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1996

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Nuclear Instruments and Methods in Physics Research A, Volume 375, (1996), pp.

ABS 61-ABS 62

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## Transverse effects in UV FELs

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In an ultraviolet Free Electron Laser (UV FEL), the electron beam can be approximately the same size as the optical mode. Studied are the effects of electron beam size and betatron focusing on gain.

To study the effects of electron beam size on gain, three dimensional transverse FEL simulations,  $(x, y, \tau)$ , where  $\tau = ct/L$  is the dimensionless time and  $L$  is the undulator length, were used. The simulations use the pendulum equation to describe the electrons' motion and the optical wave equation to describe the optical fields [1]. Fig. 1 shows the result of a simulation with dimensionless current density  $j = 30$ , standard deviation of a Gaussian spread in electron phase velocities  $\sigma_G = 1.3$ , standard deviation  $\sigma_\theta = 1.0$  due to a Gaussian spread in electron injection angles, and dimensionless electron beam size  $\sigma_e = 0.45$  which contributes to a phase velocity spread  $\sigma_v = 0.2$  [1]. Harmonic betatron oscillations are incorporated with dimensionless betatron frequency  $\omega_\beta$  [1]. For the parameters used,  $\omega_\beta = 1.0$ , meaning the electrons execute approximately one sixth of an oscillation as they traverse the undulator [2]. The dimensionless field strength  $|a(x, n)|$  at the end of the undulator is plotted against the transverse coordinate  $x$  at each pass  $n$  in the top left plot, and  $|a(x, y)$ ,

the field strength at the mirror, is plotted at top right. The middle plot is of  $|a(x, \tau)|$ , the field strength along the optical path between the mirrors. Also shown are the final positions of a random sampling of electrons along the beam path in the undulator. The beam radius to optical mode radius ratio at the mode waist is  $\sigma_e/w_0 = 0.8$  and the dimensionless Rayleigh length  $z_0 = w_0^2$  [1]. Many electrons in the "tails" of the optical mode don't fully participate in the interaction, leading to a gain reduction. The four plots along bottom show the electron distribution at each pass  $f(v, n)$ , the final electron phase space positions  $(\zeta, \nu)$ , and the power,  $P(n)$ , and gain,  $\ln(1 + G(n))$ , evolutions.

In Fig. 2, the dimensionless Rayleigh range  $z_0$  was varied over the range 0.1 to 1.0 while plotting normalized gain  $G/0.135jF$ , where  $F = \pi\sigma_e^2/(z_0 + 1/12z_0)$  is the average filling factor. The Rayleigh range used in Fig. 1 is  $z_0 = 0.3$ , corresponding to the minimum gain in Fig. 2. For a Gaussian optical mode and filament electron beam,  $z_0 = 1/\sqrt{12} \approx 0.3$  corresponds to minimum mode volume and maximum gain [1]. However,  $z_0 \approx 0.3$  gives the minimum gain for the UVFEL where the optical mode volume must be large enough to envelope the whole

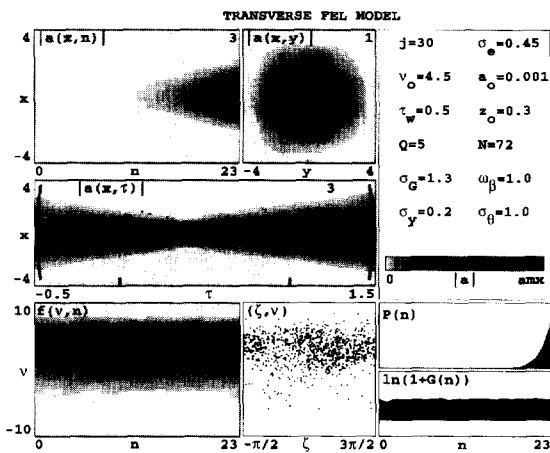


Fig. 1. 3-dimensional simulation of the UV FEL showing gain reduction when the optical mode volume is minimized.

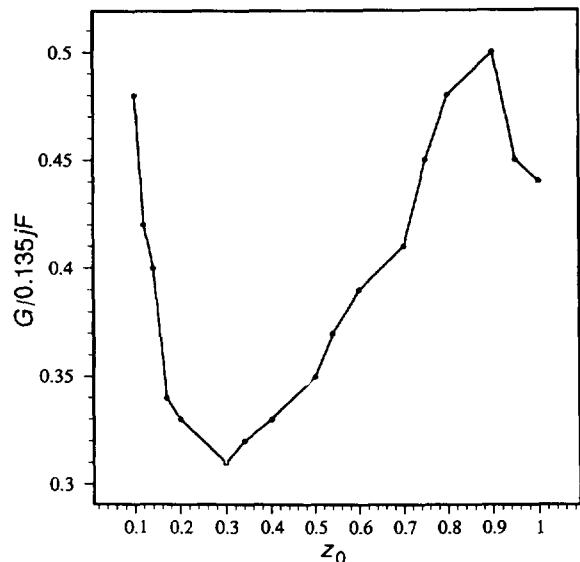


Fig. 2. Plot of normalized gain versus Rayleigh range for the UV FEL.

