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The free electron laser interaction with a short-Rayleigh-length optical mode

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Abstract

High-power, short-wavelength free electron lasers (FELs) can make use of a short-Rayleigh-length (SRL) optical mode in order to reduce the intensity on resonator mirrors. The conventional FEL interaction attempts to optimize the coupling between the electron beam and optical mode by minimizing the optical mode volume around the electron beam. In contrast, the SRL FEL focuses optical power in a small region of the undulator, which accelerates the electron bunching process. As a result, the fundamental FEL interaction is significantly altered with a rapidly changing optical field and phase along the undulator. Advantages and disadvantages of FELs designed with an SRL optical mode are discussed.

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Keywords: Free electron laser; Optical resonator

1. Introduction

In the free electron laser (FEL), a laser beam is amplified by a co-propagating, relativistic electron beam passing through an undulator [1]. The optical field is transverse to the velocity component of the electrons along the undulator axis so that coupling, or exchange of energy, is small. The transverse electron oscillations caused by the undulator field give rise to the small coupling, but must be extended over many undulator periods in order to obtain useful energy exchange resulting in electron bunching. An alternative to the conventional FEL interaction is to focus the

laser beam to a small waist over a short distance in the middle of the undulator. The short interaction distance diminishes coupling, but the stronger optical field at the focus increases the interaction. These competing effects can still result in significant energy exchange. This alternative can be achieved with a short-Rayleigh-length (SRL) FEL [2] where large-diameter resonator mirrors focus the laser beam of wavelength λ to a small waist radius w_0 so that the Rayleigh length $Z_0 = \pi w_0^2 / \lambda$ is much smaller than the undulator length $L = N\lambda_0$. Typically FELs use a resonator with a Rayleigh length of $Z_0 = L/(12)^{1/2}$, which minimizes the optical mode volume around the electron beam.

The motivation to use an FEL with a short Rayleigh length resonator arises from the need to reduce the intensity on the mirrors in a high-power

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compact design [3]. An example is a MW laser with a resonator about $S \approx 12$ m in length. In order to reduce the mirror intensity to below a limit of about 200 kW/cm^2 , the mirror spot radius must be several centimeters. A single Gaussian mode with this spot size at the mirrors has a Rayleigh length of only a few centimeters and a waist of $w_0 \approx 0.1$ mm. The single Gaussian mode is desirable so that the electron beam (which is the gain medium) can interact over the whole wavefront amplifying a single mode.

2. FEL dynamics

The SRL FEL interaction is significantly altered from that of the conventional FEL. As the laser beam propagates along the undulator, diffraction determines the evolution of the dimensionless optical electric field amplitude and phase [1]

$$a(\tau) = \frac{a_0}{(1 + (\tau - \tau_w)^2 / z_0^2)^{1/2}}$$

$$\phi(\tau) = -\tan^{-1}((\tau - \tau_w) / z_0)$$

at the center of the wavefront. The dimensionless field amplitude $a(\tau)$ is proportional to the laser electric field amplitude, a_0 is the field amplitude at the mode waist ($\tau = \tau_w$), $\tau = ct/L = z/L$ is the dimensionless time of interaction along the undulator length, and $z_0 = Z_0/L = \pi w_0^2 / L\lambda$ is the dimensionless Rayleigh length [1]. The microscopic evolution of an electron at the center of the co-propagating optical mode traveling along the undulator with N periods is best understood by reference to the pendulum equation $\ddot{\zeta} = \dot{v} = a(\tau) \cos(\zeta + \phi(\tau))$ where $\zeta = (k + k_0)z - \omega t$ is the dimensionless electron phase, $v = L[(k + k_0)\beta_z - k] \approx 4\pi N(\gamma - \gamma_R) / \gamma_R$ is the dimensionless electron phase velocity, γ_R is the Lorentz factor for the resonant electron energy, $k = 2\pi/\lambda = \omega/c$ is the optical wavenumber, $k_0 = 2\pi/\lambda_0$, and time derivatives are taken with respect to dimensionless time τ [1]. The pendulum phase space (ζ, v) is divided into open and closed orbits by the separatrix $v_s^2 = 2a(\tau)(1 + \sin(\zeta_s + \phi(\tau)))$. The height of the separatrix along the v -axis is $2(a(\tau))^{1/2}$ while the position along the ζ -axis is determined by the optical phase $\phi(\tau)$. The separatrix height increases by a factor

$(1 + (2z_0)^{-2})^{1/4} \approx (2z_0)^{-1/2}$ from the ends to the middle of the undulator when the Rayleigh length is small. The rate of electron bunching and energy exchange is determined by the field amplitude $a(\tau)$ during the interaction time $\tau = 0 \rightarrow 1$ along the undulator length. The field amplitude increases by the factor $(1 + (2z_0)^{-2})^{1/2} \approx 1/(2z_0)$ from the ends to the middle of the undulator.

