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Abstract— **This paper develops the theory for waveform-diverse moving-target synthetic-aperture radar. We assume that the targets are moving linearly, but we allow an arbitrary flight path and (almost) arbitrary waveforms. We consider the monostatic case, in which a single antenna phase center is used for both transmitting and receiving. We include the case of waveforms whose duration is sufficiently long that the targets and/or platform move appreciably while the data is being collected.**

I. Introduction

This paper considers waveform-diverse synthetic-aperture radar in the case when there are multiple moving targets in the scene. We consider a pulsed system traversing a circular flight path, and assume that the targets are moving linearly. We consider the monostatic case, in which a single antenna phase center is used for both transmitting and receiving. The problem is formulated in terms of forming an image in phase space, where the independent variables include not only position but also the vector velocity.

We include the case of waveforms whose duration is sufficiently long that the targets and/or platform move appreciably while the data is being collected. Figure 1 shows the regime of validity of the start-stop approximation for X-band. This figure plots $v \leq \lambda/T$, where v is the relative velocity, λ is the wavelength at the center frequency, and T is the waveform duration. For shorter wavelengths, the curve moves towards the axes, and the region of validity is smaller. We see that the start-stop approximation is invalid for high-frequency systems or long-duration waveforms or high-velocity targets. For example, a target moving 30 m/sec (67 mph) moves 3 mm in 100 μsec and 3 m in .1 sec, distances that could easily be comparable to the system wavelength. The issue also arises when low-power, long-duration waveforms are used.

This paper is an extension of the work [1], which showed how to combine the temporal, spectral, and spatial attributes of radar data. In particular, the theory developed in this paper shows how to combine fast-time Doppler and range measurements made from different spatial locations. This approach can be used, for example, for SAR and ISAR imaging when relative velocities are large enough so that target returns at each look are Doppler-shifted. Alternatively, this theory shows how to include spatial considerations into classical radar ambiguity theory. In addition, this approach provides a connection between SAR and Moving Target Indicator (MTI) radar.

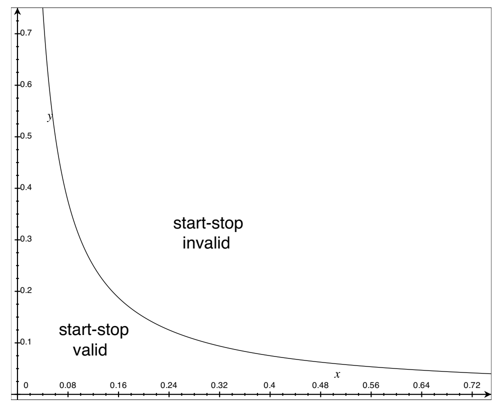

Fig. 1. This shows the region of validity for the start-stop approximation for X-band (wavelength 3cm). The horizontal axis represents the waveform duration, and the vertical axis shows the relative velocity. For shorter wavelengths, the curve moves towards the axes, and the region of validity is smaller.

SAR for moving targets has been studied in previous work: In [2], Fienup analyzed the phase perturbations caused by moving targets and showed how the motion affects the image. The work [5] identifies ambiguities in four-dimensional space that result from attempting to image moving targets from a sensor moving along a straight flight path. The patent [4] and papers [6], [7], [8] all use the start-stop approximation to identify moving targets from their phase history. The paper [3] uses a fluid model to impose conservation of mass on a distribution of moving scatterers, and a Kalman tracker to improve the image adaptively.

In this abstract, we outline a derivation for the phase-space point-spread function for a pulsed waveform-diverse spotlight SAR system traversing a circular flight path. We show that this point-spread function can be written in terms of the ordinary radar ambiguity function, evaluated at certain arguments.

II. Model for radar data

If the target reflectivity in its own reference frame is denoted by $Q(\boldsymbol{x})$, then the reflectivity of the target moving with velocity $\boldsymbol{v}$ is $Q_{\boldsymbol{v}}(\boldsymbol{x}-\boldsymbol{v}t) = Q(\boldsymbol{x}-\boldsymbol{v}t, \boldsymbol{v})$.

We assume that the system transmits a train of pulses of the form

$$\sum_m f_m(t - T_m) \qquad m = 0, 1, 2, \dots \tag{1}$$

where the delay between successive pulses is sufficiently large so that successive pulses do not overlap.

