed to find evidence for the hot hand. Tversky and Gilovich (1989) corrected and analyzed data, originally collected by Larkey et al. (1989), on streaks that occur in short periods of time among 18 of the elite players in the game. They find no evidence for the hot hand. Another study using field goal data was conducted for the website 82games.com. Among NBA players in the 2005-06 season who had at least 200 field goals attempts, 17 had a higher shooting percentage after missing the last shot than after making the last attempt by at least 10 percentage points, while no players had at least a 10-percentage-point higher shooting percentage after making the last shot than after missing the last shot.[1]

Two other interesting studies used experiments. Koehler and Conley (2003) examined 23 players in the NBA's 3-point shooting contests from 1994 to 1997. This is a quasi-experiment because there is no defender. However, players typically are rushed to get 25

[1] These statistics come from http://www.82games.com/random25.htm.

shots off in a minute, so for the last several shots, fatigue may set in or they may be rushed. Still, they find that only two of the players demonstrated any unusual streaks of success. Wardrop (1999) uses a controlled setting for a female basketball player at the University of Wisconsin. She shot 100 shots on 20 different days. Wardrop (1999) finds statistically significant evidence against the hypothesis that the shots follow a Bernoulli trial.

In summary, there is no evidence that there is a hot hand for field goals in game situations. However, there was some evidence finding significant evidence for the hot hand based on a controlled experiment and on pooling free throws for players on the Boston Celtics. It is uncertain whether the results from a controlled experiment would translate to game situations. And the strategy of pooling players without controls for the players is questionable.

A common critique of the studies has been that the statistical tests are too weak to detect significance (Miyoshi, 2000; Wardrop, 1999; Frame et al., 2003; Dorsey-Palmateer and Smith, 2004). For example, Miyoshi (2000) used simulations to show that, when certain parameters are set to realistic levels for the sensitivity tests used in Gilovich et al. (1985), then the runs test would detect only 12% of all instances of the hot hand.

In this paper, I get around the power problem by using data on all NBA players from the 2005-06 season. I pool all players into one model, while controlling for each player in a multivariate framework.

3. Methods

The data for this analysis were provided by 82games.com. The data include detailed information on all 64,698 free throws taken in the 2005-06 season. The variables include an indicator for the player, whether the player made the free throw, what sequence of a set of free throws each shot is (e.g., “1 of 1”, “1 of 2”, etc.), which quarter of the game the shot took place in, how many free throws the player made in the prior 1, 2, 3, 4, and 5 attempts, how many field goals the player made in the prior 1, 2, and 3 attempts, and how many of the past 10 free throws the team attempted were made.

For a first look at the data, I examined the 89 players who had at least 50 sets of two or more free throws. Comparing probabilities using individual-level conditional probabilities, six players had a significantly (at the 10% level) higher probability of hitting a second free throw after making the first one relative to missing the first one, while two players had significantly higher probabilities of making a free throw after a miss.

Whereas past studies typically used runs tests for individual players, I combine all players into one model in a multivariate, fixed-effects framework. Specifically I use fixed-effect logit models to estimate the effect of making previous free throws on the probability of making a given free throw. Thus, the estimates represent within-player comparisons. Such a framework provides much greater power for testing for the hot hand than previous analyses. Furthermore, it allows me to control for other factors, such as the quarter of the game during which the free throw occurs.

I use several variants of the model to address different questions and to test for the sensitivity of the results. First, I estimate whether a player is more likely to make a second free throw in a set of free throws had he made the first free throw. I estimate this model for all players (435 who remain in the regression sample). And to test whether frequent or infrequent foul-shooters are more subject to the hot hand (or streakiness), I estimate the model separately for the 213 players who had fewer than 100 free throws during the season and the 113 players who had at least 200 free throws.

The next set of models will then examine whether success on a free throw relies on success on attempts prior to the last free throw. This set of models uses the first free throw in a set as the outcome, which reduces potential problems from a possible correlation between free throws in a set due to being in the same game situation and level of fatigue. In these models, I examine how success on up to the last 5 free throws affects the probability of making the current free throw.

4. Results

Table 1 presents the marginal effects and their standard errors for the main set of models. In these models, I examine the second of all sets of two or three free throws. I test whether a player is more likely to make the second free throw if he had made the first free throw. The only other control variables are quarter dummy variables. The first column shows the results for all players without fixed effects, with all 28,240 second free throws analyzed for 485 different players. The next three models add player fixed effects to the model. The fixed-effect model with all players, in the second column, loses 172 observations for 50 players who made all or made none of their free throws. The third column shows the results for those 213 players who had fewer than 100 free throws during the season; and the fourth column shows the results for the 113 players who had more than 200 free throws during the season. The comparison of the estimates shows whether regular free throw shooters are more or less likely to be subject to the hot hand than infrequent free throw shooters.

The evidence for all players and for the frequent and infrequent free throw shooters all provides evidence for the hot hand. Not controlling for player fixed effects, the estimated effect of hitting the first free throw is a 6.4-percentage-point increase in the probability of hitting the second free throw. However, when player fixed effects are included, hitting the first free is estimated to raise the probability that a player will hit the second free throw by an estimated 2.9 percentage points for all players. Thus, it appears that not including player fixed effects, as Waldrop (1995) had not done, could cause an upward bias in the estimated "hot hand" effect. Separating players by free throw frequency, hitting the first free throw is estimated to increase the probability of hitting the second shot by 2.7 percentage points for infrequent shooters, and by 5.0 percentage points for frequent shooters. The estimates are statistically significant at the 5-percent level for infrequent shooters and at the one-percent level for all players and frequent shooters. However, the difference in the estimates between the frequent and infrequent shooters is not statistically significant.