In the conventional FEL with $z_0 \approx 1/\sqrt{12} \approx 0.3$, changes in the optical field amplitude $a(\tau)$ and phase $\phi(\tau)$ are modest and are often considered negligible in FEL analysis. However, there are already noticeable effects of diffraction in the FEL interaction. A numerical simulation integrates the electron equations of motion as they interact with the diffracting field described by $a(\tau)$ and $\phi(\tau)$ [1]. While the physics of the FEL interaction is best understood from the pendulum equation above, fully self-consistent electron dynamics are used in the integration. Fig. 1 shows the phase space evolution $(\zeta(\tau), v(\tau))$ of 20 sample electrons in a section of the electron beam about one wavelength of light long. At the beginning of the undulator ($\tau = 0$) with $N = 20$ periods, the electron positions are drawn as light brown. As they interact with the laser field, their positions are drawn darker brown up to the end of the undulator ($\tau = 1$). The sample electrons show that more electrons lose energy to the laser beam than gain energy, so that the laser beam is amplified. This process leads to coherent electron bunching in a region of phase space where

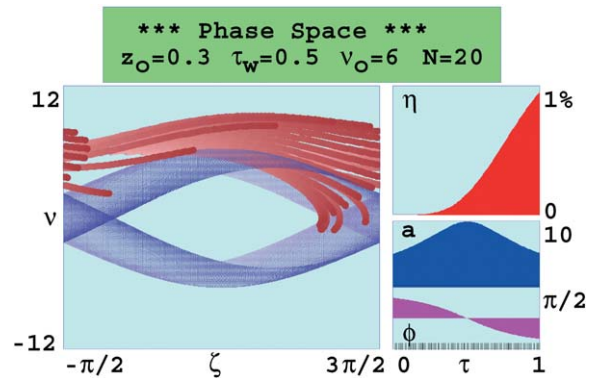


Fig. 1. The phase space evolution of the electrons and laser light for conventional Rayleigh length $z_0 = 0.3$.

the phase space paths decrease the phase velocities. Shown in red at the right in Fig. 1 is the evolution of the electron beam's single-pass efficiency $\eta(\tau) = \langle v_0 - v(\tau) \rangle / 4\pi N$, where $v_0 = v(0) = 6$ is the initial electron phase velocity, and $\langle \dots \rangle$ represents an average over all electrons. At the beginning of the undulator when there is no bunching, the efficiency is zero, increasing to about 1% at the end of the undulator. At the middle right in blue is shown the evolution of the laser field amplitude $a(\tau)$, which increases to a modestly strong value of $a(1/2) = 10$ at the center of the undulator where the optical mode is focused. Below in purple is shown the accompanying optical phase evolution $\phi(\tau)$, which decreases through the mode focus at $\tau = \tau_w = 1/2$. The separatrix in phase space is drawn in light blue when the electrons are at the beginning of the undulator ($\tau = 0$) to darker blue as the electrons move along the undulator to $\tau = 1$. The evolving separatrix shows the effect of both the evolving optical amplitude and phase. The height of the separatrix can be seen to increase to a maximum height of $2(a(1/2))^{1/2} \approx 2\pi$ in the middle of the undulator, while the evolving optical phase $\phi(\tau)$ shifts the separatrix to the right.

In the SRL FEL with small values of $z_0 \approx 0.1 \rightarrow 0.03$, diffraction effects will more significantly alter the optical field amplitude and phase, and the FEL interaction characteristics. Fig. 2 shows the phase space evolution for Rayleigh length $z_0 = 0.03$ focused at the middle