We denote the antenna path by $\boldsymbol{\gamma}(t)$. Then, neglecting polarization and multiple scattering, and neglecting the change in antenna position between the transmit time and the receive time, the field $\mathcal{E}_m^\circ$ received at $\boldsymbol{\gamma}(t)$ due to the mth pulse can be written

$$\mathcal{E}_m^\circ(t) \propto \int f_m(\phi_m^\circ(t,\boldsymbol{z},\boldsymbol{v}))\; Q(\boldsymbol{z},\boldsymbol{v}) d^3z d^3v_m \qquad (2)$$

where the phase is

$$\phi_m^\circ(t,\boldsymbol{z},\boldsymbol{v}) = \alpha_{\boldsymbol{v},m}\left(t - R_m^\circ(\boldsymbol{z},\boldsymbol{v})/c\right) - R_m^\circ(\boldsymbol{z},\boldsymbol{v})/c - T_m \qquad (3)$$

where the Doppler scale factor is

$$\alpha_{\boldsymbol{v},m} = \frac{1+\widehat{\boldsymbol{\gamma}}_m \cdot \boldsymbol{v}/c}{1-\widehat{\boldsymbol{\gamma}}_m \cdot \boldsymbol{v}/c} \qquad (4)$$

with $\widehat{\boldsymbol{\gamma}}_m = \boldsymbol{\gamma}(T_m)/R$, $R = |\boldsymbol{\gamma}(T_m)|$ for circular SAR, and

$$R_m^\circ(\boldsymbol{z},\boldsymbol{v}_m) = R - \widehat{\boldsymbol{\gamma}}_m \cdot \boldsymbol{z}\ . \qquad (5)$$

III. Image Formation

We form an image by weighted matched filtering:

$$I(\boldsymbol{p},\boldsymbol{u}) \propto \sum_m \int f_m^*\left(\phi_m(t,\boldsymbol{p},\boldsymbol{u})\right) \alpha_{\boldsymbol{u},m}(1-\widehat{\boldsymbol{\gamma}}_m \cdot \boldsymbol{v}/c)\mathcal{E}_m^\circ(t)dt \qquad (6)$$

IV. Image Analysis

Using the model (2) in (6) gives rise to

$$I(\boldsymbol{p},\boldsymbol{u}) = \int K(\boldsymbol{p},\boldsymbol{u};\boldsymbol{z},\boldsymbol{v}) Q(\boldsymbol{z},\boldsymbol{v}) d^3z d^3v \qquad (7)$$

where the point-spread function K is

$$K(\boldsymbol{u},\boldsymbol{p};\boldsymbol{z},\boldsymbol{v}) = \sum_m \int f_m^*\left(\phi_m^\circ(t,\boldsymbol{p},\boldsymbol{u})\right) f_m(\phi_m^\circ(t,\boldsymbol{z},\boldsymbol{v})) \times \frac{1-\widehat{\boldsymbol{\gamma}}_m \cdot \boldsymbol{u}/c}{1-\widehat{\boldsymbol{\gamma}}_m \cdot \boldsymbol{v}/c}\alpha_{\boldsymbol{u},m}dt. \qquad (8)$$

If we make the change of variables $t \mapsto t' = \phi_m^\circ$, we obtain

$$K(\boldsymbol{p},\boldsymbol{u};\boldsymbol{z},\boldsymbol{v}) = \sum_m \mathcal{A}_m\left(\frac{\alpha_{\boldsymbol{u},m}}{\alpha_{\boldsymbol{v},m}}, \Delta\tau_m(\boldsymbol{p},\boldsymbol{u};\boldsymbol{z},\boldsymbol{v})\right) \times \left|\frac{\alpha_{\boldsymbol{u},m}}{\alpha_{\boldsymbol{v},m}}\right| \frac{1-\widehat{\boldsymbol{\gamma}}_m \cdot \boldsymbol{u}/c}{1-\widehat{\boldsymbol{\gamma}}_m \cdot \boldsymbol{v}/c}, \qquad (9)$$

where

$$\mathcal{A}_m(\sigma,\tau) = \int f_m^*(\sigma t - \tau) f_m(t) dt \qquad (10)$$

is the wideband ambiguity function [9] and where

$$\Delta\tau_m(\boldsymbol{p},\boldsymbol{u}_m;\boldsymbol{z},\boldsymbol{v}_m) = \frac{\alpha_{\boldsymbol{u},m}}{c}\left[\frac{R_m^\circ(\boldsymbol{z},\boldsymbol{v})}{\alpha_{\boldsymbol{v},m}} - \frac{R_m^\circ(\boldsymbol{p},\boldsymbol{u})}{\alpha_{\boldsymbol{u},m}} + R_m^\circ(\boldsymbol{z},\boldsymbol{v}) - R_m^\circ(\boldsymbol{p},\boldsymbol{u}) + \frac{T_m}{\alpha_{\boldsymbol{v},m}} - \frac{T_m}{\alpha_{\boldsymbol{u},m}}\right]. \qquad (11)$$

If the waveforms f_m have thumbtack ambiguity functions, then both downrange position and downrange velocity can be obtained at each look m, and the variation in aspects as m varies (*i.e.,* over the synthetic aperture) provides cross-range information.

V. Conclusion

We see from (9) that the phase-space point-spread function is a weighted sum of ordinary radar ambiguity functions, evaluated at arguments that depend on the difference in positions and velocities.

Acknowledgment

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