As mentioned earlier, one potential criticism of examining the correlation of success for free throws in a set of two free throws is that the situation of the set of free throws could affect the probability of success for both free throws. I can reduce this potential problem by estimating how success on prior free throw sets could affect the probability that a person makes a free throw.

In Table 2, I examine the first free throw (of a single free throw or a set of free throws). Being the first free throw, the correlation to the previous free throws due to being in a similar situation (fatigue and game pressure) should be significantly lower. The first column shows how making the prior free throw affects the probability of making the current free throw, with the sample for this model being all first free throws when the player had at least one prior free throw in the game. The next column estimates how the number of free throws made in the prior two attempts affects the probability of making the current free throw, with the sample this time requiring that the player had at least two prior free throws. The following columns are for the number of made free throws in the prior 3, 4, and 5 attempts.

The estimated effect of making the prior free throw is a 1.9-percentage-point increase in the probability of making the current free throw, which is significant at the 10-percent level. This estimate is smaller than the 2.9-percentage-point effect from the prior free throw within the same set of free throws.

The estimate for the number of made free throws in the prior two attempts is significant at the 5-percent level. Making one of the prior two free throws increases the probability of making a free throw (relative to not making any of the prior two attempts) by 1.5 percentage points, while making both of the last two free throws increases the probability by 3.0 percentage points. The estimates for the number of free throws made in the last 3, 4, and 5 attempts were all insignificant.

Interestingly, a free throw following a made field goal with a foul (an "And-one" situation) does not have a significant effect on the probability of making the free throw. One application of the hot hand, or an adrenaline effect, would be that making a shot as one is getting fouled is indicative of shooting well or having adrenaline. However, it does not appear to be correlated with success on the foul line.

<u>Caveats</u>

One drawback of the approach in this study is that a "cold hand" could potentially drive the results. That is, the finding of a higher probability of making a shot after making the previous shot could be the result of a player having periods in which he made a low percentage of shots but not any particular periods in which he made an extraordinarily high percentage of shots. So, the evidence presented in this paper may be for the existence of the “cold hand” and not necessarily the “hot hand.”

5. Conclusions

Previous studies testing for the hot hand have mostly been based on testing individuals. Thus, many studies have not had sufficient power to detect any evidence for the hot hand. The only convincing study that had detected the hot hand was based on a controlled experimental setting, which may not translate well to a game situation.

In this study, I use a pooled, multivariate framework to provide greater power in testing for the hot hand in free throw shooting among NBA players. In contrast to past studies for game situations, I find strong evidence for the hot hand, as hitting the prior free throw increases the probability of making a free throw by 2 to 3 percentage points. The hot hand appears to be stronger for infrequent than for frequent foul-shooters. Furthermore, whereas success on the prior two free throws increases the probability of making a free throw, success on free throws prior to that apparently does not matter.

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Table 1: Marginal effects from fixed-effect logit models

(Sample = second of two free throws)

Dependent variable = whether made the foul shot

	No player fixed effects	Player fixed effects		
	All	All	Less than 100 free throws in season	At least 200 free throws in season
made last free throw	0.064***	0.029***	0.027**	0.050***
	(0.006)	(0.008)	(0.011)	(0.019)
2nd quarter	-0.001	-0.003	0.011	-0.038
	(0.008)	(0.011)	(0.014)	(0.030)
3rd quarter	-0.011	-0.019*	-0.004	-0.070**
	(0.008)	(0.011)	(0.014)	(0.030)
4th quarter	-0.005	-0.020*	-0.011	-0.038
	(0.007)	(0.011)	(0.014)	(0.029)
# observations	28,240	28,068	4149	17,225
# players	485	435	213	113

Table 2: Marginal effects from fixed-effect logit models

(Sample = first free throw in a set and having at least 1, 2, 3, 4, or 5 prior free throw attempts in the game)

Dependent variable = whether made the foul shot

	Had at least 1 prior free throw attempt	Had at least 2 prior free throw attempts	Had at least 3 prior free throw attempts	Had at least 4 prior free throw attempts	Had at least 5 prior free throw attempts
made last free throw	0.019* (0.010)				
# free throws made in last 2 attempts		0.015** (0.007)			
# free throws made in last 3 attempts			0.010 (0.008)		
# free throws made in last 4 attempts				0.008 (0.007)	
# free throws made in last 5 attempts					0.000 (0.008)
2nd quarter	-0.027 (0.017)	-0.039** (0.018)	-0.008 (0.029)	-0.003 (0.033)	-0.002 (0.046)
3rd quarter	0.000 (0.016)	-0.006 (0.018)	0.024 (0.026)	0.031 (0.030)	0.038 (0.040)
4th quarter	-0.006 (0.016)	-0.013 (0.017)	0.006 (0.026)	0.015 (0.029)	0.020 (0.039)
"And-one" (following a made field goal)	0.008 (0.012)	0.002 (0.012)	0.002 (0.016)	-0.004 (0.017)	-0.015 (0.022)
# observations	19,516	17,458	10,733	8794	5609
# players	383	373	283	260	200