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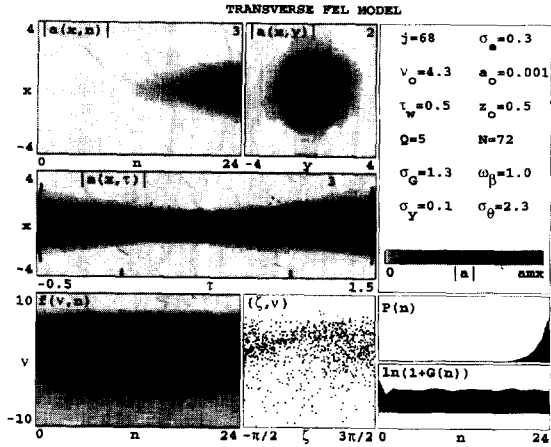


Fig. 3. 3-dimensional simulation of the UV FEL showing gain reduction due to large spread in electron injection angles.

electron beam. Decreasing the Rayleigh range below  $z_0 = 0.3$  causes the mode to expand quickly away from the mode waist, making the mode large at the ends of the undulator and increases gain. Increasing the Rayleigh range makes the mode large everywhere by increasing the mode waist radius  $w_0$  and increases gain. As  $z_0$  gets larger than 0.9, gain is reduced because the optical mode is getting much larger than the electron beam. For a larger electron beam, the optimum  $z_0$  would increase further beyond  $z_0 \approx 0.9$ .

Since  $\omega_\beta = 1.0$ , the electron's betatron trajectories are

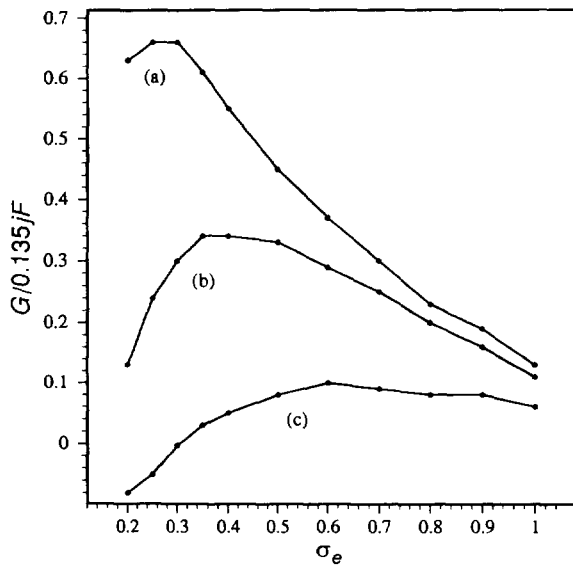


Fig. 4. Plots of normalized gain versus dimensionless beam radius for the UV FEL using three different normalized emittances: (a)  $\epsilon_n = 4\pi$ , (b)  $\epsilon_n = 8\pi$ , and (c)  $\epsilon_n = 16\pi$  mm mrad.

nearly straight lines allowing the electrons to be focused in order to keep them inside the optical mode along the length of the undulator. However, with dimensionless emittance  $\epsilon = \sigma_e \sqrt{\sigma_\theta}$  fixed, decreasing the dimensionless electron beam waist radius  $\sigma_e = r_e \sqrt{\pi/L\lambda}$  increases the dimensionless angular spread  $\sigma_\theta = 2\pi N \gamma^2 \delta \theta^2 / (1 + K^2)$  [1]. As shown in Fig. 3, if the electron beam radius is made too small, angular spread increases the spread of electron beam phase velocities and gain is reduced. The parameters used in Fig. 3,  $\sigma_e = 0.3$  and  $\sigma_\theta = 2.3$ , correspond to an electron beam with normalized emittance  $\epsilon_n = 8\pi$  mm mrad. The plot of  $|a(x, \tau)|$  shows that the electrons are focused to a small waist at  $\tau = 0.5$ , but the large angular spread causes the beam to grow larger than the optical mode at the ends of the undulator. The plots of electron distribution and final phase space are dominated by the exponential distribution in phase velocity caused by the Gaussian spread in injection angles determined by the large  $\sigma_\theta$ .

To find the optimum electron beam radius for a given emittance  $\epsilon = \sigma_e \sqrt{\sigma_\theta}$ , the normalized gain is plotted as the dimensionless electron beam radius was varied over the range  $\sigma_e = 0.2$  to 1.0, keeping peak current constant. Fig. 4 shows the normalized gain versus  $\sigma_e$  for three different normalized emittances: (a)  $4\pi$ , (b)  $8\pi$ , and (c)  $16\pi$  mm mrad. Fig. 3 corresponds to the point on curve (b) at  $\sigma_e = 0.3$ , where it is apparent that making the electron beam radius too small has slightly reduced normalized gain from its peak value at  $\sigma_e = 0.35$ . The curves show that decreasing the dimensionless electron beam radius from  $\sigma_e = 1.0$  increases the normalized gain as more electrons are enveloped in the optical mode. If the electron beam is made too small, the resulting large angular spread degrades the FEL interaction, leading to a reduction in normalized gain. Note that with poor emittance as in curve (c), beam size has little effect on gain due to large angular spread at any beam radius. If emittance is improved as in (a), electron beam size becomes more important, and the beam radius for maximum gain becomes smaller.

**Acknowledgements**

The authors wish to thank CEBAF and the Naval Postgraduate School for their support in this work.

**References**

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 [2] G. Neil and S. Benson, Continuous Electron Beam Accelerator Facility, private communication.