of an $N = 20$ period undulator ($\tau_w = 1/2$) to a peak field strength of $a_0 = 70$. This field strength is representative of the dimensionless field in a MW FEL using the SRL design. The 20 sample electrons start at phase velocity $v_0 = 9$ for optimum efficiency with this Rayleigh length. The most dramatic difference from Fig. 1 is the sharp increase in the optical field amplitude (shown in blue) and the rapid change in the optical phase (shown in purple) due to the tight mode focus at the middle of the undulator. For small values of z_0 , the field values at each end of the undulator are given by $a(0) = a(1) \approx 2z_0 a_0 \approx 4$ so that the separatrix height at the ends of the undulator is $2(a(0))^{1/2} = 2(a(1))^{1/2} \approx 4$. The separatrix height increases by a factor of $(2z_0)^{-1/2} \approx 4$ to about $2(a(1/2))^{1/2} \approx 16$ at the middle of the undulator. All electrons are initially outside the separatrix in what would be open orbits for a constant field amplitude and phase. But with the focused field of the SRL FEL, some electrons become trapped in closed orbits when “grabbed” by the “ballooning” separatrix at the mode focus. The “grabbing” of just a part of the beam also bunches electrons in phase space. Fig. 2 shows a rapid increase in efficiency at the mode focus, followed by continued energy extraction until the end of the interaction. The result is a final efficiency of close to 1% with an induced fractional energy spread of $\Delta v / (4\pi N) \approx 7\%$, where Δv is the induced spread in electron phase velocities (full-width).

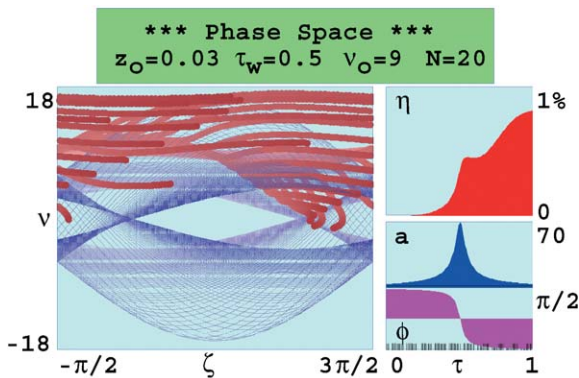


Fig. 2. The phase space evolution of the electrons and laser light for short Rayleigh length $z_0 = 0.03$.

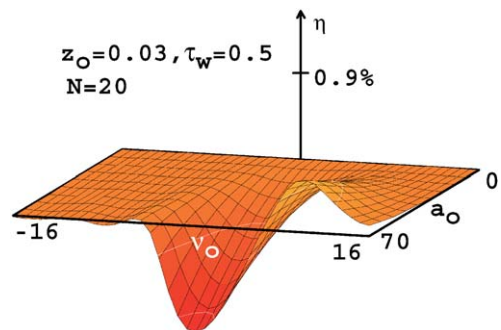


Fig. 3. For a short Rayleigh length of $z_0 = 0.03$, the efficiency map $\eta(a_0, v_0)$ shows a peak value of $\eta \approx 0.9\%$ occurring at $v_0 \approx 10$.

3. The efficiency map

The simple simulation model described above can be used to explore the dependence of the final efficiency η on the optical field strength a_0 at the mode focus and the phase velocity v_0 . Searching through values of v_0 is equivalent to searching through values of optical wavelength or electron beam energy in a small range around resonance ($v_0 = 0$). Searching through values of a_0 from weak fields ($a_0 \leq \pi$) to strong fields ($a_0 \gg \pi$) follows the increase in power in an FEL oscillator. Fig. 3 shows the final efficiency map $\eta(a_0, v_0)$ in a SRL FEL with $z_0 = 0.03$, $\tau_w = 0.5$, and $N = 20$ undulator periods. The efficiency at each of the grid points on the map is determined by a simulation as in Fig. 2. In the mapping of efficiencies, the optical field strength increases from weak fields $a_0 \ll \pi$ to a maximum of $a_0 = 70$ which is well into the strong field region occurring at FEL saturation [1]. The field values are representative of the peak fields in a MW FEL where a SRL might be necessary. The peak efficiency is found to be $\eta \approx 0.9\%$, occurring at $v_0 \approx 10$. The region of positive efficiency, in which energy is extracted from the electron beam to the laser field, is above resonance ($v_0 = 0$), increasing in stronger fields a_0 . At lower values of v_0 , there is a region of negative efficiency where the electrons take energy from the laser beam.

4. Concluding remarks

The model used here shows that the effect of the short Rayleigh length is to reduce the peak available efficiency, but nevertheless allow operation at practical efficiencies of a few percent. The efficiency maps were used to explore the peak available efficiency as the position of the mode focus is varied from $\tau_w = 0.1$ (near the beginning of the undulator) to $\tau_w = 0.9$ (near the end of the undulator). A maximum efficiency is found to be broadly peaked around $\tau_w \approx 0.4$. Surprisingly, the maximum value of efficiency only decreases down to $\eta \approx 0.8\%$ over the range of values $\tau_w \approx 0.3–0.6$.

Acknowledgements

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